Electromagnetic effects in strongly interacting matter

Gergely Endrődi

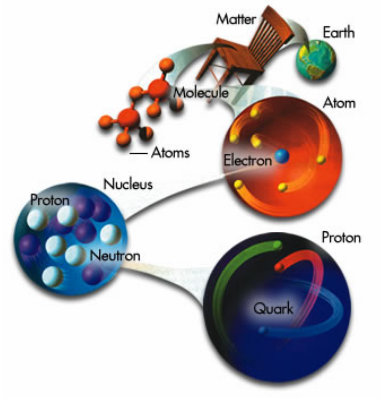
University of Bielefeld

UNIVERSITÄT BIELEFELD

CRC-TR 211
Strong-interaction matter under extreme conditions

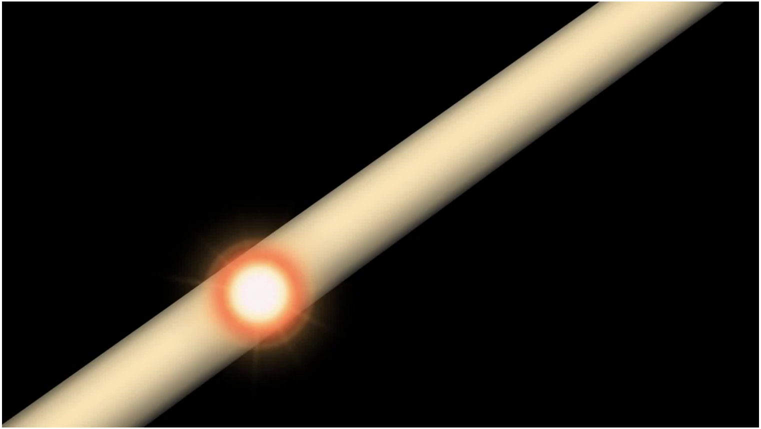
Ortvay Seminar @ ELTE Budapest
May 18, 2023

Strong force

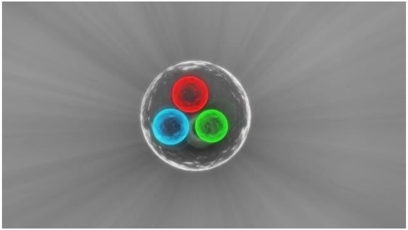


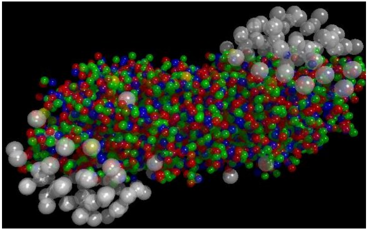
- quarks and gluons "confined" in the proton

Collision experiments [CERN outreach]



Cold versus hot





- two distinct phases of matter

  cold, confined vs. hot, deconfined
- phase transition in between
- theory of quarks and gluons: Quantum ChromoDynamics

what is the nature of these phases?

what is the reason behind confinement and deconfinement?

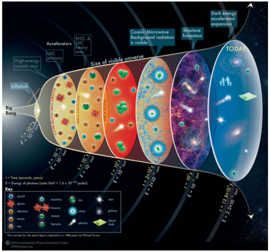
Outline

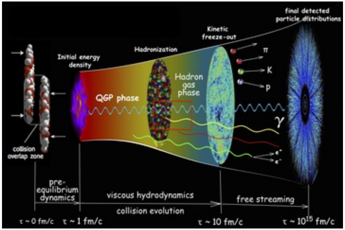
- why? – quarks and gluons in the Universe around us
- how? – quarks and gluons on a supercomputer
- selected lattice QCD results
  - QCD in magnetic fields – phase diagram
  - QCD in electric fields – equilibrium and linear response
  - axion-photon coupling
- summary

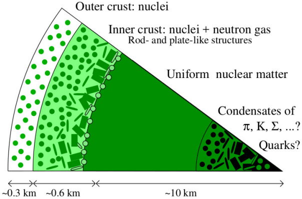
Quarks and gluons in the Universe around us

