Causality violations in simulations of heavy ion collisions

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Goal for this presentation: Overview of causality conditions in hydrodynamics theory and understand the application of these conditions in heavy-ion collisions.

PHYSICAL REVIEW C **105**, L061901 (2022)

Letter

Causality violations in realistic simulations of heavy-ion collisions

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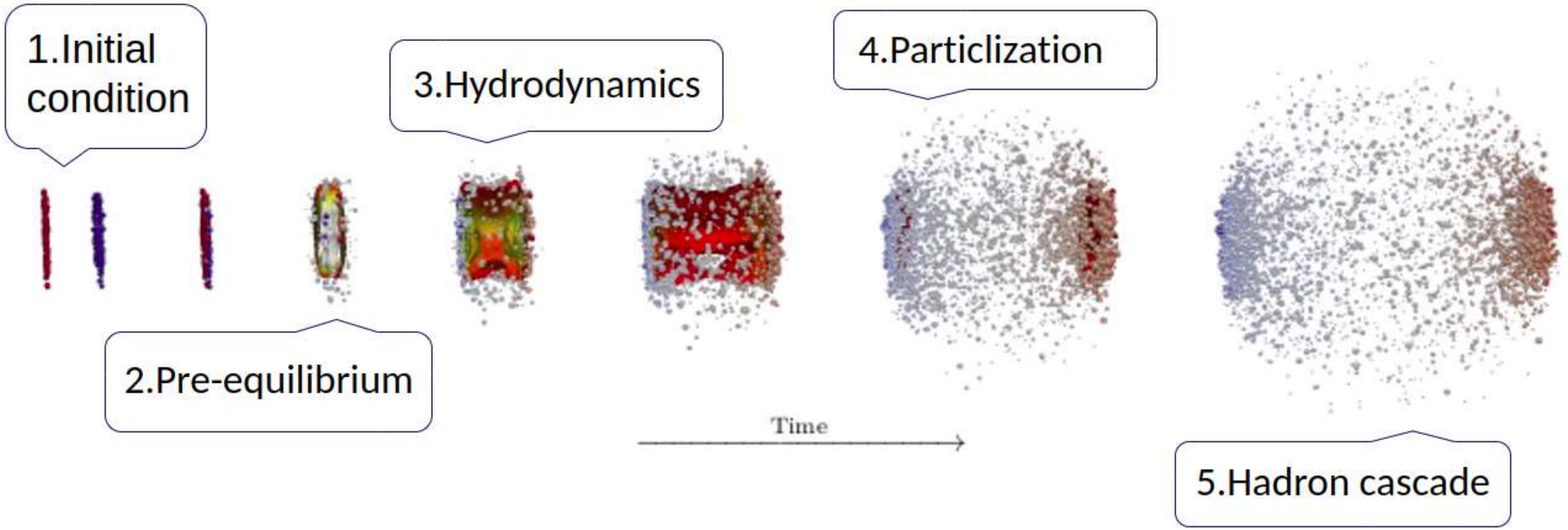
Causality is violated in the early stages of state-of-the-art heavy-ion hydrodynamic simulations. Such violations are present in up to 75% of the fluid cells in the initial time and only after 2–3 fm/c of evolution do we find that 50% of the fluid cells are definitely causal. Superluminal propagation reaches up to 15% the speed of light in some of the fluid cells. The inclusion of pre-equilibrium evolution significantly reduces the number of acausal cells. Our findings suggests that relativistic causality may place constraints on the available parameter space of heavy-ion collision simulations when factored into more thorough statistical analyses.

Simulation of heavy ion collision



Hydrodynamics to describe the QGP: relativistic, viscous and out of equilibrium

Ideal Hydrodynamics

Perturbations

$$\epsilon = \epsilon_0 + \delta\epsilon$$

$$u^\mu = u_0^\mu + \delta u^\mu$$

$$u_\nu \nabla_\mu T^{\mu\nu}_{(0)} = 0$$

$$\Delta^\rho_\nu \nabla_\mu T^{\mu\nu}_{(0)} = 0$$

$$u_\nu \nabla_\mu \epsilon + (\epsilon + P)\nabla_\mu u^\mu = 0$$

$$(\epsilon + P) u^\mu \nabla_\mu u^\rho + c_s^2 \nabla_\mu \epsilon = 0$$

Use group velocity concept

$$v_g \equiv \left|\frac{d\omega}{dk}\right| = c_s \leq 1$$

Condition for causality

ROMATSCHKE, Paul; ROMATSCHKE, Ulrike. Relativistic fluid dynamics in and out of equilibrium: and applications to relativistic nuclear collisions. 2019.

Viscous Hydrodynamics – Navier Stokes (First order theory)

$$\nabla_\mu T^{\mu\nu} = 0$$

Shear and bulk viscosities

$$\pi^{\mu\nu} = -\eta\sigma^{\mu\nu}$$
$$\Pi = \zeta\lambda^{\perp}_{\lambda}u^{\lambda}$$

$$v_g = \left|\frac{d\omega}{dk}\right| = 2\gamma_\eta k$$

Grows without bound

Unlimited velocity, acausal theory

Maxwell-Cataneo proposal

$$\tau_\pi D\pi^{\mu\nu} + \pi^{\mu\nu} = -\eta\sigma^{\mu\nu} + \ldots$$
$$\tau_\Pi D\Pi + \Pi = \zeta\lambda^{\perp}_{\alpha}u^{\alpha} + \ldots$$

Relaxation term

ROMATSCHKE, Paul; ROMATSCHKE, Ulrike. Relativistic fluid dynamics in and out of equilibrium: and applications to relativistic nuclear collisions. 2019.

