QCD matter in extreme conditions

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BIELEFELD

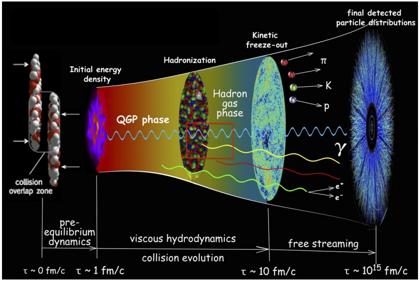
CRC–TR 211
Strong-interaction matter
under extreme conditions

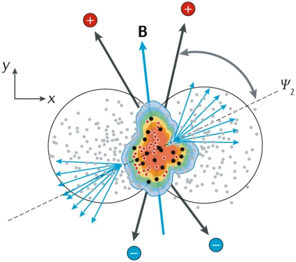
KHuK Annual Meeting
Bad Honnef, December 9, 2022

Introduction

Quarks and gluons in extreme conditions

- heavy ion collisions $T \lesssim 10^{12}\,^{\circ}C = 200\ \mathrm{MeV}$, $n \lesssim 0.12\ \mathrm{fm}^{-3}$
  $B \lesssim 10^{19}\ \mathrm{G} = 0.3\ \mathrm{GeV}^2/e$





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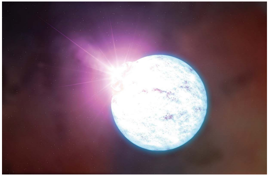
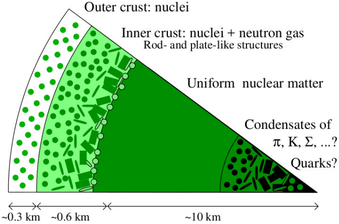
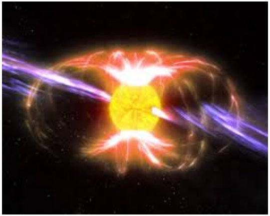
- neutron stars $T \lesssim 1$ keV, $n \lesssim 2$ fm^{-3}
  magnetars $B \lesssim 10^{15}$ G







🔗 Lattimer, Nature Astronomy 2019

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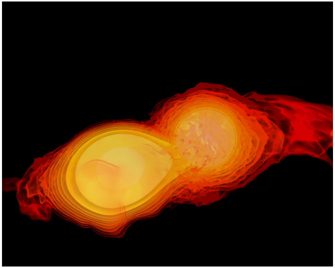

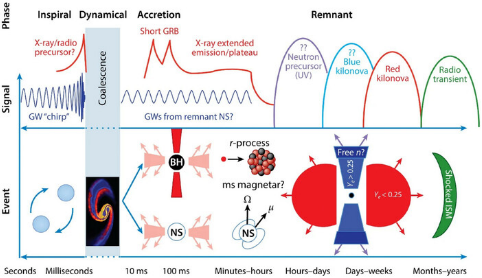

astro.uni-frankfurt.de/rezzolla

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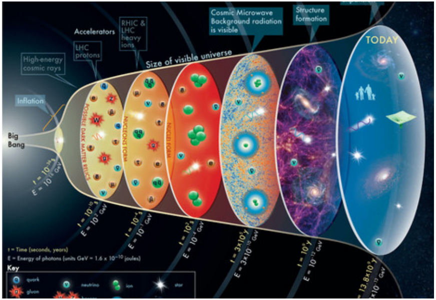
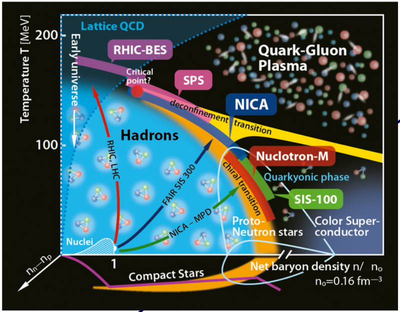
- eary universe, QCD epoch $T \lesssim 200$ MeV
  standard scenario: $n \approx 0$

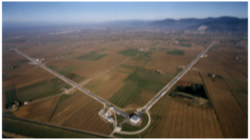


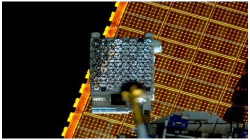
Major experimental and observational campaigns



