Hot QCD in electric fields

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Gauge Topology, Flux Tubes And Holographic Models:
The Intricate Dynamics Of QCD In Vacuum And Extreme Environments

ECT* Trento, May 26th 2022

Outline

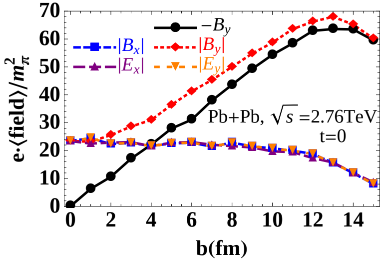
- introduction
- equilibrium with electric fields
- Schwinger vs. Weldon
- imaginary fields on the lattice
- summary

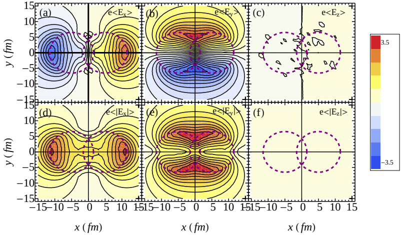
Introduction

Electromagnetic fields for QCD

- electromagnetic fields in the early stage of heavy-ion collisions reaching m_π^2 and well beyond

  Deng et al. '12

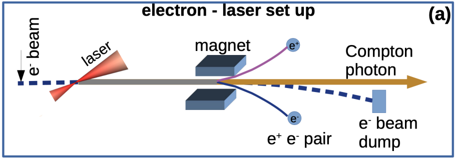


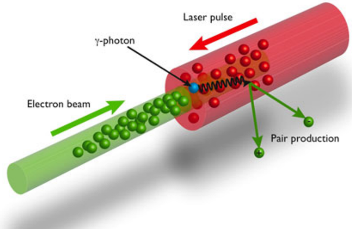


- impact of electric field enhanced for asymmetric systems (for example Cu+Au at RHIC) Voronyuk et al. '14

Electromagnetic fields for QED

- high-intensity laser experiments
  electromagnetic fields reaching m_e^2 and beyond
  CoRELS, ELI, SEL, MP3
- electron-laser beam collisions
  E320 experiment at SLAC
  LUXE experiment at DESY Abramowicz et al. '21

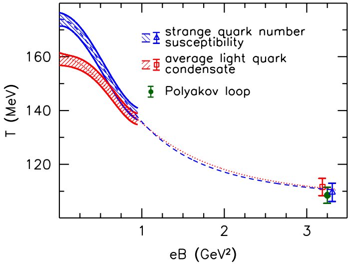


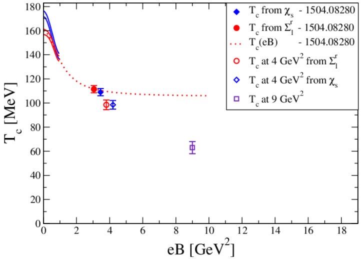


ELI outreach, Mattias Marklund

Magnetic fields

- dynamical chiral symmetry breaking at $B > 0$ in QED
  Gusynin et al. '95
- magnetic catalysis of chiral condensate in QCD (low T)
  Shovkovy '13
- inverse magnetic catalysis in QCD ($T \approx T_c$)
  Bruckmann, Endrődi, Kovács '13
- QCD phase diagram Bali et al. '11 Endrődi '15 D'Elia et al. '21





- magnetic susceptibility Bonati et al. '13 Bali et al. '20

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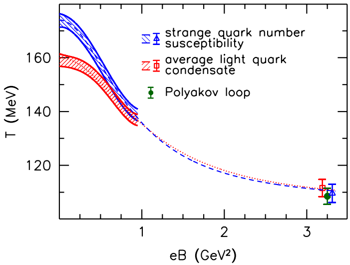

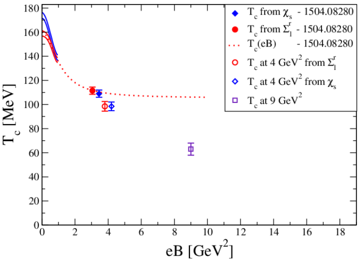

- magnetic susceptibility Bonati et al. '13 Bali et al. '20
- what about electric fields?

