Strongly interacting matter in intense electromagnetic fields

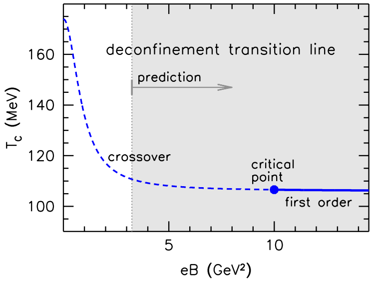
Gergely Endrődi

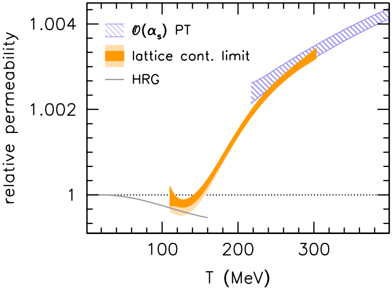
University of Bielefeld

Physics in Intense Fields 2022
September 2, 2022

Appetizer

strongly interacting matter in magnetic fields:
fundamental phase diagram and permeability
from first-principles lattice QCD simulations





Endrődi '15

D'Elia, Maio, Sanfilippo, Stanzione '21

Bali, Endrődi, Piemonte '20

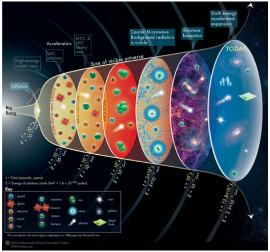
Outline

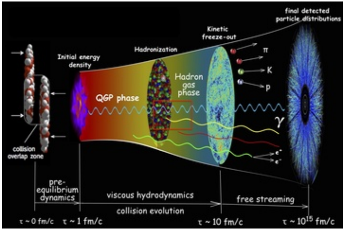
- introduction:
  strongly interacting matter in strong electromagnetic fields
  see also J. Andersen Fri 08:30
- lattice simulation techniques
- phase diagram and permeability
- electric background fields
- summary

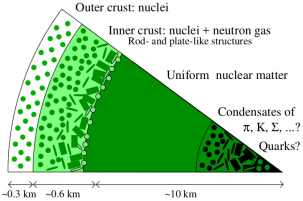
Introduction

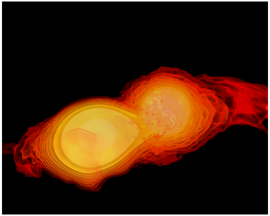
Extreme environments

- hot and/or dense strongly interacting matter in
  - QCD epoch of early Universe
  - heavy-ion collisions
  - neutron stars and their mergers

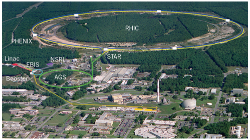


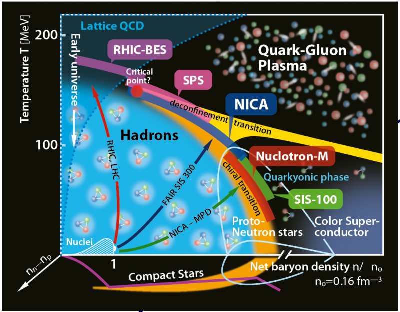




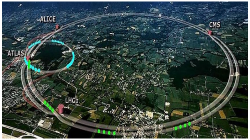


Major experimental and observational campaigns

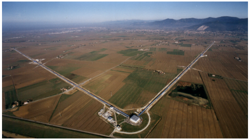


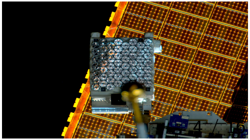


Observational astronomy



Heavy ion collisions





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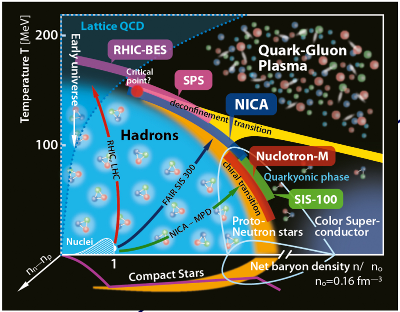

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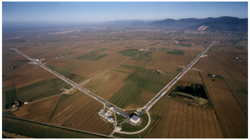

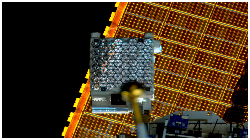

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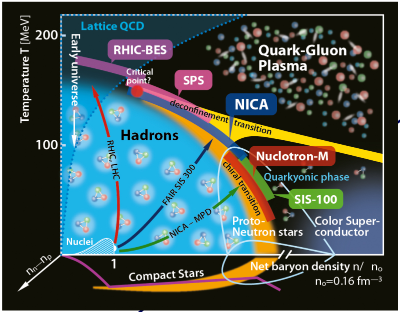

