Magnetized QCD on the lattice

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Preface:
QCD phases and equation of state

The phases of QCD

- phases of QCD characterized by approx. order parameters
- quark condensate $\bar{\psi}\psi$ (chiral symmetry breaking)
- Polyakov loop P (deconfinement)

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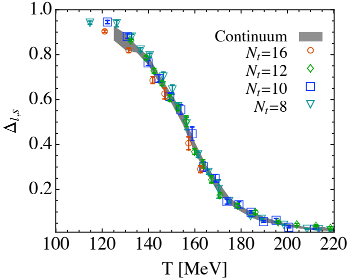

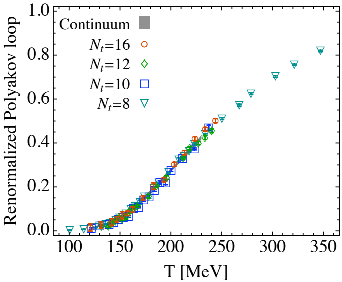

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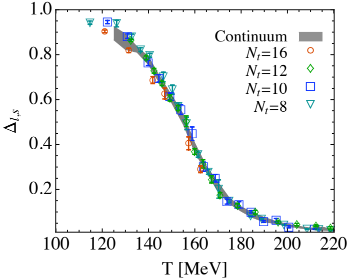

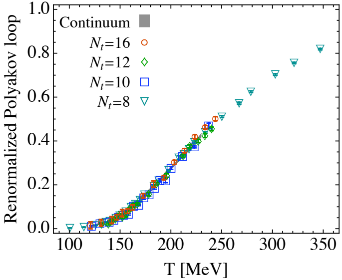

- crossover Aoki et al. '06 Bhattacharya et al. '14

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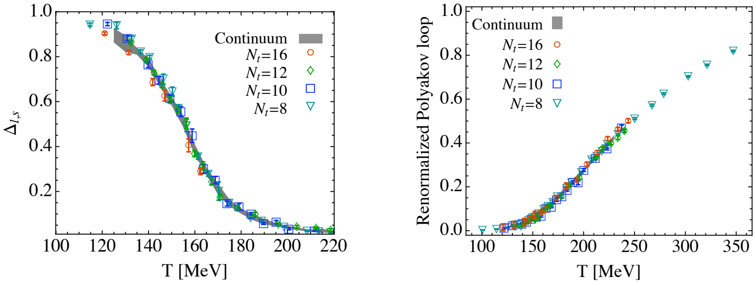
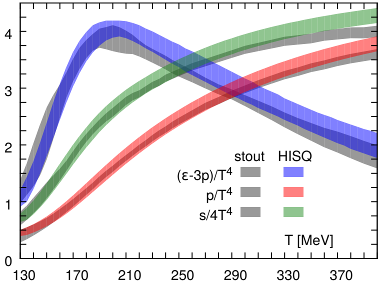


- crossover Aoki et al. '06 Bhattacharya et al. '14
- $T_c \leftrightarrow$ inflection point Bazavov et al. '18

Equation of state of QCD

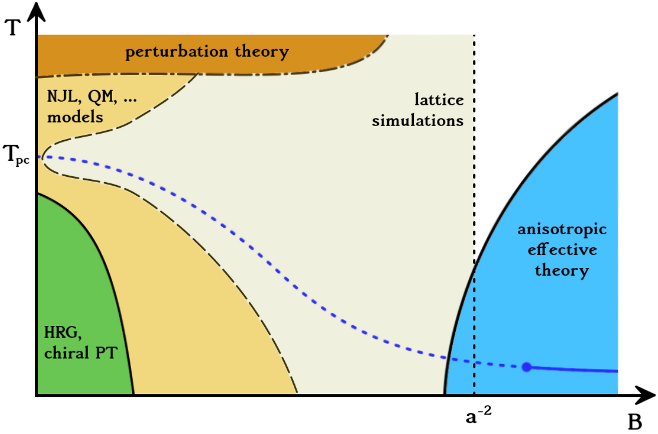
- equilibrium description $\epsilon(p)$ of QCD matter
- encoded in, for example, $p(T)$



Bazavov et al. '14   Borsányi et al. '13

Phase diagram

- approaches: effective theories, low-energy models, lattice simulations, perturbation theory



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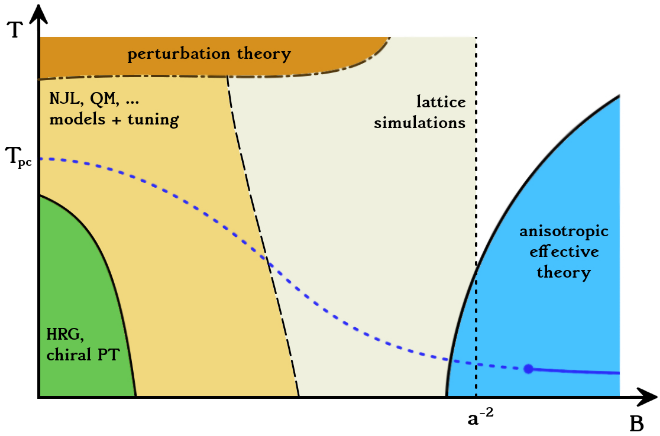
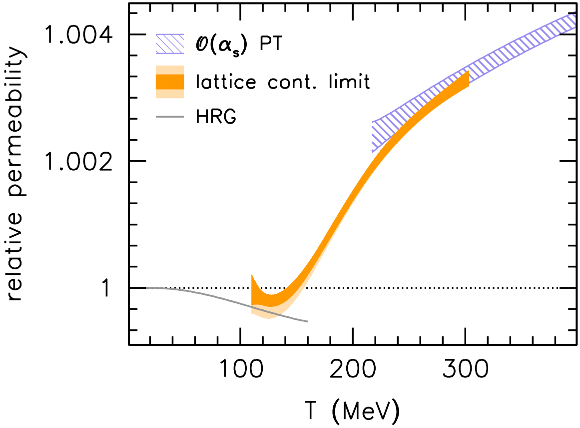


- tuning necessary for low-energy models

Permeability



- deviation to unity gives $\mathcal{O}(B^2)$ contribution to EoS

Outline

- why magnetic fields?
- lattice approach
- phase diagram
  - magnetic catalysis and inverse catalysis
  - new developments about the mass-dependence
  - large B limit
  - PNJL model and improvement
- permeability
  - magnetic flux quantization
  - current-current correlators
- summary

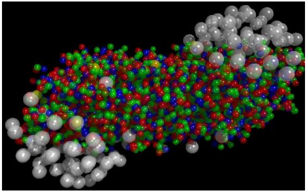
Why magnetic fields?

