Magnetic susceptibility of QCD matter

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in collaboration with

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Hot problems of strong interactions
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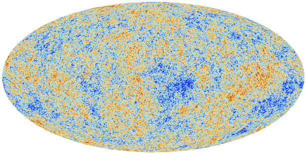
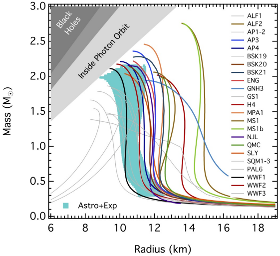
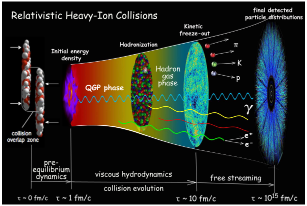
Preface: equation of state

Equation of state of QCD

- equilibrium description of QCD matter: $\epsilon(p)$
  - evolution of the early universe (Friedmann equations)
  - mass-radius relation of neutron stars (TOV equations)
  - evolution of heavy-ion collisions







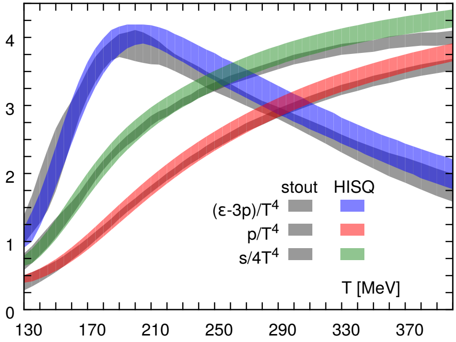
Özel, Freire '16

Shen, Heinz '16

- in each setting, strong magnetic fields may be present
  Kharzeev, Landsteiner, Schmitt, Yee '14

Equation of state of QCD

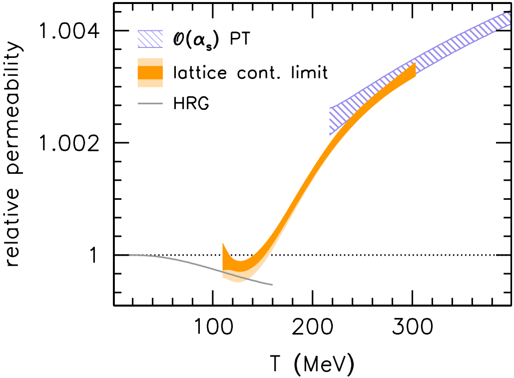
- at zero magnetic field, EoS is encoded in $p(T)$ calculated using lattice QCD simulations



Bazavov et al. '14 Borsányi et al. '13

Permeability

- contribution to EoS for small magnetic fields
  also accessible on the lattice



Bali, Endrődi, Piemonte '20

Outline

- introduction: permeability and magnetic fields in lattice simulations
- new technique: current-current correlators
- connection to HRG model and perturbation theory
- spin- and angular momentum-terms
- summary

Introduction

Susceptibility and permeability

- leading-order dependence of matter free energy density on B

$$\chi = -\left.\frac{\partial^2 f}{\partial (eB)^2}\right|_{B=0}$$

  from this the $\mathcal{O}(B^2)$ equation of state can be reconstructed

- total free energy

$$f^{\rm tot} = -\chi \cdot \frac{(eB)^2}{2} + \frac{B^2}{2} = \frac{B^2}{2\mu}$$

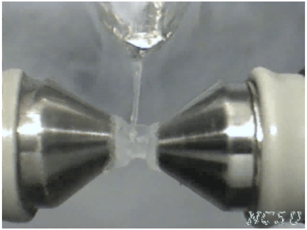
- permeability Landau-Lifschitz Vol 8.

$$\mu = \frac{1}{1 - e^2\chi}$$

- $\mu > 1 \; (\chi > 0)$: paramagnetism
  $\mu < 1 \; (\chi < 0)$: diamagnetism

Paramagnets and diamagnets

- paramagnets: attracted to magnetic field
- diamagnets: repel magnetic field



paramagnet: liquid oxygen

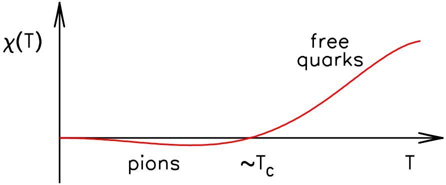
NCSU physics demonstrations

diamagnet: frog

Berry, Geim '10 (Ig Nobel prize)

