UNIVERSITÄT BIELEFELD

Spectral and Transport properties of the Quark Gluon Plasma from Lattice QCD

Olaf Kaczmarek

University of Bielefeld

[L. Altenkort, OK, R. Larsen, S. Mukherjee, P. Petreczky, H.T. Shu, S. Stendebach,
Heavy Quark Diffusion from 2+1 Flavor Lattice QCD, PRL 130 (2023) 231902]

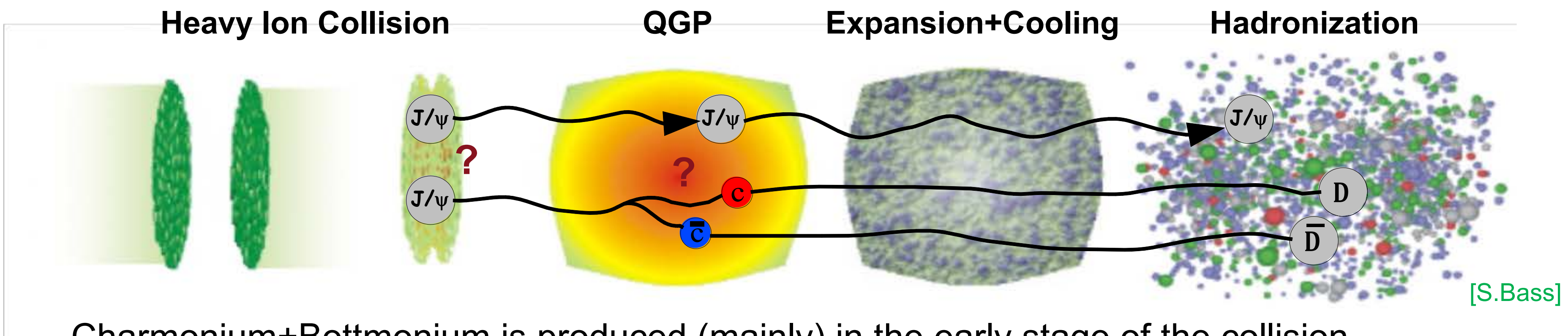
[L. Altenkort, D. de la Cruz, OK, R. Larsen, G.D. Moore, S. Mukherjee, P. Petreczky, H.T. Shu, S. Stendebach
Quark Mass Dependence of Heavy Quark Diffusion Coefficient from Lattice QCD, arXiv:2311.01525]

[L. Altenkort, A.M. Eller, OK, L. Mazur, G.D. Moore,
Heavy quark momentum diffusion from the lattice using gradient flow, PRD103 (2021) 014511]

[A.Francis, OK, M. Laine, T. Neuhaus, H. Ohno,
Nonperturbative estimate of the heavy quark momentum diffusion coefficient, PRD92(2015)116003]

NHR-Computational Physics Symposium 2023
Online, 03.11.2023

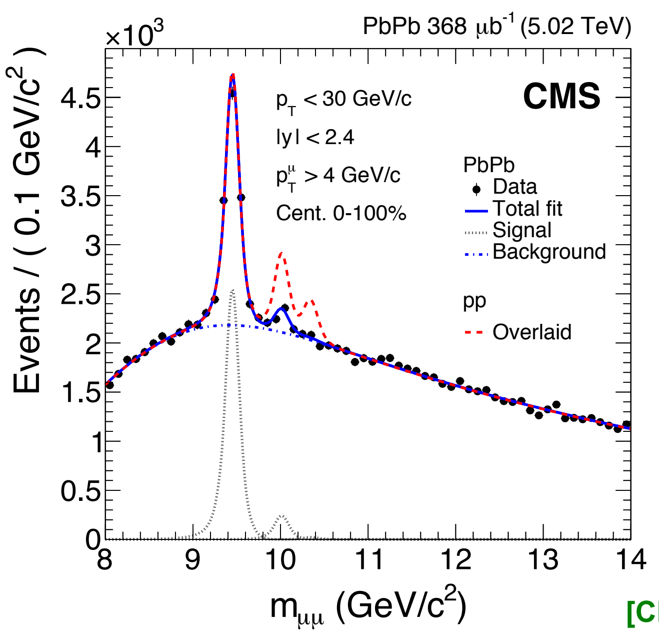
CRC-TR 211
Strong-interaction matter under extreme conditions



