Background magnetic fields and the QCD phase diagram

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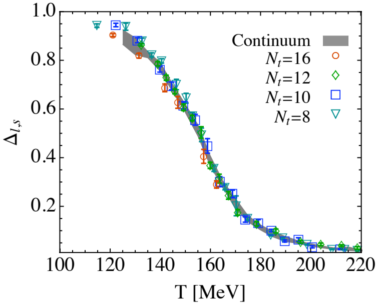
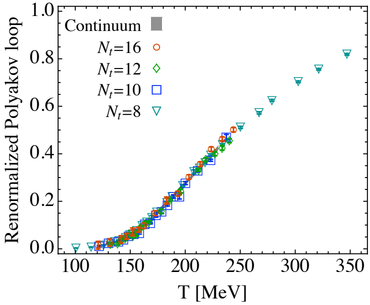
Preface

The phases of QCD

- phases of QCD characterized by the order parameters
- quark condensate $\bar{\psi}\psi$ (chiral symmetry breaking)
- Polyakov loop P (deconfinement)
- $T_c \leftrightarrow$ inflection point





lattice results from [Borsányi et al. '10]

remember talk [C. Ratti, Monday]

QCD with background magnetic fields

- QCD Lagrangian with electrodynamic fields

$$\mathcal{L}_{\rm QCD} = \mathcal{L}_{\rm gluon}(\mathcal{A}_\mu) + \sum_f \bar{\psi}_f \gamma_\mu (\partial_\mu + i\mathcal{A}_\mu + i q_f A_\mu + m_f)\psi_f$$

- constant background field

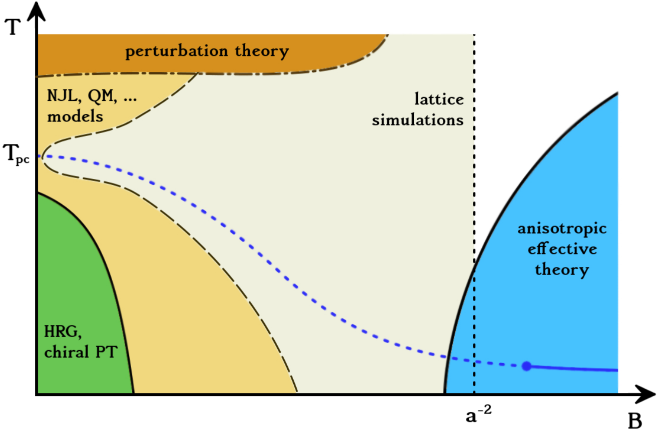
$$A_0 = 0, \qquad \nabla \times \mathbf{A} = \mathbf{B}$$

- fluctuations of the photon field negligible if B is strong
- electric charges

$$q_u = \frac{2e}{3}, \qquad q_d = -\frac{e}{3}, \qquad q_s = -\frac{e}{3}$$

Qualitative chart of applicability

- approaches: effective theories, low-energy models, lattice simulations, perturbation theory



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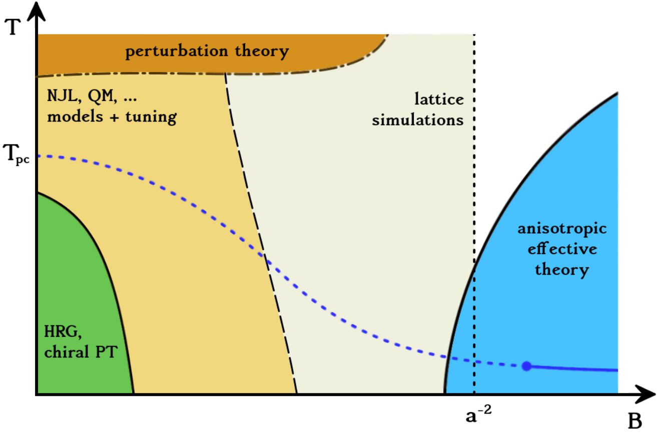


- tuning necessary for low-energy models

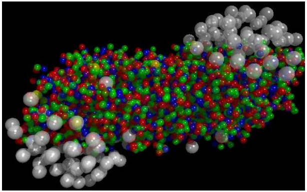
Outline

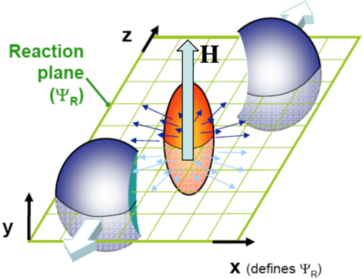
- applications
  - heavy-ion collisions
  - magnetars
  - early universe
- mechanisms
  - magnetic catalysis
  - inverse magnetic catalysis
  - models vs. lattice results
- large B limit
- outlook