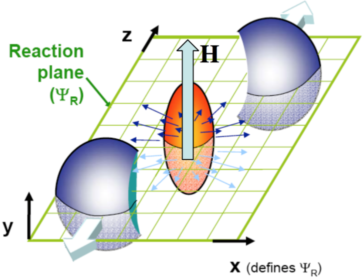
Magnetic fields

- off-central heavy-ion collisions Kharzeev, McLerran, Warringa '07

  impact: chiral magnetic effect, anisotropies, elliptic flow ...

  Fukushima '12 Kharzeev, Landsteiner, Schmitt, Yee '14





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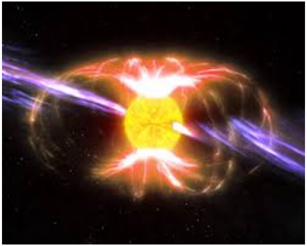

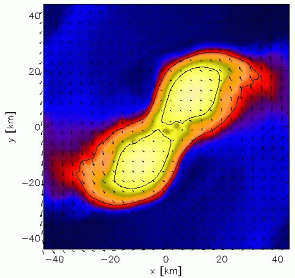

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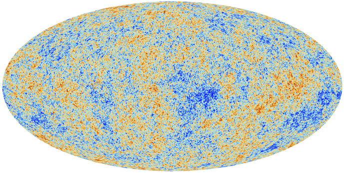
- in the early universe, generated through phase transition in electroweak epoch ✐ Vachaspati '91 ✐ Enqvist, Olesen '93



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- strength: $B \approx 10^{15}$ T $\approx 10^{20} B_{\mathrm{earth}} \approx 5 m_\pi^2$
  $\rightsquigarrow$ competition between strong force and electromagnetism

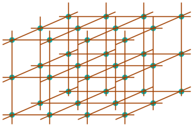
Lattice approach

Path integral and lattice field theory

- path integral [Feynman '48]

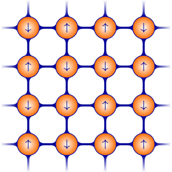
$$\mathcal{Z} = \int \mathcal{D}A_\mu \, \mathcal{D}\bar{\psi} \, \mathcal{D}\psi \, \exp\left(- \int \mathrm{d}^4x \, \mathcal{L}_{\mathrm{QCD}}(x) \right)$$

- discretize spacetime on a lattice with spacing a [Wilson '74]



- Monte-Carlo algorithms to generate configurations

  like in the 2D Ising model:



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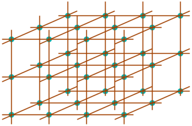


- Monte-Carlo algorithms to generate configurations with $\sim 10^9$ variables $\rightsquigarrow$ high-performance computing

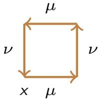
Lattice discretization

- path integral over quarks analytically

$$\mathcal{Z} = \int \mathcal{D}U_\mu \exp\left(-S_{\rm gluon}\right) \det(\not{D} + m)$$

- work with $\mathrm{SU}(3)$ link variables $U_\mu = \exp(iaA_\mu) \equiv \xrightarrow[\mu]{}$
- gluon action ($\beta = 6/g_s^2$)

$$S_{\rm gluon} \sim \beta \sum_x \sum_{\mu,\nu} \mathrm{Re\,Tr}$$



- staggered Dirac operator

$$\not{D} \sim \sum_\mu \eta_\mu \left[\xrightarrow[\mu]{} - \xleftarrow[\mu]{} \right]$$

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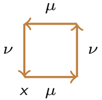
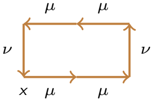
$$S_{\text{gluon}} \sim \beta \sum_x \sum_{\mu,\nu} \mathrm{Re}\,\mathrm{Tr}\Big[c_1 \; [\text{plaquette}] + c_2 \; [\text{rectangle}] \Big]$$





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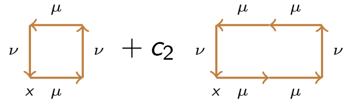

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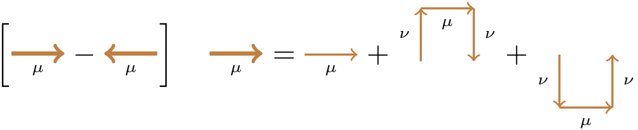


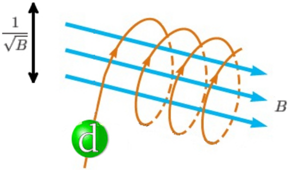
Phase diagram – review

Magnetic catalysis, free quarks

- chiral condensate ↔ spectral density around 0 Banks,Casher '80

$$\bar{\psi}\psi \sim \mathrm{tr}\,(\not{D} + m)^{-1} \xrightarrow{m\to 0} \rho(0)$$

- for free quarks, ρ is determined by Landau levels:



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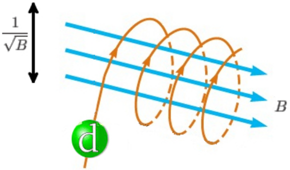

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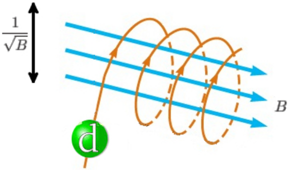

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- magnetic catalysis: $\bar{\psi}\psi$ is enhanced by B