Theories like Israel-Stewart: DNMR (Second order theory)

$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + 2\tau_\pi \pi_\alpha^{\langle\mu}\omega^{\nu\rangle\alpha} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \varphi_7 \pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha} - \tau_{\pi\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu} + \varphi_6\Pi\pi^{\mu\nu}$$

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta - \delta_{\Pi\Pi}\Pi\theta + \varphi_1\Pi^2 + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu} + \varphi_3\pi^{\mu\nu}\pi_{\mu\nu}$$

Relaxation term

11 second order transport coefficients

$$\tau_\pi, \delta_{\pi\pi}, \phi_7, \tau_{\pi\pi}, \lambda_{\pi\Pi}, \phi_6, \tau_\Pi, \delta_{\Pi\Pi}, \phi_1, \lambda_{\Pi\pi}, \phi_3$$

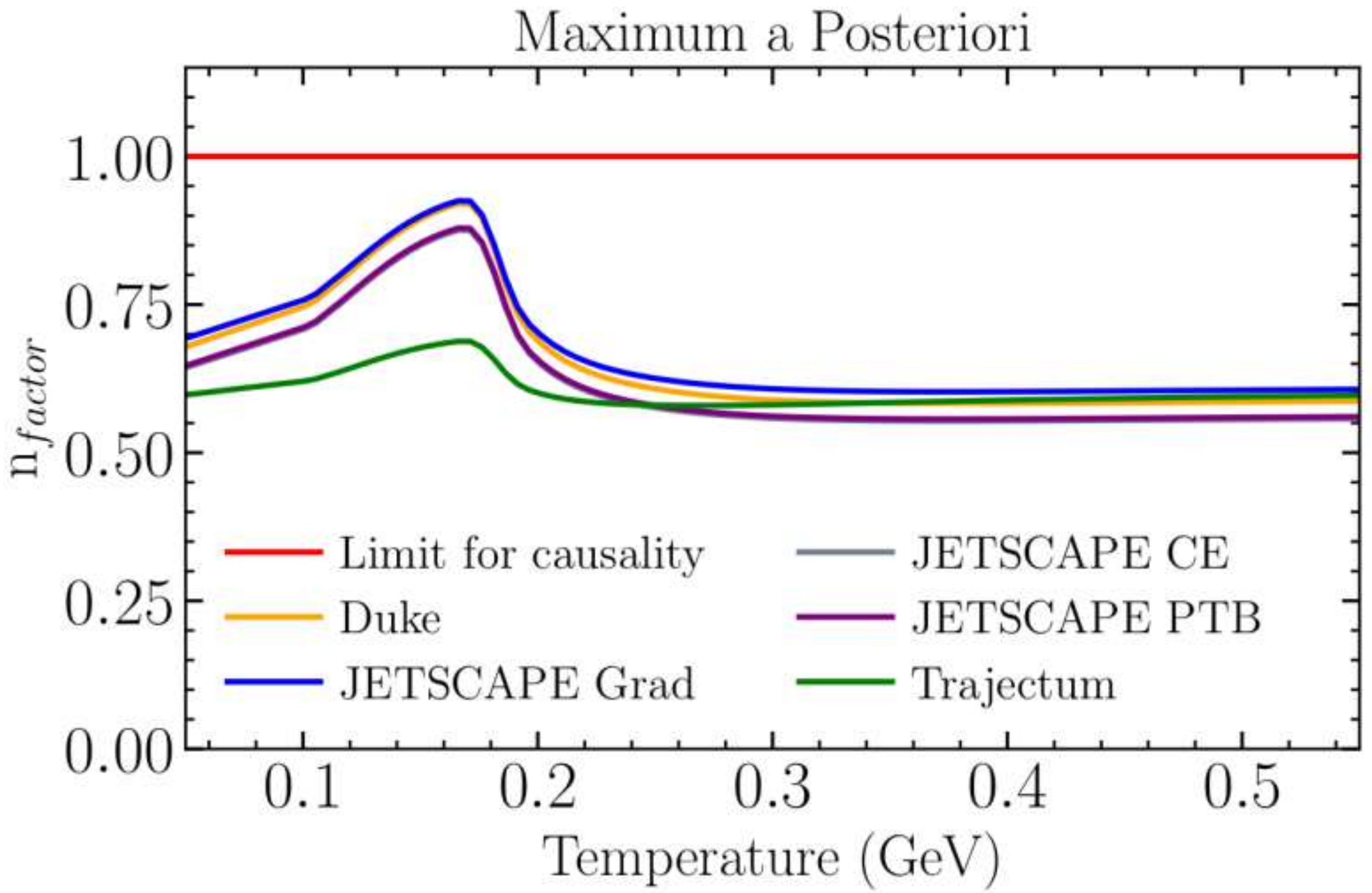
Linear causality condition

$$c_s^2 + \frac{4}{3}\frac{\eta}{\tau_\pi(\epsilon + p)} + \frac{\zeta}{\tau_\Pi(\epsilon + p)} \leq 1$$

DENICOL, G. S.; JEON, S.; GALE, C. Transport coefficients of bulk viscous pressure in the 14-moment approximation. **Physical Review C**, v. 90, n. 2, p. 024912, 2014.

PU, Shi; KOIDE, Tomoi; RISCHKE, Dirk H. Does stability of relativistic dissipative fluid dynamics imply causality?. **Physical Review D**, v. 81, n. 11, p. 114039, 2010.

$$n_{\text{factor}} \equiv c_s^2 + \frac{4}{3}\frac{\eta}{\tau_\pi(\epsilon + p)} + \frac{\zeta}{\tau_\Pi(\epsilon + p)} \leq 1$$



KRUPCZAK, R. et al. Causality violations in simulations of large and small heavy-ion collisions. **arXiv preprint arXiv:2311.02210**, 2023.