Applications

Applications: heavy-ion collisions

- off-central events generate magnetic fields
  [Kharzeev, McLerran, Warringa '07]

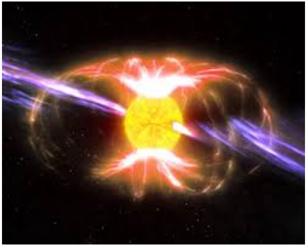


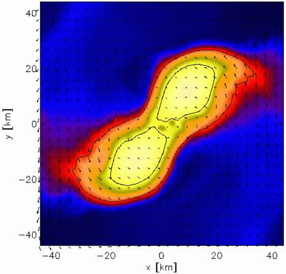


- strength: $B = 10^{15}$ T $\approx 10^{20} B_{\rm earth} \approx 5 m_\pi^2$
- impact: chiral magnetic effect, anisotropies, elliptic flow, hydrodynamics with B, ...
  reviews: [Fukushima '12] [Kharzeev, Landsteiner, Schmitt, Yee '14] [Kharzeev '15]

Applications: magnetars

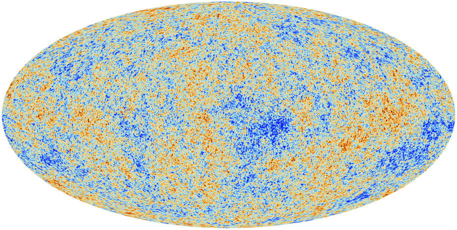
- neutron stars with strong surface magnetic fields [Duncan, Thompson '92]





- strength on surface: $B = 10^{10}$ T
- strength in core: $B = 10^{14}$ T $\approx 10^{19} B_{\mathrm{earth}} \approx 0.5 m_\pi^2$
- impact: convectional processes, mass-radius relation, gravitational collapse/merger [Anderson et al '08], ...

Applications: early universe



- large-scale intergalactic magnetic fields 10 μG $= 10^{-9}$ T
- origin in the early universe
- generation through a phase transition: electroweak epoch $B \approx 10^{19}$ T [Vachaspati '91, Enqvist, Olesen '93]

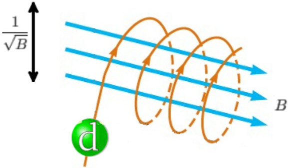
Effect of magnetic fields: zero temperature

Magnetic catalysis in a nutshell

- chiral condensate $\leftrightarrow$ spectral density around 0 [Banks, Casher '80]

$$\bar{\psi}\psi \propto \rho(0)$$

- large magnetic fields reduce dimensionality $3+1 \to 1+1$ and induce degeneracy $\propto B$

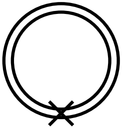


- in the chiral limit, to maintain $\bar{\psi}\psi > 0$ [Gusynin et al '96]

$B = 0$	$\rho(p) \sim p^2 \mathrm{d}p$	"strong interaction is needed"
$B \gg m^2$	$\rho(p) \sim B\,\mathrm{d}p$	"the weakest interaction suffices"

Magnetic catalysis: chiral PT

- charged pions in a magnetic field
- Schwinger propagator [Schwinger '51]



- one-loop diagram with scalar insertion gives [Cohen, Werbos '07]

$$\frac{\bar\psi\psi - \bar\psi\psi_{B=0}}{\bar\psi\psi_{B=0}} = \frac{1}{16\pi^2 f_\pi^2}\int_0^\infty \frac{\mathrm{d}s}{s^2} e^{-m_\pi^2 s}\left(\frac{eBs}{\sinh(eBs)} - 1\right)$$
$$= \frac{1}{2}\beta_1^{\mathrm{scalar}} \cdot \frac{(eB)^2}{f_\pi^2 m_\pi^2} + \mathcal{O}((eB)^4)$$

- leading order in B with universal coefficient [Endrődi '13] $\beta_1^{\mathrm{scalar}} = 1/(48\pi^2) > 0$

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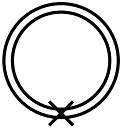

