QCD in magnetic fields: from the butterfly to the phase diagram

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I would like to thank

Gunnar Bali, Falk Bruckmann, Martha Constantinou,
Marios Costa, Zoltán Fodor, Sándor Katz, Tamás Kovács,
Stefan Krieg, Haris Panagopoulos, Andreas Schäfer, Kálmán Szabó

and all of my other collaborators and friends

Motivation

for *significant contributions to our understanding of QCD matter in strong magnetic fields and to QCD thermodynamics.*

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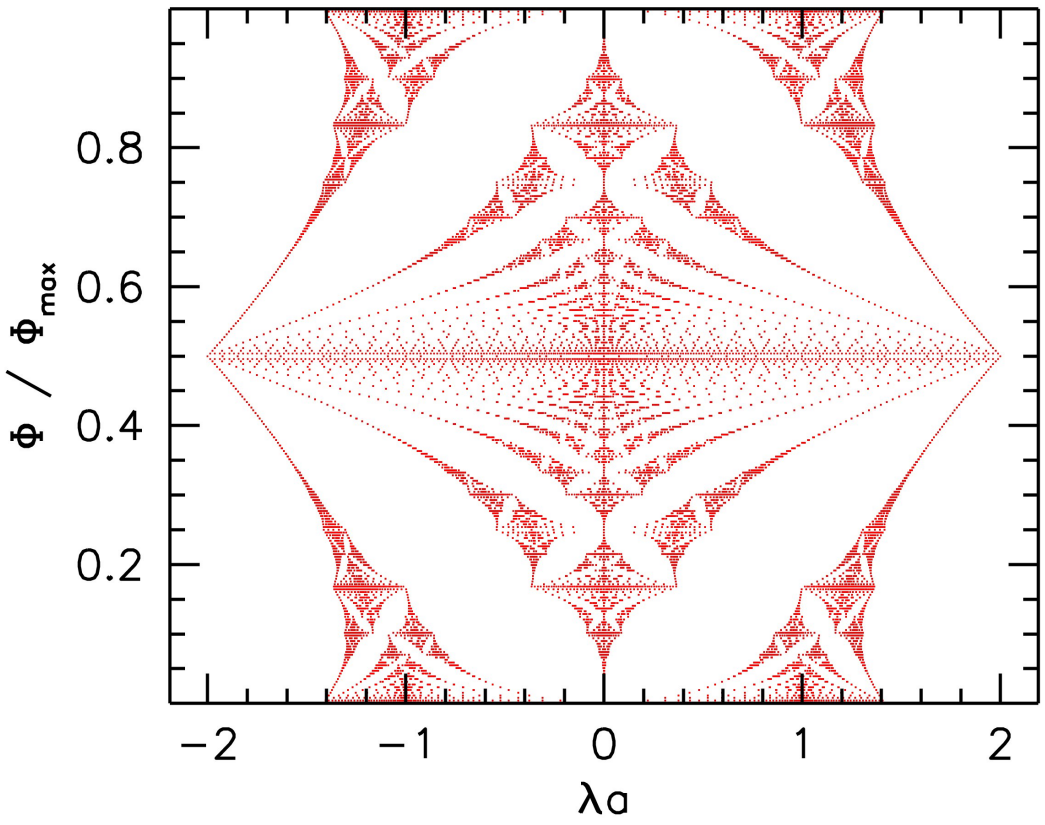

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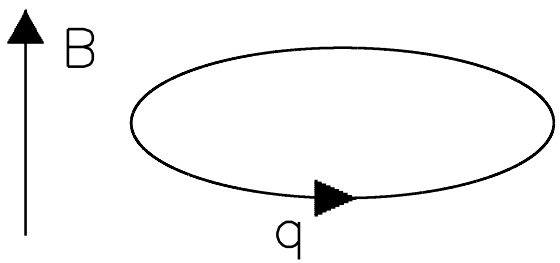
- magnetic response: connects various fields in physics

From the butterfly to the phase diagram

1: the butterfly

Landau versus Bloch

- free quark (electron)
- exposed to magnetic field in continuum space: Landau orbits



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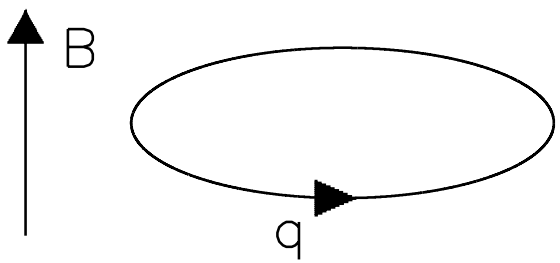

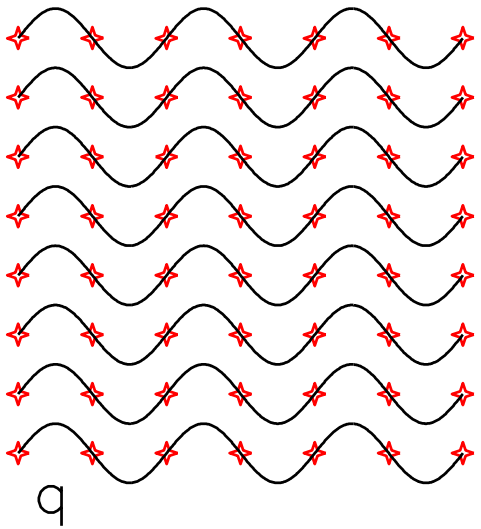

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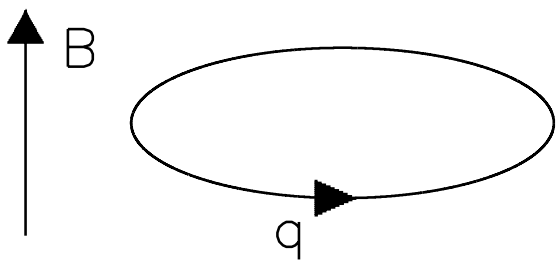
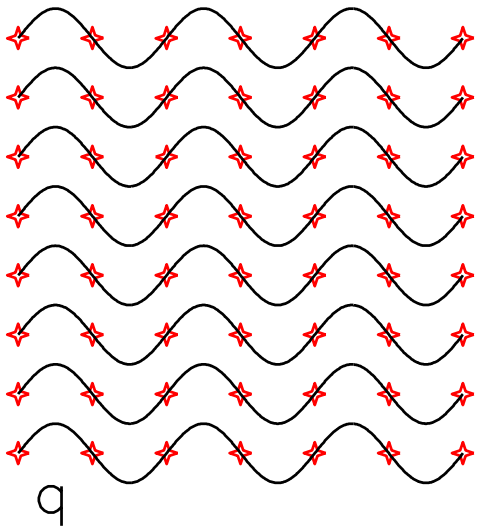
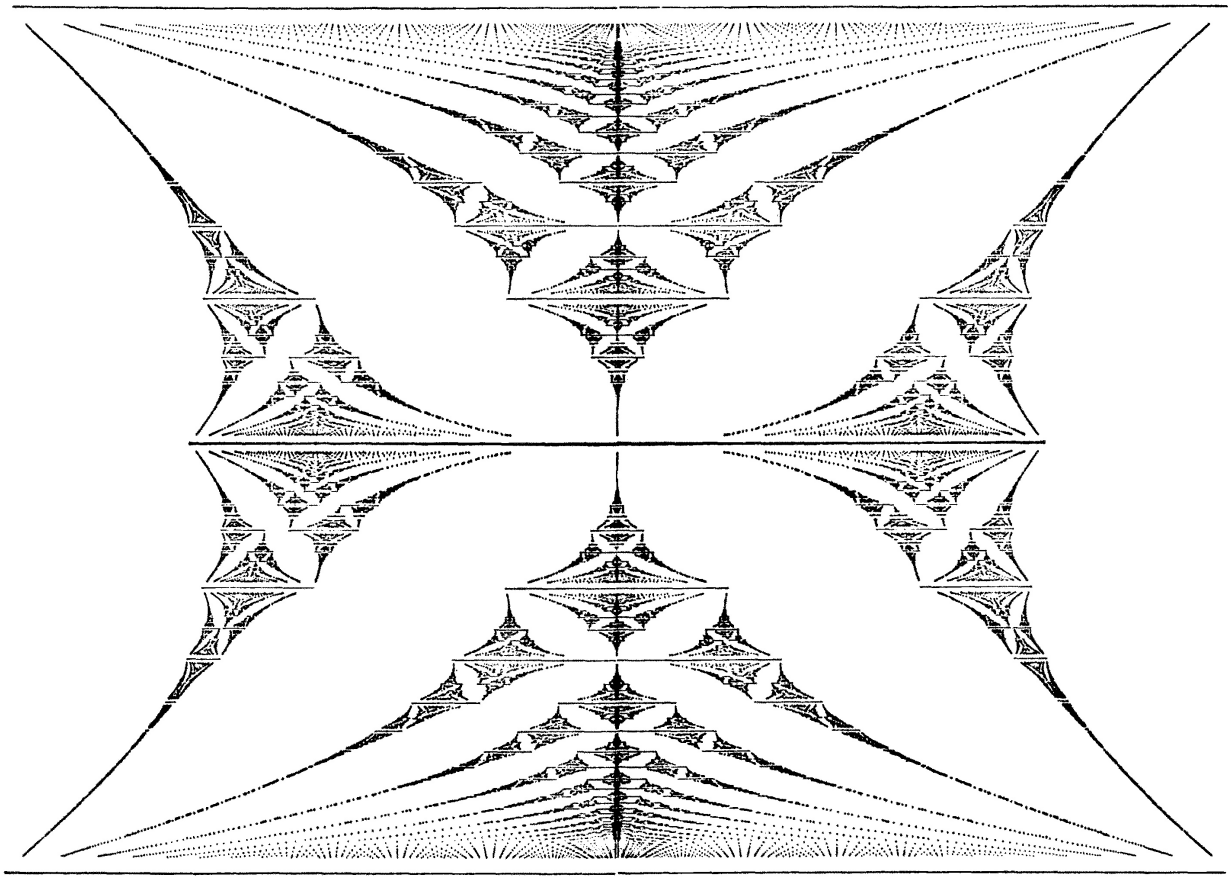