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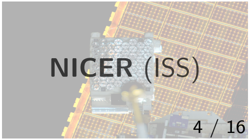
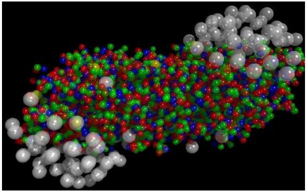
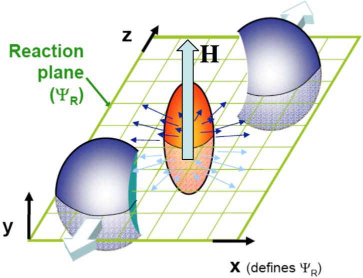


Magnetic fields

- off-central heavy-ion collisions Kharzeev, McLerran, Warringa '07
  impact: chiral magnetic effect, anisotropies, elliptic flow . . .
  Fukushima '12 Kharzeev, Landsteiner, Schmitt, Yee '14





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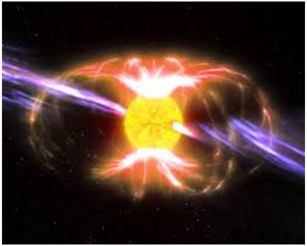

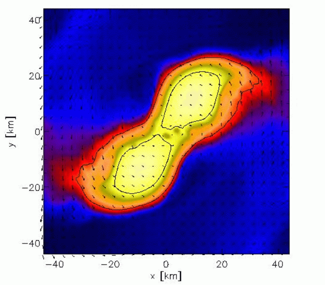

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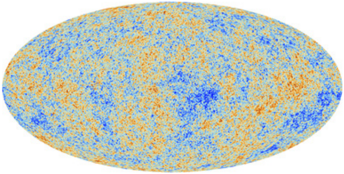

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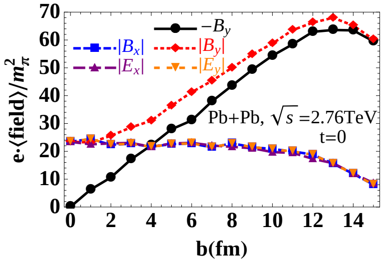
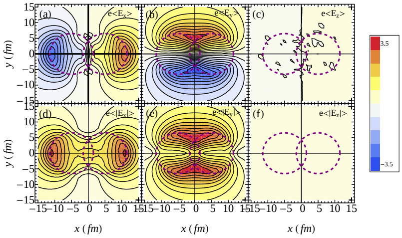
- strength: $B \approx 10^{15}$ T $\approx 10^{20} B_{\rm earth} \approx 5 m_\pi^2$
  $\rightsquigarrow$ competition between strong force and electromagnetism

Electromagnetic fields: heavy ion collisions

- electromagnetic fields in the early stage of heavy-ion collisions reaching m_π^2 and well beyond
  Deng et al. '12





- most probably short-lived fields Huang '15
- impact of electric field enhanced for asymmetric systems (for example Cu+Au at RHIC) Voronyuk et al. '14

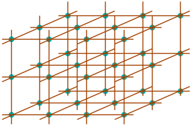
Lattice simulations

Path integral and lattice field theory

- path integral 🔗 Feynman Rev. Mod. Phys. '48

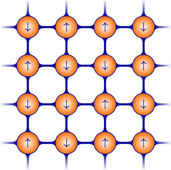
$$\mathcal{Z} = \int \mathcal{D}A_\mu \, \mathcal{D}\bar{\psi} \, \mathcal{D}\psi \; \exp\left(- \int \mathrm{d}^4x \, \mathcal{L}_{\mathrm{QCD}}(x) \right)$$

- discretize spacetime on a lattice with spacing a
  🔗 Wilson PRD '74



- Monte-Carlo algorithms to generate configurations

  like in the 2D Ising model:



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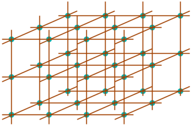
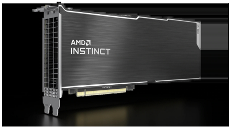


- Monte-Carlo algorithms to generate configurations with $\sim 10^9$ variables $\rightsquigarrow$ high-performance computing
  🔗 nvidia.com 🔗 amd.com



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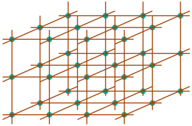

- Monte-Carlo algorithms to generate configurations
- works only if path integral weight is positive
  otherwise: sign (complex action) problem