Observational astronomy

Heavy ion collisions





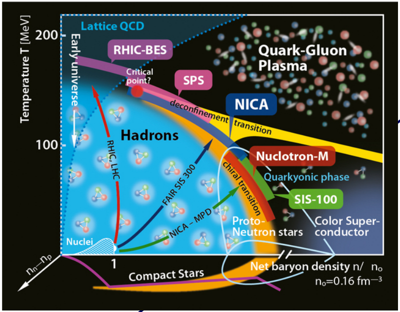
Major experimental and observational campaigns

J-PARC (Japan)

NICA (Russia)

FAIR (Germany)

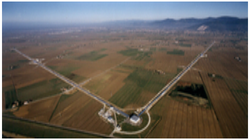
RHIC (USA)

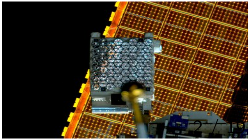


LHC (CERN)

Observational astronomy

Heavy ion collisions





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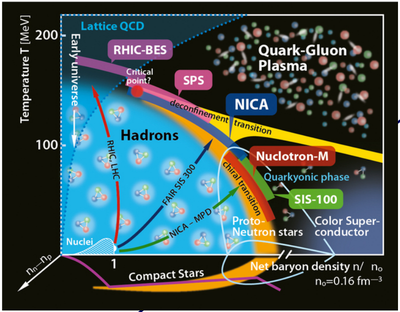
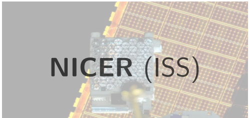


LHC (CERN)

SKA (AU, RSA)

Heavy ion collisions

(Adv.) **VIRGO** (IT-FR)



NICER (ISS)

QCD phase diagram(s)

Phase diagram

- control parameters: T, $n \leftrightarrow \mu$, B

  $\mu_{\{u,d,s\}}$ / $\mu_{\{B,Q,S\}}$ / $\mu_{\{B,I,S\}}$

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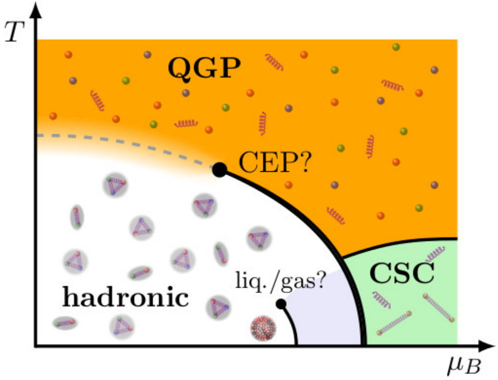

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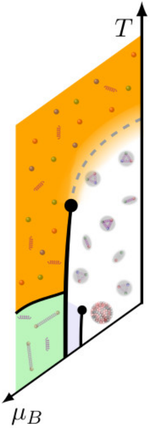

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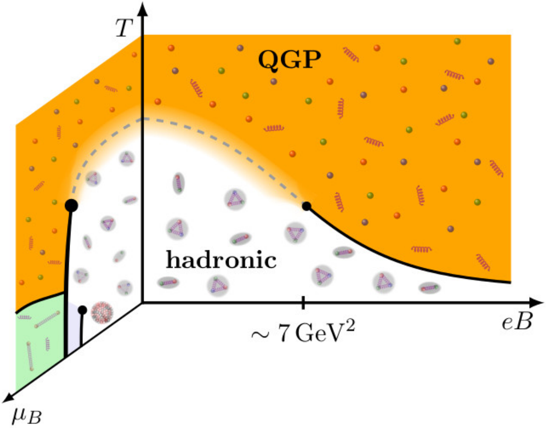

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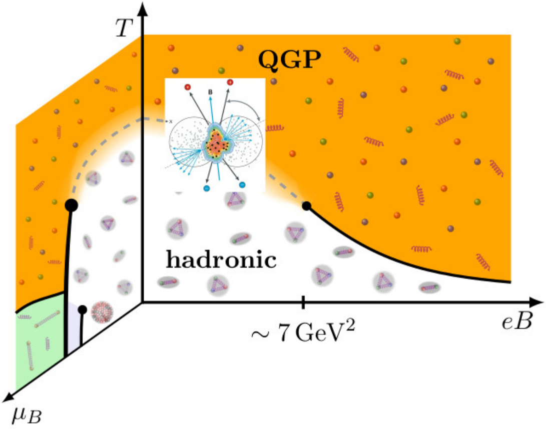

