Magnetic phase diagram of QCD: current status from the lattice

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Magnetic Fields in Hadron Physics, 10. May 2016

in collaboration with

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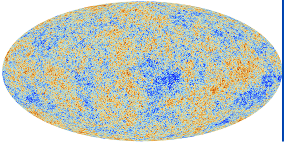
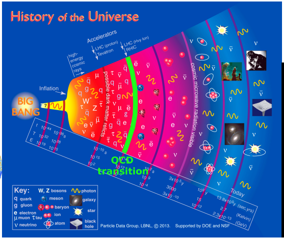
A. Schäfer, K. Szabó

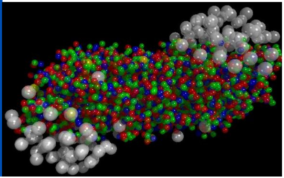
Motivation

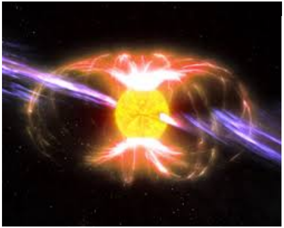
Magnetic fields in QCD - why?

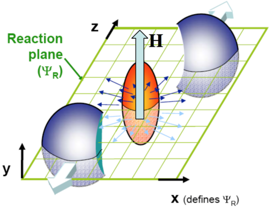
- this you all know . . .

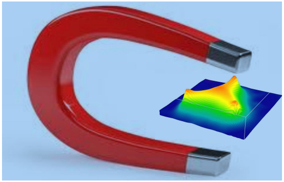






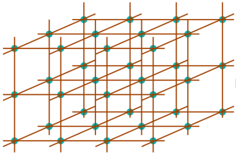
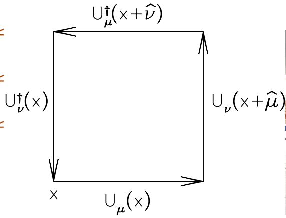


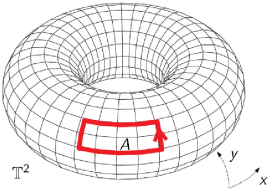


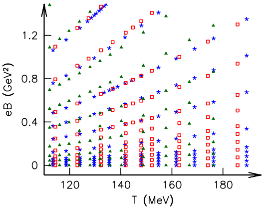


Magnetic fields in QCD - how?

