QCD matter with isospin-asymmetry

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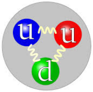
Outline

- introduction: QCD with isospin asymmetry
- setup
  - pion condensation
- results
  - phase boundary for pion condensation
  - deconfinement
  - QCD phase diagram
  - further applications
- conclusions

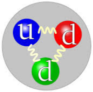
Introduction

Isospin chemical potential

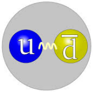
- isospin density $n_I = n_u - n_d$
- $n_I < 0 \rightarrow$ excess of neutrons over protons
  $\rightarrow$ excess of π^- over π^+



p^+

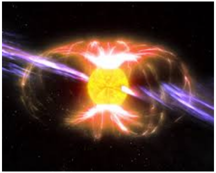


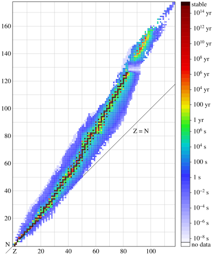
n



π^+

- applications
  - neutron stars
  - heavy-ion collisions (RHIC isobaric runs)





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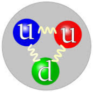

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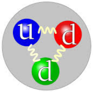

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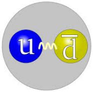


π^+

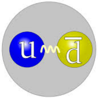
- chemical potentials (2-flavor)

$$\mu_B = 3(\mu_u + \mu_d)/2 \qquad \mu_I = (\mu_u - \mu_d)/2$$

- here: zero baryon number but nonzero isospin

$$\mu_u = \mu_I \qquad \mu_d = -\mu_I$$

Methods

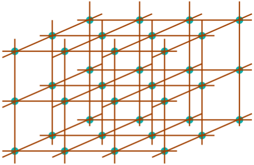


- QCD at low energies $\approx$ pions
  chiral perturbation theory
- on the level of charged pions: $\mu_\pi = 2\mu_I$

at zero temperature	$\mu_\pi < m_\pi$	vacuum state
	$\mu_\pi = m_\pi$	Bose-Einstein condensation

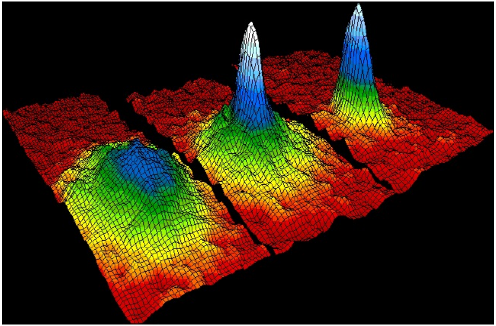
- on the level of quarks: lattice simulations

$$\mathcal{L}_{\rm QCD} = \bar{\psi} M \psi + \frac{1}{4} \mathrm{tr}\, F_{\mu\nu} F_{\mu\nu}$$



Bose-Einstein condensate

- accumulation of bosonic particles in lowest energy state



[Anderson et al '95 JILA-NIST/University of Colorado]

- velocity distribution of Ru atoms at low temperature
  $\rightarrow$ peak at zero velocity (zero energy)

Bose-Einstein condensate

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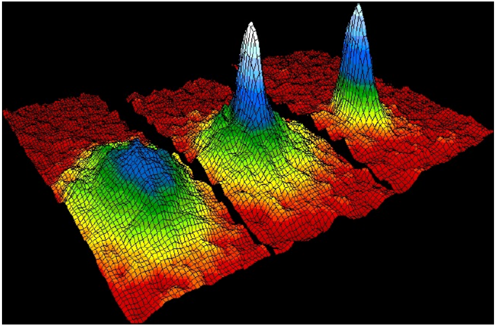

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- velocity distribution of Ru atoms at low temperature
  $\rightarrow$ peak at zero velocity (zero energy)
- phase transition, spontaneous symmetry breaking

