UNIVERSITÄT BIELEFELD

The heavy quark diffusion coefficient from 2+1 flavor lattice QCD

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[L. Altenkort, OK, R. Larsen, S. Mukherjee, P. Petreczky, H.T. Shu, S. Stendebach,
Heavy Quark Diffusion from 2+1 Flavor Lattice QCD, arXiv:2302.08501]

[L. Altenkort, A.M. Eller, OK, L. Mazur, G.D. Moore,
Heavy quark momentum diffusion from the lattice using gradient flow, PRD103 (2021) 014511]

[A.Francis, OK, M. Laine, T. Neuhaus, H. Ohno,
Nonperturbative estimate of the heavy quark momentum diffusion coefficient, PRD92(2015)116003]

11th International Conference on
Hard and Electromagnetic Probes of High-Energy Nuclear Collisions
Aschaffenburg, 29.03.2023

CRC-TR 211
Strong-interaction matter under extreme conditions

Vector-meson spectral function – hard to separate different scales

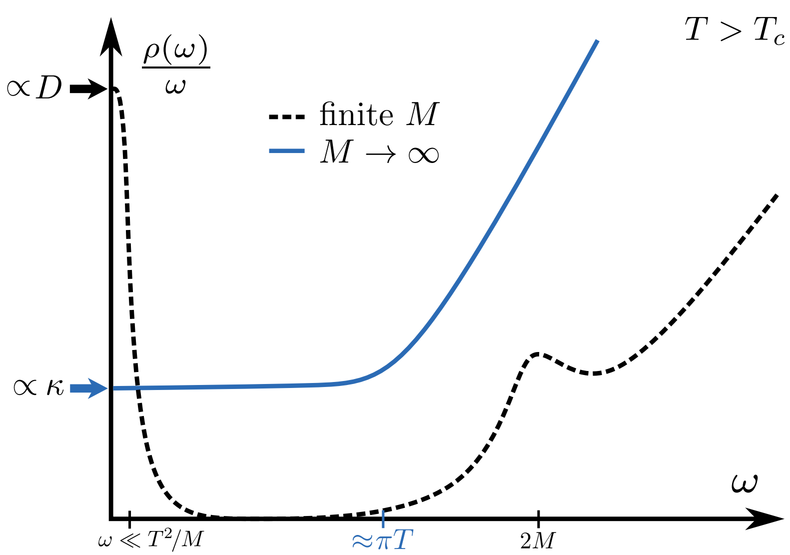
$$G(\tau,\vec{p},T) = \int_0^\infty \frac{d\omega}{2\pi} \rho(\omega,\vec{p},T) K(\tau,\omega,T)$$

$$K(\tau,\omega,T) = \frac{\cosh\left(\omega(\tau - \frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

Different contributions and scales enter in the spectral function

- continuum at large frequencies
- possible bound states at intermediate frequencies
- transport contributions at small frequencies
- in addition cut-off effects on the lattice

Spectral functions in the QGP



difficult to extract D_s from vector meson correlation fct.

$$G_{\mu\nu}(\tau,\vec{x}) = \langle J_\mu(\tau,\vec{x}) J_\nu^\dagger(0,\vec{0})\rangle$$
$$J_\mu(\tau,\vec{x}) = 2\kappa Z_V \bar{\psi}(\tau,\vec{x})\Gamma_\mu \psi(\tau,\vec{x})$$

→ narrow transport peak hard to resolve

→ large lattices and continuum extrapolation needed

→ use perturbation theory to constrain the UV behavior

easier to extract heavy quark momentum diffusion coefficient κ in the heavy quark mass limit

→ smooth $\omega \to 0$ limit expected

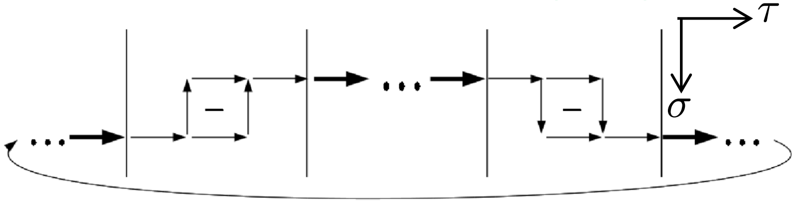
Heavy Quark Effective Theory (HQET) in the large quark mass limit

for a single quark in medium

leads to a (pure gluonic) "color-electric correlator"

[J.Casalderrey-Solana, D.Teaney, PRD74(2006)085012,
S.Caron-Huot,M.Laine,G.D. Moore,JHEP04(2009)053]

$$G_E(\tau) \equiv -\frac{1}{3}\sum_{i=1}^{3} \frac{\left\langle \mathrm{Re\,Tr}\left[U(\tfrac{1}{T};\tau)\, g\, E_i(\tau,\mathbf{0})\, U(\tau;0)\, g\, E_i(0,\mathbf{0})\right]\right\rangle}{\left\langle \mathrm{Re\,Tr}[U(\tfrac{1}{T};0)]\right\rangle}$$



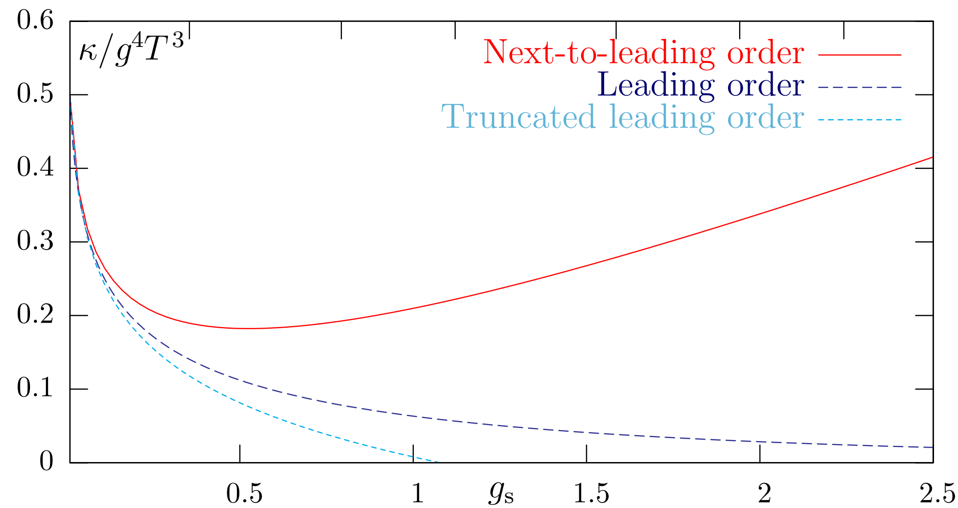
Smooth limit expected from NLO PT

[Caron-Huot, M.Laine, G.Moore, JHEP 0904 (2009) 053]

$$\kappa = \lim_{\omega\to 0} \frac{2T\rho_E(\omega)}{\omega}$$

NLO perturbative calculation:

