"APPROXIMATION AND CONCENTRATION OF MEASURE IN HIGH DIMENSIONS AND APPLICATIONS"
BIELEFELD, JUNE 10–14
TITLES AND ABSTRACTS

Mindaugas Bloznelis

Title. Connectivity thresholds of random community affiliation networks

Abstract. Let $e_1, e_2, \ldots$ be a sequence of edges of the complete graph K_n selected independently and uniformly at random. Given m, the first m elements $e_1, \ldots, e_m$ define random graph on M edges, where M is the (random) number of distinct elements. By conditioning on M one obtains the well known Erdős-Rényi random graph $G(n, M)$. In the talk I will consider a more general random graph G, defined by a sequence $G_1, \ldots, G_m$ of iid random subgraphs of K_n: the vertex set of G is that of K_n and the edge set is the union of the edge sets of $G_1, \ldots, G_m$. One can interpret G as a network of overlapping communities $G_1, \ldots, G_m$. The questions addressed are: the connectivity threshold, the phase transition in the size of the giant component, the asymptotics of counts of small dense subgraphs of G as $n, m \to +\infty$.

Devraj Duggal

Title. On Spherical Covariance Representations

Abstract. We first motive the study of covariance representations by surveying preceding results in the Gauss space. Their spherical counterparts are then derived thereby allowing applications to the spherical concentration phenomenon. Lastly, we will briefly mention an extension to a larger class of Probability measures. This talk is based on joint work with Sergey Bobkov.

Anna Gusakova

Title. Concentration inequalities for Poisson U-statistics

Abstract. Let η be a Poisson point process on a general measurable space. A Poisson functional is a random variable $F(\eta)$, such that almost surely we have $F(\eta) = f(\eta)$ for some measurable veal valued function f on the space of counting measures. Poisson functionals have been intensively studied within last years and they play an important role in stochastic geometry since many important geometric functionals of stochastic geometry models are in fact Poisson functionals. Poisson U-statistic is an example of Poisson functional, which has particularly nice structure. In this talk we present concentration inequalities for Poisson U-statistics under some rather mild conditions. We will discuss their optimality and consider a few applications to stochastic geometry models.

Jonas Jalowy

Title. Evolution of zeros of random polynomials under differential operators: isotropic distributions and connection to free probability

Abstract. Start with a random polynomial with independent coefficients and look at the empirical distribution of its complex zeros. How do these zeros evolve, when we apply the heat flow operator (or other differential operators) to the polynomial? In one example of Weyl polynomials undergoing the heat flow, the limiting zero distribution evolves from the circular law into the elliptic law until it collapses to the Wigner semicircle law. In this talk, I will present the general limiting zero distribution and various descriptions for the dynamics of the zeros. Even though the problem seems innocent at first glance, it features fascinating phenomena and surprising connections to other areas such as free probability, random matrices, (optimal) transport, particle systems and PDE's. Focussing on the free probability perspective, we will see that repeated differentiation corresponds to the free convolution semigroup of the initial distribution and that the effect of the heat flow can be interpreted as increasing a semicircle component in the real direction and decreasing it in the imaginary direction. The theory will be accompanied by illustrative simulations and some intriguing conjectures. This is based on joint work with Brian Hall, Ching Wei Ho and Zakhar Kabluchko.

Zakhar Kabluchko

Title. Evolution of zeros of polynomials under differential operators: real-rooted polynomials and finite free probability

Abstract. Start with a high-degree real-rooted polynomial and look at the empirical distribution of its zeros. How do these zeros evolve if we apply the heat flow operator (or other differential operators) to the polynomial? While the talk of Jonas Jalowy focuses on the case of polynomials with complex roots, we shall consider real-rooted polynomials and discuss connections to finite free probability.

Egor Kosov

Title. Regularity properties of linear and polynomial functionals of random vectors

Abstract. In the talk we present several results about Besov regularity of random variables that are polynomials in random vectors on R^n with sufficiently good densities (e.g. densities of bounded variation, logarithmically concave densities, Gaussian densities). We also show how regularity properties are connected with the small ball probability for random variables of such type. We particularly focus on the case of random vectors with independent coordinates and linear functionals, generalizing the results of Rudelson-Vershynin and Bobkov-Chistyakov about the maximum of the distribution density.

