Electromagnetic effects in thermal QCD

Gergely Endrődi

University of Bielefeld

Chirality, Vorticity and Magnetic fields in HIC

UCAS Beijing

July 16, 2023

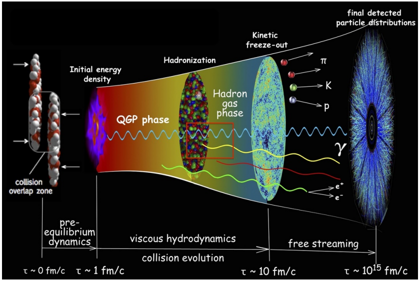
Outline

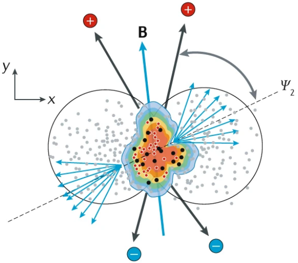
- introduction: lattice QCD thermodynamics
- QCD + $\mathbf{B}$
  phase diagram and linear response
- QCD + $\mathbf{B}(x)$
  impact on thermodynamics
- QCD + $\mathbf{E}$
  equilibrium and linear reponse
- QCD + $\mathbf{E} \cdot \mathbf{B}$
  axion-photon coupling
- summary

Introduction

Quarks and gluons in extreme conditions

- heavy ion collisions $T \lesssim 10^{12}\,^\circ C = 200$ MeV, $n \lesssim 0.12$ fm^{-3}
  $B \lesssim 10^{19}$ G $= 0.3$ GeV$^2/e$

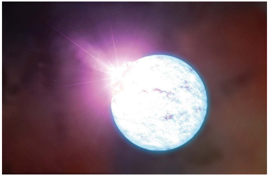


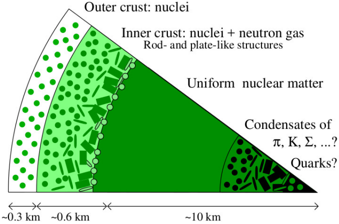


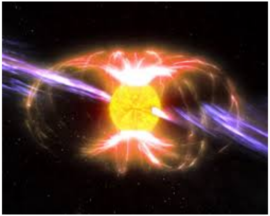
Kharzeev, Liao Nature 2021

Quarks and gluons in extreme conditions

- heavy ion collisions $T \lesssim 10^{12}\,^{\circ}C = 200$ MeV, $n \lesssim 0.12$ fm^{-3}
  $B \lesssim 10^{19}$ G $= 0.3$ GeV$^2/e$
- neutron stars $T \lesssim 1$ keV, $n \lesssim 2$ fm^{-3}
  magnetars $B \lesssim 10^{15}$ G



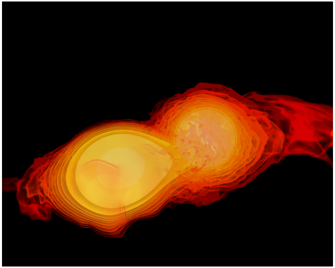


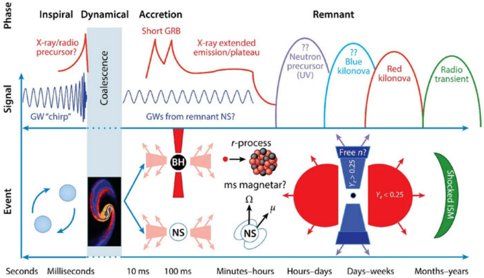


Lattimer, Nature Astronomy 2019

Quarks and gluons in extreme conditions

- heavy ion collisions $T \lesssim 10^{12}\,°C = 200$ MeV, $n \lesssim 0.12$ fm^{-3}
  $B \lesssim 10^{19}$ G $= 0.3$ GeV$^2/e$
- neutron stars $T \lesssim 1$ keV, $n \lesssim 2$ fm^{-3}
  magnetars $B \lesssim 10^{15}$ G
- neutron star mergers $T \lesssim 50$ MeV

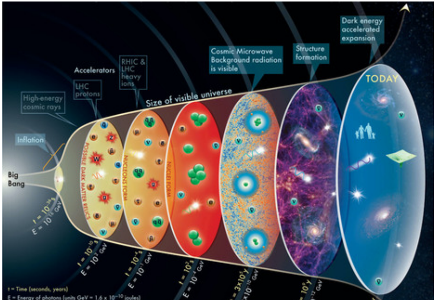




astro.uni-frankfurt.de/rezzolla

Quarks and gluons in extreme conditions

- heavy ion collisions $T \lesssim 10^{12}\,{}^\circ C = 200$ MeV, $n \lesssim 0.12$ fm^{-3}
  $B \lesssim 10^{19}$ G $= 0.3$ GeV$^2/e$
- neutron stars $T \lesssim 1$ keV, $n \lesssim 2$ fm^{-3}
  magnetars $B \lesssim 10^{15}$ G
- neutron star mergers $T \lesssim 50$ MeV
- eary universe, QCD epoch $T \lesssim 200$ MeV
  standard scenario: $n \approx 0$
  B from electroweak epoch Vachaspati '91 Enqvist, Olesen '93

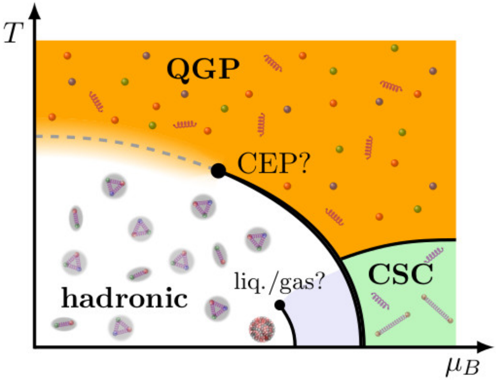


Phase diagram

- control parameters: T, $n \leftrightarrow \mu$, B

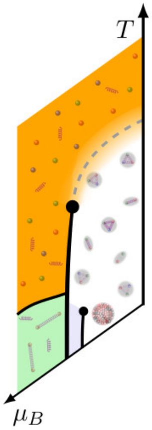
Phase diagram

- control parameters: T, $n \leftrightarrow \mu$, B
- ~~well-known~~ famous phase diagram