  Gusynin, Miransky, Shovkovy '96 Shovkovy '13

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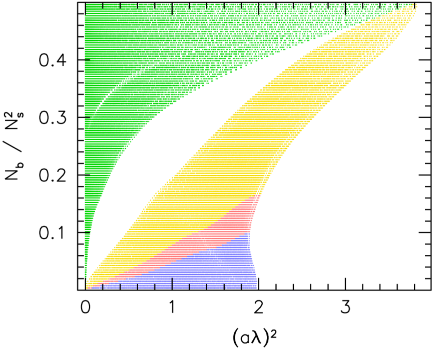

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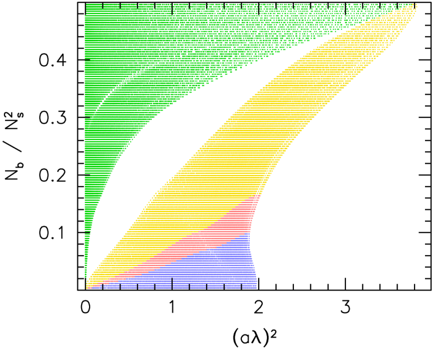

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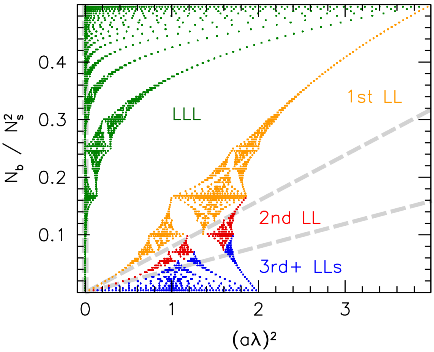

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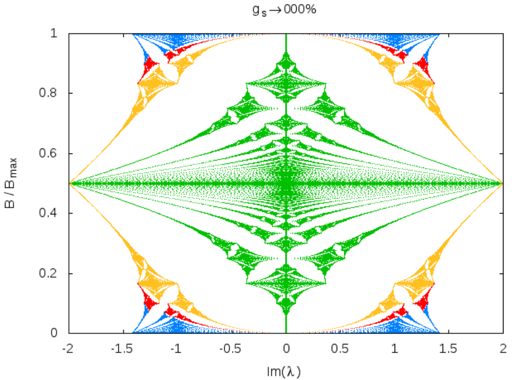
- side remark: free case solution on the lattice $\leftrightarrow$ Hofstadter's butterfly (solid state physics model) Hofstadter '76

Magnetic catalysis, full QCD



Sea quarks in a magnetic field

- effect of B in full QCD Bruckmann, Endrődi, Kovács '13
  - direct (valence) effect $B \leftrightarrow q_f$
  - indirect (sea) effect $B \leftrightarrow q_f \leftrightarrow g$

$$\left\langle \bar{\psi}\psi(B) \right\rangle \propto \int \mathcal{D}A_\mu \, e^{-S_g} \underbrace{\det(\not{D}(B,A)+m)}_{\text{sea}} \underbrace{\mathrm{Tr}\left[(\not{D}(B,A)+m)^{-1}\right]}_{\text{valence}}$$

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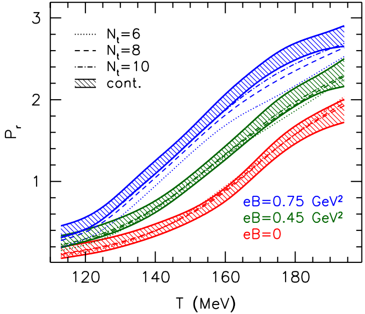

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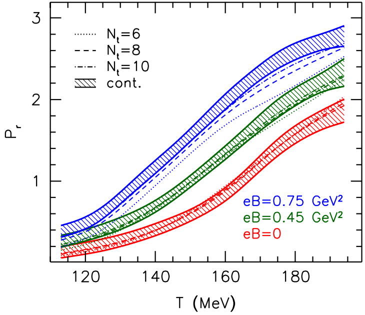

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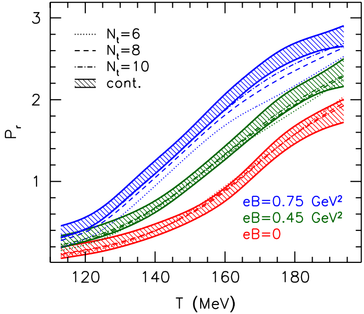
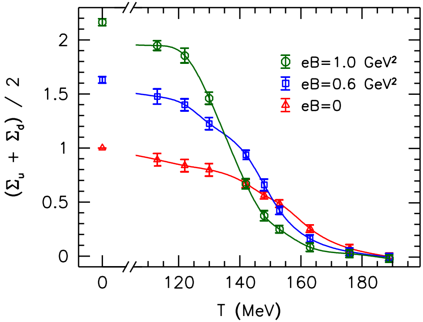


- P anticorrelates with condensate
- sea effect reduces $\left\langle \bar\psi\psi \right\rangle$

Phase diagram for $B > 0$

- physical m_π, staggered quarks, continuum limit
  Bali, Bruckmann, Endrődi, Fodor, Katz et al. '11 '12



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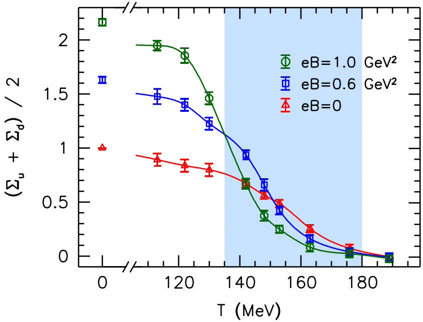

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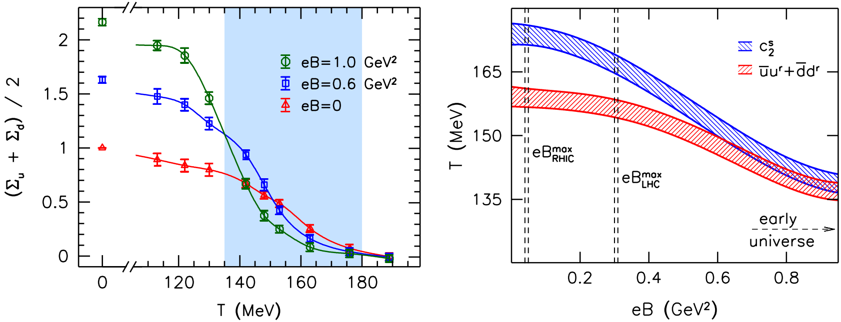