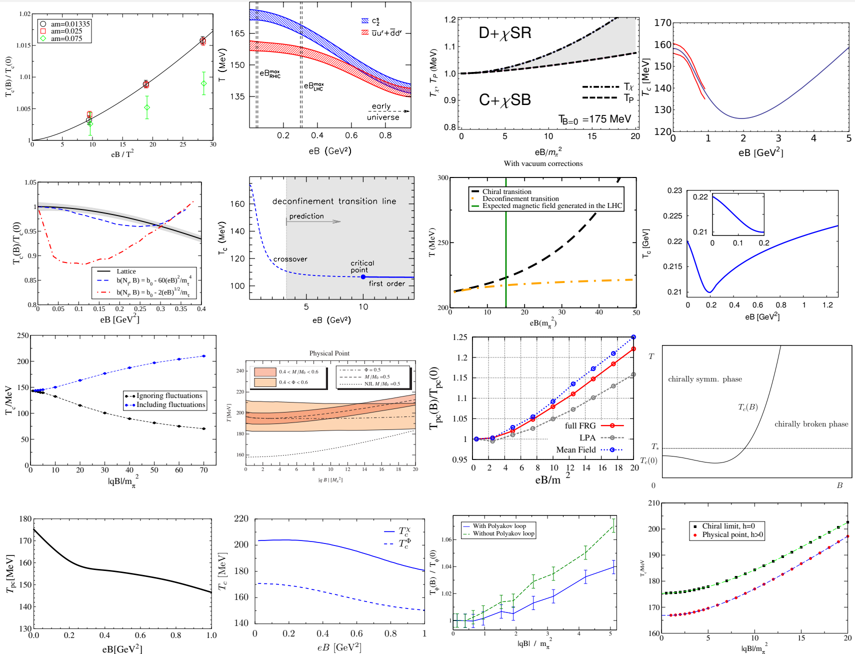
- this Gunnar has just explained

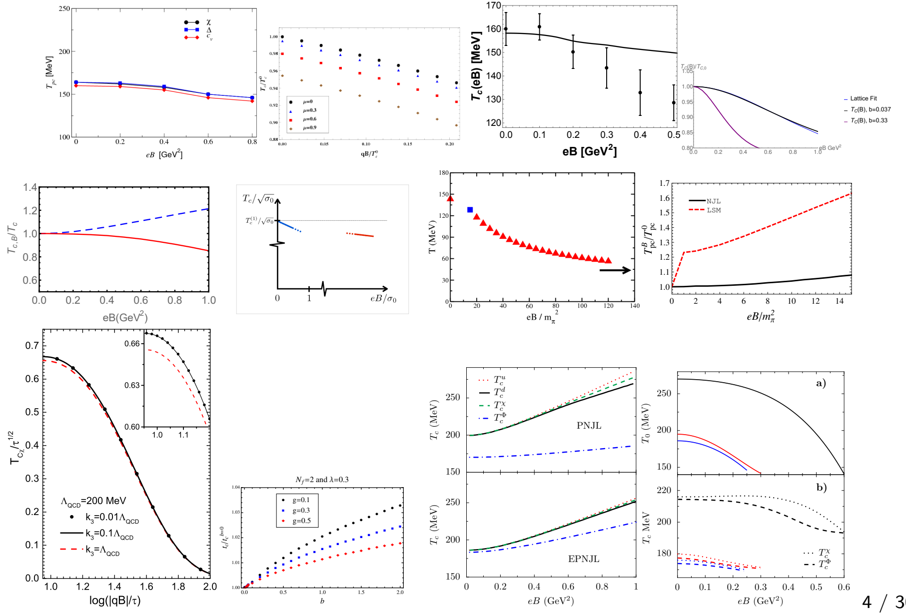





Find your plot



Find your plot, continued



Outline

- QCD transition at $B > 0$
  - magnetic catalysis
  - inverse magnetic catalysis
  - brief history of phase diagrams
  - open questions
- most recent lattice results
  - full QCD for strong magnetic fields
  - effective theory for $B \to \infty$ limit
- mechanism behind inverse catalysis
- outlook and conclusions

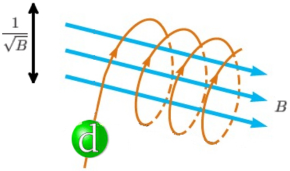
Magnetic catalysis

Magnetic catalysis explained

- chiral condensate $\leftrightarrow$ spectral density around 0 [Banks, Casher '80]

$$\left\langle \bar{\psi}\psi \right\rangle \propto \rho(0)$$

- large magnetic fields reduce dimensionality $3+1 \to 1+1$ and induce degeneracy $\propto B$

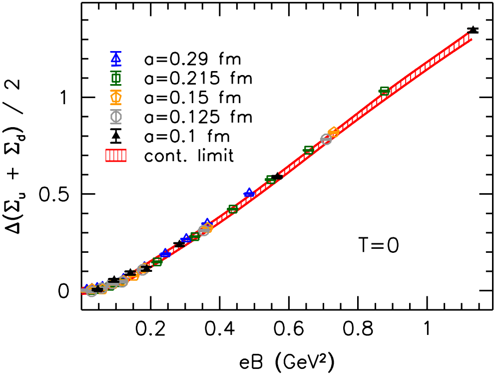


- in the chiral limit, to maintain $\left\langle \bar{\psi}\psi \right\rangle > 0$ [Gusynin et al '96]

$B = 0$	$\rho(p) \sim p^2 \mathrm{d}p$	"strong interaction is needed"
$B \gg m^2$	$\rho(p) \sim B\,\mathrm{d}p$	"the weakest interaction suffices"

Magnetic catalysis – zero temperature

- MC at zero temperature is a robust concept:
  χPT, NJL, AdS-CFT, linear σ model, lattice QCD , . . .



lattice QCD, physical m_π, continuum limit

[Bali,Bruckmann,Endrődi,Fodor,Katz,Schäfer '12]

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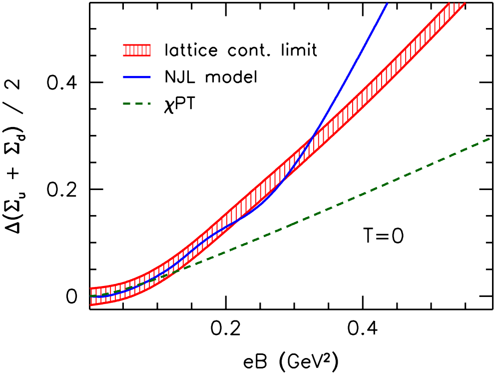

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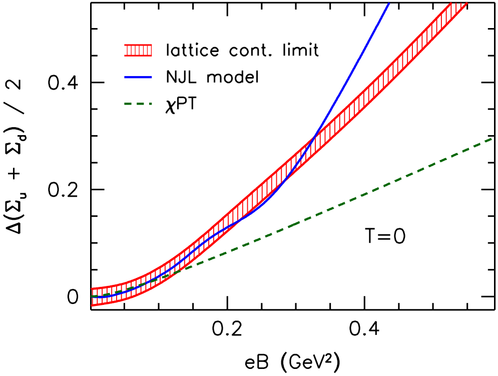

lattice QCD, physical m_π, continuum limit
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- improve NJL model further, cf. [Krein, yesterday]

Inverse magnetic catalysis

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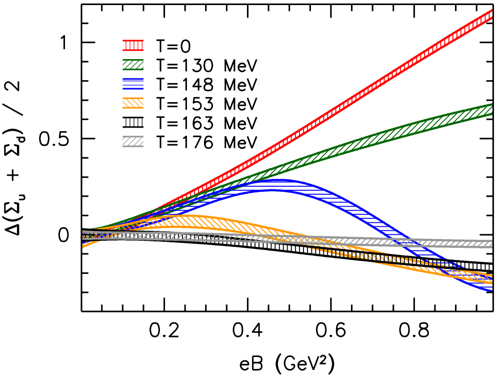
  [Bali,Bruckmann,Endrődi,Fodor,Katz,Schäfer '12]



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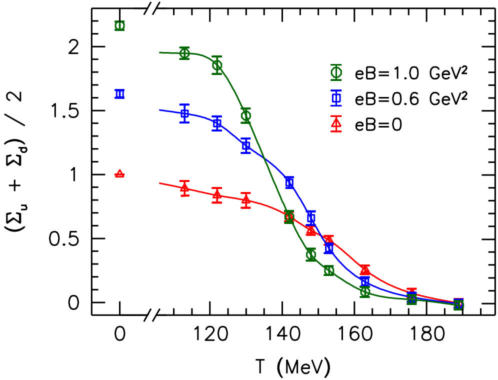

