QCD matter in strong magnetic and electric fields

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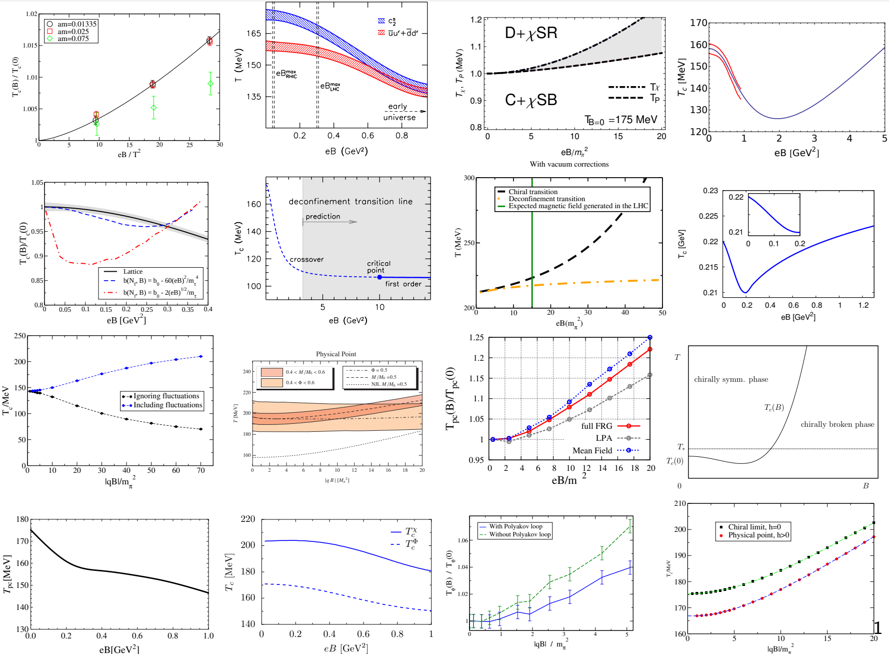
University of Bielefeld

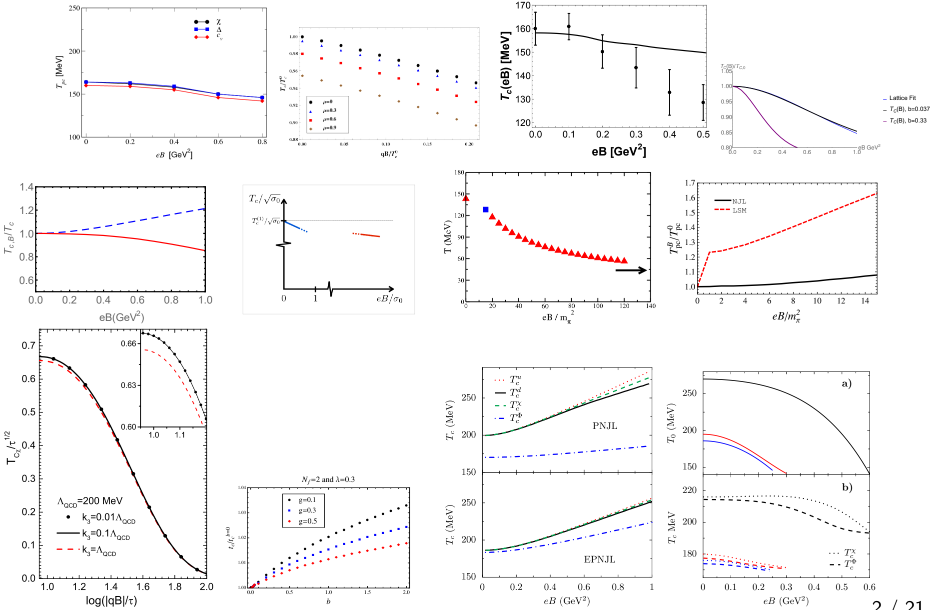
Electromagnetic Effects in Strongly Interacting Matter
@ ICTP SAIFR Sao Paulo
October 28 2022