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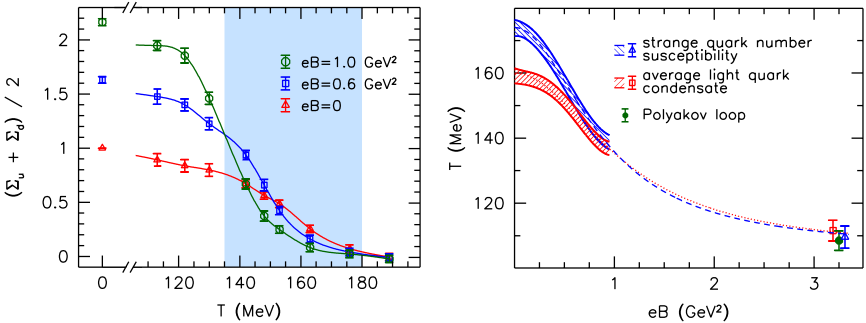

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Quark mass dependence

IMC $\stackrel{?}{=}$ $T_c(B) \searrow$

- early lattice simulations: 🔗 D'Elia, Mukherjee, Sanfilippo, '10
  heavier quarks + lattice artefacts = no IMC, $T_c(B) \nearrow$

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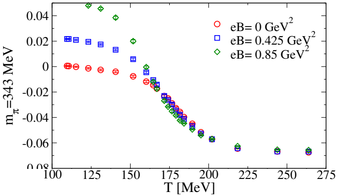

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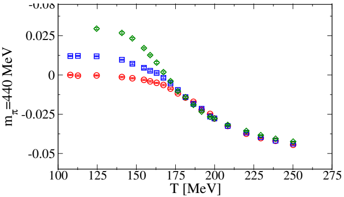

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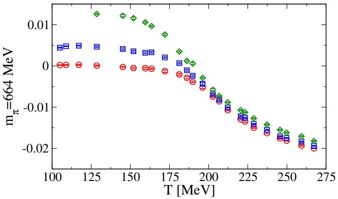

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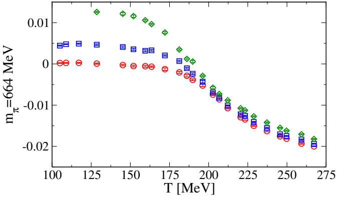

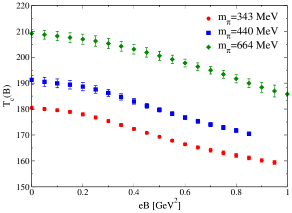

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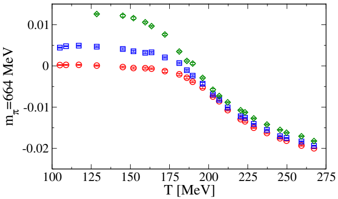

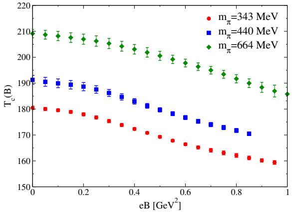

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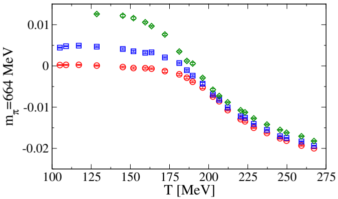
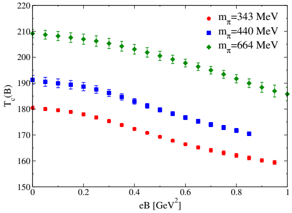
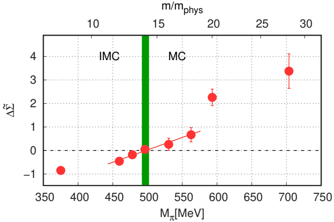


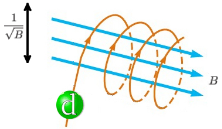


- IMC $\neq$ $T_c(B) \searrow$
- no IMC $m_\pi \gtrsim 500$ MeV
  🔗 Endrődi, Giordano et al. '19



Large B limit

- full QCD simulations only possible for $eB \ll 1/a^2$
- calculate effective theory for $eB \gg \Lambda^2_{\mathrm{QCD}}, T^2$
- B breaks rotational symmetry
  and effectively reduces dimensionality



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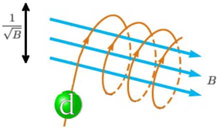

- quarks decouple and gluons inherit spatial anisotropy: [Endrődi '15], see also [Miransky, Shovkovy '02]

$$\mathcal{L}_{\mathrm{QCD}} \xrightarrow{B\to\infty} \mathrm{tr}\,\mathcal{B}_z^2 + \mathrm{tr}\,\mathcal{B}_{x,y}^2 + \infty \cdot \mathrm{tr}\,\mathcal{E}_z^2 + \mathrm{tr}\,\mathcal{E}_{x,y}^2$$

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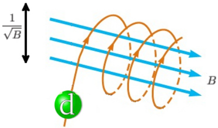

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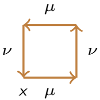
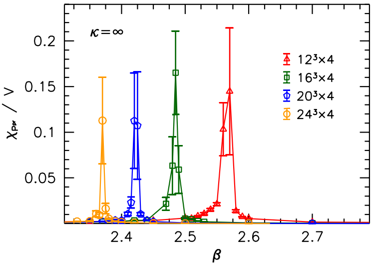
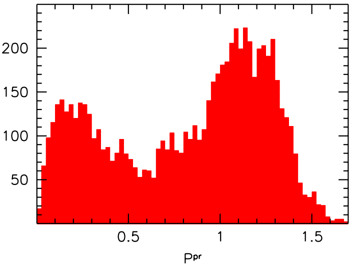
- $S_{\rm gluon} \sim \sum_x \sum_{\mu,\nu} \mathrm{Re\,Tr}$  $\cdot \begin{cases} \infty & \mu,\nu = z,t \\ \beta & \text{otherwise} \end{cases}$