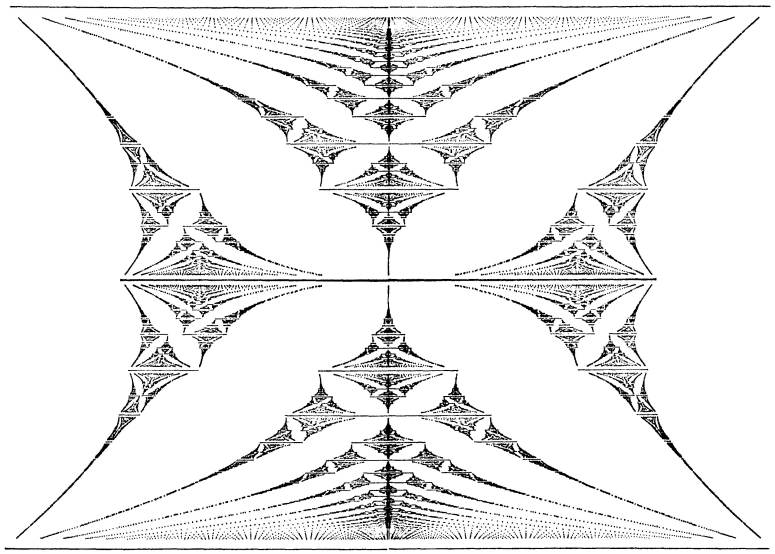
- what happens if the two are combined?

Hofstadter's butterfly [Hofstadter '76]



- Bloch electrons immersed in a magnetic field
  energy levels versus magnetic flux $\Phi = a^2 qB/2\pi$

Hofstadter's butterfly [Hofstadter '76]



- true fractal structure (if the lattice is infinite)
- energies accumulate into bands if flux $\Phi \in \mathbb{Q}$ ($2\pi/a^2$ and qB are commensurable)
- energies isomorphic to the Cantor set if $\Phi \notin \mathbb{Q}$ ($2\pi/a^2$ and qB are incommensurable)

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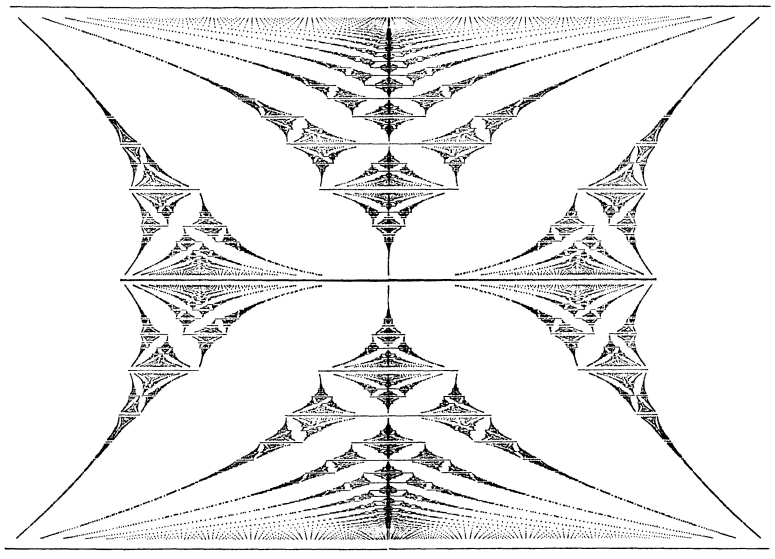

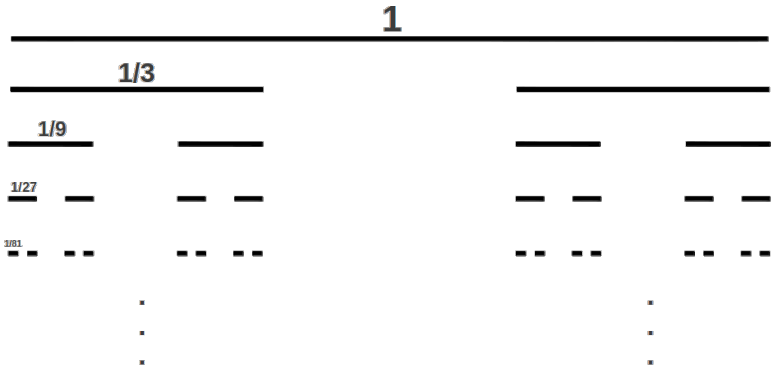

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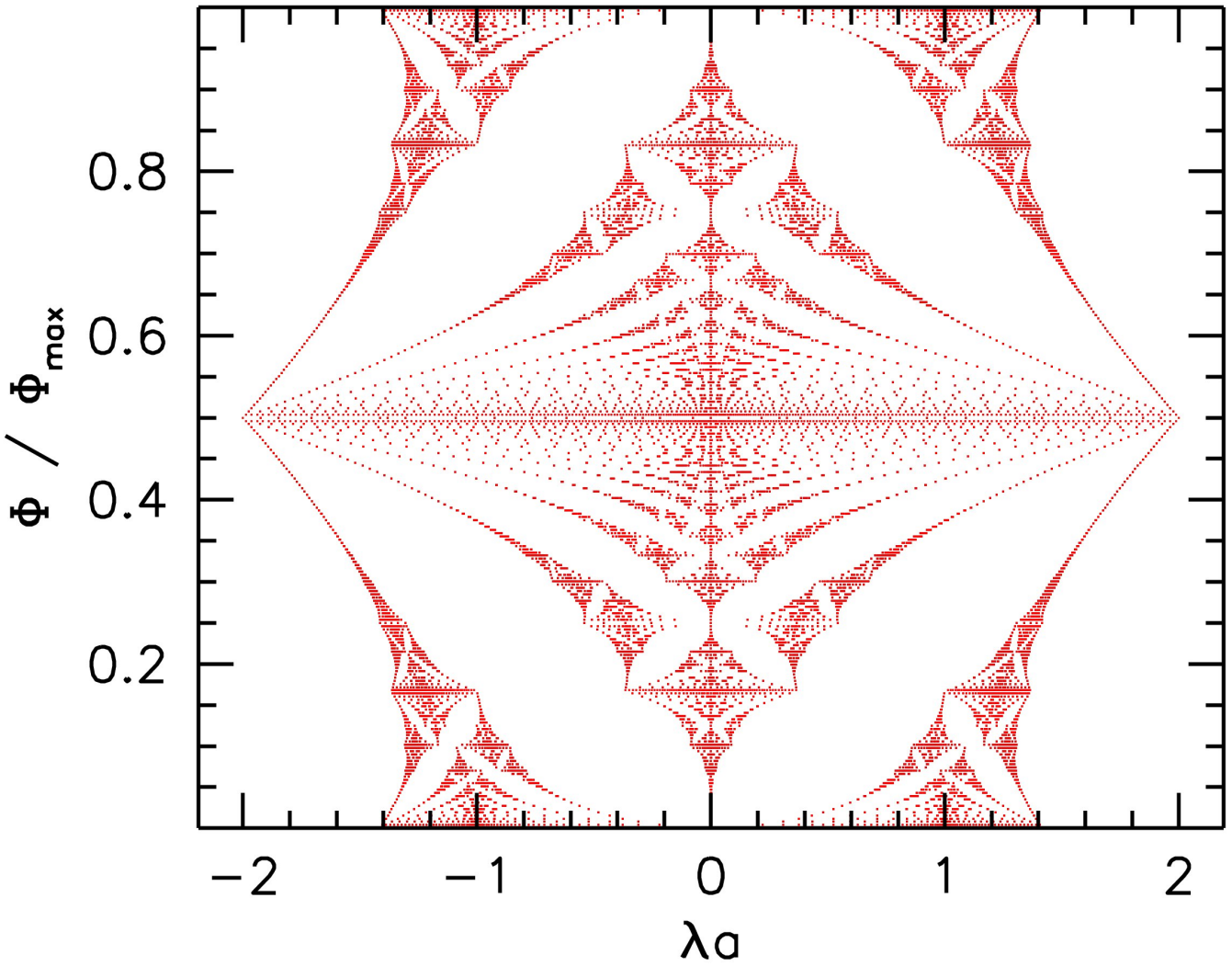
Hofstadter's butterfly: free quarks on the lattice



- Dirac equation for quarks in a magnetic field in 2D
  [GE, unpublished]