$T > 0$ ✓
$N > 0$ ×
$B > 0$ ✓
$E > 0$ ×

Phase diagram

The phases of QCD

- most important symmetry: chiral symmetry
- order parameter: quark condensate $\bar{\psi}\psi = \frac{\partial \log \mathcal{Z}}{\partial m}$

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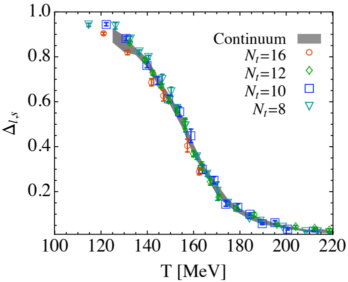

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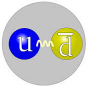

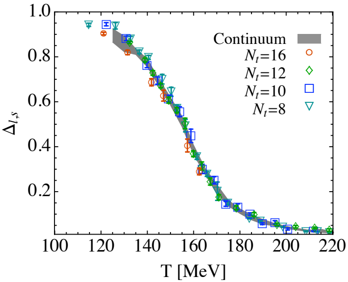

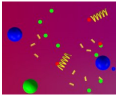

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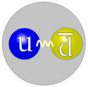

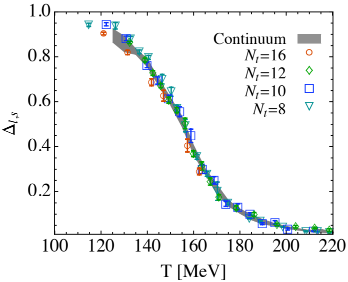

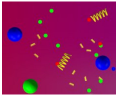

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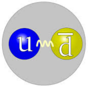

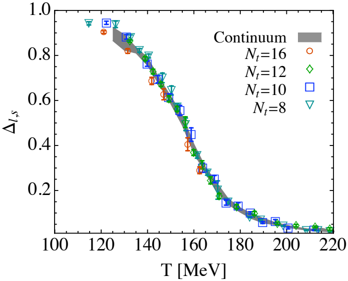

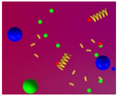

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- $T_c \leftrightarrow$ inflection point Bazavov et al. '18

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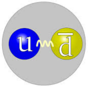

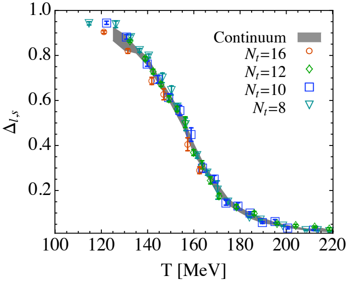

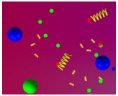

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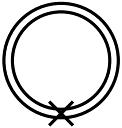
- impact of B on quark condensate?

Magnetic catalysis

- simplest theory: massive charged fermion in a magnetic field
- Schwinger propagator Schwinger '51



- one-loop diagram with scalar insertion gives

$$\bar{\psi}\psi - \bar{\psi}\psi_{B=0} = \frac{m}{4\pi^2}\int_0^\infty \frac{\mathrm{d}s}{s} e^{-m_\pi^2 s}\left[B\coth(Bs) - \frac{1}{s}\right]$$
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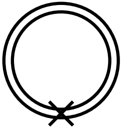

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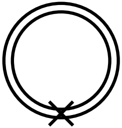

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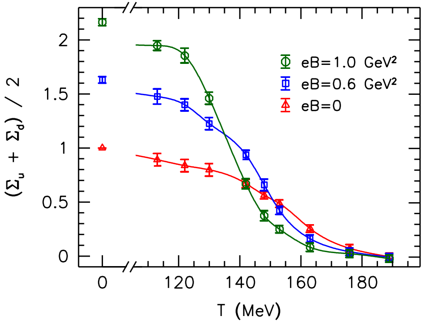
- more on this in the review Shovkovy '12

Inverse catalysis and phase diagram

- physical m_π, staggered quarks, continuum limit
  Bali, Bruckmann, Endrődi, Fodor, Katz et al. '11   '12



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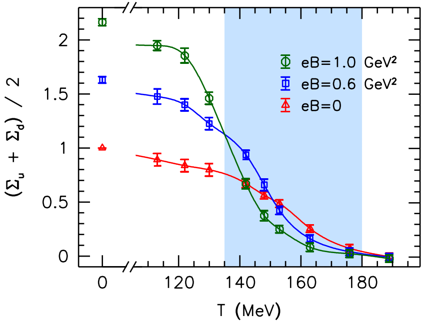