Lattice setup

- partition function

$$\mathcal{Z} = \int \mathcal{D}U \; \underbrace{\det \mathcal{M}_\ell}_{\text{light quarks}} \; \underbrace{\det \mathcal{M}_s}_{\text{strange quark}} \; \underbrace{e^{-S_\mathrm{g}}}_{\text{gluons}}$$

$$\mathcal{M}_\ell = \begin{pmatrix} \not{D}(\mu_I) + m_\ell & \lambda\gamma_5 \\ -\lambda\gamma_5 & \not{D}(-\mu_I) + m_\ell \end{pmatrix} \qquad \mathcal{M}_s = \not{D}(0) + m_s$$

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- isospin chemical potential for the light quarks

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- zero strangeness

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- degenerate light quark masses

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- isospin chemical potential for the light quarks
- zero strangeness
- degenerate light quark masses
- pionic source: explicit symmetry breaking
  necessary for spontaneous symmetry breaking in finite volume
  needs to be extrapolated $\lambda \to 0$

Pion condensate from the lattice

Singular value representation

- pion condensate

$$\langle \pi^{\pm} \rangle = \frac{T}{V} \frac{\partial \log \mathcal{Z}}{\partial \lambda} = \ldots = \frac{T}{V} \left\langle \mathrm{Tr} \frac{2\lambda}{|\not{D}(\mu_I) + m_\ell|^2 + \lambda^2} \right\rangle$$

- singular values

$$|\not{D}(\mu_I) + m|^2 \, \psi_i = \xi_i^2 \, \psi_i$$

- spectral representation [Brandt, Endrődi 1611.06758]

$$\langle \pi^{\pm} \rangle = \frac{T}{V} \left\langle \sum_i \frac{2\lambda}{\xi_i^2 + \lambda^2} \right\rangle \xrightarrow{V \to \infty} \int \mathrm{d}\xi \, \langle \rho(\xi) \rangle \, \frac{2\lambda}{\xi^2 + \lambda^2} \xrightarrow{\lambda \to 0} \pi \, \langle \rho(0) \rangle$$

first derived in [Kanazawa, Wettig, Yamamoto '11]

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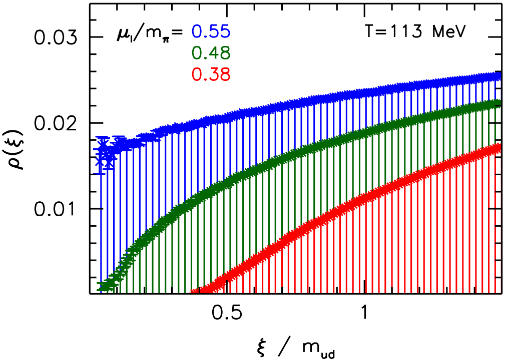
- compare to Banks-Casher-relation at $\mu_I = 0$

Singular value density

- spectral densities at $\lambda/m = 0.17$



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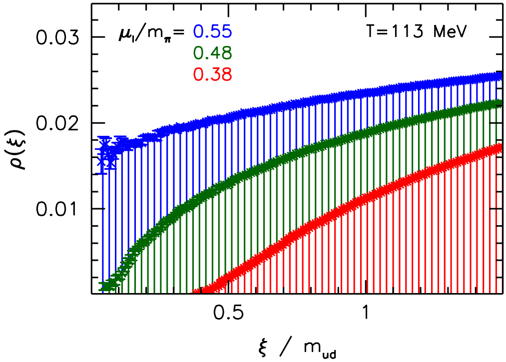

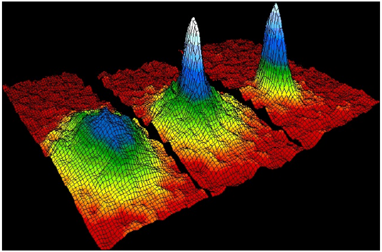

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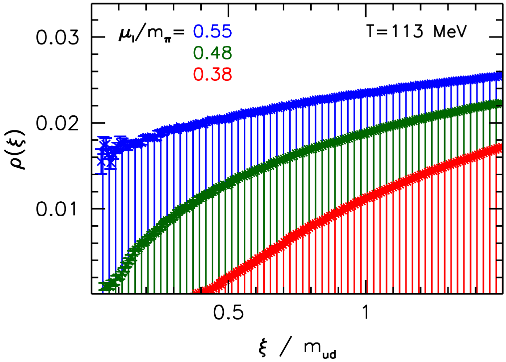
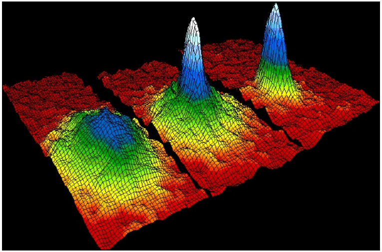


