QCD matter in extreme environments: isospin-asymmetry and electromagnetic fields

Gergely Endrődi

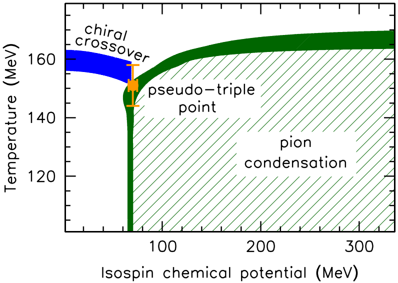
University of Bielefeld

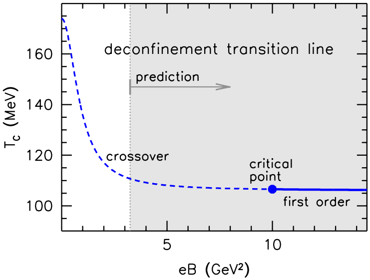
Universität Bielefeld

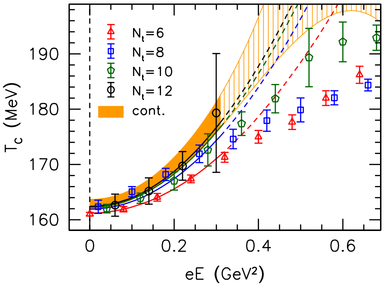
CRC–TR 211
Strong-interaction matter under extreme conditions

Seminar at Helsinki Institute of Physics
November 28, 2023

fundamental phase diagrams of QCD
with possible phenomenological implications







Brandt, Endrődi, Schmalzbauer ’18
Brandt, Endrődi ’19

Endrődi ’15

D’Elia, Maio, Sanfilippo, Stanzione ’21
Endrődi, Markó ’23

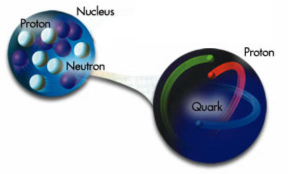
Outline

- introduction: strongly interacting matter in
  - strong electromagnetic fields
  - nonzero isospin density
- lattice simulation techniques
- phase diagrams: current status
- application: cosmic trajectory
- electric fields in equilibrium
- summary

Introduction

Strong interactions

- explain 99.9% of visible matter in the Universe



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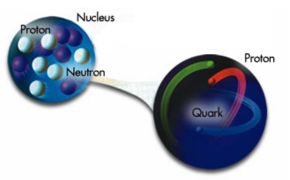

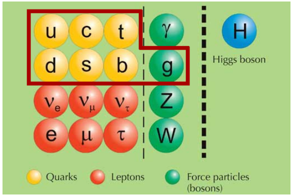

- elementary particles: quarks and gluons

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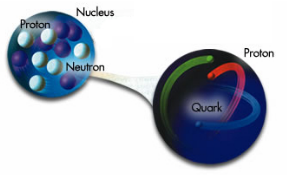

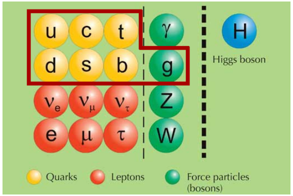

- elementary particles: quarks and gluons
- elementary fields: $\psi(x)$ and $A_\mu(x)$ enter the QCD Lagrangian

$$\mathcal{L}_{\mathrm{QCD}} = \frac{1}{4}\,\mathrm{Tr}\,F_{\mu\nu}(g_s, A)^2 + \bar{\psi}[\gamma_\mu(\partial_\mu + i g_s A_\mu) + m]\psi$$

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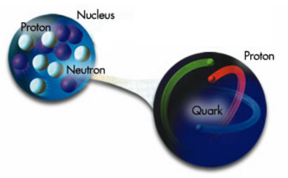

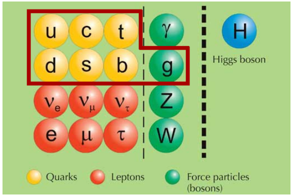

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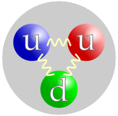
  $m_u,\ m_d \approx 3 - 5$ MeV, $\quad m_p = 938$ MeV



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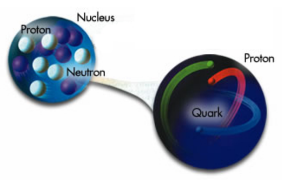

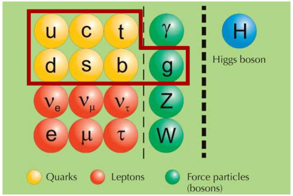

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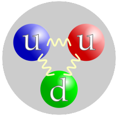
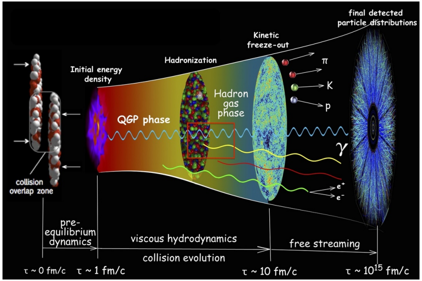
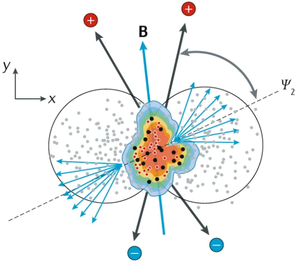


- asymptotic freedom at high energy scales $\rightsquigarrow$ deconfinement

Quarks and gluons in extreme conditions

- heavy ion collisions $T \lesssim 10^{12}\,^{\circ}C = 200$ MeV, $n \lesssim 0.12$ fm^{-3}
  $B \lesssim 10^{19}$ G $= 0.3$ GeV$^2/e$





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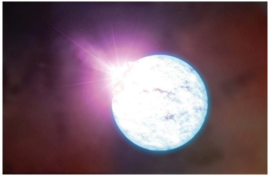

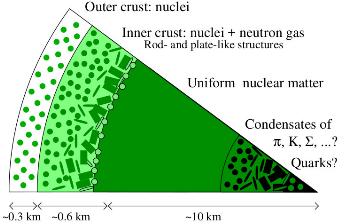

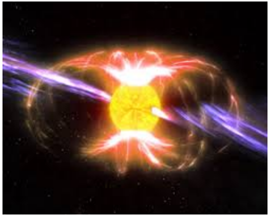

Lattimer, Nature Astronomy 2019

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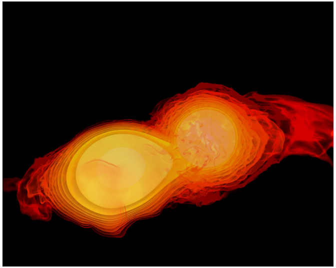

