Highly relativistic nucleus-nucleus collisions: The central rapidity region

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Introduction

Motivation:

- QGP is produced
- Insight into transition from QGP to hadrons
- Early universe

Questions:

- How to interpret these collisions?
- What experimental signatures can be used to deduce sth. about the state of matter?

Definitions

Assuming constant velocities v of fluid elements for the longitudinal expansion:

$$z(t) = vt \Rightarrow v(z,t) = \frac{z}{t}$$

Natural variables for describing this system will be:

$$\tau = \sqrt{t^2 - z^2}$$

$$y = \mathrm{artanh}\left(\frac{z}{ct}\right) = \frac{1}{2}\ln\left(\frac{1+z/ct}{1-z/ct}\right) = \frac{1}{2}\ln\left(\frac{1+v_z/c}{1-v_z/c}\right) \rightarrow \frac{1}{2}\ln\left(\frac{1+\cos\theta}{1-\cos\theta}\right) \text{ for } |v| \rightarrow c$$

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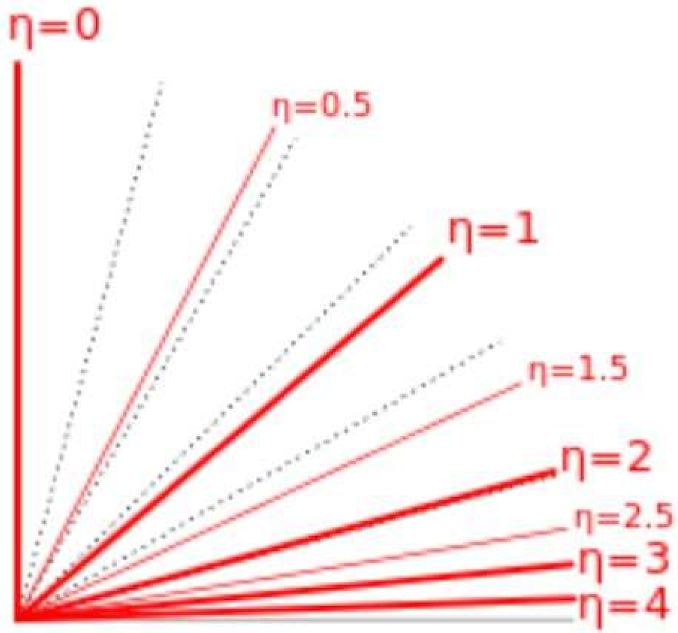

[Pseudorapidity - Wikipedia]

Space-Time Evolution

- Start from nucleon-nucleon collisions
- Generalize to nucleus-nucleus collisions

What we will find out:

- Symmetries of the system
- Estimates of various quantities
- Proper time dependence of temperature etc.

Assumptions:

1. Central-rapidity-plateau for $\frac{dN}{dy}$ in hadron-hadron collisions (large angles)
2. Central-rapidity-plateau of similar height in nucleon-nucleus collisions
3. There exists a leading baryon-effect