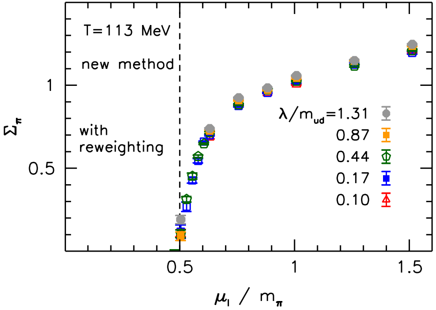


- compare density of states around zero 'energy' ($\xi \approx 0$) to velocity distribution around zero
- Bose-Einstein condensation!

Phase diagram

Pion condensate

- repeat this analysis for many different T and μ_I



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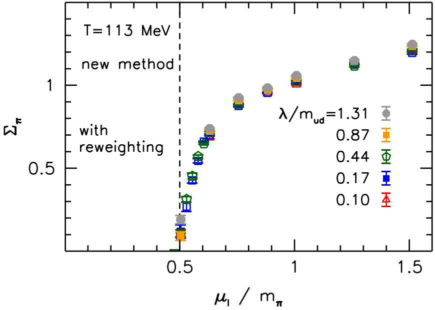
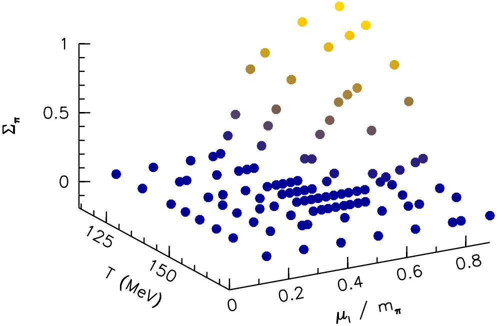
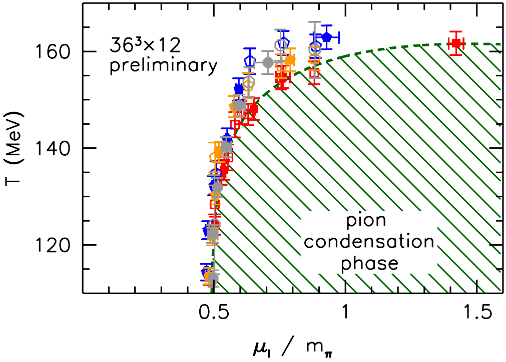




Phase boundary

- interpolate $\rho(0)$ as function of μ_I to find phase boundary



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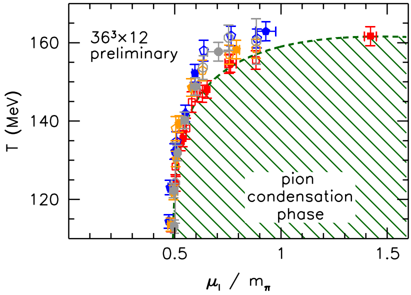

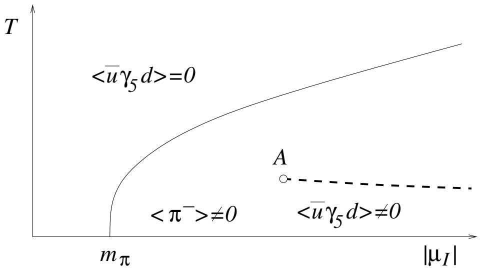

- compare to expectations from χPT [Son, Stephanov '00]

