Phase diagram of isospin-asymmetric QCD: direct results vs Taylor expansion

Gergely Endrődi

Goethe University of Frankfurt

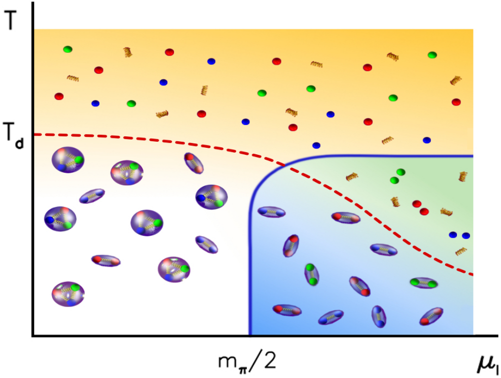
in collaboration with
Bastian Brandt, Sebastian Schmalzbauer

Lattice seminar @ HUB, 26. November 2018

Outline

- introduction: QCD with isospin
  - pion condensation
  - spontaneous vs explicit symmetry breaking
- extrapolations in the explicit breaking parameter
  - singular value representation
  - leading-order reweighting
  - Banks-Casher-type improvements
- results
  - phase diagram
  - comparison to Taylor expansion
- summary

Main result: phase diagram



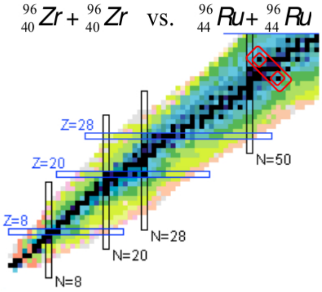
[Brandt, Endrődi, Schmalzbauer 1712.08190]

Introduction

Introduction

- isospin density $n_I \equiv n_u - n_d$
- $n_I < 0 \rightarrow$ excess of neutrons over protons
  $\rightarrow$ excess of π^- over π^+
- why relevant?

  $\rightarrow$ heavy-ion collisions, in particular for isobar runs



[RHIC isobar program, B. Müller]

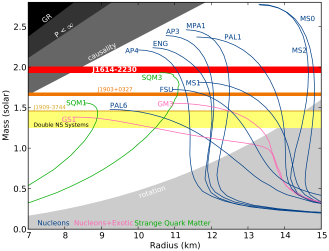
Introduction

- isospin density $n_I \equiv n_u - n_d$
- $n_I < 0 \rightarrow$ excess of neutrons over protons
  $\rightarrow$ excess of π^- over π^+
- why relevant?

  $\rightarrow$ heavy-ion collisions, in particular for isobar runs

  $\rightarrow$ neutron star interiors and composition

[Georgia Tech (Caltech Media Assets)]



[Demorest et al '10]

Isospin chemical potential

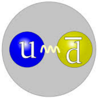
- in the grand canonical ensemble
- quark chemical potentials (3-flavor)

$$\mu_u = \frac{\mu_B}{3} + \mu_I \qquad \mu_d = \frac{\mu_B}{3} - \mu_I \qquad \mu_s = \frac{\mu_B}{3} - \mu_S$$

- zero baryon number, zero strangeness, but nonzero isospin

$$\mu_u = \mu_I \qquad \mu_d = -\mu_I \qquad \mu_s = 0$$

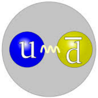
- pion chemical potential $\mu_\pi = \mu_u - \mu_d = 2\mu_I$



- isospin density $n_I = n_u - n_d$

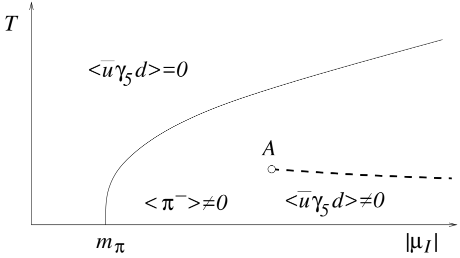
Pion condensation

- QCD at low energies ≈ pions
  chiral perturbation theory
- chemical potential for charged pions: μ_π



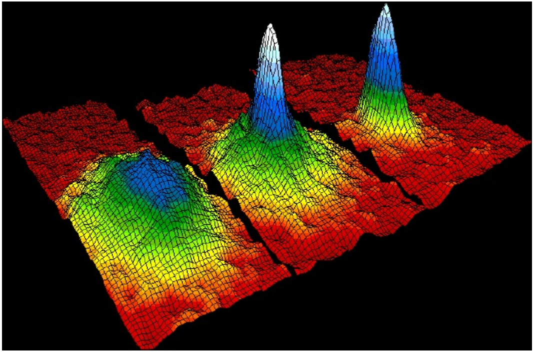
at zero temperature	$\mu_\pi < m_\pi$	vacuum state
	$\mu_\pi \geq m_\pi$	Bose-Einstein condensation

[Son, Stephanov '00]



Bose-Einstein condensate

- accumulation of bosonic particles in lowest energy state



[Anderson et al '95 JILA-NIST/University of Colorado]

- velocity distribution of Ru atoms at low temperature
  $\rightarrow$ peak at zero velocity (zero energy)

BEC in lattice QCD

Symmetry breaking

- QCD with light quark matrix

$$M = \not{D} + m_{ud}\mathbb{1}$$

- chiral symmetry (flavor-nontrivial)

$$\mathrm{SU}(2)_V$$

Symmetry breaking

- QCD with light quark matrix

$$M = \not{D} + m_{ud}\mathbb{1} + \mu_I \gamma_0 \tau_3$$

- chiral symmetry (flavor-nontrivial)

$$\mathrm{SU}(2)_V \rightarrow \mathrm{U}(1)_{\tau_3}$$

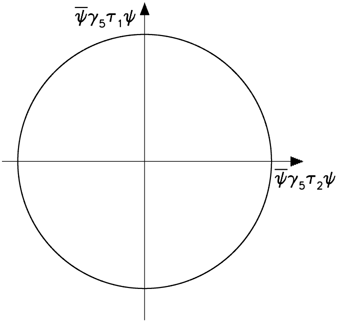
Symmetry breaking

- QCD with light quark matrix

$$M = \not{D} + m_{ud}\mathbb{1} + \mu_I \gamma_0 \tau_3$$

- chiral symmetry (flavor-nontrivial)