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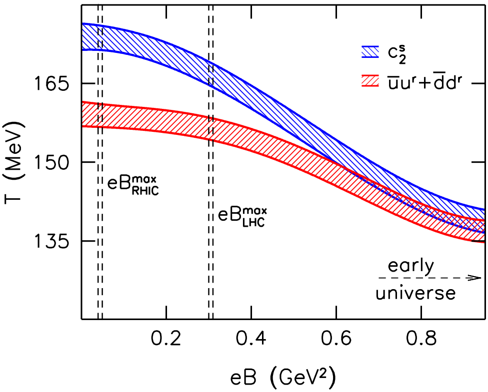
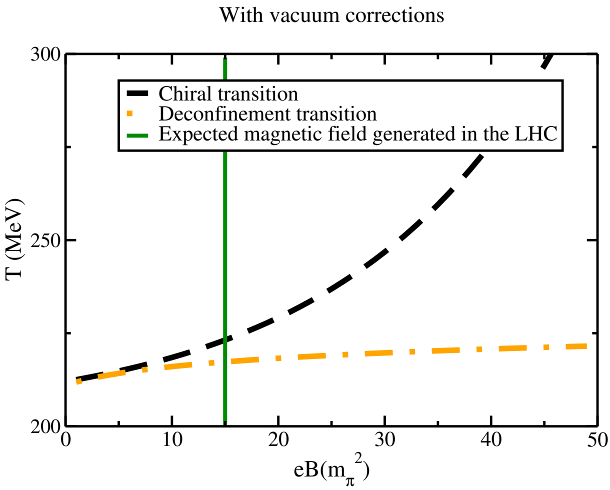
- valence and sea effects compete and around T_c the sea wins
- inflection point shifts to left $\rightarrow$ T_c is reduced



Phase diagram

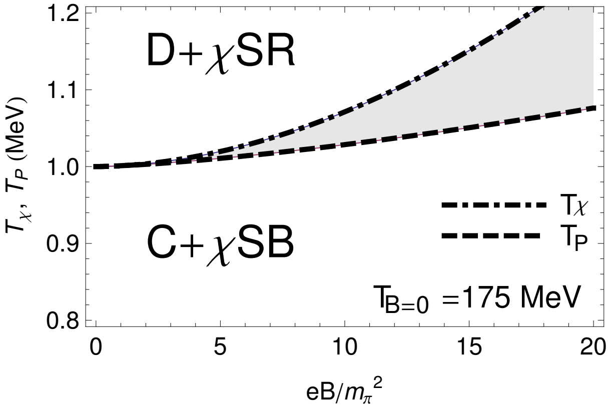
A brief history of $B - T$ phase diagrams

- 2010: linear σ model [Mizher, Chernodub, Fraga]



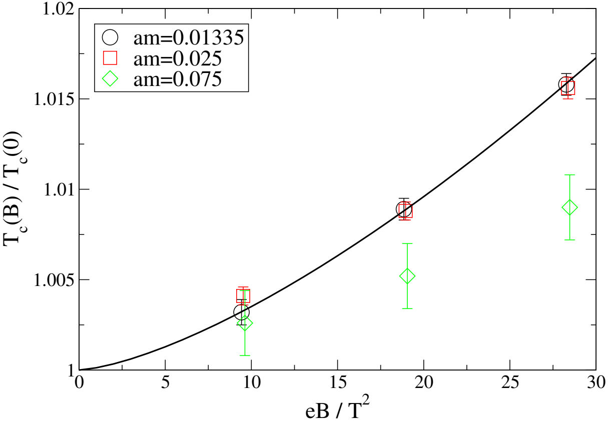
A brief history of $B - T$ phase diagrams

- 2010: PNJL model [Gatto, Ruggieri]



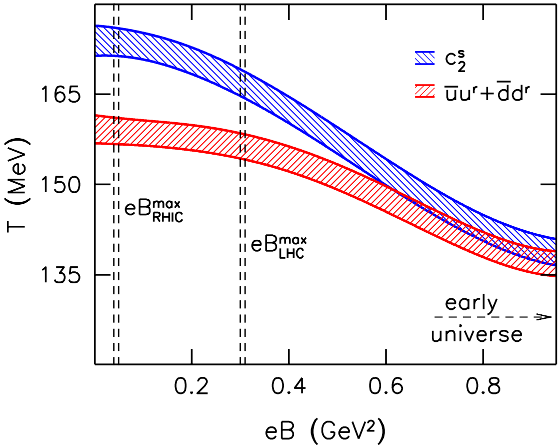
A brief history of $B - T$ phase diagrams

- 2010: lattice, coarse, heavy [D'Elia, Mukherjee, Sanfilippo]



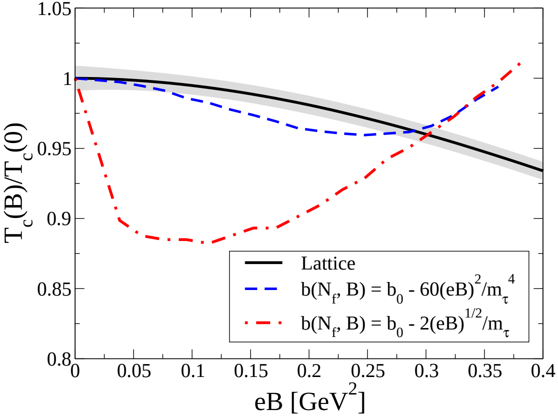
A brief history of $B - T$ phase diagrams