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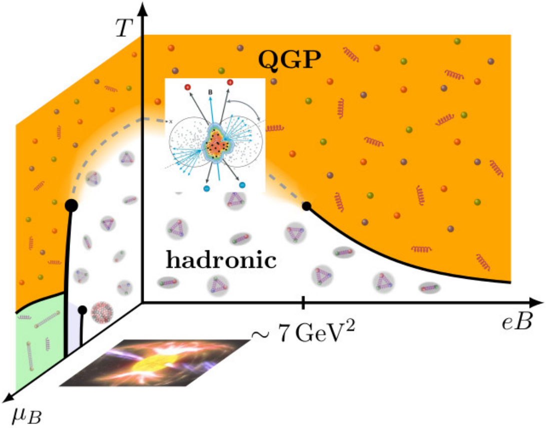

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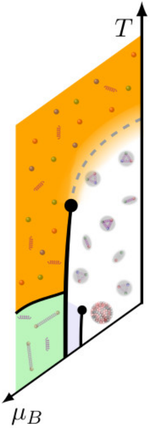

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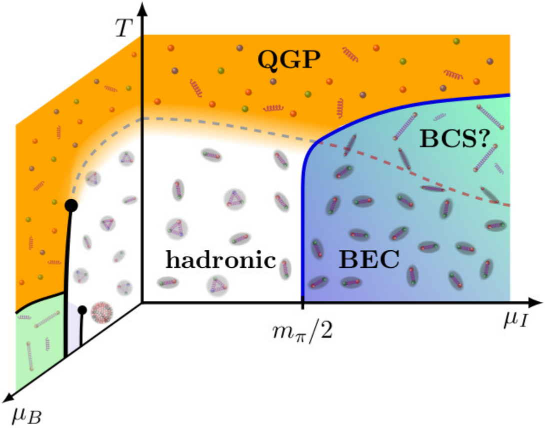

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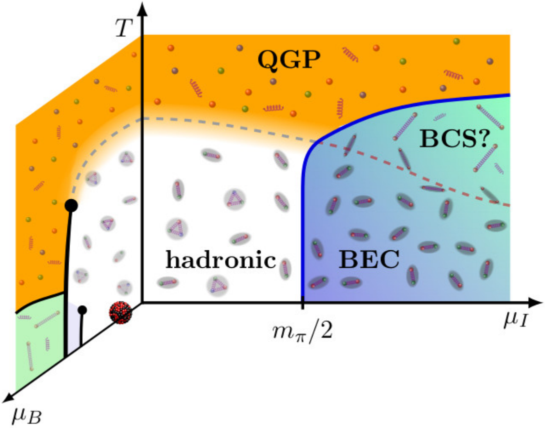

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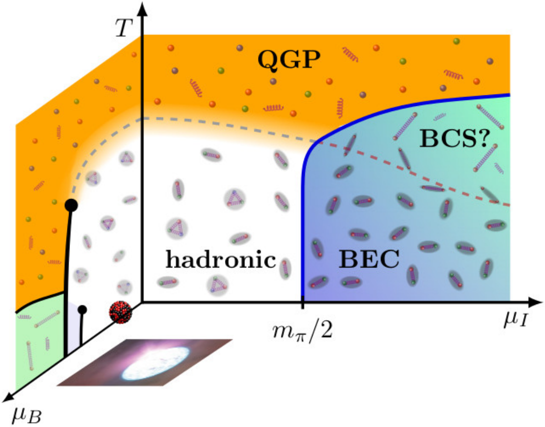

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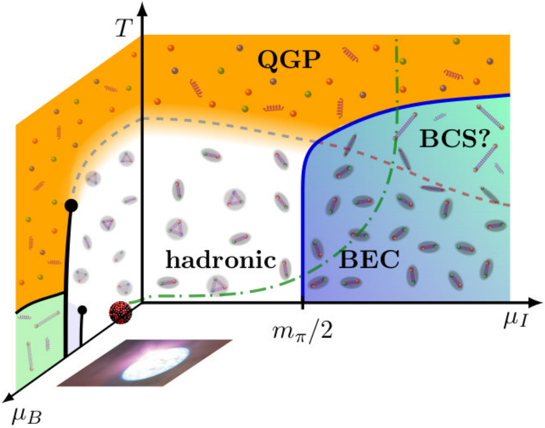