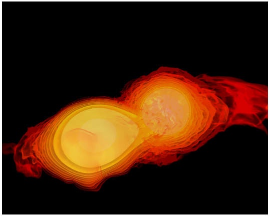
Extreme environments

- hot and/or dense strongly interacting matter in
  - QCD epoch of early Universe
  - heavy-ion collisions
  - neutron stars and their mergers



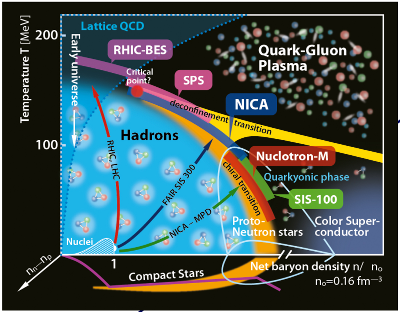


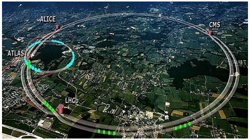




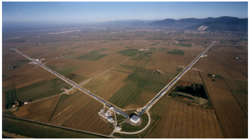
Major experimental and observational campaigns

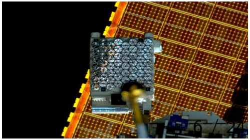
Observational astronomy





Heavy ion collisions





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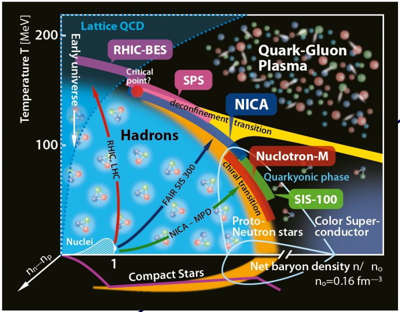

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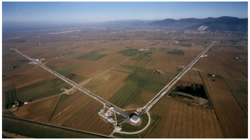

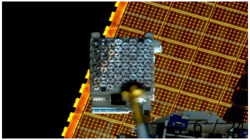

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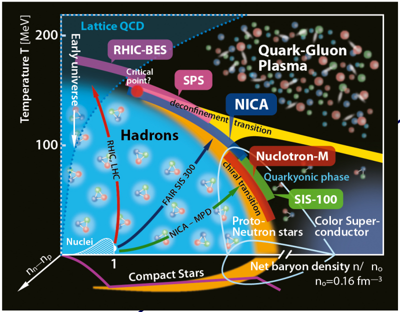

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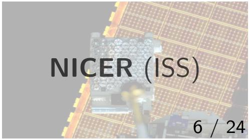
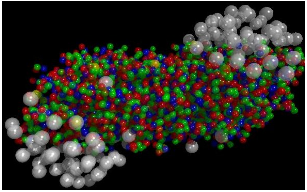
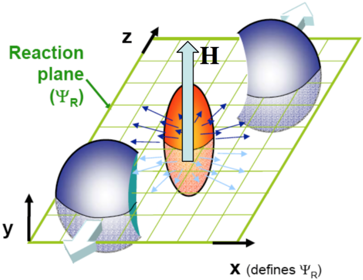


Magnetic fields

- off-central heavy-ion collisions Kharzeev, McLerran, Warringa '07
  impact: chiral magnetic effect, anisotropies, elliptic flow . . .
  Fukushima '12 Kharzeev, Landsteiner, Schmitt, Yee '14





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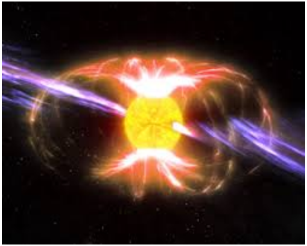

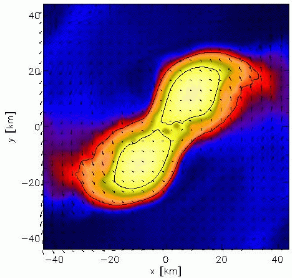

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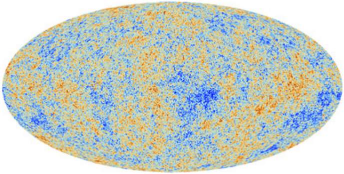
- in the early universe, generated through phase transition in electroweak epoch Vachaspati '91 Enqvist, Olesen '93



