QCD in extreme conditions on the lattice

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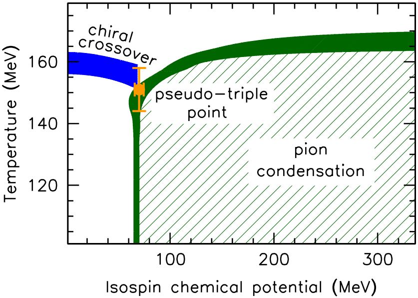
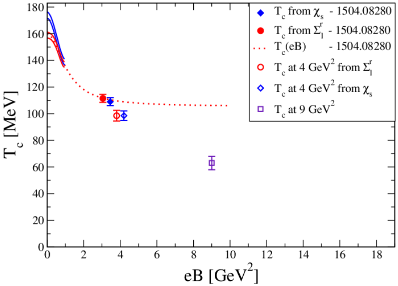
18th International Conference on QCD in Extreme Conditions
NTNU Trondheim, July 27, 2022

Appetizer

fundamental phase diagrams of QCD
with possible phenomenological implications





🔗 Brandt, Endrődi, Schmalzbauer ’18

🔗 Brandt, Endrődi ’19

🔗 D’Elia, Maio, Sanfilippo, Stanzione ’21

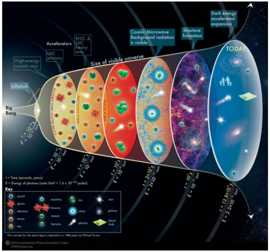
Outline

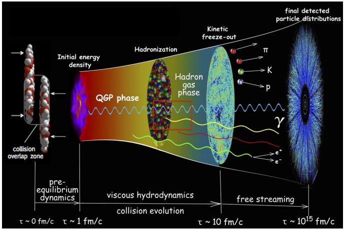
- introduction: strongly interacting matter in
  - strong electromagnetic fields
  - nonzero isospin density
- lattice simulation techniques
- phase diagram: current status
- application: cosmic trajectory
- beyond constant magnetic fields
- summary

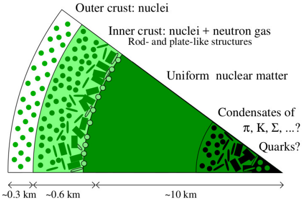
Introduction

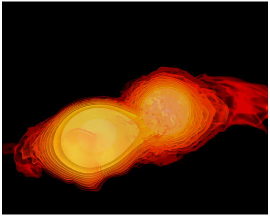
Extreme environments

- hot and/or dense strongly interacting matter in
  - QCD epoch of early Universe
  - heavy-ion collisions
  - neutron stars and their mergers



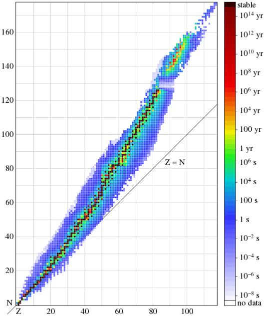


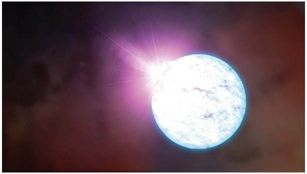




Isospin asymmetry: nuclei and neutron stars

- isospin asymmetry: $n_I \propto n_u - n_d$
  creates p^+-n asymmetry, excites π^+

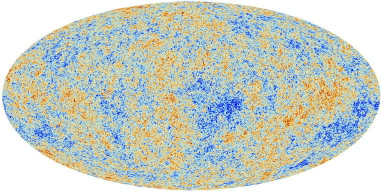




- proton to nucleon ratio in nuclei $\frac{Z}{A} \approx 0.4$
  but: 'neutron skin' near surface
- proton to nucleon ratio in interior of neutron stars $\frac{Z}{A} \approx 0.025$

Isospin asymmetry: cosmology

- early Universe characterized by charge neutrality $n_Q = 0$,
  (almost perfect) baryon symmetry $n_B = 0$
  but lepton number n_L only weakly constrained by observations
  Oldengott, Schwarz '17



- weak equilibrium

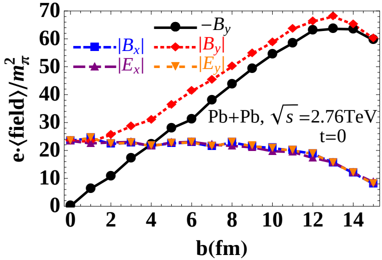
$$d \leftrightarrow u\, e^- \, \bar{\nu}_e$$