Theories like Israel-Stewart: DNMR

Nonlinear causality conditions are more complicated...

PHYSICAL REVIEW LETTERS **126,** 222301 (2021)

Nonlinear Constraints on Relativistic Fluids Far from Equilibrium

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New constraints are found that must necessarily hold for Israel-Stewart-like theories of fluid dynamics to be causal far away from equilibrium. Conditions that are sufficient to ensure causality, local existence, and uniqueness of solutions in these theories are also presented. Our results hold in the full nonlinear regime, taking into account bulk and shear viscosities (at zero chemical potential), without any simplifying symmetry or near-equilibrium assumptions. Our findings provide fundamental constraints on the magnitude of viscous corrections in fluid dynamics far from equilibrium.

DOI: 10.1103/PhysRevLett.126.222301

Sufficient conditions

Necessary conditions

$$(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{1}{2}\tau_{\pi\pi}|\Lambda_1| \geq 0,$$

$$\varepsilon + P + \Pi - \frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{\tau_{\pi\pi}}{4\tau_\pi}\Lambda_3 \geq 0,$$

$$\frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) + \frac{\tau_{\pi\pi}}{4\tau_\pi}(\Lambda_a + \Lambda_d) \geq 0, \quad a \neq d,$$

$$\varepsilon + P + \Pi + \Lambda_a - \frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{\tau_{\pi\pi}}{4\tau_\pi}(\Lambda_d + \Lambda_a) \geq 0, \quad a \neq d$$

$$\frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) + \frac{\tau_{\pi\pi}}{2\tau_\pi}\Lambda_d + \frac{1}{6\tau_\pi}\left[2\eta + \lambda_{\pi\Pi}\Pi + (6\delta_{\pi\pi} - \tau_{\pi\pi})\Lambda_d\right] + \frac{\zeta + \delta_{\Pi\Pi}\Pi + \lambda_{\Pi\pi}\Lambda_d}{\tau_\Pi} + (\varepsilon + P + \Pi + \Lambda_d)c_s^2 \geq 0$$

$$\varepsilon + P + \Pi + \Lambda_d - \frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{\tau_{\pi\pi}}{2\tau_\pi}\Lambda_d - \frac{1}{6\tau_\pi}\left[2\eta + \lambda_{\pi\Pi}\Pi + (6\delta_{\pi\pi} - \tau_{\pi\pi})\Lambda_d\right] - \frac{\zeta + \delta_{\Pi\Pi}\Pi + \lambda_{\Pi\pi}\Lambda_d}{\tau_\Pi} - (\varepsilon + P + \Pi + \Lambda_d)c_s^2 \geq 0$$

Sufficient conditions:

$$(\varepsilon + P + \Pi - |\Lambda_1|) - \frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{\tau_{\pi\pi}}{2\tau_\pi}\Lambda_3 \geq 0$$

$$(2\eta + \lambda_{\pi\Pi}\Pi) - \tau_{\pi\pi}|\Lambda_1| > 0,$$

$$\tau_{\pi\pi} \leq 6\delta_{\pi\pi},$$

$$\frac{\lambda_{\Pi\pi}}{\tau_\Pi} + c_s^2 - \frac{\tau_{\pi\pi}}{12\tau_\pi} \geq 0,$$

$$\frac{1}{3\tau_\pi}\left[4\eta + 2\lambda_{\pi\Pi}\Pi + (3\delta_{\pi\pi} + \tau_{\pi\pi})\Lambda_3\right] + \frac{\zeta + \delta_{\Pi\Pi}\Pi + \lambda_{\Pi\pi}\Lambda_3}{\tau_\Pi} + |\Lambda_1| + \Lambda_3 c_s^2 + \frac{\frac{12\delta_{\pi\pi} - \tau_{\pi\pi}}{12\tau_\pi}\left(\frac{\lambda_{\Pi\pi\pi}}{\tau_\Pi} + c_s^2 - \frac{\tau_{\pi\pi}}{12\tau_\pi}\right)(\Lambda_3 + |\Lambda_1|)^2}{\varepsilon + P + \Pi - |\Lambda_1| - \frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{\tau_{\pi\pi}}{2\tau_{\pi\pi}}\Lambda_3} \leq (\varepsilon + P + \Pi)(1 - c_s^2),$$