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- strength: $B \approx 10^{15}$ T $\approx 10^{20} B_{\rm earth} \approx 5 m_\pi^2$

  $\rightsquigarrow$ competition between strong force and electromagnetism

Quarks and gluons on a supercomputer

Particles and fields

- particles are excitations of fields

string	$\sim$ field
amplitude of vibration	$\sim$ # of particles

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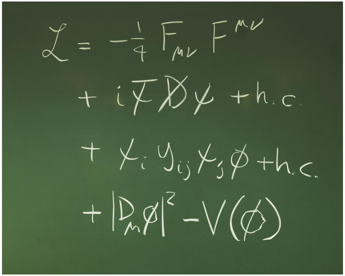
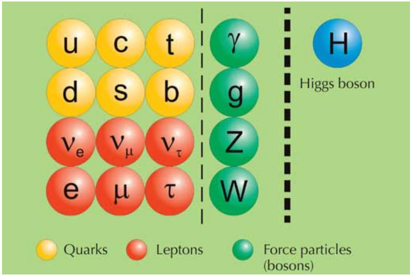
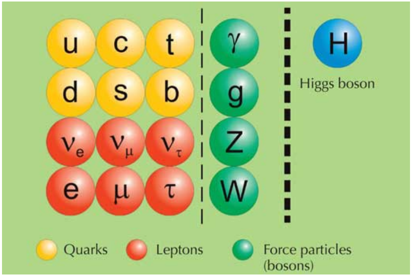
- Standard Model





Standard Model and QCD



- the Standard Model involves three types of interactions

force	mediator particle	strength
electromagnetic	γ	$\alpha = 1/137$
weak	Z, W	$\alpha_w \sim 10^{-6}$
strong	g	$\alpha_s \sim 1$

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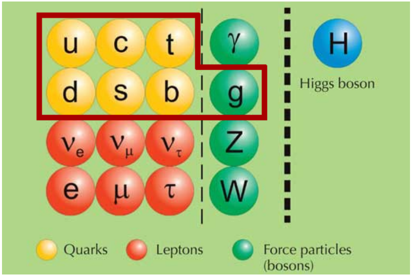

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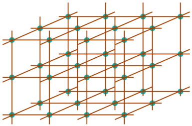
- **strong sector**: Quantum Chromodynamics (QCD)

Path integral and lattice field theory

- path integral 🔗 Feynman '48

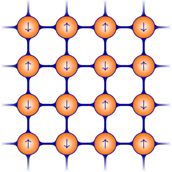
$$\mathcal{Z} = \int \mathcal{D}A_\mu \, \mathcal{D}\bar{\psi} \, \mathcal{D}\psi \, \exp\left(- \int d^4x \, \mathcal{L}_{\text{QCD}}(x) \right)$$

- discretize spacetime on a lattice with spacing a
  🔗 Wilson '74



- Monte-Carlo algorithms to generate configurations

  like in the 2D Ising model:



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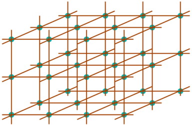
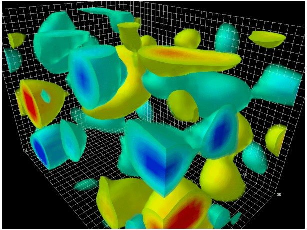


- Monte-Carlo algorithms to generate configurations
  🔗 animation courtesy D. Leinweber

  with field configurations



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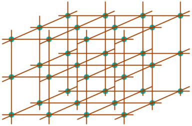
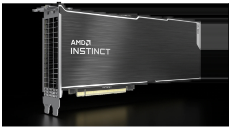


- Monte-Carlo algorithms to generate configurations with $\sim 10^9$ variables $\rightsquigarrow$ high-performance computing
  nvidia.com amd.com