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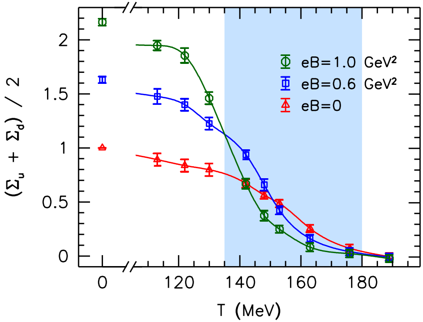

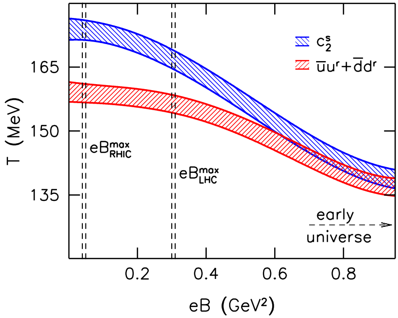

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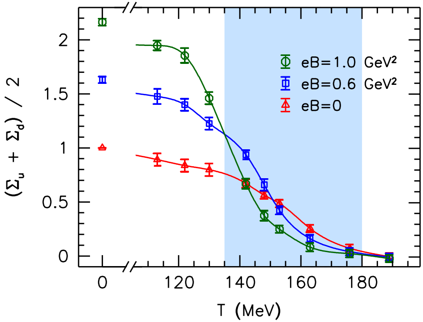

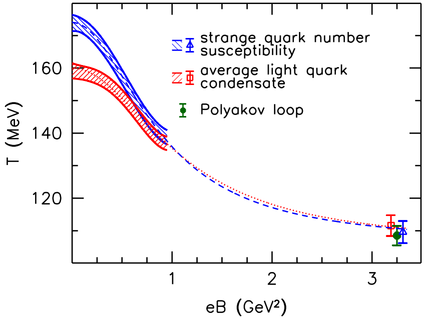

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Phase diagram and critical point

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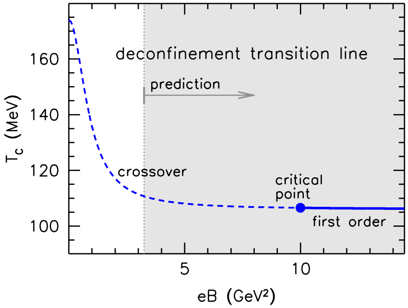
- location of critical point based on extrapolation from $0 < eB \lesssim 3$ GeV^2 $\Rightarrow$ $\boxed{eB_c \approx 10(2)\ \text{GeV}^2}$ Endrődi '15



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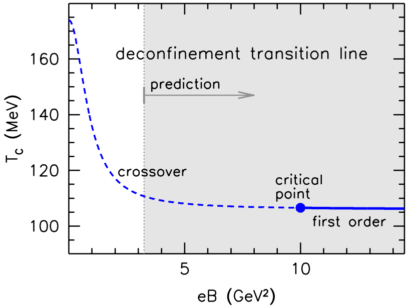
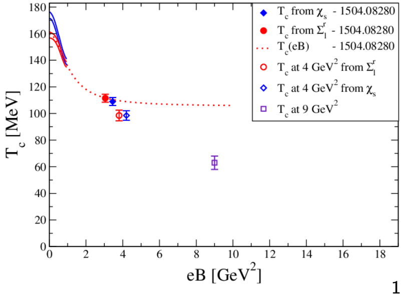
- simulating up to $eB \approx 9$ GeV^2 $\Rightarrow$ $\boxed{4\ \text{GeV}^2 < eB_c < 9\ \text{GeV}^2}$

  D'Elia, Maio, Sanfilippo, Stanzione '21

  Andersen, Naylor, Tranberg '16





Permeability

Susceptibility and permeability

- leading-order dependence of free energy density on B

$$\chi = -\left.\frac{\partial^2 f}{\partial (eB)^2}\right|_{B=0}$$

- permeability Landau-Lifschitz Vol 8. $\mu = (1 - e^2\chi)^{-1}$
- $\mu > 1$ $(\chi > 0)$: paramagnetism
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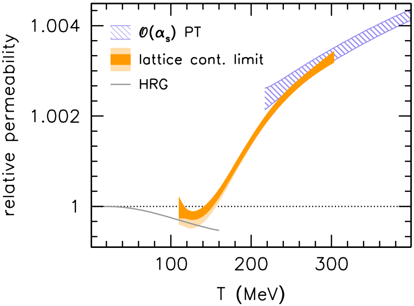

Bali, Endrődi, Piemonte '20

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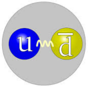