Preface

Find your plot ICTP Sao Paulo '16



Find your plot, continued ICTP Sao Paulo ’16



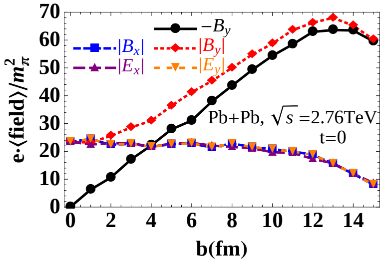
Outline

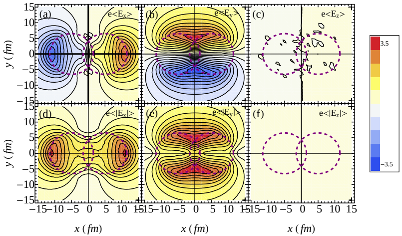
- impact of strong magnetic fields on QCD
  - homogeneous fields: new developments
  - inhomogeneous fields: novel effects 🔍 Dean Valois Fri 09:30
  - anomalous transport: new developments
- impact of strong electric fields on QCD
- summary

Introduction

Electromagnetic fields for QCD

- electromagnetic fields in the early stage of heavy-ion collisions reaching m_π^2 and well beyond 🔗 Deng et al. '12

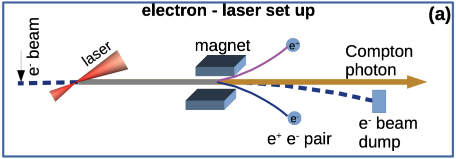


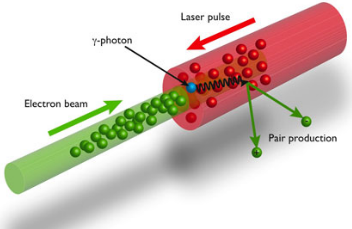


- fields are most probably short-lived 🔗 Huang '15
- fields are highly inhomogeneous
- strong electric components event by event
  ⇝ asymmetric systems as Cu+Au @ RHIC 🔗 Voronyuk et al. '14

Electromagnetic fields for QED

- high-intensity laser experiments
  electromagnetic fields up to m_e^2 and beyond   Fedotov et al. '22
  CoRELS, ELI, SEL, MP3
- electron-laser beam collisions
  E320 experiment at SLAC
  LUXE experiment at DESY   Abramowicz et al. '21



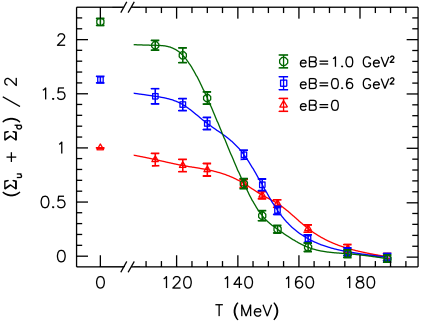


ELI outreach, Mattias Marklund

Phase diagram: new developments

Inverse catalysis and phase diagram

- physical m_π, staggered quarks, continuum limit
  Bali, Bruckmann, Endrődi, Fodor, Katz et al. '11 '12



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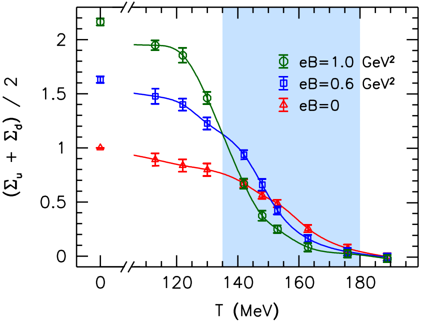

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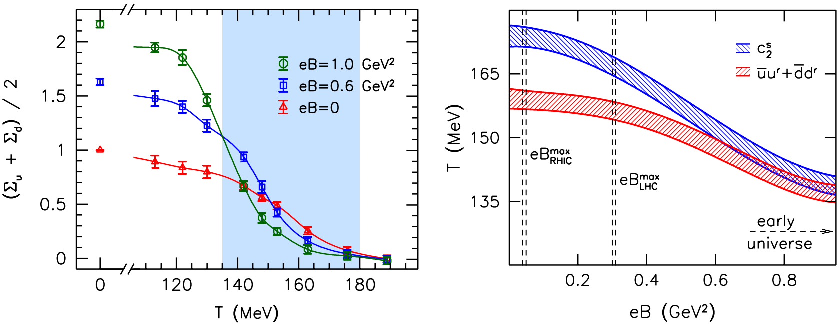