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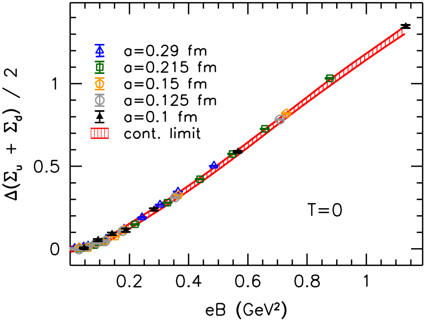
Magnetic catalysis: lattice simulations

- numerical simulation of the path integral

$$\mathcal{Z} = \int \mathcal{D}U \, \mathcal{D}\bar{\psi} \, \mathcal{D}\psi \, \exp(-S_{\mathrm{QCD}})$$

- obtain condensate from $\bar{\psi}\psi = \frac{1}{V_4} \frac{\partial \log \mathcal{Z}}{\partial m}$

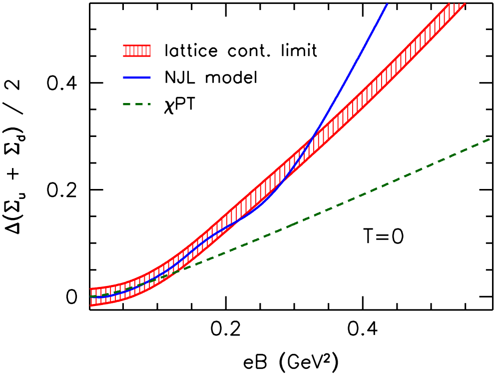


- physical m_π, continuum limit

[Bali,Bruckmann,Endrődi,Fodor,Katz,Schäfer '12]

Magnetic catalysis – zero temperature

- magnetic catalysis at zero temperature is a robust concept: χPT, NJL, AdS-CFT, linear σ model, lattice QCD, . . .
  [Andersen, Naylor '14]



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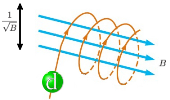
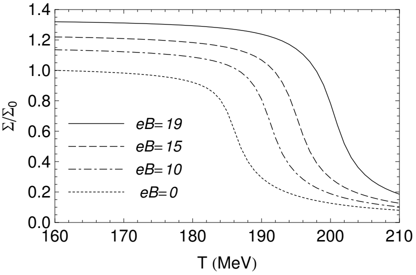
Effect of magnetic fields: nonzero temperature

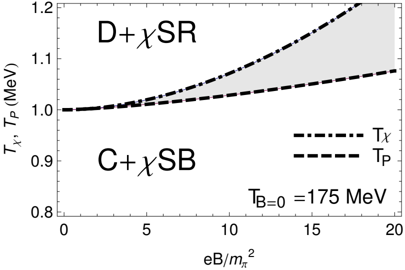
Phase diagram: models

- recall catalysis argument $\bar{\psi}\psi \propto \rho(0)$



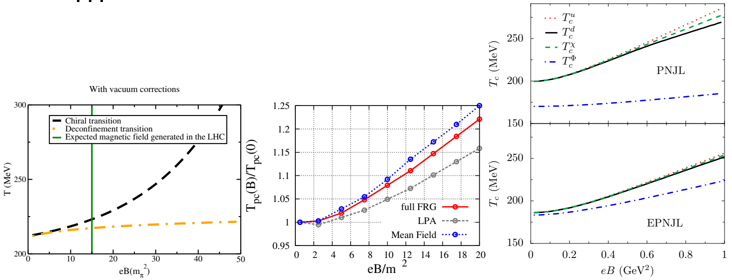
- model calculations at $T > 0$:
  - magnetic catalysis for all T
  - $T_c(B)$ increases
- for example the PNJL model [Gatto, Ruggieri '11]





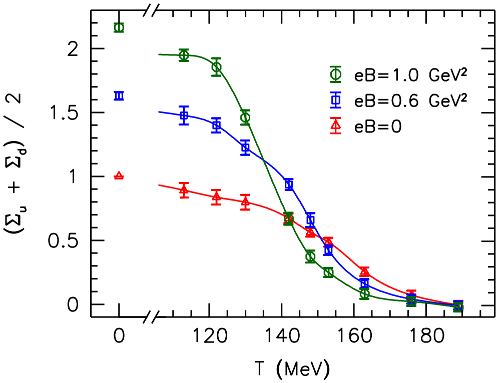
Phase diagram: models

- majority of low-energy models give the same qualitative result
  - linear sigma model + Polyakov loop [Mizher, Chernodub, Fraga '10]
  - quark-meson model + functional renormalization group [Kamikado, Kanazawa '13]
  - NJL model + Polyakov loop [Ferreira, Costa, Menezes, Providencia, Scoccola '13]
  - ...