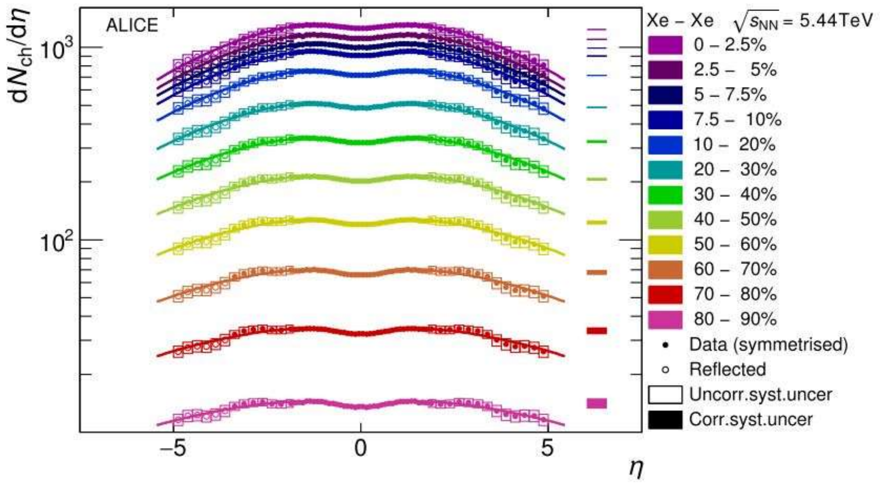

Fig. 1. Charged-particle pseudorapidity density for 12 centrality classes over a broad η range in Xe–Xe collisions at $\sqrt{s_{NN}} = 5.44$ TeV. Boxes around the points reflect the total systematic uncertainties, while the filled squares on the right reflect the normalisation uncertainty from the centrality determination. Statistical errors are negligible. The reflection (open circles) of the $3.5 < \eta < 5$ values around $\eta = 0$ is also shown. The lines correspond to fits to a gaussian distribution in rapidity multiplied by an effective Jacobian of transformation from η to y.

[Centrality and pseudorapidity dependence of the charged-particle multiplicity density in Xe–Xe collisions at sNN=5.44TeV - ScienceDirect]

Consider nucleus-nucleus collisions:

- Rest frame of one of the nuclei
- Assume that as the Lorentz-contracted nucleus passes through the resting nucleus all of the nucleons are struck
- Assume that the secondary nucleon of each collision has a momentum distribution as if it was hit in isolation (several hundred MeV)

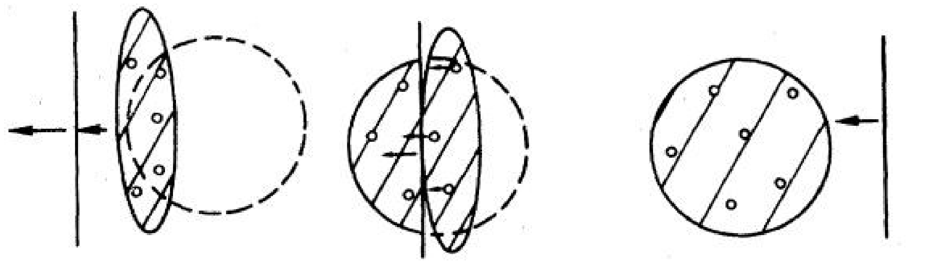

FIG. 1. Schematic of the evolution of a compressed "baryon fireball" in nucleus-nucleus collisions, according to the mechanism of Anishetty, Koehler, and McLerran (Ref. 8).

Now consider center-of-mass frame:

- Now 2 receding pancakes at velocity close to c that carry the baryon number
- Temporarily replace one projectile by a single nucleon:

1. The plateau assumption for nucleon-nucleus collisions implies similar particle production as in nucleon-nucleon collisions
2. At the Super Proton Synchrotron collider:

$$\frac{dN_{ch}}{dy} \sim 3, \text{ guessing } \langle E \rangle \sim 400\text{MeV and } \frac{N_{neutral}}{N_{ch}} \sim 0.5$$
$$\Rightarrow \frac{d\langle E \rangle}{dy} \sim 3 \times 0.4 \times 1.5 = 1.8\text{GeV}$$

If the single nucelon is replaced by a dilute gas of nucleons the energy production should be additive

Using this to estimate the initial energy density between the pancakes:

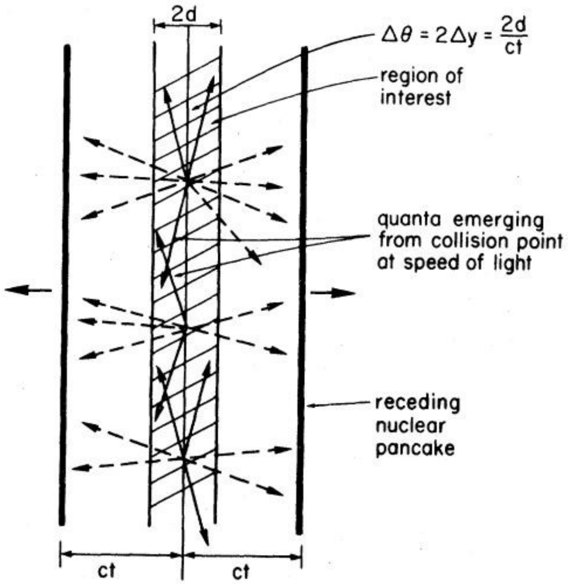

FIG. 2. Geometry for the initial state of centrally produced plasma in nucleus-nucleus collisions.

