The phases of hot/dense/magnetized QCD from the lattice

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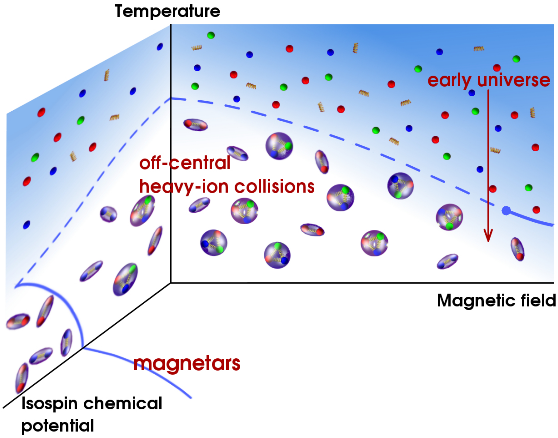
QCD phase diagram



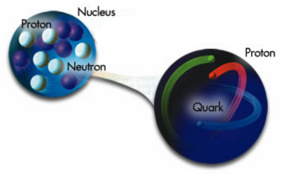
Outline

- relevance of background magnetic fields and isospin asymmetries
- hot/magnetized QCD
  - including magnetic fields
  - including very strong magnetic fields
  - results: phase diagram
- hot/asymmetric QCD
  - including the isospin asymmetry
  - pion condensation
  - results: phase diagram
- further applications
- conclusions

Introduction

Strong interactions

- explain 99.9% of visible matter in the Universe



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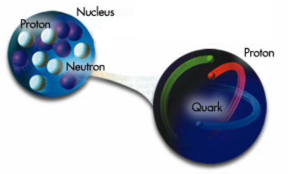

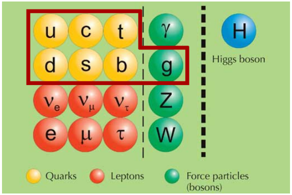

- elementary particles: quarks and gluons

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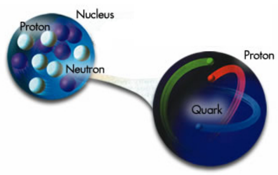

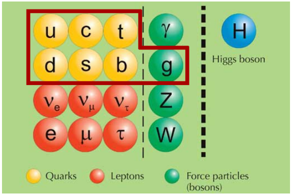

- elementary particles: quarks and gluons
- elementary fields: $\psi(x)$ and $A_\mu(x)$
- QCD Lagrangian

$$\mathcal{L}_{\rm QCD} = \frac{1}{4} \mathrm{Tr}\, F_{\mu\nu}(g_s, A)^2 + \bar{\psi}[\gamma_\mu(\partial_\mu + i g_s A_\mu) + m]\psi$$

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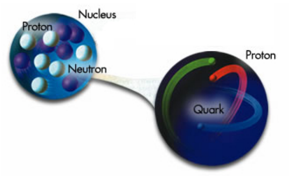

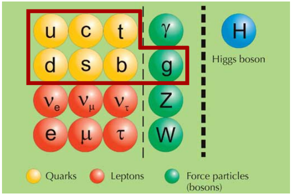

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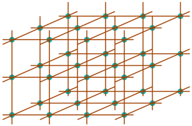
- $g_s = \mathcal{O}(1) \rightsquigarrow$ non-perturbative physics

Path integral and lattice field theory

- path integral [Feynman '48]

$$\mathcal{Z} = \int \mathcal{D}A_\mu \, \mathcal{D}\bar{\psi} \, \mathcal{D}\psi \, \exp\left(- \int \mathrm{d}^4 x \, \mathcal{L}_{\mathrm{QCD}}(x) \right)$$

- discretize spacetime on a lattice with spacing a [Wilson '74]



- Monte-Carlo algorithms to generate configurations
- 10^9-dimensional integrals ⇝ high-performance computing

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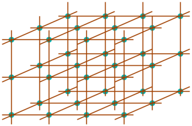
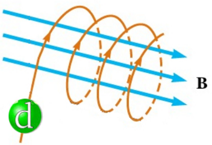


- Monte-Carlo algorithms to generate configurations
- 10^9-dimensional integrals $\rightsquigarrow$ high-performance computing

QCD and external parameters

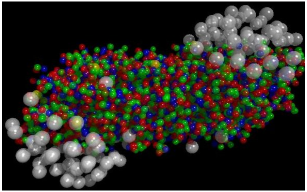
- running coupling $g_s(E)$
- relevant parameters that control the energy scale:
  - temperature T: excites all states
  - baryon density $n_B \propto n_u + n_d$: excites p^+ and n
  - isospin asymmetry $n_I \propto n_u - n_d$:
    creates p^+-n asymmetry, excites π^+
  - background magnetic field B: forces quarks on Landau levels

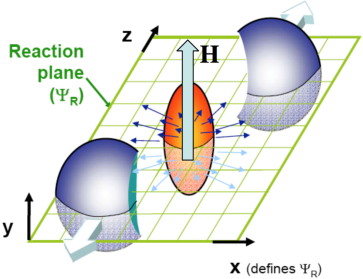


(chemical potentials conjugate to densities: μ_B, μ_I)

Magnetic fields: heavy-ion collisions

- off-central events generate magnetic fields
  [Kharzeev, McLerran, Warringa '07]

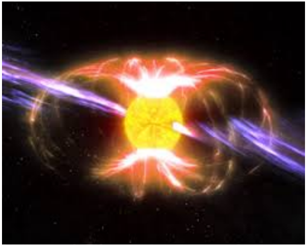


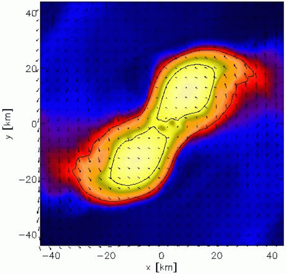


- strength: $B = 10^{15}$ T $\approx 10^{20} B_{\rm earth} \approx 5 m_\pi^2$
- impact: chiral magnetic effect, anisotropies, elliptic flow, hydrodynamics with B, . . .
  reviews: [Fukushima '12] [Kharzeev, Landsteiner, Schmitt, Yee '14] [Kharzeev '15]

Magnetic fields magnetars

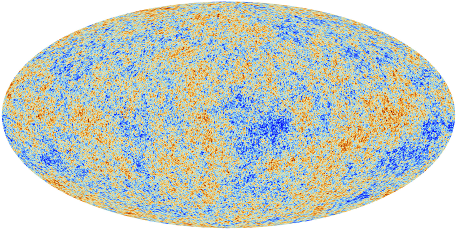
- neutron stars with strong surface magnetic fields [Duncan, Thompson '92]





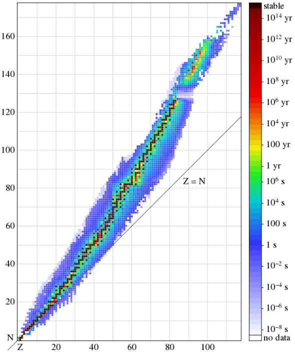
- strength on surface: $B = 10^{10}$ T
- strength in core: $B = 10^{14...16}$ T $\approx 10^{19...21} B_{\text{earth}}$
- impact: convectional processes, mass-radius relation, gravitational collapse/merger [Anderson et al '08], ...

