QCD in external magnetic fields

Gergely Endrődi

Goethe University of Frankfurt

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in collaboration with

Gunnar Bali, Falk Bruckmann, Zoltán Fodor, Matteo Giordano,
Sándor Katz, Tamás Kovács, Stefan Krieg, Ferenc Pittler,
Andreas Schäfer, Kálmán Szabó, Jacob Wellnhofer

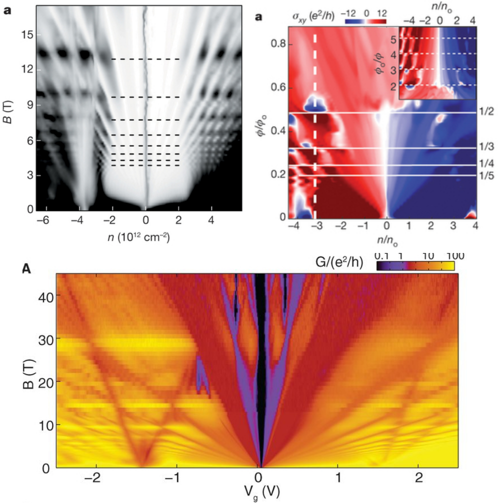
Outline

- preface: magnetic fields and lattices
- lattice quantum chromodynamics
  - role of Landau levels
  - order parameter
  - magnetic catalysis
- nonzero temperature
  - phase diagram
  - applications
- summary

Magnetic fields and lattices

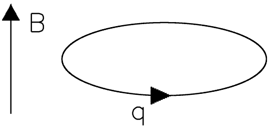
Experiments

- conductance of graphene lattices in strong magnetic fields
  [Ponomarenko et al Nature 497 ’13, Dean et al Nature 497 ’13]
  [Hunt et al Science 340 ’13]



Landau versus Bloch

- free charged particle
  - exposed to magnetic field in continuum space: Landau orbits



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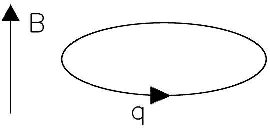
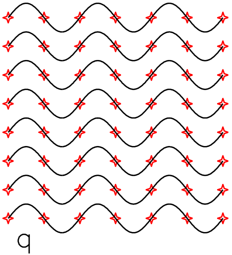
  - on a (crystal) lattice: Bloch waves





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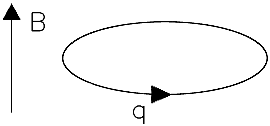
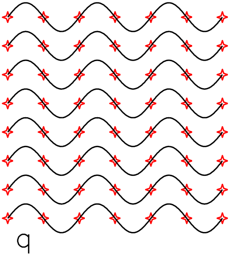
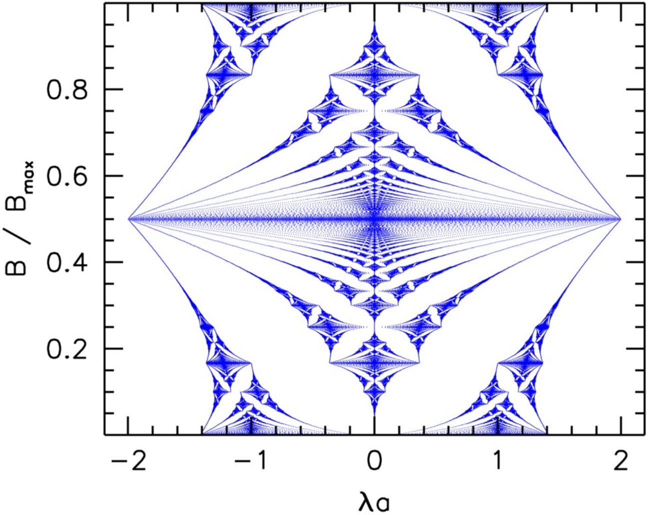