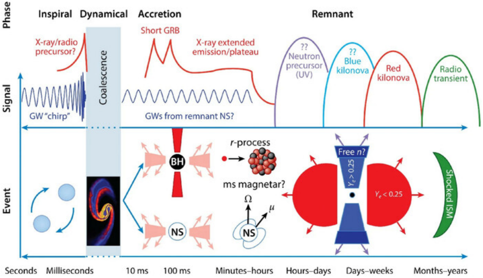

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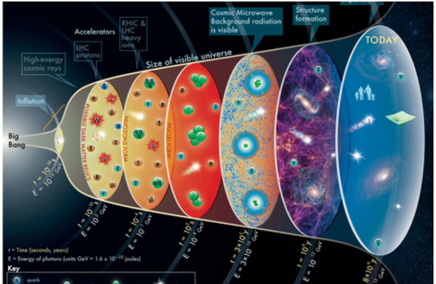
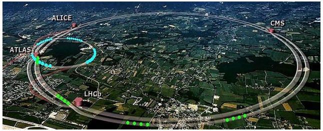
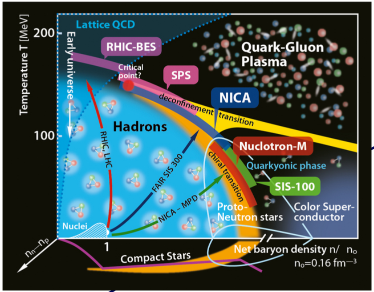
- eary universe, QCD epoch $T \lesssim 200$ MeV
  standard scenario: $n \approx 0$



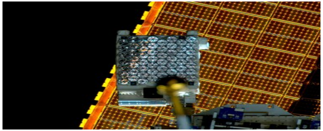
Major experimental and observational campaigns





Observational astronomy

Heavy ion collisions



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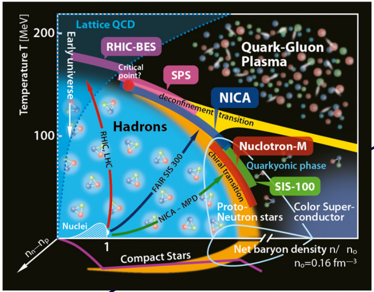

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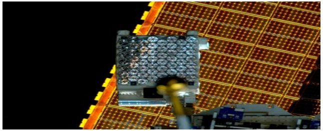

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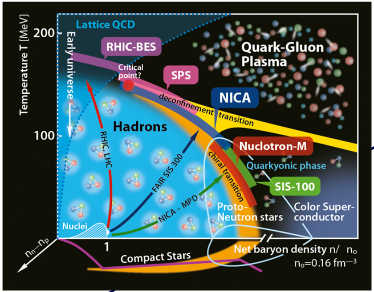

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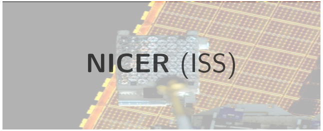

QCD phase diagram(s)

Phase diagram

- control parameters: T, $n \leftrightarrow \mu$, B $\qquad \mu_{\{u,d,s\}}$ / $\mu_{\{B,Q,S\}}$ / $\mu_{\{B,I,S\}}$

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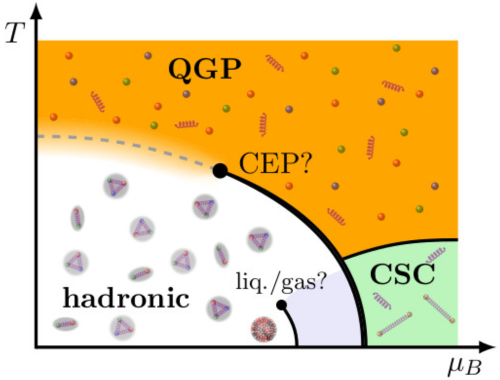

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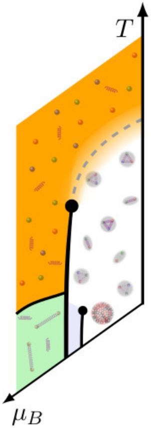

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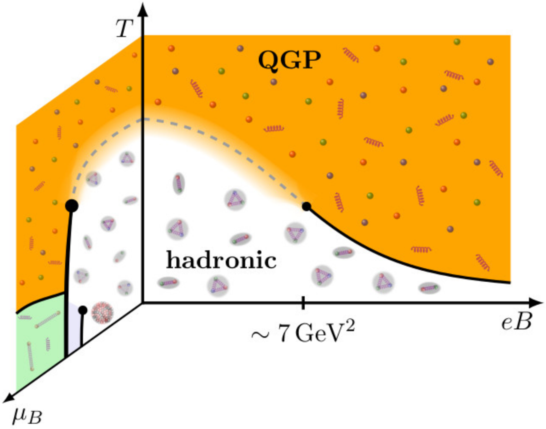

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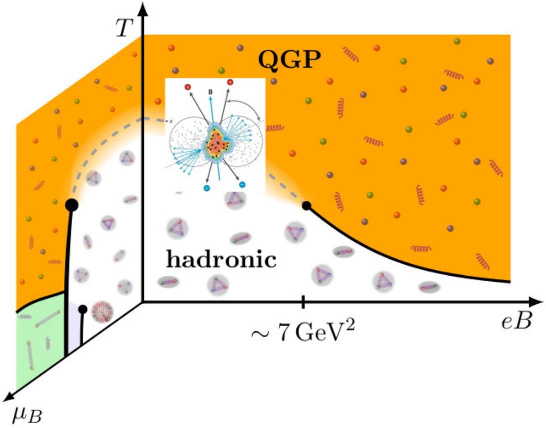

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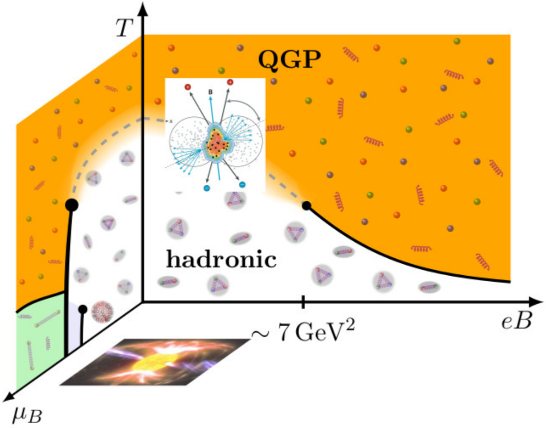