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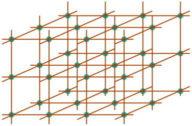

- Monte-Carlo algorithms to generate configurations
- works only if path integral weight is positive
  otherwise: sign (complex action) problem

$T > 0$ ✓
$N > 0$ ×
$B > 0$ ✓
$E > 0$ ×

Phase diagram

The phases of QCD

- most important symmetry: chiral symmetry
- order parameter: quark condensate $\bar{\psi}\psi = \frac{\partial \log \mathcal{Z}}{\partial m}$

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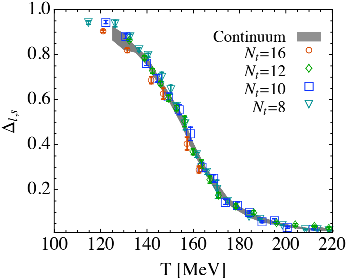
Borsányi et al. '10



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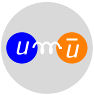

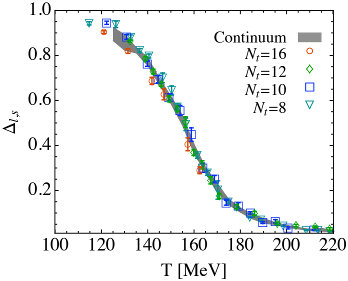

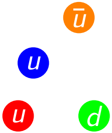

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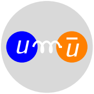

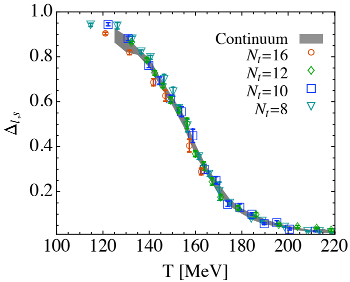

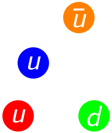

- crossover Aoki et al. '06 Bhattacharya et al. '14

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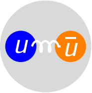

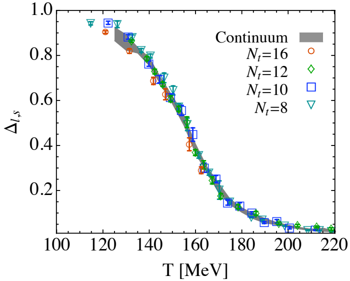

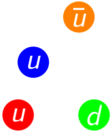

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- $T_c \leftrightarrow$ inflection point Borsányi et al. '10 Bazavov et al. '18

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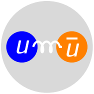

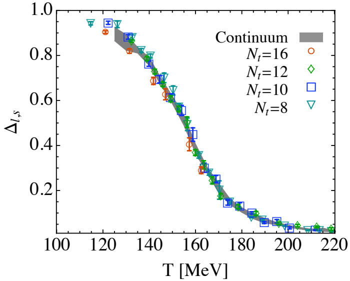

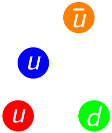

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- impact of B on quark condensate?

Magnetic catalysis

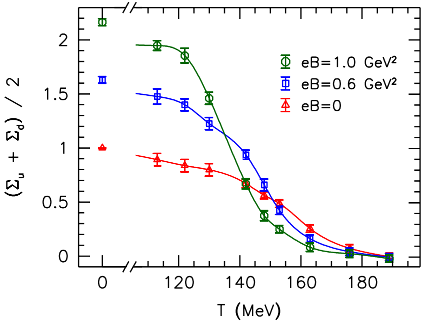
- magnetic field treated as classical background field (no back-reaction)
- magnetic field aligns quark magnetic moments $q\mathbf{S} \parallel \mathbf{B}$
- spin-zero composite states preferred

"magnetic catalysis" Gusynin, Miransky, Shovkovy '95

- can be related to positivity of QED β-function Endrődi '13
- magnetic catalysis in various settings Shovkovy '12
- effective theories, QCD models predicted magnetic catalysis for all temperatures Andersen, Naylor, Tranberg '16