$$\mathrm{SU}(2)_V \to \mathrm{U}(1)_{\tau_3}$$



- spontaneously broken by a pion condensate

$$\left\langle \bar{\psi}\gamma_5\tau_{1,2}\psi \right\rangle$$

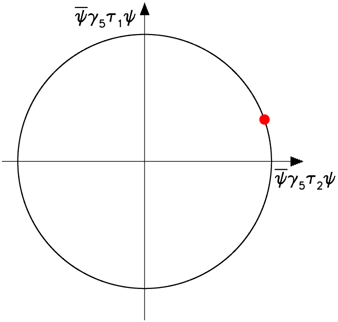
Symmetry breaking

- QCD with light quark matrix

$$M = \not{D} + m_{ud}\mathbb{1} + \mu_I \gamma_0 \tau_3$$

- chiral symmetry (flavor-nontrivial)

$$\mathrm{SU}(2)_V \to \mathrm{U}(1)_{\tau_3}$$



- spontaneously broken by a pion condensate

$$\left\langle \bar{\psi}\gamma_5\tau_{1,2}\psi \right\rangle$$

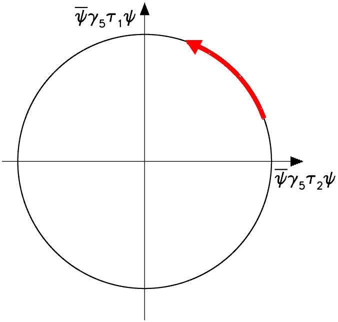
Symmetry breaking

- QCD with light quark matrix

$$M = \not{D} + m_{ud}\mathbb{1} + \mu_I \gamma_0 \tau_3$$

- chiral symmetry (flavor-nontrivial)

$$\mathrm{SU}(2)_V \to \mathrm{U}(1)_{\tau_3}$$



- spontaneously broken by a pion condensate

$$\left\langle \bar{\psi}\gamma_5\tau_{1,2}\psi \right\rangle$$

- a Goldstone mode appears

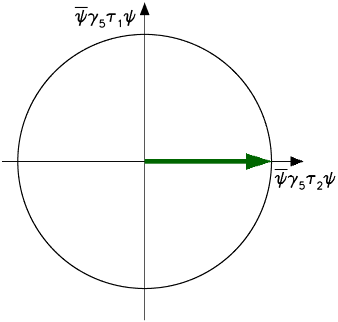
Symmetry breaking

- QCD with light quark matrix

$$M = \not{D} + m_{ud}\mathbb{1} + \mu_I\gamma_0\tau_3 + i\lambda\gamma_5\tau_2$$

- chiral symmetry (flavor-nontrivial)

$$\mathrm{SU}(2)_V \to \mathrm{U}(1)_{\tau_3} \to \varnothing$$



- spontaneously broken by a pion condensate

$$\left\langle \bar{\psi}\gamma_5\tau_{1,2}\psi \right\rangle$$

- a Goldstone mode appears
- add small explicit breaking

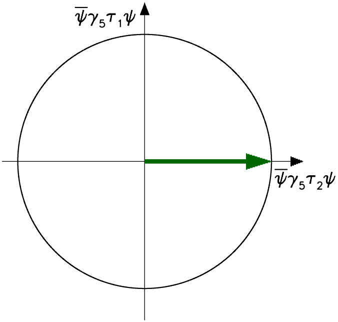
Symmetry breaking

- QCD with light quark matrix

$$M = \not{D} + m_{ud}\mathbb{1} + \mu_I \gamma_0 \tau_3 + i\lambda\gamma_5\tau_2$$

- chiral symmetry (flavor-nontrivial)

$$\mathrm{SU}(2)_V \to \mathrm{U}(1)_{\tau_3} \to \varnothing$$



- spontaneously broken by a pion condensate

$$\left\langle \bar{\psi}\gamma_5\tau_{1,2}\psi \right\rangle$$

- a Goldstone mode appears
- add small explicit breaking

- extrapolate results $\lambda \to 0$

Dictionary

	pion condensation	
pattern	$\mathrm{U}(1)_{\tau_3} \to \varnothing$	
coset	$\mathrm{U}(1)$	
Goldstones	1	
spontaneous	$\left\langle \bar{\psi}\gamma_5\tau_2\psi \right\rangle$	
explicit	$\lambda \to 0$	

Dictionary

	pion condensation	vacuum chiral symmetry breaking
pattern	$\mathrm{U}(1)_{\tau_3} \to \varnothing$	
coset	$\mathrm{U}(1)$	
Goldstones	1	
spontaneous	$\left\langle \bar{\psi}\gamma_5\tau_2\psi \right\rangle$	
explicit	$\lambda \to 0$	

Dictionary

	pion condensation	vacuum chiral symmetry breaking
pattern	$\mathrm{U}(1)_{\tau_3} \to \varnothing$	$\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \to \mathrm{SU}(2)_V$
coset	$\mathrm{U}(1)$	
Goldstones	1	
spontaneous	$\left\langle \bar{\psi}\gamma_5\tau_2\psi \right\rangle$	
explicit	$\lambda \to 0$	

Dictionary

	pion condensation	vacuum chiral symmetry breaking
pattern	$\mathrm{U}(1)_{\tau_3} \to \varnothing$	$\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \to \mathrm{SU}(2)_V$
coset	$\mathrm{U}(1)$	$\mathrm{SU}(2)_A$
Goldstones	1	
spontaneous	$\left\langle \bar{\psi}\gamma_5\tau_2\psi \right\rangle$	
explicit	$\lambda \to 0$	

Dictionary

	pion condensation	vacuum chiral symmetry breaking
pattern	$\mathrm{U}(1)_{\tau_3} \to \varnothing$	$\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \to \mathrm{SU}(2)_V$
coset	$\mathrm{U}(1)$	$\mathrm{SU}(2)_A$
Goldstones	1	3
spontaneous	$\left\langle \bar{\psi}\gamma_5\tau_2\psi \right\rangle$	
explicit	$\lambda \to 0$	

Dictionary

	pion condensation	vacuum chiral symmetry breaking
pattern	$\mathrm{U}(1)_{\tau_3} \to \varnothing$	$\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \to \mathrm{SU}(2)_V$
coset	$\mathrm{U}(1)$	$\mathrm{SU}(2)_A$
Goldstones	1	3
spontaneous	$\left\langle \bar{\psi}\gamma_5\tau_2\psi \right\rangle$	$\left\langle \bar{\psi}\psi \right\rangle$
explicit	$\lambda \to 0$	