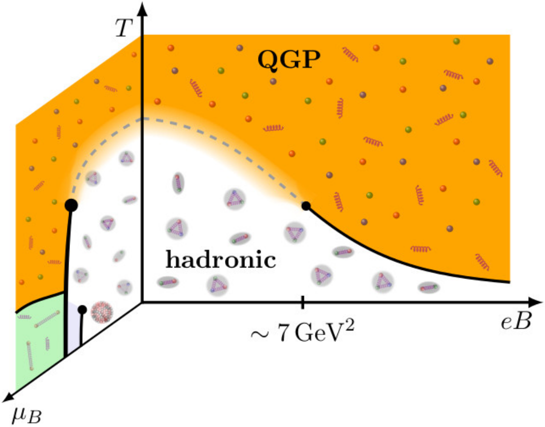
Phase diagram

- control parameters: T, $n \leftrightarrow \mu$, B
- ~~well-known~~ famous phase diagram



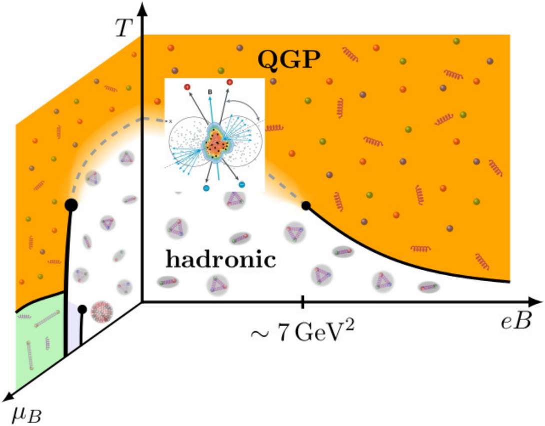
Phase diagram

- control parameters: T, $n \leftrightarrow \mu$, B
- ~~well-known~~ famous phase diagram
- well-known, less famous phase diagram



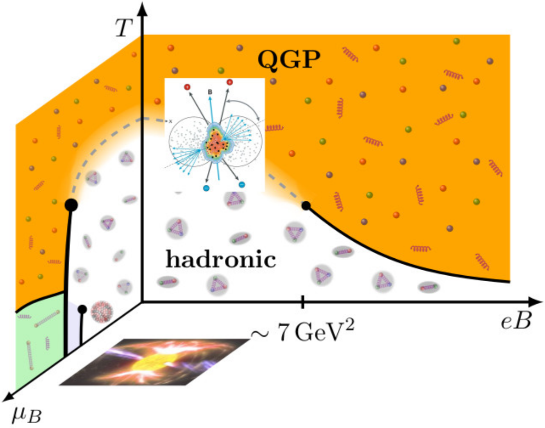
Phase diagram

- control parameters: T, $n \leftrightarrow \mu$, B
- ~~well-known~~ famous phase diagram
- well-known, less famous phase diagram



Phase diagram

- control parameters: T, $n \leftrightarrow \mu$, B
- ~~well-known~~ famous phase diagram
- well-known, less famous phase diagram

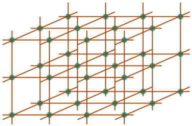


Path integral and lattice field theory

- path integral Feynman '48 for equilibrium QFT

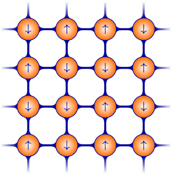
$$\mathcal{Z} = \int \mathcal{D}A_\mu \, \mathcal{D}\bar{\psi} \, \mathcal{D}\psi \, \exp\left(-\int \mathrm{d}^4x \, \mathcal{L}_{\mathrm{QCD}}(x) \right)$$

- discretize spacetime on a lattice with spacing a
  Wilson '74



- Monte-Carlo algorithms to generate configurations

  like in the 2D Ising model:

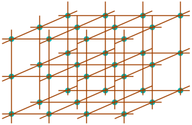


Path integral and lattice field theory

- path integral Feynman '48 for equilibrium QFT

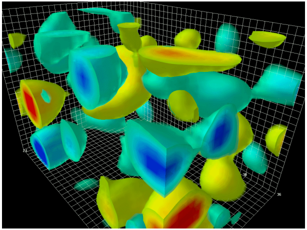
$$\mathcal{Z} = \int \mathcal{D}A_\mu \, \mathcal{D}\bar{\psi} \, \mathcal{D}\psi \; \exp\left(- \int \mathrm{d}^4 x \, \mathcal{L}_{\mathrm{QCD}}(x) \right)$$

- discretize spacetime on a lattice with spacing a
  Wilson '74



- Monte-Carlo algorithms to generate configurations
  animation courtesy D. Leinweber

  with field configurations

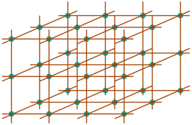


Path integral and lattice field theory

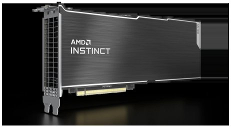
- path integral Feynman '48 for equilibrium QFT

$$\mathcal{Z} = \int \mathcal{D}A_\mu \, \mathcal{D}\bar{\psi} \, \mathcal{D}\psi \, \exp\left(- \int d^4x \, \mathcal{L}_{\mathrm{QCD}}(x) \right)$$

- discretize spacetime on a lattice with spacing a
  Wilson '74



- Monte-Carlo algorithms to generate configurations with $\sim 10^9$ variables $\rightsquigarrow$ high-performance computing
  nvidia.com amd.com