Holger Kösters

Title. Spectral distributions of products of independent random matrices

Abstract. The spectral distributions of products of independent random matrices have found a lot of attention in the last years. In my talk I will discuss some results for certain invariant random matrices which generalize products of independent Ginibre matrices. In particular, I will point out some connections to free probability.

Franz Lehner

Title. Free Integral Calculus

Abstract. Subordination functions in free probability are closely related to non-commutative conditional expectations. We provide closed formulas for the conditional expectation of resolvents of arbitrary non-commutative polynomials in free random variables onto the subalgebras generated subsets of the variables. More precisely, given a linearization of the resolvent we compute a linearization of its conditional expectation. The coefficients of the expressions obtained in this process involve certain Boolean cumulant functionals which can be computed by solving a system of matrix equations. On the way towards the main result we introduce a non-commutative differential calculus which allows to evaluate conditional expectations and the said Boolean cumulant functionals. Joint work with K. Szpojankowski `https://arxiv.org/abs/2311.04039`

James Melbourne

Title. An entropic interpretation of anti-concentration and non-central sections of convex bodies

Abstract. We extend Bobkov and Chistyakov's (2015) upper bounds on concentration functions of sums of independent random variables to a multivariate setting and explore connections to Renyi entropy power inequalities. The approach is based on pointwise estimates on densities of sums of independent random vectors uniform on centered Euclidean balls. In this vein, we also obtain sharp lower bounds on volumes of noncentral sections of isotropic convex bodies. The work is joint with Tomasz Tkocz and Katarzyna Wyczesany.

Alexey Naumov

Title. Rosenthal-type inequalities for linear statistics of Markov chains

Abstract. In this paper, we establish novel deviation bounds for additive functionals of geometrically ergodic Markov chains similar to Rosenthal and Bernstein-type inequalities for sums of independent random variables. We pay special attention to the dependence of our bounds on the mixing time of the corresponding chain. More precisely, we establish explicit bounds that are linked to the constants from the martingale version of the Rosenthal inequality, as well as the constants that characterize the mixing properties of the underlying Markov kernel. Finally, our proof technique is, up to our knowledge, new and based on a recurrent application of the Poisson decomposition.

Leonie Neufeld

Title. Weighted Sums in Free Probability Theory

Abstract. Given free identically distributed self-adjoint random variables $X_1, \dots, X_n$ satisfying certain moment constraints, the free analog of the Berry-Esseen theorem asserts that the distribution of the normalized sum $S_n = n^{-1/2}\sum_{i=1}^n X_i$ converges weakly to Wigner's semicircle law with a rate of convergence of order $n^{-1/2}$ measured with respect to the Kolmogorov distance. Replacing the sum S_n by the weighted sum $S_\theta = \sum_{i=1}^n \theta_i X_i$ for certain vectors $\theta = (\theta_1, \dots, \theta_n)$ taken from the unit sphere, we show that the rate of convergence to Wigner's semicircle law can be improved to the order n^{-1}. This establishes a free analog of a result proven by Klartag and Sodin in 2012 in classical probability theory.

Leonid Pastur

Title. On Random Matrices Arising in Deep Neural Networks

Abstract. We will consider the distribution of singular values for the product of random matrices related to the analysis of deep neural networks. The matrices are similar to the product of sample covariance matrices of statistics, but an important difference is that in statistics the population covariance matrices are assumed to be non-random or random but independent of the random data matrix, while now they are certain functions of the random data matrices (matrices of synaptic weights in the terminology of deep neural networks). The problem was treated recently by J. Pennington et al. assuming that the weight matrices are either Gaussian or unitary and using the methods of free probability theory. Since, however, free probability theory deals with population covariance matrices that do not depend on data matrices, its applicability to this case must be justified. We will use a version of the random matrix theory techniques to provide a justification in the general case where the entries of weight matrices are independent identically distributed random variables with zero mean and finite fourth moment. We will also comment on the case where the weight matrices are unitary and on the free probability meaning of these results.