Dictionary

	pion condensation	vacuum chiral symmetry breaking
pattern	$\mathrm{U}(1)_{\tau_3} \to \varnothing$	$\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \to \mathrm{SU}(2)_V$
coset	$\mathrm{U}(1)$	$\mathrm{SU}(2)_A$
Goldstones	1	3
spontaneous	$\left\langle \bar{\psi}\gamma_5\tau_2\psi \right\rangle$	$\left\langle \bar{\psi}\psi \right\rangle$
explicit	$\lambda \to 0$	$m \to 0$

Dictionary

	pion condensation	vacuum chiral symmetry breaking
pattern	$\mathrm{U}(1)_{\tau_3} \to \varnothing$	$\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \to \mathrm{SU}(2)_V$
coset	$\mathrm{U}(1)$	$\mathrm{SU}(2)_A$
Goldstones	1	3
spontaneous	$\left\langle \bar{\psi}\gamma_5\tau_2\psi \right\rangle$	$\left\langle \bar{\psi}\psi \right\rangle$
explicit	$\lambda \to 0$	$m \to 0$

- long story short: pion condensation $1/3$ as challenging as the chiral limit of the QCD vacuum

Simulation with $\lambda > 0$

- staggered light quark matrix with $\eta_5 = (-1)^{n_x+n_y+n_z+n_t}$

$$M = \begin{pmatrix} \not{D}_\mu + m & \lambda\eta_5 \\ -\lambda\eta_5 & \not{D}_{-\mu} + m \end{pmatrix}$$

- we have $\gamma_5\tau_1$-hermiticity

$$\eta_5\tau_1 M \tau_1 \eta_5 = M^\dagger$$

- determinant is real and positive

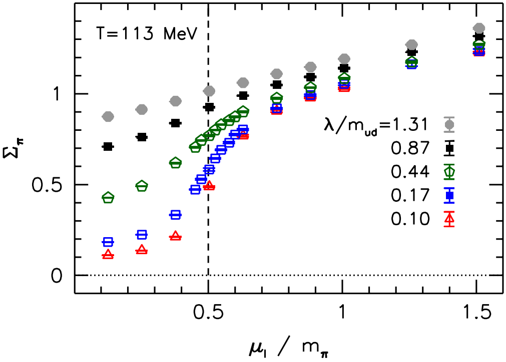
$$\det M = \det(|\not{D}_\mu + m|^2 + \lambda^2)$$

- early studies [Kogut, Sinclair '02] [de Forcrand, Stephanov, Wenger '07] [Endrődi '14] with unimproved action
- here: $N_f = 2 + 1$ rooted stout-smeared staggered quarks + tree-level Symanzik improved gluons

Pion condensate on the lattice

- traditional method [Kogut, Sinclair '02]
  measure full operator at nonzero λ (via noisy estimators)

$$\Sigma_\pi \propto \left\langle \mathrm{Tr} M^{-1} \eta_5 \tau_2 \right\rangle$$

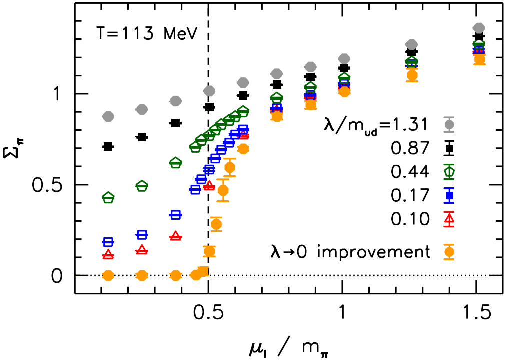


- extrapolation $\lambda \to 0$ very steep

Pion condensate on the lattice

- traditional method [Kogut, Sinclair '02]
  measure full operator at nonzero λ (via noisy estimators)

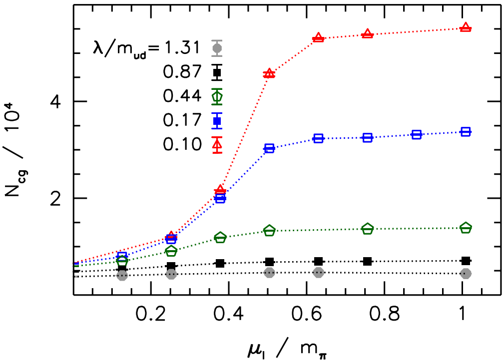
$$\Sigma_\pi \propto \left\langle \mathrm{Tr} M^{-1} \eta_5 \tau_2 \right\rangle$$



- extrapolation $\lambda \to 0$ very steep
- new method to etract $\lambda = 0$ limit

Computational cost

- computational cost for inverting M grows as $\lambda \to 0$



- iteration count diverges if a massless mode is present
  $\rightsquigarrow$ alternative definition of pion condensation
- additionally, reduced step-size necessary due to enhanced fluctuations in fermionic force

Improved λ-extrapolation

Singular value representation

- singular values

$$|\not{D}_\mu + m|^2 \, \psi_i = \xi_i^2 \, \psi_i$$

independent of Dirac eigenvalues due to $[\not{D}_\mu, \not{D}_\mu^\dagger] \neq 0$

- pion condensate

$$\Sigma_\pi = \frac{\partial}{\partial \lambda} \log\det(|\not{D}_\mu + m|^2 + \lambda^2) = \mathrm{Tr}\,\frac{2\lambda}{|\not{D}_\mu + m|^2 + \lambda^2}$$

- spectral representation

$$\Sigma_\pi = \frac{T}{V} \sum_i \frac{2\lambda}{\xi_i^2 + \lambda^2} = \int \mathrm{d}\xi \, \rho(\xi) \, \frac{2\lambda}{\xi^2 + \lambda^2} \xrightarrow{\lambda \to 0} \pi \rho(0)$$

first derived in [Kanazawa, Wettig, Yamamoto '11]

Singular value representation

- singular values

$$|\not{D}_\mu + m|^2 \, \psi_i = \xi_i^2 \, \psi_i$$

independent of Dirac eigenvalues due to $[\not{D}_\mu, \not{D}_\mu^\dagger] \neq 0$

- pion condensate

$$\Sigma_\pi = \frac{\partial}{\partial \lambda} \log \det(|\not{D}_\mu + m|^2 + \lambda^2) = \mathrm{Tr} \frac{2\lambda}{|\not{D}_\mu + m|^2 + \lambda^2}$$