Methods to study QCD thermodynamics

Lattice simulations

- Euclidean QCD path integral over gauge field $\mathcal{A}$

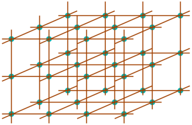
$$\mathcal{Z} = \int \mathcal{D}\mathcal{A}\, e^{-S_g[\mathcal{A}]}\, \det[\not{D}[\mathcal{A}] + m]$$

- Monte-Carlo simulations need: $\det[\not{D} + m] \in \mathbb{R}^+$

$$\text{need } \Gamma: \qquad \Gamma \not{D}\, \Gamma^\dagger = \not{D}^\dagger, \quad \Gamma^\dagger \Gamma = 1$$

- $\exists\, \Gamma$ ✓

  $B,\ \mu_I,\ i\mu_B,\ iE$



continuum limit to recover full theory

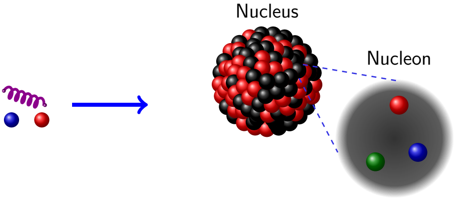
- $\nexists\, \Gamma$ × complex action (sign) problem

  $\mu_B,\ E$

Functional renormalization group

- renormalization group flow from UV to IR

  🔗 Kadanoff '66 🔗 Wilson '71
- for QCD: from quarks and gluons to hadrons and nuclei



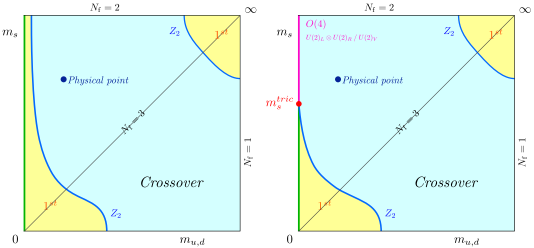
  via successive integration of high-momentum modes

  Wetterich equation 🔗 Wetterich '92
- exact flow equation, access to complete phase diagram ✓
- requires approximations (truncations, Ansätze) to solve ×

Thermodynamics at $\mu_B = 0$

Chiral limit at zero density

- transition at physical quark masses is a crossover
  Aoki et al. '06 Bhattacharya et al. '14



- chiral limit: expect $1^{\mathrm{st}}_{N_f=3}$ and $1^{\mathrm{st}}_{N_f=2}/2^{\mathrm{nd}}_{N_f=2}$ depending on $\mathrm{U}_A(1)$ restoration Pisarski, Wilczek '84

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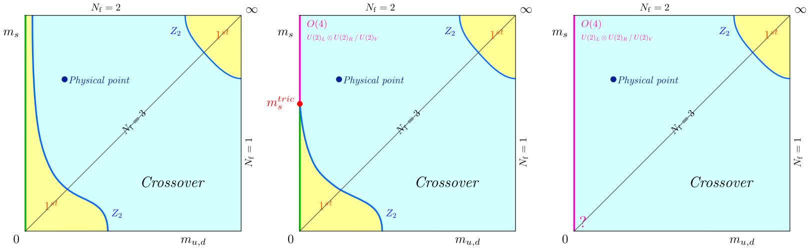

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- lattice exploiting tricritical scaling in N_f: $2^{\mathrm{nd}}_{N_f=2,3}$

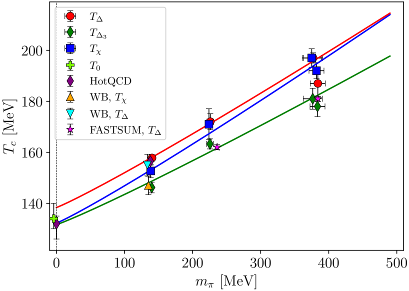
  Cuteri, Philipsen, Sciarra '21

  FRG including 't Hooft coupling $2^{\mathrm{nd}}_{N_f=2}$ Braun et al. '20

Chiral limit at zero density

- scaling of pseudocritical temperature gives:
  $T_c(m_{ud} = 0, m_s^{\mathrm{phys}}) = 132^{+3}_{-6}$ MeV 🔗 Ding et al. '19
  compare 🔗 Kotov et al. '21 🔗 Borsányi et al. '20 🔗 Aarts et al. '20