Inverse catalysis and phase diagram

- physical m_π, staggered quarks, continuum limit
  Bali, Bruckmann, Endrődi, Fodor, Katz et al. '11 '12



- magnetic catalysis at low T

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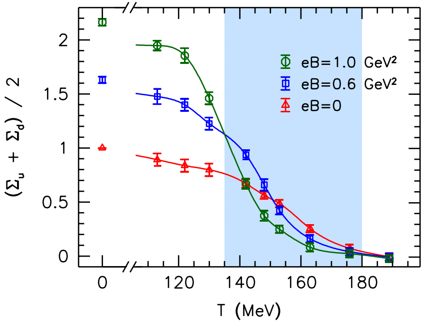

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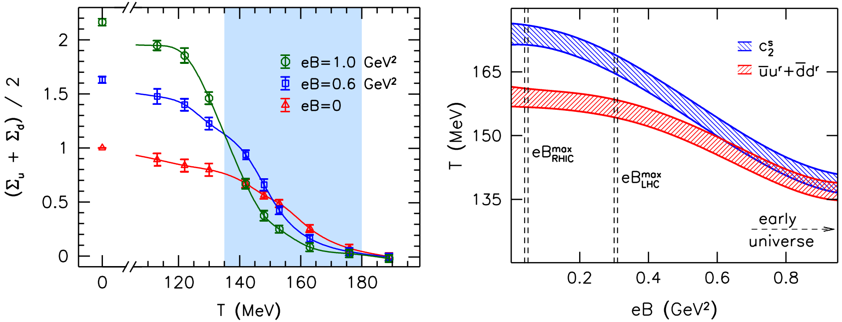

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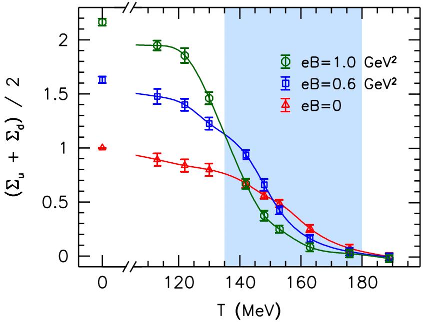

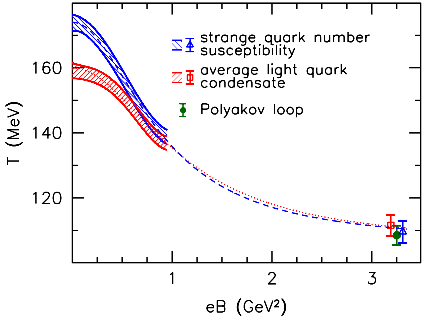

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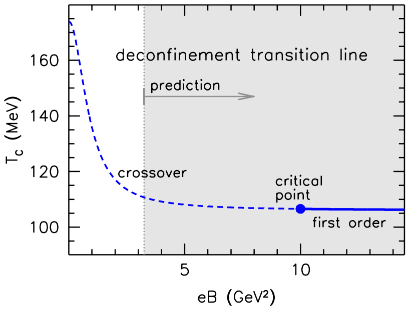
Phase diagram and critical point

- effective theory of QCD at $B \to \infty$: first-order deconfinement transition $\Rightarrow$ critical point! Miransky, Shovkovy '02
- location of critical point estimated from numerical simulations $eB_c \approx 10(2)$ GeV^2 Endrődi '15