- spectral representation

$$\Sigma_\pi = \frac{T}{V} \sum_i \frac{2\lambda}{\xi_i^2 + \lambda^2} = \int \mathrm{d}\xi \, \rho(\xi) \, \frac{2\lambda}{\xi^2 + \lambda^2} \xrightarrow{\lambda \to 0} \pi \rho(0)$$

first derived in [Kanazawa, Wettig, Yamamoto '11]

- compare to Banks-Casher-relation at $\mu_I = 0$

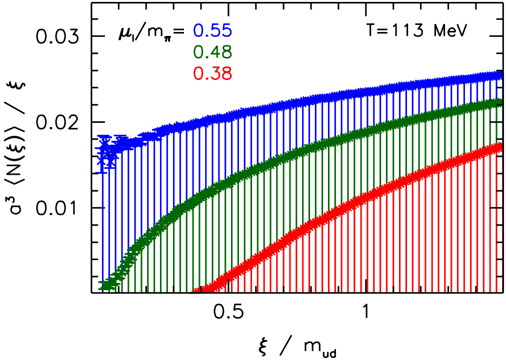
Dictionary

	pion condensation	vacuum chiral symmetry breaking
pattern	$\mathrm{U}(1)_{\tau_3} \to \varnothing$	$\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \to \mathrm{SU}(2)_V$
coset	$\mathrm{U}(1)$	$\mathrm{SU}(2)_A$
Goldstones	1	3
spontaneous	$\left\langle \bar{\psi}\gamma_5\tau_2\psi \right\rangle$	$\left\langle \bar{\psi}\psi \right\rangle$
explicit	$\lambda \to 0$	$m \to 0$
Banks-Casher	$\rho^{\lvert \not{D}_\mu + m \rvert^2}(0)$	$\rho^{\not{D}}(0)$

Singular value density

- integrated spectral density

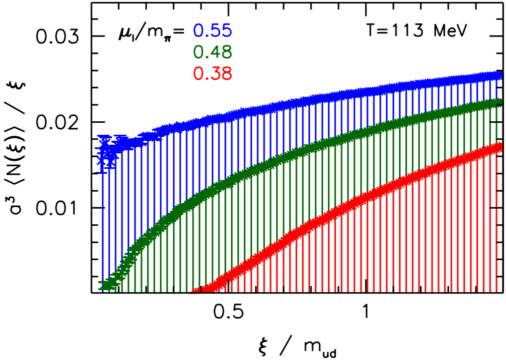
$$N(\xi) = \int_0^{\xi} d\xi' \rho(\xi'), \qquad \rho(0) = \lim_{\xi \to 0} N(\xi)/\xi$$

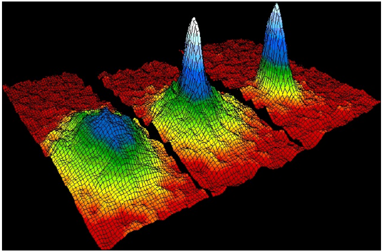


Singular value density

- integrated spectral density

$$N(\xi) = \int_0^{\xi} d\xi' \rho(\xi'), \qquad \rho(0) = \lim_{\xi \to 0} N(\xi)/\xi$$

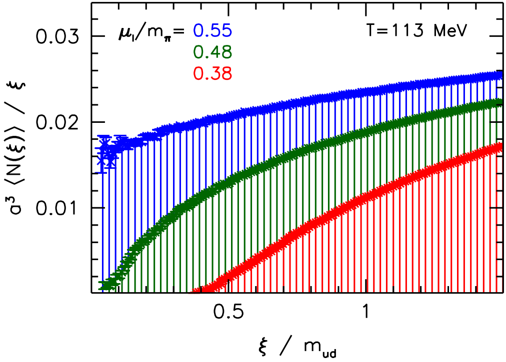


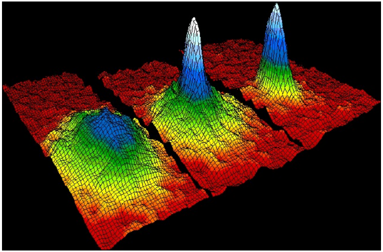


- compare $\rho(0)$ to velocity distribution around zero

- integrated spectral density

$$N(\xi) = \int_0^\xi d\xi' \rho(\xi'), \qquad \rho(0) = \lim_{\xi \to 0} N(\xi)/\xi$$





- compare $\rho(0)$ to velocity distribution around zero
- Bose-Einstein condensation!

Reweighting

- reweighting factor

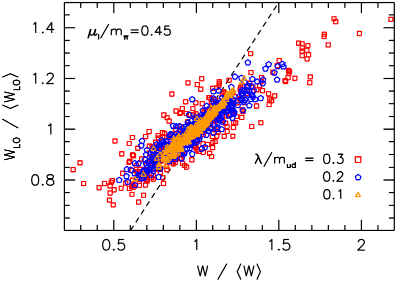
$$W = \frac{\det(|\not{D}_\mu + m|^2)}{\det(|\not{D}_\mu + m|^2 + \lambda^2)}$$

- but λ is small, so expand in it:

$$W_{\rm LO} = \exp\left[-\lambda V_4 \cdot \Sigma_\pi\right]$$

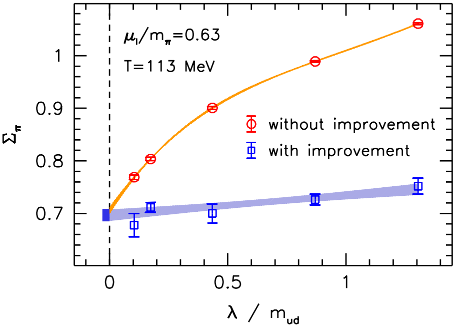
$$\langle \mathcal{O} \rangle_{\rm rew} = \frac{\langle \mathcal{O} W_{\rm LO} \rangle}{\langle W_{\rm LO} \rangle} + \text{ higher orders in } \lambda$$

