Staggered anomalous transport

Gergely Endrődi

University of Bielefeld

in collaboration with Eduardo Garnacho, Bastian Brandt,
Francesca Cuteri, Gergely Markó

Novel 2023
MITP Mainz, March 9, 2023

Outline

- introduction: anomalous transport phenomena
- implementation for staggered quarks
- results and discussion

Introduction: anomalous transport

Anomalous transport

- usual transport:
  vector current due to electric field

$$\langle \vec{J} \rangle = \sigma \cdot \vec{E}$$

- chiral magnetic effect (CME): Fukushima, Kharzeev, Warringa '08
  vector current due to chirality and magnetic field

$$\langle \vec{J} \rangle = \sigma_{\mathrm{CME}} \cdot \vec{B}$$

- chiral separation effect (CSE):
  axial current due to baryon number and magnetic field

$$\langle \vec{J}_5 \rangle = \sigma_{\mathrm{CSE}} \cdot \vec{B}$$

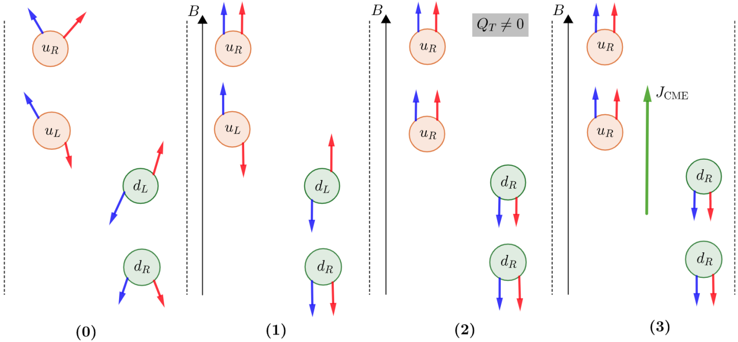
Phenomenological and theoretical relevance

- experimental observation of CME in condensed matter systems 🔗 Li, Kharzeev, Zhan et al '14
- experimental searches for CME and related observables in heavy-ion collisions 🔗 STAR collaboration '21
- serves as indirect way to probe topological fluctuations and CP-odd domains in heavy-ion collisions
- recent review: 🔗 Kharzeev, Liao, Voloshin, Wang '16

- CME and CSE are observables sensitive to chirality and thus optimal to test fermionic lattice discretizations

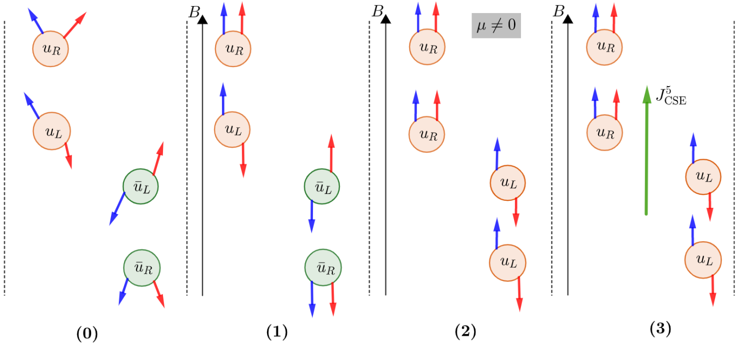
General (handwaving) argument

- spin, momentum CME



General (handwaving) argument

- spin, momentum CSE



More precise argument

- Lorentz covariance, P symmetry for CSE ($\langle \vec{J}_5 \rangle \propto \vec{B} \parallel \vec{e}_3$)

$$\langle J_{5\alpha} \rangle \propto \qquad\qquad F_{\gamma\delta}$$

More precise argument

- Lorentz covariance, P symmetry for CSE ($\langle \vec{J_5} \rangle \propto \vec{B} \parallel \vec{e}_3$)

$$\langle J_{5\alpha} \rangle \propto \epsilon_{\alpha\beta\gamma\delta} A_\beta \cdot F_{\gamma\delta}$$