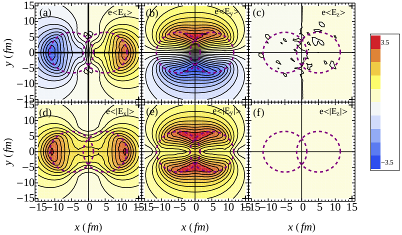
large $n_L \leftrightarrow$ large isospin asymmetry Abuki, Brauner, Warringa '09

Electromagnetic fields: heavy ion collisions

- electromagnetic fields in the early stage of heavy-ion collisions reaching m_π^2 and well beyond

  🔗 Deng et al. '12





- most probably short-lived fields 🔗 Huang '15
- impact of electric field enhanced for asymmetric systems (for example Cu+Au at RHIC) 🔗 Voronyuk et al. '14

Lattice simulations

Monte Carlo simulations

- Euclidean QCD path integral over gauge field $\mathcal{A}$

$$\mathcal{Z} = \int \mathcal{D}\mathcal{A}\, e^{-S_g[\mathcal{A}]} \det[\not{D}[\mathcal{A}] + m]$$

- Monte-Carlo simulations need: $\det[\not{D} + m] \in \mathbb{R}^+$
  for that one needs Γ so that

$$\Gamma \not{D}\, \Gamma^\dagger = \not{D}^\dagger, \quad \Gamma^\dagger \Gamma = 1$$

$$\det[\not{D}+m] = \det[\Gamma^\dagger\Gamma(\not{D}+m)] = \det[\Gamma(\not{D}+m)\Gamma^\dagger] = \det[\not{D}^\dagger+m] = \det[\not{D}+m]^*$$

- usually positivity can also be shown
- such a Γ exists: no complex action problem

Complex actions vs. real actions

- two-flavor Dirac operator ($\mathcal{A}_\mu$: $\mathrm{SU}(3)$ field, A_μ: $\mathrm{U}(1)$ field) (remember Wick rotation $A_4 = -iA_0$)

$$\not{D}[\mathcal{A}, A] = \gamma_\mu(\partial_\mu + i\mathcal{A}_\mu) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + i\gamma_\mu \begin{pmatrix} A_\mu^u & 0 \\ 0 & A_\mu^d \end{pmatrix}$$

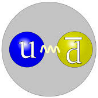
- pure QCD: $A_\mu = 0$ — $\Gamma = \gamma_5$ ✓
- magnetic field: $A_2^f = q_f B x_1$ — $\Gamma = \gamma_5$ ✓
- imaginary baryon chem. pot.: $A_4^u = A_4^d = \mu$ — $\Gamma = \gamma_5$ ✓
- imaginary electric field: $A_4^f = q_f E x_1$ — $\Gamma = \gamma_5$ ✓
- real baryon chem. pot.: $iA_4^u = iA_4^d = \mu$ — $\lightning$
- real electric field: $iA_4^f = q_f E x_1$ — $\lightning$
- real isospin chem. pot.: $iA_4^u = -iA_4^d = \mu_I$ — $\Gamma = \gamma_5 \tau_1$ ✓

Son, Stephanov '01

Pion condensation

Pion condensation

- isospin chemical potential: $\mu_u = \mu_I$, $\mu_d = -\mu_I$, $\mu_s = 0$
- QCD at low energies ≈ pions
  chiral perturbation theory



- charged pion chemical potential: $\mu_\pi = 2\mu_I$

at zero temperature $\mu_\pi < m_\pi$ vacuum state
$\mu_\pi \geq m_\pi$ Bose-Einstein condensation

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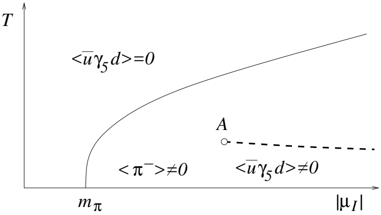

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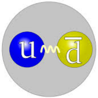

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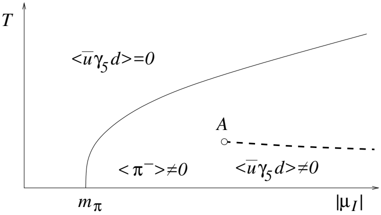

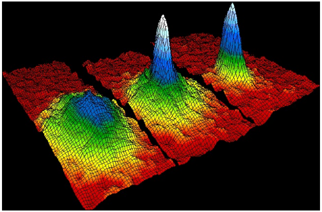

trapped Rb atoms at low temperature

Anderson et al '95 JILA-NIST/University of Colorado

Symmetry breaking

- QCD with light quark matrix

$$M = \not{D} + m_{ud}\mathbb{1}$$

- chiral symmetry (flavor-nontrivial)

$$\mathrm{SU}(2)_V$$

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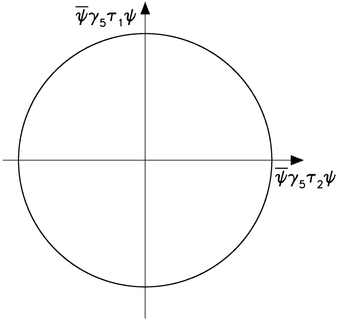