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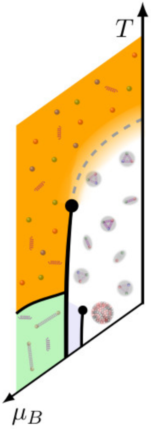

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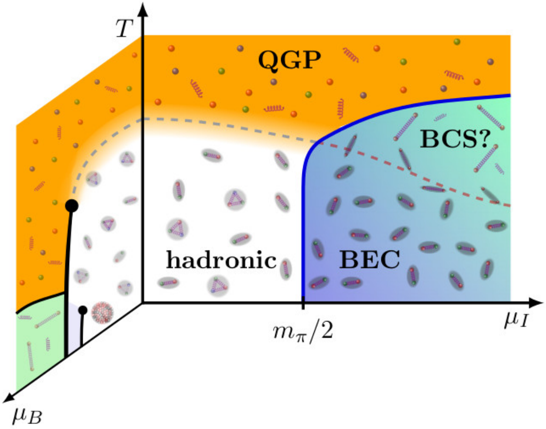

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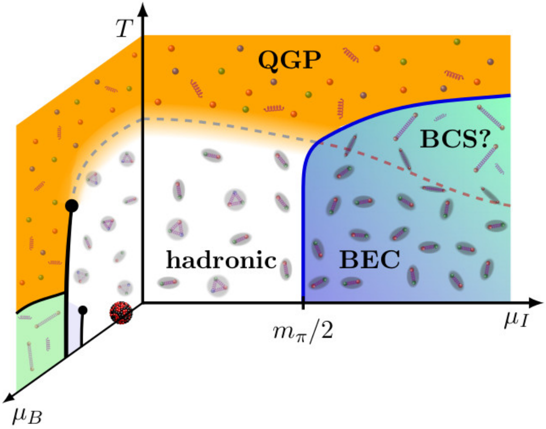

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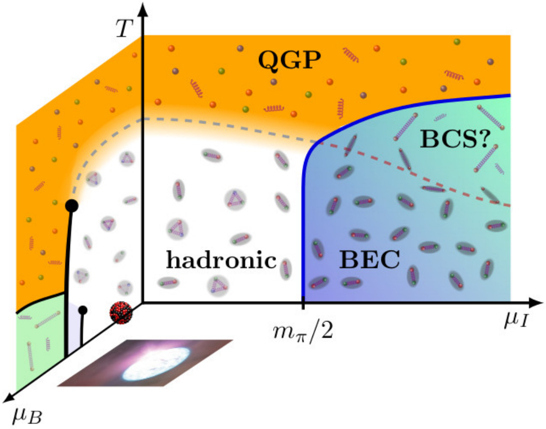

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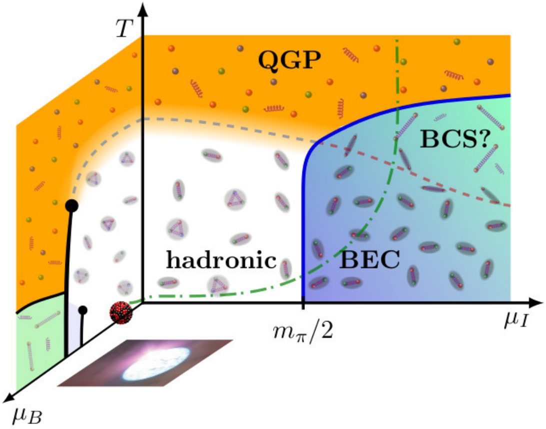


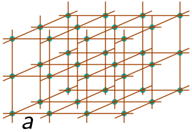
Lattice QCD simulations

Lattice simulations

- path integral Feynman '48

$$\mathcal{Z} = \int \mathcal{D}A_\mu \, \mathcal{D}\bar{\psi} \, \mathcal{D}\psi \, \exp\left(- \int \mathrm{d}^4x \, \mathcal{L}_{\mathrm{QCD}}(x) \right)$$

- discretize QCD action on space-time lattice Wilson '74

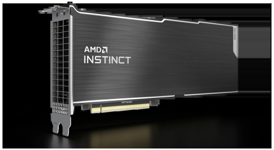


continuum limit $a \to 0$ in a fixed physical volume: $N \to \infty$

- dimensionality of lattice path integral: 10^{9-10} $\rightsquigarrow$ computationally very demanding

SuperMUC-NG

nvidia.com



amd.com

Bielefeld GPU cluster

Monte Carlo simulations

- Euclidean QCD path integral over gauge field $\mathcal{A}$

$$\mathcal{Z} = \int \mathcal{D}\mathcal{A}\, e^{-S_g[\mathcal{A}]} \det[\not{D}[\mathcal{A}] + m]$$

- Monte-Carlo simulations need: $\det[\not{D} + m] \in \mathbb{R}^+$
  for that one needs Γ so that

$$\Gamma \not{D} \Gamma^\dagger = \not{D}^\dagger, \quad \Gamma^\dagger \Gamma = 1$$

$$\det[\not{D} + m] = \det[\Gamma^\dagger \Gamma (\not{D} + m)] = \det[\Gamma (\not{D} + m) \Gamma^\dagger] = \det[\not{D}^\dagger + m] = \det[\not{D} + m]^*$$

- usually positivity can also be shown
- such a Γ exists: no complex action problem

Complex actions vs. real actions

- two-flavor Dirac operator ($\mathcal{A}_\mu$: $\mathrm{SU}(3)$ field, A_μ: $\mathrm{U}(1)$ field) (remember Wick rotation $A_4 = -iA_0$)

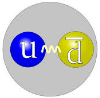
$$\not{D}[\mathcal{A}, A] = \gamma_\mu(\partial_\mu + i\mathcal{A}_\mu)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + i\gamma_\mu\begin{pmatrix}A_\mu^u & 0\\ 0 & A_\mu^d\end{pmatrix}$$