involves a baryon chemical potential

$$\langle J_{53} \rangle = C_{\mathrm{CSE}} \cdot \mu \cdot B$$

with $\mu \cdot J_4$ in the action

More precise argument

- Lorentz covariance, P symmetry for CSE ($\langle \vec{J}_5 \rangle \propto \vec{B} \parallel \vec{e}_3$)

  $$\langle J_{5\alpha} \rangle \propto \epsilon_{\alpha\beta\gamma\delta} \, A_\beta \cdot F_{\gamma\delta}$$

  involves a baryon chemical potential

  $$\langle J_{53} \rangle = C_{\mathrm{CSE}} \cdot \mu \cdot B$$

  with $\mu \cdot J_4$ in the action
- Lorentz covariance, P symmetry for CME ($\langle \vec{J} \rangle \propto \vec{B} \parallel \vec{e}_3$)

  $$\langle J_\alpha \rangle \propto \epsilon_{\alpha\beta\gamma\delta} \, A_{5\beta} \cdot F_{\gamma\delta}$$

  involves a 'chiral' chemical potential

  $$\langle J_3 \rangle = C_{\mathrm{CME}} \cdot \mu_5 \cdot B$$

  with $\mu_5 \cdot J_{54}$ in the action

Ward identities

- we consider two degenerate light quark flavors (mass m)
- vector Ward identity

$$\partial_\alpha J_\alpha = 0$$

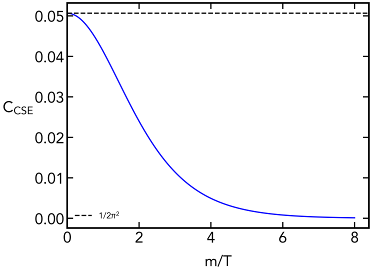
- axial Ward identity

$$\partial_\alpha J_{5\alpha} = 2m\, P_5 + q_{\mathrm{top}}$$

thus μ_5 is no true chemical potential but merely an external parameter

Analytical results in the free case

- CSE for non-interacting fermions (no gluons, just magnetic field) Metlitski, Zhitnitsky '05



- CME for non-interacting fermions

  Fukushima, Kharzeev, Warringa '08 Sheng, Rischke, Vasak, Wang

$$C_{\mathrm{CME}} = \frac{1}{2\pi^2}$$

Staggered implementation

Staggered vector current

- consider standard staggered Dirac operator (c.f. Hölbling)

$$M = \not{D} + m = \frac{1}{2} \sum_{\alpha} \eta_{\alpha} \left[V_{\alpha} - V_{\alpha}^{\dagger} \right] + m$$

Staggered vector current

- consider standard staggered Dirac operator (c.f. Hölbling)

$$M = \not{D} + m = \frac{1}{2} \sum_{\alpha} \eta_\alpha \left[V_\alpha - V_\alpha^\dagger \right] + m$$

- vector operator Golterman, Smit '84 Patel, Sharpe '92

$$\Gamma_\alpha = \frac{1}{2} \eta_\alpha \left[V_\alpha + V_\alpha^\dagger \right]$$

- vector current

$$\langle J_3 \rangle = \left\langle \mathrm{tr} \left(\frac{\Gamma_3}{M} \right) \right\rangle$$

satisfies VWI i.e. it is conserved Sharatchandra, Thun, Weisz '81

Staggered vector current

- consider standard staggered Dirac operator (c.f. Hölbling)

$$M = \not{D} + m = \frac{1}{2} \sum_{\alpha} \eta_\alpha \left[V_\alpha - V_\alpha^\dagger \right] + m$$

- vector operator Golterman, Smit '84 Patel, Sharpe '92

$$\Gamma_\alpha = \frac{1}{2} \eta_\alpha \left[V_\alpha + V_\alpha^\dagger \right]$$