[Caron-Huot, G. Moore, JHEP 0802 (2008) 081]



→ large correction towards strong interactions

→ non-perturbative lattice methods required

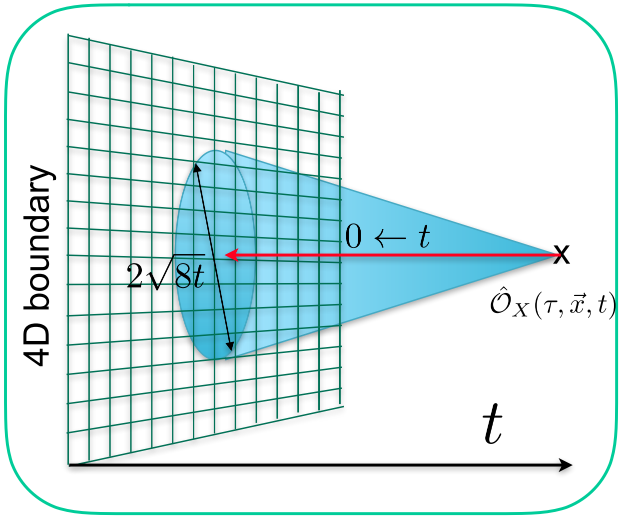
Gradient Flow as a noise reduction method

Gradient flow - *diffusion* equation for the gauge fields along extra dimension, *flow-time t*

[M. Lüscher, 2010]

$$\frac{\partial}{\partial t}A_\mu(t,x) = -\frac{\partial S_{\mathrm{YM}}}{\partial A_\mu}$$

$$A_\mu(t=0,x) = A_\mu(x)$$



- continuous smearing of the gauge fields, effective smearing radius: $r_{\mathrm{smear}} \sim \sqrt{8t}$
- gauge fields become smooth and renormalized
- no UV divergences at finite flow-time *t* → operators of flowed fields are renormalized
- UV fluctuations effectively reduces → noise reduction technique
- Applicable in quenched and full QCD → methods developed in quenched studies

What is the flow time dependence of correlation functions?

How to perform the continuum and t→0 limit correctly?

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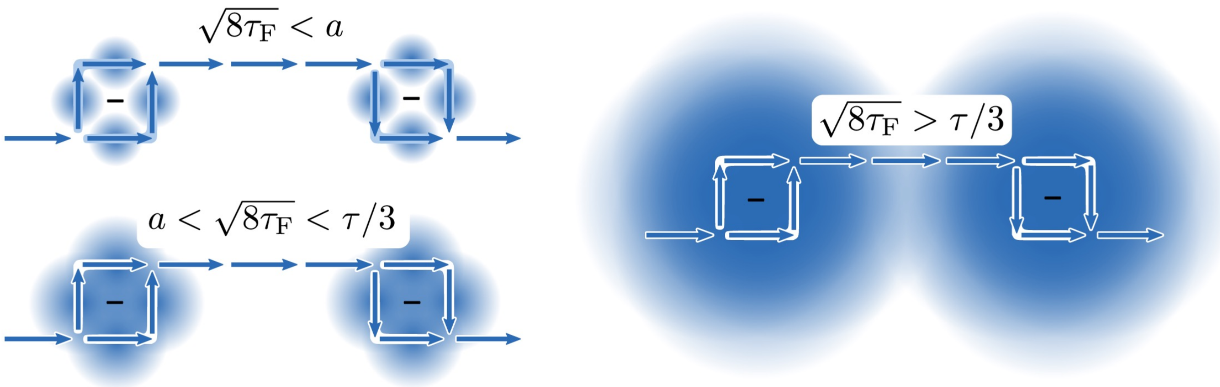

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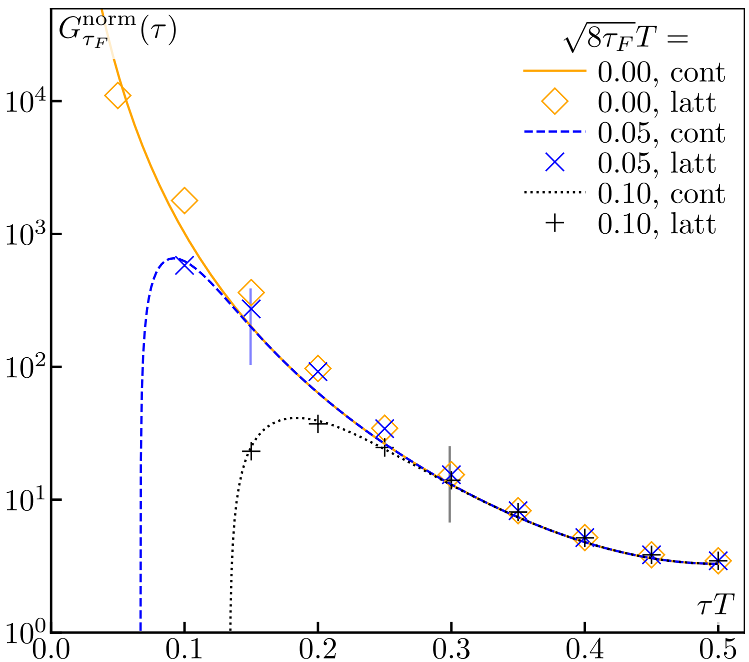
LO perturbative limits

for the flow-time dependence:

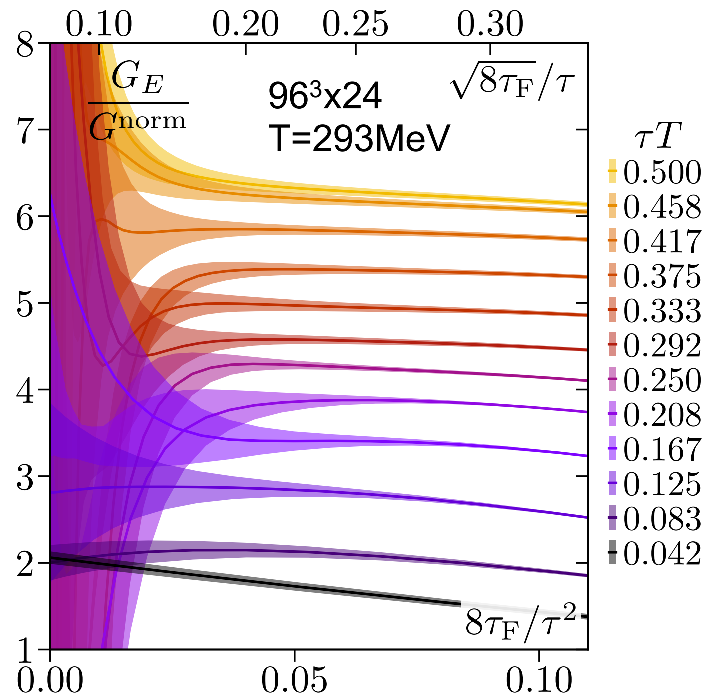
$$\tilde{\tau}_f < 0.1136(\tau T)^2$$



[A.M Eller, G.D. Moore, PRD97 (2018) 114507]