- 2011: lattice, cont.limit, physical
  [Bali, Bruckmann, Endrődi, Fodor, Katz, Krieg, Schäfer, Szabó]



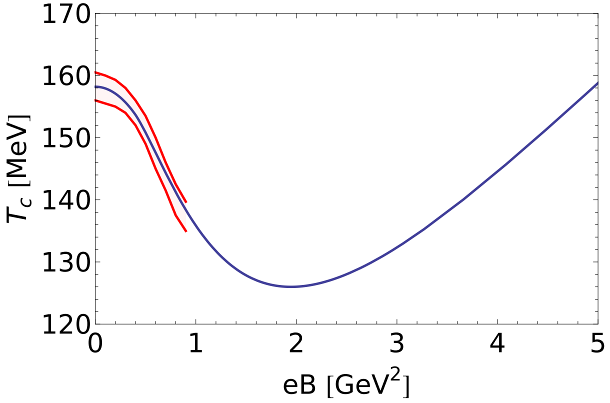
A brief history of $B - T$ phase diagrams

- 2014: parameterized models [Fraga, Mintz, Schaffner-Bielich]



A brief history of $B - T$ phase diagrams

- 2014: FRG+NJL [Müller, Pawlowski]



Open questions

- for $eB < 1$ GeV^2 the phase diagram is known from lattice
  - $T_c(B)$ monotonously decreases
  - the transition is an analytic crossover
- what happens for $eB > 1$ GeV^2?
  - is there a turning point, where $T_c(B)$ starts increasing?
  - is there a splitting between the chiral/deconfinement transitions?
  - is there a splitting between the up/down chiral transitions?
  - does the transition become a real phase transition?

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- significance: guiding effective theories and low-energy models
- aim: answer these questions using lattice simulations

Strategy

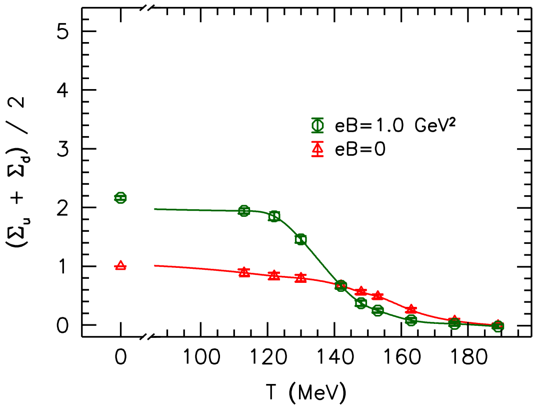
- largest possible field on a finite lattice is

$$eB_{\mathrm{max}} \approx a^{-2} \quad \Rightarrow \quad eB_{\mathrm{max}}/T^2 \approx N_t^2$$

- how to go even beyond?
  - exploit that eB is the largest scale and calculate the relevant effective theory
- strategy [Endrődi 1504.08280]:
  - simulate full QCD at $eB = 3.25$ GeV^2
  - simulate the effective theory at $B \to \infty$

Lattice results – full QCD

Quark condensates



- average of up and down quark condensates:
  T_c=inflection point

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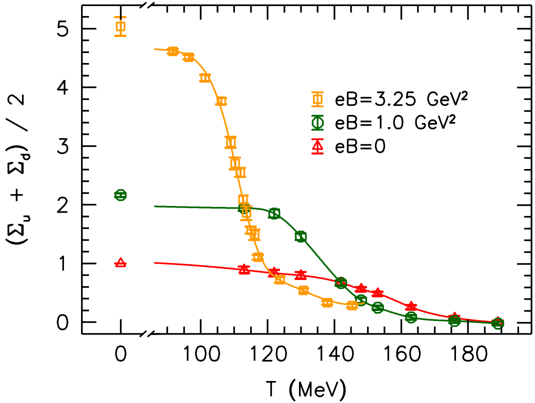

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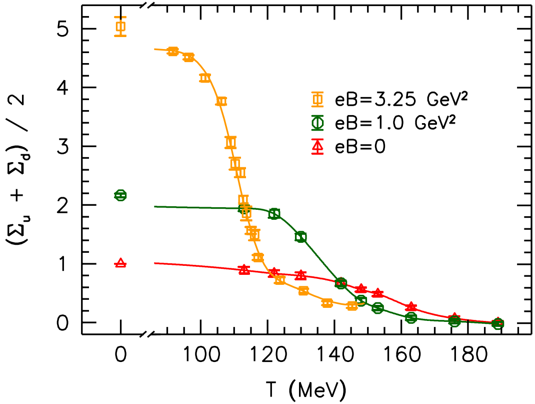
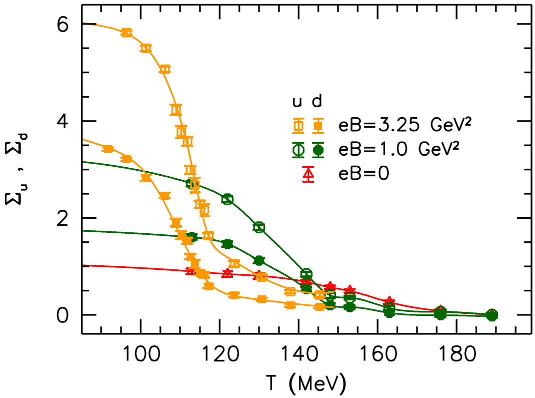


- average of up and down quark condensates: T_c=inflection point
  - is there a turning point, where $T_c(B)$ starts increasing? No.