- scatter plot: $W_{\rm LO}$ vs. W on small lattices



Comparison between old and new methods

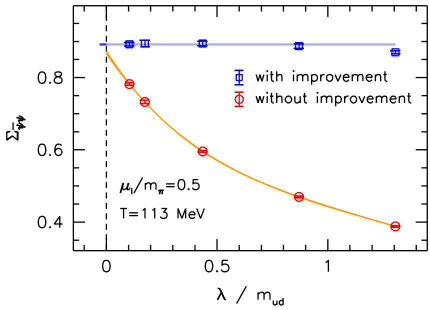
- improvement is crucial for reliable $\lambda \to 0$ extrapolation

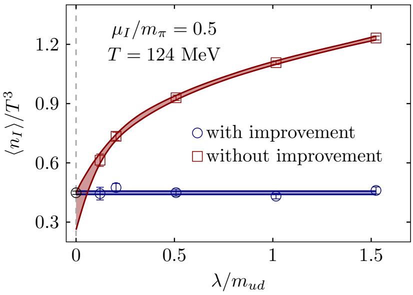


New method for other observables

- spectral representation for quark condensate

$$\bar{\psi}\psi = \frac{2T}{V}\sum_i \mathrm{Re}\,\frac{\langle\psi_i|(\not{D}_\mu + m)^\dagger|\psi_i\rangle}{\xi_i^2+\lambda^2} \qquad n_I = \frac{2T}{V}\sum_i \mathrm{Re}\,\frac{\langle\psi_i|(\not{D}_\mu + m)^\dagger \not{D}'_\mu|\psi_i\rangle}{\xi_i^2+\lambda^2}$$



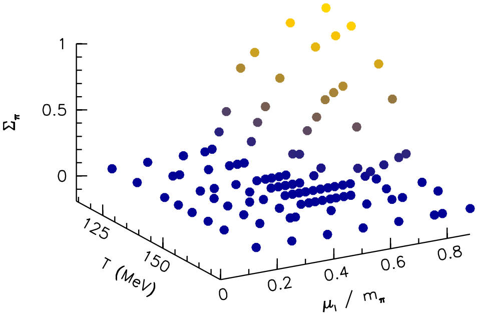


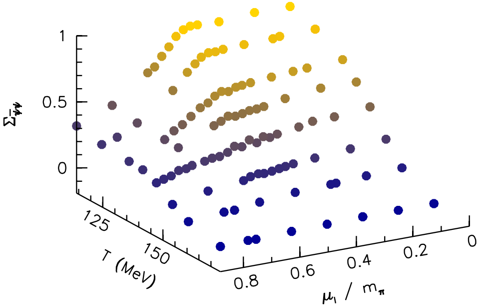
- improvement crucial for reliable $\lambda \to 0$ extrapolation

Results: phase diagram [1712.08190]

Condensates

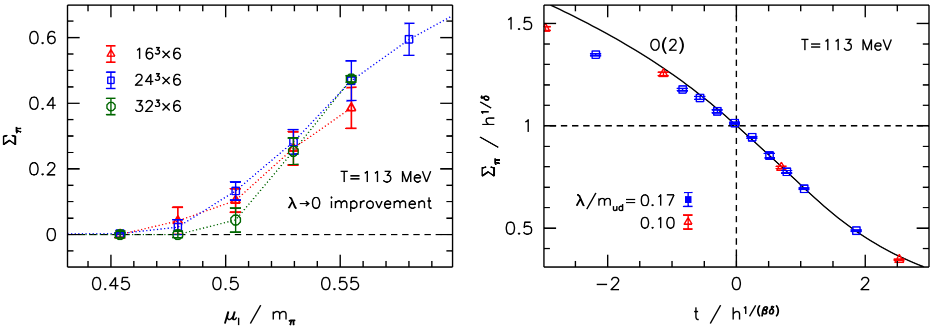
- pion and chiral condensate after $\lambda \to 0$ extrapolation





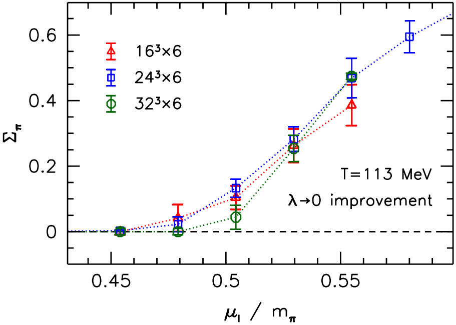
- read off chiral crossover $T_{pc}(\mu_I)$ and pion condensation boundary $\mu_{I,c}(T)$

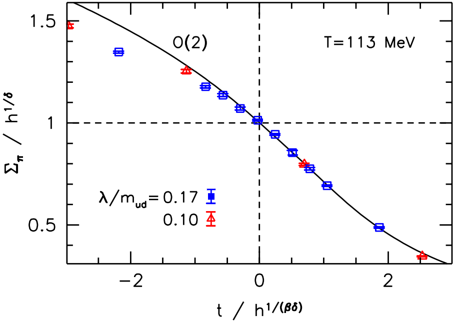
Order of the transition



- volume scaling of order parameter shows typical sharpening
- collapse according to $\mathrm{O}(2)$ critical exponents [Ejiri et al '09]

Order of the transition

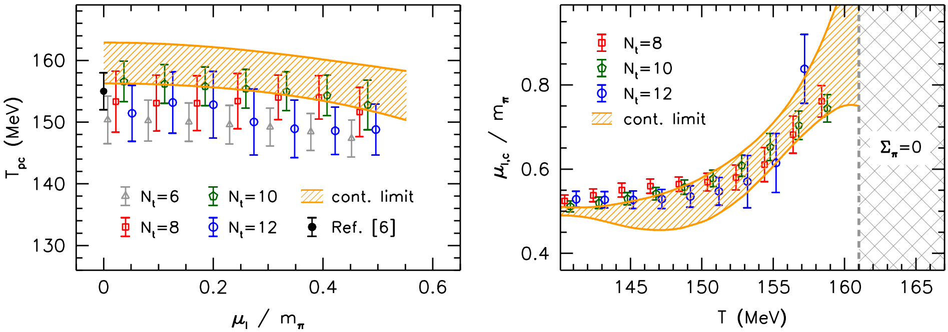




- volume scaling of order parameter shows typical sharpening
- collapse according to $\mathrm{O}(2)$ critical exponents [Ejiri et al '09]
- indications for a second order phase transition at $\mu_I \approx m_\pi/2$, in the $\mathrm{O}(2)$ universality class