2+1-flavor lattice QCD results on the flow

dependence of the color-electric correlator:



[L. Altenkort, OK, R. Larsen, et al., arXiv:2302.08501]

Effective reduction of UV fluctuations → good noise reduction technique

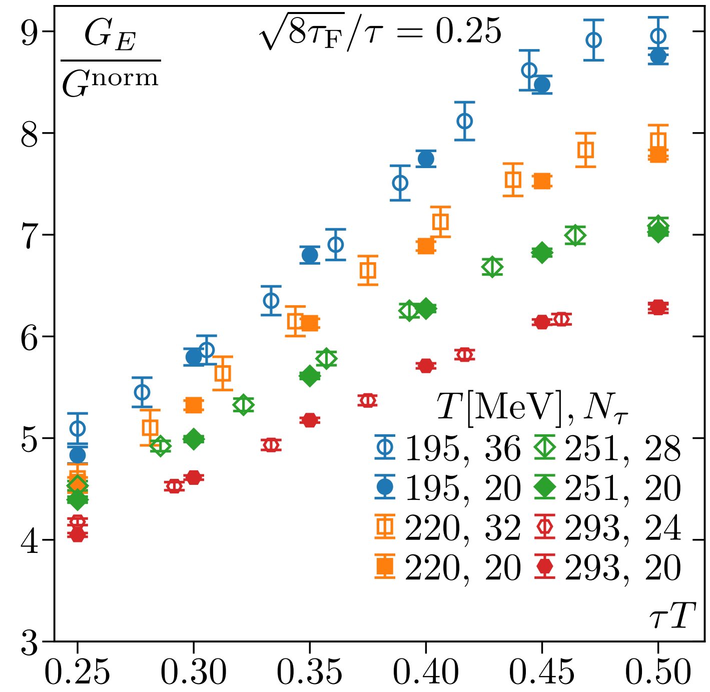
Signal gets destroyed at flow times above the perturbative estimate

Linear behavior at intermediate flow times

2+1-flavor lattice QCD on large and fine isotropic lattices at four temperatures above T_c

- HISQ action with physical strange quark mass and m_s/m_l=5 ($m_\pi \approx 300\ MeV$)
- using gradient flow method to improve the signal

T [MeV]	T/T_c	a [fm]	β	N_σ	N_τ	# conf.
195	1.09	0.0505	7.570	64	20	5899
		0.0421	7.777	64	24	3435
		0.0280	8.249	96	36	2256
220	1.22	0.0449	7.704	64	20	7923
		0.0374	7.913	64	24	2715
		0.0280	8.249	96	32	912
251	1.40	0.0393	7.857	64	20	6786
		0.0327	8.068	64	24	5325
		0.0280	8.249	96	28	1680
293	1.63	0.0336	8.036	64	20	6534
		0.0306	8.147	64	22	9101
		0.0280	8.249	96	24	688



1) perform the continuum limit, a$\rightarrow$ 0 $\leftrightarrow$ $N_t \rightarrow \infty$

2) perform the flow time to zero limit of the continuum correlators

3) determine κ in the continuum using an Ansatz for the spectral fct. $\rho(\omega)$

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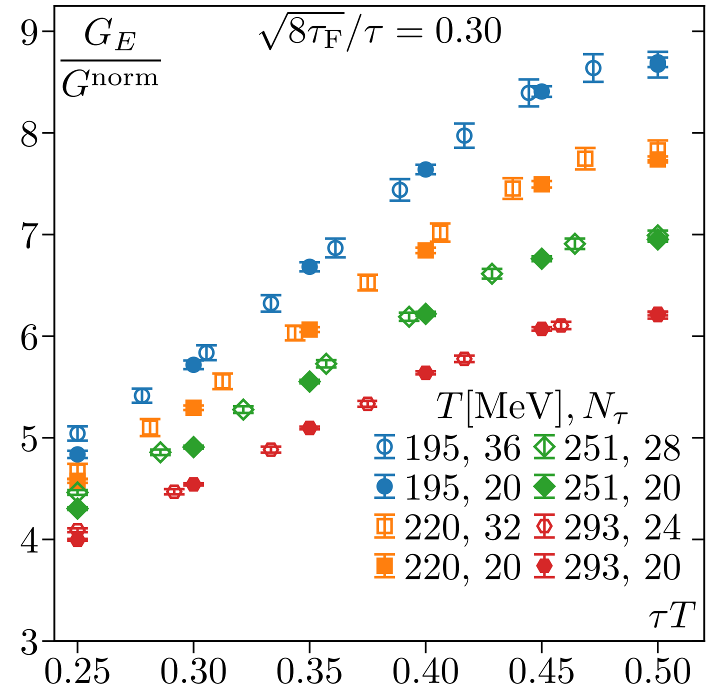