Quark condensates



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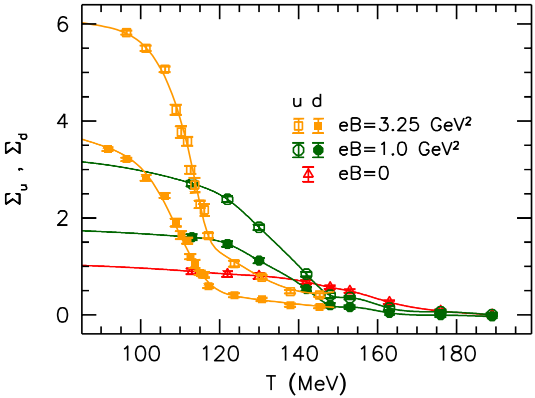
Quark condensates



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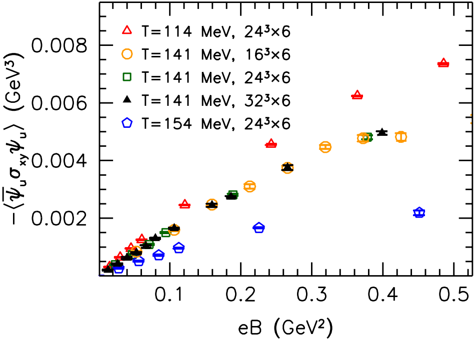
Spin polarization

- new expectation value induced by B [Ioffe, Smilga '84]

$$\left\langle \bar{\psi}_f \sigma_{xy} \psi_f \right\rangle = \tau \cdot q_f F_{xy} = \tau \cdot q_f B$$



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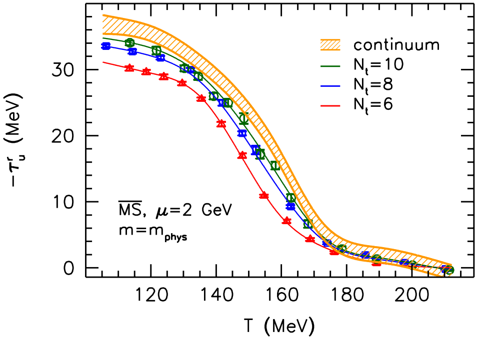
- the two condensates evaporate simultaneously
$T_c^{\bar{\psi}\sigma\psi} = T_c^{\bar{\psi}\psi}(B=0) \approx 155$ MeV [Bali et al '12]



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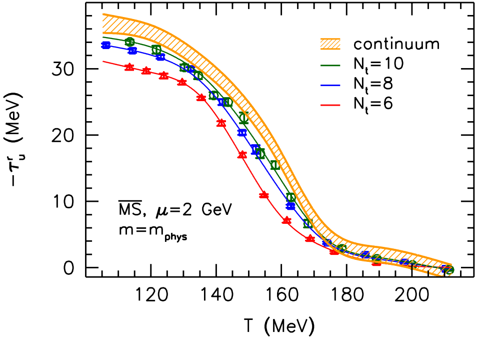
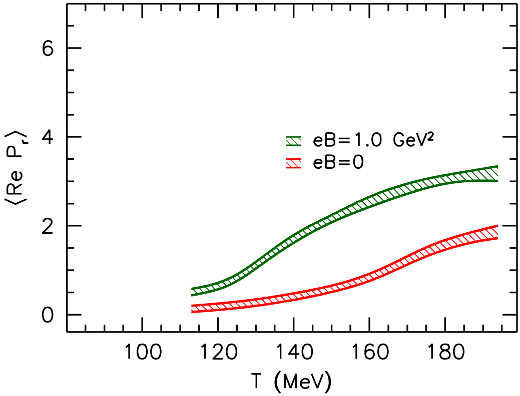


- reproduced in the NJL model extended with σ_{xy}-channels
[Ferrer, Incera, Portillo, Quiroz '14]

Polyakov loop



- Polyakov loop: T_c=inflection point

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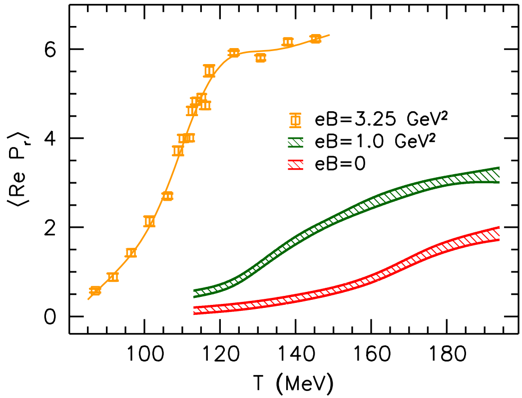

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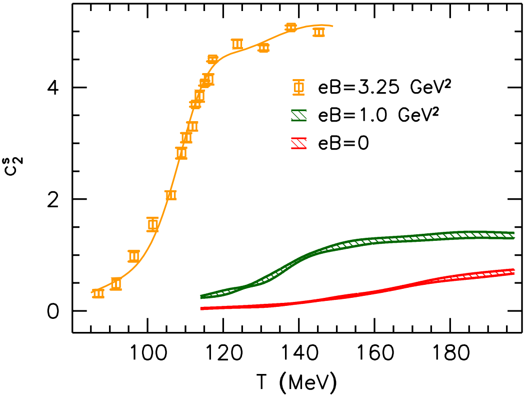
Strange quark number susceptibility