First-order transition

- order parameter is the Polyakov loop Endrődi '15





- Polyakov loop susceptibility peak height scales with V
- histogram shows double peak-structure at T_c

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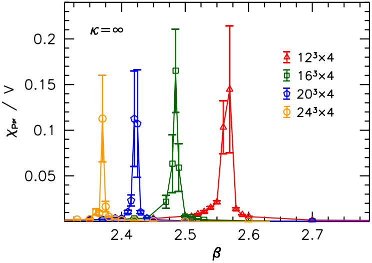

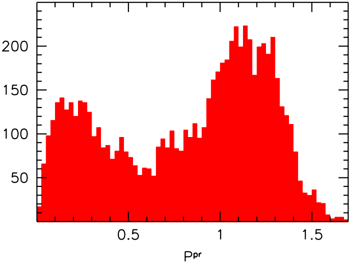

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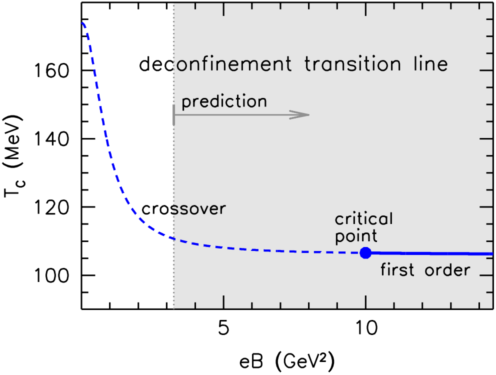
Phase diagram



- location of a critical point, estimated via the narrowing of susceptibility peaks in full QCD Endrődi '15

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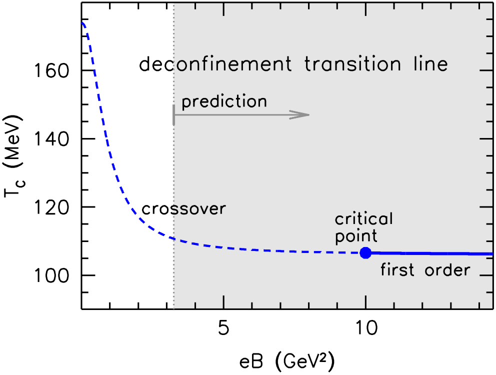

- location of a critical point, estimated via the narrowing of susceptibility peaks in full QCD Endrődi '15
- $B \to \infty$ limit is unaffected by quark masses
  $\Rightarrow$ consistent with mass-independence of $T_c(B) \searrow$

Model approaches

Low-energy models

- model calculations predict the opposite phase diagram
  Andersen, Naylor, Tranberg '14
  - no inverse magnetic catalysis for any T
  - $T_c(B)$ increases

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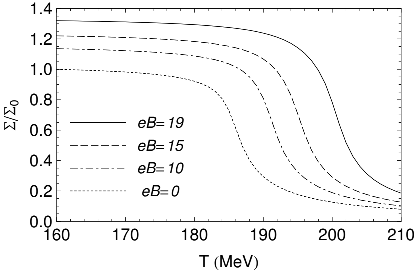
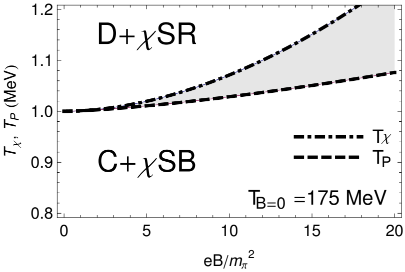
- one out of the many examples: the PNJL model
  Gatto, Ruggieri '11





Improving the PNJL model

- parameter G (four-fermion coupling)
- provide lattice input at $T = 0$, $B > 0$ to define physical $G(B)$
  Endrődi, Markó '19
- input = constituent quark mass (lattice: from baryon masses)

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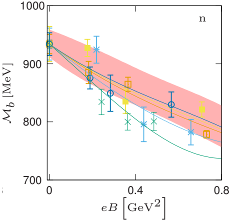

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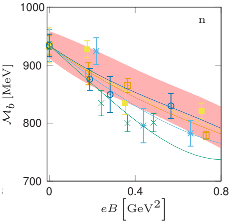

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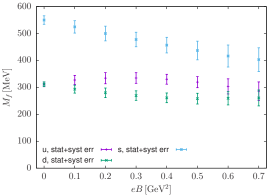

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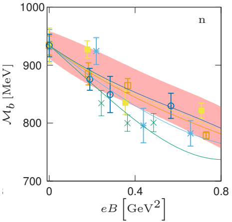

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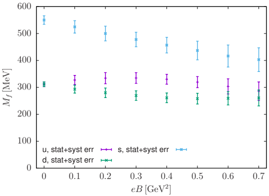

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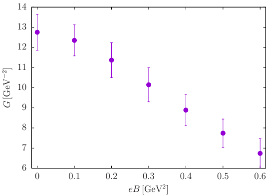

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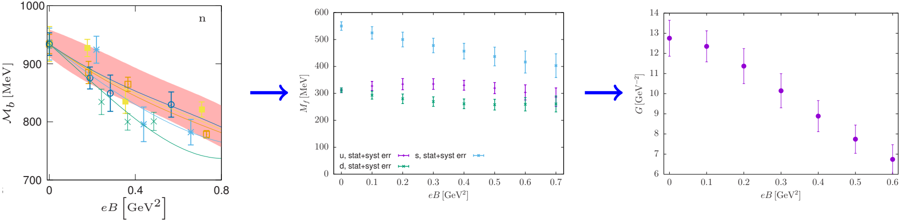

- to achieve roughly B-independent constituent quark masses, $G(B)$ needs to decrease

Improving the PNJL model

- compare standard and improved PNJL model

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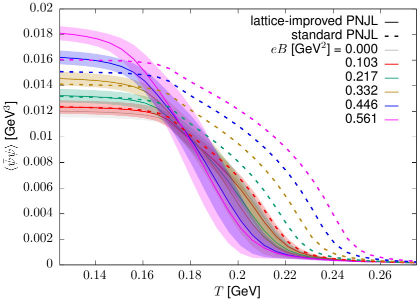