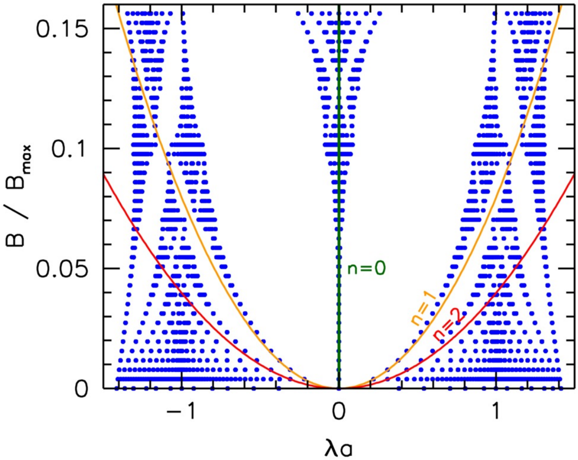
- what happens if the two are combined?

Hofstadter's butterfly [Hofstadter '76]



- relativistic electrons in a magnetic field in 2D [Endrődi '14]
  $B_{\mathrm{max}} \propto 1/a^2$

Hofstadter's butterfly at low magnetic fields



- Landau levels are visible at low B [Endrődi '14]
  $\lambda^2 = 2n \cdot B$

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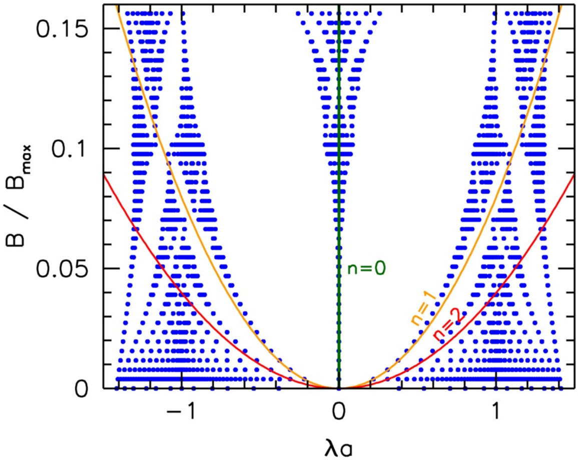

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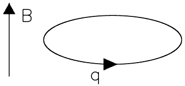

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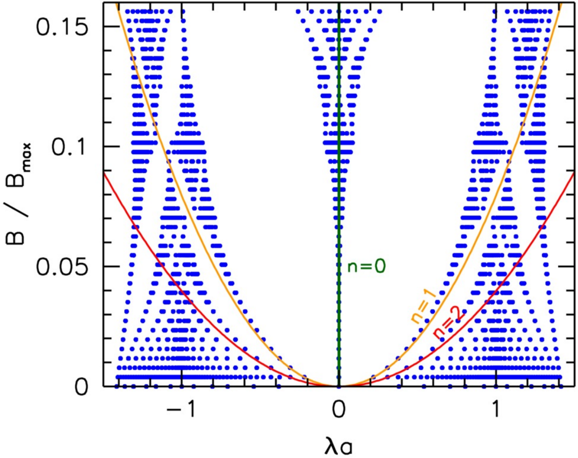

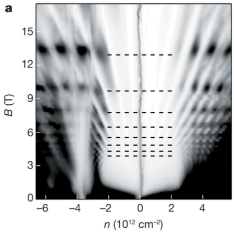

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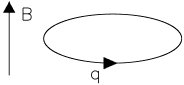

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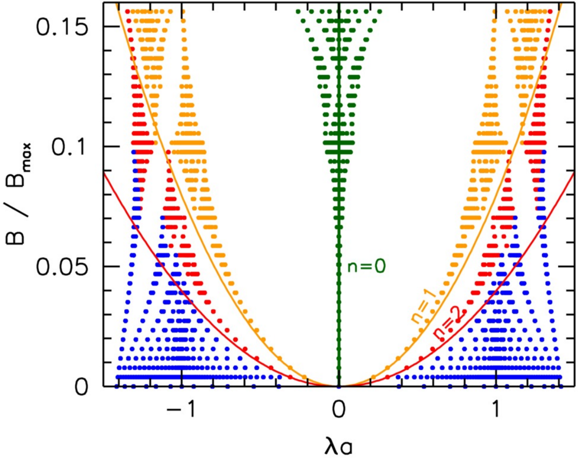