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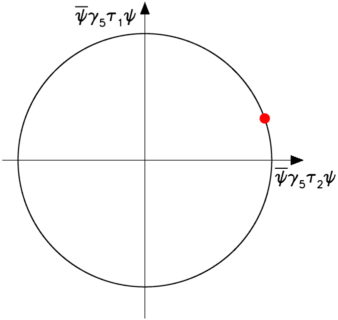

- spontaneously broken by a pion condensate

$$\left\langle \bar{\psi}\gamma_5\tau_{1,2}\psi \right\rangle = \left\langle \bar{u}\gamma_5 d \pm \bar{d}\gamma_5 u \right\rangle$$

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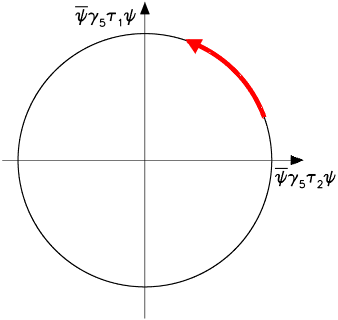

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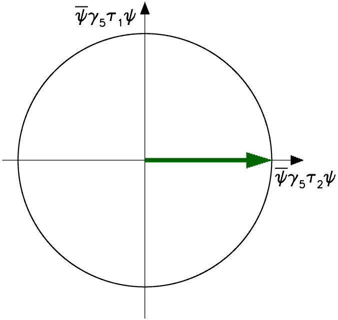
$$\mathrm{SU}(2)_V \to \mathrm{U}(1)_{\tau_3} \to \varnothing$$



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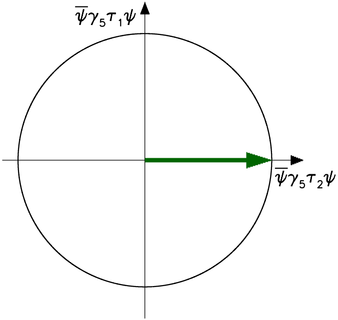

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- extrapolate results $\lambda \to 0$

Dictionary

	pion condensation	vacuum chiral symmetry breaking
pattern	$\mathrm{U}(1)_{\tau_3} \to \varnothing$	$\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \to \mathrm{SU}(2)_V$
coset	$\mathrm{U}(1)$	$\mathrm{SU}(2)_A$
Goldstones	1	3
spontaneous	$\left\langle \bar{\psi}\gamma_5\tau_2\psi \right\rangle$	$\left\langle \bar{\psi}\psi \right\rangle$
explicit	$= \partial \log \mathcal{Z} / \partial\lambda$	$= \partial \log \mathcal{Z} / \partial m$
limit	$\lambda \to 0$	$m \to 0$

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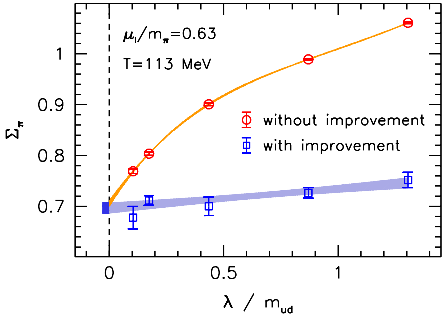
limit	$\lambda \to 0$	$m \to 0$

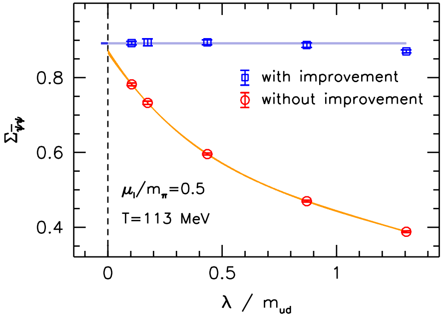
- long story short: pion condensation $1/3$ as challenging as the chiral limit of the QCD vacuum

Phase diagram: nonzero isospin

Extrapolation to $\lambda = 0$

- improvement is crucial for reliable $\lambda \to 0$ extrapolation
  Brandt, Endrődi, Schmalzbauer '17   Brandt, Endrődi '19