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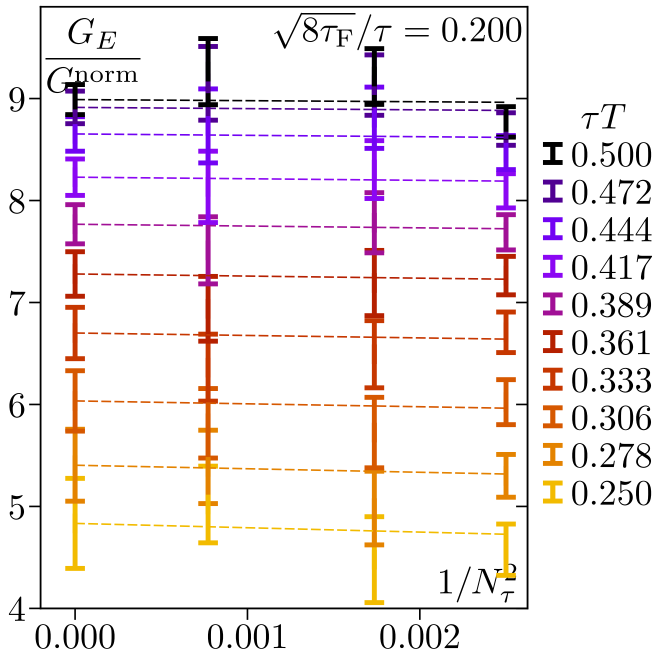
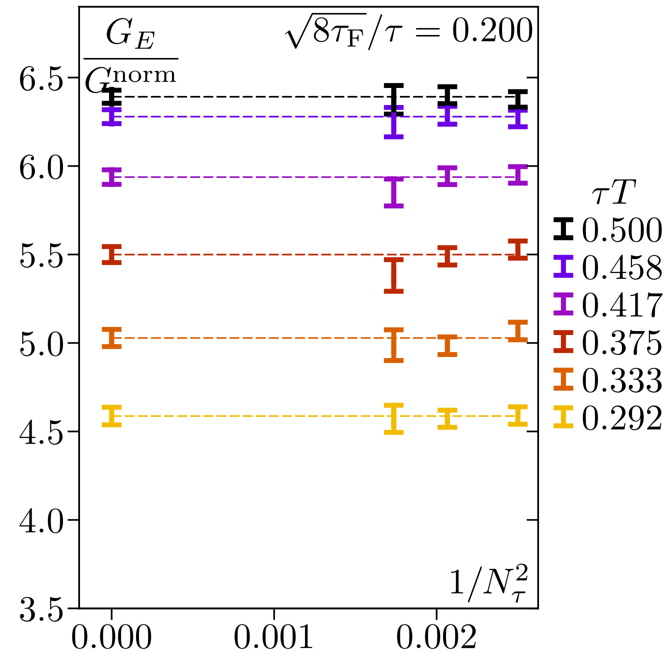
Gradient Flow method – 1) a→0 limit at fixed flow time

- cut-off effects get reduced with increasing flow time
- continuum limit, *a*→*0 (*N_t→ ∞*)*, at fixed physical flow time:

T=195MeV N_t=36,24,20



T=293MeV N_t=24,22,20

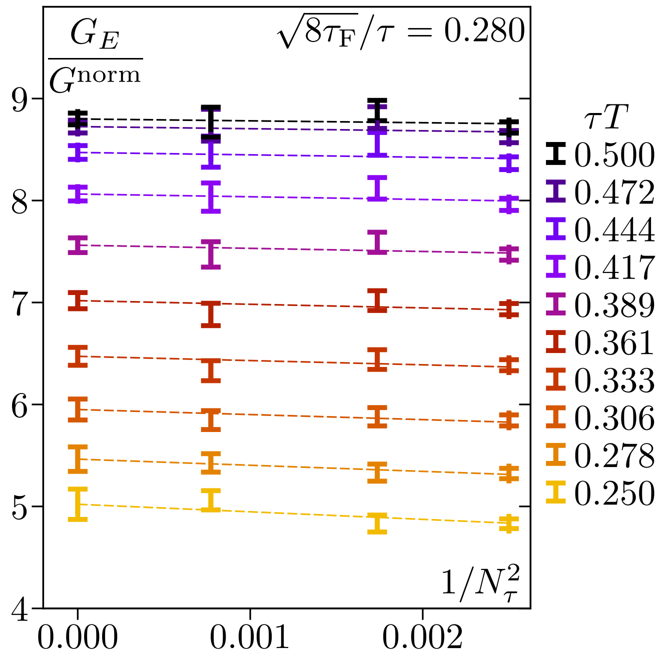


- well defined continuum correlators for different finite flow times

next step: flow time to zero extrapolation of continuum correlators

- cut-off effects get reduced with increasing flow time
- continuum limit, *a*→*0* ($N_t \to \infty$), at fixed physical flow time:

T=195MeV N_t=36,24,20



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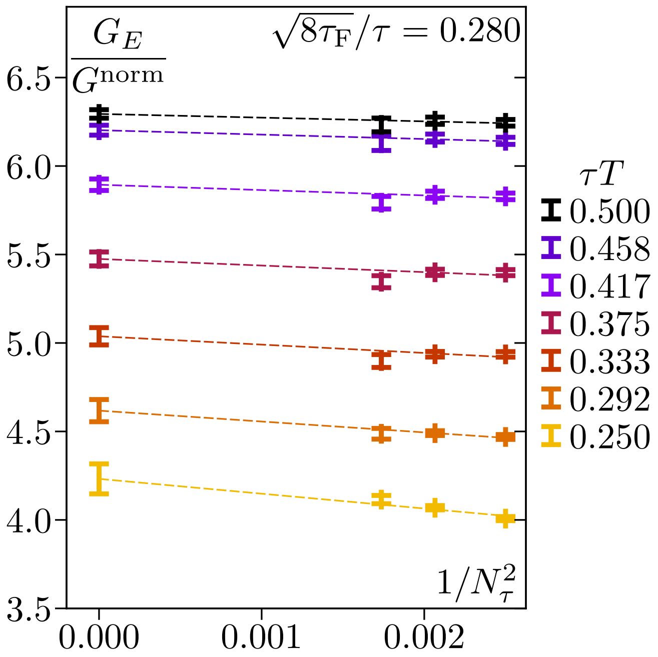


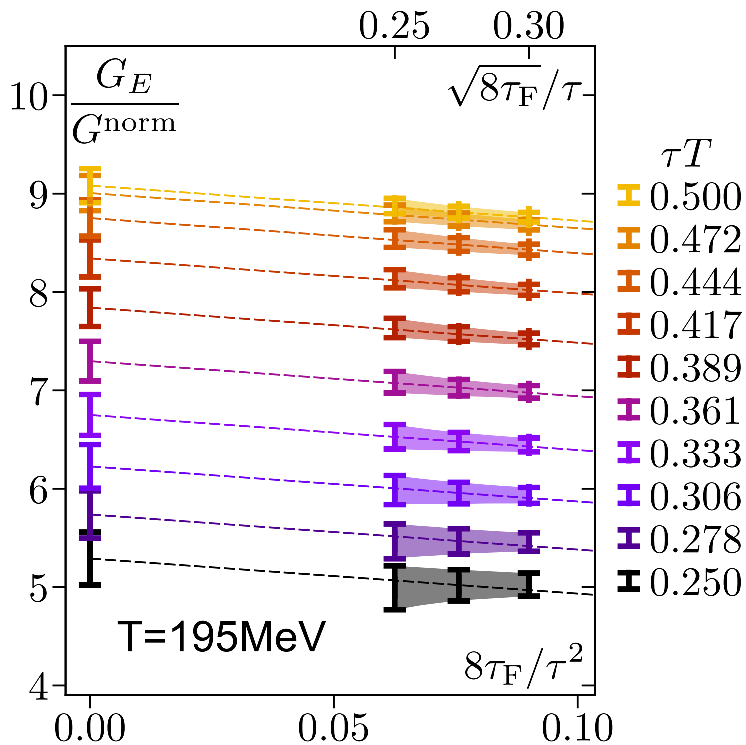
- well defined continuum correlators for different finite flow times