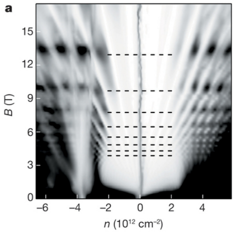

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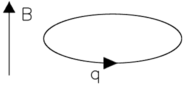


Translation to lattice QCD

From quantum mechanics to QCD

- non-interacting electrons $\rightsquigarrow$ strongly interacting quarks
  smears out butterfly
- quantum mechanics $\rightsquigarrow$ quantum field theory with path integral

$$\mathcal{Z} = \int \mathcal{D}A_\mu \, \mathcal{D}\bar{\psi} \, \mathcal{D}\psi \, e^{-S}$$

$$S = \int \mathrm{d}^4 x \left[\frac{1}{4} \mathrm{Tr} \mathcal{F}_{\mu\nu} \mathcal{F}_{\mu\nu} + \bar{\psi}(\not{D} + m)\psi \right], \qquad \not{D} = \gamma_\mu(\partial_\mu + i g_s \mathcal{A}_\mu + i q A_\mu)$$

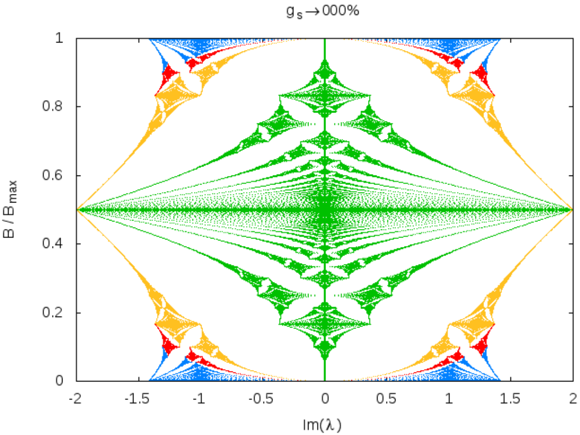
- crystal $\rightsquigarrow$ lattice as a regulator

$$p_{\mathrm{max}} \approx 1/a$$

continuum limit: $a \to 0$

- energies (measureable) $\rightsquigarrow$ Dirac eigenvalues
  (not measureable)

The butterfly in lattice QCD



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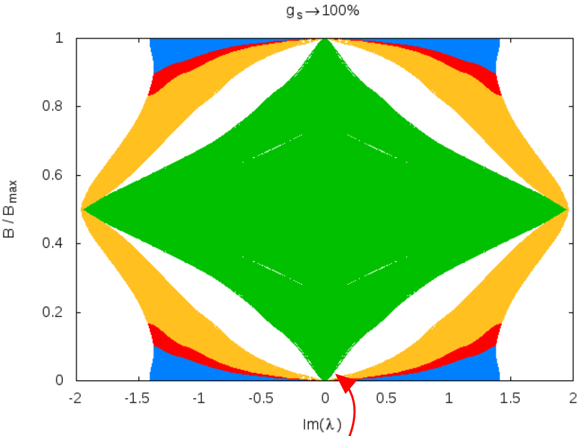

- continuum limit $a \to 0$
  $\Rightarrow$ lowest Landau-level can be separated in full QCD
  (related to topology) [Bruckmann, Endrődi et al '16]