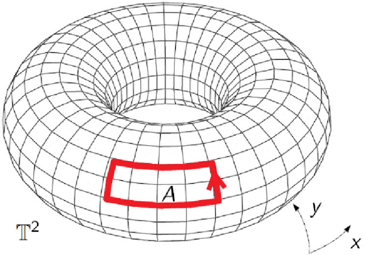
Magnetic susceptibility – expectations

- in the vacuum $\mu = 1$, so $\chi = 0$
- spins align with B, so free quarks are paramagnetic
- orbital angular momentum anti-aligns with B (Lenz's law), so free pions are diamagnetic



On the lattice: flux quantization problem

Magnetic field on the torus



torus $\mathbb{T}^2$
with surface area $L_x L_y$

D'Elia, Negro '11

- phase factor along path: $\varphi_\mathcal{C} = \exp(iq \oint_\mathcal{C} \mathrm{d}x_\mu A_\mu)$
- Stokes:
$$\varphi_\mathcal{C} = \exp(iq \iint_A \mathrm{d}\sigma B) = \exp(iqB \cdot A)$$
but also
$$\varphi_\mathcal{C} = \exp(-iq \iint_{\mathbb{T}^2 - A} \mathrm{d}\sigma B) = \exp(-iqB \cdot (L_x L_y - A))$$
- consistent if 't Hooft '79 Hashimi, Wiese '08
$$\exp(iqBL_x L_y) = 1 \quad \rightarrow \quad qBL_x L_y = 2\pi \cdot N_b, \quad N_b \in \mathbb{Z}$$

Flux quantization

- flux quantization in finite volume

$$eB = \frac{6\pi \cdot N_b}{L_x L_y}, \qquad N_b = 0, 1, \ldots$$

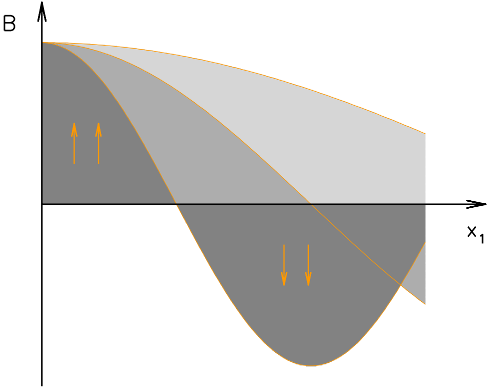
$\Rightarrow \chi$ via differentiation wrt. B is ill-defined

- workarounds:
  - calculate $f(N_b)$ in a sufficiently large volume and differentiate numerically Bonati et al. '13 Bali et al. '14
    ⚡ computationally expensive
  - replace constant B by 'half-half setup' with zero flux, differentiation is allowed Levkova, DeTar '13
    ⚡ introduces large finite size effects
  - relate χ to pressure differences Bali et al. '13
    ⚡ needs anisotropic lattices
  - new method: express χ as an operator in the thermodynamic limit Bali, Endrődi, Piemonte '20

New method: sketch

Oscillatory background fields

- approach constant B via oscillatory fields



$$B(x) = B \cdot \cos(p_1 x_1)$$

Current-current correlator method

- vector potential interacts with current

$$\mathcal{Z} = \int \mathcal{D}U\mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left[S_0 + i\int \mathrm{d}^4x\, A_\mu j_\mu\right], \qquad j_\mu = \sum_f q_f \bar{\psi}\gamma_\mu\psi$$

- susceptibility at finite p_1

$$B(x_1) = B\cdot\cos(p_1x_1), \qquad A_2(x_1) = B\cdot\frac{\sin(p_1x_1)}{p_1}$$

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$$\chi^{(p_1)} = -\left. \frac{\partial^2 f}{\partial (eB)^2} \right|_{B=0} = -\frac{T}{V} \int \mathrm{d}^4 x \, \mathrm{d}^4 y \, \frac{\sin(p_1 x_1) \sin(p_1 y_1)}{p_1^2} \, \langle j_2(x) j_2(y) \rangle$$