- vector current

$$\langle J_3 \rangle = \left\langle \mathrm{tr} \left(\frac{\Gamma_3}{M} \right) \right\rangle = \left. \frac{\partial \log \mathcal{Z}}{\partial \mu_3} \right|_{\mu_3 = 0}$$

  satisfies VWI i.e. it is conserved Sharatchandra, Thun, Weisz '81

- 'spatial chemical potential' enters $\not{D}$ as Hasenfratz, Karsch '83

$$\frac{1}{2} \eta_3 \left[V_3 e^{\mu_3} - V_3^\dagger e^{-\mu_3} \right]$$

Staggered axial current

- axial vector operator

$$\Gamma_{5\alpha} = \frac{1}{3!} \sum_{\beta\gamma\delta} \epsilon_{\alpha\beta\gamma\delta} \Gamma_\beta \Gamma_\gamma \Gamma_\delta$$

- axial vector current

$$\langle J_{53} \rangle = \left\langle \mathrm{tr}\left(\frac{\Gamma_1 \Gamma_2 \Gamma_4}{M} \right) \right\rangle + \text{perm.}$$

satisfies AWI Sharatchandra, Thun, Weisz '81

- pseudoscalar operator

$$\Gamma_5 = \frac{1}{4!} \sum_{\alpha\beta\gamma\delta} \epsilon_{\alpha\beta\gamma\delta} \Gamma_\alpha \Gamma_\beta \Gamma_\gamma \Gamma_\delta$$

Staggered axial current

- axial vector operator

$$\Gamma_{5\alpha} = \frac{1}{3!} \sum_{\beta\gamma\delta} \epsilon_{\alpha\beta\gamma\delta} \Gamma_\beta \Gamma_\gamma \Gamma_\delta$$

- axial vector current

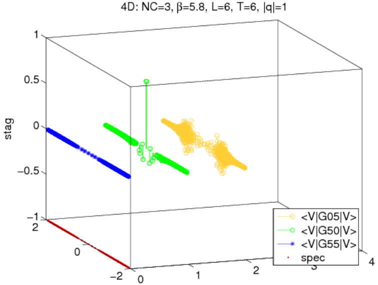
$$\langle J_{53} \rangle = \left\langle \mathrm{tr}\left(\frac{\Gamma_1 \Gamma_2 \Gamma_4}{M} \right) \right\rangle + \text{perm.}$$

satisfies AWI Sharatchandra, Thun, Weisz '81

- pseudoscalar operator sensitive to topology (c.f. Dürr)

$$\Gamma_5 = \frac{1}{4!} \sum_{\alpha\beta\gamma\delta} \epsilon_{\alpha\beta\gamma\delta} \Gamma_\alpha \Gamma_\beta \Gamma_\gamma \Gamma_\delta$$

Dürr '13



Chiral separation effect

- remember $\langle J_{53} \rangle = C_{\mathrm{CSE}} \cdot \mu \cdot B$ and $J_{53} \propto \mathrm{tr}\left(\frac{\Gamma_1 \Gamma_2 \Gamma_4}{M}\right)$

Chiral separation effect

- remember $\langle J_{53} \rangle = C_{\mathrm{CSE}} \cdot \mu \cdot B$ and $J_{53} \propto \mathrm{tr}\left(\frac{\Gamma_1 \Gamma_2 \Gamma_4}{M}\right)$
- Taylor expansion

$$C_{\mathrm{CSE}} \cdot B = \left.\frac{\partial \langle J_{53} \rangle}{\partial \mu}\right|_{\mu=0}$$

B-derivative numerically (flux quantization, cf. Hands)

Chiral separation effect

- remember $\langle J_{53} \rangle = C_{\mathrm{CSE}} \cdot \mu \cdot B$ and $J_{53} \propto \mathrm{tr}\left(\frac{\Gamma_1 \Gamma_2 \Gamma_4}{M}\right)$
- Taylor expansion

$$C_{\mathrm{CSE}} \cdot B = \left.\frac{\partial \langle J_{53} \rangle}{\partial \mu}\right|_{\mu=0}$$