Transition temperatures

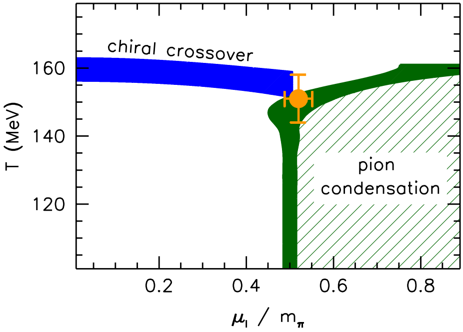
- T_{pc}: inflection point of chiral condensate
- $\mu_{I,c}$: boundary of $\Sigma_\pi > 0$ region
- continuum limit based on $N_t = 6, 8, 10, 12$



Phase diagram

- meeting point of chiral crossover and pion condensation boundary: *pseudo-triple* point

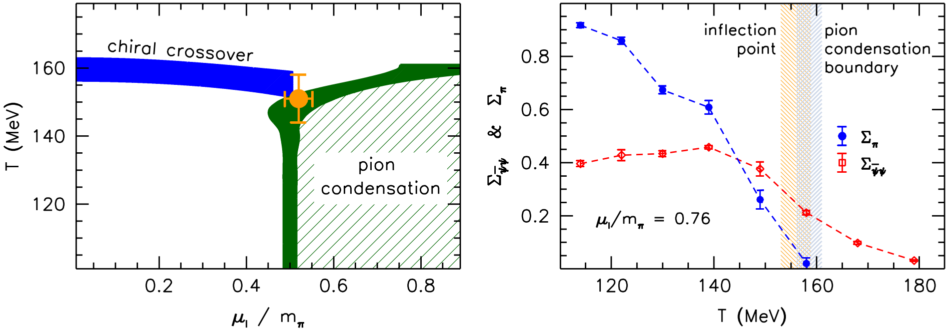
  at $T_{pt} = 151(7)$ MeV, $\mu_{I,pt} = 70(5)$ MeV



Phase diagram

- meeting point of chiral crossover and pion condensation boundary: *pseudo-triple* point

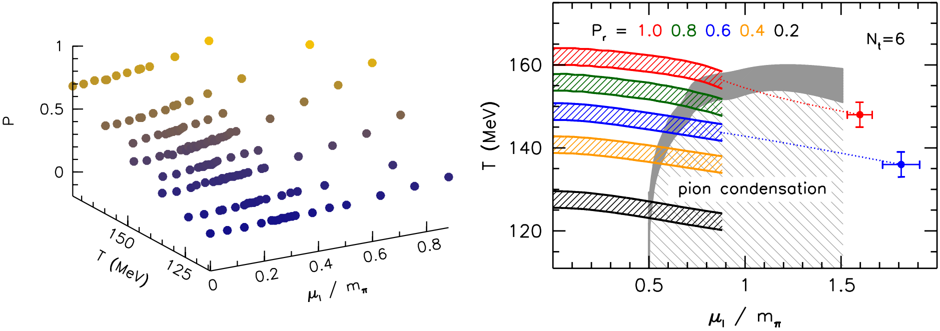
  at $T_{pt} = 151(7)$ MeV, $\mu_{I,pt} = 70(5)$ MeV



- the two transitions coincide beyond the pseudo-triple point

Phase diagram

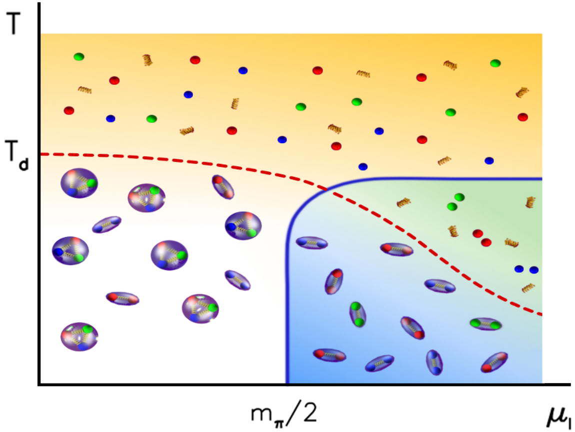
- Polyakov loop as measure for deconfinement



- no significant response in P on pion condensation
- deconfinement crossover persists in pion condensed phase
  ⇝ BCS superconductivity [Son, Stephanov '01]

Phase diagram

- favored phase diagram schematically:
  hadronic, quark-gluon plasma, BEC, BCS phases



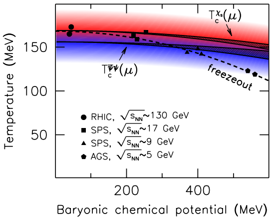
Taylor expansion method

Taylor expansion method

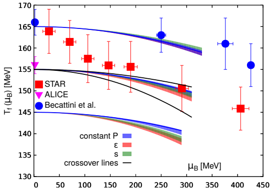
- overcome sign problem at $\mu_B > 0$
- reconstruct observable $\mathcal{O}(\mu_B)$ via

$$\mathcal{O}(\mu_B) = \sum_{i=0}^{\infty} c_i^{\mathcal{O}} \mu_B^i$$

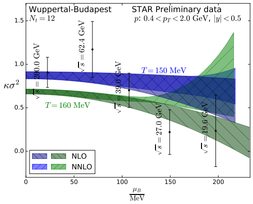
- routinely used for phase diagram and for EoS



[Endrődi et al '11]



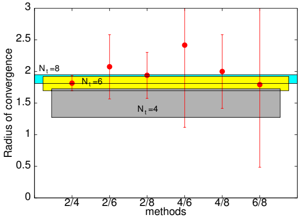
[HotQCD '17]



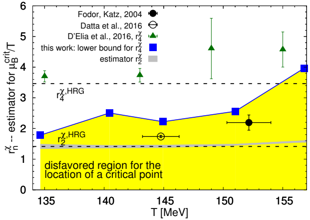
[BMWc '18]

Radius of convergence

- reconstruction via Taylor expansion only works for analytic functions
- radius of convergence marks nearest singularity
- used to investigate the QCD critical endpoint



[Datta, Gavai, Gupta '17]



[HotQCD '17]

Taylor expansion for nonzero isospin

- comparison between full results and expansion is possible
- our choice for the observable:

$$\langle n_I \rangle = \frac{T}{V} \frac{\partial \log \mathcal{Z}}{\partial \mu_I}$$