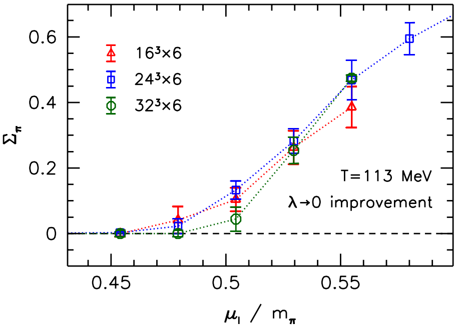


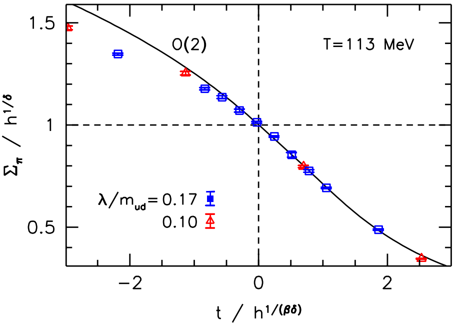


$$\Sigma_\pi = \frac{T}{V}\frac{\partial \log \mathcal{Z}}{\partial \lambda}$$

$$\Sigma_{\bar\psi\psi} = \frac{T}{V}\frac{\partial \log \mathcal{Z}}{\partial m_{ud}}$$

Order of the transition





- volume scaling of order parameter shows typical sharpening
- collapse according to O(2) critical exponents Ejiri et al '09

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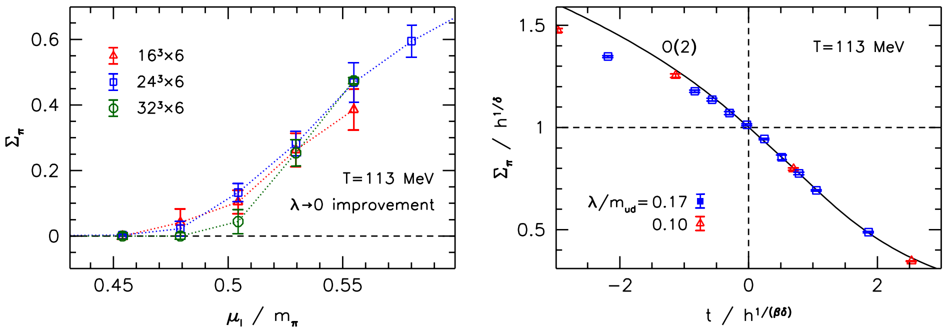
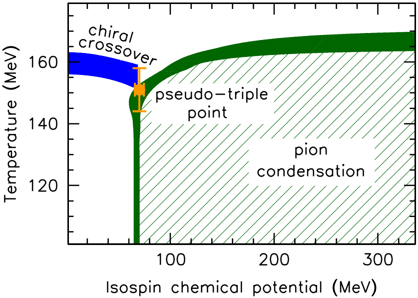


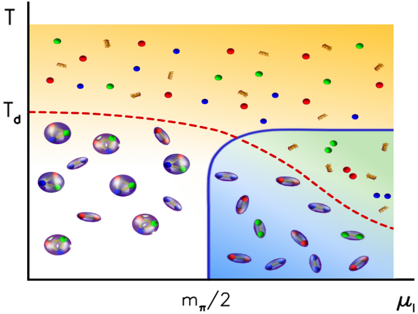
- volume scaling of order parameter shows typical sharpening
- collapse according to $\mathrm{O}(2)$ critical exponents Ejiri et al '09
- indications for a second order phase transition at $\mu_I = m_\pi/2$, in the $\mathrm{O}(2)$ universality class

Phase diagram

- phases in the $T - \mu_I$ phase diagram: hadronic (confined), quark-gluon plasma (deconfined), pion condensation (confined), BCS (deconfined)

  Brandt, Endrődi, Schmalzbauer '17 Brandt, Endrődi '19





- comparison to effective models, χPT, Q2CD, ...

  Adhikari et al. '18 Zhokhov et al. '19 Adhikari et al. '20

  Boz et al. '20 Astrakhantsev et al. '20

Equation of state: nonzero isospin

Equation of state

- equilibrium description of matter

$$\epsilon(p)$$

relevant for:

  - neutron star physics (TOV equations)
  - cosmology, evolution of early Universe (Friedmann equation)
  - heavy-ion collision phenomenology (charge fluctuations)

- thermodynamic relations

$$p = \frac{T}{V}\log\mathcal{Z}, \qquad s = \frac{\partial p}{\partial T}, \qquad n_I = \frac{\partial p}{\partial \mu_I}, \qquad \epsilon = -p + Ts + \mu_I n_I$$

$$I = \epsilon - 3p, \qquad c_s^2 = \left.\frac{\partial p}{\partial \epsilon}\right|_{s/n_I}$$