$$E = N\frac{d\langle E\rangle}{dy}\Delta y = N\frac{d\langle E\rangle}{dy}\frac{1}{2}\left[\frac{2d}{t}\right] \quad \Rightarrow \quad \epsilon \approx \frac{N}{\mathscr{A}}\frac{d\langle E\rangle}{dy}\frac{1}{2t}$$

In the case of real ion-ion collisions we must replace the number of incident nucleons per unit area $N/\mathscr{A}$ by some effective elementary area $d_0{}^2$,

$$\frac{N}{\mathscr{A}} \leftrightarrow d_0{}^{-2} . \qquad 0.3 \lesssim d_0 \lesssim 1.0 \text{ fm}$$

$$\epsilon \approx \frac{1 \text{ GeV}}{t d_0{}^2} \Rightarrow \epsilon_0 \approx 1-10 \text{ GeV/fm}^3$$ For initial time of 1fm

initial density of quanta ρ_0 of $\sim$2–20 fm^{-3}

On time scales of 5-10 fm/c, we assume local thermal equilibrium. When the expansion begins, the 1/t dependence of the energy density does not apply anymore. It is to be understood as the initial condition for the hydrodynamic flow.

- Now Lorentz boost this system with $\gamma \sim 3$
- Again 2 Lorentz contracted pancakes
- For nucleon-nucleus collisions: same large angle particle production as in the previous frame, because of boost invariance
- Therefore same initial conditions
- So, initial conditions at 1fm/c are boost invariant
- The subsequent evolution will also have this symmetry

Description of the evoulution

Defining $\epsilon(x), p(x), T(x), u^\mu \text{ with } u_\mu u^\mu = 1$

$$\Rightarrow T_{\mu\nu} = (\epsilon + p)u_\mu u_\nu - g_{\mu\nu} p \quad \text{with} \quad \partial^\mu T_{\mu\nu} = 0$$

For early times, we assume no transverse flow, so:

$$\epsilon(\tau, y), p(\tau, y), T(\tau, y), u = (u_0(\tau, y), 0, 0, u_z(\tau, y))$$

Boost invariance: $\epsilon(\tau), p(\tau), T(\tau), u = \frac{1}{\tau}(t, 0, 0, z) \quad \Rightarrow \quad \frac{d\epsilon}{d\tau} = -\frac{\epsilon - p}{\tau}$

Entropy density: $$\partial_\tau s = -\frac{s}{\tau} \Rightarrow s(x) \propto \frac{1}{\tau} \Rightarrow \frac{dS}{dy} = \frac{d}{dy}\int d^3x s(x) \propto \tau s(x) = const$$

Since particle production rate is also constant: $$\frac{dS}{dy} \propto \frac{dN}{dy}$$

Using $$\left[\frac{dS}{dy}\right]_{nucleus-nucleus} = \frac{1}{d_0^2}\pi(1.2A^{\frac{1}{3}}\mathrm{fm})^2\left[\frac{dS}{dy}\right]_{pp} \Rightarrow \frac{\left[\frac{dN_\pi}{dy}\right]_{AA}}{\left[\frac{dN_\pi}{dy}\right]_{pp}} \sim 4\left[\frac{\mathrm{fm}}{d_0}\right]^2 A^{\frac{2}{3}}$$

For U-U collisions: $$\frac{dN_\pi}{dy} \sim 800\left[\frac{\mathrm{fm}}{d_0}\right]^2$$

Other general features: $$\epsilon \text{ and } p \sim \tau^{-\frac{4}{3}}, \quad \epsilon \propto T^4 \Rightarrow T \sim \tau^{-\frac{1}{3}}, \quad \frac{1}{T}\frac{dT}{d\tau} = -\frac{v_s^2}{\tau}$$

Transverse Flow

- Rarefaction front propagating inwards

- Radial coordinate of the front:

$$\rho(t) = R - \int_0^{\sqrt{t^2 - z^2}} v_s(t')dt'$$

- Equation of state is also dealt with

Summary

- Boost invariance
- Velocity profile
- Baryon number is found in the pancakes
- Constant entropy per unit rapidity
- Proper time dependence of the temperature etc. as long as there is 1D-flow
- Transverse flow