

JOHANNES LENHARD, MARTIN CARRIER (Department of Philosophy, Bielefeld University)

Mathematics as a Tool



Johannes Lenhard does research in philosophy of science with a particular focus on the history and philosophy of mathematics and statistics. During the last years his research has concentrated on simulation modeling. He argues that it can be philosophically characterized as a new type of mathematical modeling. Currently, he works in a ZiF cooperation group on 'Mathematics As a Tool'.



Martin Carrier has worked on various fields in the philosophy of science—History of Early Modern Physical Theory, Theory Change: Problems of Methodological Comparison and Confirmation Theory, Space-Time Philosophy. His present chief research field is the methodology of applied research and the methodological changes imposed on science by the pressure of practice. Philosophy of science often focuses on the characteristics of fundamental research without taking into account that a large part of scientific research today is commissioned research, industrial research, or applied research, performed so as to accomplish short-term practical goals. In such instances, the aims of research do not grow out of the smooth development of a discipline but are shaped by non-scientific problems, and the relevant time-frames are narrow. Kinds of bias may emerge under such conditions that are lacking in basic research but merit closer philosophical scrutiny.

Mathematik als Werkzeug

Unsere Kooperationsgruppe untersucht die Mathematik in ihrem Gebrauch als Werkzeug, weil wir an der Nützlichkeit der Mathematik interessiert sind, und zwar in solchen Bereichen, in denen die fundamentalen Gegenstände und ihre Beziehungen gerade nicht selbst schon von mathematischem Charakter sind und nicht mathematisch formulierten Naturgesetzen folgen. Eine Vielzahl wissenschaftlicher Disziplinen – von der Klimawissenschaft über die Epidemiologie bis zur Linguistik – arbeitet verstärkt mit mathematischen (insbesondere Computer-) Modellen, ohne freilich dem klassischen Bild der mathematischen Physik zu folgen. Wie lässt sich die Funktion der Mathematik dort charakterisieren?

Die Mathematik bietet einen Zugang zu allgemeinen Strukturen unabhängig von deren physikalischem (oder ontologischem) Charakter. Man kann die Mathematik als Instrument gebrauchen, so unsere These, um neue und sehr praktische Einsichten zu gewinnen in Gebieten, obwohl deren grundlegende Prinzipien sich einer Mathematisierung widersetzen. Anders als in der klassischen Konzeption der modernen Wissenschaft, gemäß der es bei der Mathematisierung darum geht, das »Buch der Natur« (Galilei) zu entziffern, geht es bei der Mathematisierung in unserem Sinne eher um praktische, instrumentelle Vorzüge.

Der folgende Aufsatz beginnt mit einem philosophisch-historischen Teil, der unseren Beitrag in die Diskussion um die Rolle der Mathematisierung in den Wissenschaften einordnet. Im zweiten, systematisch orientierten Teil werden fünf Merkmale der Mathematik als Werkzeug hervorgehoben, unter ihnen die Vermittlung zwischen Theorie und Beobachtungen, datengetriebene Forschung und die Rolle von Idealisierungen.

What is meant when mathematics is called a tool? More than once, we received the spontaneous reply that the phrase would be inadequate because mathematics is much more than a tool. Rather than claiming that ‘mathematics is merely a tool’, the title of our ZiF cooperation group shall indicate that it is fruitful to look at mathematics insofar as it is used as a tool in the sciences.

An investigation that is guided by this perspective will have to take into account two inter-related questions. How is this tool actually used and what characterizes mathematics as a tool? Both a philosophical conception and the consideration of actual practices are relevant. Then, however, we have to deal with a tension between empirical and normative viewpoints. On the one hand, the status and impact of mathematics should be explored by reconstructing scientific practice rather than by relying on general philosophical conceptions. The latter dominated philosophy of science a couple of decades ago. On the other hand, one should not uncritically accept every hype, but rather insist on critical reflection. Both aspects taken together circumscribe the rationale of our group.

That mathematics plays an important role in many sciences is a commonly accepted fact. For a long time, leading disciplines of science featured or aimed at establishing ‘fundamental’ mathematization. In a nutshell, the use of mathematics is understood as an important facet of what makes the ‘hard’ sciences hard.