Equation of state on the lattice

- integral method to calculate differences

$$n_I = \frac{T}{V}\frac{\partial \log \mathcal{Z}}{\partial \mu_I}, \qquad p(T,\mu_I) - p(T,0) = \int_0^{\mu_I} \mathrm{d}\mu_I'\, n_I(\mu_I')$$

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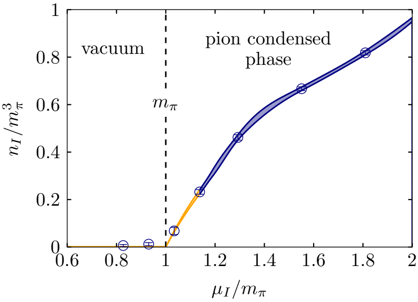
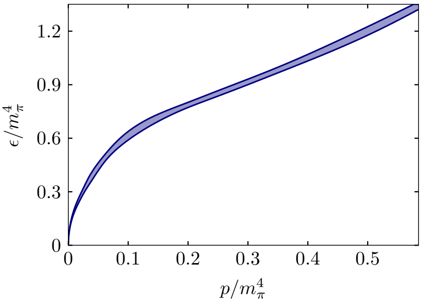
- results at $T \approx 0$

  Brandt, Endrődi, Fraga, Hippert, Schaffner-Bielich, Schmalzbauer '18





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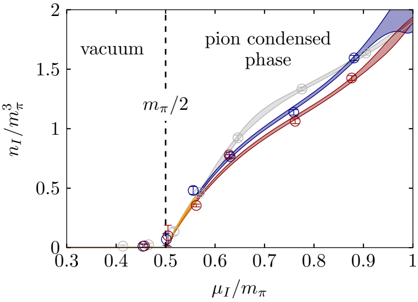

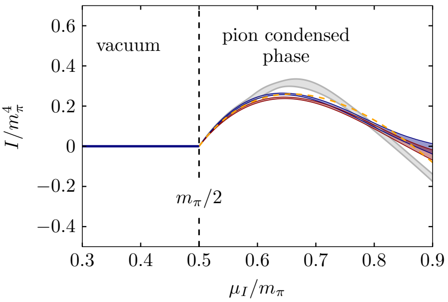

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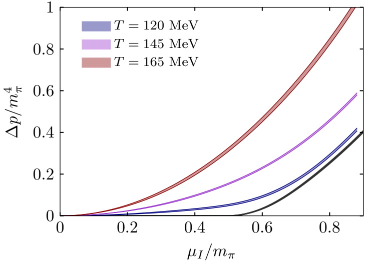
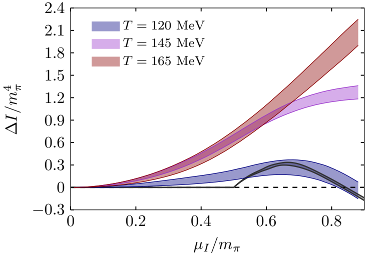
- interaction measure negative at high μ_I, low T





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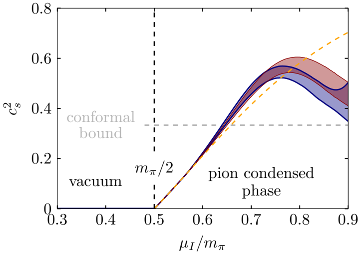

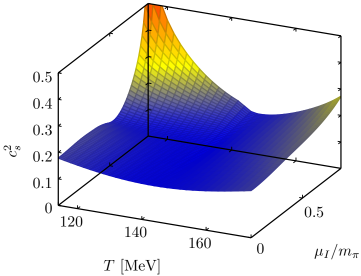

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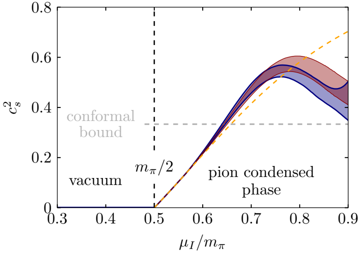
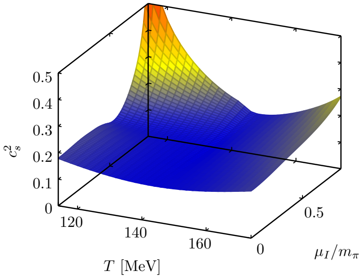