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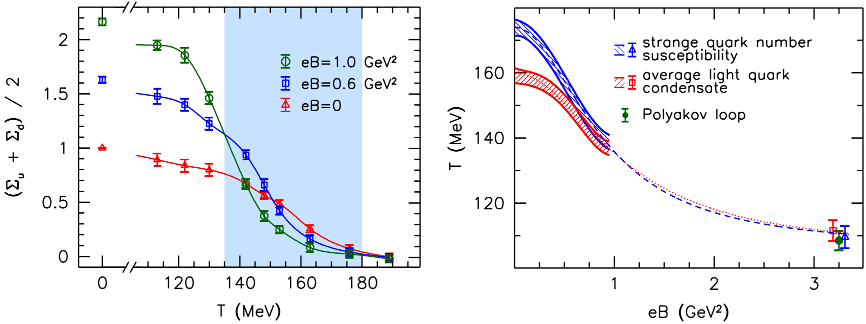

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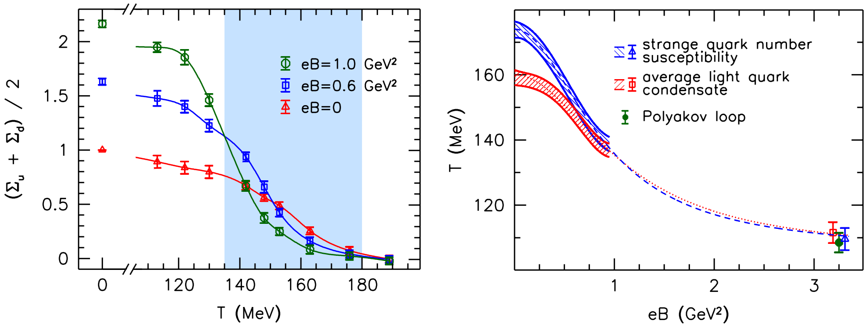

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Phase diagram and critical point

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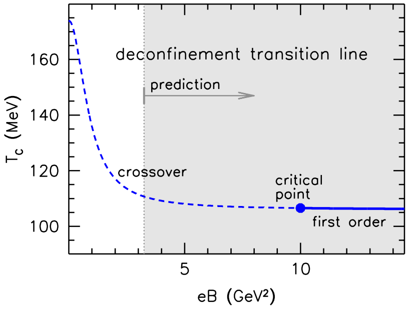

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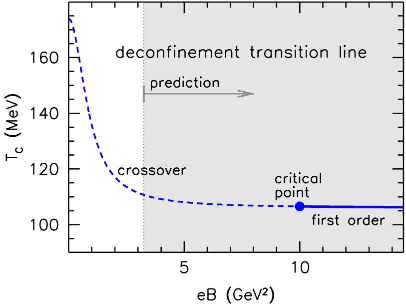

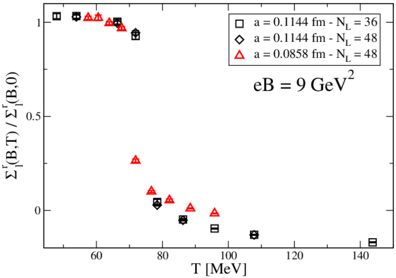

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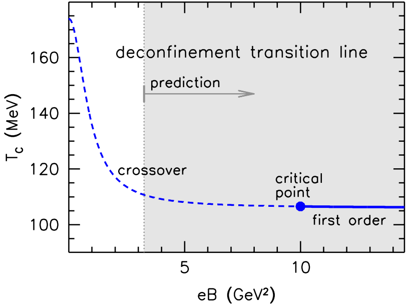
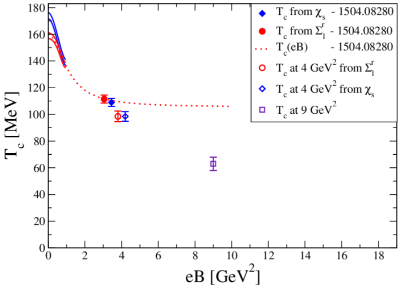


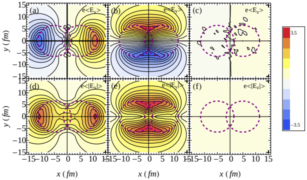


Phase diagram and critical point

- as suspected, QCD has a critical point for large homogeneous B
- first ever lattice evidence for first-order phase transition for QCD at physical masses and physical parameters!
- B_c most probably too large for phenomenological relevance (?)