Hofstadter's butterfly: free quarks on the lattice

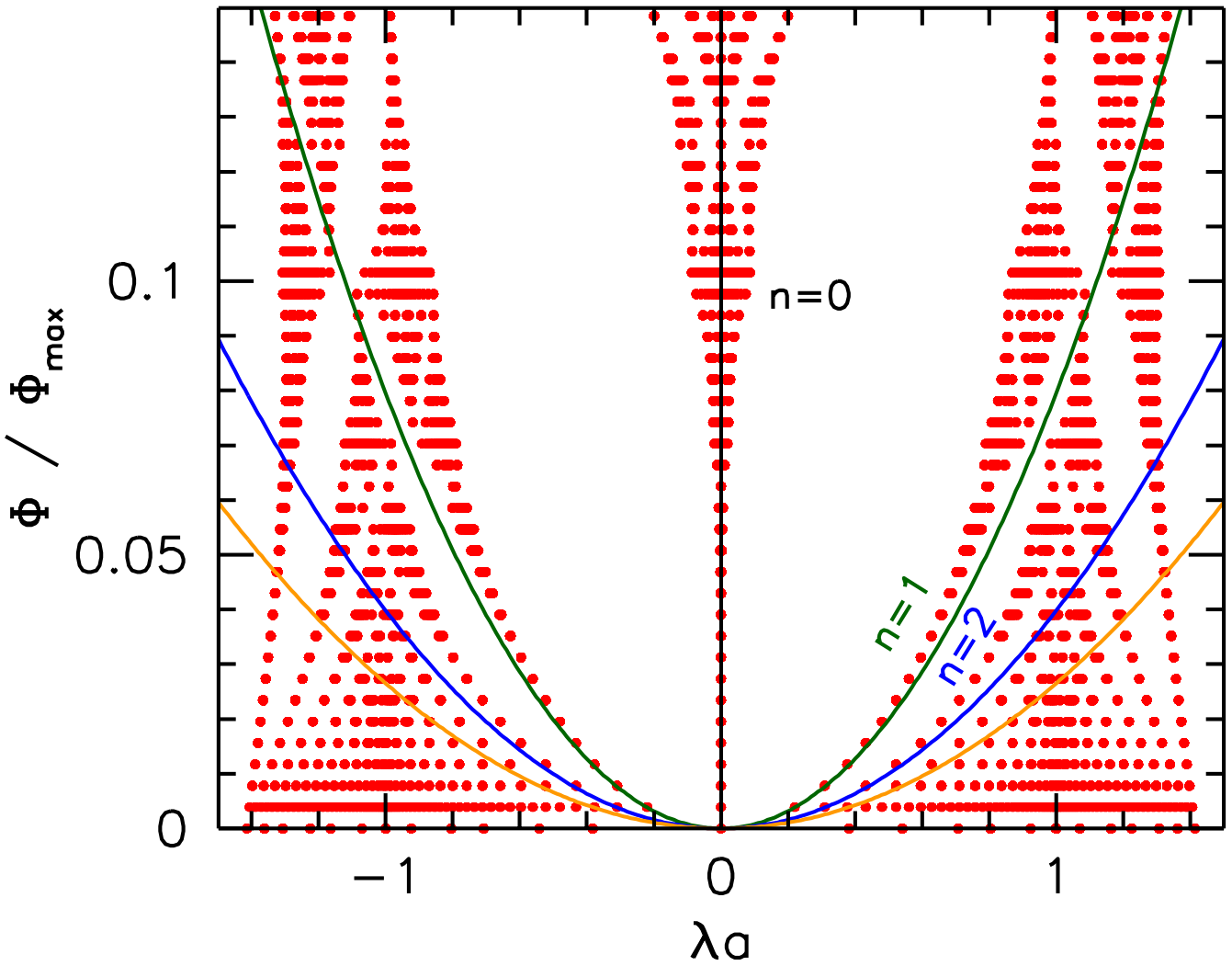
- crystal in solid state physics

  versus

  regulator in quantum field theory
- $\Phi = a^2 \cdot qB/2\pi \xrightarrow{a \to 0} 0$

  $\Rightarrow$ butterfly disappears in the continuum limit
- but, low-Φ behavior contains continuum physics

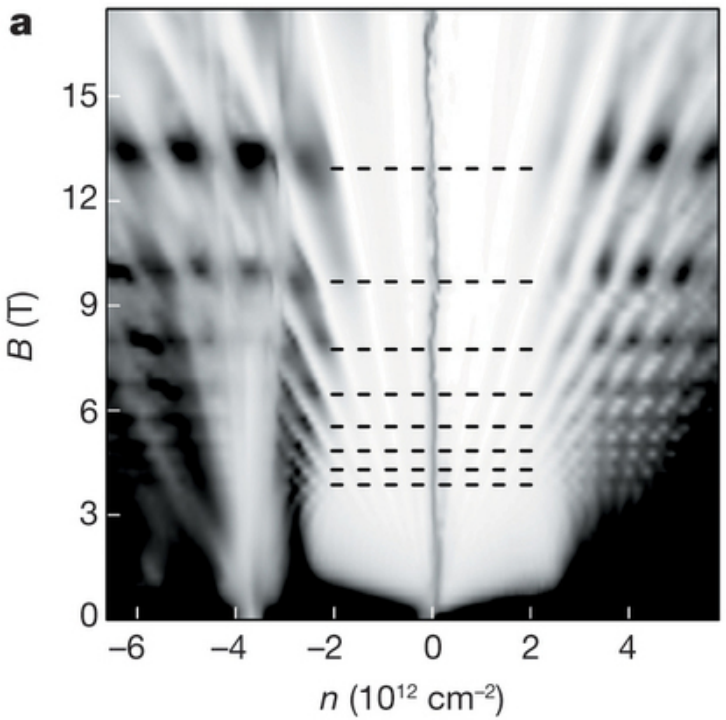
Hofstadter's butterfly: free quarks on the lattice



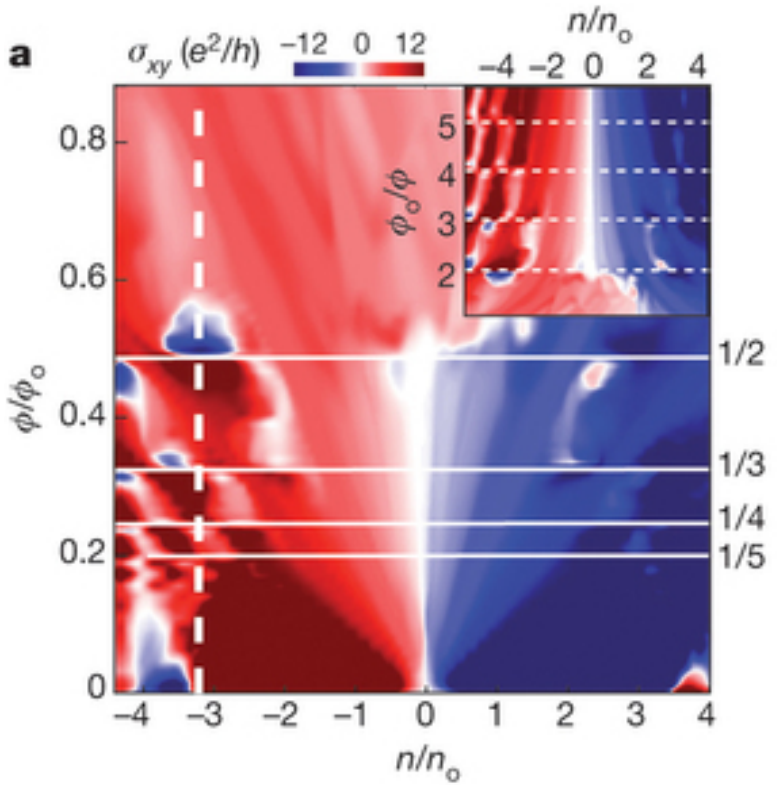
- Landau levels at low $B \rightarrow$ dissolve into bands [GE, unpublished]

Hofstadter's butterfly: experiments

- "catching the butterfly"



[Ponomarenko et al '13]



[Dean et al '13]

2: the condensate

Hofstadter's butterfly: impact on QCD [GE, unpublished]

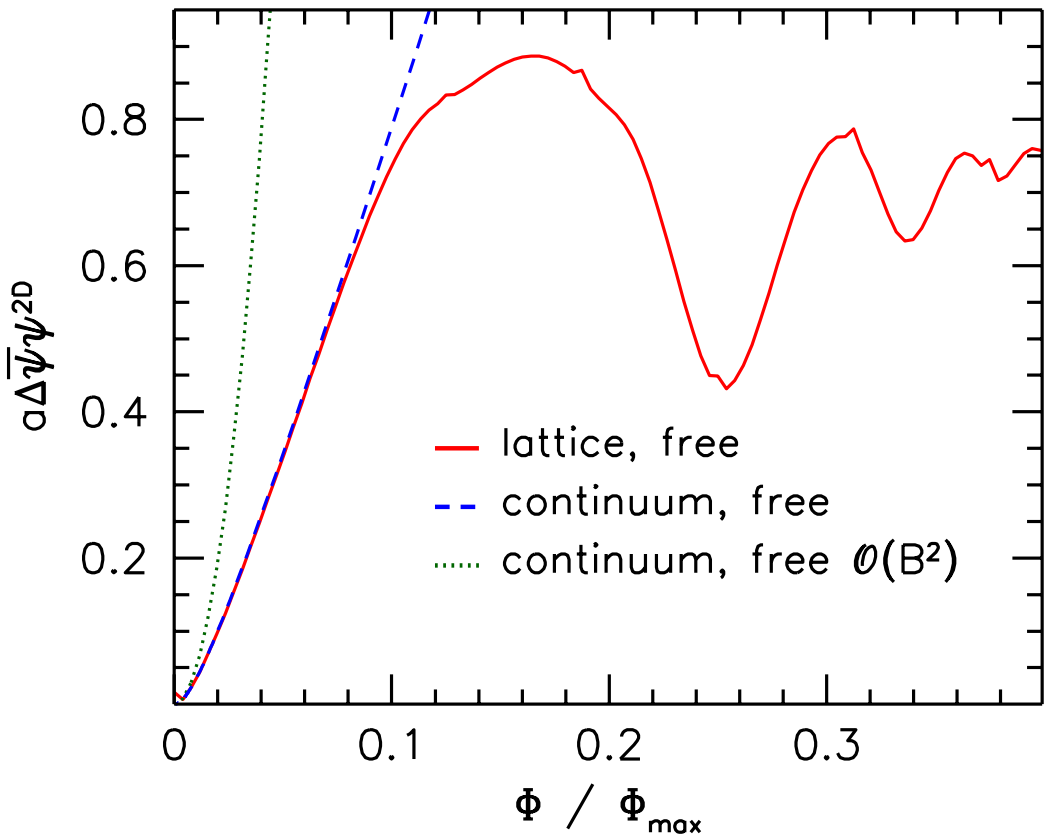
- the butterfly disappears in the continuum limit, but its wings around $a^2 qB = 2\pi\Phi \approx 0$ contain physical information
- in contrast to electron energies, Dirac eigenvalues cannot be measured
- physical observable composed of the eigenvalues: condensate of quarks with mass m

$$\bar{\psi}\psi^{\mathrm{2D}} = \sum_{\lambda} \frac{m}{\lambda^2 + m^2}$$

- nonzero quark mass washes out the fractal structure up to $qB \propto m^2$

  $\rightarrow$ animation

Hofstadter's butterfly and the condensate



- in the continuum: $\bar{\psi}\psi^{2\mathrm{D}} \propto \beta_1 \cdot B^2 + \mathcal{O}(B^4)$
  due to electric charge renormalization [GE 1301.1307]
  $\Rightarrow$ tendency dictated by $\beta_1 > 0$ (no asympt. freedom in QED)

