Lattice QCD in rotating frames

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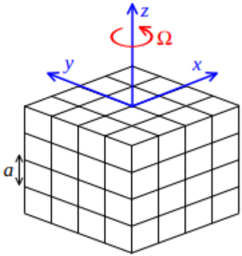
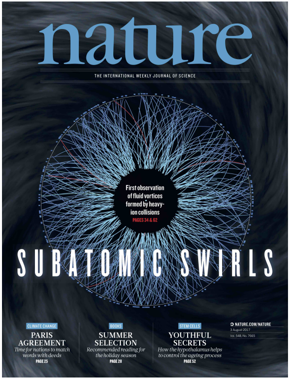
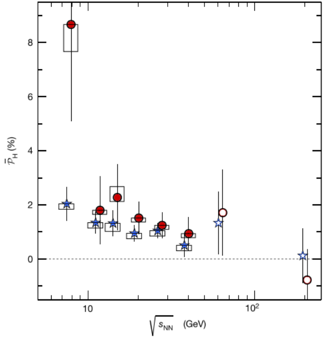


Why rotation?

- $\Lambda, \bar{\Lambda}$ Polarization 🔗 Star '17 Rotating QGP $\Omega \sim 6$ MeV
- Hydrodynamics simulations $\Omega \sim (20 - 40)$ MeV 🔗 Jiang, Lin, Liao '17





Lattice QCD in a nutshell

- Partition function

$$\mathcal{Z} = \int \mathcal{D}\mathcal{U}\mathcal{D}\bar{\psi}\mathcal{D}\psi\, e^{-S_E} = \int \mathcal{D}\mathcal{U} \; \det M \; e^{-S_G}$$

with S_E the Wick-rotated, finite temperature QCD action

$$S_E = S_G + S_F = \int_0^{1/T} \mathrm{d}\tau \int \mathrm{d}^3 x \left[\frac{\mathrm{Tr}\, F^2}{2g^2} + \sum_f \bar{\psi}_f \underbrace{(\not{D} + m_f)}_{\equiv M} \psi_f \right]$$

- Observables

$$\langle \mathcal{O} \rangle = \int \mathcal{D}\mathcal{U} \; \det M \; e^{-S_G} \, \mathcal{O}$$

- Discretize the action, with **lattice spacing** a, and use **importance sampling Monte Carlo** integration to evaluate the path integral

- Gluons live on the links!

$$A_\mu(x) \in \mathsf{su}(3) \to U_\mu(n) \in \mathsf{SU}(3)$$

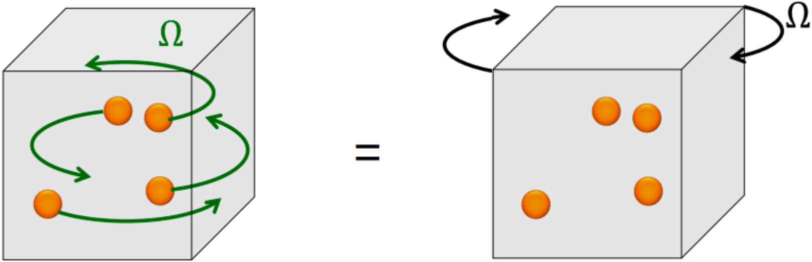
$$\frac{1}{2g^2}\int \mathrm{d}^4x \; \mathrm{Tr}\, F_{\mu\nu}(x)F_{\mu\nu}(x) \to \frac{2}{g^2}\sum_n \sum_{\mu<\nu} \mathrm{Re}\,\mathrm{Tr}\left[1 - U_{\mu\nu}(n)\right]$$

with the **plaquette**

$$U_{\mu\nu}(n) = U_\mu(n)\, U_\nu(n+\hat{\mu})\, U_\mu(n+\hat{\nu})^\dagger\, U_\nu(n)^\dagger$$

- If we have $\mu, E, \theta \ldots \Rightarrow \det M$ (or e^{-S}) $\in \mathbb{C}$ **sign problem**

- Simulate *rigidly* rotating state:



External field

Rotating frame of reference

- In relativistic theories: rotating frame = curved space-time

- Choose z as rotation axis: $\theta = -\Omega t$
- New coordinates:

$$
\begin{aligned}
t &= t' \\
x &= x' \cos\left(\Omega t'\right) - y' \sin\left(\Omega t'\right) \\
y &= x' \cos\left(\Omega t'\right) + y' \sin\left(\Omega t'\right) \\
z &= z'
\end{aligned}
$$

- New metric:

$$
g^{M}_{\mu\nu} = \begin{pmatrix} 1 - r^2\Omega^2 & y\Omega & -x\Omega & 0 \\ y\Omega & -1 & 0 & 0 \\ -x\Omega & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}
$$

with $r = \sqrt{x^2 + y^2}$

- Metric is independent on $t \to$ Hamiltonian is conserved

$$H = \int \mathrm{d}V \sqrt{g_{00}} \epsilon(\vec{r}) = \frac{1}{2g^2} \int \mathrm{d}^3x \sqrt{\gamma} \sqrt{g_{00}}\, g^{\alpha\beta} g^{\sigma\rho}\, \mathrm{Tr}\left[F_{\alpha\sigma} F_{\beta\rho}\right]$$