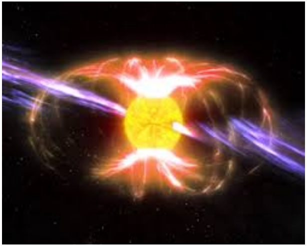
Magnetic fields: early universe



- large-scale intergalactic magnetic fields 10 μG $= 10^{-9}$ T
- origin in the early universe
- generation through a phase transition: electroweak epoch $B \approx 10^{19}$ T [Vachaspati '91, Enqvist, Olesen '93]

Isospin asymmetry: nuclei and neutron stars





- neutron to nucleon ratio in nuclei $\frac{Z}{A} \approx 0.4$
  but: 'neutron skin' near surface
- neutron to nucleon ratio in interior of neutron stars $\frac{Z}{A} \approx 0.025$

Order parameters

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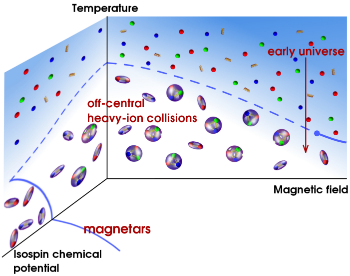


- quark condensate $\bar{\psi}\psi$ (chiral symmetry breaking)

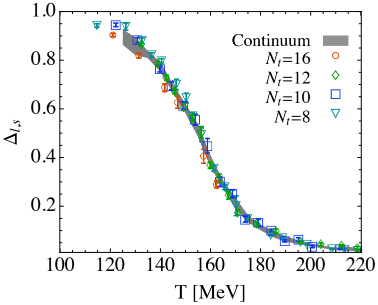
$$\bar{\psi}\psi = \frac{T}{V}\frac{\partial \log \mathcal{Z}}{\partial m} = \left\langle \mathrm{Tr}\,\frac{1}{\not{D} + m} \right\rangle$$

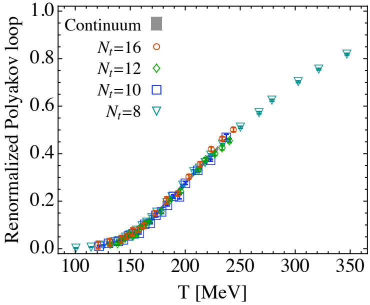
- Polyakov loop P (deconfinement)

$$P = \left\langle \mathrm{Tr}\,\mathcal{P}\exp\int_0^{1/T} A_4(\mathbf{x},\tau)\,\mathrm{d}\tau \right\rangle$$

Order parameters

- $T_c \leftrightarrow$ inflection point
- nature of phase transition $\leftrightarrow$ singularity in slope at T_c





[Aoki, Endrődi et al '06, Borsányi et al. '10]

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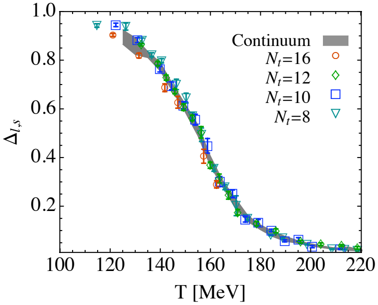
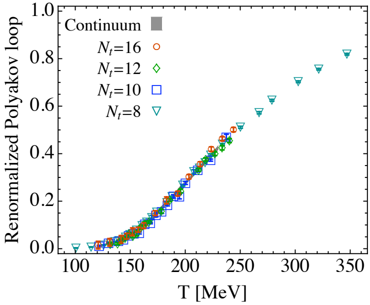




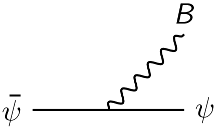
[Aoki, Endrődi et al '06, Borsányi et al. '10]

- analytical crossover

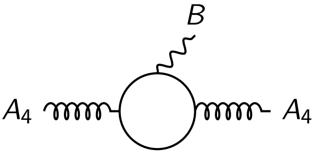
Background magnetic fields

Impact of magnetic fields

- on quark condensate: primary



- on Polyakov loop: secondary

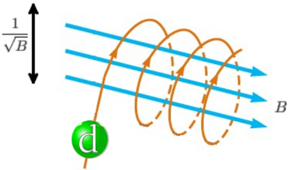


Magnetic catalysis

- chiral condensate $\leftrightarrow$ spectral density around 0 [Banks, Casher '80]

$$\bar{\psi}\psi \sim \mathrm{tr} \not{D}^{-1} \propto \rho(0)$$

- large magnetic fields reduce dimensionality $3+1 \to 1+1$ and induce degeneracy $\propto B$



- to maintain $\bar{\psi}\psi > 0$ [Gusynin et al '96]

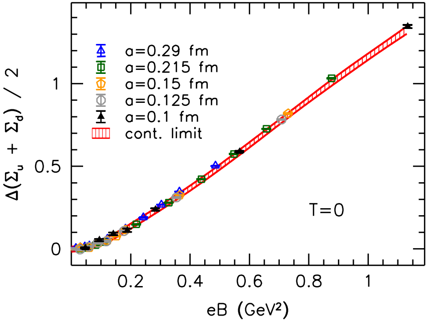
$B = 0$	$\rho(p)\mathrm{d}p \sim Tp^2\mathrm{d}p$	"strong interaction is needed"
$B \gg m^2$	$\rho(p)\mathrm{d}p \sim TB\,\mathrm{d}p$	"the weakest interaction suffices"

Magnetic catalysis: lattice simulations

- numerical simulation of the path integral

$$\mathcal{Z} = \int \mathcal{D}U\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\ \exp(-S_{\rm QCD})$$

- obtain condensate from $\bar{\psi}\psi = \frac{1}{V_4}\frac{\partial \log \mathcal{Z}}{\partial m}$