Beyond constant fields: inhomogeneities

- remember HIC: inhomogeneous magnetic fields



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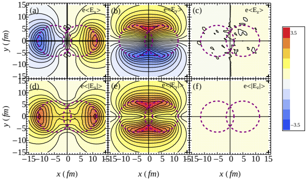

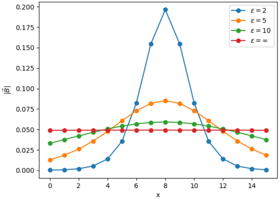

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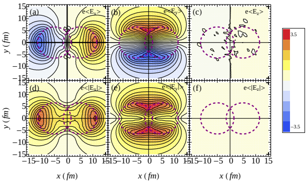

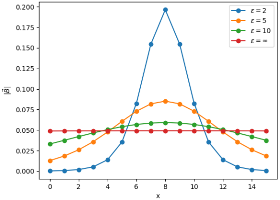

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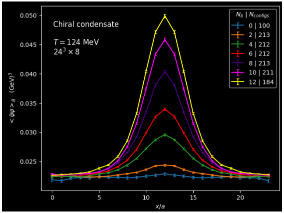
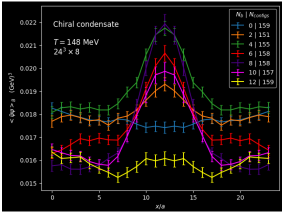
- impact on quark condensate Dean Valois Fri 09:30





Anomalous transport: new developments

Anomalous conductivities PRELIMINARY

- status 2022: still no continuum extrapolated results for C_{CSE} and C_{CME} at physical quark masses
- our approach 🔗 Garnacho, Endrődi et al. in preparation

$$\left.\frac{\partial \langle j_{53} \rangle_B}{\partial \mu}\right|_{\mu=0} = C_{\mathrm{CSE}} \cdot eB \qquad \left.\frac{\partial \langle j_{3} \rangle_B}{\partial \mu_5}\right|_{\mu_5=0} = C_{\mathrm{CME}} \cdot eB$$

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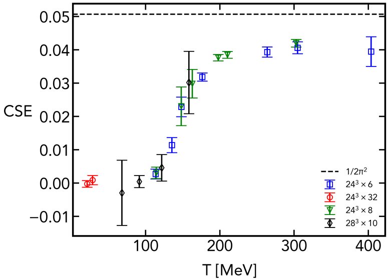

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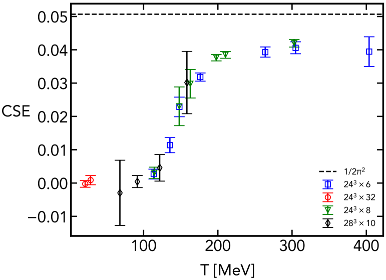

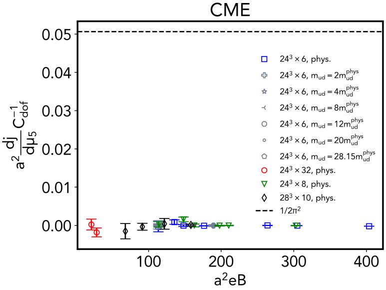

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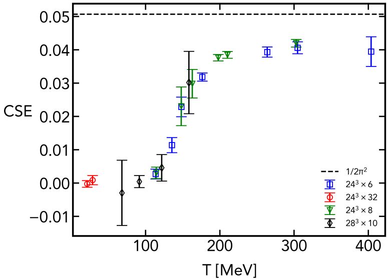
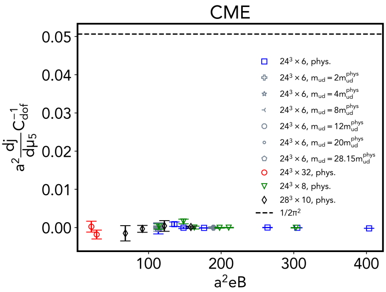




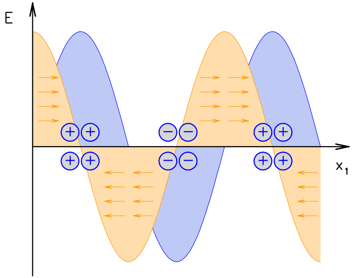
- CSE ✓ CME ?