Equilibrium in presence of electric fields?

Preliminaries

- electromagnetic field E, B from vector potential

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \qquad E_i = F_{0i}, \qquad B_i = \epsilon_{ijk} F_{jk}/2$$

- fermions couple to A_μ via charge q

$$\not{D} = \gamma^\mu(\partial_\mu + iqA_\mu)$$

- E, B treated as classical background fields: <u>no back-reaction</u>
- we are interested in leading response: susceptibilities

$$\xi = \left.\frac{\partial^2\Omega}{\partial E^2}\right|_{E=0}, \qquad \chi = \left.\frac{\partial^2\Omega}{\partial B^2}\right|_{B=0}$$

- only thermal effects (no Schwinger pair creation)

$$\Omega = \left.\frac{T}{V}\log\mathcal{Z}\right|_T - \left.\frac{T}{V}\log\mathcal{Z}\right|_{T=0}$$

Equilibrium

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- but an equilibrium can be defined:

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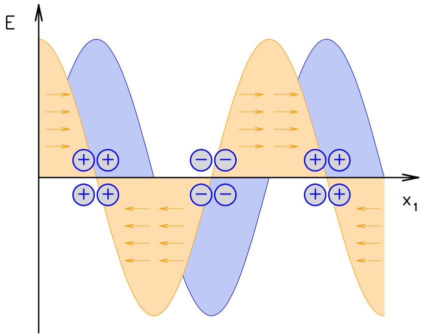

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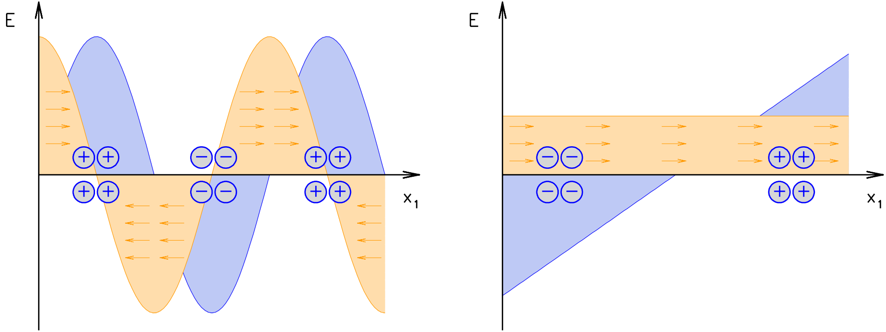

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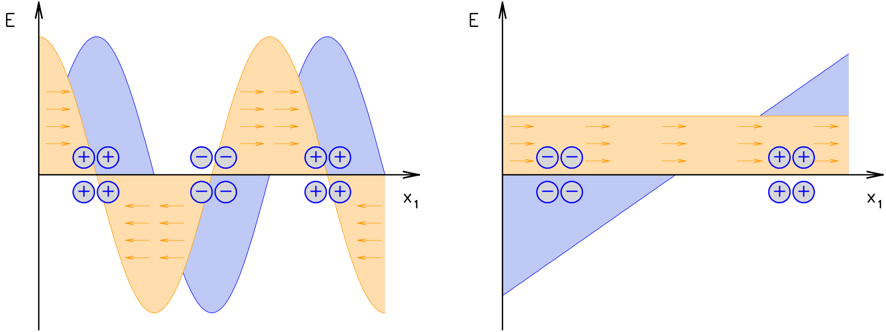
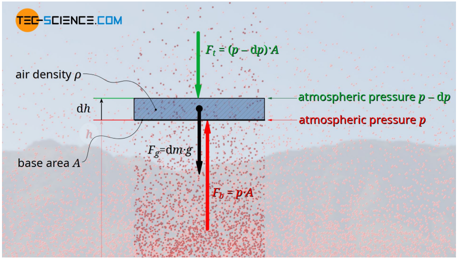


- the two become equivalent for large wavelengths

Analogy: barometric distribution

- recall barometric formula above ‘flat earth’ tec-science.com



- gravitational force $\leftrightarrow$ electric force
- atmospheric pressure $\leftrightarrow$ fermionic degeneracy pressure

  Bo-Sture Skagerstam, private comm.