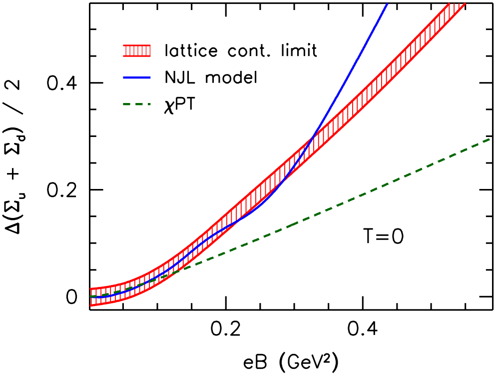


- physical m_π, continuum limit

  [Bali,Bruckmann,Endrődi,Fodor,Katz,Schäfer '12]

Magnetic catalysis – zero temperature

- magnetic catalysis at zero temperature is a robust concept: χPT, NJL, AdS-CFT, linear σ model, lattice QCD, . . .
  [Andersen, Naylor '14]



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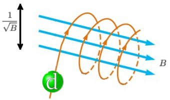
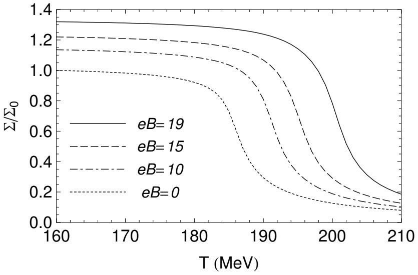
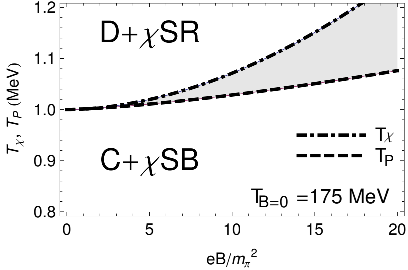
Effect of magnetic fields: nonzero temperature

Phase diagram: models



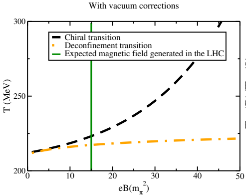
- recall catalysis argument $\bar{\psi}\psi \propto \rho(0)$
- model calculations at $T > 0$:
  - magnetic catalysis for all T
  - $T_c(B)$ increases
- for example the PNJL model [Gatto, Ruggieri '11]

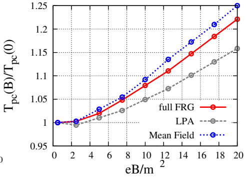


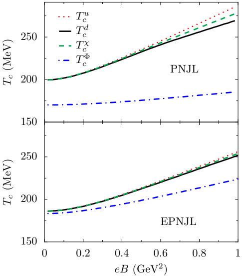


Phase diagram: models

- majority of low-energy models give the same qualitative result
  - linear sigma model + Polyakov loop [Mizher, Chernodub, Fraga '10]
  - quark-meson model + functional renormalization group [Kamikado, Kanazawa '13]
  - NJL model + Polyakov loop [Ferreira, Costa, Menezes, Providencia, Scoccola '13]
  - ...

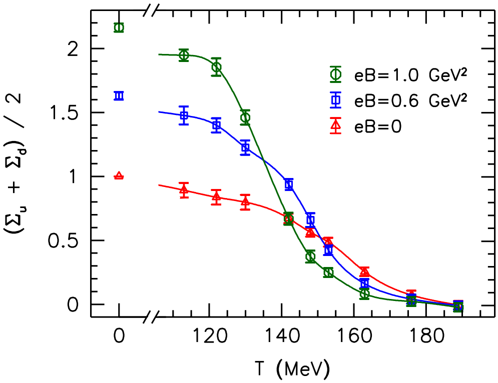






Phase diagram: lattice simulations

- lattice QCD, physical m_π, continuum limit
  [Bali,Bruckmann,Endrődi,Fodor,Katz,Krieg,Schäfer,Szabó '11, '12]



- surprise: magnetic catalysis turns into inverse magnetic catalysis (IMC) around T_c [Bruckmann, Endrődi, Kovács '13]

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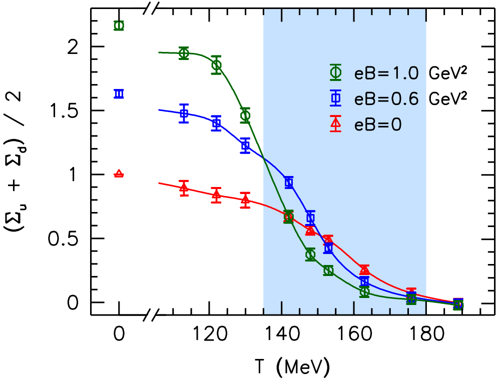

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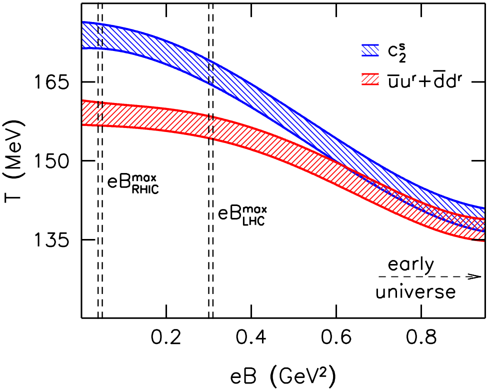

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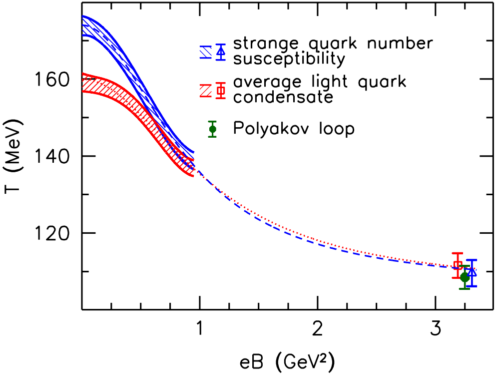
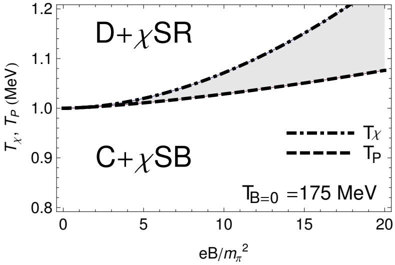
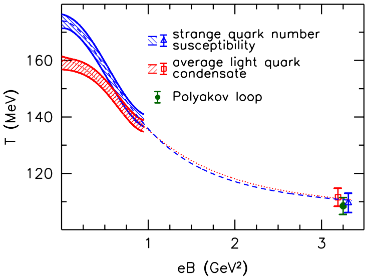