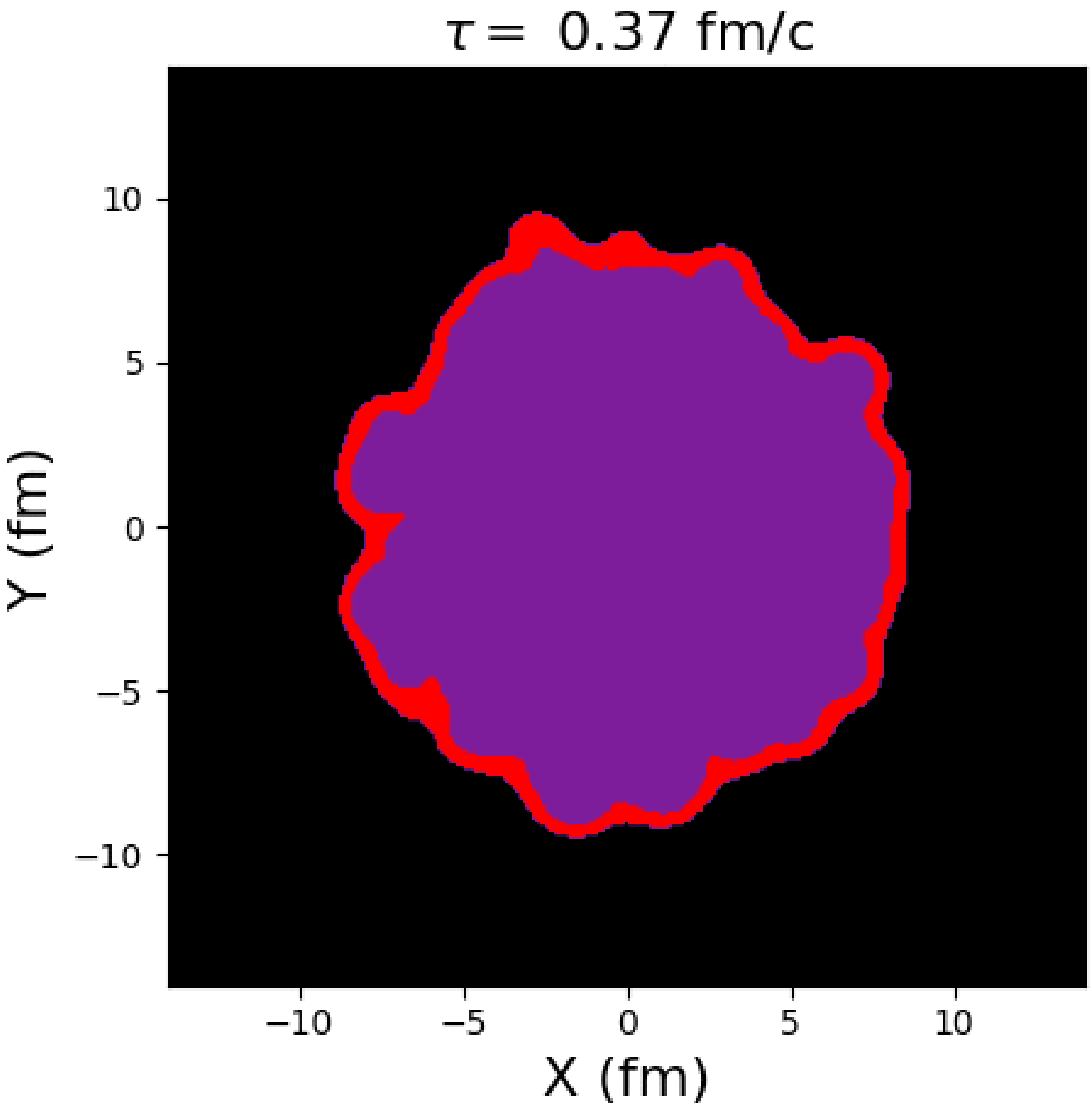
$$\frac{1}{6\tau_\pi}\left[2\eta + \lambda_{\pi\Pi}\Pi + (\tau_{\pi\pi} - 6\delta_{\pi\pi})|\Lambda_1|\right] + \frac{\zeta + \delta_{\Pi\Pi}\Pi - \lambda_{\Pi\pi}|\Lambda_1|}{\tau_\Pi} + (\varepsilon + P + \Pi - |\Lambda_1|)c_s^2 \geq 0,$$

$$1 \geq \frac{\frac{12\delta_{\pi\pi} - \tau_{\pi\pi}}{12\tau_{\pi\pi}}\left(\frac{\lambda_{\Pi\pi}}{\tau_\Pi} + c_s^2 - \frac{\tau_{\pi\pi}}{12\tau_\pi}\right)(\Lambda_3 + |\Lambda_1|)^2}{\left[\frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{\tau_{\pi\pi}}{2\tau_\pi}|\Lambda_1|\right]^2},$$

$$\frac{1}{3\tau_\pi}\left[4\eta + 2\lambda_{\pi\Pi}\Pi - (3\delta_{\pi\pi} + \tau_{\pi\pi})|\Lambda_1|\right] + \frac{\zeta + \delta_{\Pi\Pi}\Pi - \lambda_{\Pi\pi}|\Lambda_1|}{\tau_\Pi} + (\varepsilon + P + \Pi - |\Lambda_1|)c_s^2 \geq \frac{(\varepsilon + P + \Pi + \Lambda_2)(\varepsilon + P + \Pi + \Lambda_3)}{3(\varepsilon + P + \Pi - |\Lambda_1|)} \times \left\{1 + \frac{2\left[\frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) + \frac{\tau_{\pi\pi}}{2\tau_\pi}\Lambda_3\right]}{\varepsilon + P + \Pi - |\Lambda_1|}\right\},$$

BEMFICA, Fábio S. et al. Nonlinear constraints on relativistic fluids far from equilibrium. **Physical review letters**, v. 126, n. 22, p. 222301, 2021.