Phase diagram: lattice simulations

- lattice QCD, physical m_π, continuum limit
  [Bali,Bruckmann,Endrődi,Fodor,Katz,Krieg,Schäfer,Szabó '11, '12]



- surprise: magnetic catalysis turns into inverse magnetic catalysis (IMC) around T_c [Bruckmann, Endrődi, Kovács '13]

Phase diagram: lattice simulations

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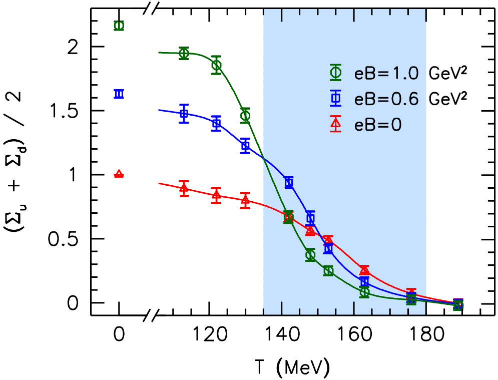

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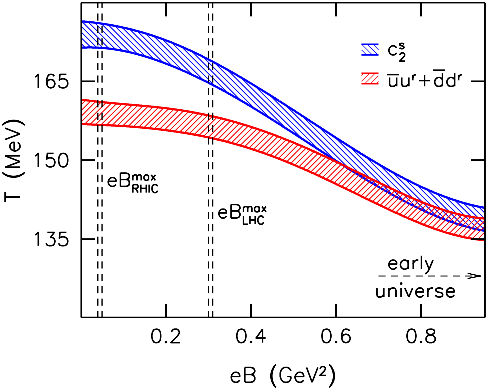
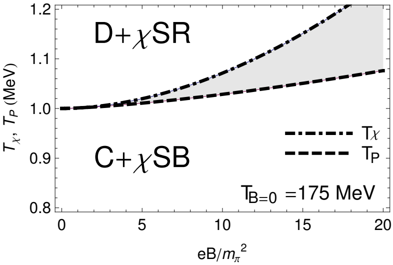
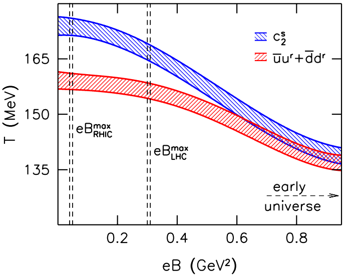
Phase diagram: lattice simulations

- impact on the QCD phase diagram
  [Bali, Bruckmann, Endrődi, Fodor, Katz, Krieg, Schäfer, Szabó '11]



Phase diagram: comparison





	model	lattice
$T_c(B)$	increases	decreases
$T_c^{(P)}$ and $T_c^{(\bar\psi\psi)}$	diverge	converge
condensate	magnetic catalysis $\forall T$	inverse catalysis $T \approx T_c$

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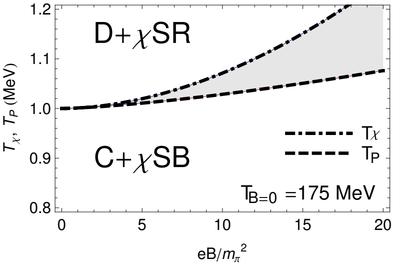

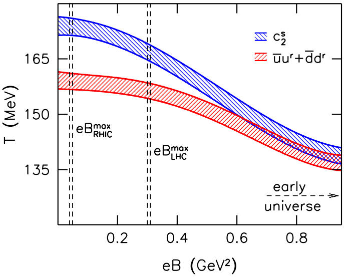

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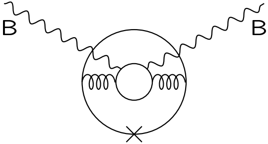
- models and lattice simulations are as different as can be