- inverse catalysis emerges in transition region

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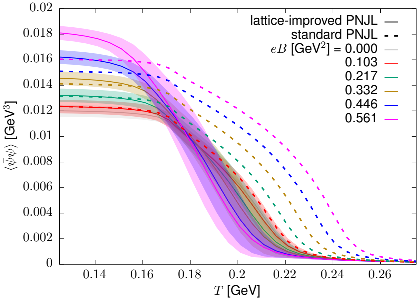

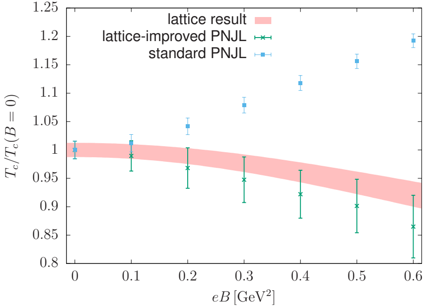

- inverse catalysis emerges in transition region
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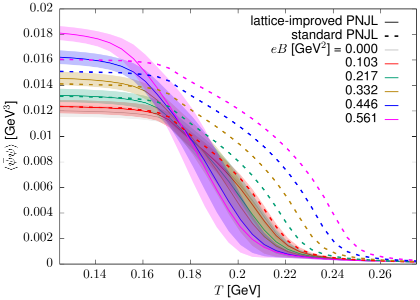
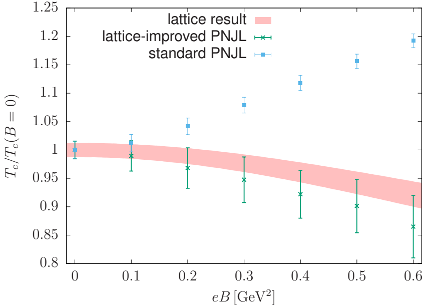
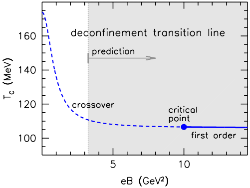




- inverse catalysis emerges in transition region
- $T_c(B)$ decreases
- perfect agreement with lattice results

Phase diagram – summary

- phase diagram for strong background magnetic fields



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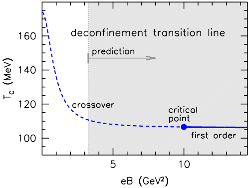
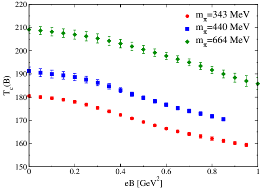
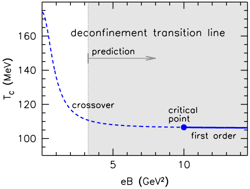


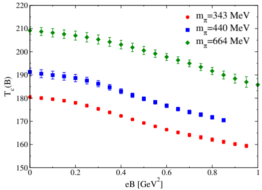
- $T_c(B)$ similar for heavier quarks IMC only present for light quarks

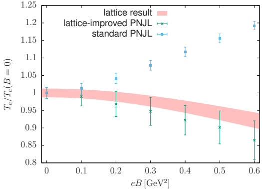


Phase diagram – summary

- phase diagram for strong background magnetic fields
- $T_c(B)$ similar for heavier quarks
  IMC only present for light quarks
- PNJL model can be improved using only $T = 0$ lattice input







Equation of state – a new method to calculate the permeability

Susceptibility and permeability

- leading-order dependence of matter free energy density on B

$$\chi = -\left.\frac{\partial^2 f}{\partial(eB)^2}\right|_{B=0}$$

from this the $\mathcal{O}(B^2)$ equation of state can be reconstructed

- total free energy

$$f^{\mathrm{tot}} = -\chi \cdot \frac{(eB)^2}{2} + \frac{B^2}{2} = \frac{B^2}{2\mu}$$

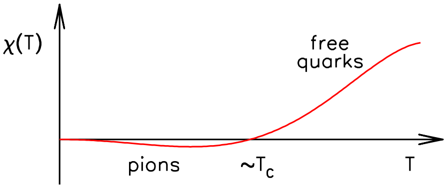
- permeability 🔗 Landau-Lifschitz Vol 8.

$$\mu = \frac{1}{1 - e^2\chi}$$

- $\mu > 1$ $(\chi > 0)$: paramagnetism
  $\mu < 1$ $(\chi < 0)$: diamagnetism

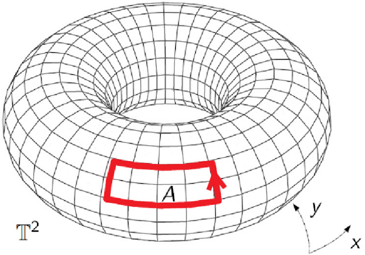
Magnetic susceptibility – expectations

- in the vacuum $\mu = 1$, so $\chi = 0$
- spins align with B, so free quarks are paramagnetic
- orbital angular momentum anti-aligns with B (Lenz's law), so free pions are diamagnetic



Flux quantization problem

Magnetic field on the torus



torus $\mathbb{T}^2$
with surface area $L_x L_y$

D'Elia, Negro '11

- phase factor along path: $\varphi_{\mathcal{C}} = \exp(iq \oint_{\mathcal{C}} \mathrm{d}x_\mu A_\mu)$
- Stokes:
$$\varphi_{\mathcal{C}} = \exp(iq \iint_A \mathrm{d}\sigma B) = \exp(iqB \cdot A)$$
but also
$$\varphi_{\mathcal{C}} = \exp(-iq \iint_{\mathbb{T}^2 - A} \mathrm{d}\sigma B) = \exp(-iqB \cdot (L_x L_y - A))$$
- consistent if 't Hooft '79 Hashimi, Wiese '08

$$\exp(iqBL_x L_y) = 1 \quad \rightarrow \quad qBL_x L_y = 2\pi \cdot N_b, \quad N_b \in \mathbb{Z}$$