Joscha Prochno

Title. The probabilistic behavior of lacunary sums

Abstract. It is known through classical works of Kac, Salem, Zygmund, Erdös and Gal that lacunary sums behave in several ways like sums of independent random variables, satisfying, for instance, a central limit theorem or a law of the iterated logarithm. We present some recent results on their large deviation behavior, which show that on this scale, contrary to the scale of the CLT or the LIL, the LDP is sensitive to the arithmetic properties of the underlying Hadamard gap sequence. If time allows, we shall briefly discuss some recent results regarding the optimality of Diophantine conditions in the law of the iterated logarithm for lacunary systems.

Cyril Roberto

Title. Conditional Entropy Power Inequality and stability of log-supermodular densities under convolution

Abstract. We recall the classical Entropy power Inequality and (briefely) present the idea of the proof by Carlen and Soffer (after Shannon and Stam). Then we present a conditional form of such an inequality based on the notion of log-supermodularity. In particular we show that a log-supermodular density remains log-supermodular after convolution by product of log-concave densities, confirming a conjecture by Zartash and Robeva. Joint with Mokshay Madiman and james Melbourne.

Mark Rudelson

Title. A large deviation inequality for the rank of a random matrix

Abstract. We obtain the optimal estimate for the probability that an n by n random matrix with i.i.d. entries has rank at most n-k, for any k which is less than $n^{1/2}$. This estimate allows to prove a conjecture of Feige and Lellouche from computer science.

Mikhail Sodin

Title. Random Weierstrass zeta-function

Abstract. We describe a construction of random meromorphic functions with prescribed simple poles with unit residues at a given planar stationary point process. We characterize those stationary processes with finite second moment for which, after subtracting the mean, the random function becomes stationary. These random meromorphic functions can be viewed as random analogues of the Weierstrass zeta function from the theory of elliptic functions, or equivalently as electric fields generated by an infinite random distribution of point charges. The talk will be based on joint work with Oren Yakir and Aron Wennman (arXiv, 2022).

Marta Strzelecka

Title. Operator ℓ_p to ℓ_q norms of random matrices with iid entries

Abstract. Although the behaviour of the spectrum of a random matrix X with iid entries $X_{i,j}$ is now well understood, until now not much was known about the non-asymptotic behaviour of operator norms from ℓ_p to ℓ_q (where $1 \le p, q \le \infty$) of such matrices when (p, q) differs from $(2, 2)$. In the talk we present our recent result (joint with R. Latala) valid under a mild regularity assumption that the entries $X_{i,j}$ satisfy

$$\|X_{i,j}\|_{2\rho} \le \alpha \|X_{i,j}\|_{\rho} \qquad \text{for every } \rho \ge 1$$

with some constant $\alpha \ge 1$. We show that under this assumption the expectation of the operator norm of X from ℓ_p^n to ℓ_q^m is comparable, up to a constant depending only on α, to

$$m^{1/q} \sup_{t \in B_p^n} \Big\| \sum_{j=1}^{n} t_j X_{1,j} \Big\|_{q \wedge \log m} + n^{1/p^*} \sup_{s \in B_{q^*}^m} \Big\| \sum_{i=1}^{m} s_i X_{i,1} \Big\|_{p^* \wedge \log n}.$$

In the case of square matrices (i.e., when $m = n$), this quantity is comparable to

$$\begin{cases} n^{1/q+1/p^*-1/2} \|X_{1,1}\|_2, & p^*, q \le 2, \\ n^{1/(p^* \wedge q)} \|X_{1,1}\|_{p^* \wedge q \wedge \log n}, & p^* \vee q \ge 2. \end{cases}$$

Formulas of this type exist also in the rectangular case if the entries $X_{i,j}$ are: Gaussian, Weibullian, log-concave tailed, or log-convex tailed.

Michał Strzelecki

Title. The s-numbers of Schatten class embeddings

Abstract. Roughly speaking, the s-numbers of a linear operator between two Banach spaces are certain quantities which indicate how compact this operator is. One can introduce various sequences of s-numbers (classical examples include the approximation and Gelfand numbers of an operator). Then one can wonder what the asymptotic behavior of these s-numbers is in the case when we have a nice and simple linear operator between two nice and classical Banach spaces.

In the talk I will introduce all the necessary preliminaries about s-numbers and then give examples of techniques which can be used to find the asymptotic behavior of the s-numbers of the embeddings $\ell_p^N \to \ell_q^N$ (some probabilistic accents will appear here). In the last part we will discuss what changes when we look at the s-numbers of the embeddings of Schatten classes. Based on joint work with Joscha Prochno.