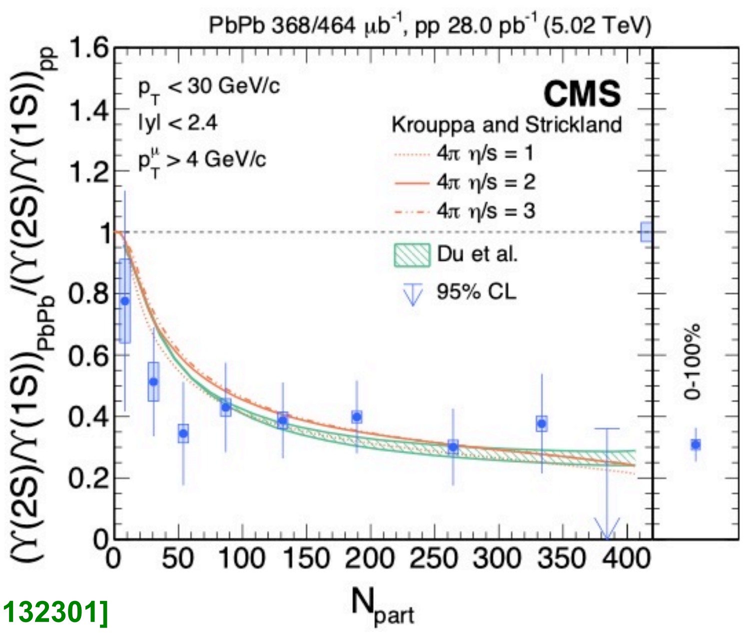
Charmonium+Bottmonium is produced (mainly) in the early stage of the collision

Depending on the Dissociation Temperature

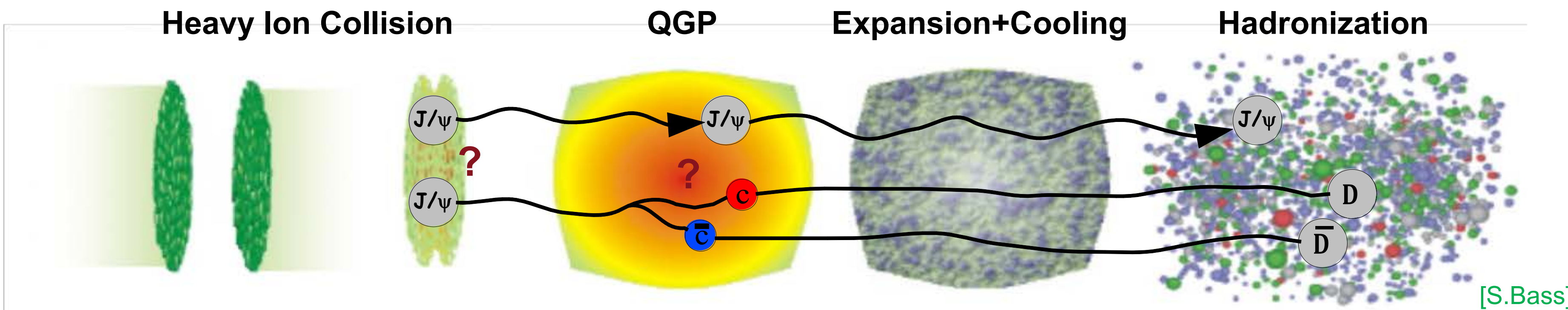
- remain as bound states in the whole evolution
- release their constituents in the plasma



Sequential suppression for bottomonium observed at CMS



[CMS collaboration, PRL120 (2018) 132301]