  $N_f = 3$: $T_c(m_{ud} = m_s = 0) = 98^{+3}_{-6}$ MeV 🔗 Dini et al. '21



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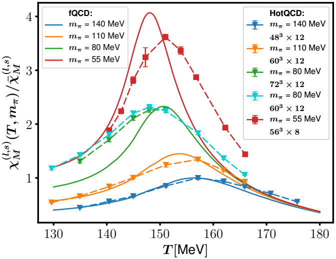
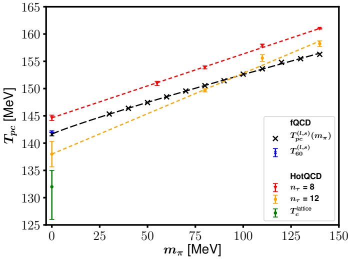
- direct comparison between FRG and lattice

  Braun et al. '20 Ding et al. '19

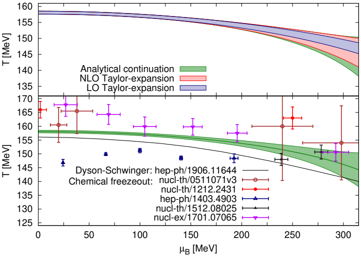


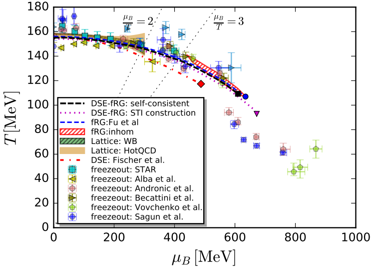


Thermodynamics at $\mu_B > 0$

Phase diagram in the $T - \mu_B$ plane

- analytical continuation of lattice results at $i\mu_B > 0$ consistency with Taylor expansion 🔗 Borsányi et al. '20
- functional methods prefer critical endpoint
- FRG: 🔗 Fu, Pawlowski, Rennecke '20 🔗 Gao, Pawlowski '20 🔗 Otto, Busch, Schaefer '22
  DSE: including meson backcoupling effects 🔗 Gunkel, Fischer '21





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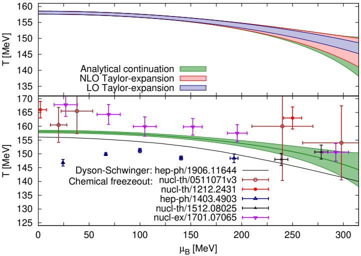
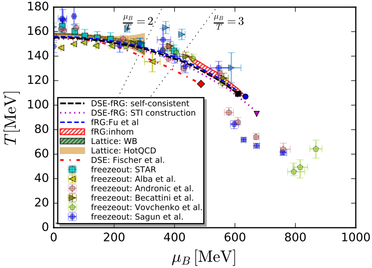




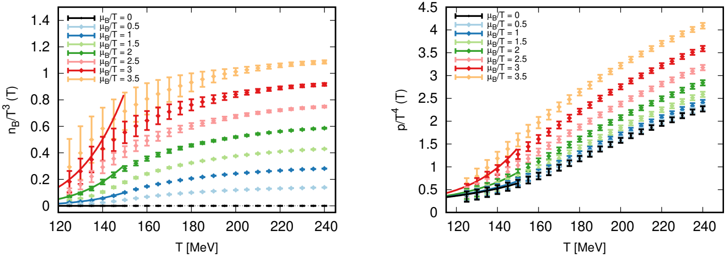
- inhomogeneous instability at large μ_B?