Phase diagram: comparison





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$T_c(B)$	increases	decreases
$T_c^{(P)}$ and $T_c^{(\bar{\psi}\psi)}$	diverge	converge
condensate	magnetic catalysis $\forall T$	inverse catalysis $T \approx T_c$

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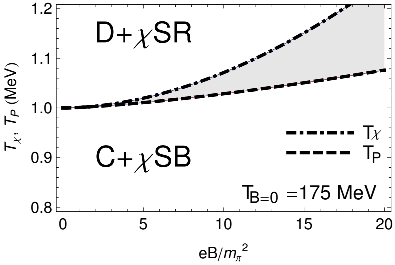

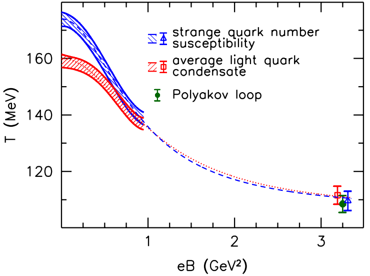

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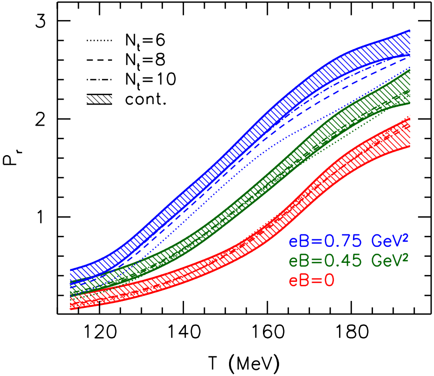
- models and lattice simulations are as different as can be

Inverse magnetic catalysis

- related to secondary effect of B on gluons

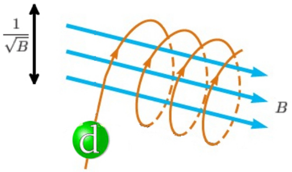


- encoded in effective potential for P in models
- tune this potential or coupling constants of models with B
  [Fraga et al. '13, Ferreira et al. '14, Ayala et al. '15, '18 ...]

Large B: anisotropic effective theory

Large B limit

- what happens to $\mathcal{L}_{\rm QCD}$ at $eB \gg \Lambda^2_{\rm QCD}, T^2$?
  - first guess: asymptotic freedom says $\alpha_s \to 0$ i.e. complete decoupling of quarks and gluons
  - but: B breaks rotational symmetry and effectively reduces the dimension of the theory for quarks

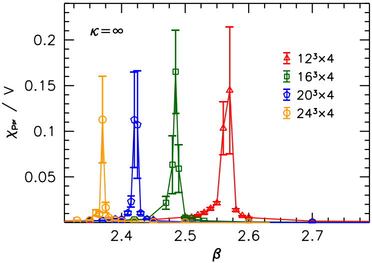


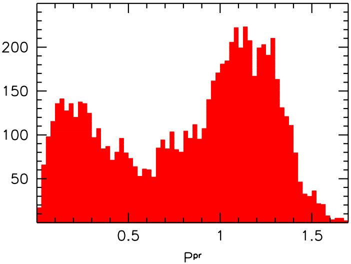
- gluons also inherit this spatial anisotropy, $\kappa(B) \propto B$
  [Miransky, Shovkovy 2002; Endrődi 1504.08280]

$$\mathcal{L}_{\rm QCD} \xrightarrow{B\to\infty} \mathrm{tr}\,\mathcal{B}_\parallel^2 + \mathrm{tr}\,\mathcal{B}_\perp^2 + [1+\kappa(B)]\,\mathrm{tr}\,\mathcal{E}_\parallel^2 + \mathrm{tr}\,\mathcal{E}_\perp^2$$

Simulating the anisotropic effective theory

- pure (but anisotropic) gauge theory: can be simulated on the lattice [Endrődi 1504.08280]





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- histogram shows double peak-structure at T_c

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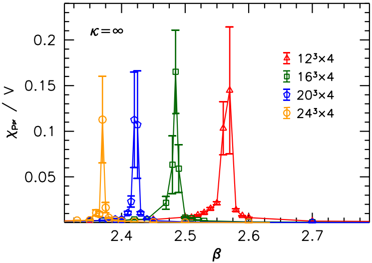
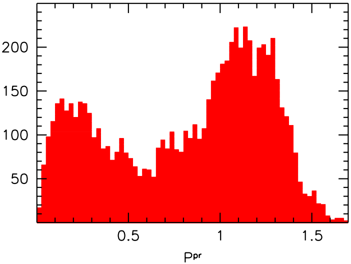




- Polyakov loop susceptibility peak height scales with V
- histogram shows double peak-structure at T_c
- the transition is of first order

Critical point

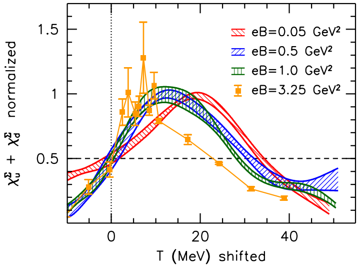
- analytical crossover for $0 \leq eB \leq 3.25$ GeV^2

  first-order transition for $B \to \infty$

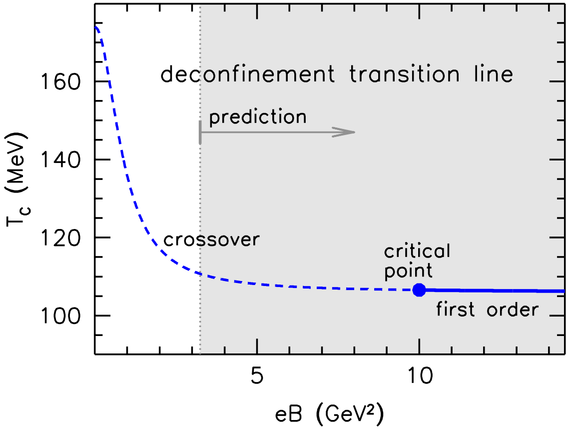
  ▸ there must be a critical point in between [Cohen, Yamamoto '13]

- estimate: extrapolate width of susceptibility peak to 0

$eB_{\mathrm{CP}} \approx 10(2)$ GeV^2



Phase diagram



Isospin asymmetry

Isospin chemical potential

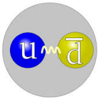
- quark chemical potentials (3-flavor)

$$\mu_u = \frac{\mu_B}{3} + \mu_I \qquad \mu_d = \frac{\mu_B}{3} - \mu_I \qquad \mu_s = \frac{\mu_B}{3} - \mu_S$$