Testing the causality conditions in the heavy ion simulations



CAUSAL

Necessary ✓

Sufficient ✓

INDETERMINATE

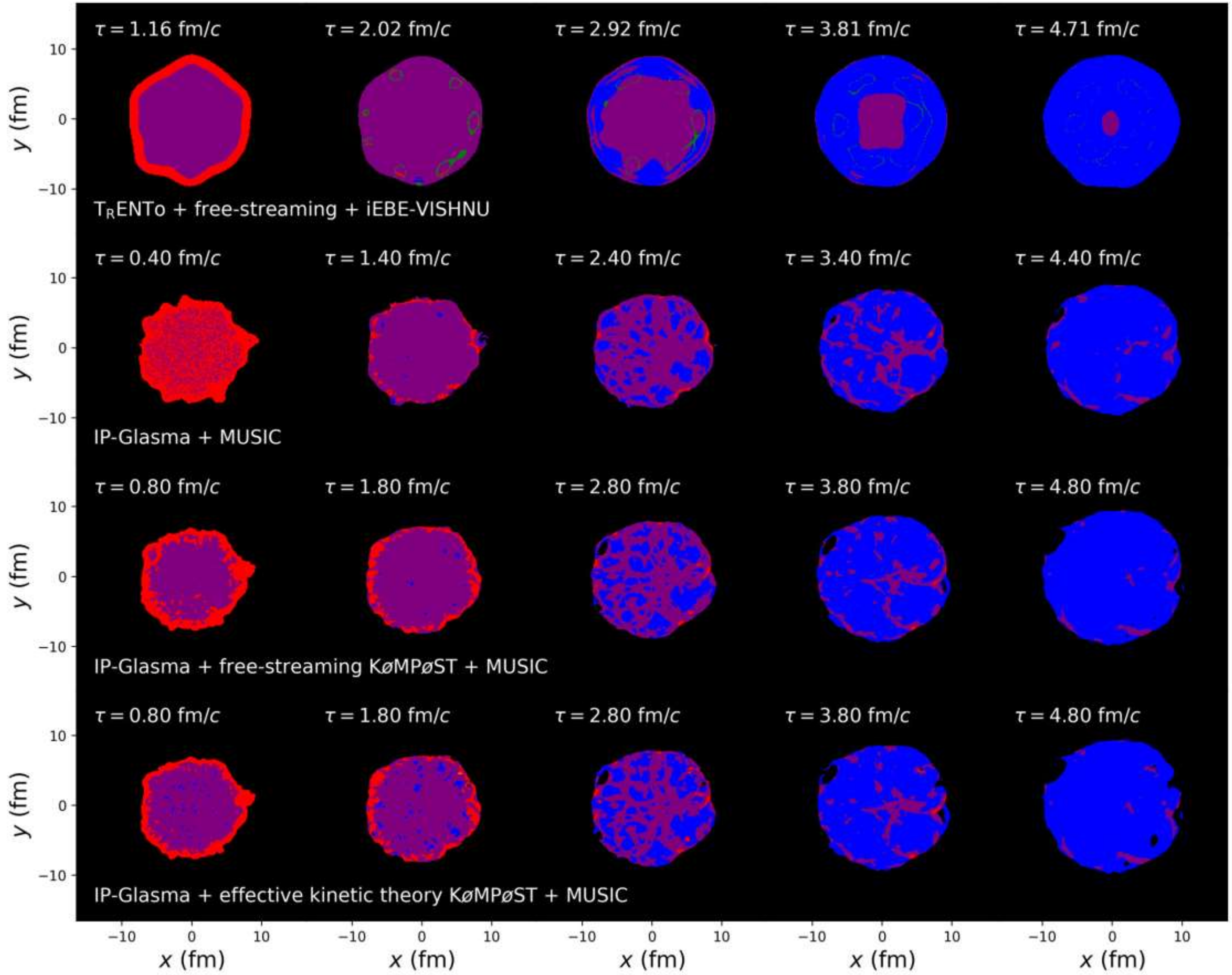
Necessary ✓

Sufficient ✗

ACAUSAL

Sufficient ✗

Necessary ✗



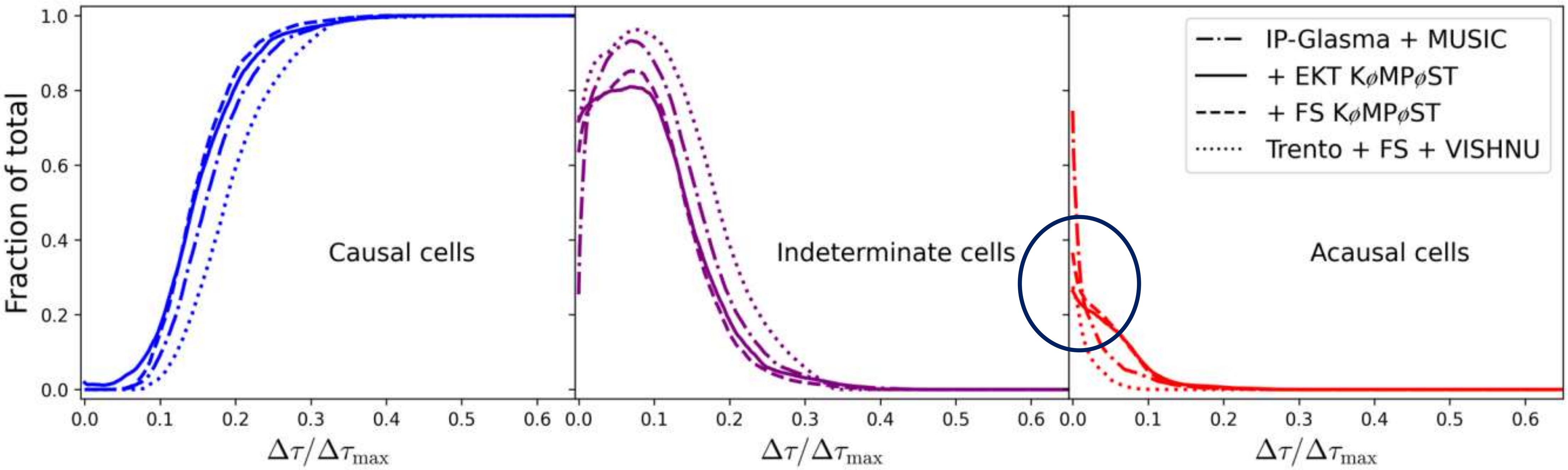
ACAUSAL

INDETERMINATE

CAUSAL

Conclusion:
causality violation seems to be always present

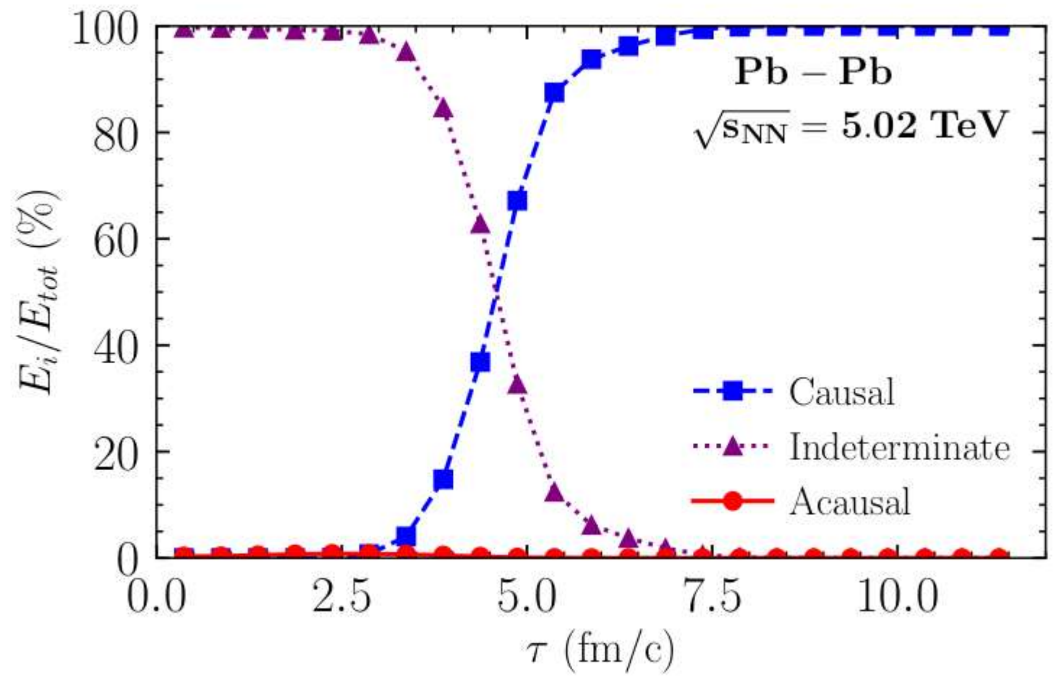
PLUMBERG, Christopher et al. Causality violations in realistic simulations of heavy-ion collisions. **Physical Review C**, v. 105, n. 6, p. L061901, 2022.