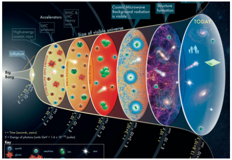


- EoS can get very stiff inside pion condensation phase
- comparison: χPT, models Adhikari et al. '21 Avancini et al. '19

Cosmological implications

Cosmic trajectories

- early Universe



- conservation equations for isentropic expansion

$$\frac{n_B}{s} = b, \quad \frac{n_Q}{s} = 0, \quad \frac{n_{L_\alpha}}{s} = l_\alpha \quad (\alpha \in \{e, \mu, \tau\})$$

- parameters: $T, \quad \mu_B, \quad \mu_Q, \quad \mu_{L_\alpha}$
- experimental constraints Planck coll. '15 Oldengott, Schwarz '17

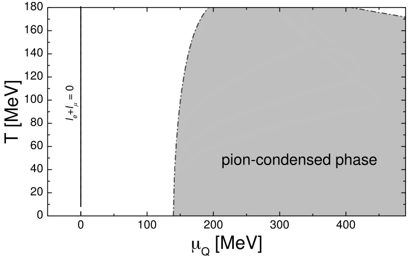
$$b = (8.60 \pm 0.06) \cdot 10^{-11}, \qquad \boxed{|l_e + l_\mu + l_\tau| < 0.012}$$

(the individual l_α may have opposite signs)

- $n_Q = 0$ with $l_e > 0$ allows equilibrium of e^-, ν_e, π^+

Cosmic trajectories

- cosmic trajectory $T(\mu_Q)$ is solved for
- standard scenario ($l_\alpha = 0$): $\mu_Q = 0$ for all T



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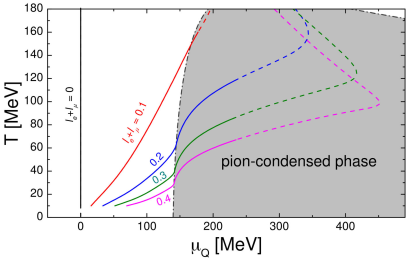


- cosmic trajectory enters BEC phase for lepton asymmetries allowed by observations

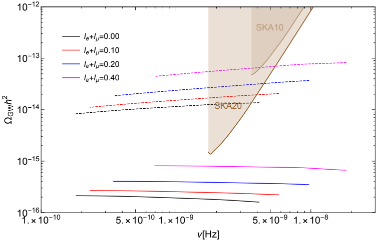
  Vovchenko, Brandt, Cuteri, Endrődi, Hajkarim, Schaffner-Bielich '20

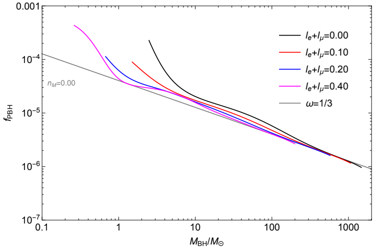
- condition for pion condensation to occur:

$$|l_e + l_\mu + l_\tau| < 0.012 \qquad |l_e + l_\mu| \gtrsim 0.1$$

Signatures of the condensed phase

- relic density of primordial gravitational waves is enhanced with respect to amplitude at $l_e + l_\mu = 0$
- fraction of primordial black holes with mass below one solar mass is enhanced





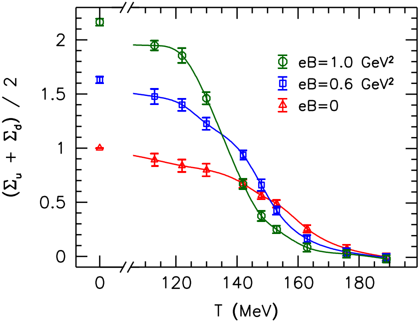
Vovchenko, Brandt, Cuteri, Endrődi, Hajkarim, Schaffner-Bielich '20

- to be detected experimentally (SKA)

Phase diagram: electromagnetic fields

Inverse catalysis and phase diagram

- physical m_π, staggered quarks, continuum limit
  Bali, Bruckmann, Endrődi, Fodor, Katz et al. '11 '12



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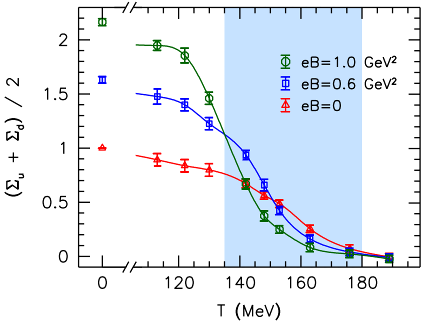