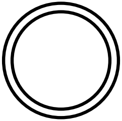
Susceptibilities

Schwinger's approach

- Schwinger propagator for constant electromagnetic fields
  Schwinger '51

$S_{\alpha\beta}(E, B) =$ 

- effective action analytically at one loop order at $T > 0$
  Loewe, Rojas '92 Elmfors, Skagerstam '95 Gies '98

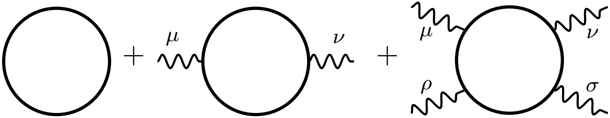
$\Omega(E, B) =$ 

- subsequent expansion in E and B

$$\Omega = \chi \cdot \frac{B^2}{2} + \xi \cdot \frac{E^2}{2} + \mathcal{O}(E^4, B^4, E^2 B^2)$$

Weldon's approach

- free energy expansion in arbitrary background field A_μ

$\Omega(A) =$  $+ \mathcal{O}(A^6)$

- quadratic order: vacuum polarization diagram

$$\Pi_{\mu\nu}(k) = (k^2 g_{\mu\nu} - k_\mu k_\nu) \cdot \Pi(k^2)$$

at $T > 0$

$$\Pi_{\mu\nu}(\omega, |\mathbf{k}|) = P_{\mu\nu} \cdot \Pi_T(\omega, |\mathbf{k}|) + Q_{\mu\nu} \cdot \Pi_L(\omega, |\mathbf{k}|)$$

- susceptibilities from Π_T and Π_L
for massless fermions 🔗 Weldon '82

One object, two approaches

- Schwinger's approach

Euler-Heisenberg Lagrangians, light-by-light scattering, pair production at $E > 0$, photon splitting at $B > 0$, birefringence . . . laser physics Gies '00 Dunne '04
thermodynamics in QED / QCD models mainly at $B > 0$
Miransky, Shovkovy '15

- Weldon's approach

perturbation theory in hot QCD
hard thermal loops Braaten, Pisarski '91 Blaizot, Iancu '01
QGP transport . . . Arnold, Moore, Yaffe '03

One object, two approaches

spoiler:

- magnetic susceptibility

$$\chi_{\text{Schwinger}} = \chi_{\text{Weldon}}$$

- electric susceptibility

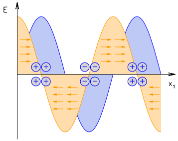
$$\xi_{\text{Schwinger}} \neq \xi_{\text{Weldon}}$$

Results: susceptibilities

Susceptibilities a la Weldon

- consider background $A_\mu(x_1) \leftrightarrow \widetilde{A}_\mu(k_1)$
- susceptibilities for static, homogeneous fields are obtained as

$$\xi = \lim_{k_1 \to 0} \frac{\Pi_{00}(\omega = 0, k_1)}{k_1^2}, \qquad \chi = \lim_{k_1 \to 0} \frac{\Pi_{22}(\omega = 0, k_1)}{k_1^2}$$

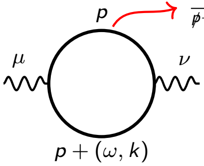


Bali, Endrődi, Piemonte '20

- we generalize Weldon's calculation for $m > 0$

$$\frac{1}{\not{p}+m+i\epsilon} + (\not{p}+m)\frac{2\pi i\delta(p^2-m^2)}{e^{|p_0|/T}+1}$$