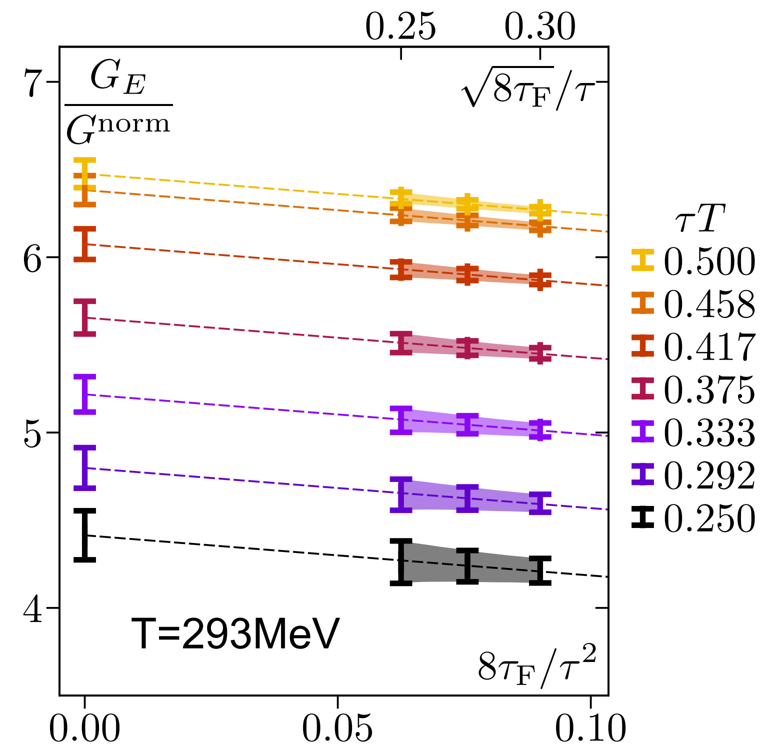
next step: flow time to zero extrapolation of continuum correlators

Continuum limit, *a→0 ($N_t \to \infty$)*, at fixed physical flow time:

followed by

Flow time limit, *t→0*, for each distance:



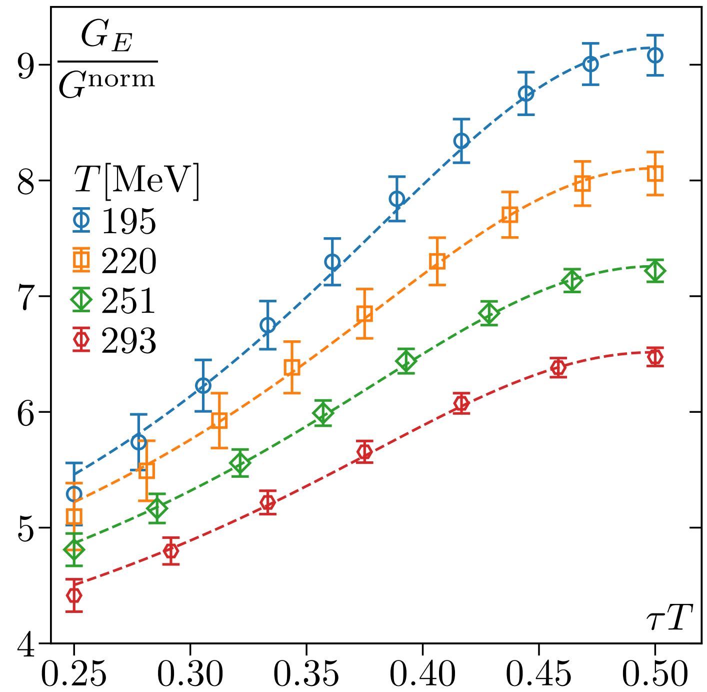


→ well defined continuum and flow time extrapolation

→ well defined renormalized correlation function

Continuum extrapolated color-electric correlation function from

2+1-flavor lattice QCD at four temperatures above T_c



Determine κ in the continuum using various Ansätze for the spectral function $\rho(\omega)$

fitted to the continuum extrapolated correlation functions

$$G(\tau,\vec{p},T) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\rho(\omega,\vec{p},T)K(\tau,\omega,T) \qquad K(\tau,\omega,T) = \frac{\cosh\left(\omega(\tau-\frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

Spectral function models with correct asymptotic behavior

$$\rho_{\rm UV}(\omega) = \frac{g^2(\bar{\mu}_\omega) C_F \omega^3}{6\pi}$$

modeling corrections to ρ_{IR} in various ways

$$\rho_{\rm IR}(\omega) = \frac{\kappa\omega}{2T}$$

Label	$\rho_{\rm model}$	μ	Fit parameters
$\max_{\rm LO}$ $\max_{\rm NLO}$	$\max(\Phi_{\rm IR}, \Phi_{\rm UV})$	$\max(\mu_{\rm eff}, \omega)$ $\max(\mu_{\rm eff}, \mu_{\rm opt})$	$\kappa/T^3, K$
${\rm smax}_{\rm LO}$ ${\rm smax}_{\rm NLO}$	$\sqrt{\Phi_{\rm IR}^2 + \Phi_{\rm UV}^2}$	$\max(\mu_{\rm eff}, \omega)$ $\max(\mu_{\rm eff}, \mu_{\rm opt})$	$\kappa/T^3, K$
${\rm plaw}_{\rm LO}$ ${\rm plaw}_{\rm NLO}$	$\theta(\omega_{\rm IR}-\omega)\Phi_{\rm IR}+$ $\theta(\omega-\omega_{\rm IR})\theta(\omega_{\rm UV}-\omega)p(\omega)+$ $\theta(\omega-\omega_{\rm UV})\Phi_{\rm UV}$	$\max(\mu_{\rm eff}, \omega)$ $\max(\mu_{\rm eff}, \mu_{\rm opt})$	$\kappa/T^3, K$