A threefold correspondence for free quarks

wings of Hofstadter's butterfly at low magnetic fields

$\simeq$

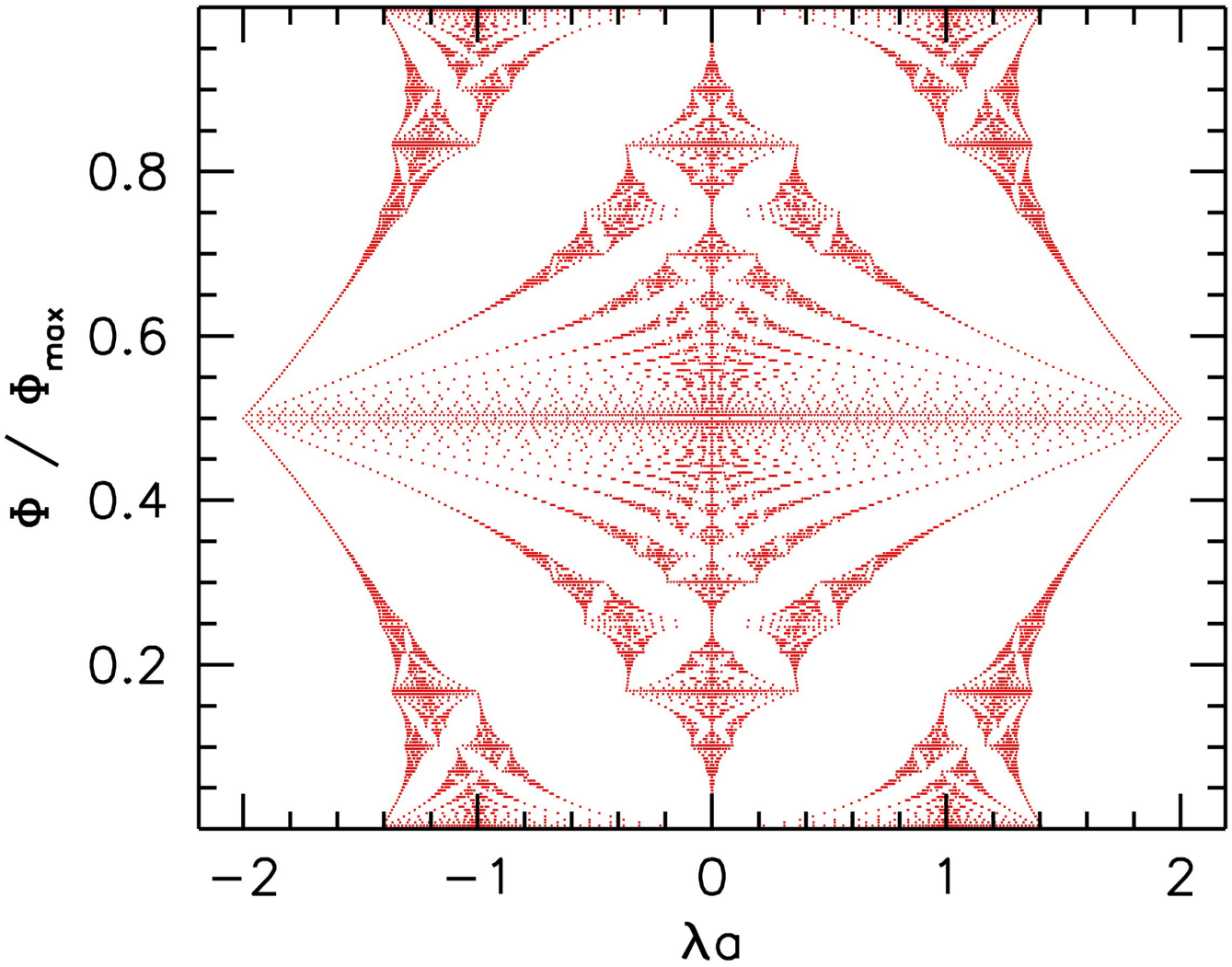
no asymptotic freedom in QED

$\simeq$

enhancement of quark condensate by magnetic fields

3: the QCD phase diagram

Hofstadter's butterfly: impact on QCD [GE, unpublished]



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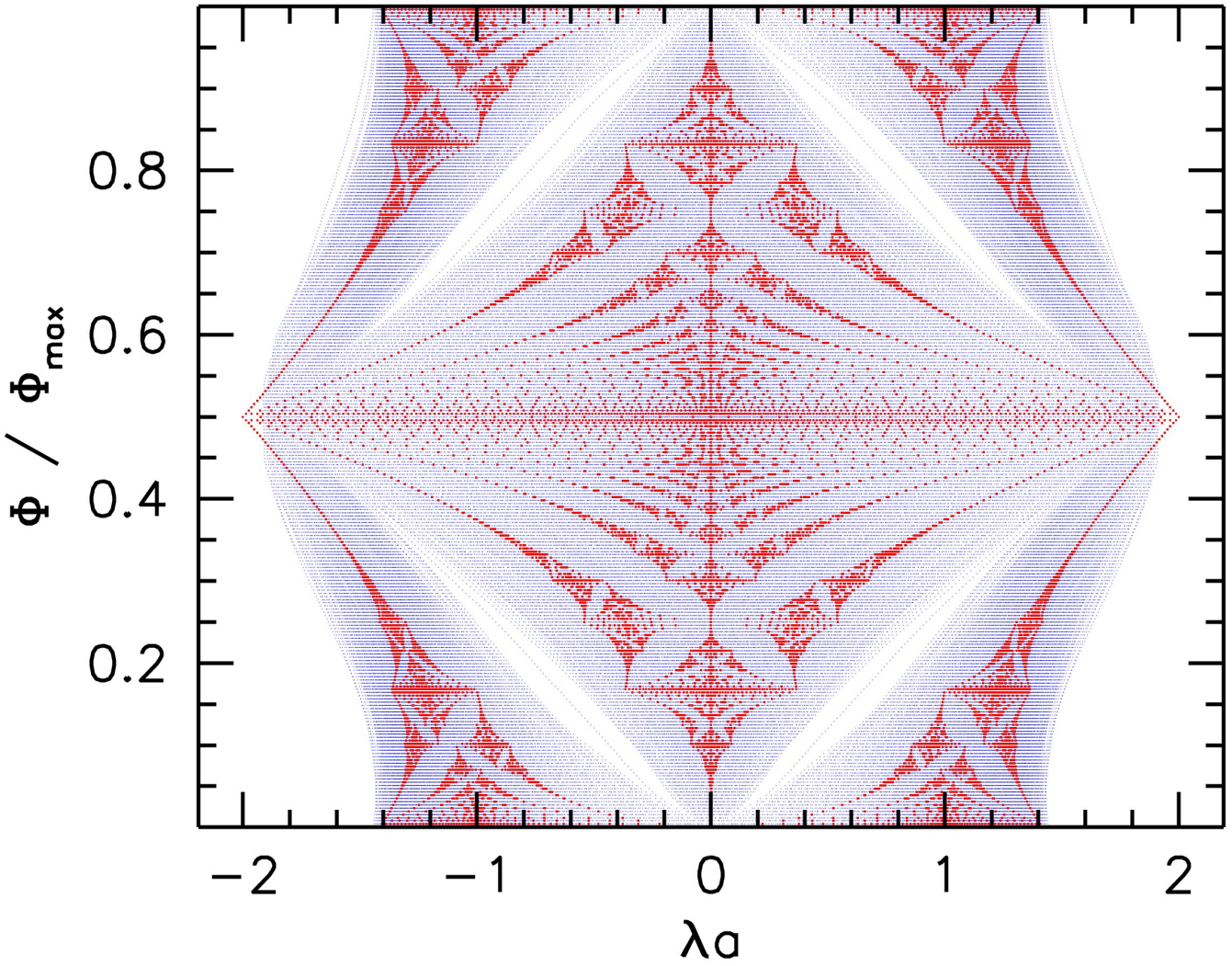
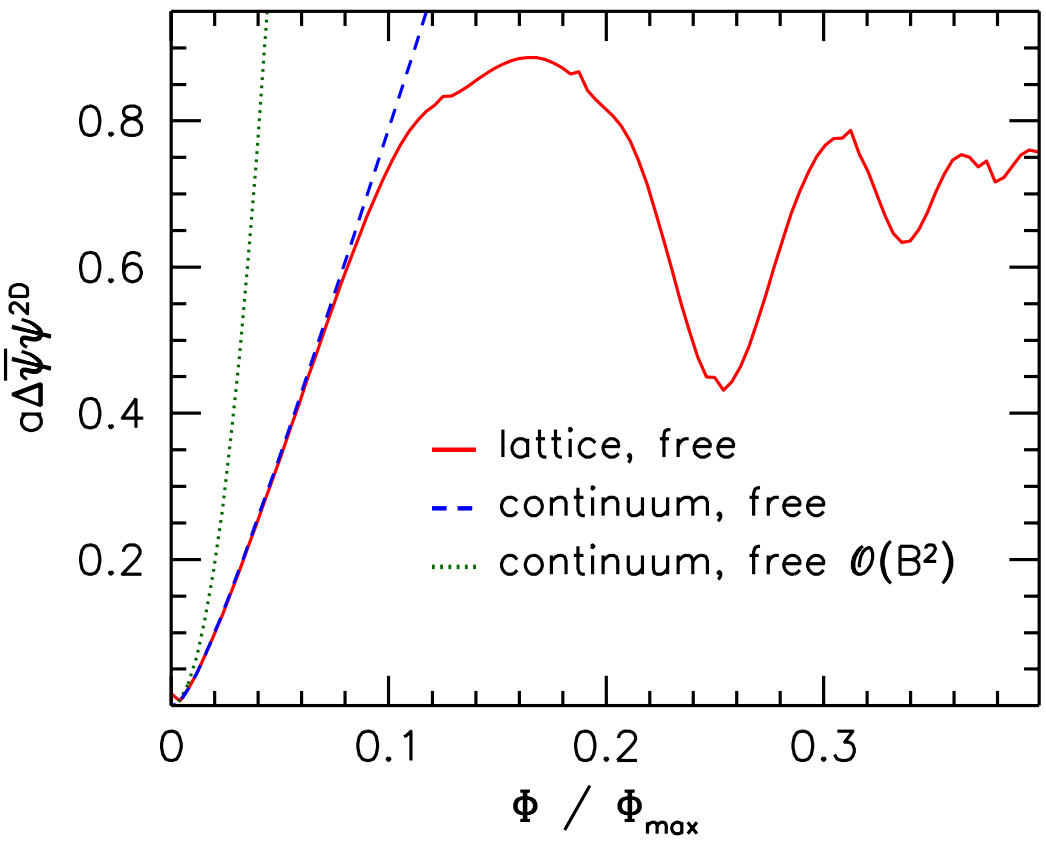