The phases of QCD

Chiral condensate

- physical observable in QCD:
  condensate of quarks with mass m

$$\bar{\psi}\psi = \sum_{\lambda} \frac{m}{\lambda^2 + m^2}$$

- for $m = 0$: order parameter for chiral symmetry breaking

$$\mathrm{SU_L}(N_f) \times \mathrm{SU_R}(N_f) \quad \xrightarrow{\bar{\psi}\psi} \quad \mathrm{SU_{LR}}(N_f)$$

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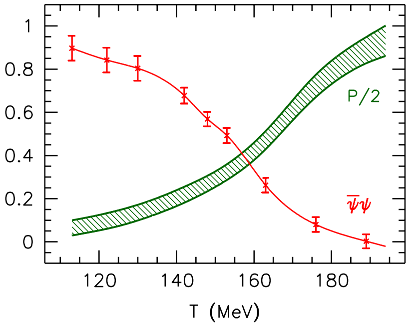
Phases of QCD

- chiral symmetry breaking and confinement go hand in hand

$$\bar{\psi}\psi \quad \Leftrightarrow \quad P$$

- can be probed by the temperature



[Borsányi et al '10, Bruckmann, Endrődi et al '13]

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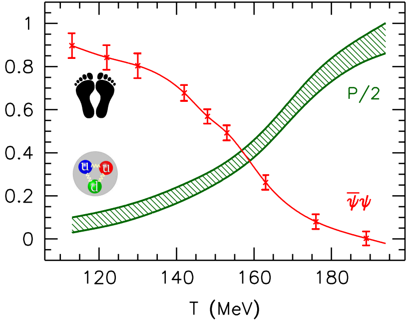

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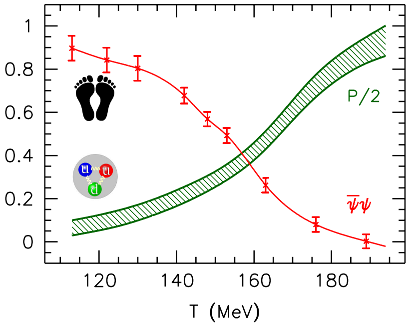
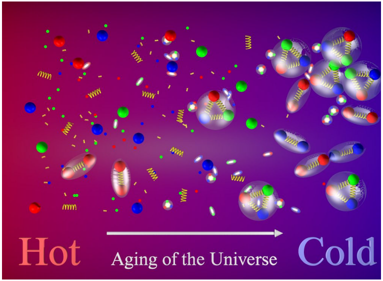
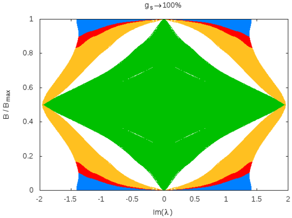




[Borsányi et al '10, Bruckmann, Endrődi et al '13] [Aoki, Endrődi et al '06]

Condensate in magnetic fields

Landau-levels



lowest Landau-level:

- λ well below other eigenvalues
- multiplicity is proportional to magnetic flux $\Phi = B \cdot L^2$

higher Landau-levels:

- $\lambda > \sqrt{2B}$

- if B sufficiently large, only lowest LL matters

Chiral condensate

- physical observable in QCD:
  condensate of quarks with mass m

$$\bar{\psi}\psi = \sum_{\lambda} \frac{m}{\lambda^2 + m^2}$$

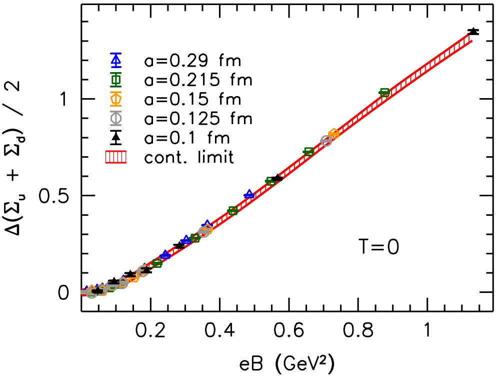
- for $B \gg m^2$, lowest LL approximation

$$\bar{\psi}\psi \sim BL^2 \sum_{\lambda \in \text{ lowest LL}} \frac{m}{\lambda^2 + m^2}$$