- zero baryon number, zero strangeness, but nonzero isospin

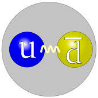
$$\mu_u = \mu_I \qquad \mu_d = -\mu_I \qquad \mu_s = 0$$

- pion chemical potential $\mu_\pi = \mu_u - \mu_d = 2\mu_I$



- isospin density $n_I = n_u - n_d$

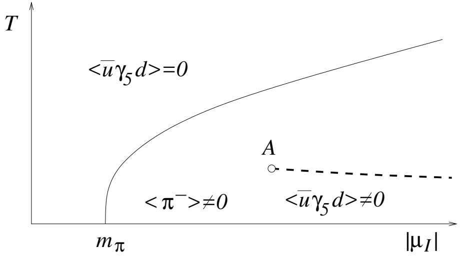
Pion condensation



- QCD at low energies ≈ pions
  chiral perturbation theory
- chemical potential for charged pions: μ_π

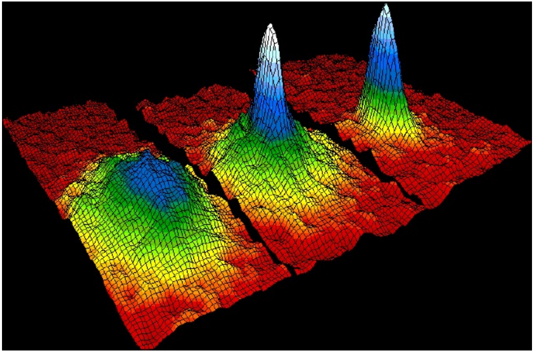
at zero temperature	$\mu_\pi < m_\pi$	vacuum state
	$\mu_\pi \geq m_\pi$	Bose-Einstein condensation

[Son, Stephanov '00]



Bose-Einstein condensate

- accumulation of bosonic particles in lowest energy state



[Anderson et al '95 JILA-NIST/University of Colorado]

- velocity distribution of Ru atoms at low temperature
  $\rightarrow$ peak at zero velocity (zero energy)

Setup on the lattice

[Brandt, Endrődi, Schmalzbauer '17]

Symmetry breaking

- QCD with light quark matrix

$$M = \not{D} + m_{ud}\mathbb{1}$$

- chiral symmetry (flavor-nontrivial)

$$\mathrm{SU}(2)_V$$

Symmetry breaking

- QCD with light quark matrix

$$M = \not{D} + m_{ud}\mathbb{1} + \mu_I \gamma_0 \tau_3$$

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$$\mathrm{SU}(2)_V \rightarrow \mathrm{U}(1)_{\tau_3}$$

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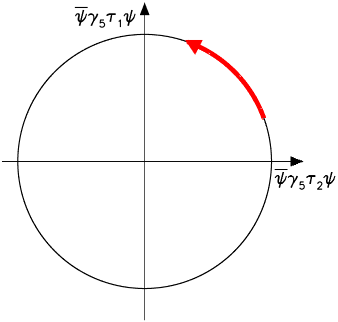


- spontaneously broken by a pion condensate

$$\langle\pi^\pm\rangle = \left\langle\bar{\psi}\gamma_5\tau_{1,2}\psi\right\rangle$$

- a Goldstone mode appears

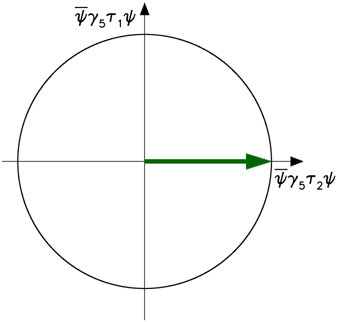
Symmetry breaking

- QCD with light quark matrix

$$M = \not{D} + m_{ud}\mathbb{1} + \mu_I\gamma_0\tau_3 + i\lambda\gamma_5\tau_2$$

- chiral symmetry (flavor-nontrivial)

$$\mathrm{SU}(2)_V \to \mathrm{U}(1)_{\tau_3} \to \varnothing$$



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Symmetry breaking

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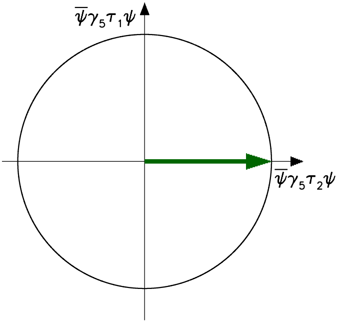

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- a Goldstone mode appears
- add small explicit breaking
- extrapolate results $\lambda \to 0$

Simulation details

- staggered light quark matrix with $\eta_5 = (-1)^{n_x+n_y+n_z+n_t}$

$$M = \begin{pmatrix} \not{D}(\mu_I) + m & \lambda\eta_5 \\ -\lambda\eta_5 & \not{D}(-\mu_I) + m \end{pmatrix}$$

- we have $\gamma_5\tau_1$-hermiticity

$$\eta_5\tau_1 M \tau_1 \eta_5 = M^\dagger$$

- determinant is real and positive

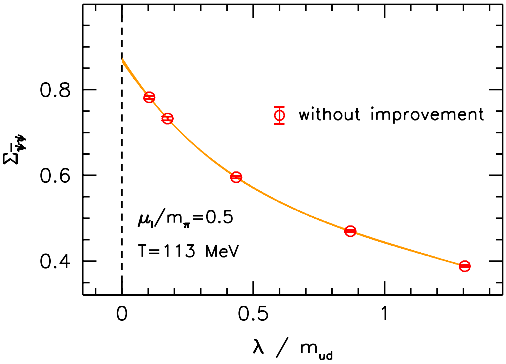
$$\det M = \det(|\not{D}(\mu_I) + m|^2 + \lambda^2) > 0$$

- partition function via Monte-Carlo

$$\mathcal{Z} = \int \mathcal{D}U \, \underbrace{\det(|\not{D}(\mu_I) + m|^2 + \lambda^2)}_{\text{light quarks}} \, \underbrace{\det(\not{D}(0) + m_s)}_{\text{strange quark}} \, \underbrace{e^{-S_g}}_{\text{gluons}}$$