Alexander Tikhomirov

Title. Limit theorems for the spectrum of random block matrices

Abstract. We consider the limiting behavior of the empirical spectral distribution function of random block structure matrices. In particular, we consider the asymptotic behavior of the spectrum distribution of circular block matrices with blocks of higher dimensional, as well as the adjacency matrices of bipartite random graphs with weights.

Philipp Tuchel

Title. Limit theorems for the volume of random projections and sections of ℓ_p^N-balls

Abstract. We consider N-dimensional unit balls $\mathbb{B}_p^N$ generated by the p-norm. For each N we sample a random subspace E_N of a fixed dimension $m \in \mathbb{N}$ and consider the volume and other characteristics of $\mathbb{B}_p^N$ projected onto E_N or intersected with E_N. In this setting we prove central limit theorems, moderate and large deviation principles in high dimensions, that is, as $N \to \infty$. As an application we derive geometric properties such as the exact high-dimensional first-order asymptotics of the intrinsic volumes of $\mathbb{B}_p^N$.

Vladimir Ulyanov

Title. On Approximations of Sums of Locally Dependent Random Variables and its Applications

Abstract. Let $(X_i, i \in J)$ be a family of locally dependent nonnegative integer-valued random variables, and consider the sum $W = \sum_{i\in J} X_i$. We first establish a general upper bound for $d_{TV}(W, M)$ using Stein's method, where the target variable M is either the mixture of Poisson distribution and binomial or negative binomial distribution. As applications, we attain $O(|J|^{-1})$ optimal error bounds for (k_1, k_2)-runs and k-runs. Our results are significant improvements of the existing results of order $O(|J|^{-0.5})$. Moreover, using a recent result of Bobkov and Ulyanov (2022) on a refined central limit theorem for integer-valued independent summands we obtain asymptotic expansions for the distribution function of W. The talk is based on preprint, see `https://doi.org/10.48550/arXiv.2209.09770`

Martin Wahl

Title. A kernel-based analysis of Laplacian Eigenmaps

Abstract. Laplacian Eigenmaps and Diffusion Maps are nonlinear dimensionality reduction methods that use the eigenvalues and eigenvectors of (un)normalized graph Laplacians. Both methods are applied when the data is sampled from a low-dimensional manifold, embedded in a high-dimensional Euclidean space. From a mathematical perspective, the main problem is to understand these empirical Laplacians as spectral approximations of the underlying Laplace-Beltrami operator. In this talk, we study Laplacian Eigenmaps through the lens of kernel PCA, and consider the heat kernel as reproducing kernel feature map. This leads to novel points of view and allows to leverage results for empirical covariance operators in infinite dimensions.

Andrei Zaitsev

Title. Estimates of the proximity of successive convolutions of the probability distributions on the convex sets and in the Prokhorov distance

Abstract. Let $X_1, \dots, X_n, \dots$ be independent identically distributed d-dimensional random vectors with common distribution F. Then $S_n = X_1 + \cdots + X_n$ has distribution F^n (degree is understood in the sense of convolutions). Let

$$\rho(F, G) = \sup_A |F\{A\} - G\{A\}|,$$

where the supremum is taken over all convex subsets of $\mathbb{R}^d$. Basic result is as follows. For any nontrivial distribution F there is $c(F)$ such that

$$\rho(F^n, F^{n+1}) \le \frac{c(F)}{\sqrt{n}}$$

for any natural n. The distribution F is considered trivial if it is concentrated on a hyperplane that does not contain the origin. Clearly, for such F

$$\rho(F^n, F^{n+1}) = 1.$$

A similar result is obtained for the Prokhorov distance between distributions normalized by the square root of n.

Dmitry Zaporozhets

Title. Pair correlations of sequences

Abstract. The presentation is based on work in progress jointly with V. V. Kapustin. We will derive an estimate for the growth of the variance of sequences that satisfy the property of zeros of Riemann's zeta function, discovered by Montgomery, thereby obtaining a deterministic analog of the corresponding result for the sine process. Additionally, a number of open questions related to this topic will be proposed.