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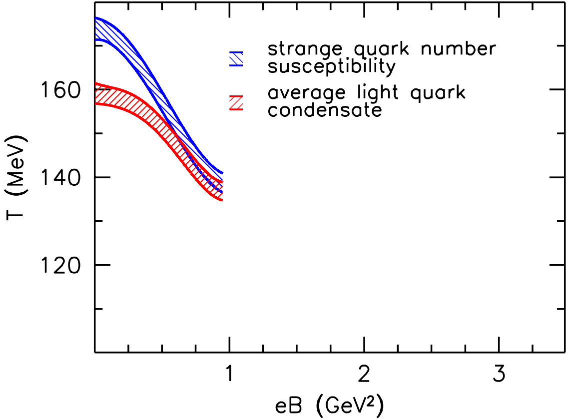
Phase diagram



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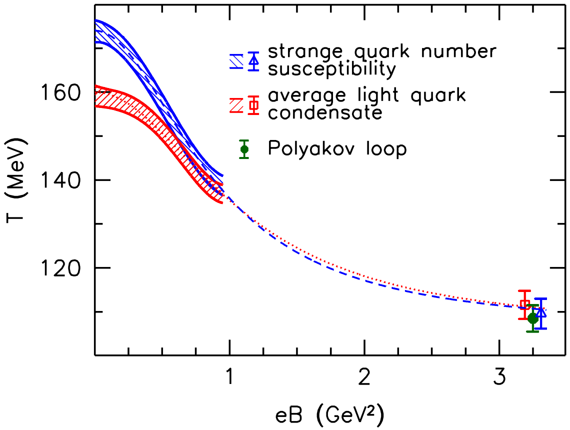
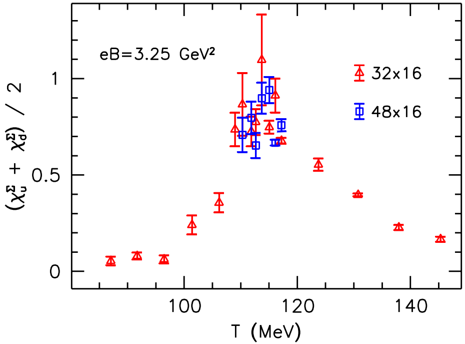


- summarizing T_c from all observables at $eB = 3.25$ GeV^2

Nature of transition: chiral susceptibility



- peak height independent of volume $\rightarrow$ analytic crossover (real phase transition would show singularity as $V \rightarrow \infty$)

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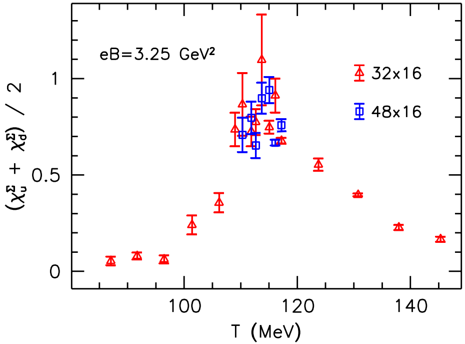

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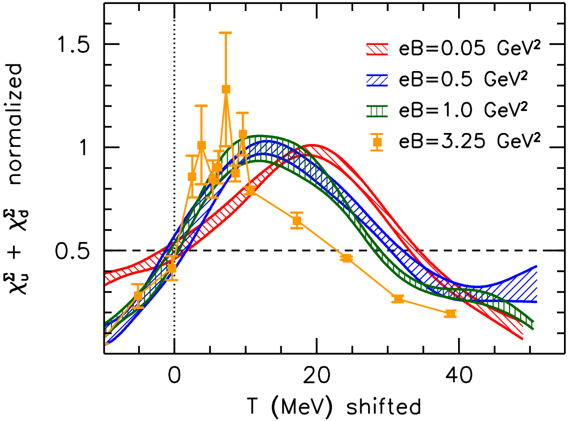
like  and not like 

Strength of the transition

- is there a tendency for strengthening/weakening?

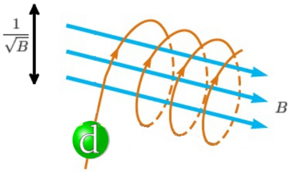


- the peak gets slowly but significantly narrower
- maybe there is a critical point at even stronger B?

Lattice results – effective theory

The effective theory

- what happens to $\mathcal{L}_{\rm QCD}$ at $eB \gg \Lambda^2_{\rm QCD}$?
  - first guess: asymptotic freedom says $\alpha_s \to 0$ i.e. complete decoupling of quarks and gluons
  - but: B breaks rotational symmetry and effectively reduces the dimension of the theory for quarks



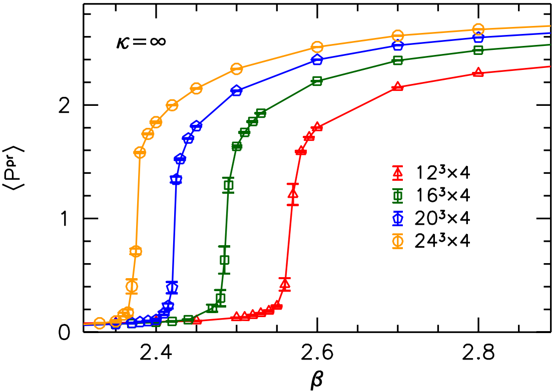
- gluons also inherit this spatial anisotropy, $\kappa(B) \propto B$
  [Miransky, Shovkovy 2002; Endrődi 1504.08280]

$$\mathcal{L}_{\rm QCD} \xrightarrow{B\to\infty} \mathrm{tr}\,\mathcal{B}_\parallel^2 + \mathrm{tr}\,\mathcal{B}_\perp^2 + [1+\kappa(B)]\,\mathrm{tr}\,\mathcal{E}_\parallel^2 + \mathrm{tr}\,\mathcal{E}_\perp^2$$