B-derivative numerically (flux quantization, cf. Hands)

- Taylor coefficient

$C_{\mathrm{CSE}} \cdot B = \langle J_{53} J_4 \rangle$

Chiral separation effect

- remember $\langle J_{53} \rangle = C_{\rm CSE} \cdot \mu \cdot B$ and $J_{53} \propto \mathrm{tr}\left(\frac{\Gamma_1\Gamma_2\Gamma_4}{M}\right)$
- Taylor expansion

$$C_{\rm CSE} \cdot B = \left.\frac{\partial \langle J_{53} \rangle}{\partial \mu}\right|_{\mu=0}$$

B-derivative numerically (flux quantization, cf. Hands)

- Taylor coefficient

$$C_{\rm CSE} \cdot B = \langle J_{53} J_4 \rangle$$

$$= \left\langle \mathrm{tr}\left(\frac{\Gamma_1\Gamma_2\Gamma_4}{M}\right) \mathrm{tr}\left(\frac{\Gamma_4}{M}\right) - \mathrm{tr}\left(\frac{\Gamma_1\Gamma_2\Gamma_4}{M}\frac{\Gamma_4}{M}\right) \right\rangle$$

Chiral separation effect

- remember $\langle J_{53} \rangle = C_{\mathrm{CSE}} \cdot \mu \cdot B$ and $J_{53} \propto \mathrm{tr}\left(\frac{\Gamma_1\Gamma_2\Gamma_4}{M}\right)$
- Taylor expansion

$$C_{\mathrm{CSE}} \cdot B = \left.\frac{\partial \langle J_{53} \rangle}{\partial \mu}\right|_{\mu=0}$$

  B-derivative numerically (flux quantization, cf. Hands)

- Taylor coefficient

$$C_{\mathrm{CSE}} \cdot B = \langle J_{53} J_4 \rangle + \left\langle \frac{\partial J_{53}}{\partial \mu} \right\rangle$$

$$= \left\langle \mathrm{tr}\left(\frac{\Gamma_1\Gamma_2\Gamma_4}{M}\right) \mathrm{tr}\left(\frac{\Gamma_4}{M}\right) - \mathrm{tr}\left(\frac{\Gamma_1\Gamma_2\Gamma_4}{M}\frac{\Gamma_4}{M}\right) \right\rangle + \left\langle \mathrm{tr}\left(\frac{1}{M}\frac{\partial \Gamma_1\Gamma_2\Gamma_4}{\partial \mu}\right) \right\rangle$$

Chiral separation effect

- remember $\langle J_{53} \rangle = C_{\mathrm{CSE}} \cdot \mu \cdot B$ and $J_{53} \propto \mathrm{tr}\left(\frac{\Gamma_1\Gamma_2\Gamma_4}{M}\right)$
- Taylor expansion

$$C_{\mathrm{CSE}} \cdot B = \left.\frac{\partial \langle J_{53} \rangle}{\partial \mu}\right|_{\mu=0}$$

  B-derivative numerically (flux quantization, cf. Hands)

- Taylor coefficient

$$\begin{aligned} C_{\mathrm{CSE}} \cdot B &= \langle J_{53} J_4 \rangle + \left\langle \frac{\partial J_{53}}{\partial \mu} \right\rangle \\ &= \left\langle \mathrm{tr}\left(\frac{\Gamma_1\Gamma_2\Gamma_4}{M}\right) \mathrm{tr}\left(\frac{\Gamma_4}{M}\right) - \mathrm{tr}\left(\frac{\Gamma_1\Gamma_2\Gamma_4}{M}\frac{\Gamma_4}{M}\right)\right\rangle + \left\langle \mathrm{tr}\left(\frac{1}{M}\frac{\partial \Gamma_1\Gamma_2\Gamma_4}{\partial \mu}\right)\right\rangle \\ &= \left\langle \mathrm{tr}\left(\frac{\Gamma_1\Gamma_2\Gamma_4}{M}\right) \mathrm{tr}\left(\frac{\Gamma_4}{M}\right) - \mathrm{tr}\left(\frac{\Gamma_1\Gamma_2\Gamma_4}{M}\frac{\Gamma_4}{M}\right)\right\rangle + \left\langle \mathrm{tr}\left(\frac{\Gamma_1\Gamma_2}{M}\frac{\partial \Gamma_4}{\partial \mu}\right)\right\rangle \end{aligned}$$