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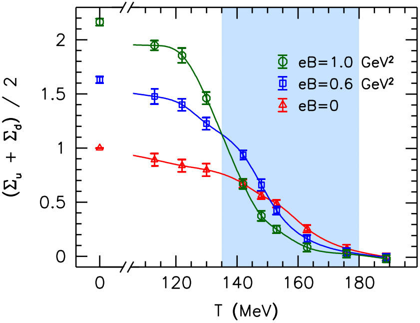

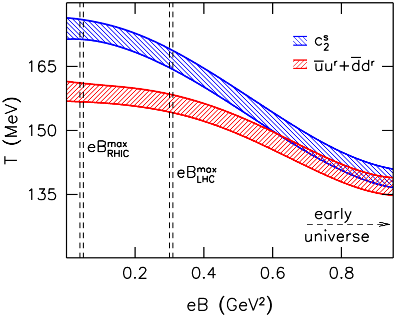

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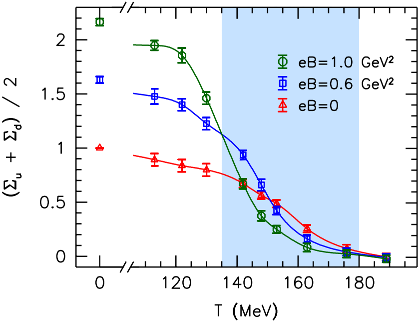

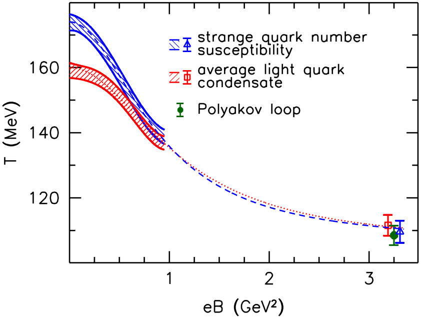

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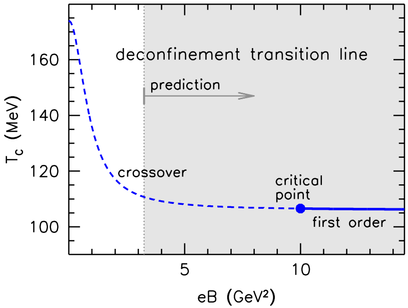
Phase diagram and critical point

- effective theory of QCD at $B \to \infty$: first-order deconfinement transition $\Rightarrow$ critical point! Miransky, Shovkovy '02
- location of critical point based on extrapolation from $0 < eB \lesssim 3\ \mathrm{GeV}^2 \Rightarrow$ $\boxed{eB_c \approx 10(2)\ \mathrm{GeV}^2}$ Endrődi '15



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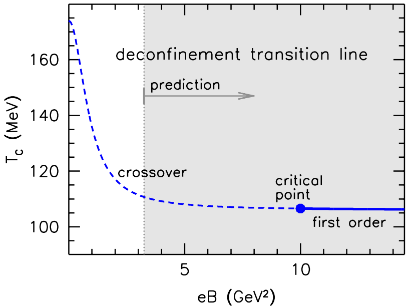
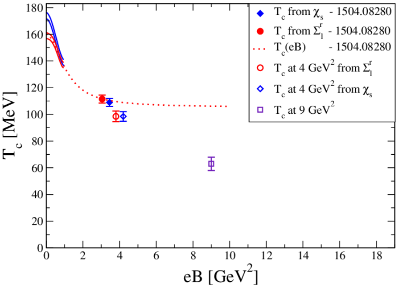
- simulating up to $eB \approx 9\ \mathrm{GeV}^2 \Rightarrow$ $\boxed{4\ \mathrm{GeV}^2 < eB_c < 9\ \mathrm{GeV}^2}$
  D'Elia, Maio, Sanfilippo, Stanzione '21

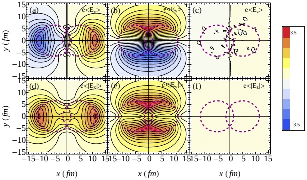




Beyond constant magnetic fields

Inhomogeneous magnetic fields

- remember HIC: inhomogeneous magnetic and electric fields



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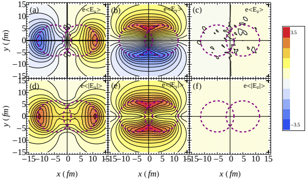

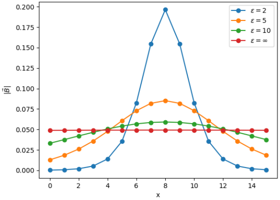

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  can be treated analytically (in absence of color interactions)

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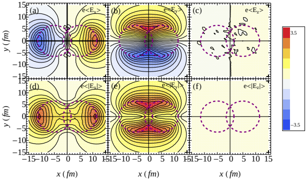