- implication: $\bar{\psi}\psi$ increases with B
  'magnetic catalysis' [Gusynin et al '95]

Magnetic catalysis at $T = 0$

- magnetic catalysis from first principles
  [Bali, Bruckmann, Endrődi, Fodor, Katz, Schäfer '12]



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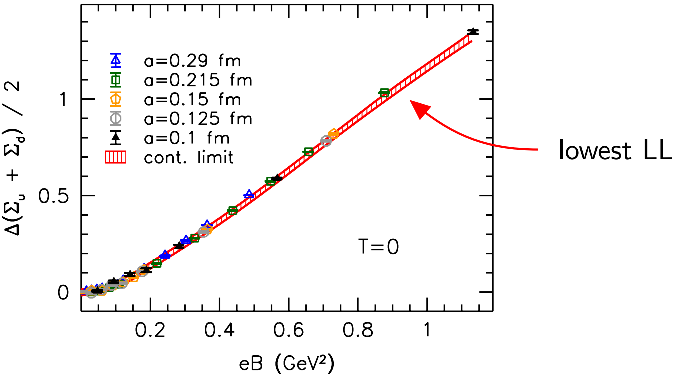

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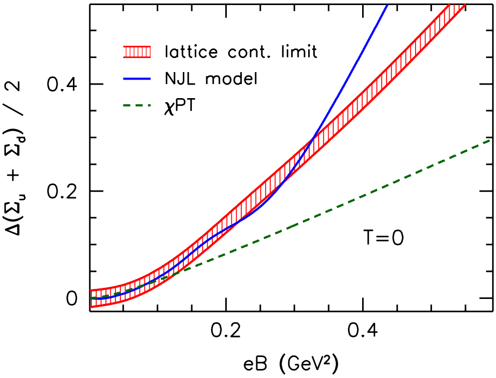


- magnetic catalysis is a robust concept: captured by NJL, χPT, AdS/CFT and further models [Andersen, Naylor '14]

Summary

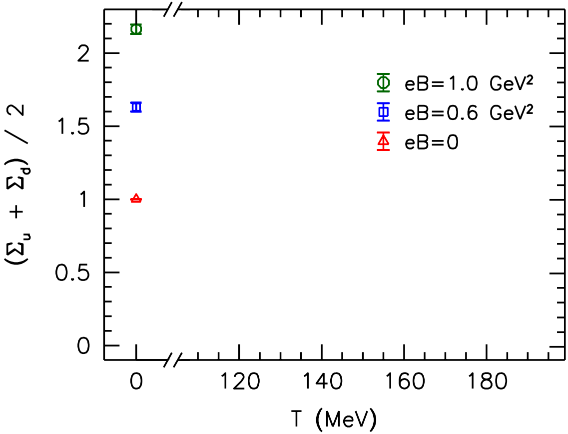
lowest Landau-level

$$\Downarrow$$

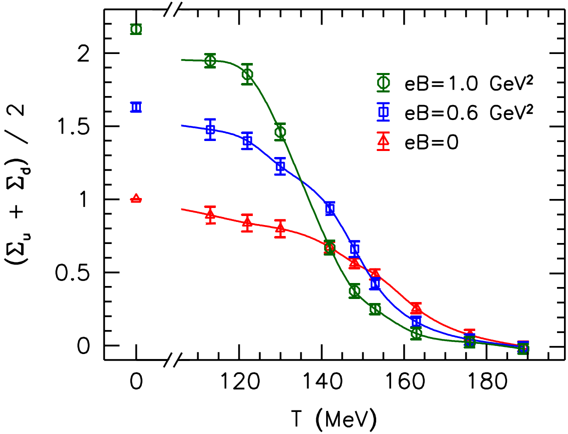
magnetic catalysis

Nonzero temperature

Condensate at nonzero temperatures



Condensate at nonzero temperatures



- going to finite temperatures changes the response qualitatively
  inverse magnetic catalysis around T_c
  [Bruckmann, Endrődi, Kovács '13]