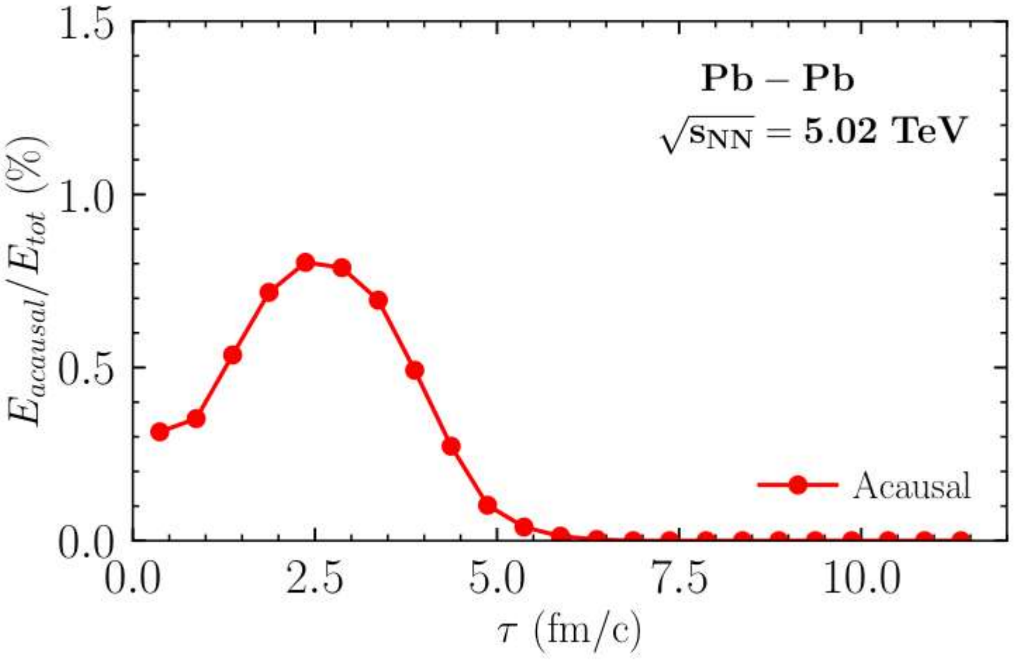


Pre-equilibrium models can significantly reduce the fraction of acausal cells in the system.

PLUMBERG, Christopher et al. Causality violations in realistic simulations of heavy-ion collisions. **Physical Review C**, v. 105, n. 6, p. L061901, 2022.

In terms of energy, it represents a small part of the system.





Trento + Free streaming + MUSIC

KRUPCZAK, R. et al. Causality violations in simulations of large and small heavy-ion collisions. **arXiv preprint arXiv:2311.02210**, 2023.