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- use trigonometric identities + translational invariance + trick

Current-current correlator method

- oscillatory susceptibility

$$\chi^{(p_1)} = \int \mathrm{d}x_1\; G(x_1)\,\frac{1-\cos(p_1 x_1)}{p_1^2}, \qquad G(x_1) = \int \mathrm{d}x_2 \mathrm{d}x_3 \mathrm{d}x_4\, \langle j_2(x) j_2(0) \rangle$$

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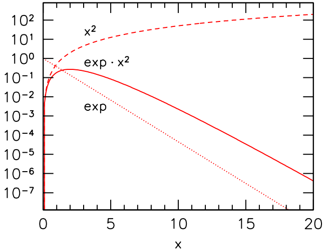
- $p_1 \to 0$ in the infinite volume

$$\chi = \int \, dx_1 \, \frac{G(x_1)}{2} \cdot x_1^2$$



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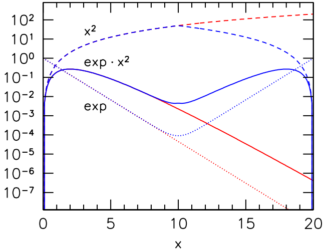
- $p_1 \overset{\sim}{\to} 0$ in finite volume

$$\chi = \int_0^L dx_1\, \frac{G(x_1)}{2} \cdot \begin{cases} x_1^2, & x_1 \le L/2 \\ (x_1 - L)^2, & x_1 > L/2 \end{cases}$$



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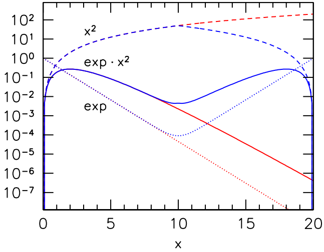
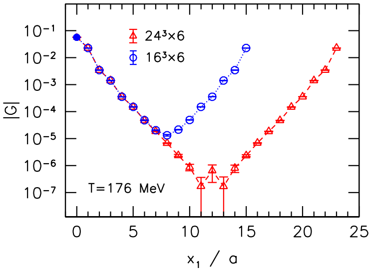


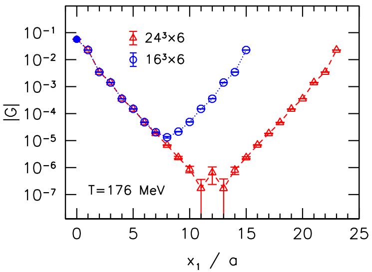
- cusp of kernel at $x_1 = L/2$ is unproblematic

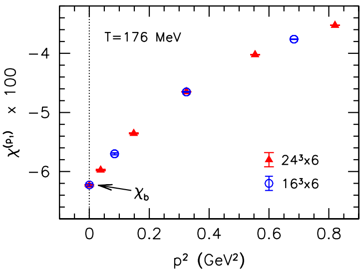
Correlators

- correlator



- correlator and its convolution with the kernels





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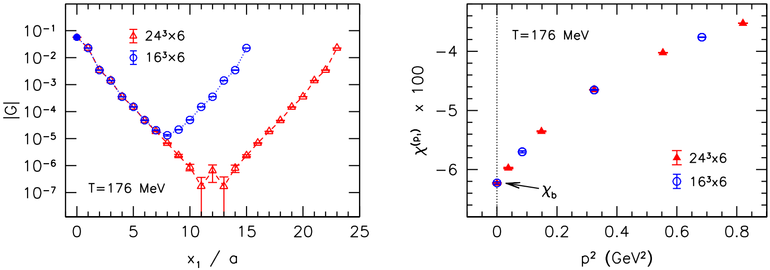