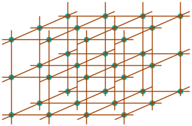


Path integral and lattice field theory

- path integral Feynman '48 for equilibrium QFT

$$\mathcal{Z} = \int \mathcal{D}A_\mu \, \mathcal{D}\bar{\psi} \, \mathcal{D}\psi \, \exp\left(- \int \mathrm{d}^4x \, \mathcal{L}_{\mathrm{QCD}}(x) \right)$$

- discretize spacetime on a lattice with spacing a
  Wilson '74



- Monte-Carlo algorithms to generate configurations
- works only if path integral weight is positive
  otherwise: sign (complex action) problem

$T > 0$ ✓
$N > 0$ ×
$B > 0$ ✓
$E > 0$ ×

Phase diagram

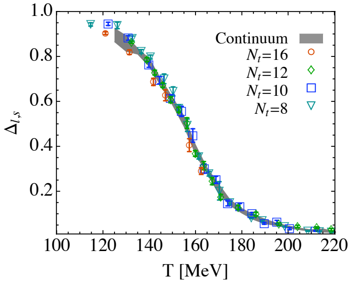
The phases of QCD

- most important symmetry: chiral symmetry
- order parameter: quark condensate $\bar{\psi}\psi = \frac{\partial \log \mathcal{Z}}{\partial m}$

The phases of QCD

- most important symmetry: chiral symmetry
- order parameter: quark condensate $\bar{\psi}\psi = \frac{\partial \log \mathcal{Z}}{\partial m}$

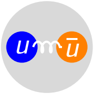
Borsányi et al. '10

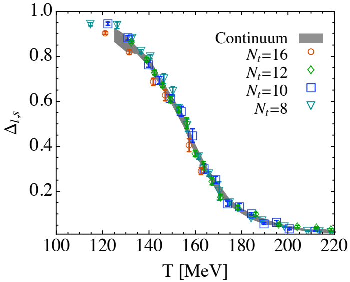


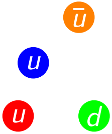
The phases of QCD

- most important symmetry: chiral symmetry
- order parameter: quark condensate $\bar{\psi}\psi = \frac{\partial \log \mathcal{Z}}{\partial m}$

Borsányi et al. '10



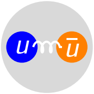


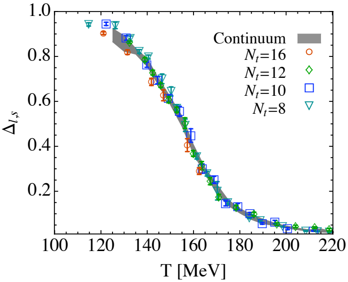


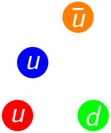
The phases of QCD

- most important symmetry: chiral symmetry
- order parameter: quark condensate $\bar{\psi}\psi = \frac{\partial \log \mathcal{Z}}{\partial m}$

Borsányi et al. '10





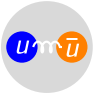


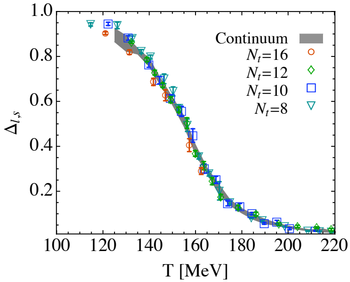
- crossover Aoki et al. '06 Bhattacharya et al. '14

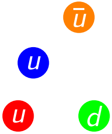
The phases of QCD

- most important symmetry: chiral symmetry
- order parameter: quark condensate $\bar{\psi}\psi = \frac{\partial \log \mathcal{Z}}{\partial m}$

Borsányi et al. '10





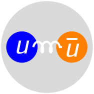


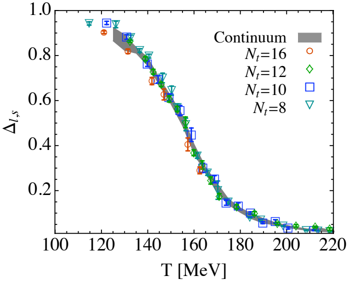
- crossover Aoki et al. '06 Bhattacharya et al. '14
- $T_c \leftrightarrow$ inflection point Borsányi et al. '10 Bazavov et al. '18

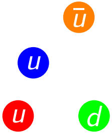
The phases of QCD

- most important symmetry: chiral symmetry
- order parameter: quark condensate $\bar{\psi}\psi = \frac{\partial \log \mathcal{Z}}{\partial m}$

Borsányi et al. '10







- crossover Aoki et al. '06 Bhattacharya et al. '14
- $T_c \leftrightarrow$ inflection point Borsányi et al. '10 Bazavov et al. '18
- impact of B on quark condensate?

Magnetic catalysis

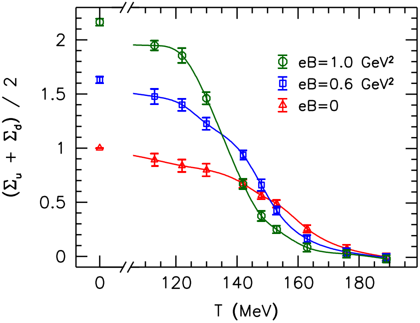
- magnetic field treated as classical background field (no back-reaction)
- magnetic field aligns quark magnetic moments $q\mathbf{S} \parallel \mathbf{B}$
- spin-zero composite states preferred

"magnetic catalysis" 🔗 Gusynin, Miransky, Shovkovy '95

- can be related to positivity of QED β-function 🔗 Endrődi '13
- magnetic catalysis in various settings 🔗 Shovkovy '12
- effective theories, QCD models predicted magnetic catalysis for all temperatures 🔗 Andersen, Naylor, Tranberg '16