Studying the conditions for causality, the most often and important condition:

$$\epsilon + P + \Pi + \Lambda_d - \frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{\tau_{\pi\pi}}{2\tau_\pi}\Lambda_d - \frac{1}{6\tau_\pi}[2\eta + \lambda_{\pi\Pi}\Pi + (6\delta_{\pi\pi} - \tau_{\pi\pi})\Lambda_d] - \frac{\zeta + \delta_{\Pi\Pi}\Pi + \lambda_{\Pi\pi}\Lambda_d}{\tau_\Pi} - (\epsilon + P + \Pi + \Lambda_d)c_s^2 \geq 0$$

Contains the linear causality condition inside: $c_s^2 + \frac{4}{3}\frac{\eta}{\tau_\pi(\epsilon + p)} + \frac{\zeta}{\tau_\Pi(\epsilon + p)} \leq 1$

Studying the conditions for causality, the most often and important condition:

$$\epsilon + P + \Pi + \Lambda_d - \frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{\tau_{\pi\pi}}{2\tau_\pi}\Lambda_d - \frac{1}{6\tau_\pi}[2\eta + \lambda_{\pi\Pi}\Pi + (6\delta_{\pi\pi} - \tau_{\pi\pi})\Lambda_d]$$
$$- \frac{\zeta + \delta_{\Pi\Pi}\Pi + \lambda_{\Pi\pi}\Lambda_d}{\tau_\Pi} - (\epsilon + P + \Pi + \Lambda_d)c_s^2 \geq 0$$

When this term is equal to zero, the acausality disappears from the system.

$$\frac{-\zeta}{\tau_\Pi} \to 0 \quad \Longrightarrow \quad \tau_\Pi = b_\Pi \frac{\zeta}{(1/3 - c_s^2)^2(\epsilon + P)}$$

Increase the bulk relaxation time

Studying the conditions for causality, the most often and important condition:

$$\epsilon + P + \Pi + \Lambda_d - \frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{\tau_{\pi\pi}}{2\tau_\pi}\Lambda_d - \frac{1}{6\tau_\pi}[2\eta + \lambda_{\pi\Pi}\Pi + (6\delta_{\pi\pi} - \tau_{\pi\pi})\Lambda_d] - \frac{\zeta + \delta_{\Pi\Pi}\Pi + \lambda_{\Pi\pi}\Lambda_d}{\tau_\Pi} - (\epsilon + P + \Pi + \Lambda_d)c_s^2 \geq 0$$

When this term is equal to zero, the acausality disappears from the system.

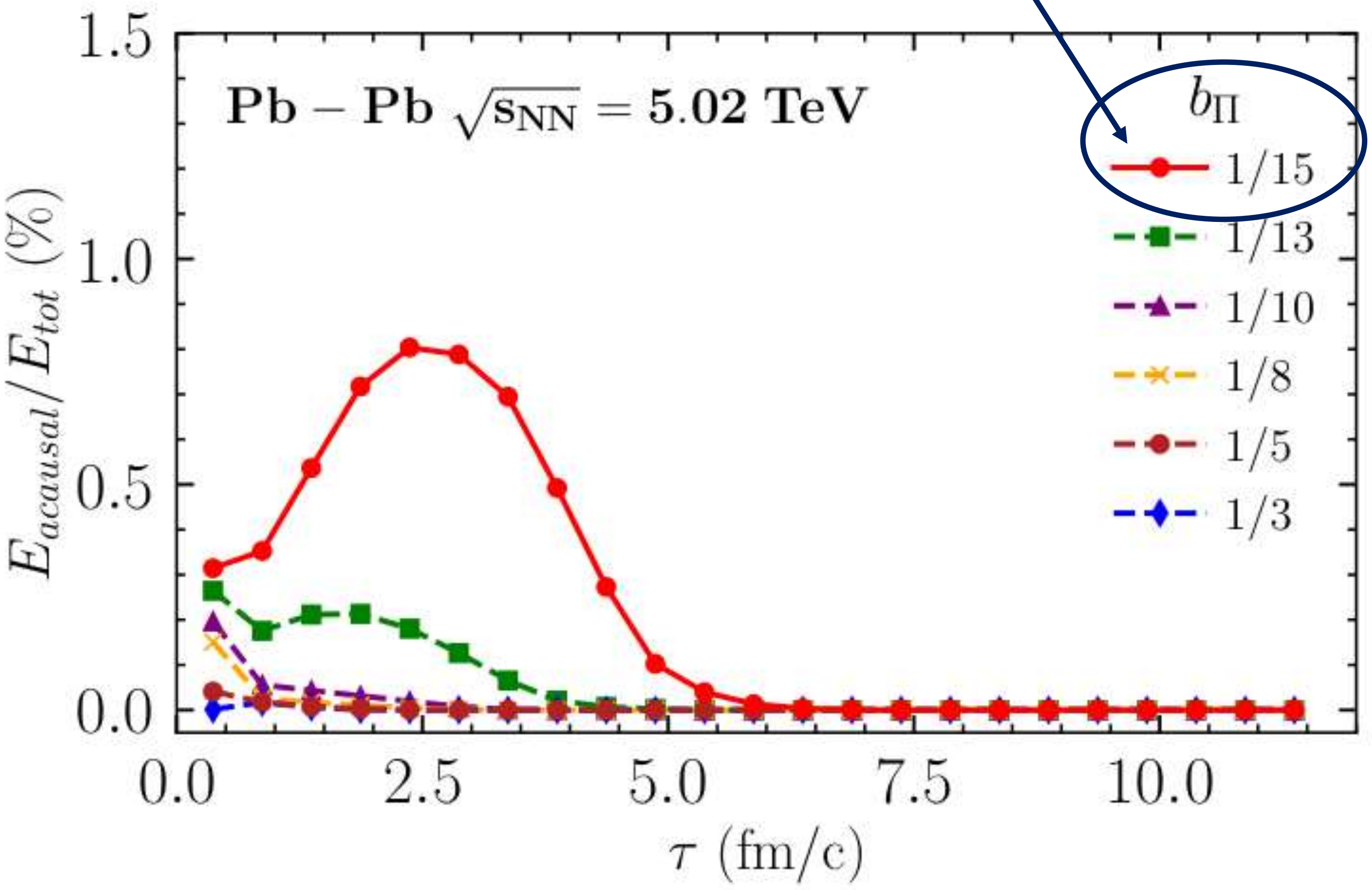
$$\frac{-\zeta}{\tau_\Pi} \to 0 \quad \Rightarrow \quad \tau_\Pi = b_\Pi \frac{\zeta}{(1/3 - c_s^2)^2(\epsilon + P)}$$