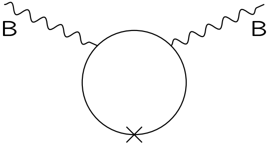
Inverse magnetic catalysis

MC and IMC

- two competing mechanisms at finite B [D'Elia, Negro '11] [Bruckmann,Endrődi,Kovács '13]
  - direct (valence) effect $B \leftrightarrow q_f$
  - indirect (sea) effect $B \leftrightarrow q_f \leftrightarrow g$

$$\left\langle \bar{\psi}\psi(B) \right\rangle \propto \int \mathcal{D}A_\mu \, e^{-S_g} \underbrace{\det(\not{D}(B,A)+m)}_{\text{sea}} \underbrace{\mathrm{Tr}\left[(\not{D}(B,A)+m)^{-1}\right]}_{\text{valence}}$$





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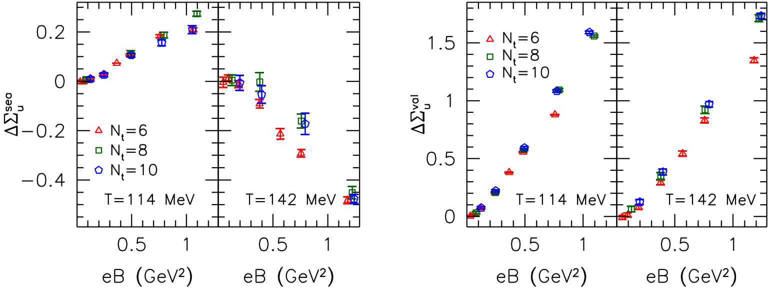
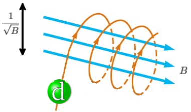
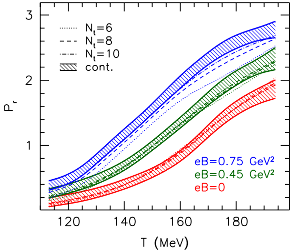


MC and IMC

- valence effect: "on a given gluon configuration, there are more low Dirac eigenvalues if B is switched on"



- sea effect: "the typical gluon configurations become different if B is switched on, and the most important difference is the change in the Polyakov loop"



What have we learned?

- simple argument behind magnetic catalysis does not work around T_c
- typical gluonic backgrounds change: indirect effect of B on neutral gluons
- model approaches lack this indirect effect
- "deconfinement is more important than chiral symmetry breaking"

Models with lattice-inspired tuning

Tuned models

- incorporate inverse catalysis in low-energy models
- strategies:
  - take model parameter $x \to x(B)$ and fit to lattice results

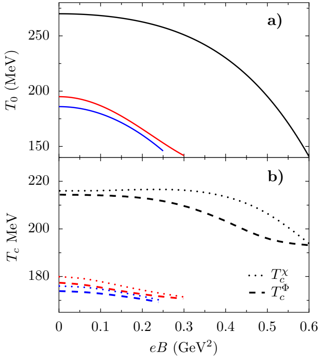
    OR
  - calculate B-dependence of parameter $x(B)$ in the model via loop corrections
- parameters:
  - T_0 of the Polyakov loop potential
  - coupling G of the NJL model
  - Yukawa coupling g in the quark-meson model
  - scalar coupling λ in the quark-meson model

Tuned models I

- (E)PNJL model with tuned T_0 [Ferreira et al. '14]

$$T_0 \to T_0(B) = T_0(0) + \zeta(eB)^2 + \xi(eB)^4$$

  - works at low B
  - turns crossover into first order phase transition at moderate B
  - gives increasing $T_c(B)$ for large B

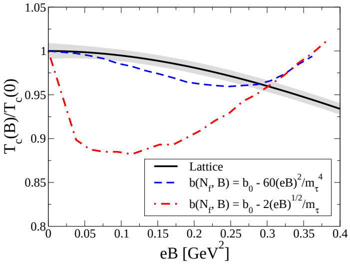


Tuned models II

- PQM model with tuned T_0 [Fraga et al. '13]

$$T_0 \to T_0(B) \text{ arbitrary}$$

  - inevitably gives increasing $T_c(B)$ for large B

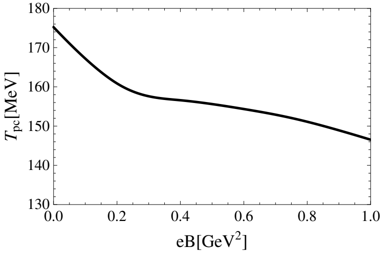


Tuned models III

- NJL model with tuned G [Farias et al. '14]

$$G \to G(B,T) = \frac{G_0}{1 + \alpha \ln(1 + \beta |eB|/\Lambda^2_{\mathrm{QCD}})} \left(1 - \gamma \frac{eBT}{\Lambda^3_{\mathrm{QCD}}}\right)$$