- finite volume effects indeed small

Correlators

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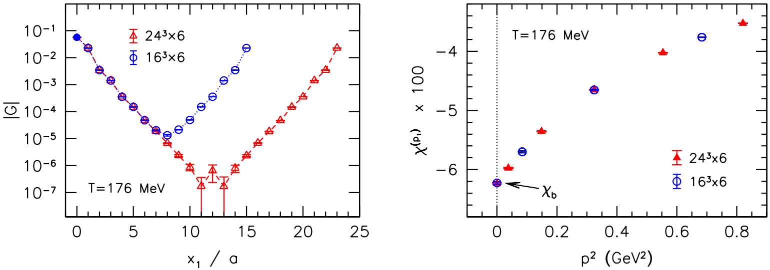
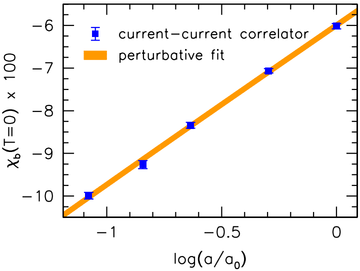


- finite volume effects indeed small
- note: $\chi^{(p)}$ analogous to vacuum polarization form factor relevant for muon $g-2$ calculations at $T=0$ Bali, Endrődi '15

Results

Zero temperature

- susceptibility contains additive divergence $\propto \log a$ due to charge renormalization Schwinger '51 Bali et al. '14



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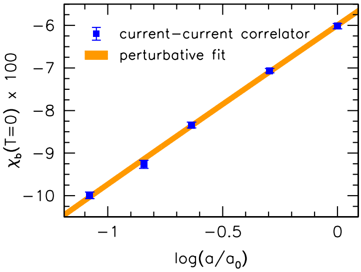

- renormalize as $\chi(T) = \chi_b(T) - \chi_b(T=0)$

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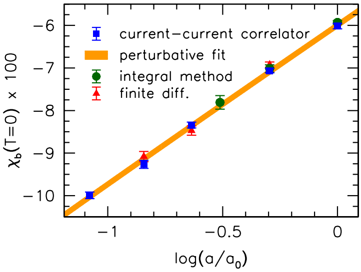
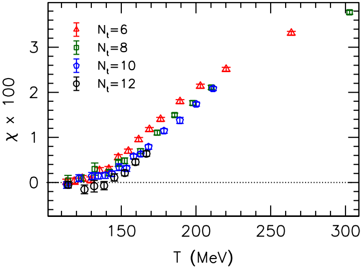


- renormalize as $\chi(T) = \chi_b(T) - \chi_b(T=0)$
- different methods in the literature agree with each other

Nonzero temperature

- continuum extrapolation using four lattice spacings



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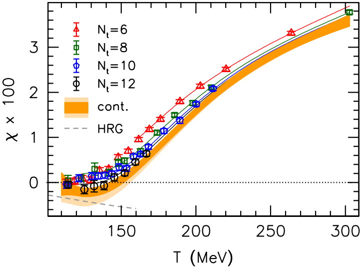

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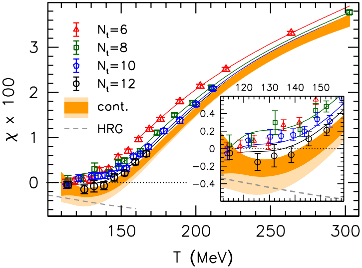

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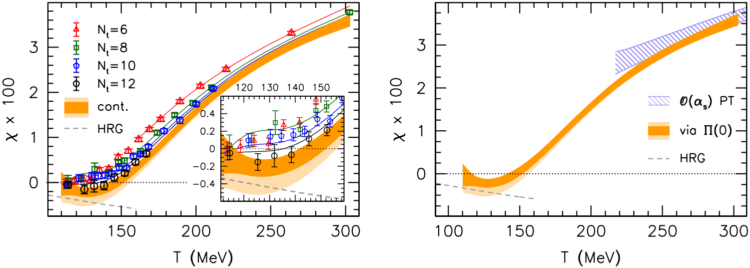

- comparison to HRG model (low T) Endrődi '13
  and to perturbation theory (high T) Bali et al. '14