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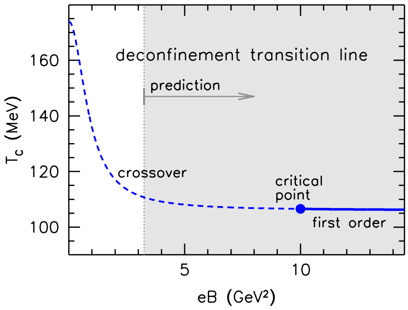
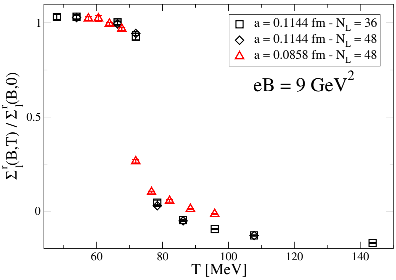
- effective theory of QCD at $B \to \infty$: first-order deconfinement transition $\Rightarrow$ critical point! Miransky, Shovkovy '02
- location of critical point estimated from numerical simulations $eB_c \approx 10(2)$ GeV^2 Endrődi '15
- recent update 4 GeV$^2 < eB_c <$ 9 GeV^2 D'Elia, Maio, Sanfilippo, Stanzione '21





Permeability

Susceptibility and permeability

- leading-order dependence of free energy density on B

$$\chi = -\left.\frac{\partial^2 f}{\partial (eB)^2}\right|_{B=0}$$

- permeability 🔗 Landau-Lifschitz Vol 8. $\mu = (1 - e^2\chi)^{-1}$
- $\mu > 1$ $(\chi > 0)$: paramagnetism
  $\mu < 1$ $(\chi < 0)$: diamagnetism

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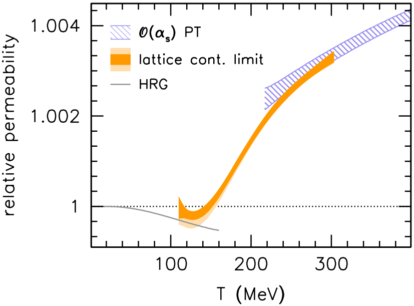

Bali, Endrődi, Piemonte '20

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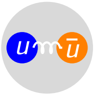

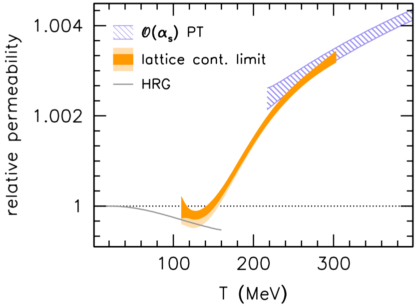

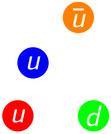

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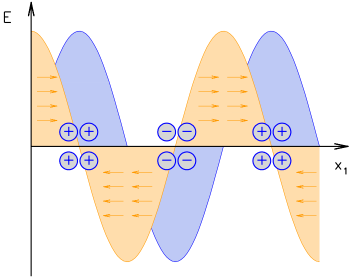
QCD in electric fields

Electric fields

- static homogeneous electric field E: charges accelerated to ∞
- equilibrium requires infrared regularization
  $\rightsquigarrow$ finite wavelength $1/k_1$



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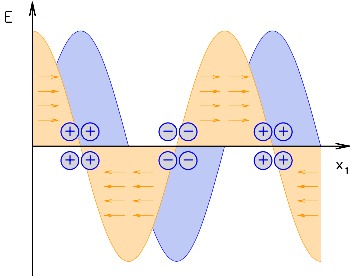
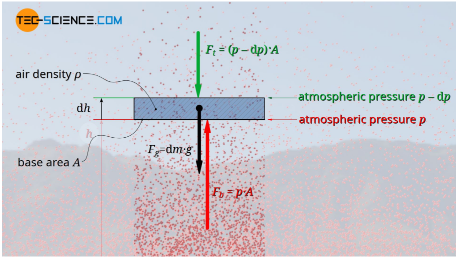


- charge distribution where electric and diffusion forces cancel
- finally take homogeneous limit $k_1 \to 0$
- we only consider thermal effects (no Schwinger pair creation)

Analogy: barometric distribution

- recall barometric formula above 'flat earth' 🔗 tec-science.com



- gravitational force ↔ electric force
- atmospheric pressure ↔ fermionic degeneracy pressure

Electric susceptibility

- leading impact of E on free energy f (permittivity)
- here: perturbative QED at nonzero T