- pure QCD: $A_\mu = 0$ — $\Gamma = \gamma_5$ ✓
- magnetic field: $A_2^f = q_f B x_1$ — $\Gamma = \gamma_5$ ✓
- imaginary baryon chem. pot.: $A_4^u = A_4^d = \mu$ — $\Gamma = \gamma_5$ ✓
- imaginary electric field: $A_4^f = q_f E x_1$ — $\Gamma = \gamma_5$ ✓
- real baryon chem. pot.: $iA_4^u = iA_4^d = \mu$ — ↯
- real electric field: $iA_4^f = q_f E x_1$ — ↯
- real isospin chem. pot.: $iA_4^u = -iA_4^d = \mu_I$ — $\Gamma = \gamma_5\tau_1$ ✓

Isospin phase diagram preface:
pion condensation

Pion condensation

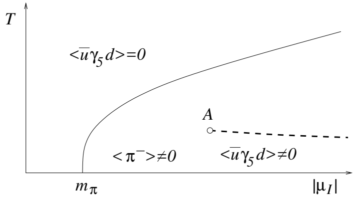
- isospin chemical potential: $\mu_u = \mu_I$, $\mu_d = -\mu_I$, $\mu_s = 0$
  - QCD at low energies $\approx$ pions
    chiral perturbation theory
  - charged pion chemical potential: $\mu_\pi = 2\mu_I$



at zero temperature $\mu_\pi < m_\pi$ vacuum state

$\mu_\pi \geq m_\pi$ Bose-Einstein condensation

Son, Stephanov '00



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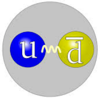

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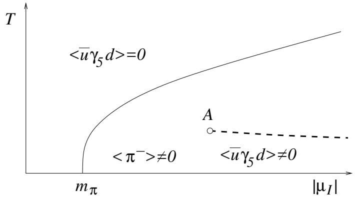

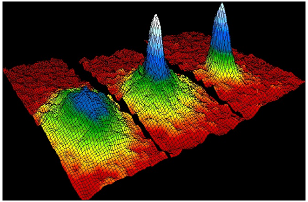

- trapped Rb atoms at low temperature Anderson et al '95 JILA-NIST/University of Colorado

Symmetry breaking

- QCD with light quark matrix

$$M = \not{D} + m_{ud}\mathbb{1}$$

- chiral symmetry (flavor-nontrivial)

$$\mathrm{SU}(2)_V$$

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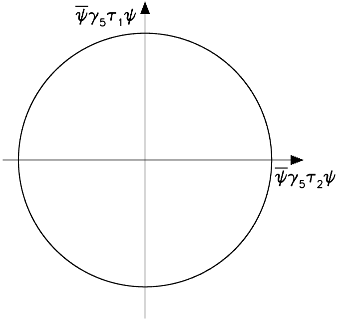

Symmetry breaking

- QCD with light quark matrix

$$M = \not{D} + m_{ud}\mathbb{1} + \mu_I \gamma_0 \tau_3$$

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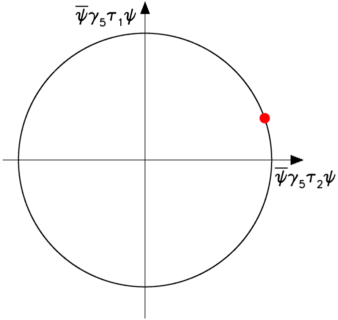

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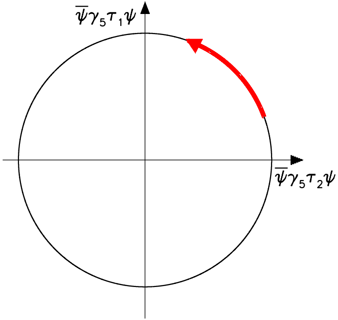

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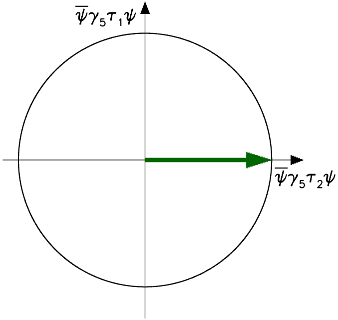

$$\mathrm{SU}(2)_V \to \mathrm{U}(1)_{\tau_3} \to \varnothing$$

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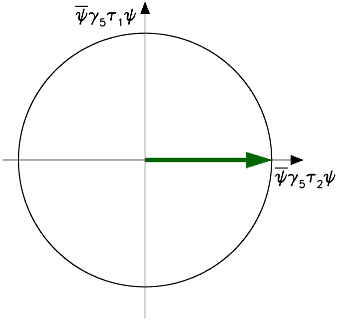

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- extrapolate results $\lambda \to 0$

Dictionary

	pion condensation	vacuum chiral symmetry breaking
pattern	$\mathrm{U}(1)_{\tau_3} \to \varnothing$	$\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \to \mathrm{SU}(2)_V$
coset	$\mathrm{U}(1)$	$\mathrm{SU}(2)_A$
Goldstones	1	3
spontaneous	$\left\langle \bar{\psi}\gamma_5\tau_2\psi \right\rangle$	$\left\langle \bar{\psi}\psi \right\rangle$
explicit	$= \partial \log \mathcal{Z} / \partial\lambda$	$= \partial \log \mathcal{Z} / \partial m$
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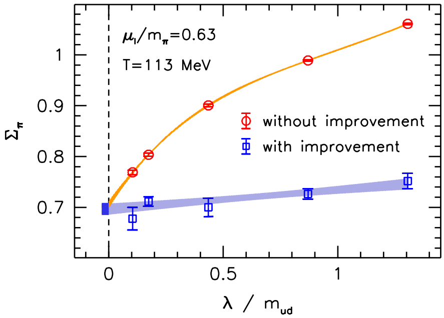
limit	$\lambda \to 0$	$m \to 0$

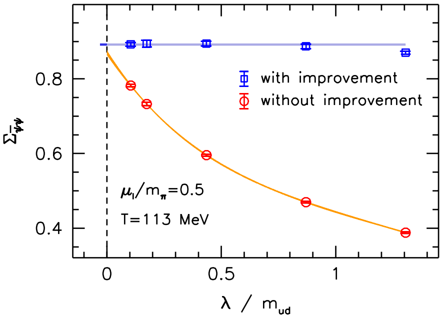
- long story short: pion condensation $1/3$ as challenging as the chiral limit of the QCD vacuum