- Dirac equation for quarks in a magnetic field in 2D with QCD interactions switched on

Hofstadter's butterfly and the condensate



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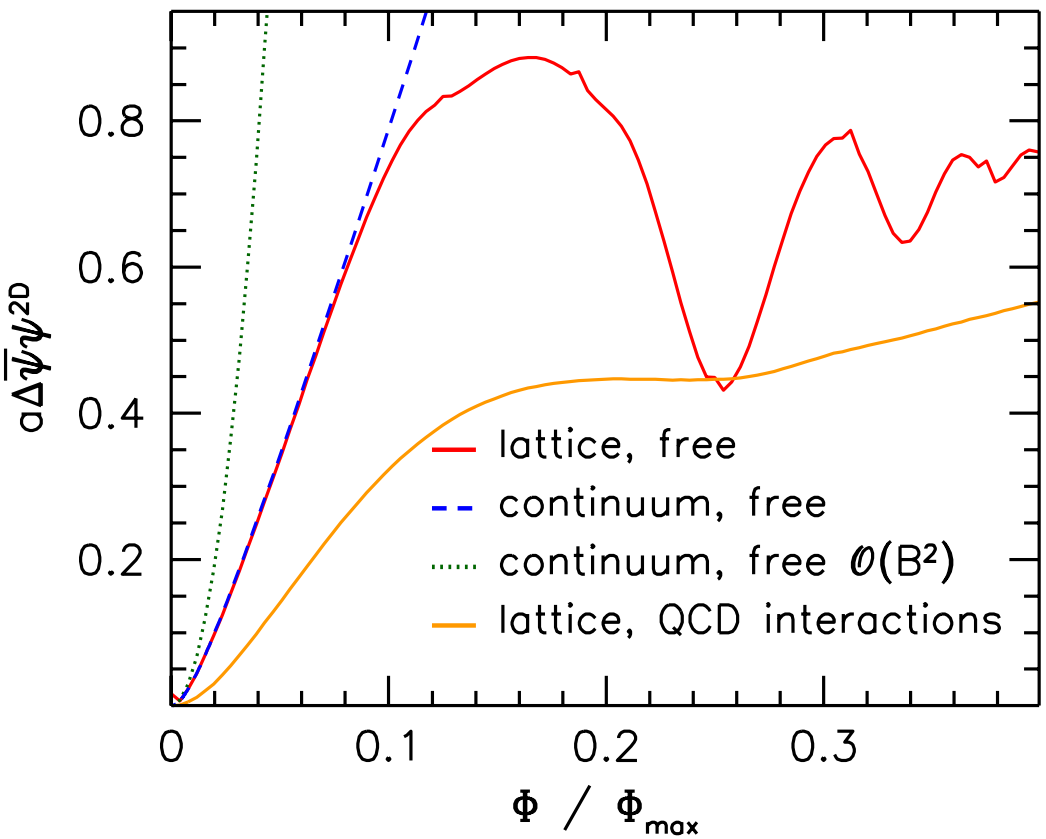

- QCD interactions wash out the fractal structure, but qualitative tendency remains

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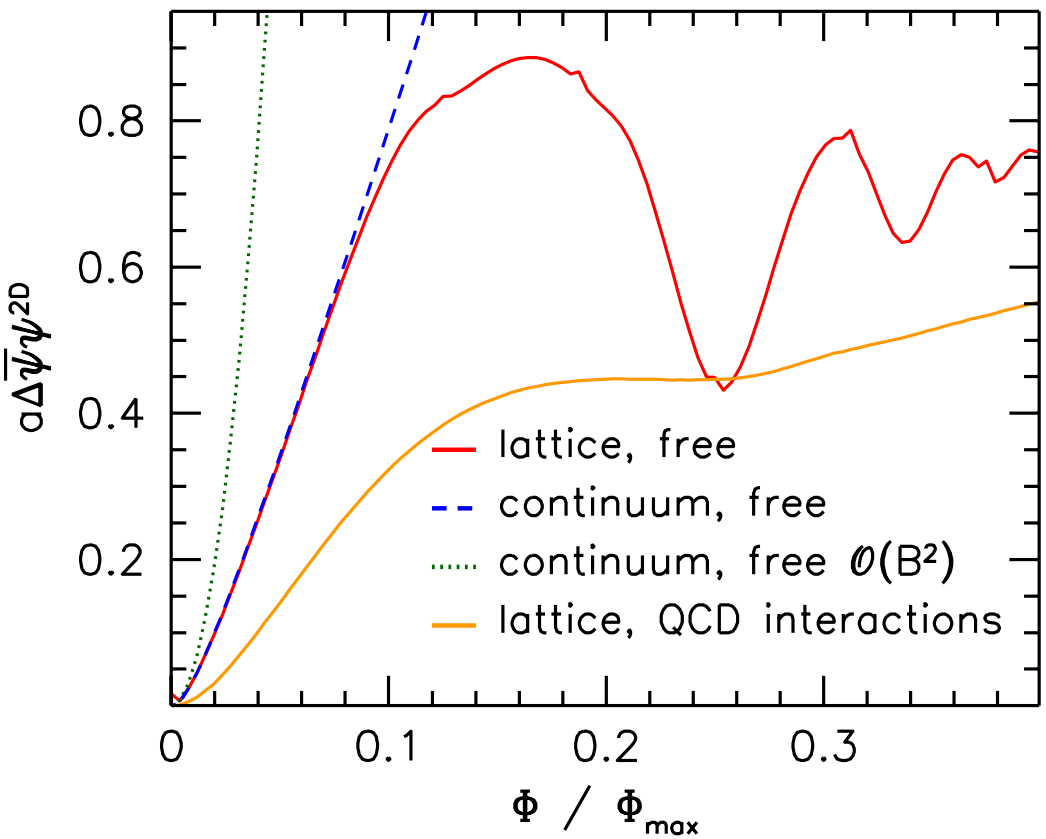
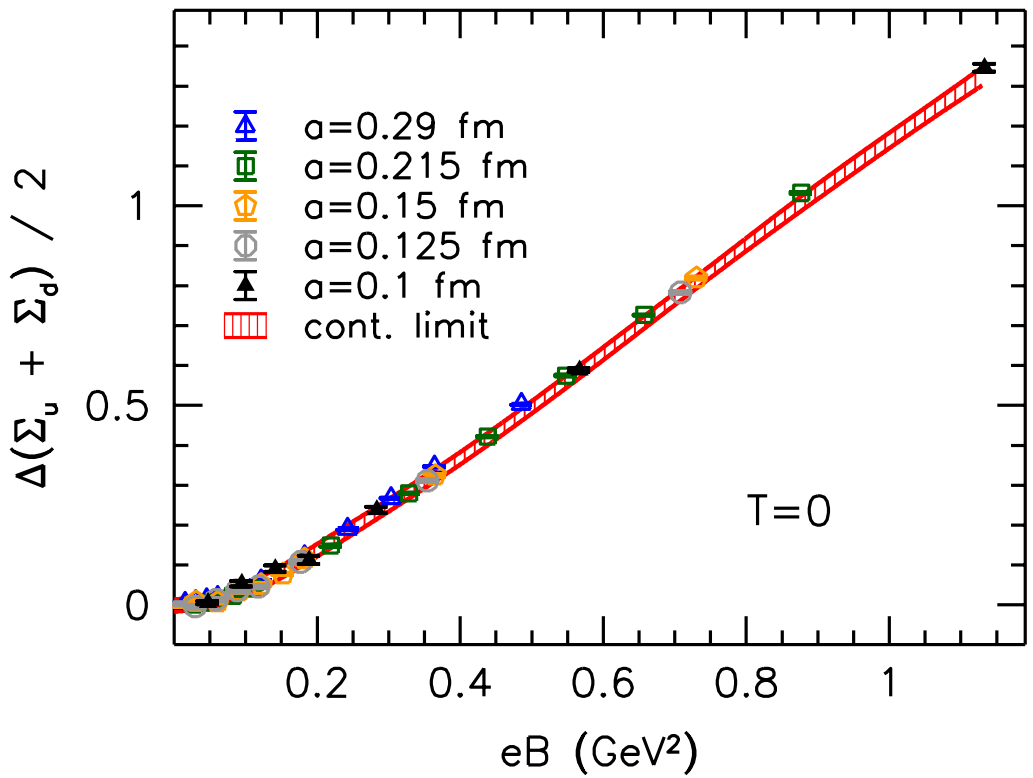