Inverse catalysis and phase diagram

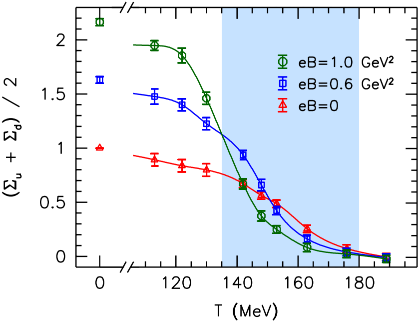
- physical m_π, staggered quarks, continuum limit
  Bali, Bruckmann, Endrődi, Fodor, Katz et al. '11 '12



- magnetic catalysis at low T

Inverse catalysis and phase diagram

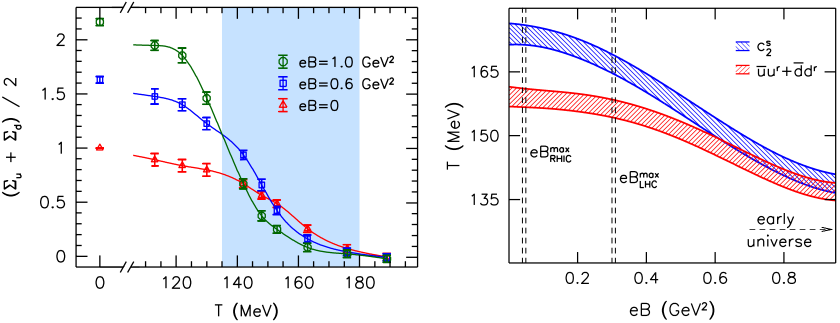
- physical m_π, staggered quarks, continuum limit
  Bali, Bruckmann, Endrődi, Fodor, Katz et al. '11 '12



- magnetic catalysis at low T
- inverse magnetic catalysis (IMC) in transition region

Inverse catalysis and phase diagram

- physical m_π, staggered quarks, continuum limit
  Bali, Bruckmann, Endrődi, Fodor, Katz et al. '11 '12



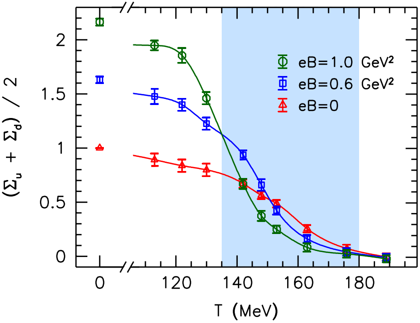
- magnetic catalysis at low T
- inverse magnetic catalysis (IMC) in transition region
- T_c is reduced by B

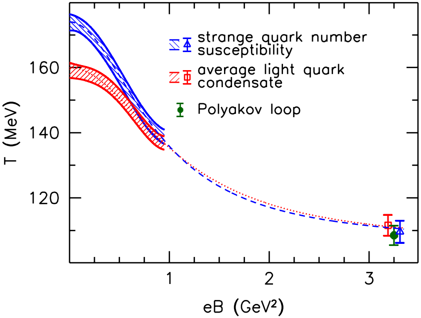
Inverse catalysis and phase diagram

- physical m_π, staggered quarks, continuum limit

  Bali, Bruckmann, Endrődi, Fodor, Katz et al. '11 '12

  Endrődi '15

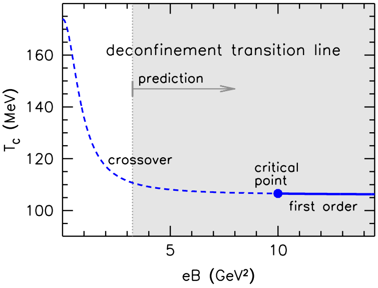




- magnetic catalysis at low T
- inverse magnetic catalysis (IMC) in transition region
- T_c is reduced by B

Phase diagram and critical point

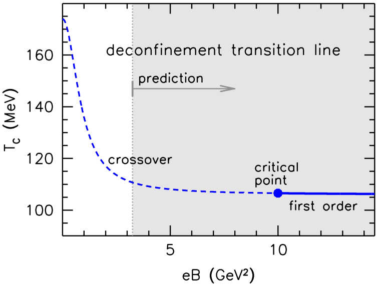
- effective theory of QCD at $B \to \infty$: first-order deconfinement transition $\Rightarrow$ critical point! Miransky, Shovkovy '02
- location of critical point estimated from numerical simulations $eB_c \approx 10(2)\ \text{GeV}^2$ Endrődi '15

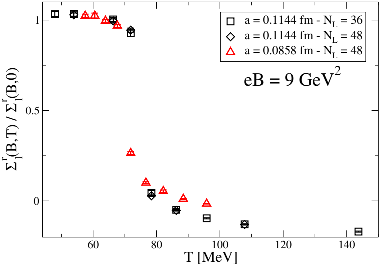


Phase diagram and critical point

- effective theory of QCD at $B \to \infty$: first-order deconfinement transition $\Rightarrow$ critical point! Miransky, Shovkovy '02
- location of critical point estimated from numerical simulations $eB_c \approx 10(2)$ GeV^2 Endrődi '15
- recent update 4 GeV^2 $< eB_c <$ 9 GeV^2

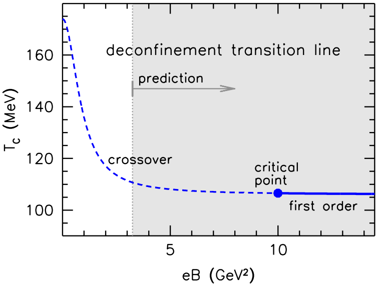
  D'Elia, Maio, Sanfilippo, Stanzione '21

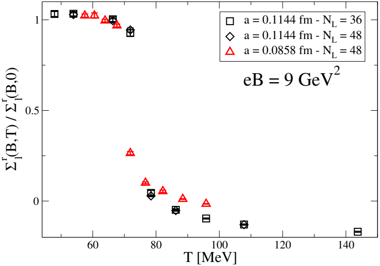




Phase diagram and critical point

- effective theory of QCD at $B \to \infty$: first-order deconfinement transition $\Rightarrow$ critical point! Miransky, Shovkovy '02
- location of critical point estimated from numerical simulations $eB_c \approx 10(2)\ \mathrm{GeV}^2$ Endrődi '15
- recent update $4\ \mathrm{GeV}^2 < eB_c < 9\ \mathrm{GeV}^2$
  D'Elia, Maio, Sanfilippo, Stanzione '21





- first ever lattice evidence for first-order phase transition for QCD at physical masses and physical parameters!