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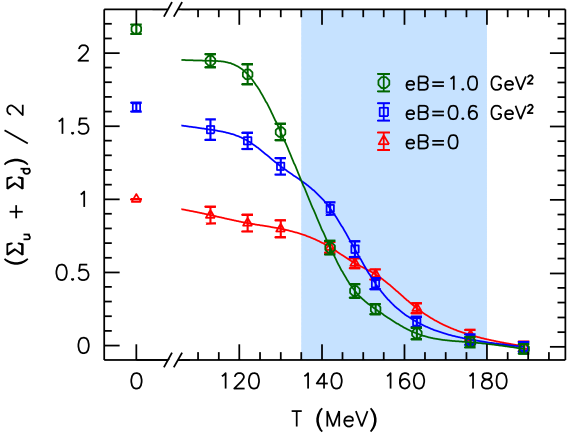

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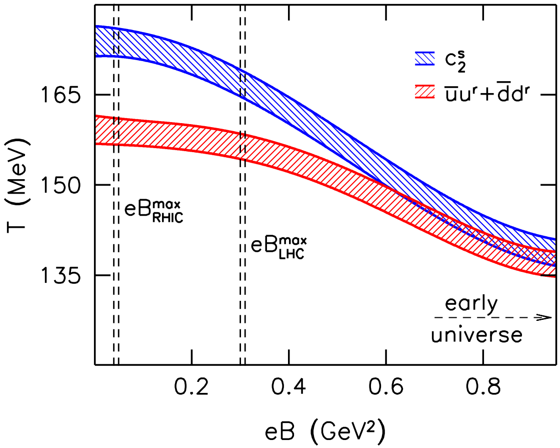
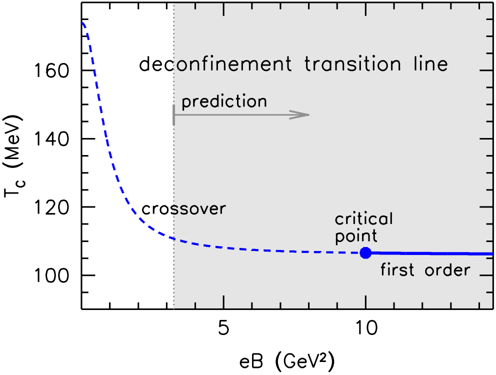
Phase diagram

- impact on the QCD phase diagram: $T_c(B)$ decreases
  [Bali, Bruckmann, Endrődi, Fodor, Katz, Krieg, Schäfer, Szabó '11]



Phase diagram for very strong fields

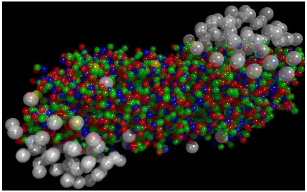
- go to even stronger B-fields
- nature of transition changes from crossover to first order
  [Endrődi '15]

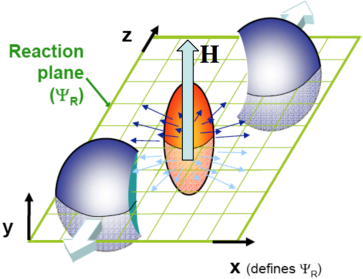


Applications

Applications: heavy-ion collisions

- off-central events generate magnetic fields
  [Kharzeev, McLerran, Warringa '07]