Electric fields

Endrődi, Markó 2208.14306

Electric fields

- static homogeneous electric field E: charges accelerated to ∞
- equilibrium requires infrared regularization
  $\rightsquigarrow$ finite wavelength $1/k_1$



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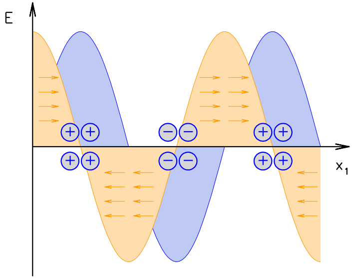

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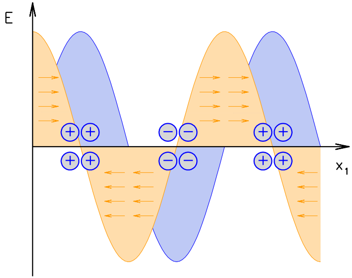

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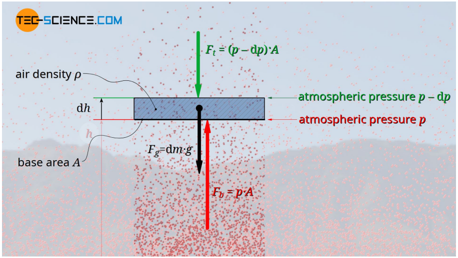
Analogy: barometric distribution

- recall barometric formula above 'flat earth' 🔗 tec-science.com



- inhomogeneous body at homogeneous temp. 🔗 Landau Vol. 5 §25
- gravitational force ↔ electric force
- atmospheric pressure ↔ fermionic degeneracy pressure
  🔗 Bo-Sture Skagerstam, private comm.

Electric susceptibility

- leading impact of E on free energy f (susceptibility)
- here: perturbative QED at nonzero T

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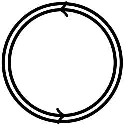
  Loewe, Rojas '92 Elmfors, Skagerstam '95 Gies '98

$f(E) =$ 

$$f = -\xi_{\text{Schwinger}} \cdot \frac{E^2}{2} + \ldots$$

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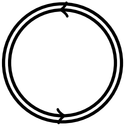
$f(E) =$ 

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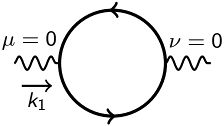

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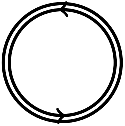

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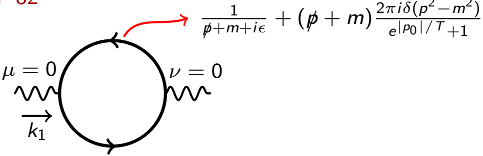

- generalize calculation to $m > 0$ Endrődi, Markó 2208.14306

One object, two approaches

- Schwinger's approach

Euler-Heisenberg Lagrangians, light-by-light scattering,
pair production at $E > 0$, photon splitting at $B > 0$, birefringence
…laser physics 🔗 Gies '00 🔗 Dunne '04

thermodynamics in QED / QCD models mainly at $B > 0$

🔗 Miransky, Shovkovy '15

- Weldon's approach

perturbation theory in hot QCD
hard thermal loops 🔗 Braaten, Pisarski '91 🔗 Blaizot, Iancu '01
QGP transport … 🔗 Arnold, Moore, Yaffe '03

Magnetic sector: equivalence

- magnetic susceptibility in high-T expansion

$$\chi_{\rm Schwinger} = \chi_{\rm Weldon} = \frac{1}{12\pi^2}\left[\log\frac{T^2}{m^2}\right] + \mathcal{O}(m^2/T^2)$$

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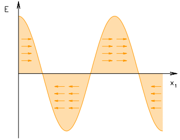
✓

Electric sector: mismatch

- singular if charge distribution is absent ($\mu = 0$)

$$\xi_{\mathrm{Weldon}}^{\mathrm{non-equi}} = \frac{T^2}{3k_1^2} + \mathcal{O}(k_1^0)$$