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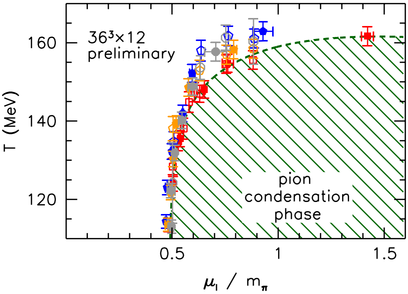
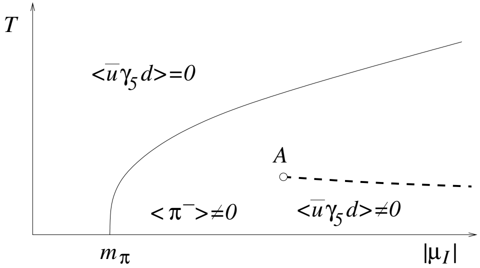
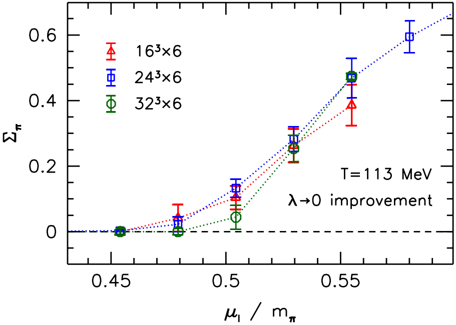
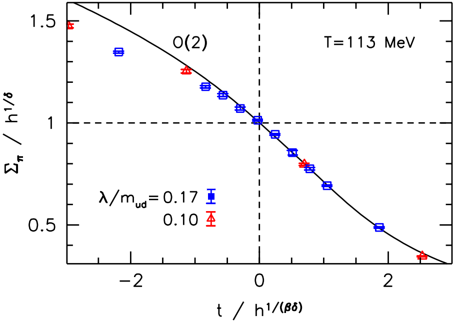




- compare to expectations from χPT [Son, Stephanov '00]
- no pion condensate above $T \approx 160$ MeV
  [Brandt, Endrődi, Schmalzbauer 1709.10487]

Order of the transition - volume scaling





- volume scaling of order parameter shows typical sharpening
- collapse according to $\mathrm{O}(2)$ critical exponents [Ejiri et al '09]

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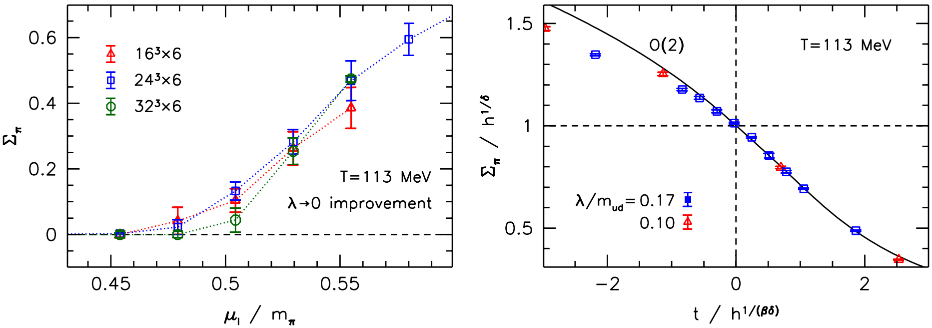
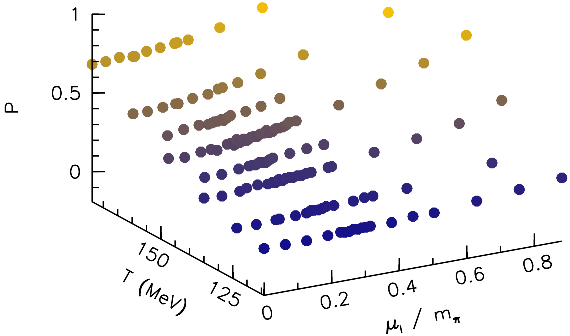


- volume scaling of order parameter shows typical sharpening
- collapse according to $\mathrm{O}(2)$ critical exponents [Ejiri et al '09]
- indications for a second order phase transition at $\mu_I = m_\pi/2$, in the $\mathrm{O}(2)$ universality class