$\Pi_{\mu\nu}(\omega, k) =$



Results: susceptibilities

- $\lim_{\omega\to 0}$; $\int \mathrm{d}p_0$; $\int \mathrm{d}\cos\theta_{\mathbf{p}}$; $\lim_{k\to 0}$; watch out for $\epsilon \to 0$; high-T expansion Endrődi, Markó, in prep.

$$\chi = \frac{1}{12\pi^2}\left[\log\frac{T^2}{m^2}\right] + \mathcal{O}(m^2/T^2)$$

$$\xi = \frac{T^2}{3k^2} - \frac{m^2}{2\pi^2 k^2} - \frac{1}{12\pi^2}\left[\log\frac{T^2}{m^2} + 1\right] + \mathcal{O}(m^2/T^2)$$

Results: susceptibilities

- magnetic susceptibility

$$\chi_{\text{Schwinger}} = \chi_{\text{Weldon}} \qquad \checkmark$$

- electric susceptibility

$$\xi_{\text{Weldon}} = \frac{1}{k^2} \frac{\partial^2 \Omega}{\partial \mu^2}\bigg|_{\mu=0} - \frac{1}{12\pi^2}\left[\log\frac{T^2}{m^2} + 1\right] + \mathcal{O}(m^2/T^2)$$

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$$\xi_{\text{Schwinger}} = - \frac{1}{12\pi^2}\left[\log\frac{T^2}{m^2} - 1\right] + \mathcal{O}(m^2/T^2)$$

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- deviation #1

ξ_{Weldon} contains $\frac{1}{k^2}\left.\frac{\partial^2\Omega}{\partial\mu^2}\right|_{\mu=0} \xrightarrow{k\to 0} \infty$

$\xi_{\text{Schwinger}}$ is finite at $k = 0$

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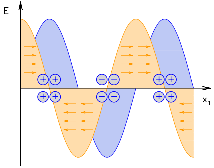
- deviation #2

$\xi_{\text{Weldon}}, \ \xi_{\text{Schwinger}}$ differ in $\dfrac{\pm 1}{12\pi^2}$

Deviation #1

Equilibria and charge distributions



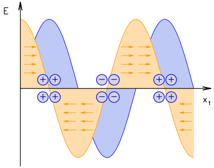
- free energy $\Omega(E, \mu)$
- Weldon: derivative evaluated at $\mu = 0$

$$\xi_{\text{Weldon}} = \left. \frac{\partial^2 \Omega(E, \mu = 0)}{\partial E^2} \right|_{E=0}$$

- instead derivative "along local equilibria"
  particle number $N_E(x_1)$ such that $\partial \mu_E(x_1)/\partial x_1 = -E$

$$\xi^{\text{equi}}_{\text{Weldon}} = \xi_{\text{Weldon}} - \frac{1}{L} \int_0^L \mathrm{d}x_1 \left. \frac{\partial^2 \Omega(0, \mu_E(x_1))}{\partial E^2} \right|_{E=0}$$

Equilibria and charge distributions



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Susceptibilities

- electric susceptibility

$$\xi^{\mathrm{equi}}_{\mathrm{Weldon}} = -\frac{1}{12\pi^2}\left[\log\frac{T^2}{m^2} + 1\right] + \mathcal{O}(m^2/T^2)$$
$$\xi_{\mathrm{Schwinger}} = -\frac{1}{12\pi^2}\left[\log\frac{T^2}{m^2} - 1\right] + \mathcal{O}(m^2/T^2)$$

- Schwinger's approach fulfills this equilibrium construction inherently
- however, the regularizations of this infrared divergence differ by a finite term