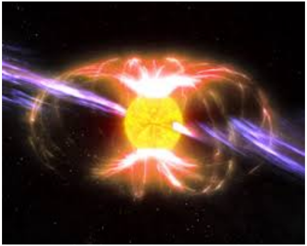


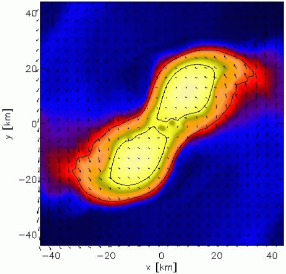


- strength: $B = 10^{15}$ T $\approx 10^{20} B_{\rm earth} \approx 5 m_\pi^2$
- impact: chiral magnetic effect, anisotropies, elliptic flow, hydrodynamics with B, ...
  reviews: [Fukushima '12] [Kharzeev, Landsteiner, Schmitt, Yee '14] [Kharzeev '15]

Applications: magnetars

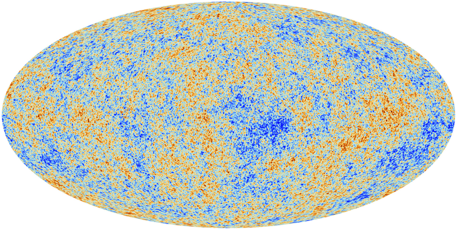
- neutron stars with strong surface magnetic fields [Duncan, Thompson '92]





- strength on surface: $B = 10^{10}$ T
- strength in core (?): $B = 10^{14}$ T $\approx 10^{19} B_{\rm earth} \approx 0.5 m_\pi^2$
- impact: convectional processes, mass-radius relation, gravitational collapse/merger [Anderson et al '08], . . .

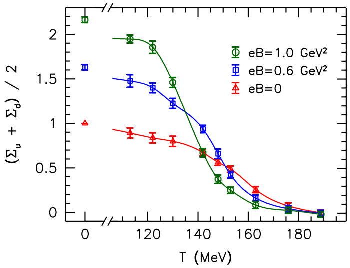
Applications: early universe

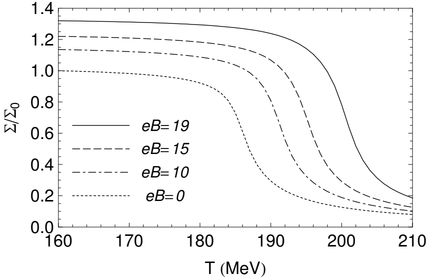


- large-scale intergalactic magnetic fields 10 μG $= 10^{-9}$ T
- origin in the early universe
- generation through a phase transition: electroweak epoch
  $B \approx 10^{19}$ T [Vachaspati '91, Enqvist, Olesen '93]

Applications: low-energy models

- effective theories / low energy models of QCD:
  incorporate relevant degrees of freedom / mechanisms
  for example PNJL model [Gatto, Ruggieri '11]





- models work at low T but not around T_c
- phase diagram just the opposite of the lattice results
- some important ingredient is missed [Andersen, Naylor '14]
  [Braun et al '14, Müller et al '15, . . .]

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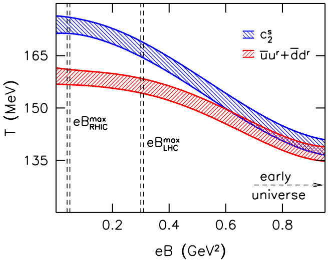