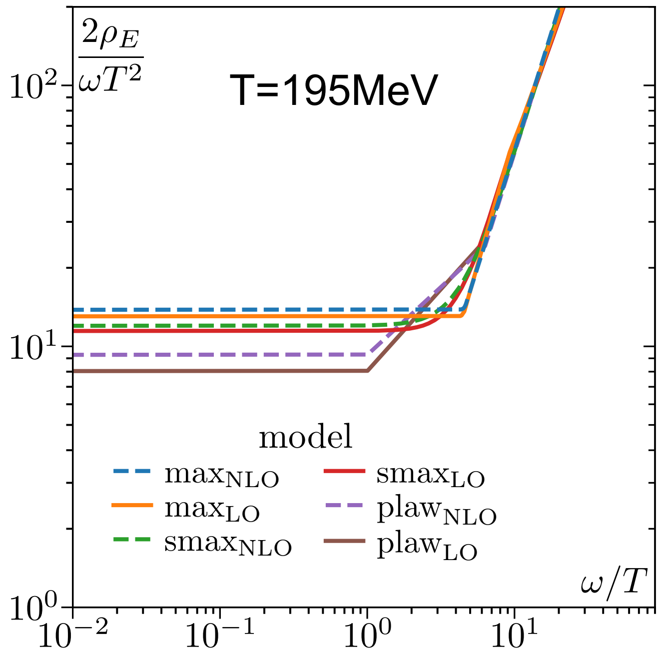
using continuum extrapolated lattice correlators

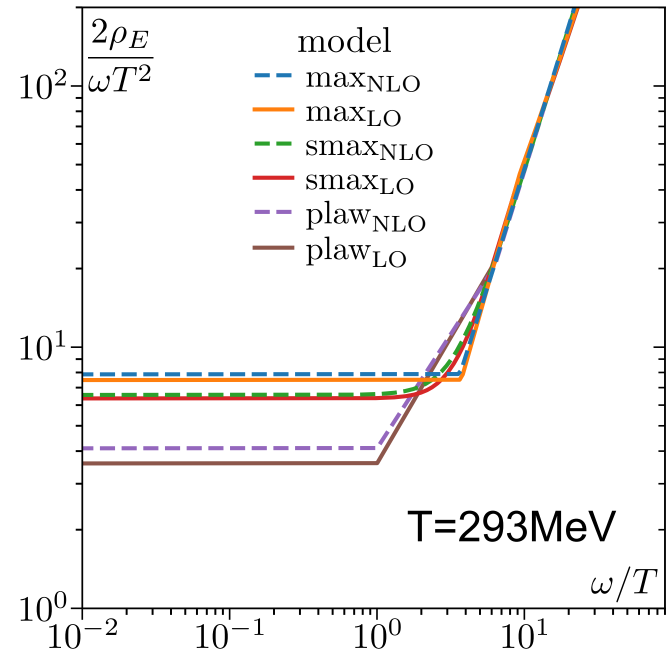
to fit the models and extract κ

$$G_{\rm model}(\tau) \equiv \int_0^\infty \frac{{\rm d}\omega}{\pi} \rho_{\rm model}(\omega) \frac{\cosh\left(\frac{1}{2} - \tau T\right)\frac{\omega}{T}}{\sinh\frac{\omega}{2T}}$$

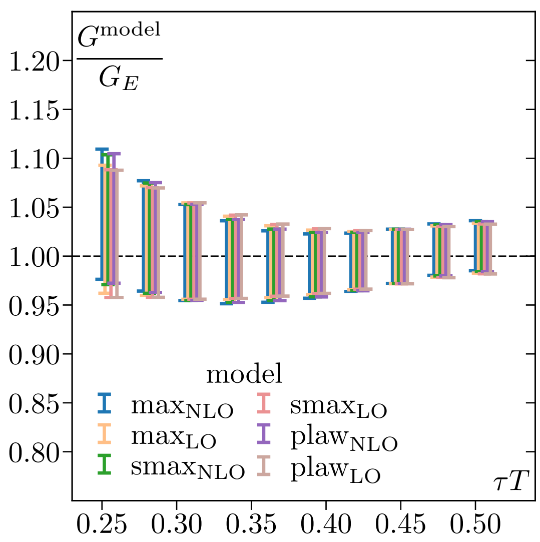
error estimates using fully bootstrapped analysis

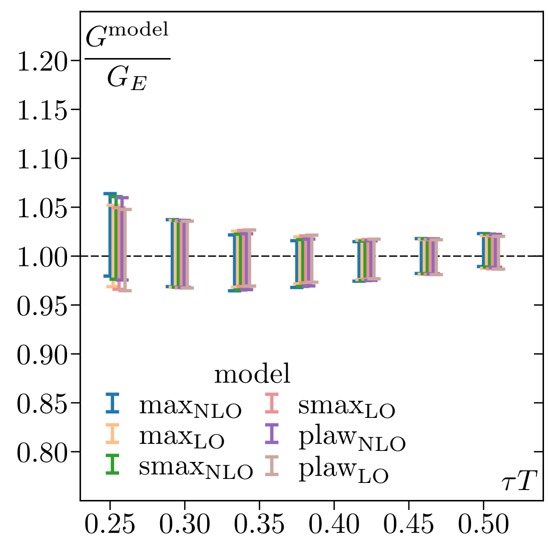
$$\kappa/T^3 = \lim_{\omega\to 0} \frac{2T\rho_{\rm E}(\omega)}{\omega}$$