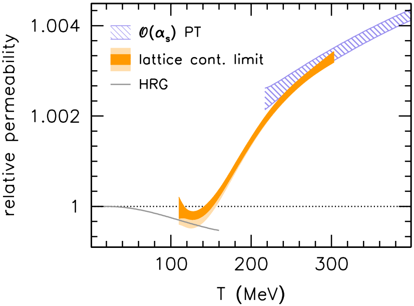

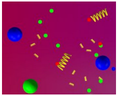

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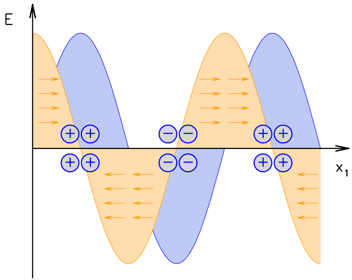
Electric fields

Endrődi, Markó 2208.14306

Electric fields

- static homogeneous electric field E: charges accelerated to ∞
- equilibrium requires infrared regularization
  $\rightsquigarrow$ finite wavelength $1/k_1$



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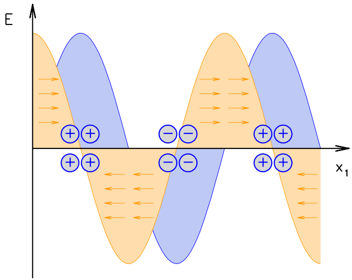

- charge distribution where electric and diffusion forces cancel
- finally take homogeneous limit $k_1 \to 0$
- we only consider thermal effects (no Schwinger pair creation)

Electric susceptibility

- leading impact of E on free energy f (permittivity)
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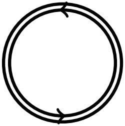
$f(E) =$ 

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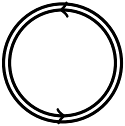
$f(E) =$ 

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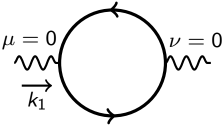

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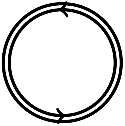
$f(E) =$ 

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$$\frac{1}{\not{p}+m+i\epsilon} + (\not{p}+m)\frac{2\pi i \delta(p^2-m^2)}{e^{|p_0|/T}+1}$$

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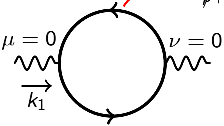
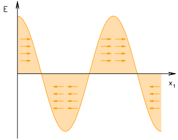


- generalize calculation to $m > 0$ Endrődi, Markó 2208.14306

Equilibrium and mismatch

- evaluated in absence of charge distribution ($\mu = 0$)

$$\xi_{\text{Weldon}}^{\text{non-equi}} = \frac{T^2}{3k_1^2} + \mathcal{O}(k_1^0)$$



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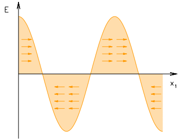
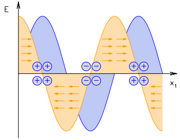


- evaluated "along local equilibria" ($N(x)$ such that $\partial\mu/\partial x = -E$)



$$\xi_{\text{Weldon}}^{\text{equi}} = -\frac{1}{12\pi^2}\left[\log\frac{T^2}{m^2} + 1\right] + \mathcal{O}(m^2/T^2)$$

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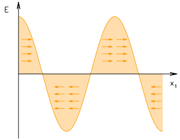

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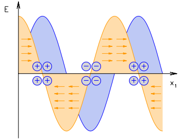

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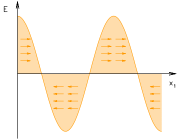

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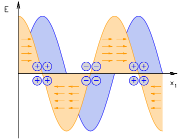

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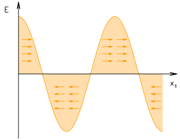

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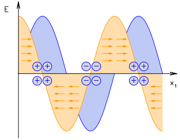

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- no mismatch for magnetic susceptibility (no displaced charges)

Summary

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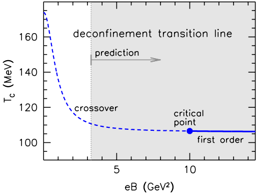
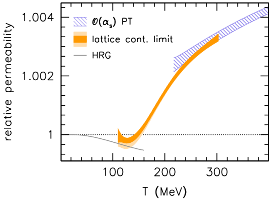
- $T - B$ phase diagram
  and the critical point



- strongly interacting matter
  changes from diamagnet to
  paramagnet as heated up



- background electric fields
  and local charge distributions

  mismatch Schwinger vs. Weldon

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