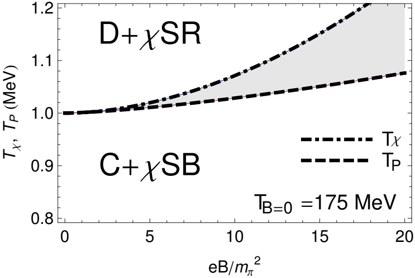

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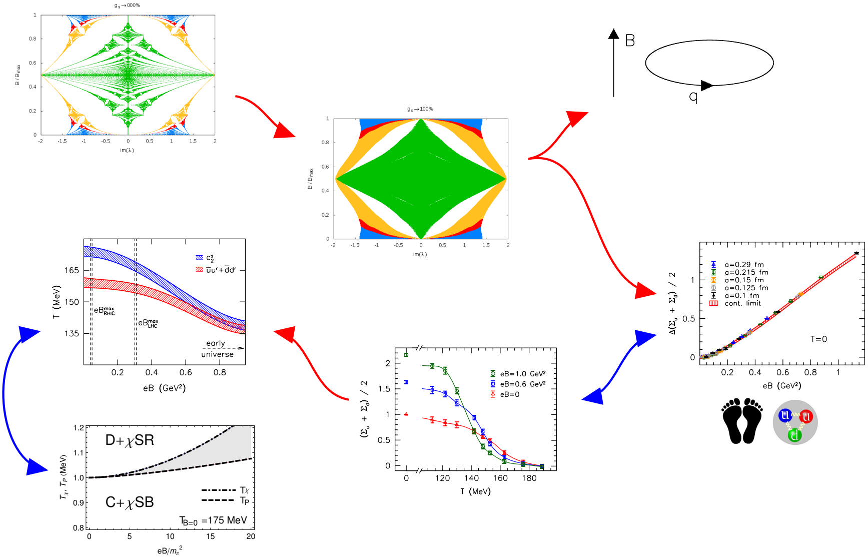
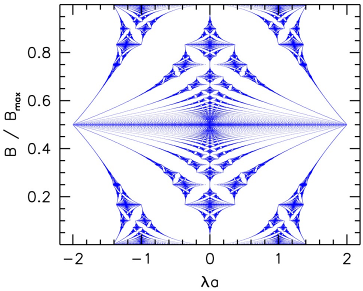
Summary



B
q
B / B max
Im(λ)
a=0.29 fm
a=0.215 fm
a=0.15 fm
a=0.125 fm
a=0.1 fm
cont. limit
T=0
eB (GeV²)
T (MeV)
early universe
eB=1.0 GeV²
eB=0.6 GeV²
eB=0
D+χSR
C+χSB
T_{B=0} =175 MeV
eB/m_π²

Backup

Hofstadter's butterfly [Hofstadter '76]



- true fractal structure (if the lattice is infinite)
- energies accumulate into bands if flux $\Phi \in \mathbb{Q}$ ($2\pi/a^2$ and qB are commensurable)
- energies isomorphic to the Cantor set if $\Phi \notin \mathbb{Q}$ ($2\pi/a^2$ and qB are incommensurable)

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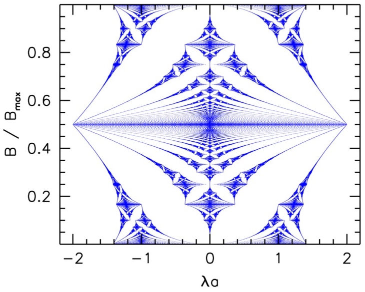

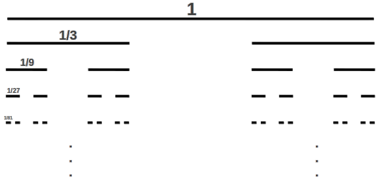

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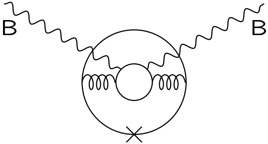
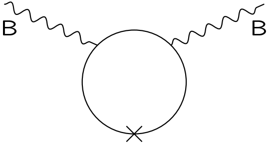
Mechanism behind MC and IMC

- two competing mechanisms at finite B
  [Bruckmann,Endrődi,Kovács '13]
  - direct (valence) effect $B \leftrightarrow q_f$
  - indirect (sea) effect $B \leftrightarrow q_f \leftrightarrow g$

$$\left\langle \bar{\psi}\psi(B) \right\rangle \propto \int \mathcal{D}A_\mu \, e^{-S_g} \underbrace{\det(\not{D}(B,A)+m)}_{\text{sea}} \underbrace{\mathrm{Tr}\left[(\not{D}(B,A)+m)^{-1}\right]}_{\text{valence}}$$





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