- compared to Taylor-expansion

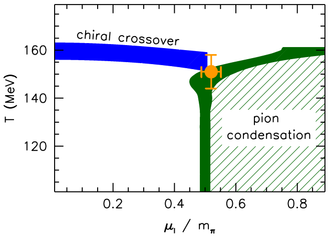
$$\frac{\langle n_I \rangle}{T^3} = \langle c_2 \rangle \cdot \frac{\mu_I}{T} + \frac{\langle c_4 \rangle}{6} \cdot \left(\frac{\mu_I}{T} \right)^3 + \mathcal{O}(\mu_I^5)$$

with $\langle c_{2,4} \rangle$ available in [Borsányi et al '12]

Results: comparison to Taylor series [1810.11045]

Breakdown at pion condensation onset

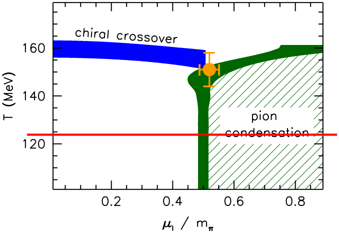
- second-order phase transition along condensation onset



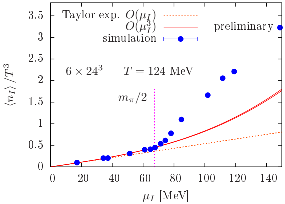
- Taylor expansion breaks down at the phase transition visualized for the $24^3 \times 6$ ensemble

Breakdown at pion condensation onset

- second-order phase transition along condensation onset

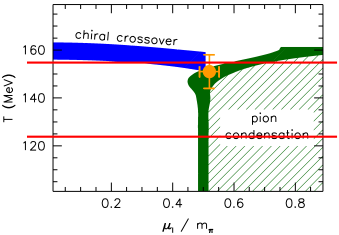


- Taylor expansion breaks down at the phase transition visualized for the $24^3 \times 6$ ensemble

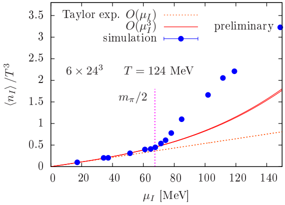


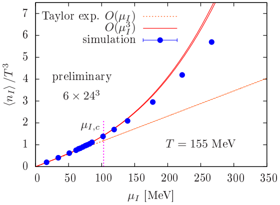
Breakdown at pion condensation onset

- second-order phase transition along condensation onset



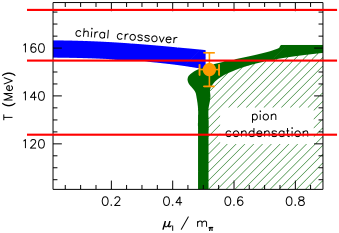
- Taylor expansion breaks down at the phase transition visualized for the $24^3 \times 6$ ensemble



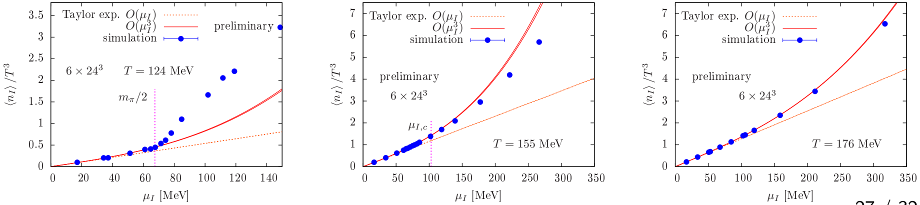


Breakdown at pion condensation onset

- second-order phase transition along condensation onset



- Taylor expansion breaks down at the phase transition visualized for the $24^3 \times 6$ ensemble



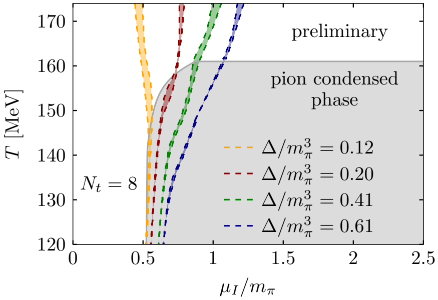
Reliability range

- quantify deviation between expanded and true values

$$\Delta^{\mathrm{LO}} = \left| \frac{\langle n_I \rangle}{T^3} - \langle c_2 \rangle \cdot \frac{\mu_I}{T} \right| \qquad \Delta^{\mathrm{NLO}} = \left| \frac{\langle n_I \rangle}{T^3} - \langle c_2 \rangle \cdot \frac{\mu_I}{T} - \frac{\langle c_4 \rangle}{6} \cdot \left(\frac{\mu_I}{T} \right)^3 \right|$$

- different behavior inside and outside pion condensed phase

leading order



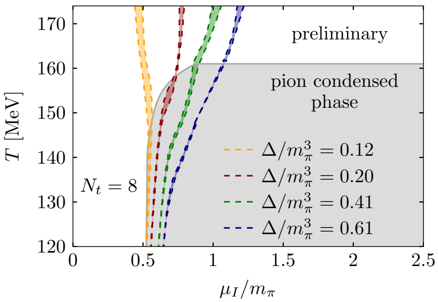
Reliability range

- quantify deviation between expanded and true values

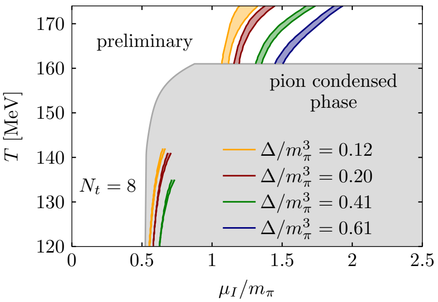
$$\Delta^{\mathrm{LO}} = \left| \frac{\langle n_I \rangle}{T^3} - \langle c_2 \rangle \cdot \frac{\mu_I}{T} \right| \qquad \Delta^{\mathrm{NLO}} = \left| \frac{\langle n_I \rangle}{T^3} - \langle c_2 \rangle \cdot \frac{\mu_I}{T} - \frac{\langle c_4 \rangle}{6} \cdot \left(\frac{\mu_I}{T} \right)^3 \right|$$