Phase diagram: nonzero isospin

Extrapolation to $\lambda = 0$

- improvement is crucial for reliable $\lambda \to 0$ extrapolation
  Brandt, Endrődi, Schmalzbauer '17 Brandt, Endrődi '19

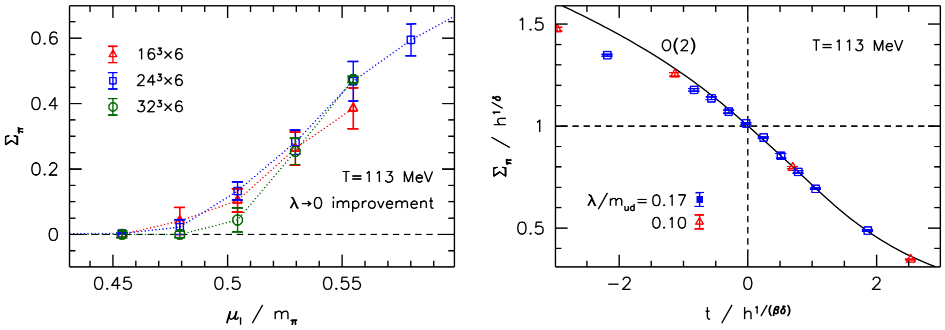




$$\Sigma_\pi = \frac{T}{V}\frac{\partial \log \mathcal{Z}}{\partial \lambda}$$

$$\Sigma_{\bar\psi\psi} = \frac{T}{V}\frac{\partial \log \mathcal{Z}}{\partial m_{ud}}$$

Order of the transition



- volume scaling of order parameter shows typical sharpening
- collapse according to $\mathrm{O}(2)$ critical exponents Ejiri et al '09

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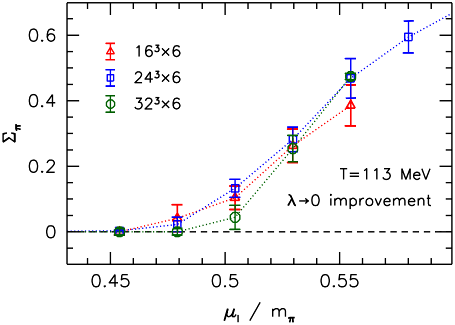
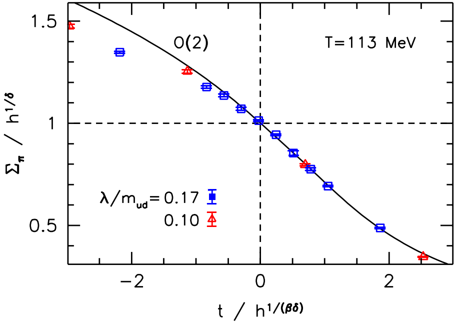


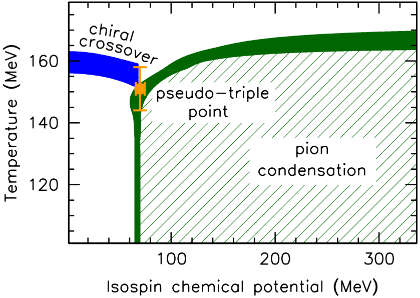


- volume scaling of order parameter shows typical sharpening
- collapse according to $\mathrm{O}(2)$ critical exponents Ejiri et al '09
- indications for a second order phase transition at $\mu_I = m_\pi/2$, in the $\mathrm{O}(2)$ universality class

Phase diagram

- phases in the $T - \mu_I$ phase diagram: hadronic (confined), quark-gluon plasma (deconfined), pion condensation (confined)

Brandt, Endrődi, Schmalzbauer '17 Brandt, Endrődi '19



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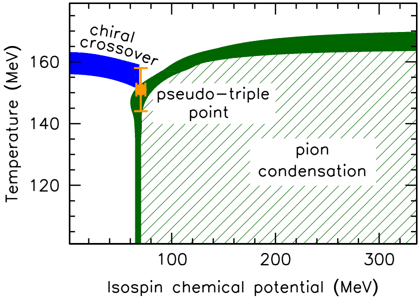

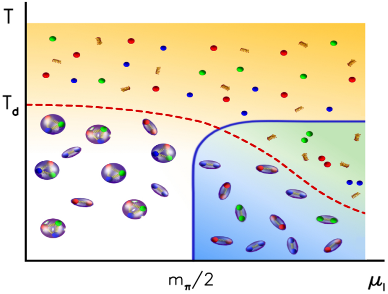

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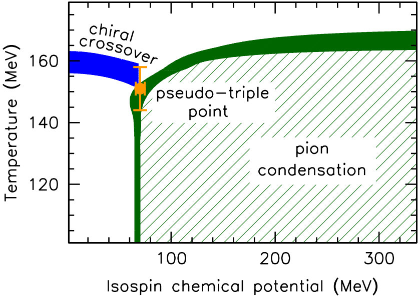

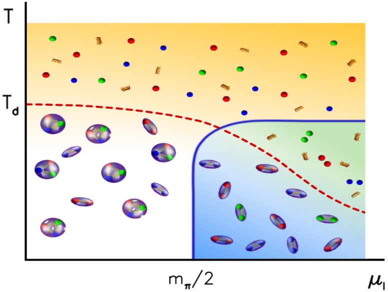

- comparison to effective models, χPT, Q2CD, ...

  🔗 Adhikari et al. '18 🔗 Zhokhov et al. '19 🔗 Adhikari et al. '20

  🔗 Boz et al. '20 🔗 Astrakhantsev et al. '20

Equation of state: nonzero isospin

Equation of state

- equilibrium description of matter

$$\epsilon(p)$$

  relevant for:
  - neutron star physics (TOV equations)
  - cosmology, evolution of early Universe (Friedmann equation)
  - heavy-ion collision phenomenology (charge fluctuations)
- thermodynamic relations

$$p = \frac{T}{V}\log \mathcal{Z}, \qquad s = \frac{\partial p}{\partial T}, \qquad n_I = \frac{\partial p}{\partial \mu_I}, \qquad \epsilon = -p + Ts + \mu_I n_I$$

$$I = \epsilon - 3p, \qquad c_s^2 = \left.\frac{\partial p}{\partial \epsilon}\right|_{s/n_I}$$