Need for improvement

- simulations at $\lambda > 0$ are far away from desired $\lambda \to 0$ limit



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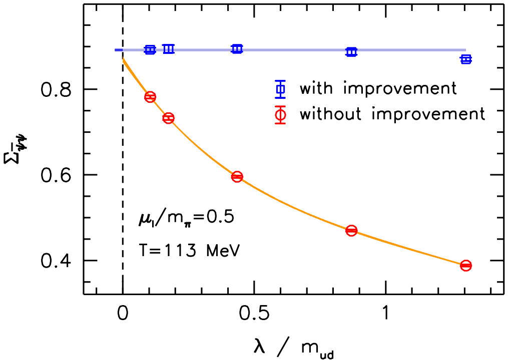

- improvement using the singular value representation of M

Singular value representation

- pion condensate

$$\Sigma_\pi = \frac{\partial}{\partial\lambda} \log\det(|\not{D}(\mu_I) + m|^2 + \lambda^2) = \mathrm{Tr}\frac{2\lambda}{|\not{D}(\mu_I) + m|^2 + \lambda^2}$$

- singular values

$$|\not{D}(\mu_I) + m|^2 \psi_i = \xi_i^2 \psi_i$$

- spectral representation

$$\Sigma_\pi = \frac{T}{V}\sum_i \frac{2\lambda}{\xi_i^2 + \lambda^2} \xrightarrow{V\to\infty} \int \mathrm{d}\xi\, \rho(\xi)\, \frac{2\lambda}{\xi^2 + \lambda^2} \xrightarrow{\lambda\to 0} \pi\rho(0)$$

first derived in [Kanazawa, Wettig, Yamamoto '11]

Singular value representation

- pion condensate

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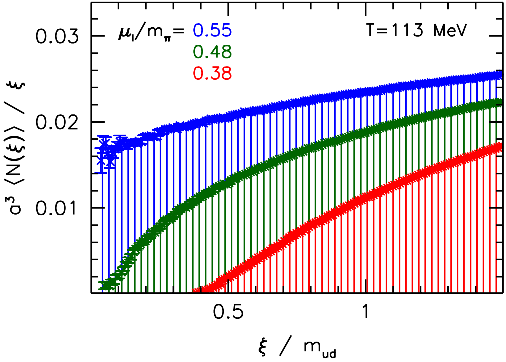
first derived in [Kanazawa, Wettig, Yamamoto '11]

- compare to Banks-Casher-relation at $\mu_I = 0$

Singular value density

- integrated spectral density

$$N(\xi) = \int_0^{\xi} d\xi' \rho(\xi'), \qquad \rho(0) = \lim_{\xi \to 0} N(\xi)/\xi$$



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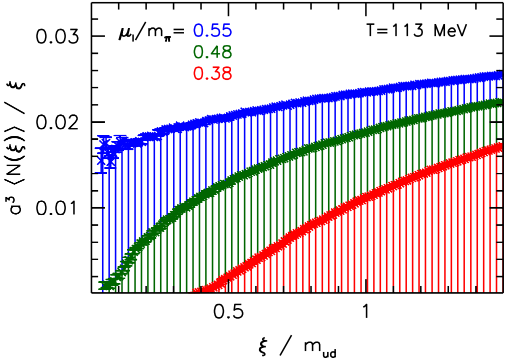

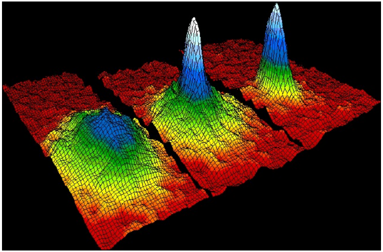

- compare $\rho(0)$ to velocity distribution around zero

Singular value density

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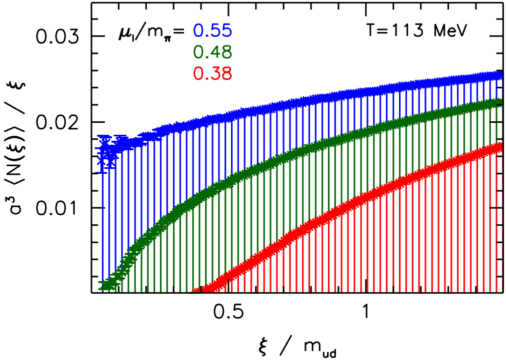
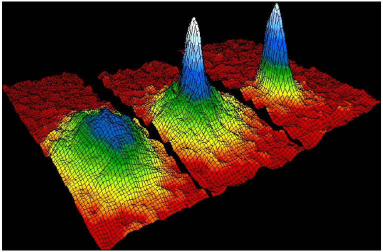


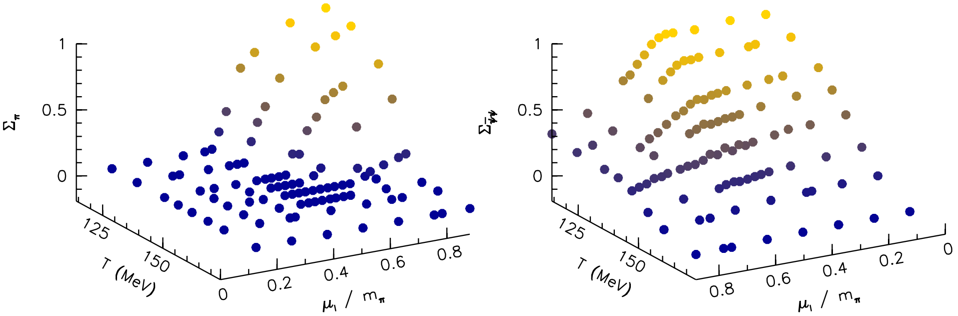


- compare $\rho(0)$ to velocity distribution around zero
- Bose-Einstein condensation!

Results: phase transition

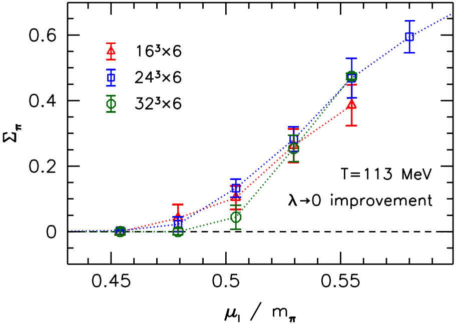
Condensates

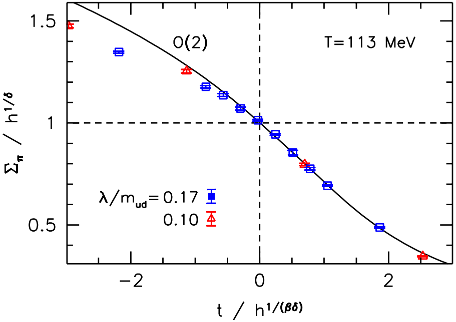
- pion and chiral condensate after $\lambda \to 0$ extrapolation