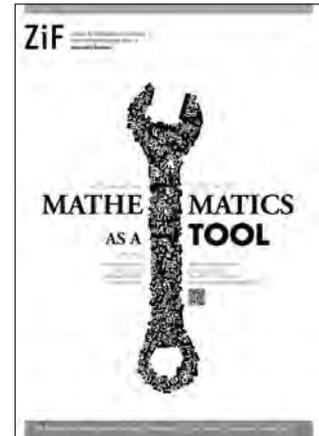
Does the use of mathematics indicate fundamental significance? Is it a necessary condition for deciphering the workings of nature? And, if so, does mathematics play a fundamental role in constituting scientific knowledge? The investigation of important questions like these has some impact on how we conceive of science and scientific rationality.

The role mathematics plays in the sciences has received different and conflicting assessments. A main distinction is the one between a strong and a weak view. Put very roughly, the former holds that mathematically formulated laws of nature refer to or reveal the rational structure of the world. The weak view denies that these fundamental laws are of an essentially mathematical character, and rather suggests that mathematics is merely a tool for systematizing observational knowledge about these laws and make additional use of these laws.

Our point is that both views are not correct, precisely because both do not take mathematics as a tool into account. Let us work out this perspective in more detail, partly in (illuminating) contrast to popular accounts of mathematics and mathematization. We understand our endeavor as a fundamentally interdisciplinary one that needs to be informed from at least three angles: the recent practice of using mathematics, history, and philosophy of science.

As a kind of preliminary result, we want to put forward a position that combines features of both, strong and weak, viewpoints. This position is supposed to characterize the use of mathematics in certain specific areas where mathematical reasoning is employed. It is not meant as a general position in the philosophy of mathematics, but rather intended to bring out characteristic features of making practical use of mathematical instruments. The position can be seen as a strong view about how mathematics functions as a tool. It is ‘strong’ insofar as it assigns an active and even shaping role to mathematics. But at the same time it refrains from any claims about the mathematical structure of the universe that is (allegedly) mirrored by mathematical theories in the physical sciences. Employing mathematics as a tool is independent of the possible mathematical structure of the objects under consideration. Hence the tool perspective is contextual rather than ontological.

When mathematics is used as a tool, it cannot be guided exclusively by internal mathematical reasoning. Instead, adequate tool-use is also a matter of the problem at hand and its context. Consequently, tool-use has to respect conditions like suitability, efficacy, optimality, and others. The notion of tool also stresses that there often is a spectrum of means to tackling a particular task that



will usually differ in how good they serve particular purposes associated with this task. The traditional philosophical viewpoint recognizes the permanent validity of mathematical theorems as a pivotal feature. The tool perspective, in contrast, underlines the inevitably provisional adequacy and validity of mathematics—any tool can be changed, made better, or lose its adequacy.

Mathematics as the Language of Nature—the Promises and Limitations of Mathematization

Let us circumscribe the traditional account of mathematics, elucidating the tool-account by way of differences. The use of mathematical laws for describing and explaining natural phenomena is among the chief epistemic achievements of the Scientific Revolution of the seventeenth century. Medieval scholarship had joined ARISTOTLE in emphasizing the difference between ideal mathematical postulates and real physical phenomena and had considered it impossible, for this reason, to accomplish a mathematical science outside of the ideal realm of celestial bodies. By contrast, the pioneers of the Scientific Revolution, such as GALILEO GALILEI, RENÉ DESCARTES, and JOHANNES KEPLER, suggested seeking for mathematical laws of nature and conceived physics as a mathematized science. GALILEO's law of freely falling bodies, DESCARTES's (or SNEL's) law of refraction and his law of inertia, or KEPLER's laws of planetary motion implemented this new idea of mathematical laws of nature. Underlying this new approach was the assumption that nature exhibits a mathematical structure. As GALILEO put it, the book of nature is written in mathematical language; or in KEPLER's words, God used geometrical figures in creating the world. In an influential historical account, ALEXANDRE KOYRÉ featured a Platonic vision of a mathematically structured nature as a methodological key element of the Scientific Revolution (KOYRÉ 1968, 1978). NEWTON's *Principia mathematica philosophia naturalis* is often regarded as an early climax of the endeavor to capture the blueprint of the universe in mathematical terms. MICHAEL MAHONEY aptly pointed out that according ontological force to mathematical structure was a significant move during the so-called Scientific Revolution that brought nature into the realm of mathematics (MAHONEY 1998).