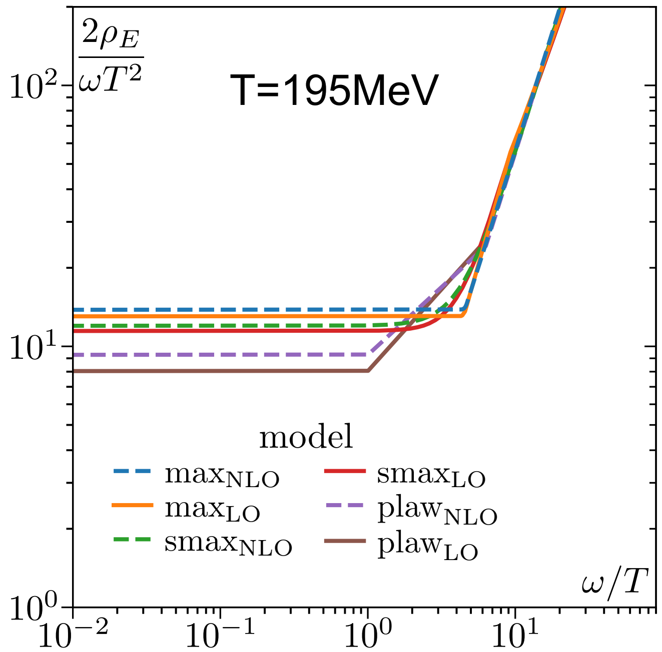


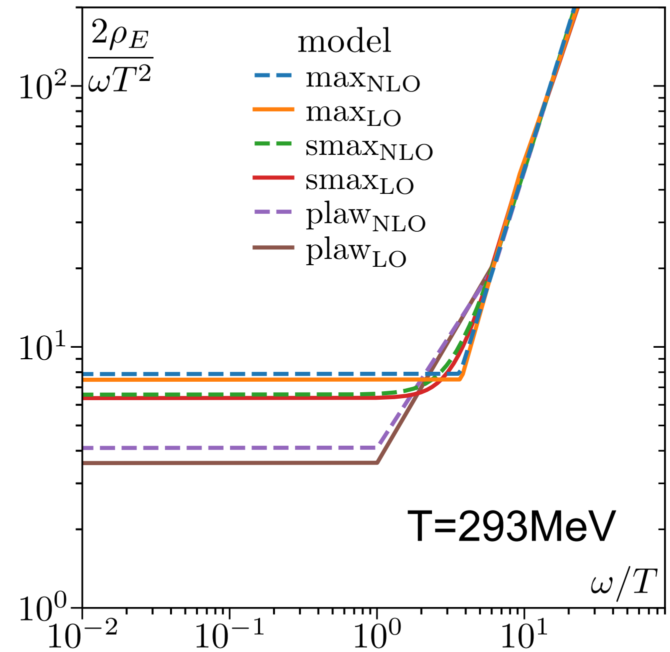


$$G_{\mathrm{model}}(\tau) \equiv \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\rho_{\mathrm{model}}(\omega)\frac{\cosh\left(\frac{1}{2}-\tau T\right)\frac{\omega}{T}}{\sinh\frac{\omega}{2T}}$$

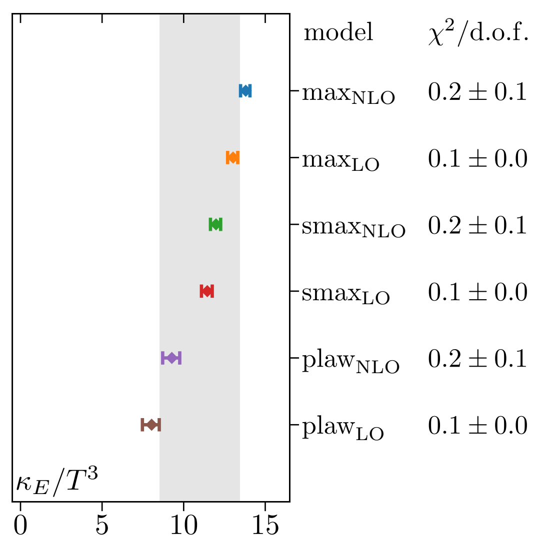


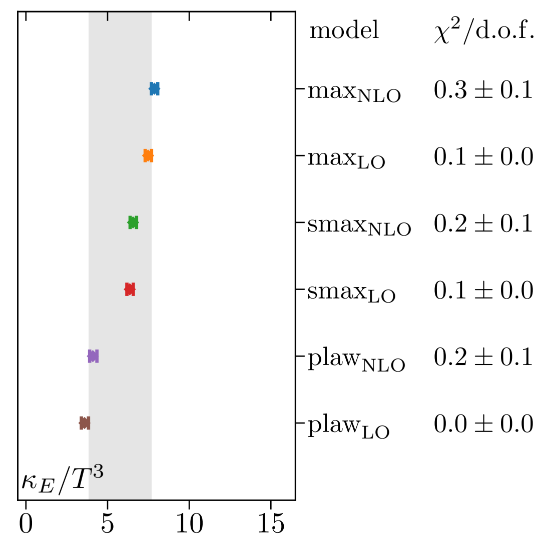






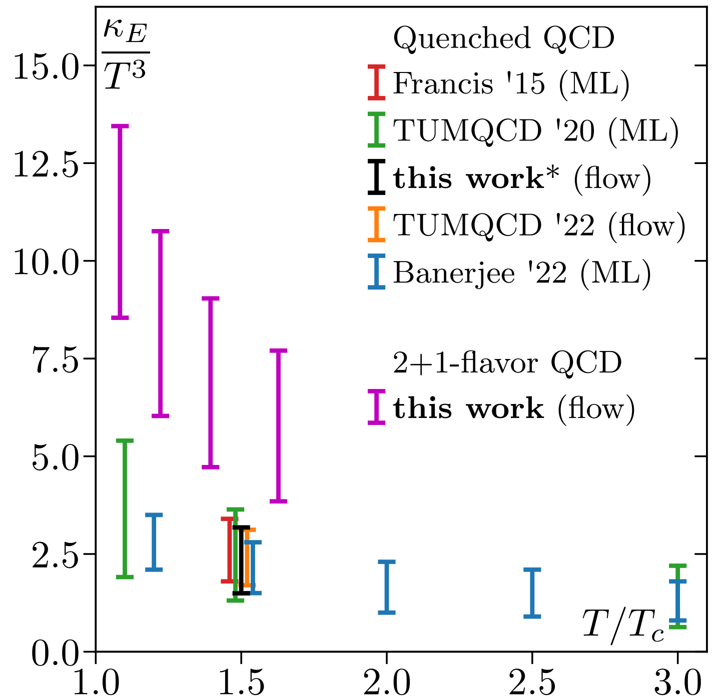
$$\kappa/T^3 = \lim_{\omega\to 0}\frac{2T\rho_{\mathrm{E}}(\omega)}{\omega}$$