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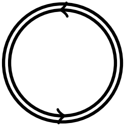
  Loewe, Rojas '92 Elmfors, Skagerstam '95 Gies '98

$f(E) =$ 

$$f = -\xi_{\rm Schwinger} \cdot \frac{E^2}{2} + \ldots$$

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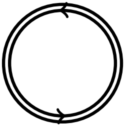
$f(E) =$ 

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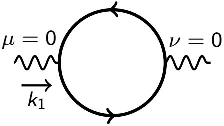

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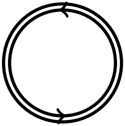
$f(E) =$ 

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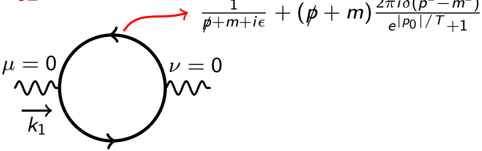
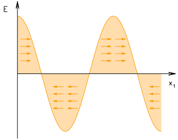


- generalize calculation to $m > 0$ Endrődi, Markó 2208.14306

Equilibrium and mismatch

- evaluated in absence of charge distribution ($\mu = 0$)

$$\xi_{\text{Weldon}}^{\text{non-equi}} = \frac{T^2}{3k_1^2} + \mathcal{O}(k_1^0)$$



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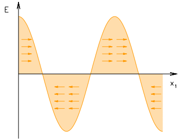
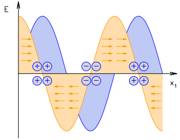


- evaluated "along local equilibria" ($N(x)$ such that $\partial\mu/\partial x = -E$)



$$\xi_{\text{Weldon}}^{\text{equi}} = -\frac{1}{12\pi^2}\left[\log\frac{T^2}{m^2} + 1\right] + \mathcal{O}(m^2/T^2)$$

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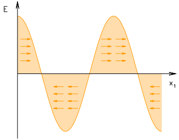

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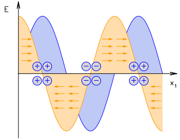

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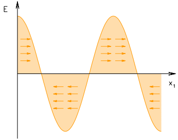

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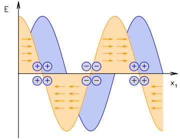

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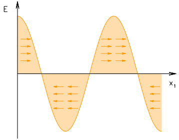

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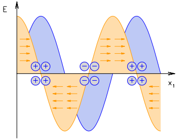

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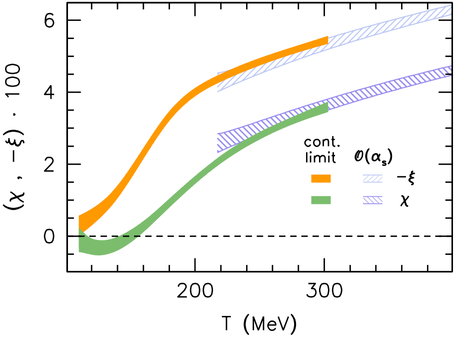
- note different ordering of limits: $V \to \infty$ vs. $E \to 0$
- no mismatch for magnetic susceptibility (no displaced charges)

Full QCD

- lattice QCD simulations for $\xi^{\text{equi}}_{\text{Weldon}}$
- continuum limits for magnetic and electric susceptibilities, compared to perturbation theory at high T



🔗 Endrődi, Markó '22

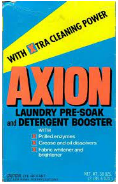
Axion-photon coupling

Axions as dark matter

- extend Standard Model with new field: axion a

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}\partial_\mu a \partial_\mu a + a\,\underbrace{\mathrm{Tr}\,\epsilon_{\mu\nu\sigma\rho} G_{\mu\nu} G_{\sigma\rho}}_{Q_{\rm top}} + a\, g_{a\gamma\gamma}\,\underbrace{\epsilon_{\mu\nu\sigma\rho} F_{\mu\nu} F_{\sigma\rho}}_{\mathbf{E}\cdot\mathbf{B}}$$