- read off chiral crossover $T_{pc}(\mu_I)$ and pion condensation boundary $\mu_{I,c}(T)$

Order of the transition





- volume scaling of order parameter shows typical sharpening
- collapse according to $\mathrm{O}(2)$ critical exponents [Ejiri et al '09]

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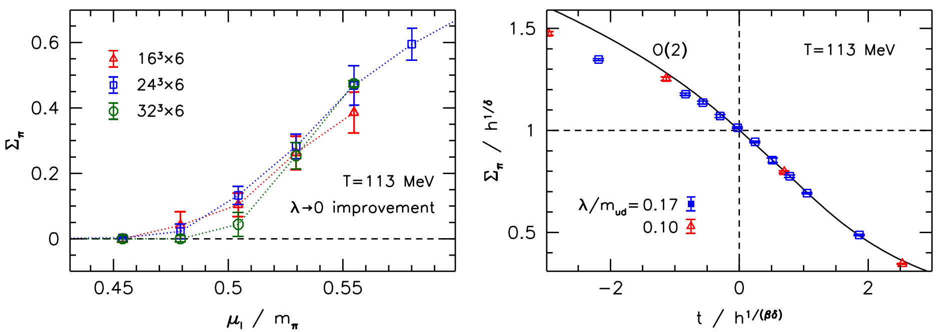
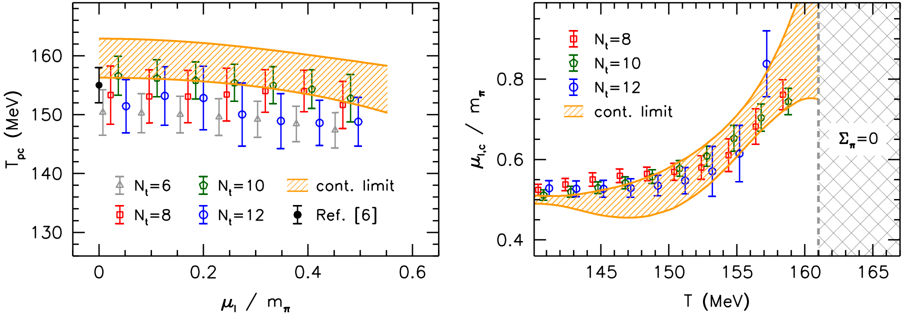


- volume scaling of order parameter shows typical sharpening
- collapse according to $\mathrm{O}(2)$ critical exponents [Ejiri et al '09]
- indications for a second order phase transition at $\mu_I = m_\pi/2$, in the $\mathrm{O}(2)$ universality class

Continuum extrapolations

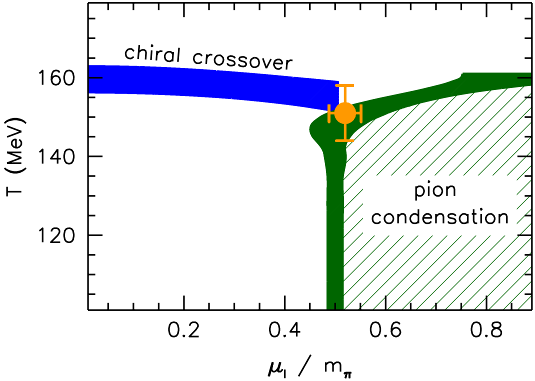
- compare (pseudo)critical temperatures for different lattice spacings $a = 1/(N_t T)$
- take continuum limit $a \to 0$ ($N_t \to \infty$)



Phase diagram

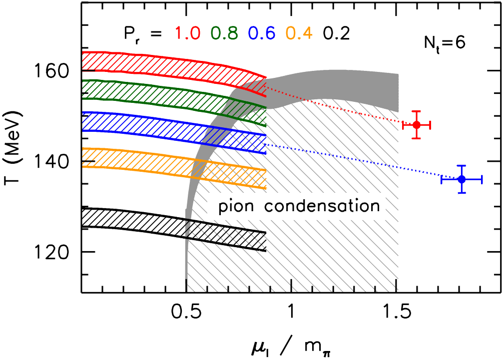
- meeting point of chiral crossover and pion condensation boundary: *pseudo-triple* point

  at $T_{pt} = 151(7)$ MeV, $\mu_{I,pt} = 70(5)$ MeV

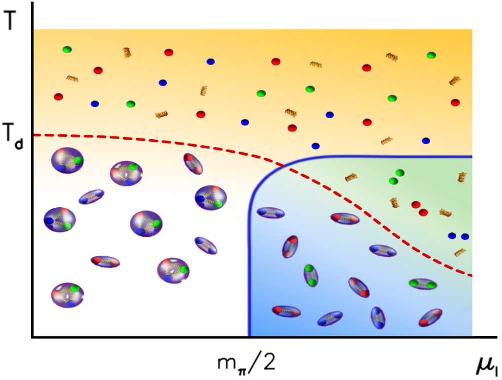


Deconfinement vs chiral symmetry breaking

- Polyakov loop contour lines apparently insensitive to pion condensation boundary
- existence of a condensed but deconfined phase?



Conjectured phase diagram



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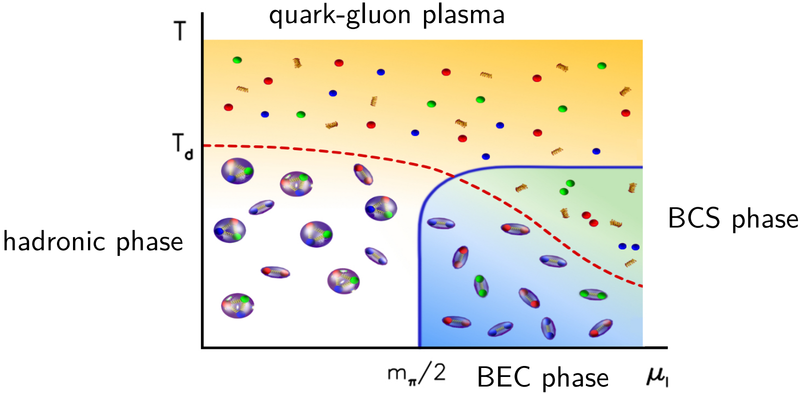