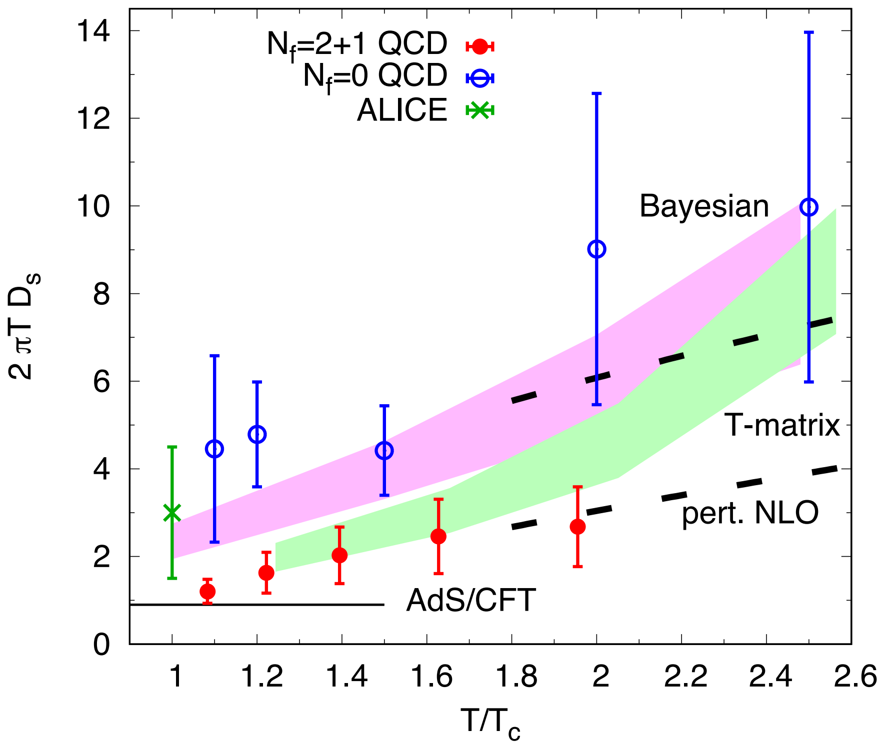




$$\frac{\kappa}{T^3} = \lim_{\omega \to 0} \frac{2T\rho_E(\omega)}{\omega}$$

$$2\pi T D = 4\pi \frac{T^3}{\kappa}$$





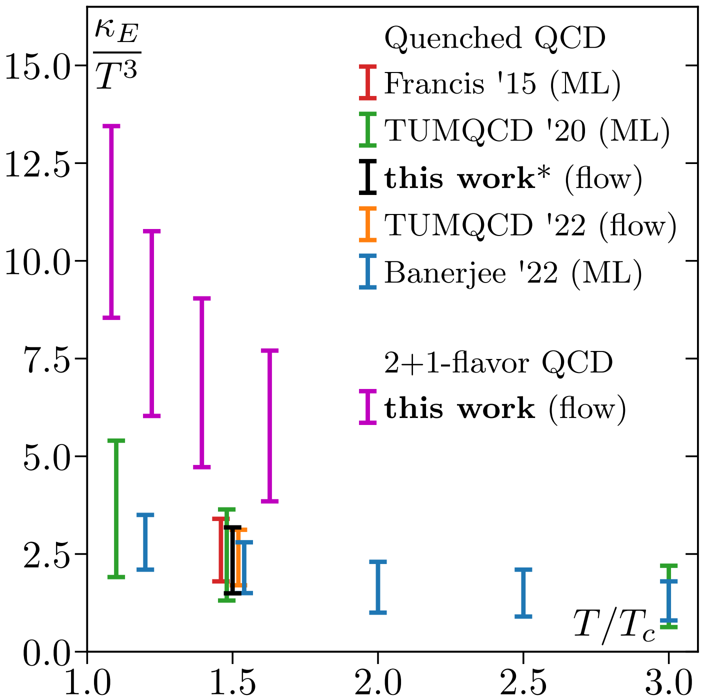
kinetic equilibration time for charm and bottom:

$$\tau_{kin}^{-1} = \eta_D = \frac{\kappa}{2M_{kin}T}\left(1 + \mathcal{O}\left(\frac{\alpha_s^{3/2}T}{M_{kin}}\right)\right)$$

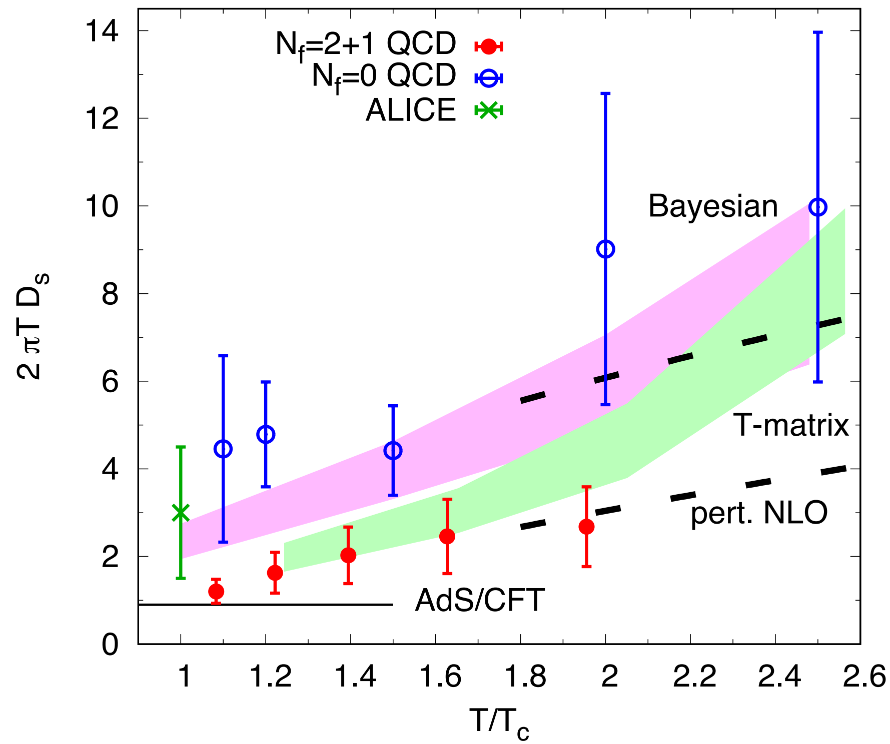
T [MeV]	κ_E/T^3	$2\pi TD$	$\tau_{\rm kin}$ [fm/c] ($M = 1.275$ GeV)	$\tau_{\rm kin}$ [fm/c] ($M = 4.18$ GeV)
195	$8.5\ldots13.4$	$0.9\ldots1.5$	$1.0\ldots1.5$	$3.2\ldots5.1$
220	$6.0\ldots10.8$	$1.2\ldots2.1$	$1.0\ldots1.7$	$3.2\ldots5.7$
251	$4.7\ldots9.0$	$1.4\ldots2.7$	$0.9\ldots1.7$	$2.9\ldots5.5$
293	$3.9\ldots7.7$	$1.6\ldots3.3$	$0.8\ldots1.5$	$2.5\ldots5.0$

Spatial heavy quark diffusion coefficient

$$\frac{\kappa}{T^3} = \lim_{\omega \to 0} \frac{2T\rho_E(\omega)}{\omega}$$



$$2\pi T D = 4\pi \frac{T^3}{\kappa}$$



Next steps:

- determine the quark mass correction: $\kappa \simeq \kappa_E + \frac{2}{3}\langle v^2 \rangle \kappa_B$, $\langle v^2 \rangle \approx \frac{3T}{M_{kin}}\left(1 - \frac{5T}{2M_{kin}}\right)$
- correction may be important for charm

[A. Bouttefeux, M. Laine, HEP 12 (2020) 150]
[M. Laine, JHEP 06 (2021) 139]

- extend to physical 2+1 flavor QCD
- determine charm and bottom quark diffusion coefficient from vector meson correlators