Equation of state

- combining Taylor expansion in μ_B and shift in T

$$\mathcal{O}(T, \mu_B) \approx \mathcal{O}(T - \kappa\mu_B^2, 0)$$

- primary observable: baryon density



🔗 Borsányi et al. '21

- at zero strangeness density, relevant for HIC 🔗 Borsányi et al. '22

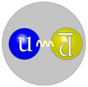
Further results

- alternative resummation schemes 🔗 Mondal et al. '21
- imaginary chemical potentials and Roberge-Weiss phase transitions 🔗 Brandt et al. '22
- QCD transition in the heavy quark/quenched limit 🔗 Borsányi et al. '22
- thermal effects on hadrons, chiral-spin symmetry 🔗 Aarts et al. '20 🔗 Glozman, Philipsen, Pisarski '22
- transport properties - photon emissivity 🔗 Cé et al. '22
- heavy quark diffusion 🔗 Brambilla et al. '22 🔗 Altenkort et al. '22

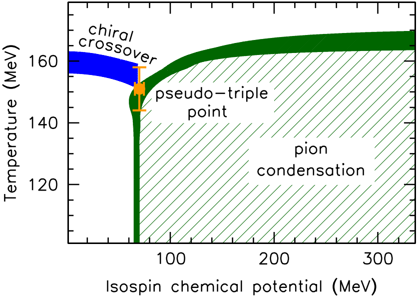
Thermodynamics at $\mu_I > 0$

Phase diagram in the $T - \mu_I$ plane

- $\mu_I = \mu_u - \mu_d$
- phases:
  hadronic, quark-gluon plasma, BEC of charged pions

  🔗 Brandt, Endrődi, Schmalzbauer '17 🔗 Brandt, Endrődi '19



- compares well to χPT at low T

  🔗 Son, Stephanov '01 🔗 Adhikari et al. '20

Equation of state on the lattice

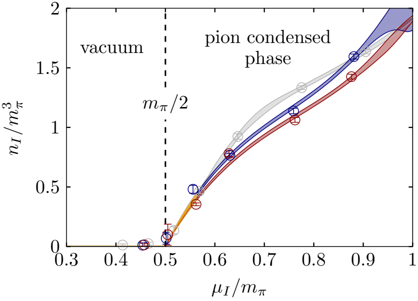
- primary observable: isospin density

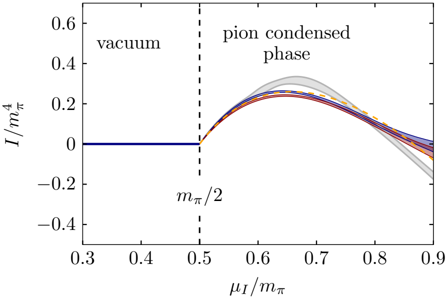
$$n_I = \frac{T}{V}\frac{\partial \log \mathcal{Z}}{\partial \mu_I}, \qquad p(T,\mu_I) - p(T,0) = \int_0^{\mu_I} \mathrm{d}\mu_I'\, n_I(\mu_I')$$

- results at $T \approx 0$

Brandt, Endrődi, Fraga, Hippert, Schaffner-Bielich, Schmalzbauer '18

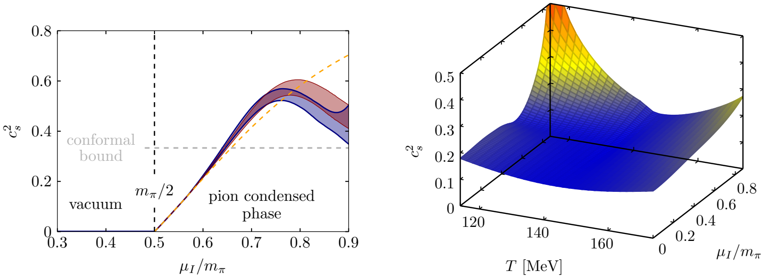
Brandt, Cuteri, Endrődi '22





Equation of state on the lattice

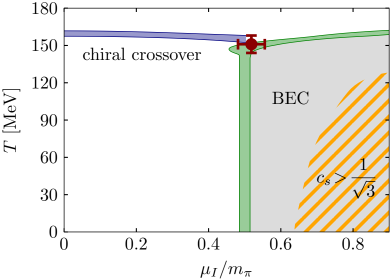
- results at $T \neq 0$ 🔗 Brandt, Cuteri, Endrődi '22
  🔗 Vovchenko, Brandt, Cuteri, Endrődi, Hajkarim, Schaffner-Bielich '20
- interaction measure peak shifts to lower T as μ_I grows
- speed of sound above $1/\sqrt{3}$ at high μ_I and intermediate T