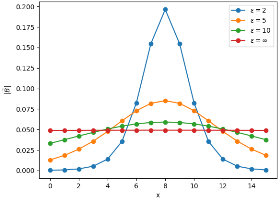

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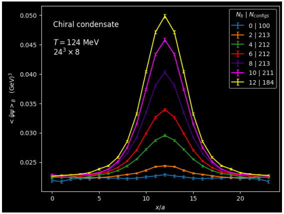
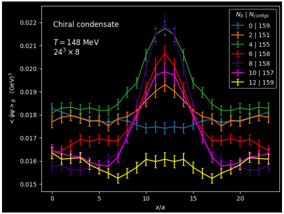
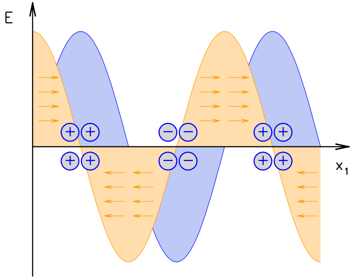
- impact on quark condensate D. Valois Wed 14:40





Electric fields

- static homogeneous electric field E: charges accelerated to ∞
- equilibrium requires infrared regularization
  $\rightsquigarrow$ finite wavelength $1/k_1$



- charge distribution where electric forces and pressure gradients cancel

Electric susceptibility

- leading impact of E on free energy f
- here: perturbative QED at nonzero T

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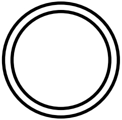
$f(E) =$  $\qquad f = -\xi \cdot \frac{E^2}{2} + \dots$

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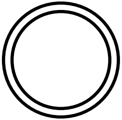
$f(E) =$  $\qquad f = -\xi \cdot \frac{E^2}{2} + \ldots$

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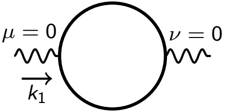
$\xi = \lim_{k_1 \to 0} \frac{-1}{k_1^2}$ 

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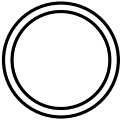
$f(E) =$  $f = -\xi \cdot \frac{E^2}{2} + \dots$

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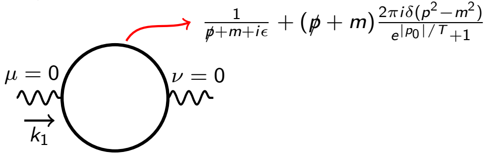
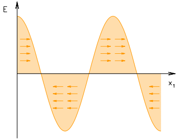
$\xi = \lim_{k_1 \to 0} \frac{-1}{k_1^2}$ 

- generalize calculation to $m > 0$ Endrődi, Markó in prep.

Equilibrium and mismatch

- evaluated in absence of charge distribution ($\mu = 0$)

$$\xi_{\text{Weldon}} \approx \frac{T^2}{k_1^2} + \text{IR finite}$$



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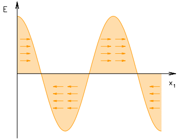
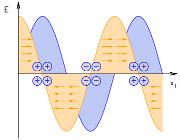


- evaluated "along local equilibria"
  ($N(x)$ such that $\partial\mu/\partial x = -E$)



$$\xi^{\text{equi}}_{\text{Weldon}} = -\frac{1}{12\pi^2}\left[\log\frac{T^2}{m^2} + 1\right] + \mathcal{O}(m^2/T^2)$$

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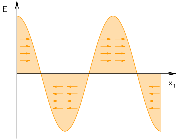

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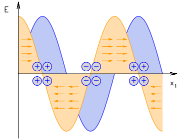

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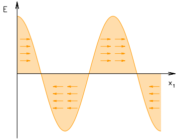

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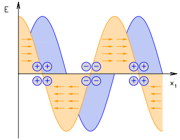

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- Schwinger's method fulfills equilibrium construction inherently but regularizations of infrared divergence differ by finite term!

Summary

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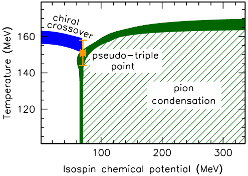
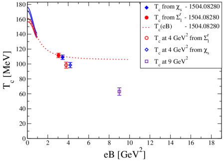
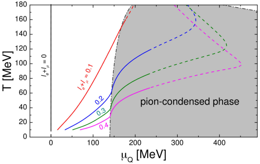
- $T - \mu_I$ phase diagram and pion condensation



- $T - B$ phase diagram and the critical point



- cosmic trajectory may enter pion condensed phase



- background electric fields and local charge distributions