- QCD interactions wash out the fractal structure, but qualitative tendency remains
- subtleties: charge renormalization, interacting case, massless limit etc. [Bali, Bruckmann, GE, Katz, Schäfer 1406.0269]

Quark condensate in 4D QCD at $T = 0$

- $\bar{\psi}\psi$ grows with B even in the interacting case in 4D
  [Bali, Bruckmann, GE, Fodor, Katz, Schäfer 1206.4205]



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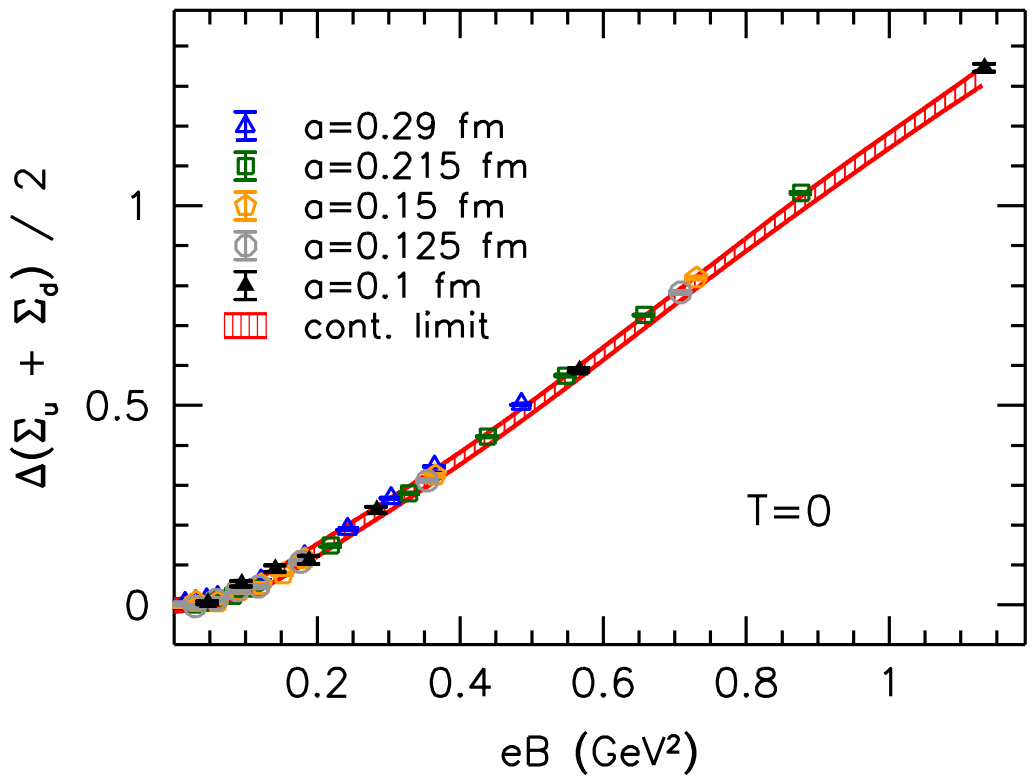

- has been long known as 'magnetic catalysis', due to dimensional reduction for $B \to \infty$ [Gusynin et al hep-ph/9509320]

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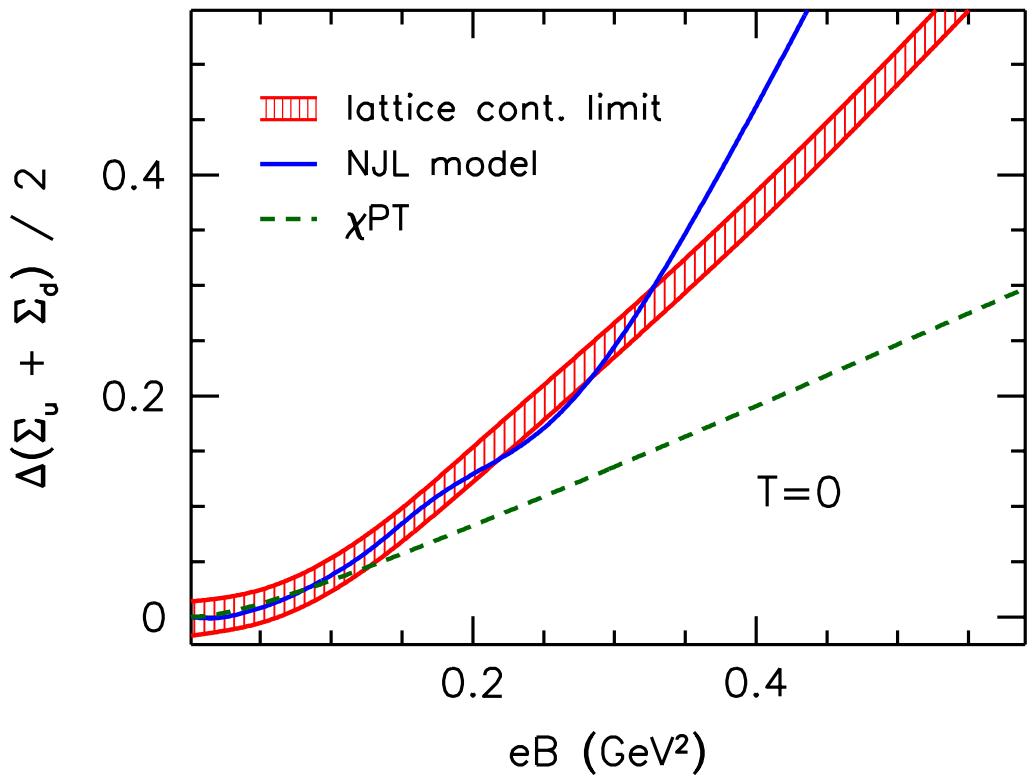
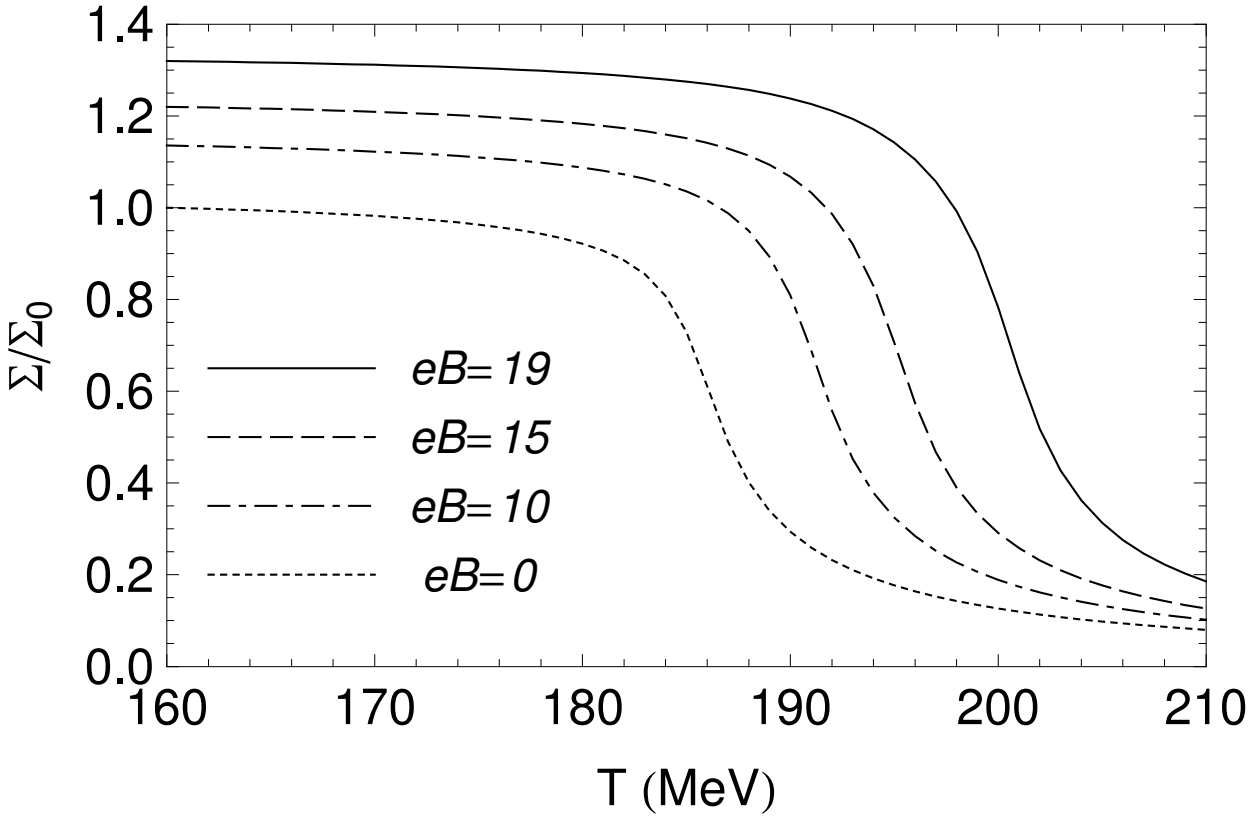


- captured by all low-energy models, e.g. χPT, NJL, ...