disconnected — connected — tadpole

Chiral magnetic effect

- analogously to CSE, here we need

$$C_{\mathrm{CME}} \cdot B = \left. \frac{\partial \langle J_3 \rangle}{\partial \mu_5} \right|_{\mu_5 = 0}$$

Chiral magnetic effect

- analogously to CSE, here we need

$$C_{\mathrm{CME}} \cdot B = \left. \frac{\partial \langle J_3 \rangle}{\partial \mu_5} \right|_{\mu_5=0}$$

- chiral chemical potential enters $\not{D}$ as

$$\frac{1}{2}\,\eta_4 \left[U_4 e^{\mu_5 \Gamma_5} - U_4^\dagger e^{-\mu_5 \Gamma_5} \right]$$

Chiral magnetic effect

- analogously to CSE, here we need

$$C_{\mathrm{CME}} \cdot B = \left.\frac{\partial \langle J_3 \rangle}{\partial \mu_5}\right|_{\mu_5=0}$$

- chiral chemical potential enters $\not{D}$ as

$$\frac{1}{2}\,\eta_4 \left[U_4 e^{\mu_5 \Gamma_5} - U_4^\dagger e^{-\mu_5 \Gamma_5} \right]$$

- swap derivatives

$$C_{\mathrm{CME}} \cdot B = \left.\frac{\partial \langle J_3 \rangle}{\partial \mu_5}\right|_{\mu_5=0} = \left.\frac{\partial \langle J_{54} \rangle}{\partial \mu_3}\right|_{\mu_3=0}$$

Chiral magnetic effect

- analogously to CSE, here we need

$$C_{\mathrm{CME}} \cdot B = \left.\frac{\partial \langle J_3 \rangle}{\partial \mu_5}\right|_{\mu_5=0}$$

- chiral chemical potential enters $\not{D}$ as

$$\frac{1}{2}\,\eta_4 \left[U_4 e^{\mu_5 \Gamma_5} - U_4^\dagger e^{-\mu_5 \Gamma_5} \right]$$

- swap derivatives

$$C_{\mathrm{CME}} \cdot B = \left.\frac{\partial \langle J_3 \rangle}{\partial \mu_5}\right|_{\mu_5=0} = \left.\frac{\partial \langle J_{54} \rangle}{\partial \mu_3}\right|_{\mu_3=0}$$

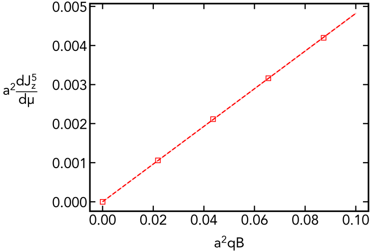
- again leads to disconnected+connected+tadpole

$$C_{\mathrm{CME}} \cdot B = \langle J_{54} J_3 \rangle + \left\langle \frac{\partial J_{54}}{\partial \mu_3} \right\rangle$$

Results

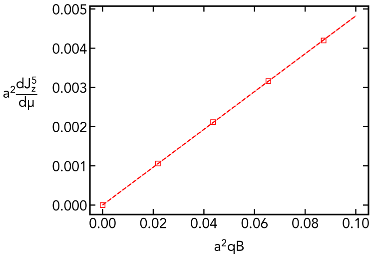
Chiral separation effect: free case

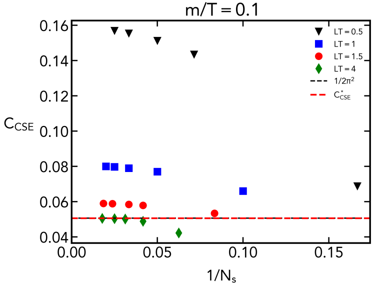
- non-interacting fermions (no gluons, just B)