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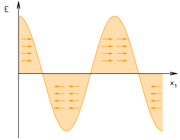
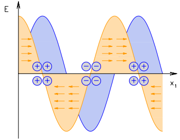


- evaluated "along local equilibria" ($N(x)$ such that $\partial\mu/\partial x = -E$)



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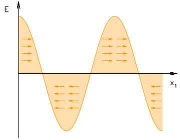

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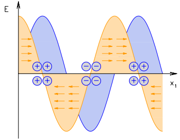

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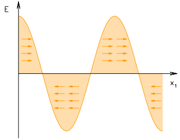
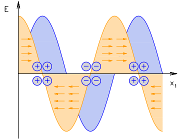


- evaluated "along local equilibria" ($N(x)$ such that $\partial\mu/\partial x = -E$)



$$\xi^{\text{equi}}_{\text{Weldon}} = -\frac{1}{12\pi^2}\left[\log\frac{T^2}{m^2} + 1\right] + \mathcal{O}(m^2/T^2)$$

$$\xi_{\text{Schwinger}} = -\frac{1}{12\pi^2}\left[\log\frac{T^2}{m^2} - 1\right] + \mathcal{O}(m^2/T^2)$$

- note different ordering of limits: $V \to \infty$ vs. $E \to 0$

Electric fields in hot QCD

- Weldon's approach requires infrared regularization (V or k_1) to implement equilibrium construction
- Schwinger's approach fulfills equilibrium construction inherently
- ordering of limits $V \to \infty$ and $E \to 0$ matters
- lattice QCD: Weldon's approach is preferred ($E > 0$ sign problem)

Impact of equilibrium construction for imaginary fields

Imaginary fields

- just like chemical potentials, electric fields are Wick-rotated non-trivially to Euclidean space
- implementing constant magnetic or constant imaginary electric fields is possible, but their 'flux' is quantized

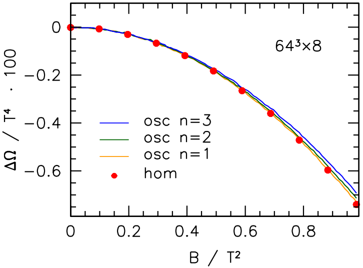
$$B \cdot L^2 = 2\pi N_B, \qquad iE \cdot L/T = 2\pi N_E, \qquad N_B, N_E \in \mathbb{Z}$$

- implementing oscillatory magnetic or oscillatory imaginary electric fields is also possible

$$B \cos\left(\frac{2\pi n x}{L}\right), \qquad iE \cos\left(\frac{2\pi n x}{L}\right), \qquad n \in \mathbb{Z}$$

Free fermions on the lattice

- magnetic fields: homogeneous (discrete B)
  vs. oscillatory (continuous B, discrete n) ✓



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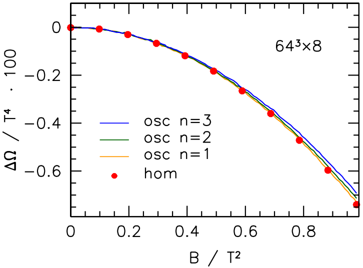
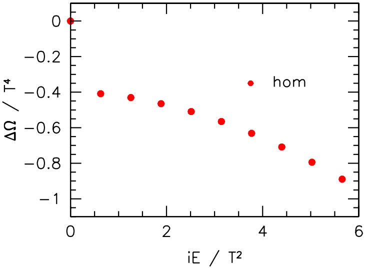
- electric fields: $A_4(x) = iEx + i\mu = iE(x - \bar{x})$
  μ-independent at $N_E > 0 \rightsquigarrow$ jump between $N_E = 0$ and $N_E > 0$





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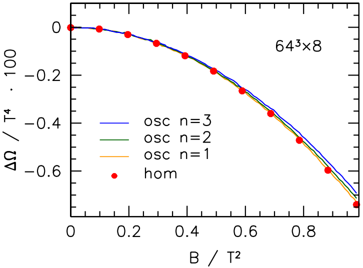