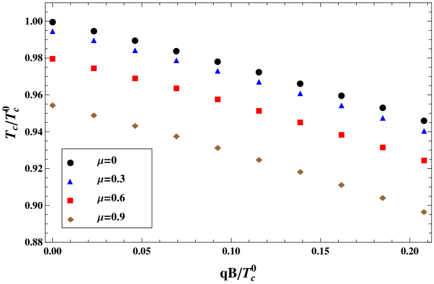
  - inspired by renormalization group running with B



Tuned models IV

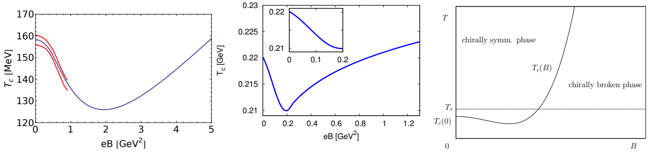
- quark-meson model with 1-loop corrections to the couplings λ and g [Ayala et al. '15]

$$\lambda \to \lambda^{1-\text{loop}}(B), \qquad g \to g^{1-\text{loop}}(B)$$



What have we learned?

- fitting a B-dependent parameter to the lattice data can help, but the model loses (most of) its predictive power
- B-dependent corrections to coupling constants improves the situation
- reduction of the coupling strength with B appears to be crucial [Ferrer et al. '14]
- including/neglecting vacuum fluctuations appears to be crucial [Mizher, Chernodub, Fraga '10, Andersen, Naylor '14]
- most studies still predict a non-monotonous $T_c(B)$ [Müller et al '15, Braun et al '15, Ilgenfritz et al '13]

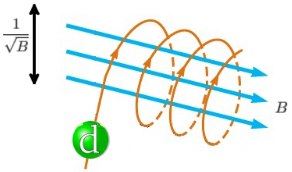


$\rightarrow$ can we check this on the lattice?

Large B: anisotropic effective theory

Large B limit

- what happens to $\mathcal{L}_{\rm QCD}$ at $eB \gg \Lambda^2_{\rm QCD}, T^2$?
  - ▶ first guess: asymptotic freedom says $\alpha_s \to 0$ i.e. complete decoupling of quarks and gluons
  - ▶ but: B breaks rotational symmetry and effectively reduces the dimension of the theory for quarks

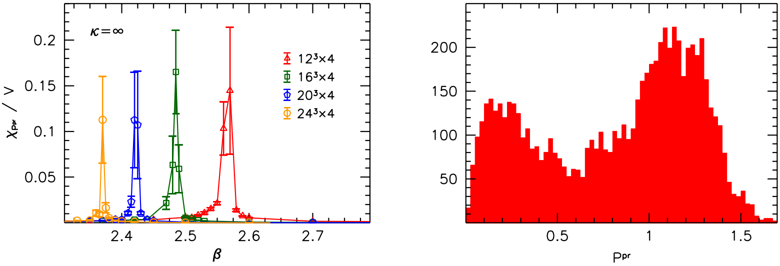


- gluons also inherit this spatial anisotropy, $\kappa(B) \propto B$
  [Miransky, Shovkovy 2002; Endrődi 1504.08280]

$$\mathcal{L}_{\rm QCD} \xrightarrow{B\to\infty} \mathrm{tr}\,\mathcal{B}_\parallel^2 + \mathrm{tr}\,\mathcal{B}_\perp^2 + [1+\kappa(B)]\,\mathrm{tr}\,\mathcal{E}_\parallel^2 + \mathrm{tr}\,\mathcal{E}_\perp^2$$

Simulating the anisotropic effective theory

- pure (but anisotropic) gauge theory: can be simulated on the lattice [Endrődi 1504.08280]



- Polyakov loop susceptibility peak height scales with V
- histogram shows double peak-structure at T_c

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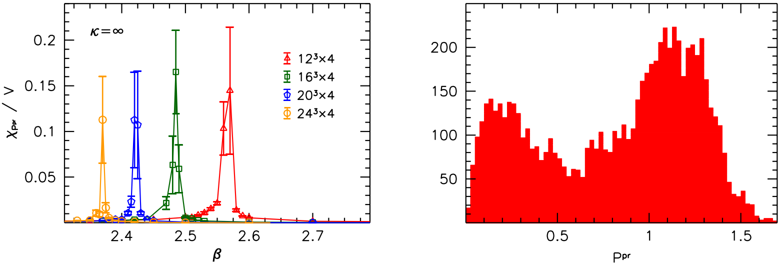
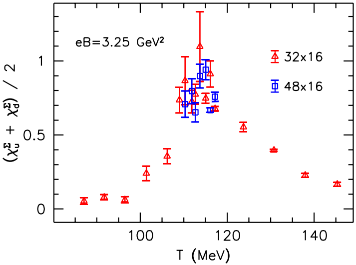