Chiral separation effect: free case

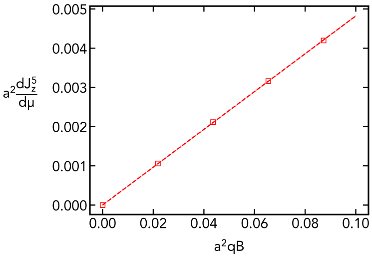
- non-interacting fermions (no gluons, just B)

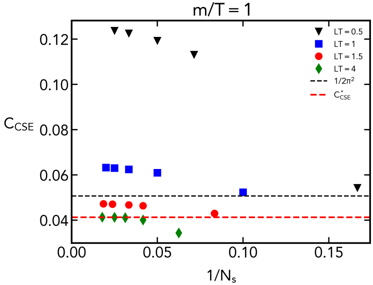




Chiral separation effect: free case

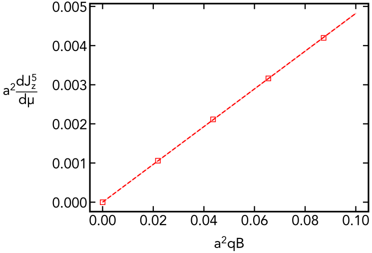
- non-interacting fermions (no gluons, just B)

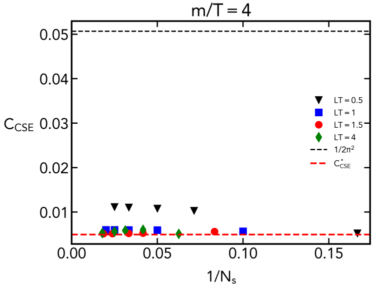




Chiral separation effect: free case

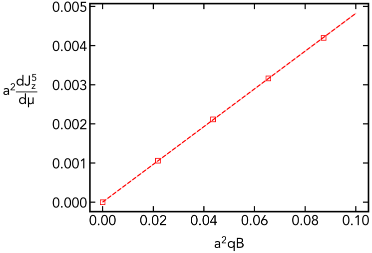
- non-interacting fermions (no gluons, just B)

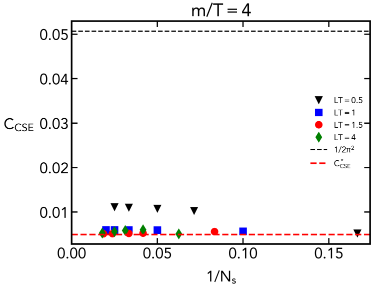




Chiral separation effect: free case

- non-interacting fermions (no gluons, just B)





- full agreement with analytical result ✓

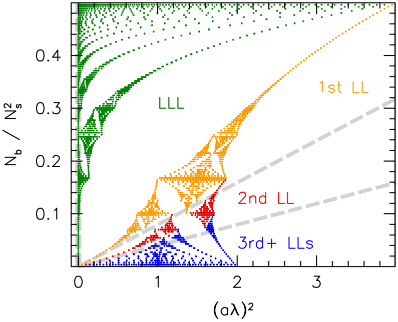
Detour: butterflies

- writing traces in the basis of $\not{D}^2 = \not{D}_{12}^2 + \not{\partial}_3^2 + \not{\partial}_4^2 \;\to\; \lambda_n^2, |\varphi_n\rangle$

$$C_{\rm CSE}\cdot B = -\sum_{n,k} \frac{\langle\varphi_n|\Gamma_{53}M^\dagger|\varphi_k\rangle}{\lambda_n^2}\frac{\langle\varphi_k|\Gamma_4 M^\dagger|\varphi_n\rangle}{\lambda_k^2} + \sum_n \frac{\langle\phi_n|\Gamma_1\Gamma_2\frac{\partial\Gamma_4}{\partial\mu}M^\dagger|\varphi_n\rangle}{\lambda_n^2}$$