Permeability

Susceptibility and permeability

- leading-order dependence of free energy density on B

$$\chi = -\left.\frac{\partial^2 f}{\partial (eB)^2}\right|_{B=0}$$

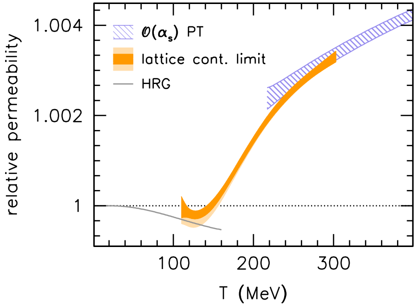
- permeability Landau-Lifschitz Vol 8. $\mu = (1 - e^2\chi)^{-1}$
- $\mu > 1$ $(\chi > 0)$: paramagnetism
  $\mu < 1$ $(\chi < 0)$: diamagnetism

Susceptibility and permeability

- leading-order dependence of free energy density on B

$$\chi = - \left. \frac{\partial^2 f}{\partial (eB)^2} \right|_{B=0}$$

- permeability Landau-Lifschitz Vol 8. $\mu = (1 - e^2\chi)^{-1}$
- $\mu > 1$ $(\chi > 0)$: paramagnetism
  $\mu < 1$ $(\chi < 0)$: diamagnetism



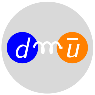
Bali, Endrődi, Piemonte '20 Bonati et al. '14

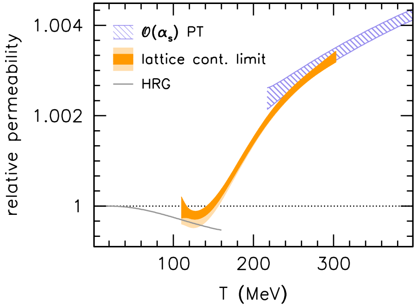
Susceptibility and permeability

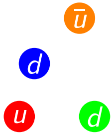
- leading-order dependence of free energy density on B

$$\chi = -\left.\frac{\partial^2 f}{\partial (eB)^2}\right|_{B=0}$$

- permeability Landau-Lifschitz Vol 8. $\mu = (1 - e^2\chi)^{-1}$
- $\mu > 1$ $(\chi > 0)$: paramagnetism
  $\mu < 1$ $(\chi < 0)$: diamagnetism





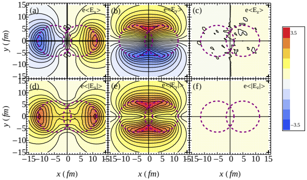


Bali, Endrődi, Piemonte '20 Bonati et al. '14

Beyond constant magnetic fields: inhomogeneities

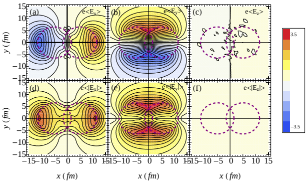
Inhomogeneous magnetic fields

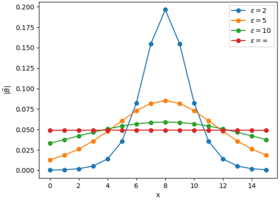
- HIC: inhomogeneous magnetic fields Deng et al. '12



Inhomogeneous magnetic fields

- HIC: inhomogeneous magnetic fields Deng et al. '12

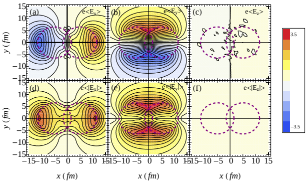


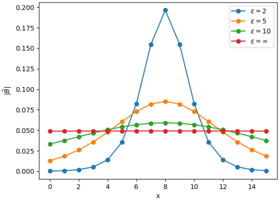


- consider profile $B(x) = B\cosh^{-2}(x/\epsilon)$ Dunne '04

Inhomogeneous magnetic fields

- HIC: inhomogeneous magnetic fields Deng et al. '12

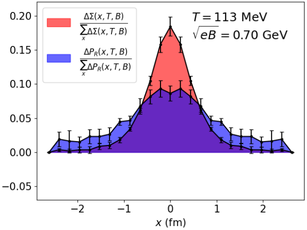


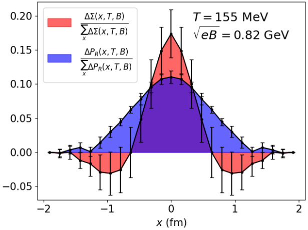


- consider profile $B(x) = B\cosh^{-2}(x/\epsilon)$ Dunne '04

- impact: condensate and Polyakov loop

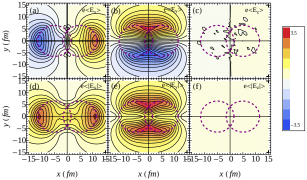
  Brandt, Cuteri, Endrődi, Markó, Sandbote, Valois '23