Flux quantization

- flux quantization in finite volume

$$eB = \frac{6\pi \cdot N_b}{L_x L_y}, \qquad N_b = 0, 1, \ldots$$

$\Rightarrow \chi$ via differentiation wrt. B is ill-defined

- workarounds:
  - calculate $f(N_b)$ in a sufficiently large volume and differentiate numerically 🔗 Bonati et al. '13 🔗 Bali et al. '14
    ↯ computationally expensive
  - replace constant B by 'half-half setup' with zero flux, differentiation is allowed 🔗 Levkova, DeTar '13
    ↯ introduces large finite size effects
  - relate χ to pressure differences 🔗 Bali et al. '13
    ↯ needs anisotropic lattices
  - new method: express χ as an operator in the thermodynamic limit 🔗 Bali, Endrődi, Piemonte '20

New method: sketch

Current-current correlator method

- vector potential interacts with current

$$i \int \mathrm{d}^4x\, A_\mu\, j_\mu, \qquad j_\mu = \sum_f q_f \bar{\psi} \gamma_\mu \psi$$

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- susceptibility at finite p_1

$$B(x_1) = \cos(p_1 x_1) \cdot B,$$

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- use trigonometric identities + translational invariance + trick

Current-current correlator method

- oscillatory susceptibility

$$\chi^{(p_1)} = \int \mathrm{d}x_1 \, \frac{1-\cos(p_1 x_1)}{p_1^2} \, G(x_1), \qquad G(x_1) = \int \mathrm{d}x_2 \mathrm{d}x_3 \mathrm{d}x_4 \, \langle j_2(x) j_2(0) \rangle$$

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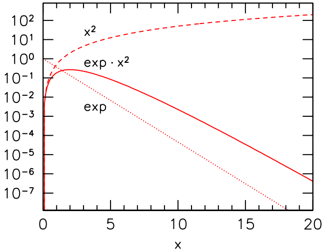
- $p_1 \to 0$ in the infinite volume

$$\chi = \int \, dx_1 \, \frac{G(x_1)}{2} \cdot x_1^2$$



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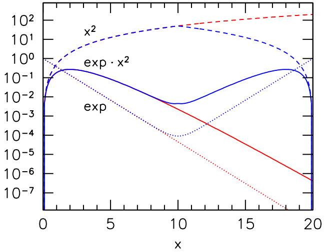

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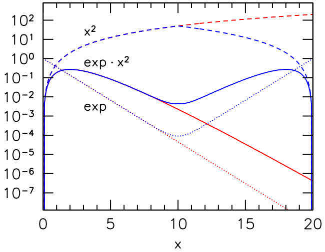
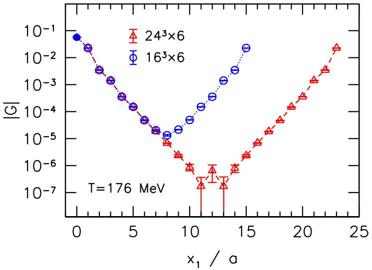
- $p_1 \overset{\sim}{\to} 0$ in finite volume

$$\chi = \int_0^L \mathrm{d}x_1 \, \frac{G(x_1)}{2} \cdot \begin{cases} x_1^2, & x_1 \leq L/2 \\ (x_1 - L)^2, & x_1 > L/2 \end{cases}$$



- cusp of kernel at $x_1 = L/2$ is unproblematic

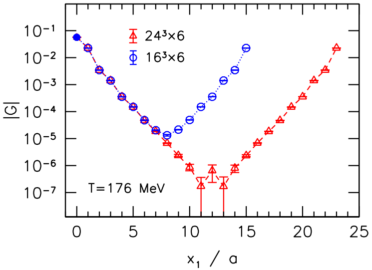
- correlator

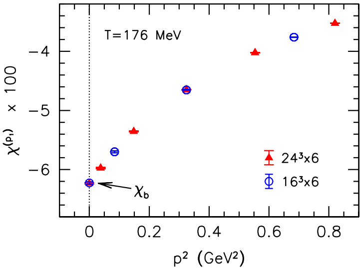


|G|
10⁻¹
10⁻²
10⁻³
10⁻⁴
10⁻⁵
10⁻⁶
10⁻⁷
24³×6
16³×6
T=176 MeV
0
5
10
15
20
25
x1 / a

Correlators

- correlator and its convolution with the kernels





Correlators

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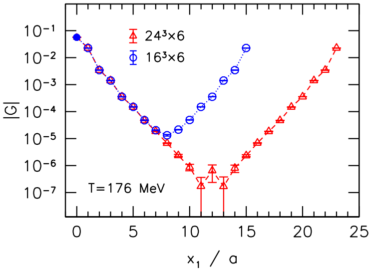
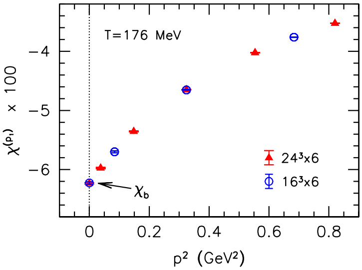
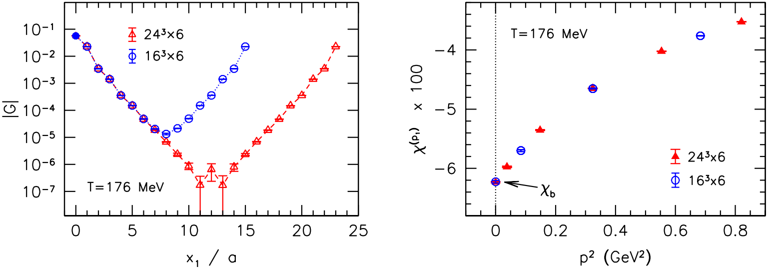




- finite volume effects indeed small

Correlators

- correlator and its convolution with the kernels

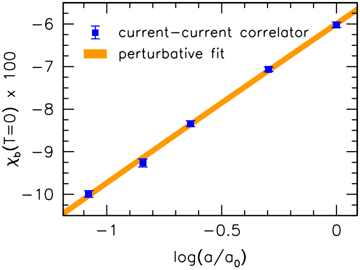


- finite volume effects indeed small
- note: $\chi^{(p)}$ analogous to vacuum polarization form factor relevant for muon $g-2$ calculations at $T=0$ Bali, Endrődi '15