- We can construct the partition function

$$Z = \mathrm{Tr} \exp\left[\beta H\right]$$

- **Ehrenfest-Tolman effect**: in gravitational fields, the temperature is not constant in space in global thermal equilibrium $\to T(r)$

$$Z = \mathrm{Tr} \exp\left[-\int \mathrm{d}V \frac{\epsilon(r)}{T(r)}\right]$$

- But! $T(r)\sqrt{g_{00}} = 1/\beta =$ const.
- Define $T(r=0) \equiv T = 1/\beta$

Important remarks

- Causality imposes $r\Omega < 1$
- System should be finite in x, y directions: not well defined thermodynamic limit
- **Boundary conditions** are very relevant (in principle)

Rotation in the Euclidean

- Wick rotation $t \to i\tau$

$$g_{\mu\nu}^{E} = \begin{pmatrix} 1 & 0 & 0 & y\Omega \\ 0 & 1 & 0 & -x\Omega \\ 0 & 0 & 1 & 0 \\ y\Omega & -x\Omega & 0 & 1 + r^2\Omega^2 \end{pmatrix}$$

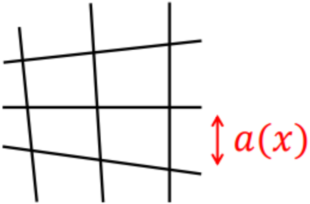
- (Complex!) action reads

$$S_G = \frac{1}{2g^2} \int \mathrm{d}^4 x [(1 - r^2\Omega^2) F_{xy}^a F_{xy}^a + (1 - y^2\Omega^2) F_{xz}^a F_{xz}^a$$
$$+ (1 - x^2\Omega^2) F_{yz}^a F_{yz}^a + F_{x\tau}^a F_{\tau}^a + F_{y\tau}^a F_{y\tau}^a + F_{z\tau}^a F_{z\tau}^a$$
$$-2iy\Omega(F_{xy}^a F_{x\tau}^a + F_{xz}^a F_{z\tau}^a) + 2ix\Omega(F_{yx}^a F_{x\tau}^a + F_{yz}^a F_{z\tau}^a) - 2xy\Omega^2 F_{zy}^a F_{xz}^a]$$

- Sign problem $\to \Omega_I = -i\Omega$ and analytic continuation

Rotation on the lattice

- Lattice $N_t \times N_z \times N_s^2$
- Causality imposes $\Omega N_s a/\sqrt{2} < 1$
- Periodic boundary conditions in t, z
- How does the discretization changes?
  How should the boundary conditions in x, y be?
- Rotation affects renormalization? Yes, but we don't discuss it



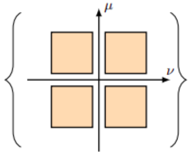
Rotation on the lattice

- Discretized version

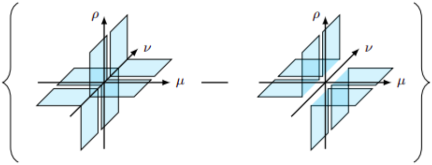
$$
\begin{aligned}
S_G = \frac{6}{g^2} \sum_n \Big[& (1 + r^2\Omega_I^2)(1 - \frac{1}{3}\,\mathrm{Re}\,\mathrm{Tr}\;\bar{U}_{xy}) \\
& + (1 + y^2\Omega_I^2)(1 - \frac{1}{3}\,\mathrm{Re}\,\mathrm{Tr}\;\bar{U}_{xz}) + (1 + x^2\Omega_I^2)(1 - \frac{1}{3}\,\mathrm{Re}\,\mathrm{Tr}\;\bar{U}_{yz}) \\
& + 3 - \frac{1}{3}\,\mathrm{Re}\,\mathrm{Tr}\big(\bar{U}_{x\tau} + \bar{U}_{y\tau} + \bar{U}_{z\tau}\big) \\
& - \frac{1}{3}\,\mathrm{Re}\,\mathrm{Tr}\Big(y\Omega_I(\bar{V}_{xy\tau} + \bar{V}_{xz\tau}) - x\Omega_I(\bar{V}_{yx\tau} + \bar{V}_{yz\tau}) + xy\Omega_I^2\,\bar{V}_{xzy}\Big)\Big]
\end{aligned}
$$

with clovers and chairs

$\bar{U}_{\mu\nu} = \frac{1}{4}$



$\bar{V}_{\mu\nu\rho} = \frac{1}{8}$



Deconfinement transition

- First order phase transition $T_c(\Omega = 0) \approx 270$ MeV
- Order parameter: Polyakov loop $\sim F_{q\bar{q}}$

$$P = \frac{1}{N_s^3} \sum_{\vec{n}} \mathrm{Tr} \left[\prod_{\tau=0}^{N_t-1} U_4(\vec{n}, \tau) \right]$$