A second vision of the Scientific Revolution consisted in connecting understanding and intervention. FRANCIS BACON and DESCARTES developed the idea of an applied science in which knowledge about natural processes provides a basis of technology. The best way to take nature into the service of humans is to elucidate her inner workings. This idea has proven highly effective for mathematical science. The precision of mathematical laws of nature makes accurate predictions possible, which in turn make such laws suitable for supporting technical intervention. When the outcome of technical procedures needs to be anticipated with high precision, mathematical laws are promising to meet such requirements.

However, the success of the mathematical sciences is not unlimited. It is true, these sciences have managed to enhance enormously their grip on complicated phenomena in the past 150 years. But: even if mathematical treatment was often able to strip off constraints of idealizations and controlled laboratory conditions and to cope with ever more complex systems, the full complexity and intricacy of real-world situations, until the present day, still poses various difficulties and obstacles to their mathematical treatment.

- An early example of a complexity problem is the so-called three-body problem that HENRI POINCARÉ demonstrated to be unsolvable around 1900. The difficulty concerns the calculation of the motion of bodies under the influence of their mutual gravitational attraction. This dynamical model proved to be so complex that no analytical solution could be derived. Consequently, in spite of the fact that the underlying mathematical laws are known comprehensively, the long-term motions that result from their rule cannot be foreseen.

- A second, similarly famous example, and an early one for the study of ‘complex systems’, goes back to the meteorologist EDWARD LORENZ. He found in the early 1960s that a given set of differential equations produced different results depending on tiny variations in the initial conditions. Even minute dissimilarities, fluctuations, and uncertainties in these initial conditions led to quite distinct subsequent states. LORENZ’s discovery is called ‘deterministic chaos’ today: the time evolution of relevant physical systems depends so sensitively on the precise initial conditions that no long-term prediction is feasible. Although the nature of the system and the laws governing its time development are known without remainder, its future course cannot be anticipated reliably. More general, systems get complex when individual entities interact on one level and their interaction leads to emergent phenomena on a higher level, especially when the investigation of the higher-level phenomena requires taking into account the details of lower-level interactions.
- In other examples of complexity, the means of mathematical modeling are insufficient for deriving useful solutions. The NAVIER-STOKES equations are believed to completely capture the behavior of fluids. But these equations cannot be solved in general; only for special cases solutions can be given. Likewise, SCHRÖDINGER’s equation is taken to account comprehensively for non-relativistic quantum phenomena. Yet already the helium atom can be treated only approximately. The complexity of the circumstances drives the mathematization of nature to its limits—at least regarding the predictions of the course of nature and the targeted interventions in this course. In sum, the examples show that the concept of complexity has many facets.

In a different vein, some fields of science appear much less suitable to mathematical treatment—quite independently of their complexity. An important area are the life sciences. Molecular biology is mostly directed at disclosing mechanisms which are governed by spatial structures. Molecules mesh with each other and occupy receptors in virtue of the geometric properties. To be sure, all these mechanisms may be based on quantum laws, and spatial features can be expressed geometrically. But, up to these days, mathematical approaches hardly contribute significantly to understanding the relevant interactions. Spatial figures, mechanisms, and feedback loops are the essential concepts. Admittedly, mathematization has led to a couple of statistical rules, like MENDEL’S laws, or yielded reaction rates of biomolecules. But it is an open question whether mathematics will be able to add important further insights to the life sciences. In short, nature seems not to welcome mathematization indiscriminately.

Thus, the adequacy of the strong view regarding the mathematization of nature seems to be constrained to rather narrow limits. Mathematical laws of nature and the option of putting them to use are restricted to a closely encircled range of sciences and phenomena. However, a look into recent sciences reveals that mathematical procedures flourish and fulfill various tasks outside this range. Fields such as molecular biology or systems biology make ample use of mathematical methods, though without embodying the vision of the strong view, i.e. without pretending to provide access to the inner workings of the pertinent mechanisms. This observation motivated us to change the perspective:

Mathematics as a Tool

Mathematical analysis can be helpful in practical respect even if the pertinent fundamental processes cannot be understood or caught productively in mathematical terms. Mathematics deals with structures and such structures can be found at various places, not alone in the makeup of bodies and their interactions. It is part of the power of mathematical methods to be able to disclose features and identify patterns in all sorts of data, independently of their nature and origin.

As a result, mathematics is essential in bringing out structures in data and making use of them, as well as in establishing links between theory and data.