Increase this parameter

Properties:
- Dimensionless parameter
- Without physical meaning
- It is not fixed by hydrodynamic theory
- This change doesn't affect the final observables

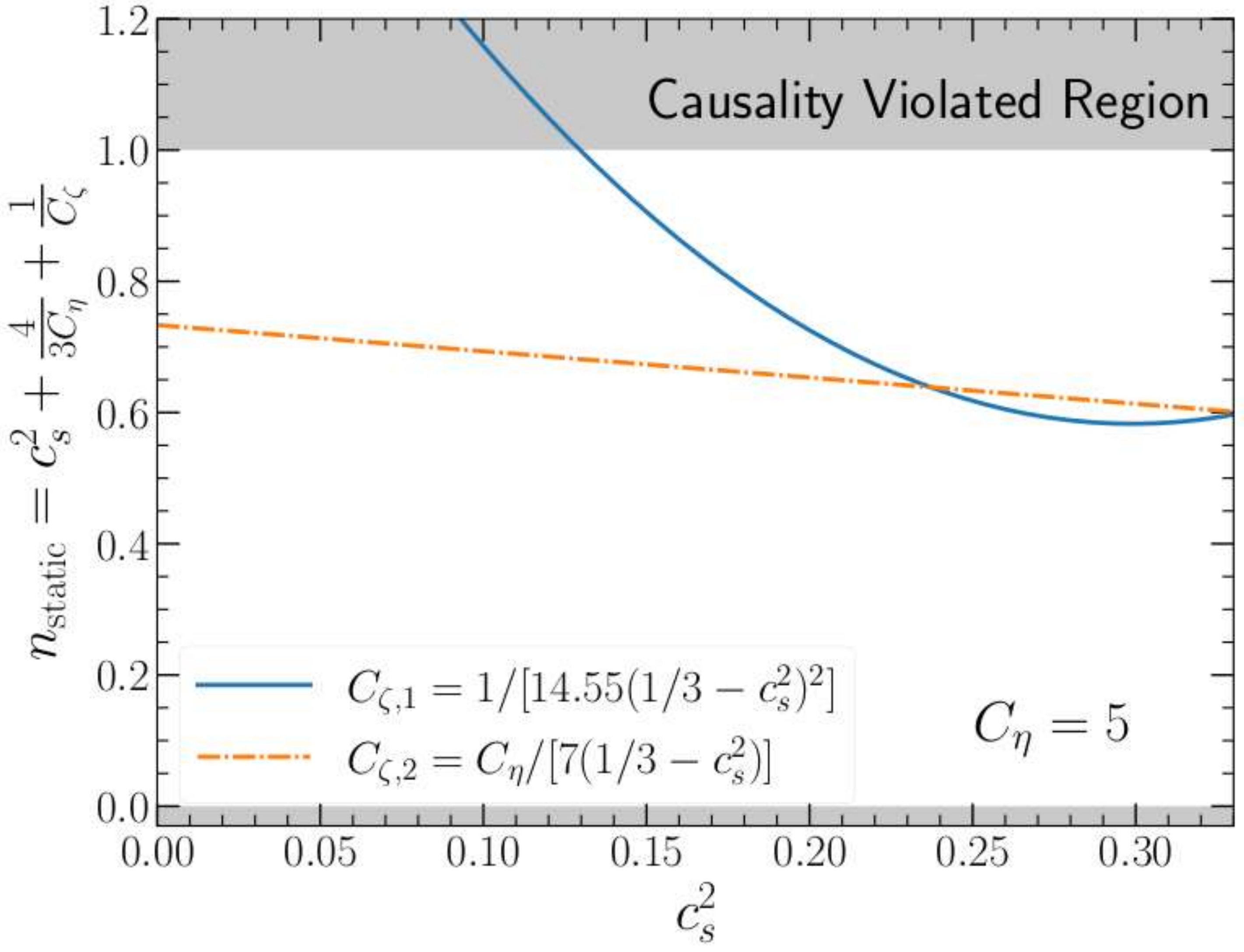
Standard value used in MUSIC (DNMR theory)

Increase this parameter, you can reduce significantly the acausality



KRUPCZAK, R. et al. Causality violations in simulations of large and small heavy-ion collisions. **arXiv preprint arXiv:2311.02210**, 2023.

Parameters related with the causality condition



$$C_\zeta \propto \frac{1}{(1/3 - c_s^2)^\alpha}$$

$$C_{\zeta,1} = \frac{b_\Pi}{(1/3 - c_s^2)^2} \longrightarrow \begin{cases} b_\Pi = 1/14.55 \\ \alpha = 2 \end{cases}$$

Kinetic approach

$$C_{\zeta,2} = \frac{b_\Pi b_\pi}{(1/3 - c_s^2)} \longrightarrow \begin{cases} b_\Pi = 1/7 \\ \alpha = 1 \end{cases}$$

Strongly coupled theory

The sound speed that comes from the equation of state has a minimum value ~0.15

CHIU, Cheng; SHEN, Chun. Exploring theoretical uncertainties in the hydrodynamic description of relativistic heavy-ion collisions. **Physical Review C**, v. 103, n. 6, p. 064901, 2021.

Conclusions...

- Acausality in heavy ion simulations is a recent topic with many open questions;
- Causality is violated in linear regime in some specific conditions, but can be controlled;
- In nonlinear regimes, the causality condition can't affirm if the system is only causal or acausal;
- Causality violations exist in heavy ion collisions independent of the computational model;
- Parametrizations for the hydrodynamics, as for example the relaxation time plays an important rule;
- The causality violation does not appear to affect significantly the final observables, which makes the issue more complex to solve.

Thank you!