Quark condensate in 4D QCD at $T > 0$: models

- low-energy models predict magnetic catalysis to persist even at high temperatures (e.g. PNJL model [Gatto et al 1012.1291])



- result: $T_c(B)$ increases

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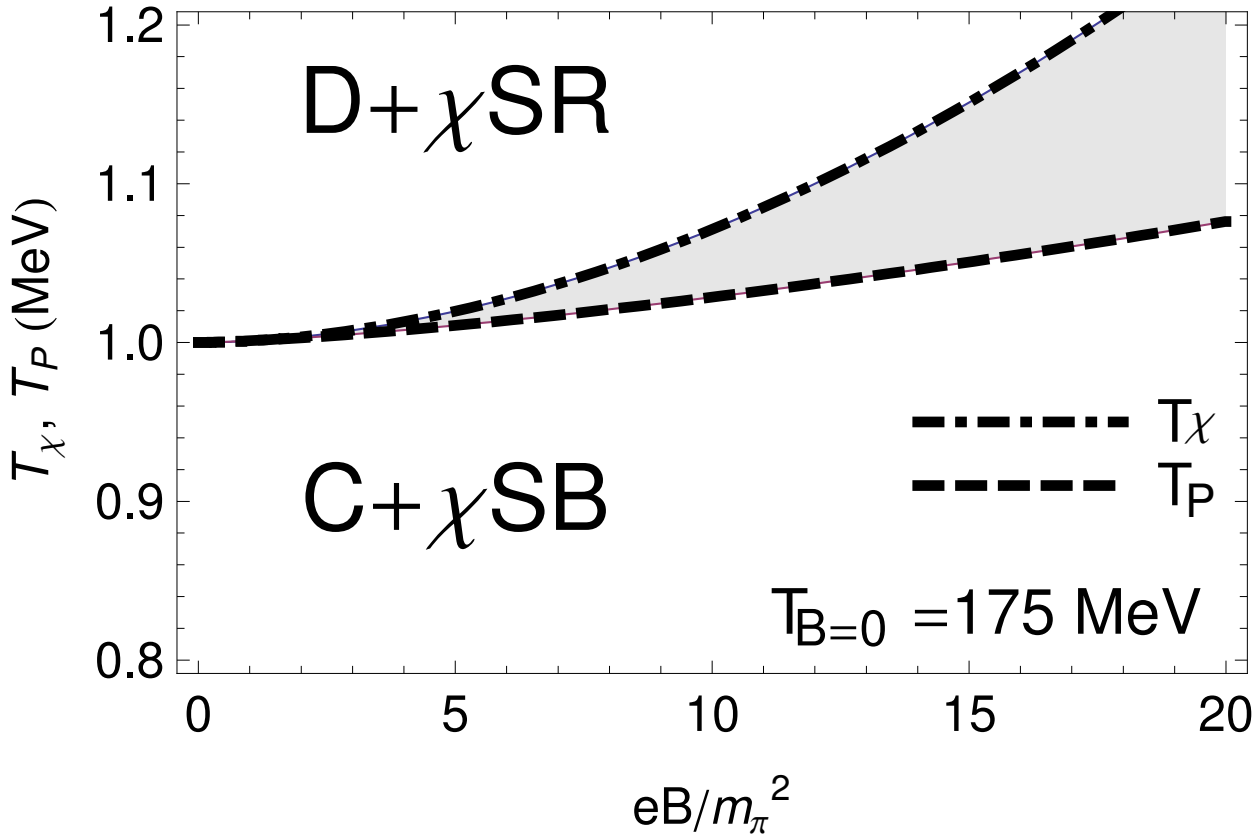
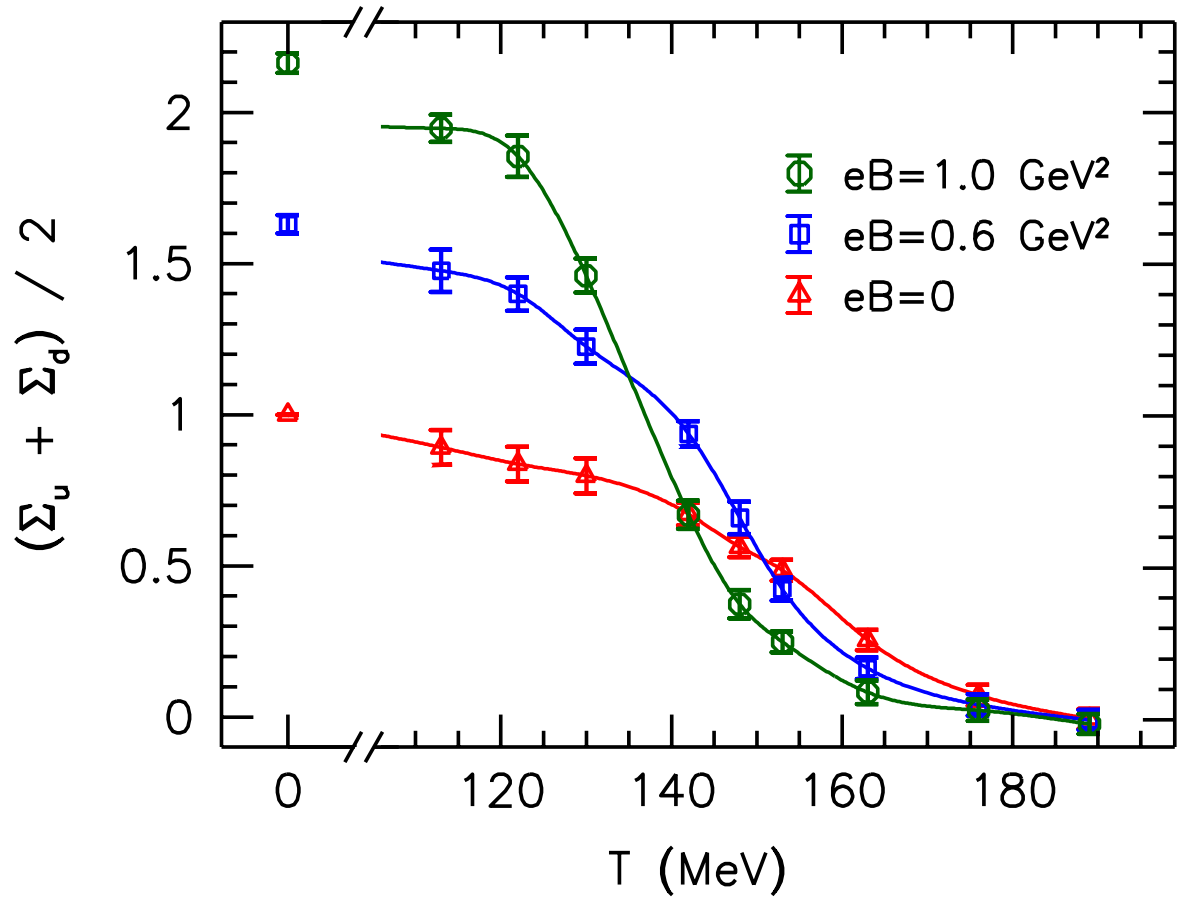


- result: $T_c(B)$ increases

Quark condensate in 4D QCD at $T > 0$: lattice

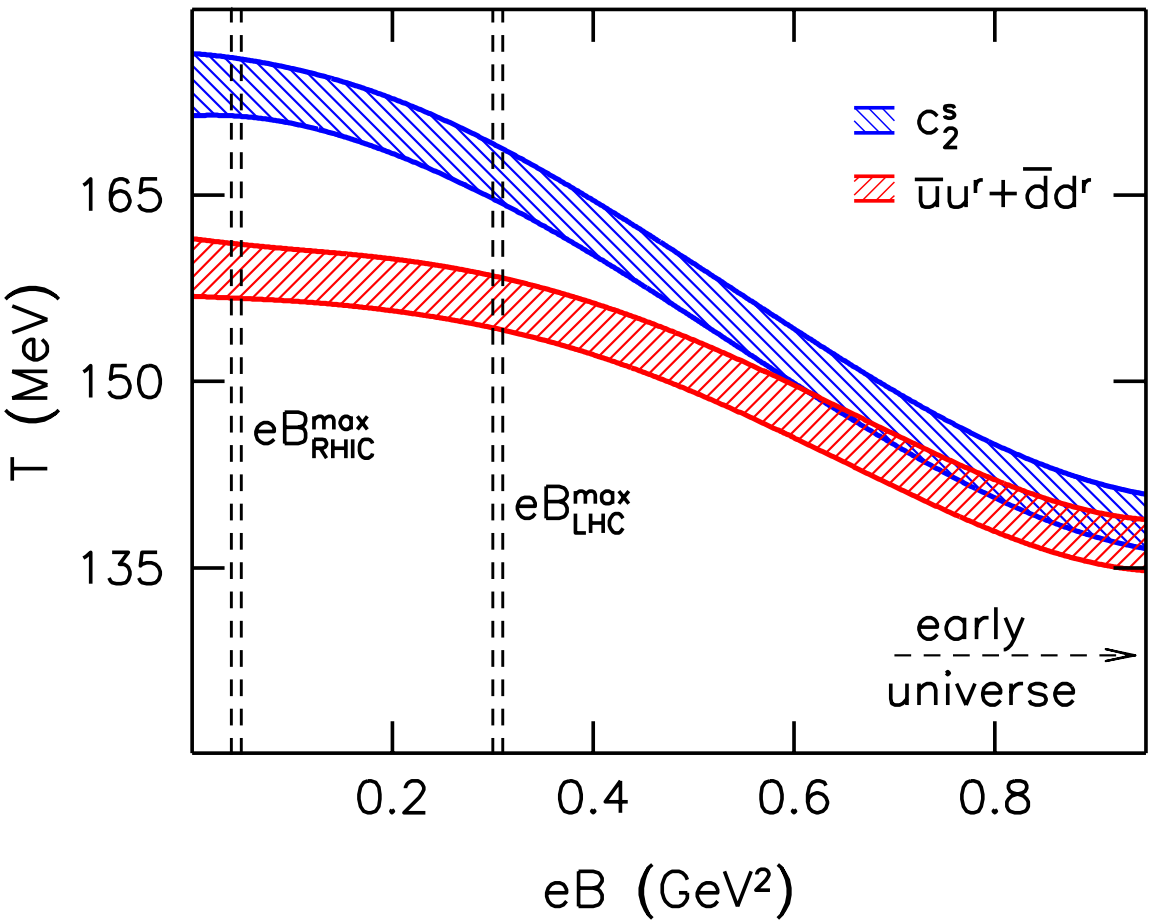
- going to finite temperatures changes the response qualitatively dubbed 'inverse magnetic catalysis' around T_c