- As to the first item, mathematics is helpful for identifying data patterns and thus makes data driven research possible. The increase of computing power in the past decades has opened up a new path toward analyzing data, a path that does not lead through constructing theories and building models.
- As to the second item, mathematics is suitable for forging links between theories and phenomena. Measurement procedures can be accounted for mathematically even if the constitution of the pertinent objects and interactions is not governed by mathematical laws. For instance, schematic screening is dependent on mathematical methods. High-throughput procedures or DNA-arrays require mathematical assistance in registering and processing the data. Automated pattern recognition or statistical analysis are indispensable for gaining access to relevant features of the data.

We speak of mathematics as a tool in order to designate the usefulness of mathematics even in areas where the fundamental objects and their interactions seem not to be governed by mathematical laws of nature. Mathematics as a general approach to disclosing and handling general structures irrespective of their physical nature can yield productive insights even for such areas. Using mathematics as a tool provides new opportunities for taking advantage of mathematical methods. Employing mathematics as an instrument in this sense can break new ground in practical respect even in fields whose basic principles are resistant to mathematization. Thus mathematization is analyzed regarding its instrumental virtues.

It is not customary to look at mathematics from this perspective. The standard account of mathematics is centered on deducibility and provability and associated with coherence and certainty. The tool account is different in that mathematics is analyzed as an instrument, and, consequently, cast for an auxiliary role. If mathematics is used as a tool, the agenda is set by other fields; mathematics helps to solve problems that emerge within these fields. Consequently, the particular internal coherence of mathematics does not by itself guarantee progress. For example, mathematical considerations like deriving new consequences from already accepted model assumptions are only a first step in considering the adequacy of a mathematical model. Further, as our group has learned (or come to know) in a number of cases, the instrumental use of mathematics often proceeds in a rough and ready way and seems to be part of endeavors that look tentative and deficient in epistemic respect. Still, it is a virtue rather than a vice of mathematics that it proves to be helpful even under conditions that leave much to be desired in epistemic respect. Using mathematics as a tool is in no way meant in a pejorative sense. It rather emphasizes the productivity of mathematics in coping with structures of arbitrary nature.

Using mathematics as a tool is by no means a recent development. On the contrary, the ancient slogan “saving the phenomena” was popular to describe the instrumental use of mathematics in astronomy. Mathematics was used to show the compatibility of observations with cosmological and physical principles and with additional auxiliary assumptions. This compatibility was established by reproducing and anticipating astronomical observations on their basis. However, many features of this mathematical account were not taken seriously as a description of nature. The narrow empirical basis of terrestrial observers was assumed to be insufficient for disclosing the true motions of the planets. As a result, in the tradition of saving the phenomena, the success of explaining and predicting celestial phenomena was not attributed to the truthful representation of the universe. Mathematics was used as a tool.

Although the tool perspective is relevant for large parts of mathematics as it occurs in the sciences, it has not yet received much scholarly attention. This might come as a surprise given the

recent practical turn in philosophy of science. One reason is that this perspective requires adopting an interdisciplinary approach that looks at mathematics in connection with other disciplines. The recent work on ‘mathematical practice’, however, primarily looks at what mathematicians do rather than focusing on how mathematics functions in a wider context (MANCOSU 2008, VAN KERKHOVE 2010).

Among the factors that contributed to eclipsing the instrumental use from the philosophical and (to a lesser extent) from the historical view is the traditional distinction between pure and applied mathematics. This distinction suggests a division of labor between the pure branch that creates and validates mathematical knowledge and the applied branch that draws particular conclusions from this body of knowledge so as to solve extra-mathematical problems. On this approach, construction and application of mathematics look like two separate activities and the applied branch is marked as being completely epistemically reliant on the pure branch. A different approach maintains the distinction between pure and applied mathematics while reversing the hierarchical relation. Mathematical structures are then claimed to be abstractions from structures in the real world on which pure mathematics would build its theories. Mathematics, accordingly, would be primarily a natural science (KITCHER 1983, MADDY 1997).

We do not share either view. The counterpoint we will develop abandons this assumption of unidirectional dependence and assumes, on the one hand, that using mathematics as a tool has an impact on the corpus of mathematical knowledge. New mathematical knowledge springs from developing methods designed to cope with practical challenges. Yet we also believe, on the other hand, that the system of mathematical knowledge shapes many practical or instrumental solutions. Mathematics and the various sciences we consider both benefit from proceeding in reciprocal dependence. Mathematics is no ready-made repository simply to be tapped. On the contrary, problems and the tools to their solution co-evolve.