- Polyakov loop susceptibility peak height scales with V
- histogram shows double peak-structure at T_c
- the transition is of first order

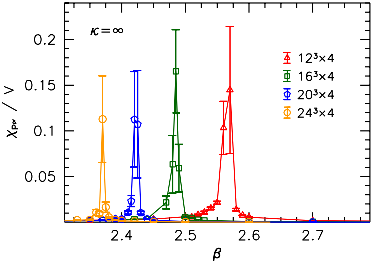
Nature of the transition

- volume-scaling of the height of the susceptibility peak

$eB = 3.25\text{GeV}^2$ $\qquad$ $eB \to \infty$



crossover



first-order

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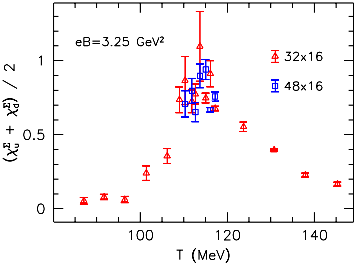
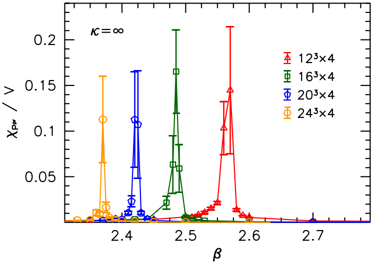




crossover $\qquad$ first-order

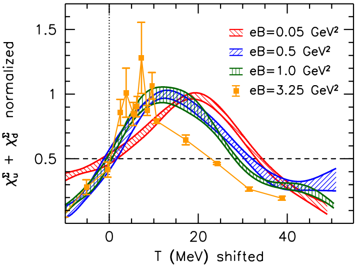
Critical point

- analytical crossover for $0 \leq eB \leq 3.25$ GeV^2
  first-order transition for $B \to \infty$

▸ there must be a critical point in between [Cohen, Yamamoto '13]

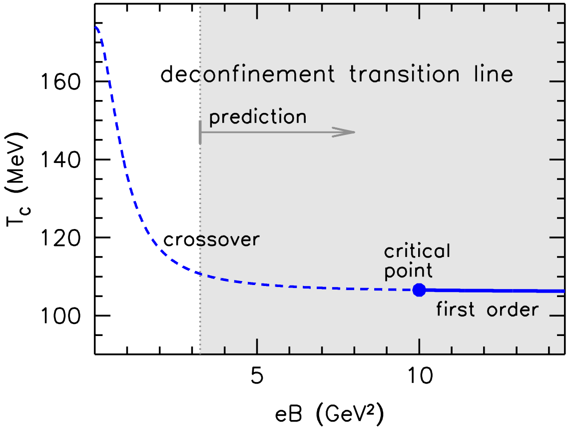
- estimate: extrapolate width of susceptibility peak to 0

$$eB_{\mathrm{CP}} = 10(2)(?)\ \mathrm{GeV}^2$$



- calculate T_c by matching the dimensionless combination $T_c w_0$ to full QCD results

Phase diagram



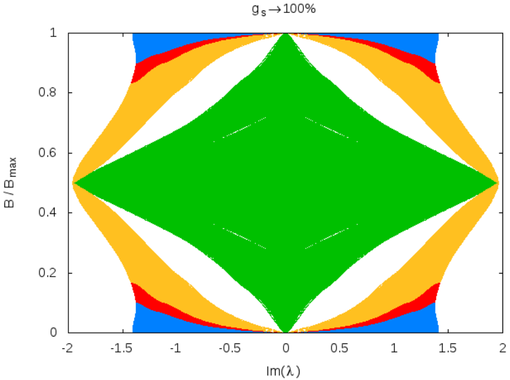
Further approaches

- holographic models [Evans, Miller, Scott '16] [Rougemont, Critelli, Noronha '16] [Dudal, Granado, Mertens '16][Li, Jia '16]
- bag model [Fraga, Palhares '12, Andersen, Naylor '14]
- large-N_c arguments [Fraga,Noronha,Palhares '13]
- QCD simulations with coarse lattices and/or heavy pions [D'Elia et al. '10] [Ilgenfritz et al. '12, '13]
- functional renormalization group methods [Braun et al '15, Müller et al '15]