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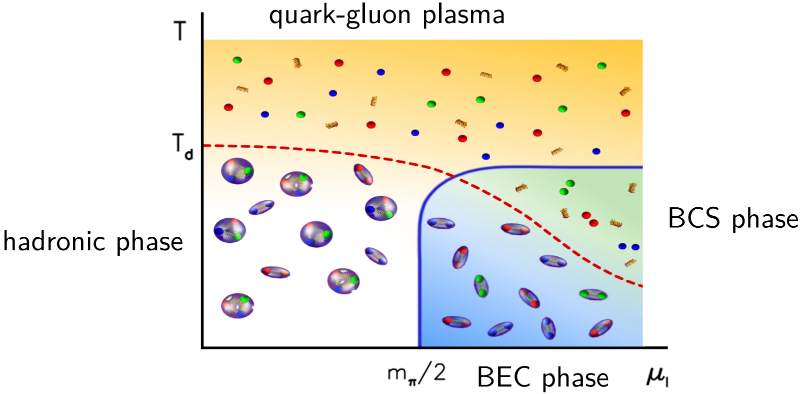
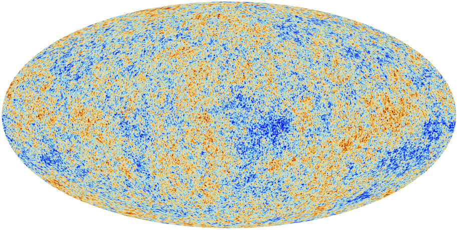


- BCS phase expected on general grounds for high μ_I
  [Son, Stephanov '00]

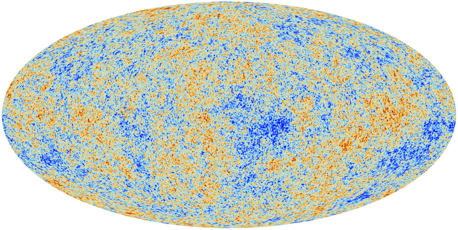
Potential applications

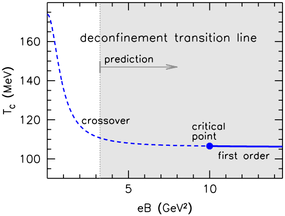
Magnetic fields in the early universe?



- large-scale intergalactic magnetic fields 10 μG $= 10^{-9}$ T origin in the early universe
- generation through a phase transition: electroweak epoch $B \approx 10^{19}$ T ≈ 600 GeV$^2/e$ [Vachaspati '91, Enqvist, Olesen '93]
- how large is B that survives until the QCD epoch?

Magnetic fields in the early universe?





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- how large is B that survives until the QCD epoch?

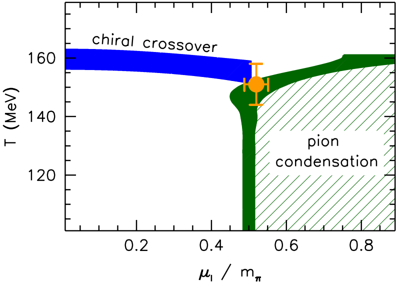
Pion condensation in the early universe?

- weak equilibrium

$$u \leftrightarrow d\, e^- \,\bar{\nu}_e$$

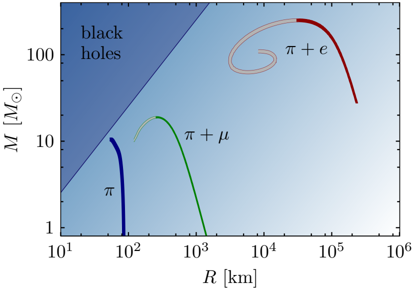
(simplicity: one family of particles)

- charge neutrality $n_Q = 0$, baryon symmetry $n_B = 0$ but nonzero lepton number $n_L \neq 0$
- chemical potentials μ_Q, μ_B and μ_L
- can $\mu_Q = 2\mu_I > m_\pi$ be reached? for sufficiently large n_L, yes [Abuki, Brauner, Warringa '09]



Pion condensation in compact stars?

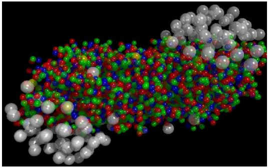
- equation of state for isospin-dense system at low T, neutralized by leptons
  [Brandt, Endrődi, Fraga, Hippert, Schaffner-Bielich, Schmalzbauer '18]
- solve Tollman-Oppenheimer-Volkov equations for gravitational stability

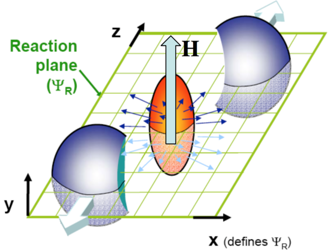


- weak decays under investigation

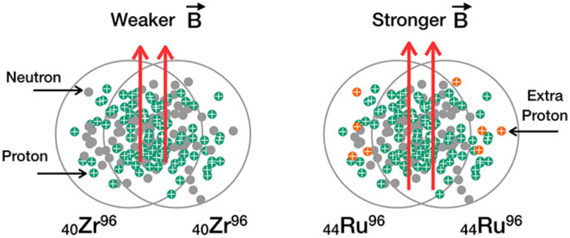
Magnetic fields in heavy-ion collisions?

- off-central events generate magnetic fields
  [Kharzeev, McLerran, Warringa '07]





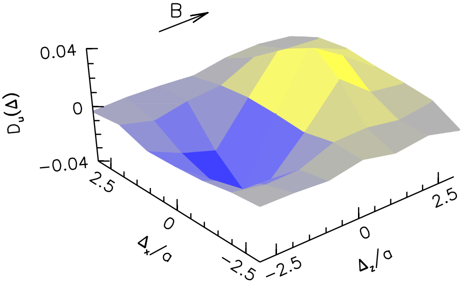
- test charge-dependence in RHIC isobar run [bnl.gov]



CME-sensitive observables for nonzero density

- correlation of topology and electric polarization
  [Bali, Bruckmann, Endrődi, Fodor, Katz, Schäfer '14]

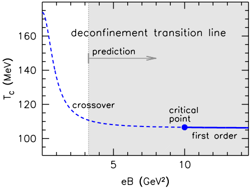
$$D_u(\Delta) = \int \mathrm{d}^4x \, \langle q_{\mathrm{top}}(x) E_z(x+\Delta) \rangle$$



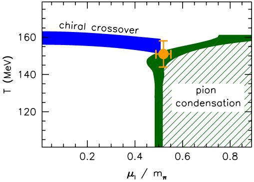
- correlations affected by isospin chemical potential?

Summary

- phase diagram for strong background magnetic fields



- phase diagram for nonzero isospin-asymmetry



- new Banks-Casher-type relation
  - ⇝ establish pion condensation
  - ⇝ improve various observables