- EoS gets very stiff inside pion condensation phase

Speed of sound

- 'supersonic' region of pion condensate
- first time that $c_s > 1/\sqrt{3}$ found in a first-principles lattice QCD calculation



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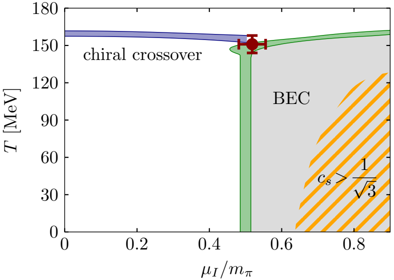
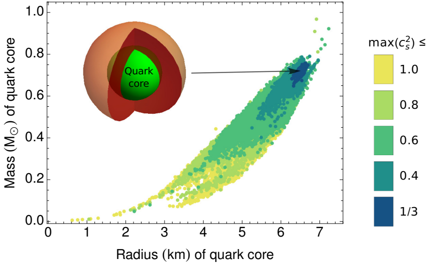
- relevance of c_s for neutron star modeling 🔗 Annala et al. ’19

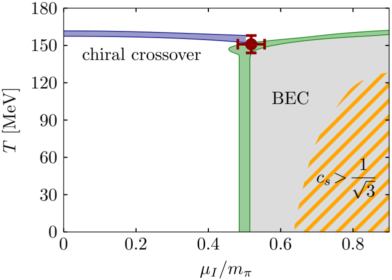


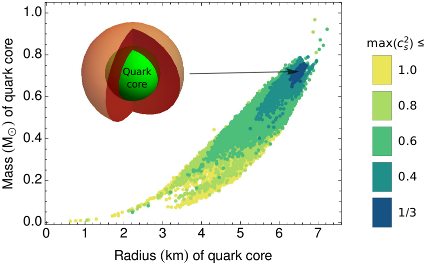


Speed of sound

- ‘supersonic’ region of pion condensate
- first time that $c_s > 1/\sqrt{3}$ found in a first-principles lattice QCD calculation
- relevance of c_s for neutron star modeling 🔗 Annala et al. ’19
- c_s at $\mu_B > 0$ from FRG and χEFT

  🔗 Braun, Schallmo ’22 🔗 Leonhardt et al. ’20

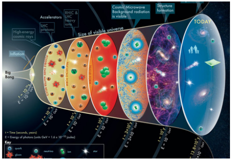




Cosmological implications

Cosmic trajectories

- early Universe



- conservation equations for isentropic expansion

$$\frac{n_B}{s} = b, \quad \frac{n_Q}{s} = 0, \quad \frac{n_{L_\alpha}}{s} = l_\alpha \quad (\alpha \in \{e, \mu, \tau\})$$

- parameters: $T, \quad \mu_B, \quad \mu_Q, \quad \mu_{L_\alpha}$
- experimental constraints 🔗 Planck coll. '15 🔗 Oldengott, Schwarz '17

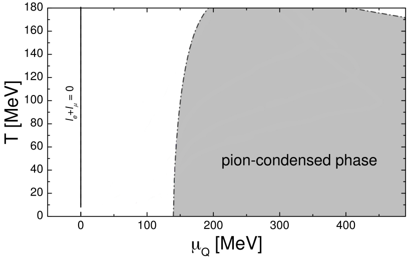
$$b = (8.60 \pm 0.06) \cdot 10^{-11}, \qquad \boxed{|l_e + l_\mu + l_\tau| < 0.012}$$

(the individual l_α may have opposite signs)

- $n_Q = 0$ with $l_e > 0$ allows equilibrium of e^-, ν_e, π^+

Cosmic trajectories

- cosmic trajectory $T(\mu_Q)$ is solved for
- standard scenario ($l_\alpha = 0$): $\mu_Q = 0$ for all T



🔗 Vovchenko, Brandt, Cuteri, Endrődi, Hajkarim, Schaffner-Bielich ’20

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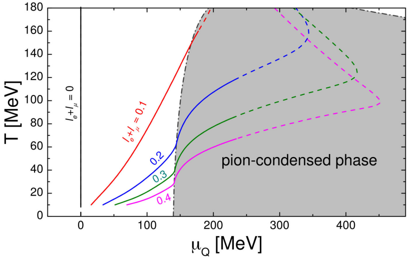