Outlook

Outlook: lowest Landau-level approximation

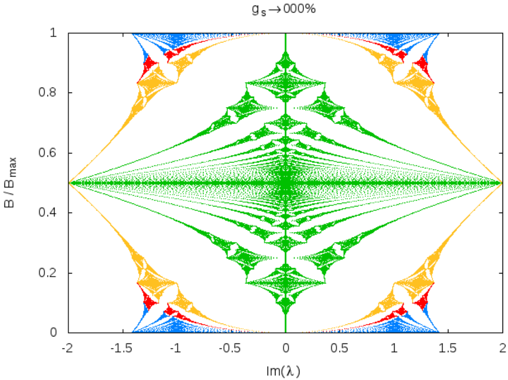
- separate lowest Landau-level on the lattice



- facilitate comparison to models

Outlook: lowest Landau-level approximation

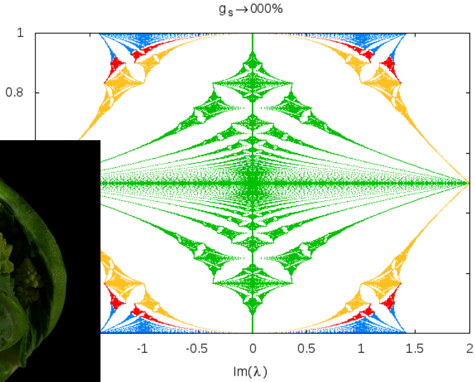
- separate lowest Landau-level on the lattice



- facilitate comparison to models
- contact to Hofstadter's butterfly [Hofstadter '76]

Outlook: lowest Landau-level approximation

- separate lowest Landau-level on the lattice



- facilitate comparison to models
- contact to Hofstadter's butterfly [Hofstadter '76]

Summary

Summary

- QCD in the presence of strong magnetic fields
- phenomenological applications

- theoretical challenge: reconcile models with lattice results
- critical point at very large magnetic fields

Backup

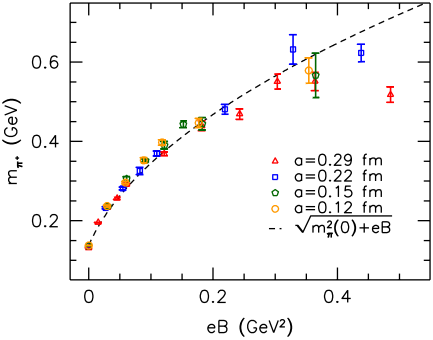
Background fields

- background field vs. fluctuating field

$$A_\mu = A_\mu^{\mathrm{const}} + A_\mu^{\mathrm{fluc}}$$

- effect on charged pion mass

$$\delta m^2_{\pi,\mathrm{fluc}} = \mathcal{O}(e^2), \qquad \delta m^2_{\pi,\mathrm{const}} = \mathcal{O}(|eB|)$$

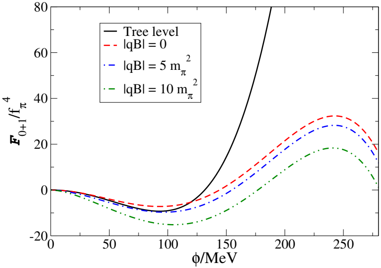


[Bali, Bruckmann, Endrődi, Fodor, Katz, Krieg, Schäfer, Szabó, '11]

Magnetic catalysis: linear sigma model

- effective potential of the model at $T = 0$

$$F = \frac{\lambda}{4}(\sigma^2 - v^2)^2 - h\sigma$$
$$+ \frac{N_c}{8\pi^2}\sum_f \int \frac{\mathrm{d}s}{s^3} e^{-(g\sigma)^2 s}\left[q_f Bs \coth(q_f Bs) - 1 - \frac{(q_f Bs)^2}{3}\right]$$
$$+ F_{B=T=0}$$



- v, h, g, λ are matched to reproduce $T = 0$ hadronic quantities
- minimize $F(\sigma)$
- minimum shifts to larger σ as B grows $\rightarrow$ magnetic catalysis [Andersen, Naylor '14]