Equation of state on the lattice

- integral method to calculate differences

$$n_I = \frac{T}{V}\frac{\partial \log \mathcal{Z}}{\partial \mu_I}, \qquad p(T,\mu_I) - p(T,0) = \int_0^{\mu_I} \mathrm{d}\mu_I'\, n_I(\mu_I')$$

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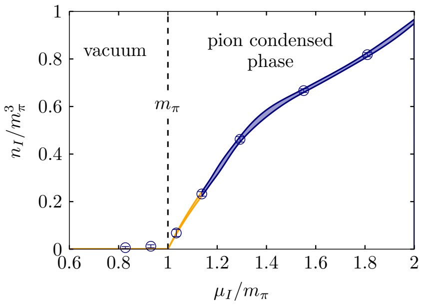

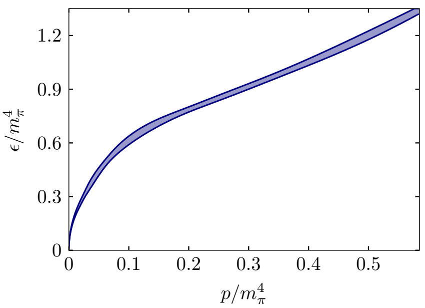

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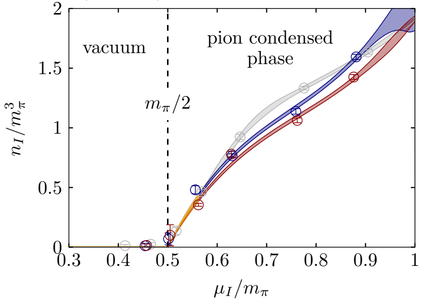
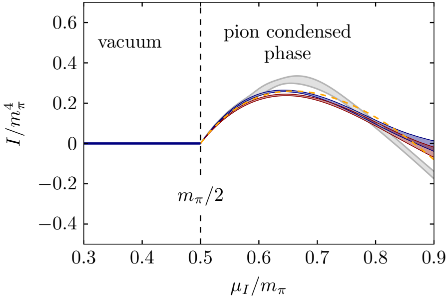
Brandt, Cuteri, Endrődi '22





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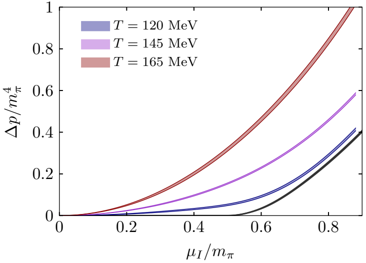

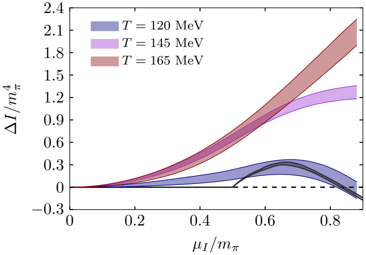

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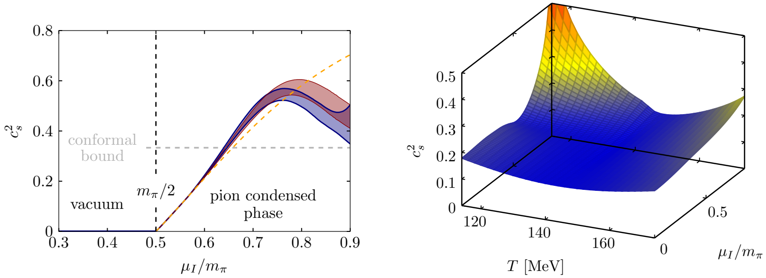

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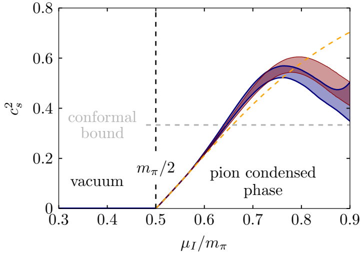
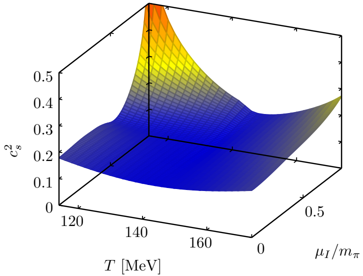




- EoS can get very stiff inside pion condensation phase
- comparison: χPT, models Adhikari et al. '21 Avancini et al. '19

Constraints for nonzero baryon density

- pressure in isospin-asymmetric matter p_I acts as upper bound for pressure of baryon-dense matter p_B Cohen '03

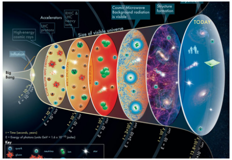
$$p_B(\mu_B) \leq p_I(\mu_I = \mu_B/3)$$

- lattice results at μ_I give non-trivial constraints for neutron star EoS
  Moore, Gorda '23 Fujimoto, Reddy '23

Cosmological implications

Cosmic trajectories

- early Universe



- conservation equations for isentropic expansion

$$\frac{n_B}{s} = b, \quad \frac{n_Q}{s} = 0, \quad \frac{n_{L_\alpha}}{s} = l_\alpha \quad (\alpha \in \{e, \mu, \tau\})$$

- parameters: $T, \quad \mu_B, \quad \mu_Q, \quad \mu_{L_\alpha}$
- experimental constraints Planck coll. '15 Oldengott, Schwarz '17

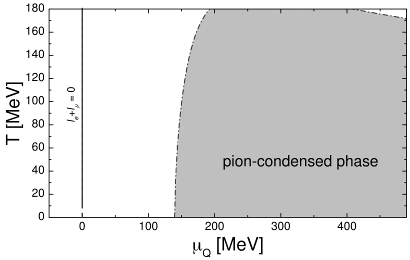
$$b = (8.60 \pm 0.06) \cdot 10^{-11}, \qquad \boxed{|l_e + l_\mu + l_\tau| < 0.012}$$

(the individual l_α may have opposite signs)

- $n_Q = 0$ with $l_e > 0$ allows equilibrium of e^-, ν_e, π^+ Abuki, Brauner, Warringa '09

Cosmic trajectories

- cosmic trajectory $T(\mu_Q)$ is solved for
- standard scenario ($l_\alpha = 0$): $\mu_Q = 0$ for all T