- different behavior inside and outside pion condensed phase

leading order

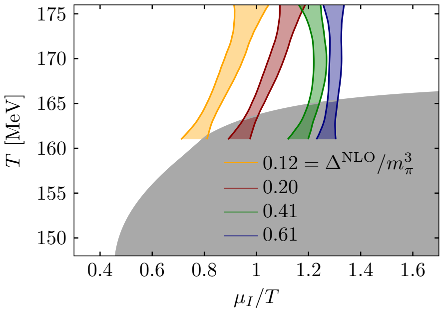


next-to-leading order



Contour lines in the continuum

- continuum extrapolation performed using $N_t = 6, 8, 10, 12$
- plot contours against μ_I/T



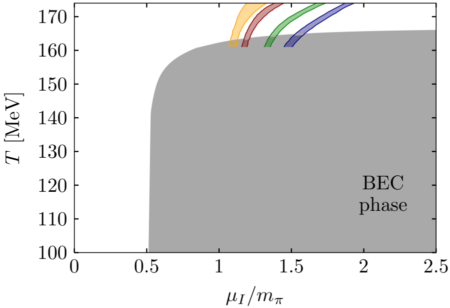
- large deviations from naively expected $\mu_I/T = $ const lines

Radius of convergence

- at low T, nearest singularity is at pion condensation onset
- radius of convergence for $\chi_I = \partial n_I / \partial \mu_I$:

$$\frac{r}{T} = \lim_{n\to\infty} \sqrt{\frac{\langle c_n \rangle}{\langle c_n + 2 \rangle} \cdot (n-1)n} \sim \sqrt{\frac{\langle c_2 \rangle}{\langle c_4 \rangle} \cdot 2}$$

using $\langle c_{2,4} \rangle$ from [Borsányi et al '12]

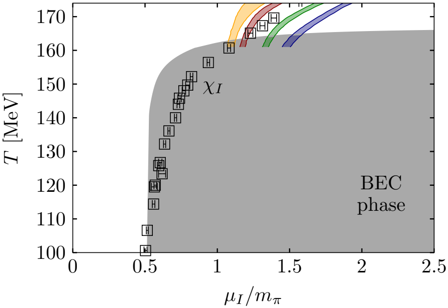


Radius of convergence

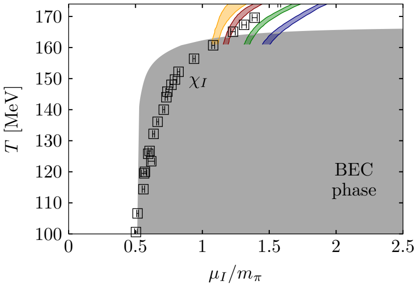
- at low T, nearest singularity is at pion condensation onset
- radius of convergence for $\chi_I = \partial n_I / \partial \mu_I$:

$$\frac{r}{T} = \lim_{n\to\infty} \sqrt{\frac{\langle c_n \rangle}{\langle c_n + 2 \rangle} \cdot (n-1)n} \sim \sqrt{\frac{\langle c_2 \rangle}{\langle c_4 \rangle} \cdot 2}$$

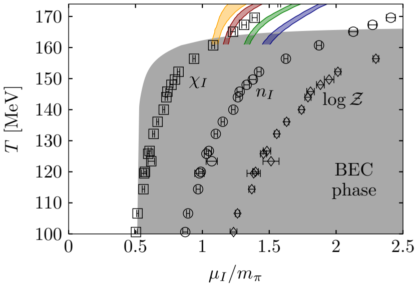
using $\langle c_{2,4} \rangle$ from [Borsányi et al '12]



Estimator from which series?

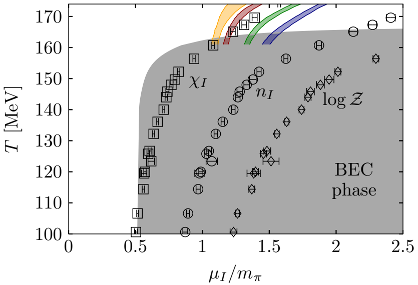


Estimator from which series?



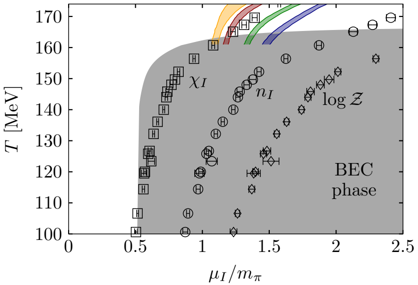
- $r\{n'_I\} = r\{n_I\} = r\{\log \mathcal{Z}\}$, but leading estimators differ as
  $1 \quad : \quad \sqrt{3} \quad : \quad \sqrt{6}$

Estimator from which series?



- $r\{n_I'\} = r\{n_I\} = r\{\log \mathcal{Z}\}$, but leading estimators differ as
  $1 \quad : \quad \sqrt{3} \quad : \quad \sqrt{6}$
  susceptibility-series seems to converge faster
  (see also [Karsch, Schaefer, Wagner, Wambach '11])

Estimator from which series?



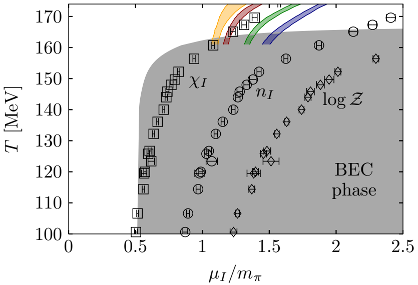
- $r\{n_I'\} = r\{n_I\} = r\{\log \mathcal{Z}\}$, but leading estimators differ as

  $1 \quad : \quad \sqrt{3} \quad : \quad \sqrt{6}$

  susceptibility-series seems to converge faster
  (see also [Karsch, Schaefer, Wagner, Wambach '11])

- need higher-order estimates

Estimator from which series?



- $r\{n_I'\} = r\{n_I\} = r\{\log \mathcal{Z}\}$, but leading estimators differ as

  $1 \quad : \quad \sqrt{3} \quad : \quad \sqrt{6}$

  susceptibility-series seems to converge faster
  (see also [Karsch, Schaefer, Wagner, Wambach '11])

- need higher-order estimates

- results may give insight to convergence properties at $\mu_B > 0$

Summary

- Bose-Einstein condensation via singular value density

  ⇝ flat extrapolation in λ

- map QCD phase diagram for nonzero isospin-asymmetry

  ⇝ detected a 2nd order phase transition in full QCD (for the first time)

- comparison to Taylor expansion around $\mu_I = 0$

  ⇝ susceptibility series optimal for convergence radius studies