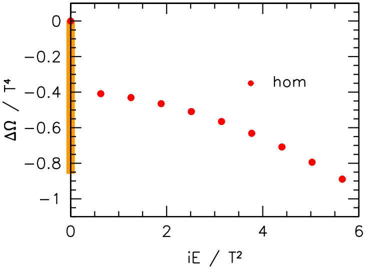

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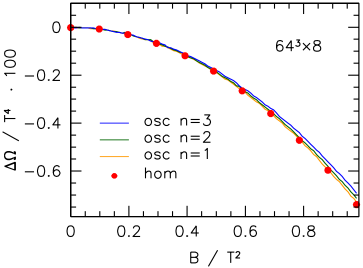

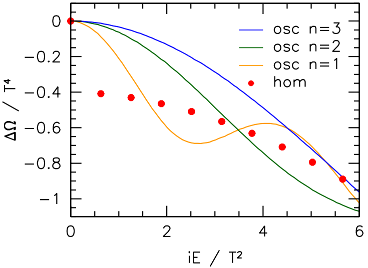

- oscillatory fields “collapse onto” mismatch

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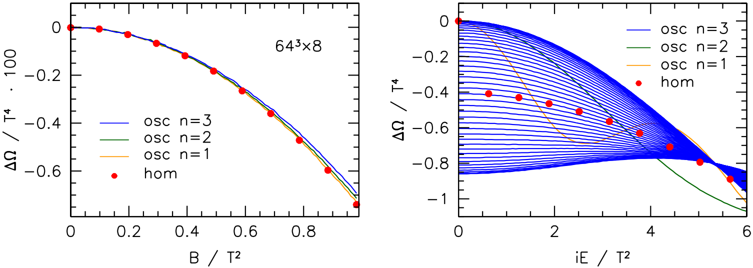

- oscillatory fields “collapse onto” mismatch
- dependence on $i\mu$ pushed towards $iE = 0$ as $n \to 0$

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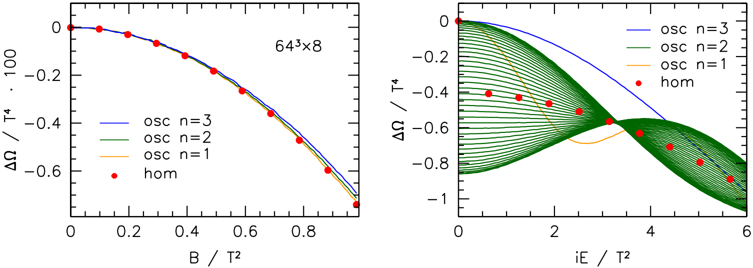

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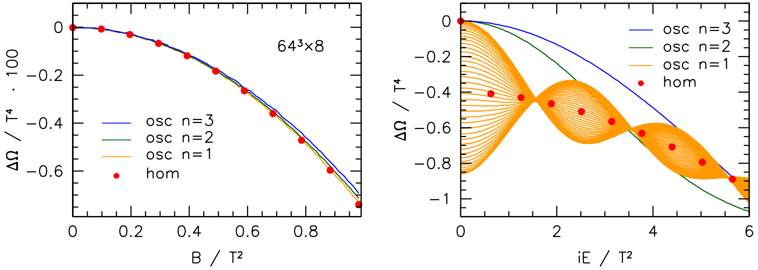

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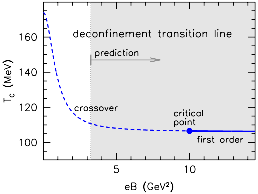
- dependence on $i\mu$ pushed towards $iE = 0$ as $n \to 0$

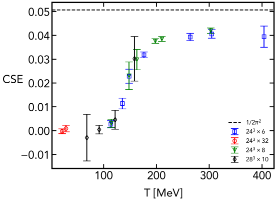
Summary

Summary

- $T - B$ phase diagram
  and the critical point



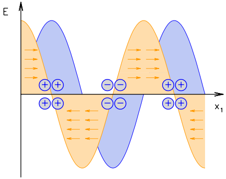
- inhomogeneous magnetic fields,
  anomalous transport



- background electric fields
  and local charge distributions

  mismatch Schwinger vs. Weldon
  already for hot QED

  🔗 Endrődi, Markó 2208.14306



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- workshop

  'Strongly interacting matter in extreme magnetic fields'
- organizers

  V. S. Timóteo, A. Ayala, D. Blaschke, G. Endrődi, R. S. Farias
- date

  25-29 September 2023
- location

  ECT* Trento, Italy