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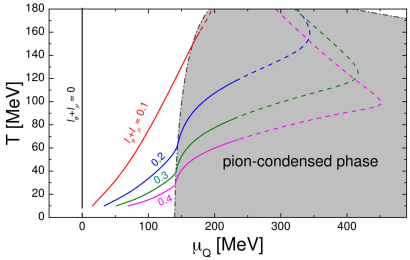
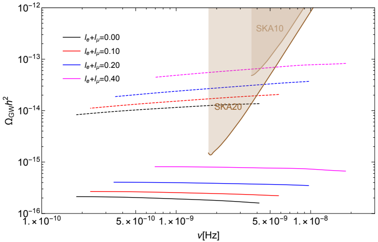


- cosmic trajectory enters BEC phase for lepton asymmetries allowed by observations Vovchenko, Brandt, Cuteri, Endrődi, Hajkarim, Schaffner-Bielich '20
- condition for pion condensation to occur:

$$|l_e + l_\mu + l_\tau| < 0.012 \qquad |l_e + l_\mu| \gtrsim 0.1$$

Signatures of the condensed phase

- relic density of primordial gravitational waves is enhanced with respect to amplitude at $l_e + l_\mu = 0$



🔗 Vovchenko, Brandt, Cuteri, Endrődi, Hajkarim, Schaffner-Bielich ’20

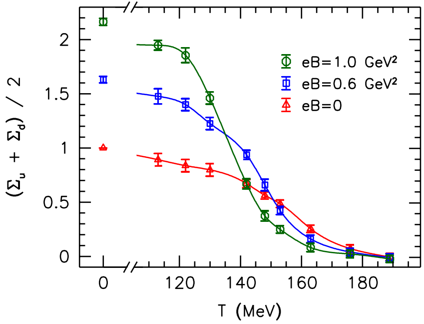
- to be detected experimentally (SKA)

Phase diagram: magnetic fields

Inverse catalysis and phase diagram

- physical m_π, staggered quarks, continuum limit

  Bali, Bruckmann, Endrődi, Fodor, Katz et al. '11 '12



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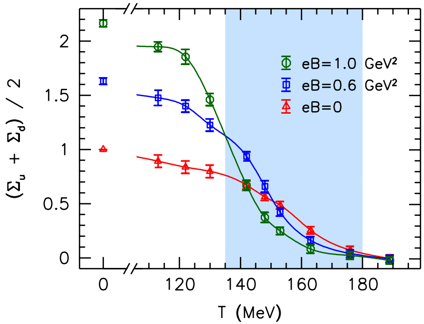

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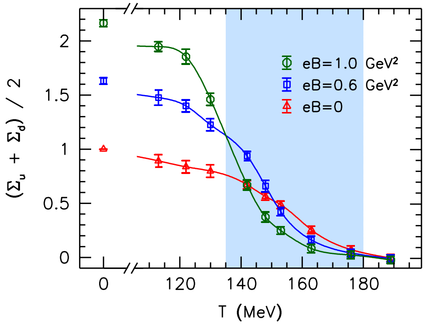

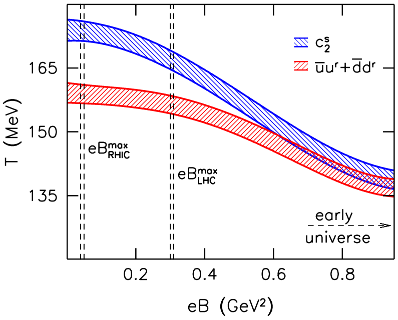

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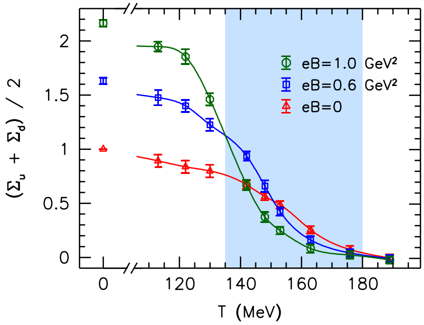

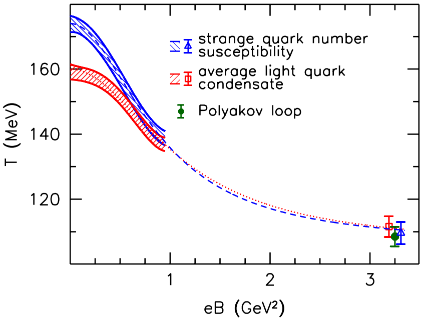

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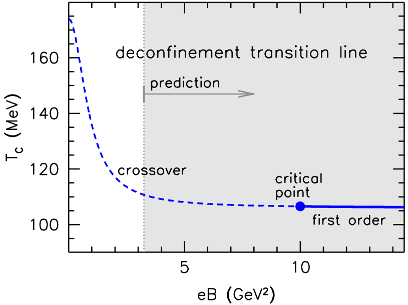
Phase diagram and critical point

- effective theory of QCD at $B \to \infty$: first-order deconfinement transition
  $\Rightarrow$ critical point! Miransky, Shovkovy '02
- location of critical point based on extrapolation from $0 < eB \lesssim 3$ GeV^2
  $\Rightarrow eB_c \approx 10(2)$ GeV^2 Endrődi '15



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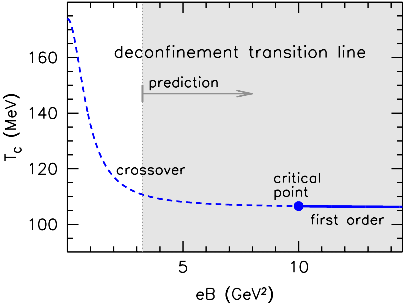
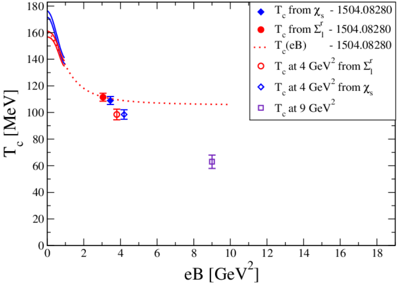
- simulating up to $eB \approx 9$ GeV^2
  $\Rightarrow 4$ GeV$^2 < eB_c < 9$ GeV^2 D'Elia, Maio, Sanfilippo, Stanzione '21