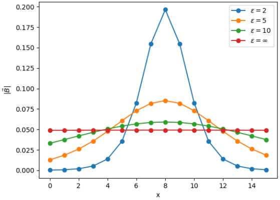




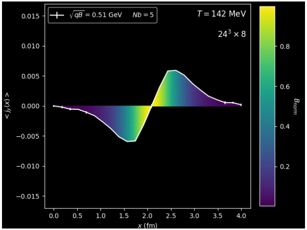
Inhomogeneous magnetic fields

- HIC: inhomogeneous magnetic fields Deng et al. '12





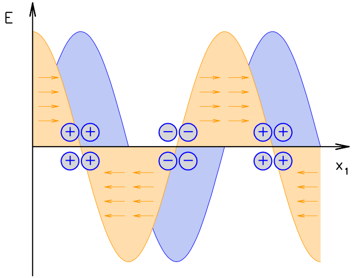
- consider profile $B(x) = B\cosh^{-2}(x/\epsilon)$ Dunne '04
- impact: electric current Valois et al. in preparation



From magnetic fields to electric fields with a detour to perturbative QED

Electric fields

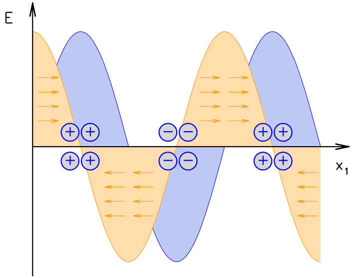
- static homogeneous electric field E: charges accelerated to ∞
- equilibrium requires infrared regularization
  $\rightsquigarrow$ finite wavelength $1/k_1$



- charge distribution where electric and diffusion forces cancel
- finally take homogeneous limit $k_1 \to 0$

Electric fields

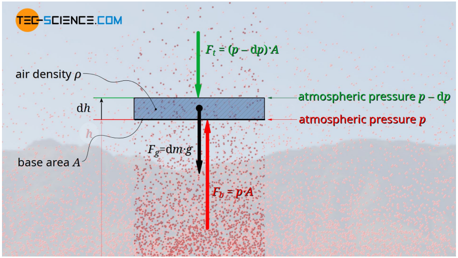
- static homogeneous electric field E: charges accelerated to ∞
- equilibrium requires infrared regularization
  $\rightsquigarrow$ finite wavelength $1/k_1$



- charge distribution where electric and diffusion forces cancel
- finally take homogeneous limit $k_1 \to 0$
- this is a thermal effect (no Schwinger pair creation)

Analogy: barometric distribution

- recall barometric formula above 'flat earth' 🔗 tec-science.com



- gravitational force $\leftrightarrow$ electric force
- atmospheric pressure $\leftrightarrow$ fermionic degeneracy pressure

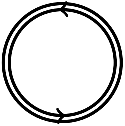
Electric susceptibility

- leading impact of E on free energy f (permittivity)
- here: perturbative QED at nonzero T

Electric susceptibility

- leading impact of E on free energy f (permittivity)
- here: perturbative QED at nonzero T
- Schwinger's approach Schwinger '51

Loewe, Rojas '92 Elmfors, Skagerstam '95 Gies '98

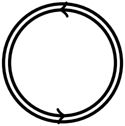
$f(E) =$ 

$$f = -\xi_{\text{Schwinger}} \cdot \frac{E^2}{2} + \ldots$$

Electric susceptibility

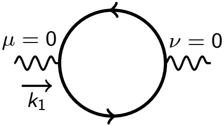
- leading impact of E on free energy f (permittivity)
- here: perturbative QED at nonzero T
- Schwinger's approach Schwinger '51

  Loewe, Rojas '92 Elmfors, Skagerstam '95 Gies '98

$f(E) =$ 

$$f = -\xi_{\text{Schwinger}} \cdot \frac{E^2}{2} + \ldots$$

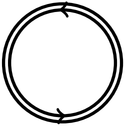
- Weldon's approach Weldon '82

$\xi_{\text{Weldon}} = \lim_{k_1 \to 0} \frac{-1}{k_1^2}$ 

Electric susceptibility

- leading impact of E on free energy f (permittivity)
- here: perturbative QED at nonzero T
- Schwinger's approach Schwinger '51

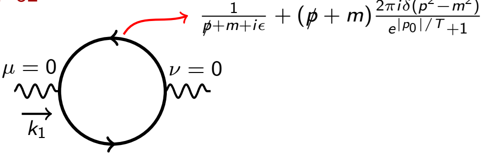
  Loewe, Rojas '92 Elmfors, Skagerstam '95 Gies '98

$$f(E) =$$ 

$$f = -\xi_{\rm Schwinger} \cdot \frac{E^2}{2} + \dots$$

- Weldon's approach Weldon '82

$$\xi_{\rm Weldon} = \lim_{k_1 \to 0} \frac{-1}{k_1^2}$$

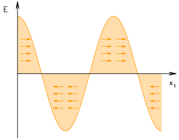


- generalize calculation to $m > 0$ Endrődi, Markó '22

Equilibrium and mismatch

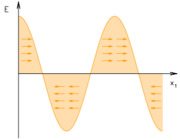
- evaluated in absence of charge distribution ($\mu = 0$)

$$\xi_{\text{Weldon}}^{\text{non-equi}} = \frac{T^2}{3k_1^2} + \mathcal{O}(k_1^0)$$



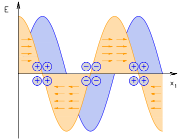
Equilibrium and mismatch

- evaluated in absence of charge distribution ($\mu = 0$)



$$\xi_{\text{Weldon}}^{\text{non-equi}} = \frac{T^2}{3k_1^2} + \mathcal{O}(k_1^0)$$

- evaluated "along local equilibria" ($N(x)$ such that $\partial\mu/\partial x = -E$)

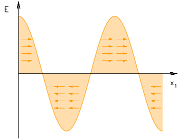


$$\xi_{\text{Weldon}}^{\text{equi}} = -\frac{1}{12\pi^2}\left[\log\frac{T^2}{m^2} + 1\right] + \mathcal{O}(m^2/T^2)$$