Impact of equilibrium construction for imaginary fields

Imaginary fields

- just like chemical potentials, electric fields are Wick-rotated non-trivially to Euclidean space
- implementing constant magnetic or constant imaginary electric fields is possible, but their 'flux' is quantized

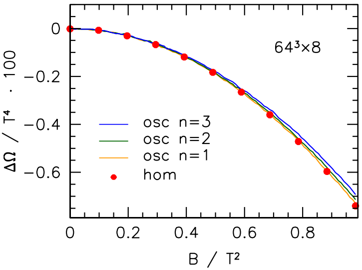
$$B \cdot L^2 = 2\pi N_B, \qquad iE \cdot L/T = 2\pi N_E, \qquad N_B, N_E \in \mathbb{Z}$$

- implementing oscillatory magnetic or oscillatory imaginary electric fields is also possible

$$B \cos\left(\frac{2\pi n x}{L}\right), \qquad iE \cos\left(\frac{2\pi n x}{L}\right), \qquad n \in \mathbb{Z}$$

Free fermions on the lattice

- magnetic fields: homogeneous (discrete B)
  vs. oscillatory (continuous B, discrete n) ✓



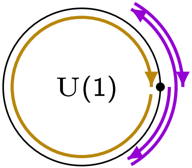
Role of chemical potentials

- at $T > 0$, partition function depends on $i\mu$ at $iE = 0$
- $\mathrm{U}(1)$ Polyakov loop $P(x_1)$
- homogeneous field

$$P(x_1) = \exp\left[i2\pi N_E x_1/L\right]$$

- oscillatory field

$$P(x_1) = \exp\left[iE_{\mathrm{osc}} \sin(kx_1)/(kT)\right]$$



cf. talk by G. Markó

- including $i\mu$ equivalent to translation for E_{hom} but not for E_{osc} therefore:

$$\Omega(iE_{\mathrm{hom}} > 0, \cancel{i\mu})$$

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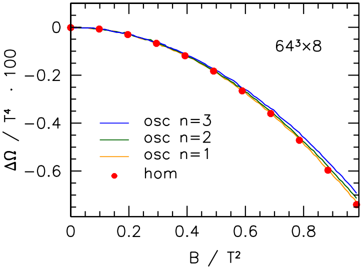
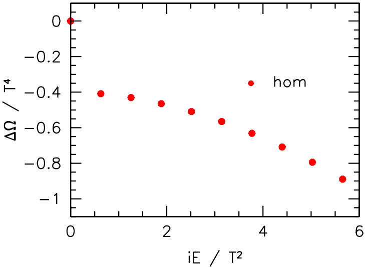
- electric fields: mismatch between $N_E > 0$ and $N_E = 0$
  $i\mu$-dependence ceases as soon as $N_E > 0$





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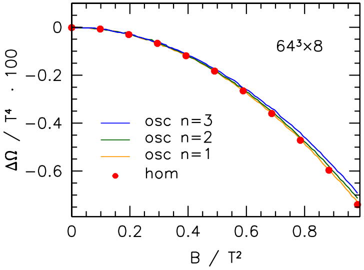

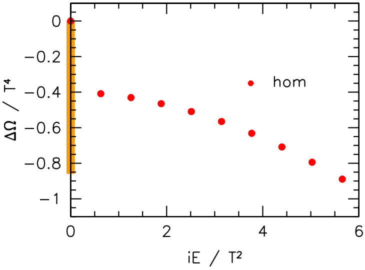

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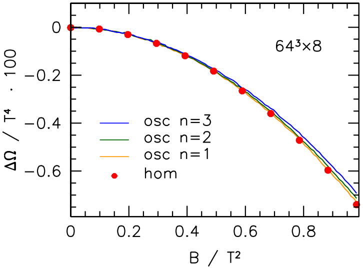