We want to propose five characteristic aspects of mathematics as a tool that exhibit significant differences to the standard account mentioned. These five partially overlapping topics will be discussed in the following section.

Five Characteristic Features

We present five features that are intended to sketch characteristic modes of using mathematics as a tool. Each of them is rich enough to justify separate treatment, but there is also much overlap among them so that in practice concrete instances usually involve more than one feature.

(i) Mathematics as a Mediator between Theory and Observation

Mathematical theories are indispensable for mediating between theory and observation. This applies to measurement, i.e., to connecting theoretical quantities with data, as well as to technology. The use of mathematics is essential for registering phenomena and for generating and shaping phenomena. In the former case, mathematics is employed as observation theory, in the latter, it is used as an instrument of intervention in the course of nature.

Mathematical theories of observation have been in use for a long time. ISAAC NEWTON measured the gravitational attraction of the Earth by bringing mathematical laws of mechanics to bear on pendulum oscillation. Measuring electric current intensity by means of a galvanometer relies on MAXWELL’s equations. Similarly, the use of mathematics for ameliorating technological intervention has a long tradition. In 1824, SADI CARNOT was the first to apply highbrow theoretical principles (caloric theory) to the operation of advanced technical devices (the steam engine). His

analysis supported the conclusion that the only way to enhance the efficiency of the steam engine was to increase the relevant temperature differences. We will not dwell on such familiar ground. We rather aim to address observation theories of a hitherto unfamiliar sort. The pertinent mathematical procedures are not tied to specific physical circumstances or phenomena. Mechanics, electrodynamics, or thermodynamics are physical theories that are employed in registering mechanical, electro-dynamical, or thermodynamical quantities. By contrast, the mathematical observation theories that interest us are independent of any particular realm of objects. Using neural networks, statistical procedures, or Bayesian formulas for surveying, exploring and interpreting the data, does not hook up with any properties of the objects under scrutiny. In other words, these mathematical procedures do not follow the causal path from the objects and their interactions to the display of certain values (as in the case of substantive observation theories). Rather, certain patterns in the data are identified irrespective of their physical nature and causal origin.

The use of mathematics as an instrument of intervention is often not strictly separated from its use as an observation theory. An example is presented by testing fertilizers in agriculture where a mathematical model is used to conceptualize the situation by defining parameters that capture the dynamics and make it accessible for experimentation. Certain parameters, for instance the fraction of a certain substance, might be varied in a systematic way. A certain way of mixing and of distributing the fertilizer may prove (economically) optimal. Then the same mathematical procedures are utilized for observing and for intervening.

The point can be generalized: The ‘quest for certainty’ has repeatedly been discussed as a characteristic feature of modern science with mathematization playing a central role in providing certainty. The instrumental perspective transforms this viewpoint. Now, mathematics is related more to a ‘quest for optimality’, providing systematic help to plan and carry out interventions.

(ii) Data-driven Research and the Use of Big Data

Generalizing these considerations leads to the phenomenon of ‘data-driven research’. Data-driven research can be contrasted with ‘model-driven research’, in which theoretical expectations or a micro-causal model distinguishes certain patterns in the data as significant. Data-driven research is different in that it starts off from the data without an underlying model that decides about their significance. For instance, in pharmaceutical research, large amounts of potential agent substances are scanned automatically (high-throughput screening). The question posed merely is whether a certain substance binds with a certain intensity to a certain receptor. Masses of data are produced by such a large-scale screening process, and the result always is that certain patterns are selected.

Notwithstanding the great hype of ‘Big Data’ that is related to the large collections of data (amazon customers, google map online users, etc.), the crucial question in methodological respect is whether data-driven research is really tantamount to shaking profoundly the heuristic and explanatory strategies in mathematical science. Our ongoing research on this point will hopefully provide a fuller account in the near future.

(iii) Tuning Models Mathematically

Theoretical mathematical models are rarely perfect. Even if a model is very good, it regularly provides a kind of mathematical skeleton that includes a number of parameters whose quantitative assignment has to be read off from empirical data. A case in point is the gravitational constant that does not follow from mathematical arguments—though it does follow mathematically that there is a gravitational constant. Such parameter evaluations are normal and not specific for using mathematics as a tool.