- S_G invariant under Z_3 symmetry, but P is not!
- At high T, Z_3 symmetry is spontaneously broken

$$\langle P \rangle = 0 : \text{ confinement}$$
$$\langle P \rangle \neq 0 : \text{ deconfinement}$$

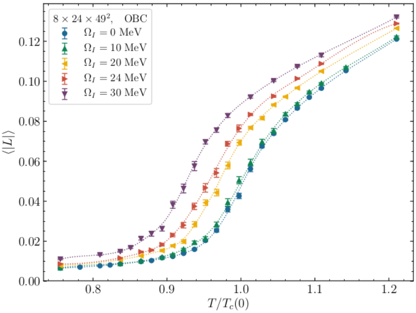
Boundary conditions in x, y

- **Periodic BC**:
  - Usual but incompatible with field of velocities
  - Respect Z_3 symmetry
- **Open BC**:
  - Any link outside of the lattice volume is excluded
  - Respect Z_3 symmetry
- **Dirichlet BC**:
  - Links on the boundary are set to unity, links going outside are excluded
  - Violate Z_3 symmetry
- Projective BC,...

Results

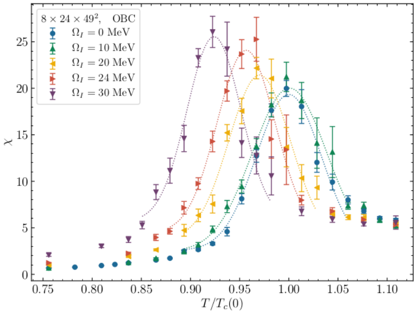
- Results mainly for NJL models
- Transition temperature *decreases* $T_c(\Omega)/T_c(0) < 1$
- What happens with gluon degrees of freedom?
- SU(3) Yang-Mills in a rotating frame
  Braguta, Kotov, Kuznedelev, Roenko '21
- T_c *increases*!

Results



$8 \times 24 \times 49^2$, OBC
$\Omega_I = 0$ MeV
$\Omega_I = 10$ MeV
$\Omega_I = 20$ MeV
$\Omega_I = 24$ MeV
$\Omega_I = 30$ MeV
$\langle |L| \rangle$
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.8
0.9
1.0
1.1
1.2
$T/T_c(0)$

(a)



$8 \times 24 \times 49^2$, OBC
$\Omega_I = 0$ MeV
$\Omega_I = 10$ MeV
$\Omega_I = 20$ MeV
$\Omega_I = 24$ MeV
$\Omega_I = 30$ MeV
χ
0
5
10
15
20
25
0.75
0.80
0.85
0.90
0.95
1.00
1.05
1.10
$T/T_c(0)$

(b)

Results

- Data well described by quadratic function with $v_I = \Omega_I (N_s - 1)a/2$

$$\frac{T_C(v_I)}{T_c(0)} = 1 - B_2 v_I^2$$

- For real rotation (analytic continuation)

$$\frac{T_C(v_I)}{T_c(0)} = 1 + B_2 v^2$$

- Results:

$$\begin{aligned} \text{OBC} &: B_2 \sim 0.7 \\ \text{PBC} &: B_2 \sim 1.3 \\ \text{DBC} &: B_2 \sim 0.5 \end{aligned}$$

- BC seem to be screened

Rotating fermions

- Covariant derivative gets modified

$$S_F = \int \sqrt{\det g_{00}}\, \bar{\psi}[\gamma^\mu (D_\mu - \Gamma_\mu) + m]\psi$$

with

$$\Gamma_\mu = -\frac{i}{4}\sigma^{ij}\omega_{\mu ij}$$

$$\sigma^{ij} = -\frac{i}{2}(\gamma^i\gamma^j - \gamma^j\gamma^i)$$

$$\omega_{\mu ij} = g_{\alpha\beta}\, e_i^\alpha (\partial_\mu e_j^\beta + \Gamma^\beta_{\nu\mu} e_j^\nu)$$

- Choosing a particular vierbein

$$S_F = \int \mathrm{d}^4 \, \bar{\psi} \left[\gamma^x D_x + \gamma^y D_y + \gamma^z D_z + \gamma^\tau \left(D_\tau + i\Omega \frac{\sigma^{12}}{2} \right) + m \right] \psi$$

with

$$\begin{aligned} \gamma^x &= \gamma^1 - y\Omega\gamma^4 \\ \gamma^y &= \gamma^2 + x\Omega\gamma^4 \\ \gamma^z &= \gamma^3 \\ \gamma^\tau &= \gamma^4 \end{aligned}$$

- Wilson fermions Yamamoto, Hirono '13
- Staggered fermions Yang, Huang '23