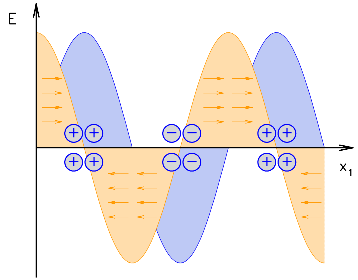




Phase diagram: electric fields with a detour to perturbative QED

Electric fields

- static homogeneous electric field E: charges accelerated to ∞
- equilibrium requires infrared regularization
  $\rightsquigarrow$ finite wavelength $1/k_1$



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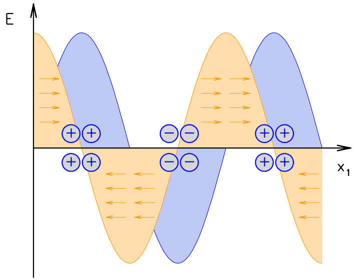
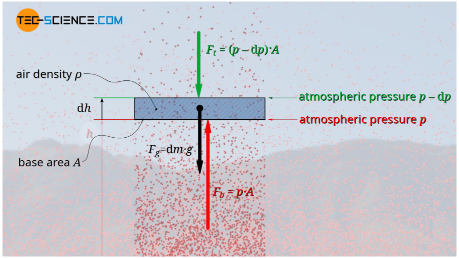


- charge distribution where electric and diffusion forces cancel
- finally take homogeneous limit $k_1 \to 0$
- this is a thermal effect (no Schwinger pair creation)

Analogy: barometric distribution

- recall barometric formula above ‘flat earth’ 🔗 tec-science.com



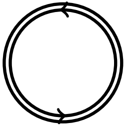
- gravitational force ↔ electric force
- atmospheric pressure ↔ fermionic degeneracy pressure

Electric susceptibility

- leading impact of E on free energy f within perturbative QED at nonzero T

Electric susceptibility

- leading impact of E on free energy f within perturbative QED at nonzero T
- Schwinger's approach 🔗 Schwinger '51 🔗 Loewe, Rojas '92 🔗 Elmfors, Skagerstam '95 🔗 Gies '98

$f(E) =$ 

$$f = -\xi_{\rm Schwinger} \cdot \frac{E^2}{2} + \ldots$$

ordering: $\lim_{E\to 0} \lim_{V\to\infty}$

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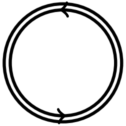
$f(E) =$  $\quad f = -\xi_{\text{Schwinger}} \cdot \frac{E^2}{2} + \ldots$

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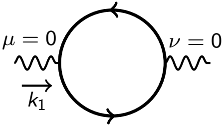
$\xi_{\text{Weldon}} = \lim_{k_1\to 0} \frac{-1}{k_1^2}$ 

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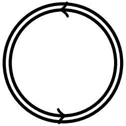
$f(E) =$  $\qquad f = -\xi_{\text{Schwinger}} \cdot \dfrac{E^2}{2} + \ldots$

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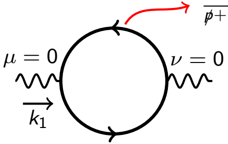

$\frac{1}{\not{p}+m+i\epsilon} + (\not{p}+m)\frac{2\pi i\delta(p^2-m^2)}{e^{|p_0|/T}+1}$

ordering: $\lim_{k_1\to 0} \lim_{E\to 0}$

Equilibrium and mismatch

- result in Schwinger's approach (high-temperature expansion)

$$\xi_{\mathrm{Schwinger}} = -\frac{1}{12\pi^2}\left[\log\frac{T^2}{m^2} - 1\right] + \mathcal{O}(m^2/T^2)$$

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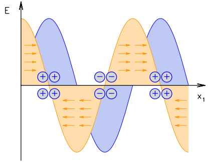

local equilibria ($N(x)$ such that $\partial\mu/\partial x = -E$)

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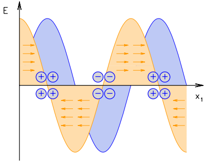

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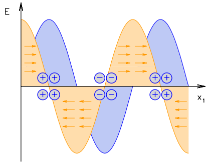

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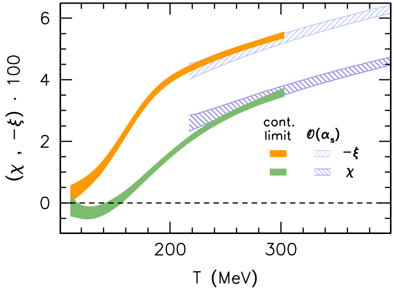
- electric susceptibility: mismatch due to different order of limits
- magnetic susceptibility: both approaches give identical results (no displaced charges, no singular thermodynamic limit)

Full QCD

- full lattice QCD simulations for ξ_{Weldon} are possible (Taylor expansion in E in finite volume)
- electric and magnetic susceptibilities (comparison to perturbation theory at high T)



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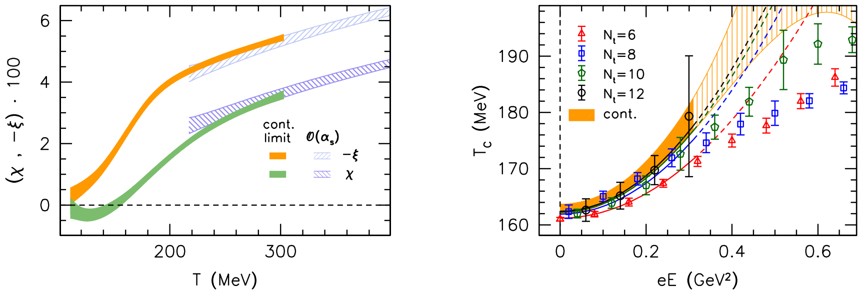
- electric and magnetic susceptibilities (comparison to perturbation theory at high T)
- phase diagram to $\mathcal{O}(E^2)$ via expansion of Polyakov loop Endrődi, Markó '23



Summary

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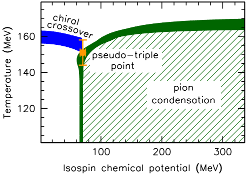
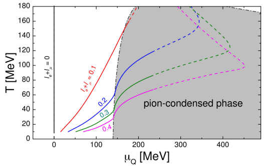
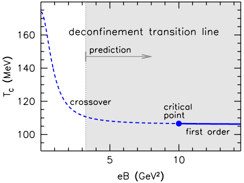
- $T - \mu_I$ phase diagram and pion condensation



- cosmic trajectory may enter pion condensed phase



- $T - B$ phase diagram and the critical point



- background electric fields and local charge distributions