  [Bruckmann, GE, Kovács 1303.3972]



Phase diagram

- impact on the QCD phase diagram: $T_c(B)$ decreases
  [Bali, Bruckmann, GE, Fodor, Katz, Krieg, Schäfer, Szabó 1111.4956]



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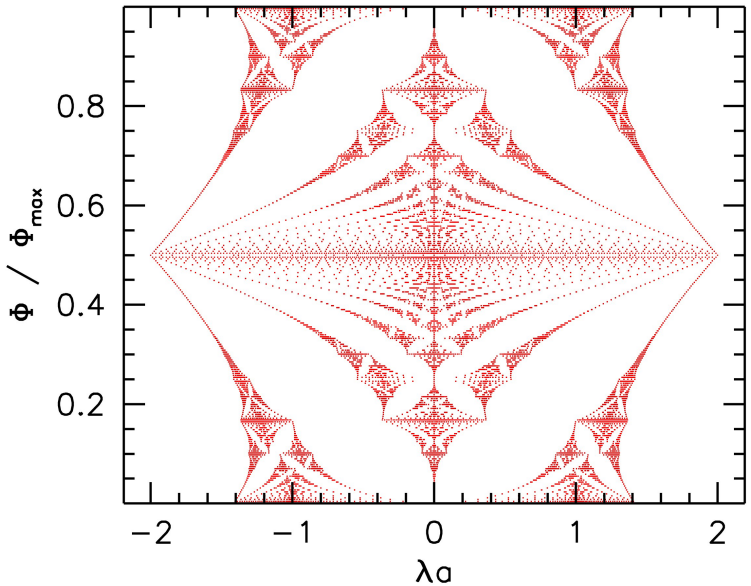
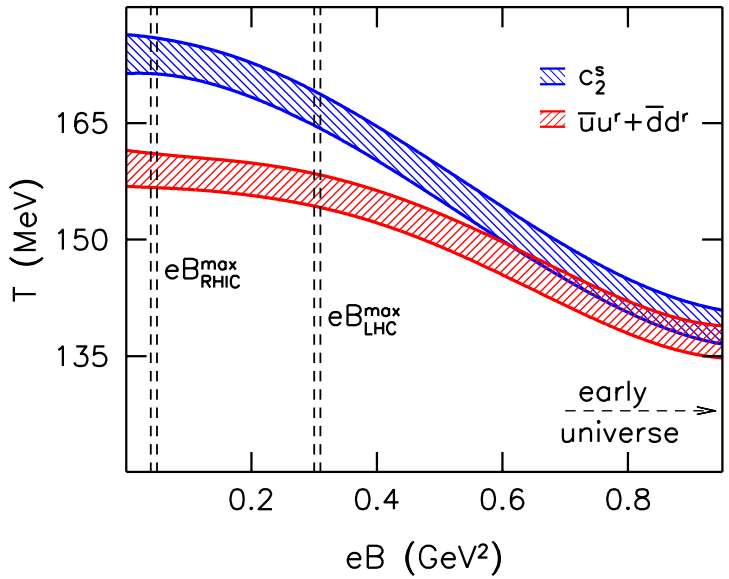


- three intertwined phenomena at $T = 0$
- wings of Hofstadter's butterfly at small B
  $\rightarrow$ solid state physics
- no asymptotic freedom in QED ($\beta_1 > 0$)
  $\rightarrow$ perturbative QFT
- magnetic catalysis of the QCD condensate
  $\rightarrow$ nonperturbative QFT, χSB

Summary II.

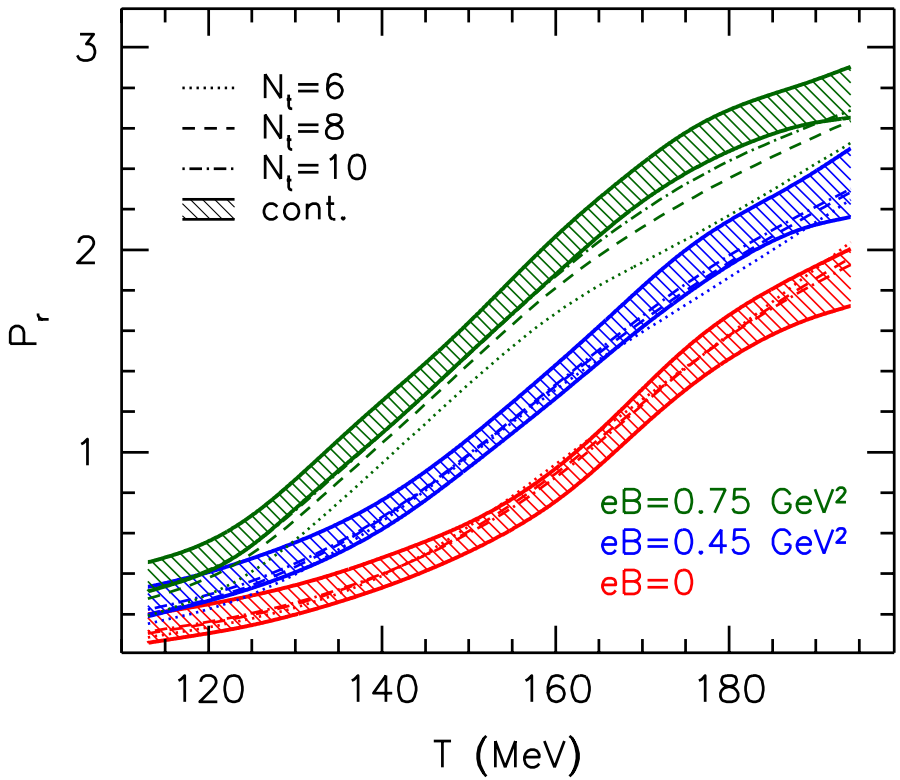


- inverse magnetic catalysis appears around $T \sim T_c$
- impact on phase diagram
- input to improve low-energy models
- relevant for phenomenology: non-central heavy-ion collisions and early universe

- more on magnetic fields: [D'Elia Sat 08:30]
  [Braguta Thu 14:35, Buividovich Fri 15:15, Kochetkov Fri 17:30, Larina poster]
  [Levkova Fri 18:10, Mariti Fri 17:50, Negro Wed 10:00, Valgushev Thu 16:15]

Inverse catalysis

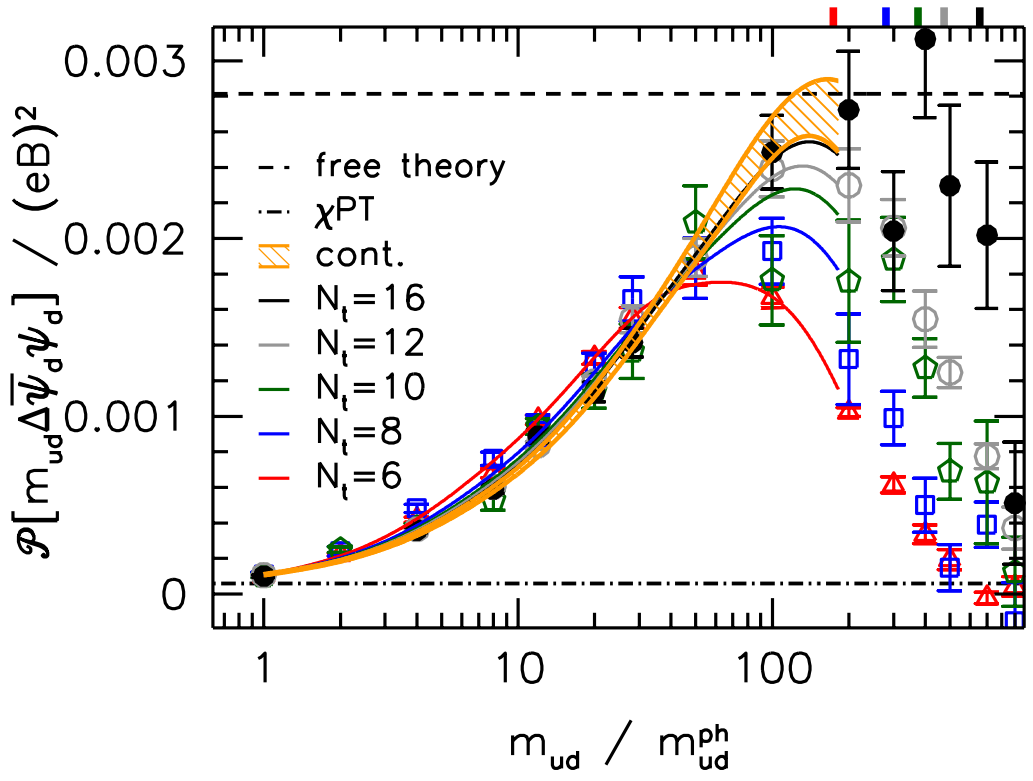
- B changes the typical gauge configurations
- most important dof: Polyakov loop



- Polyakov loop 'draws' condensate with itself
- effect only visible for light quarks [Bruckmann, GE, Kovács 1303.3972]
  (otherwise indirect effect of B on gluons is weak)

Magnetic catalysis – mass dependence

- $\bar{\psi}\psi = c \cdot (qB)^2$
- free quarks (interacting, $m \to \infty$): $c = N_c \cdot \beta_1^{\text{spinor}}$
- free pions (interacting, $m \to 0$): $c = \beta_1^{\text{scalar}}/4$



- intermediate masses: smooth dependence

  [Bali, Bruckmann, GE, Katz, Schäfer 1406.0269]