The effective theory

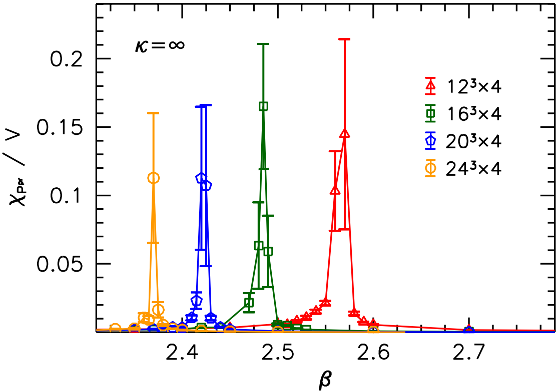
- finite κ: usual action, just multiply $z-t$ plaquettes by $(1+\kappa)$
  - large κ leads to large autocorrelation times
- $\kappa = \infty$ reduces independent degrees of freedom to
  local Polyakov loops $L_t(x, y)$ and
  local spatial Polyakov loops $L_z(x, y)$

Finite size scaling at $\kappa = \infty$



- Polyakov loop on different volumes: jump gets sharper

Finite size scaling at $\kappa = \infty$



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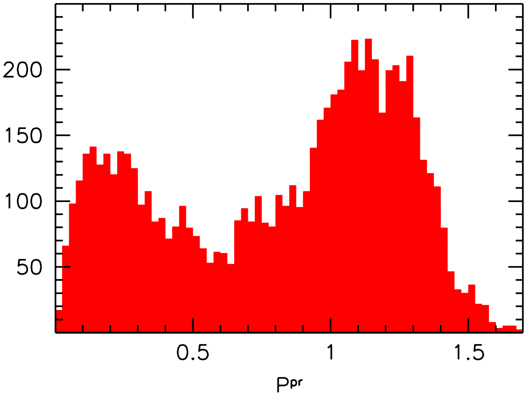

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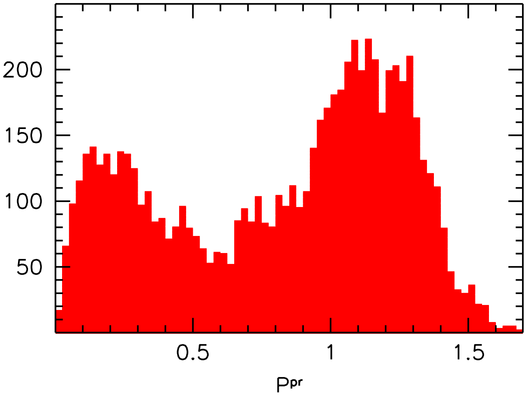


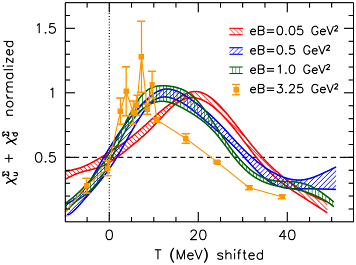
- Polyakov loop on different volumes: jump gets sharper
- Polyakov loop susceptibility peak height scales with V
- histogram shows double peak-structure at T_c
  - does the transition become a real phase transition? Yes.

Implications

Critical point

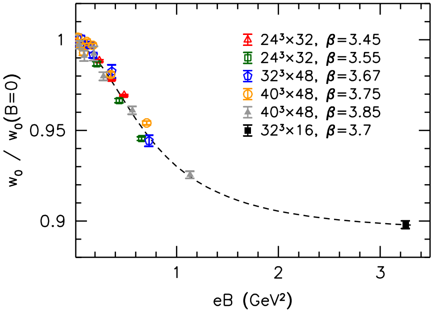
- analytical crossover for $0 \leq eB \leq 3.25$ GeV^2
  first-order transition for $B \to \infty$
  - there must be a critical point in between [Cohen, Yamamoto '13]
- estimate: extrapolate width of susceptibility peak to 0

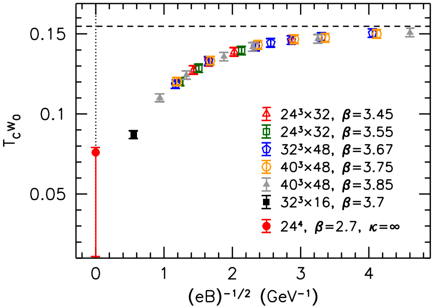
$$\underline{eB_{\rm CP} = 10(2)(?)\ \text{GeV}^2}$$