Deconfinement at high temperature

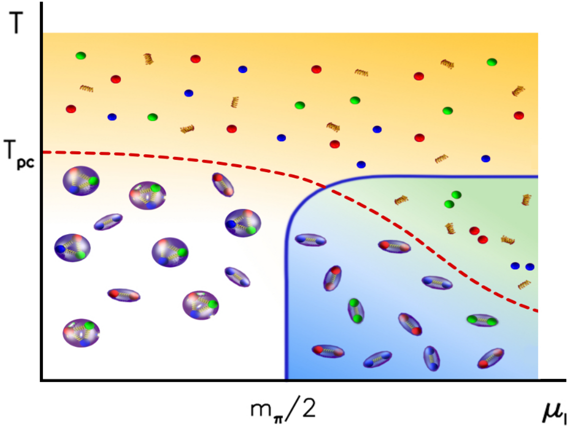
- deconfinement transition encoded in the Polyakov loop $P \sim \exp(-F_{q\bar{q}}/T)$



- deconfined matter for high μ_I: BCS superconductor with $u\bar{d}$ Cooper pairs [Son, Stephanov '02]

Phase diagram

- favored phase diagram schematically:
  hadronic, quark-gluon plasma, BEC, BCS phases



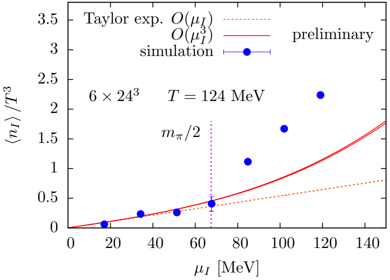
Further applications

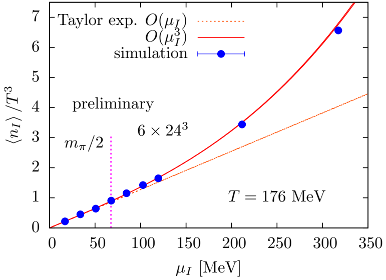
Check Taylor-expansion

- isospin density via Taylor-expansion at $\mu_I = 0$

$$n_I(\mu_I) = \chi_2^I \cdot \mu_I + \chi_4^I \cdot \mu_I^3 + \dots$$

using $\chi_{2,4}^I$ from [BMWc, 1112.4416]





- low T: breakdown of expansion at $\mu_I = m_\pi/2$
- high T: pin down validity range of LO and NLO expansion

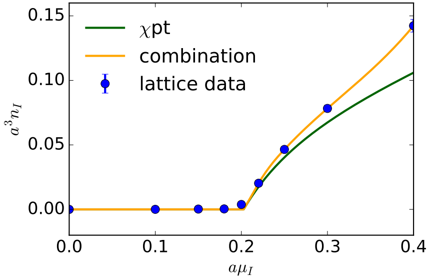
Equation of state

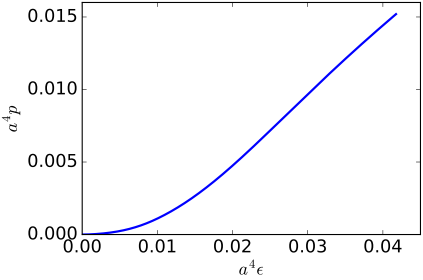
- pressure

$$p = \int_{m_\pi/2}^{\mu_I} \mathrm{d}\mu_I' \, n_I(\mu_I')$$

- energy density

$$\epsilon = -p + \mu_I n_I$$

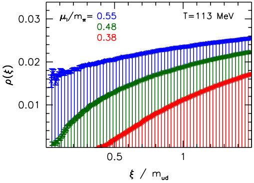




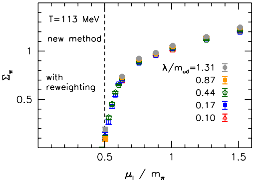
- application: mass-radius relation of compact stars

Summary

- Bose-Einstein condensation via singular value density
  $\rightsquigarrow$ flat extrapolation in λ



- established second-order phase transition at $\mu_I = m_\pi/2$



- QCD phase diagram with isospin asymmetry