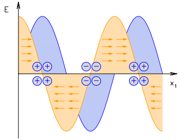
Equilibrium and mismatch

- evaluated in absence of charge distribution ($\mu = 0$)

$$\xi^{\mathrm{non-equi}}_{\mathrm{Weldon}} = \frac{T^2}{3k_1^2} + \mathcal{O}(k_1^0)$$



- evaluated "along local equilibria" ($N(x)$ such that $\partial\mu/\partial x = -E$)



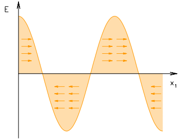
$$\xi^{\mathrm{equi}}_{\mathrm{Weldon}} = -\frac{1}{12\pi^2}\left[\log\frac{T^2}{m^2} + 1\right] + \mathcal{O}(m^2/T^2)$$

$$\xi_{\mathrm{Schwinger}} = -\frac{1}{12\pi^2}\left[\log\frac{T^2}{m^2} - 1\right] + \mathcal{O}(m^2/T^2)$$

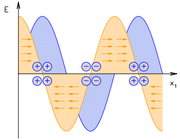
Equilibrium and mismatch

- evaluated in absence of charge distribution ($\mu = 0$)

$$\xi_{\text{Weldon}}^{\text{non-equi}} = \frac{T^2}{3k_1^2} + \mathcal{O}(k_1^0)$$



- evaluated "along local equilibria" ($N(x)$ such that $\partial\mu/\partial x = -E$)



$$\xi_{\text{Weldon}}^{\text{equi}} = -\frac{1}{12\pi^2}\left[\log\frac{T^2}{m^2} + 1\right] + \mathcal{O}(m^2/T^2)$$

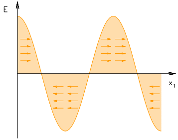
$$\xi_{\text{Schwinger}} = -\frac{1}{12\pi^2}\left[\log\frac{T^2}{m^2} - 1\right] + \mathcal{O}(m^2/T^2)$$

- note different ordering of limits: $V \to \infty$ vs. $E \to 0$

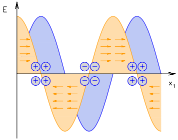
Equilibrium and mismatch

- evaluated in absence of charge distribution ($\mu = 0$)

$$\xi_{\text{Weldon}}^{\text{non-equi}} = \frac{T^2}{3k_1^2} + \mathcal{O}(k_1^0)$$



- evaluated "along local equilibria" ($N(x)$ such that $\partial\mu/\partial x = -E$)



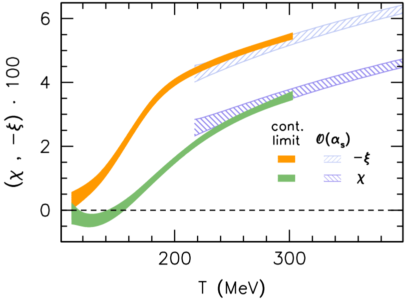
$$\xi_{\text{Weldon}}^{\text{equi}} = -\frac{1}{12\pi^2}\left[\log\frac{T^2}{m^2} + 1\right] + \mathcal{O}(m^2/T^2)$$

$$\xi_{\text{Schwinger}} = -\frac{1}{12\pi^2}\left[\log\frac{T^2}{m^2} - 1\right] + \mathcal{O}(m^2/T^2)$$

- note different ordering of limits: $V \to \infty$ vs. $E \to 0$
- no mismatch for magnetic susceptibility (no displaced charges)

Full QCD

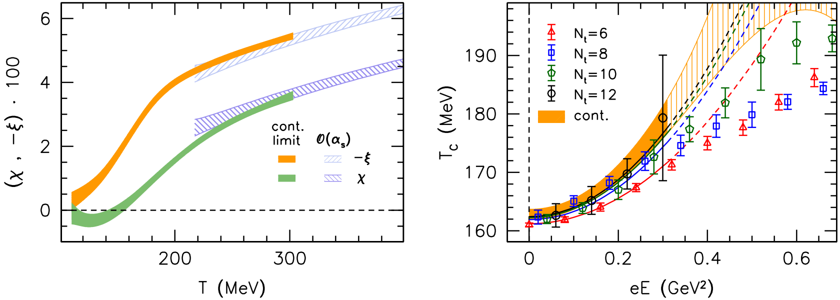
- full lattice QCD simulations for $\xi^{\mathrm{equi}}_{\mathrm{Weldon}}$ are possible
- electric and magnetic susceptibilities
  (comparison to perturbation theory at high T)



Endrődi, Markó '21; in preparation

Full QCD

- full lattice QCD simulations for $\xi^{\rm equi}_{\rm Weldon}$ are possible
- electric and magnetic susceptibilities
  (comparison to perturbation theory at high T)
- phase diagram to $\mathcal{O}(E^2)$ via expansion of Polyakov loop



Endrődi, Markó '21; in preparation

Combining magnetic and electric fields: axion-photon coupling

Magnetic and electric fields

- so far: impact of magnetic fields $\mathcal{O}(B^2)$ (and higher orders) and impact of electric fields $\mathcal{O}(E^2)$
- another quadratic combination: $\mathbf{E} \cdot \mathbf{B} \propto F_{\mu\nu}\widetilde{F}_{\mu\nu}$

Magnetic and electric fields

- so far: impact of magnetic fields $\mathcal{O}(B^2)$ (and higher orders) and impact of electric fields $\mathcal{O}(E^2)$
- another quadratic combination: $\mathbf{E} \cdot \mathbf{B} \propto F_{\mu\nu} \widetilde{F}_{\mu\nu}$
- affects CP-odd observables, in particular

$$Q_{\rm top} = \frac{1}{32\pi^2} \int \mathrm{d}^4 x \, \mathrm{Tr} \, G_{\mu\nu}(x) \widetilde{G}_{\mu\nu}(x)$$