Nonzero temperature

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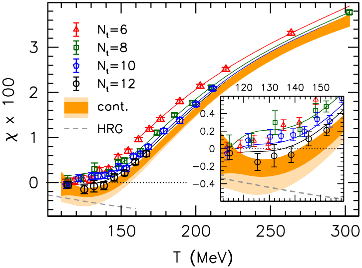
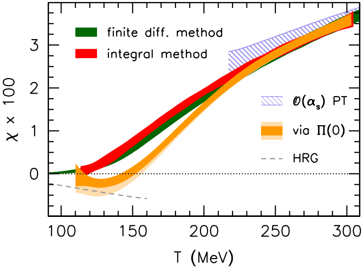
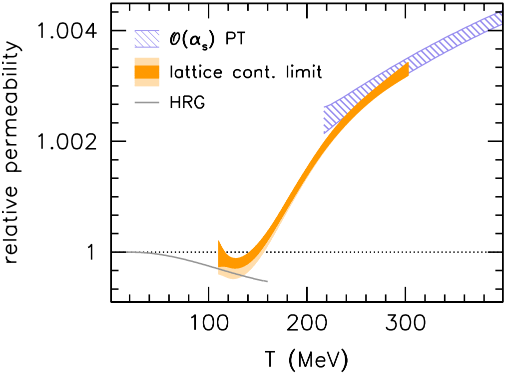




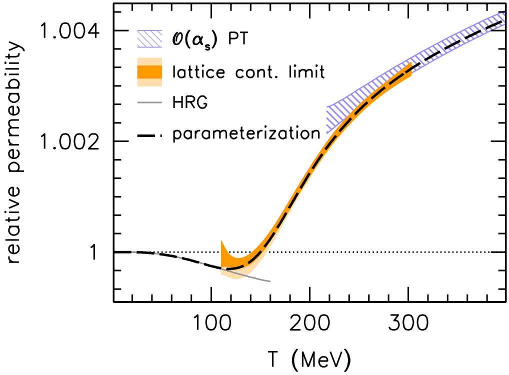
- comparison to HRG model (low T) Endrődi '13
  and to perturbation theory (high T) Bali et al. '14
- taste splitting lattice artefacts severe at low T; careful continuum extrapolation required Bali, Endrődi, Piemonte '20

Permeability



- permeability $\mu = (1 - e^2\chi)^{-1}$

Permeability

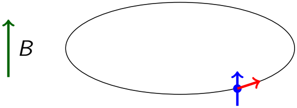


- permeability $\mu = (1 - e^2\chi)^{-1}$
- parameterization as python script, to be used in models
  https://arxiv.org/src/2004.08778v2/anc/param_EoS.py
  contains all other observables in the EoS

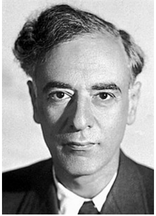
Decomposition of the susceptibility

Spin and angular momentum

- free fermion: $\chi = \chi^{\text{spin}} + \chi^{\text{ang}}$



$$\chi^{\text{spin}} : \chi^{\text{ang}} = (+3) : (-1)$$



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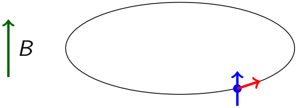

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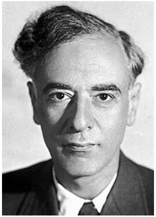


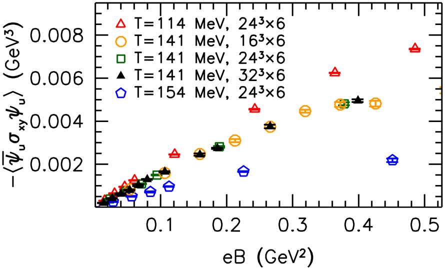
- in the interacting case it still holds

$$-\not{D}^2 = \underbrace{qB\cdot\sigma_{12}}_{\text{spin term}} + \underbrace{D_\mu^2}_{\text{angular momentum term}} \qquad \sigma_{12} = \frac{1}{2i}[\gamma_1, \gamma_2]$$

Spin term

- spin term from $\left\langle \bar{\psi}_f \sigma_{12} \psi_f \right\rangle$ Bali, Endrődi, Piemonte '20

$$\chi^{\text{spin}} = \sum_f \frac{(q_f/e)^2}{2m_f} \left[1 - \lim_{m_f^{\text{val}} \to 0} \right] \underbrace{\frac{\partial}{\partial(q_f B)} \left\langle \bar{\psi}_f \sigma_{12} \psi_f \right\rangle}_{\tau_f}$$