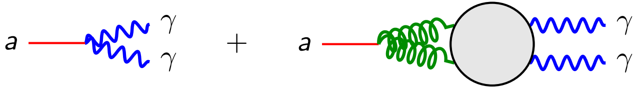
- provides solution to ‘strong CP problem’
  Peccei, Quinn ’77 Weinberg ’78 Wilczek ’78



- is a possible dark matter candidate
- extensive experimental campaign:
  haloscopes and helioscopes ADMX CAST XENON1T

Axion-photon coupling

- most relevant parameter for experimental detection
- direct coupling (model-dependent)
  plus
  indirect coupling through quark/gluon loops



- chiral perturbation theory predicts two terms of similar magnitude and opposite sign di Cortona et al. '16
- QCD contribution, for slowly varying a fields

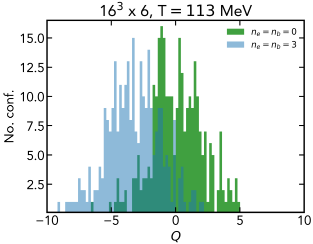
$$g_{a\gamma\gamma}^{\mathrm{QCD}} = \frac{\partial^2 \log \mathcal{Z}}{\partial a\, \partial(\mathbf{E}\cdot\mathbf{B})} = \frac{\partial \langle Q_{\mathrm{top}} \rangle}{\partial(\mathbf{E}\cdot\mathbf{B})}$$

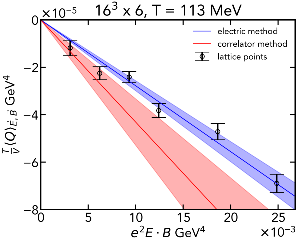
Axion-photon coupling on the lattice

- QCD contribution

$$g_{a\gamma\gamma}^{\mathrm{QCD}} = \frac{\partial \langle Q_{\mathrm{top}} \rangle}{\partial(\mathbf{E} \cdot \mathbf{B})}$$

- shift in mean topology by parallel magnetic *and* imaginary electric fields



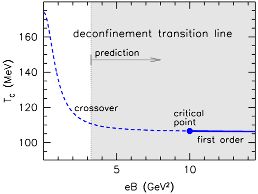


- first results for $g_{a\gamma\gamma}^{\mathrm{QCD}}$ Hernández et al. '22
- to be extrapolated to the continuum limit

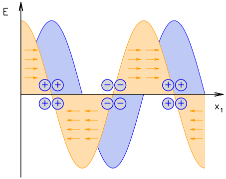
Summary

Summary

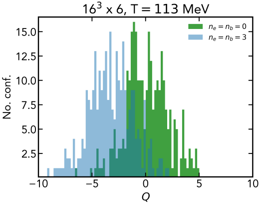
- QCD + **B**
  phase diagram
  and the critical point



- QCD + **E**
  local charge distributions
  mismatch Schwinger vs. Weldon



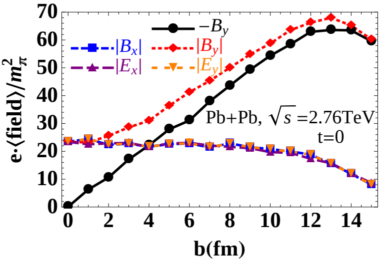
- QCD + **E** · **B**
  axion-photon coupling

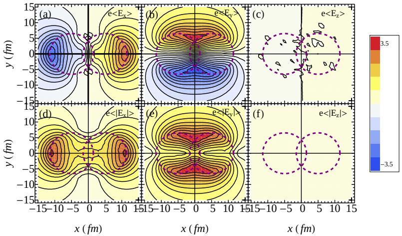


Backup

Electromagnetic fields: heavy ion collisions

- electromagnetic fields in the early stage of heavy-ion collisions reaching m_π^2 and well beyond
  Deng et al. '12





- most probably short-lived fields Huang '15
- impact of electric field enhanced for asymmetric systems (for example Cu+Au at RHIC) Voronyuk et al. '14