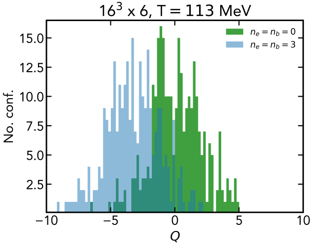
Magnetic and electric fields

- so far: impact of magnetic fields $\mathcal{O}(B^2)$ (and higher orders) and impact of electric fields $\mathcal{O}(E^2)$
- another quadratic combination: $\mathbf{E}\cdot\mathbf{B} \propto F_{\mu\nu}\widetilde{F}_{\mu\nu}$
- affects CP-odd observables, in particular

$$Q_{\rm top} = \frac{1}{32\pi^2}\int \mathrm{d}^4x\,\mathrm{Tr}\,G_{\mu\nu}(x)\widetilde{G}_{\mu\nu}(x)$$

- lattice simulations at $B>0$ and $iE>0$

D'Elia, Mariti, Negro '12   Brandt, Cuteri, Endrődi, Hernández, Markó '22



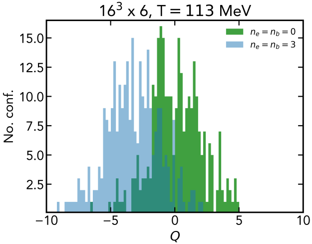
Magnetic and electric fields

- so far: impact of magnetic fields $\mathcal{O}(B^2)$ (and higher orders) and impact of electric fields $\mathcal{O}(E^2)$
- another quadratic combination: $\mathbf{E}\cdot\mathbf{B} \propto F_{\mu\nu}\widetilde{F}_{\mu\nu}$
- affects CP-odd observables, in particular

$$Q_{\rm top} = \frac{1}{32\pi^2}\int \mathrm{d}^4x\, \mathrm{Tr}\, G_{\mu\nu}(x)\widetilde{G}_{\mu\nu}(x)$$

- lattice simulations at $B > 0$ and $iE > 0$

D'Elia, Mariti, Negro '12 Brandt, Cuteri, Endrődi, Hernández, Markó '22



- what does this linear response coefficient describe?

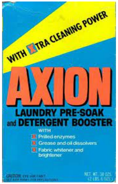
Axions as dark matter

- extend Standard Model with new field: axion a

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}\partial_\mu a\,\partial_\mu a + a\,\underbrace{\mathrm{Tr}\,G_{\mu\nu}\widetilde{G}_{\mu\nu}}_{Q_{\rm top}} + a\,g_{a\gamma\gamma}\underbrace{F_{\mu\nu}\widetilde{F}_{\mu\nu}}_{\mathbf{E}\cdot\mathbf{B}}$$

- provides solution to 'strong CP problem'

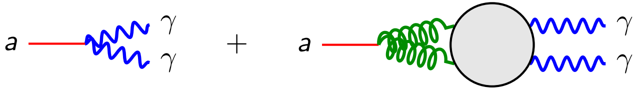
  Peccei, Quinn '77 Weinberg '78 Wilczek '78



- is a possible dark matter candidate
- extensive experimental campaign:
  haloscopes and helioscopes CAST ADMX XENON1T

Axion-photon coupling

- most relevant parameter for experimental detection
- direct coupling (model-dependent)
  plus
  indirect coupling through quark/gluon loops



- chiral perturbation theory predicts two terms of similar magnitude and opposite sign di Cortona et al. '16
- QCD contribution, for slowly varying a fields

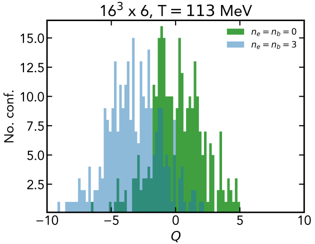
$$g_{a\gamma\gamma}^{\mathrm{QCD}} = \frac{\partial^2 \log \mathcal{Z}}{\partial a \, \partial(\mathbf{E}\cdot\mathbf{B})} = \frac{\partial \langle Q_{\mathrm{top}} \rangle}{\partial(\mathbf{E}\cdot\mathbf{B})}$$

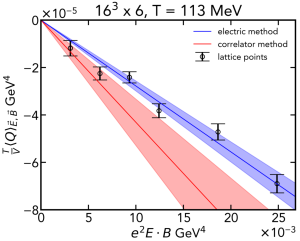
Axion-photon coupling on the lattice

- QCD contribution

$$g_{a\gamma\gamma}^{\mathrm{QCD}} = \frac{\partial \langle Q_{\mathrm{top}} \rangle}{\partial(\mathbf{E} \cdot \mathbf{B})}$$

- shift in mean topology by parallel magnetic *and* imaginary electric fields





- first results for $g_{a\gamma\gamma}^{\mathrm{QCD}}$ Brandt, Cuteri, Endrődi, Hernández, Markó '22
- to be extrapolated to the continuum limit

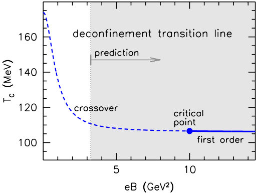
Combining magnetic fields and topology: CME, CSE

$\rightsquigarrow$ next talk by Gergely Markó

Summary

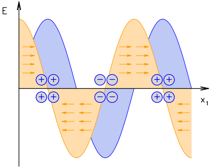
Summary

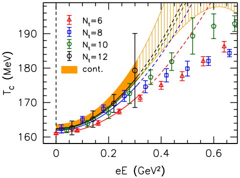
- QCD + **B**
  phase diagram and critical point



- QCD + **B**(x)
  local condensate and current

- QCD + **E**
  local charge distribution
  Schwinger vs. Weldon
  phase diagram to $\mathcal{O}(E^2)$





- QCD + **E** · **B**
  axion-photon coupling