- Hofstadter's butterfly (c.f. Hands)

Hofstadter '76 Endrődi '14



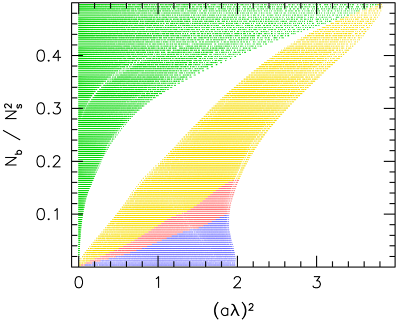
Detour: butterflies

- writing traces in the basis of $\not{D}^2 = \not{D}_{12}^2 + \not{\partial}_3^2 + \not{\partial}_4^2 \;\rightarrow\; \lambda_n^2, |\varphi_n\rangle$

$$C_{\mathrm{CSE}} \cdot B = -\sum_{n,k} \frac{\langle\varphi_n|\Gamma_{53} M^\dagger|\varphi_k\rangle}{\lambda_n^2} \frac{\langle\varphi_k|\Gamma_4 M^\dagger|\varphi_n\rangle}{\lambda_k^2} + \sum_n \frac{\langle\phi_n|\Gamma_1\Gamma_2 \frac{\partial\Gamma_4}{\partial\mu} M^\dagger|\varphi_n\rangle}{\lambda_n^2}$$

- Hofstadter's butterfly (c.f. Hands)

Hofstadter '76 Endrődi '14



on QCD configurations Bruckmann, Endrődi, Giordano et al. '17

Chiral separation effect: full QCD

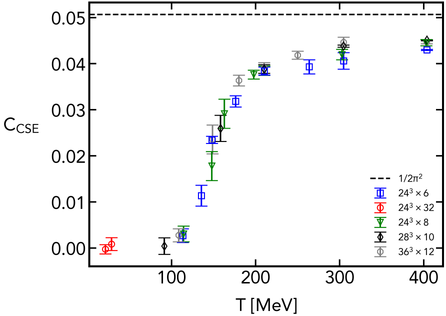
- $N_f = 1 + 1 + 1$ flavors of dynamical (rooted) staggered quarks
- physical quark masses $m_u = m_d$, m_s
- physical electric charges $q_u = -2q_d = -2q_s = 2e/3$
- $N_t = 6, 8, 10, 12$ to approach continuum limit

Garnacho, Brandt, Cuteri, Endrődi, Markó '22

Chiral separation effect: full QCD

- $N_f = 1+1+1$ flavors of dynamical (rooted) staggered quarks
- physical quark masses $m_u = m_d$, m_s
- physical electric charges $q_u = -2q_d = -2q_s = 2e/3$
- $N_t = 6$, 8, 10, 12 to approach continuum limit

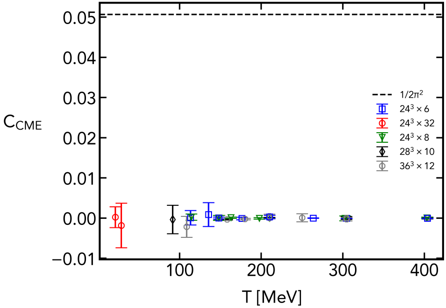
  Garnacho, Brandt, Cuteri, Endrődi, Markó '22



- compare: overlap on quenched Puhr, Buividovich '17
  Wilson/DW on staggered $\mathrm{SU}(2)$ Buividovich, Smith, von Smekal '21

Chiral magnetic effect

- same setup as above
- both for non-interacting fermions and in full QCD, $C_{\rm CME} = 0$



- compare: Wilson quenched and dynamical Yamamoto '11
  free overlap Buividovich '14

Summary

- anomalous transport phenomena involve chirality-sensitive observables
- non-trivial test of fermion discretizations
- CSE: first results in full physical QCD