However, it is a different matter when important features of models are determined by the particular problem at hand, using mathematical procedures for enriching models with refinements, adjustments, and correction factors. Tuning then plays an essential role and therefore these procedures are to be distinguished from filling numerical gaps left in theory-based models. There are many options in between where neither the mathematical structure determines the behavior nor is learning or adaptation completely open. Typically, and arguably most importantly, tuning is involved whenever the discrete structure of computer models has to be fit to theoretically described or empirically measured phenomena.

(iv) Using Alternative Routes to Solving Equations

Solving differential equations by a computer simulation proceeds by not literally solving these equations but rather by calculating values of its discretized proxy. Solutions are calculated point by point at a grid and for specific parameter values. As a result, using computational models does not simply mean to enhance the performance of mathematical models but rather changes the ways in which these models are constructed and the modes in which they operate. Digital computers require discrete versions of all relevant objects and operations. This requirement presents a kind of instrumental imperative that drives traditional mathematical modeling into new pathways.

Most notably, traditional mathematics is full of continuous quantities and relationships. Yet they need to be re-modeled as discrete entities so as to become tractable by digital computers. Typically, the discretization produces unwanted effects that then have to be neutralized by ‘artificial’ measures. An early example is the ‘artificial viscosity’ that JOHN VON NEUMANN came up with to re-introduce the possibility of very steep wavefronts after discretization of super-sonic waves (WINSBERG 2003, for a case in meteorology see LENHARD 2007).

Parameterization schemes arise from the need to find discrete counterparts to the quantities in continuous models. A telling example is the dynamics of clouds whose (micro-)physical basis is to a large part not yet known. However, cloud dynamics forms an important part of the general circulation models of the atmosphere. They involve the calculation of this dynamics from values defined at grid points. Clouds, however, are sub-grid phenomena (a typical grid may work with horizontal cells of 100 km x 100 km). Hence the effects of clouds for the whole dynamics need to be expressed as a kind of net effect at the grid points, i.e. they have to be parameterized. What are appropriate parameterization schemes? What are effective parameters that adequately summarize cloud dynamics? How are these parameters estimated reliably? These questions are interdependent and can be solved only by extensive simulation procedures that assume parameter values, observe the model dynamics, re-adapt the values, etc. Hence adequate parameterization has to find a balance between accurate description and effective manipulability. Admittedly, this point is closely related to tuning. As we said, the issues are partly overlapping.

Another example are so-called multi-scale methods. To describe the binding behavior of, say, an enzyme, researchers employ a couple of different models that are based on different theoretical approaches. An enzyme is built from amino acids and contains typically about 8000 atoms, but only a small fraction of those atoms, maybe 50, is relevant for the binding behavior. These atoms are modeled with quantum mechanical methods that provide the most detailed view and the most precise results but are computationally too expensive to cover the entire enzyme. Even for these about 50 atoms one has to reside to relatively ‘cheap’ modeling strategies, like density functional theory. The remaining majority of atoms is modeled via (classic) force fields, i.e. with so-called molecular dynamics methods. This is much coarser and ignores quantum effects, but experience has shown that this does not result in a significant loss in adequacy of model behavior.

The largest errors occur through the ways these two regimes are coupled together. The coupling cannot be determined theoretically as the two regimes are not compatible. The only solution is to introduce a tentative coupling and to adapt it so that the coupled model attains a good fit to already known cases. Such loops of adaptations, checks, and re-adaptations treat the coupling mechanism as a variable, without these loops multi-scale methods could hardly be used at all.

In short, mathematics is used as a tool for establishing a good-enough, or even optimal, compromise between accuracy and tractability.

(v) Non-Representational Idealizations

Idealizations are intimately connected to the role of mathematics in the sciences. All mathematical operations inevitably deal with objects or models that are in some sense idealized. The point is what sense of idealization is the relevant one in our context. “According to a straightforward view, we can think of idealization as a departure from complete, veridical representation of real-world phenomena” (WEISBERG 2013, 98). All mathematical models suit this description. They can be adequate without having to be completely faithful representations. The interesting point rather is in what ways they can deviate from the representational ideal.