Critical temperature

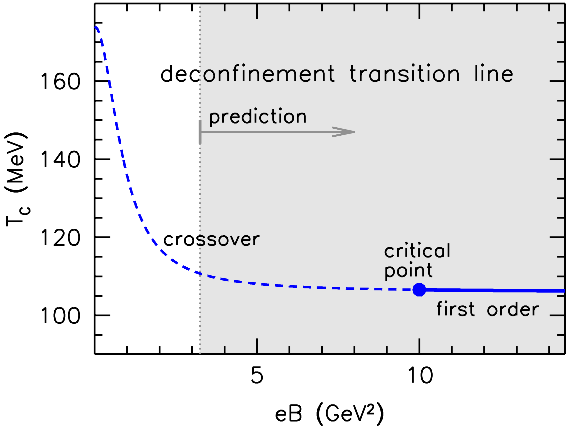
- to get $T_c(B \to \infty)$ in physical units, we need lattice scale a
  but: no a priori known dimensionful quantity at $B \to \infty$
  - attempt to use a pure gluonic quantity: w_0
    and match the combination $T_c w_0$





- *assuming that* $w_0(B)$ flattens out as $B \to \infty$
  $\to$ T_c reduces monotonously

Final conclusion

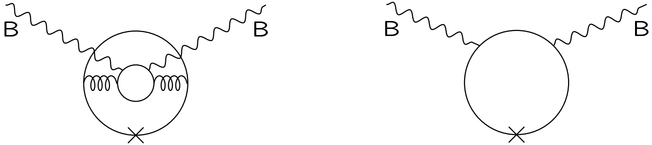


Another look at IMC

Mechanism behind MC and IMC

- two competing mechanisms at finite B
  [Bruckmann,Endrődi,Kovács '13]
  - direct (valence) effect $B \leftrightarrow q_f$
  - indirect (sea) effect $B \leftrightarrow q_f \leftrightarrow g$

$$\left\langle \bar{\psi}\psi(B) \right\rangle \propto \int \mathcal{D}A_\mu \, e^{-S_g} \underbrace{\det(\not{D}(B,A)+m)}_{\text{sea}} \underbrace{\mathrm{Tr}\left[(\not{D}(B,A)+m)^{-1}\right]}_{\text{valence}}$$



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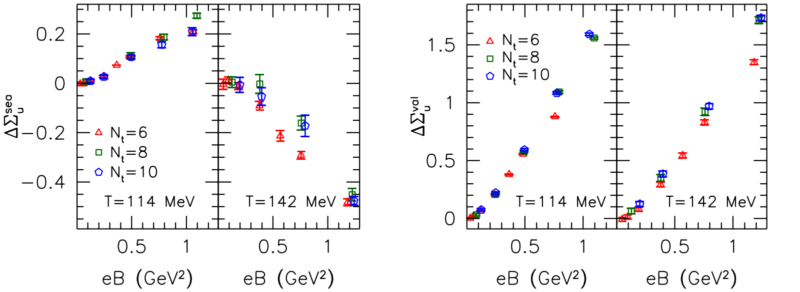
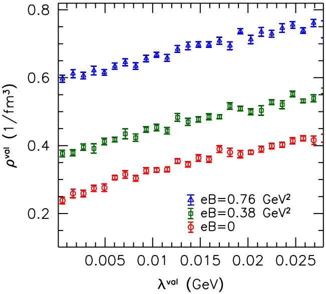


Mechanism behind MC

- valence sector: driven by the low eigenvalues of $\not{D}$

$$\left\langle \bar{\psi}\psi(B) \right\rangle \propto \int \mathcal{D}A_\mu \, e^{-S_g} \prod_i (\lambda_i^2(0) + m^2) \sum_j \frac{m}{\lambda_j^2(B) + m^2}$$



- valence sector: B creates many low eigenvalues through Landau-level degeneracy

Mechanism behind IMC

- sea sector: disfavors low eigenvalues of $\not{D}$ through det

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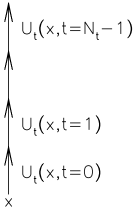

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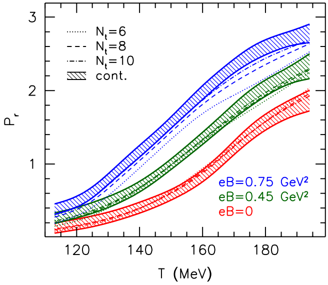

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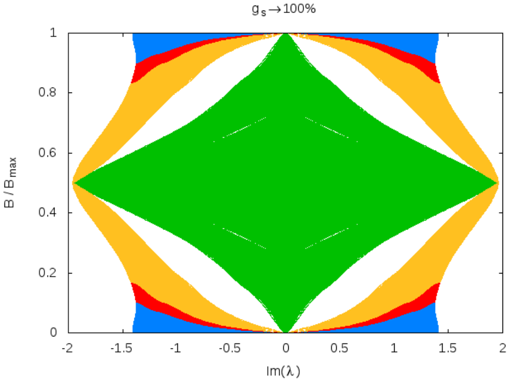
Outlook

Outlook

- separate lowest Landau-level on the lattice

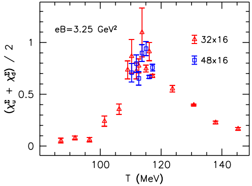


- measure how much of the effect comes from LLL and HLL
- facilitate comparison to models
- ideas?

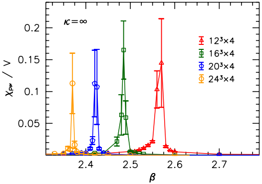
Summary

Summary

- analytic crossover even at $eB = 3.25\ \mathrm{GeV}^2$



- first-order phase transition at $B \to \infty$



- critical point, estimated location $eB_{\rm CP} = 10(2)\ \mathrm{GeV}^2$