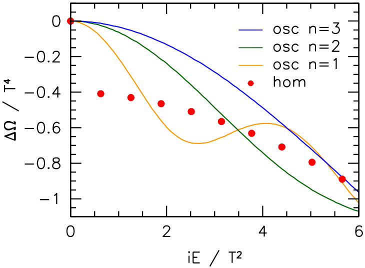

- oscillatory fields “collapse onto” mismatch

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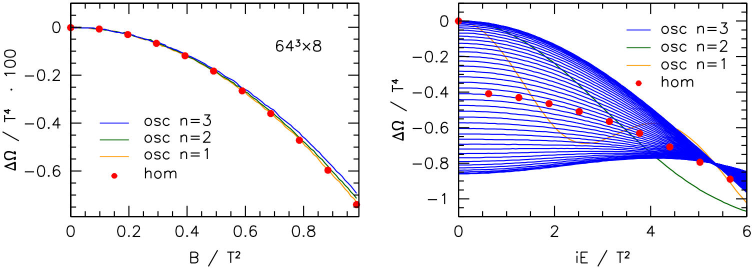

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- dependence on $i\mu$ pushed towards $iE = 0$ as $n \to 0$

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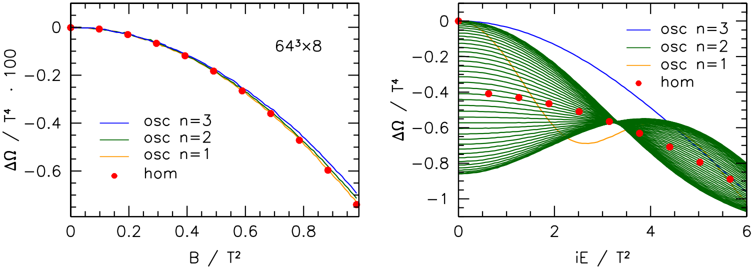

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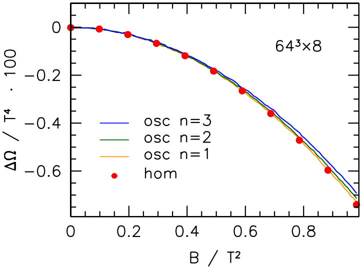

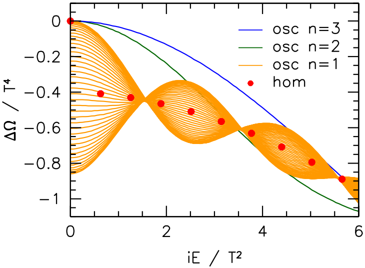

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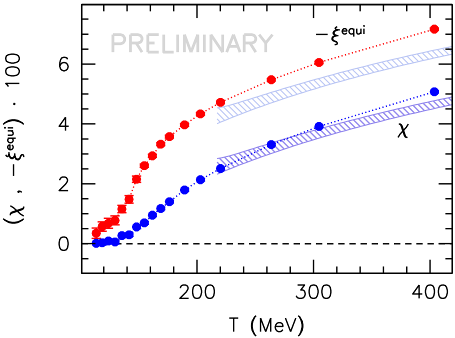
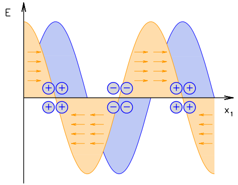
Full QCD

- stout improved staggered quarks + tree-level Symanzik gluons
  2+1 flavors at physical quark masses
  $24^3 \times 6$ lattice at different β to change T
- vacuum polarization tensors $\Pi_{00}/k_1^2 \to \xi$, $\Pi_{22}/k_1^2 \to \chi$
  Bali, Endrődi, Piemonte '20
  cf. $(g-2)_\mu$ talk by Z. Fodor
- perturbative formula including $\mathcal{O}(g_s^2)$ corrections



Summary

- equilibrium exists with nontrivial charge distribution



- two approaches to ξ disagree due to different realization of this equilibrium (and regularization of infrared divergence)

- on lattice: $i\mu$-independence at homogeneous $iE > 0$