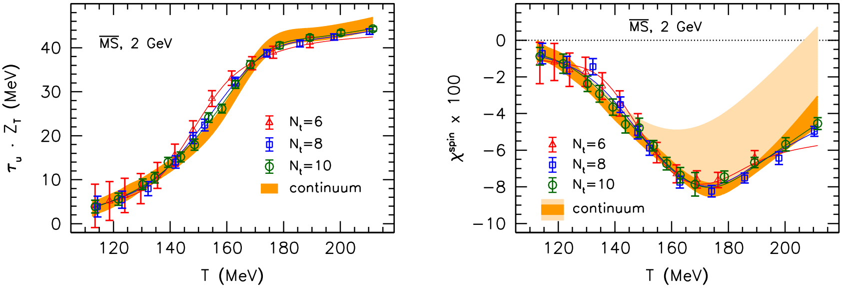
Bali et al '12

- incidentally, τ_f is related to the normalization of the leading-twist photon distribution amplitude

  Balitsky, Braun, Kolesnichenko '89

Spin term

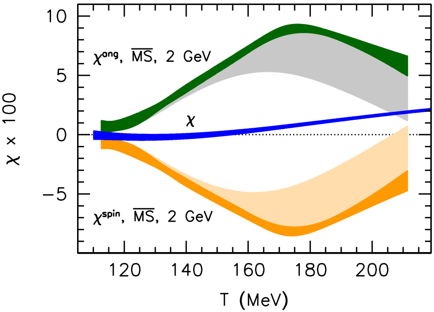
- additive renormalization by $T = 0$ subtraction (just like χ)
- multiplicative renormalization by Z_T



- light yellow band: systematic uncertainty

Decomposition

- angular momentum term indirectly $\chi^{\text{ang}} = \chi - \chi^{\text{spin}}$



- very high temperatures: $\chi^{\text{spin}} : \chi^{\text{ang}} = (+3) : (-1)$
- in QCD regime: $\chi^{\text{spin}} : \chi^{\text{ang}} \approx (-1) : (+1)$

Decomposition

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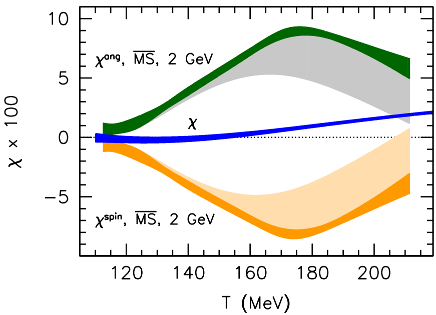
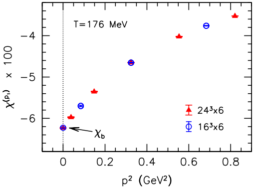


- very high temperatures: $\chi^{\rm spin} : \chi^{\rm ang} = (+3) : (-1)$
- in QCD regime: $\chi^{\rm spin} : \chi^{\rm ang} \approx (-1) : (+1)$
- interplay of confinement and spin physics

Summary

- new method to calculate χ at $B = 0$ in finite volumes



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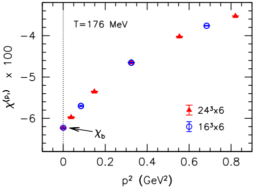
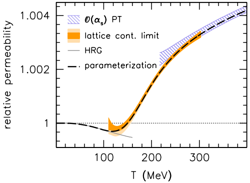


- pions are diamagnetic, QGP is paramagnetic
  parameterization of EoS(T, B)



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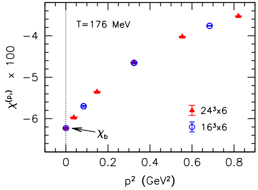

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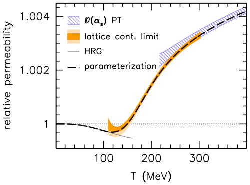

- nontrivial decomposition into spin and angular momentum: role of confinement