We want to introduce a type of idealization that cuts across the various suggestions to capture different sorts of idealized models. All these suggestions deal with object-related idealizations. Idealizations create a simpler version of the relevant objects and their relationships so that mathematical models of them become more tractable. In principle, these simplifications could be removed by de-idealizing the model step by step and thereby making the models more and more complex. However, from our perspective, the most important distinction is another one, namely, between object-related and tool-related idealizations. The latter result from the properties of the tool and make sure that the tool can be used in the first place. Tool-related idealizations exhibit no clear relation to issues of representation. The question of how adequate the tool works in a certain situation needs to be tackled independently. Philosopher of science ROBERT BATTERMAN (2010, see also the chapter in this volume) has pointed out that asymptotic limits or singularities regularly destroy the representation relation, because the objects covered by the model simply do not exist.

Synthesis

We have discussed five modes of using mathematics as a tool which partly overlap and are partly heterogeneous in kind. The instrumental use of mathematics is not governed by a single scheme. Rather, the heterogeneity of tools and of tool usages teaches a lesson about the limits of mathematization.

Though our group is still making progress and this essay is hence of a preliminary nature, we would like to highlight two preliminary findings separately: Control replaces explanation, and validation is accomplished by use.

Control often is a practical goal, for instance when a satellite has to be navigated into a stable orbit. The clincher of using mathematics as a tool lies in providing knowledge about how to control processes or systems. In the traditional understanding of the role of mathematics in the sciences, its usefulness in formulating comprehensive, unifying accounts of the phenomena is stressed. Mathematics provides the deductive links that connect the first principles with their empirical consequences and thus produces a pyramid-shaped body of knowledge. The rule of first principles over the phenomena is established by mathematical derivations. In the present context of mathematics as a tool, however, the deductive links are often tenuous, restricted to a narrow

realm of conditions, forged in an ad-hoc manner, or intransparent. As a result, the explanatory power conferred on the principles by applying mathematics is considerably weakened. Yet the power of prediction need not be reduced at the same time. On the contrary, accurate predictions are an essential criterion for judging the quality of mathematical tools. One of the conclusions to be drawn from such consideration is that predictive power may not be completely dependent on insights into the inner workings of nature. Instead, predictive power is often established by using mathematics as an instrument. Consequently, the validation of such tools is accomplished by examining their practical achievements. The pivot of validation is practical use.

We think a general lesson can be learned from regarding mathematics as a tool. Namely, it challenges the philosophical viewpoint that modern science is characterized by a uniform method and also calls into question the role mathematics plays in arguments about science and rational order.

Acknowledgement

We would like to mention that this essay profited greatly from the discussion in our cooperation group that includes PHILIPPE BLANCHARD, JÜRGEN JOST, and MICHAEL RÖCKNER.

Cited literature

- BATTERMAN, ROBERT W. (2010) On the Explanatory Role of Mathematics in Empirical Science, *British Journal for the Philosophy of Science* 61, 1–25.
- KITCHER, PHILIP (1983) *The Nature of Mathematical Knowledge*, New York: Oxford University Press.
- KOYRÉ, ALEXANDRE (1968) *Newtonian Studies*, Chicago: University of Chicago Press.
- KOYRÉ, ALEXANDRE (1978) *Galileo Studies*, Hassocks: Harvester Press.
- LENHARD, JOHANNES (2007) Computer Simulation: The Cooperation Between Experimenting and Modeling, *Philosophy of Science* 74, 176–194.
- MADDY, PENELOPE (1997) *Naturalism in Mathematics*, Oxford: Clarendon Press.
- MAHONEY, MICHAEL S. (1998) *The Mathematical Realm of Nature*, in D. E. GARBER et al.(eds.), *Cambridge History of Seventeenth-Century Philosophy*, Cambridge: Cambridge University Press, Vol. I, pp. 702–55.
- MANCOSU, PAOLO (ed.) (2008) *The Philosophy of Mathematical Practice*, Oxford: Oxford University Press.
- VAN KERKHOVE, BART, JONAS DE VUYST, and JEAN PAUL VAN BENDEGEM (eds.) (2010) *Philosophical Perspectives on Mathematical Practice*, London: College Publications.
- WEISBERG, MICHAEL (2013) *Simulation and Similarity: Using Models to Understand the World*, Oxford: Oxford University Press.
- WINSBERG, ERIC (2003) Simulated Experiments: Methodology for a Virtual World, *Philosophy of Science* 70, 105–125.