Vovchenko, Brandt, Cuteri, Endrődi, Hajkarim, Schaffner-Bielich '20

- condition for pion condensation to occur:

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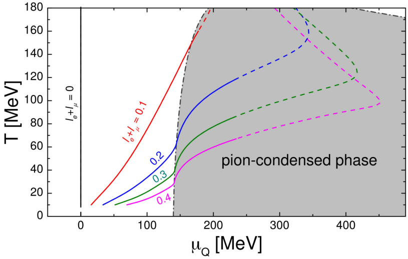

Vovchenko, Brandt, Cuteri, Endrődi, Hajkarim, Schaffner-Bielich '20

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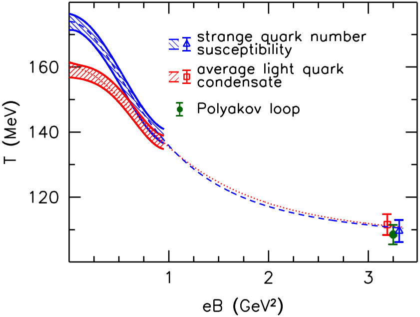
- enhanced primordial grav. waves (SKA)

Thermodynamics at $B > 0$

Phase diagram and critical point

- physical m_π, staggered quarks, continuum limit

  🔗 Bali, Bruckmann, Endrődi, Fodor, Katz et al. '11   🔗 '12   🔗 Endrődi '15



Phase diagram and critical point

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- transition strengthens $\Rightarrow$ critical point at $eB_c \approx 10(2)$ GeV^2
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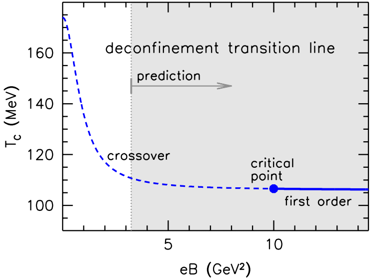

Phase diagram and critical point

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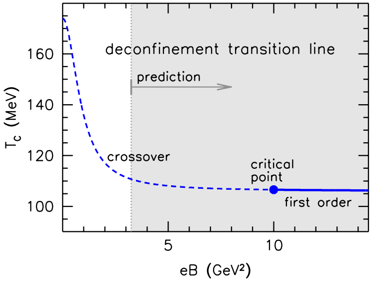
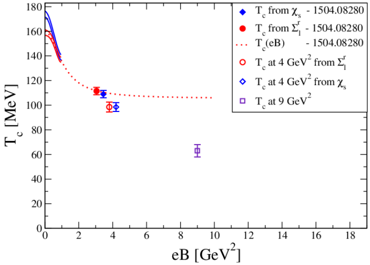
  🔗 Endrődi '15
- simulating up to $eB \approx 9$ GeV^2 $\Rightarrow$ 4 GeV$^2 < eB_c <$ 9 GeV^2
  🔗 D'Elia, Maio, Sanfilippo, Stanzione '21





Further results on magnetic fields

- fluctuations of conserved charges at $B > 0$, $T > 0$

  🔗 Ding et al. '21

- anomalous transport phenomena at $B > 0$

  🔗 Astrakhantsev et al. '20 🔗 Brandt, Cuteri, Endrődi, Garnacho, Markó '22

- magnetic susceptibility

  🔗 Buividovich, Smith, von Smekal '21

beyond homogeneous magnetic fields

- inhomogeneous magnetic fields

  🔗 Valois et al. '21

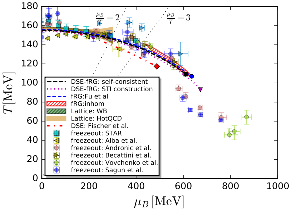
- electric background fields

  🔗 Endrődi, Markó '22

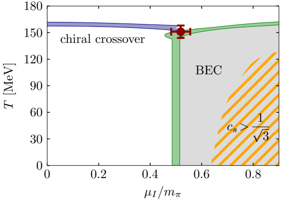
Summary

Summary

- closing down on the $\mu_B > 0$ critical endpoint:
  lattice, FRG, DSE



- $T - \mu_I$ phase diagram
  (supersonic) pion condensation
  possible impact on cosmology



- $T - B$ phase diagram
  and the critical point