Results

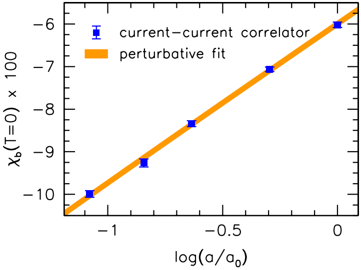
Zero temperature

- susceptibility contains additive divergence $\propto \log a$ due to charge renormalization 🔗 Schwinger '51 🔗 Bali et al. '14



Zero temperature

- susceptibility contains additive divergence $\propto \log a$ due to charge renormalization Schwinger '51 Bali et al. '14



- renormalize as $\chi(T) = \chi_b(T) - \chi_b(T=0)$

Zero temperature

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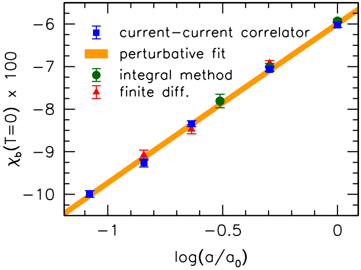
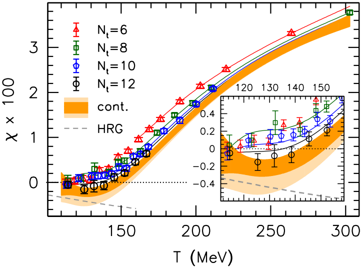


- renormalize as $\chi(T) = \chi_b(T) - \chi_b(T = 0)$
- different methods in the literature agree with each other

Nonzero temperature

- continuum extrapolation using four lattice spacings



Nonzero temperature

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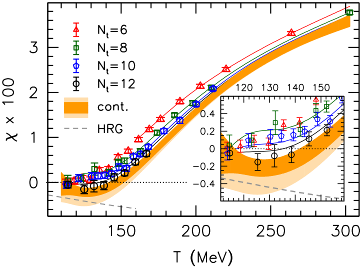

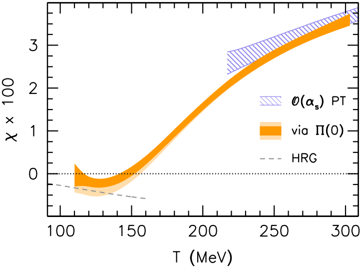

- comparison to HRG model (low T) Endrődi '13
  and to perturbation theory (high T) Bali et al. '14

Nonzero temperature

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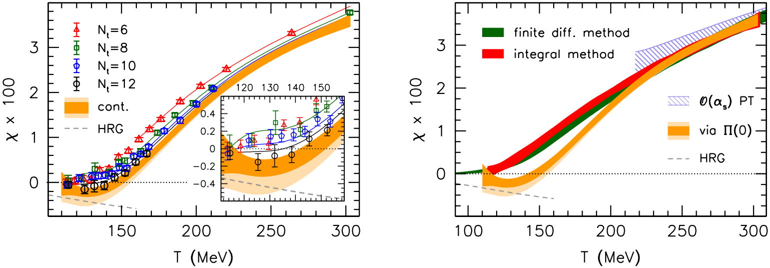
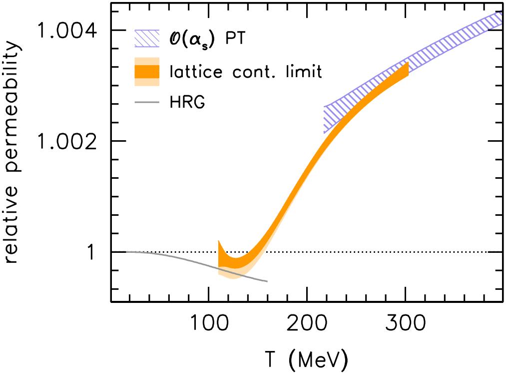


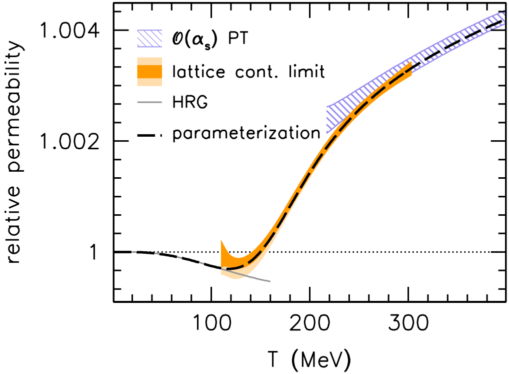
- comparison to HRG model (low T) Endrődi '13
  and to perturbation theory (high T) Bali et al. '14
- taste splitting lattice artefacts severe at low T; careful continuum extrapolation required Bali, Endrődi, Piemonte '20

Permeability



- permeability $\mu = (1 - e^2\chi)^{-1}$

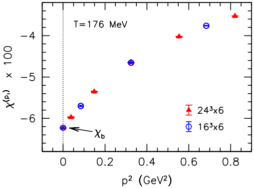
Permeability



- permeability $\mu = (1 - e^2\chi)^{-1}$
- parameterization as python script, to be used in models
  https://arxiv.org/src/2004.08778v2/anc/param_EoS.py
  contains all other observables in the EoS

Permeability – summary

- avoid flux quantization issue; susceptibility as smooth limit in finite volumes



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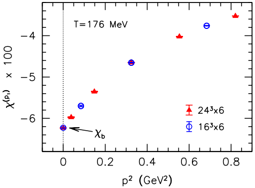
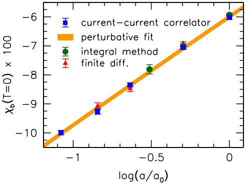


- zero-temperature subtraction of additive divergences



Permeability – summary

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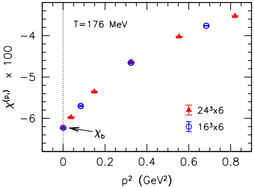

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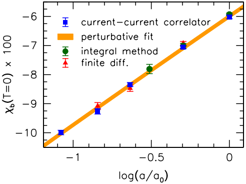

- pions are diamagnetic, QGP is paramagnetic