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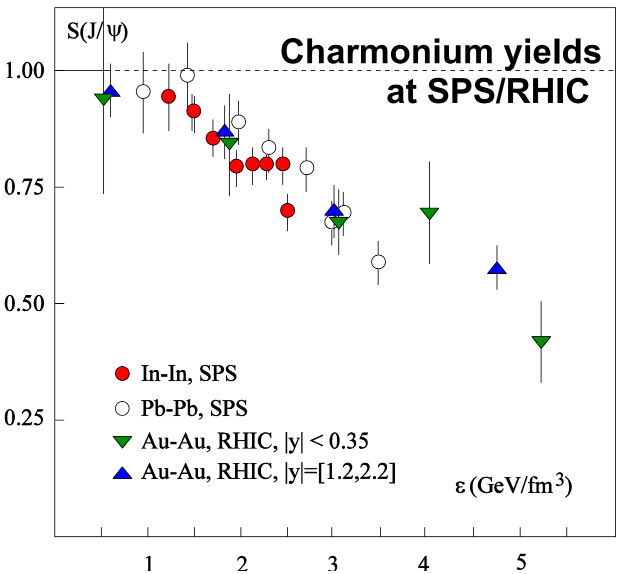
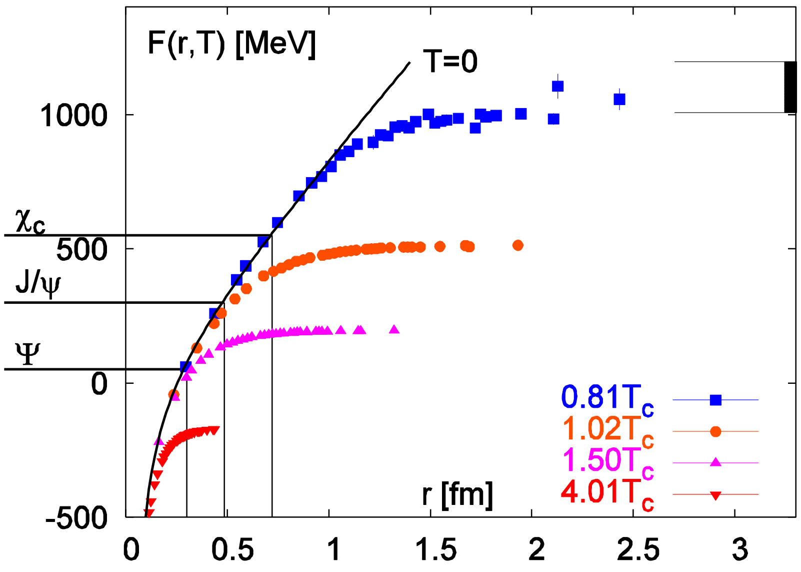
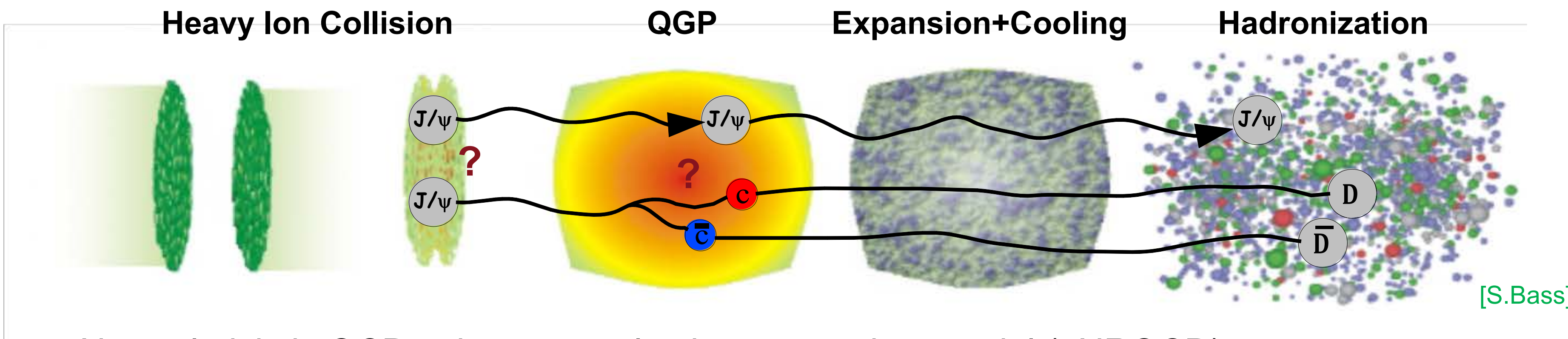
[Kaczmarek, Zantow, 2005]



First estimates on Dissociation Temperatures

from detailed knowledge of Heavy Quark Free Energies and Potential Models

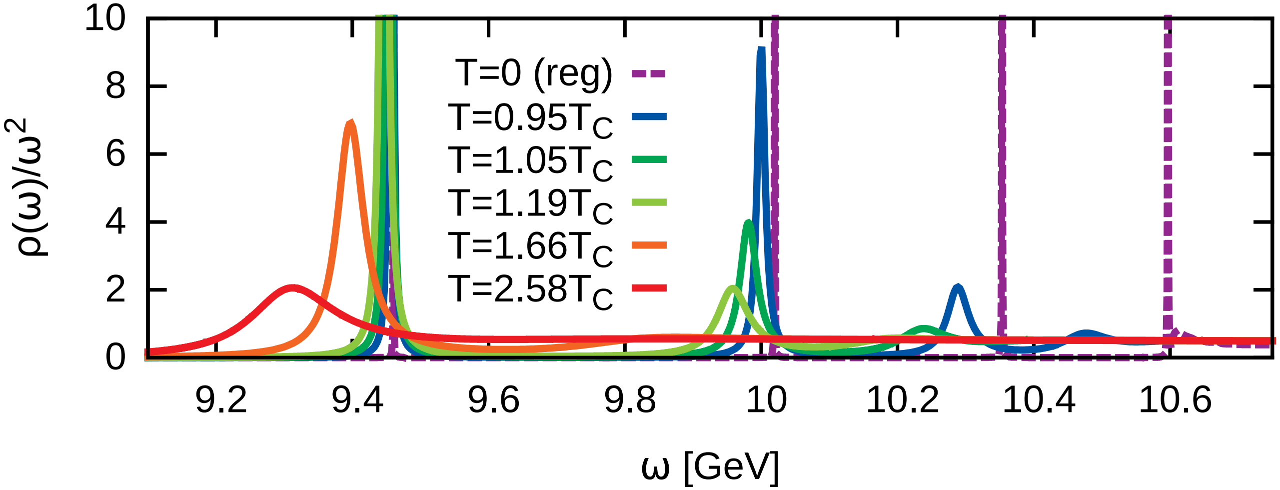




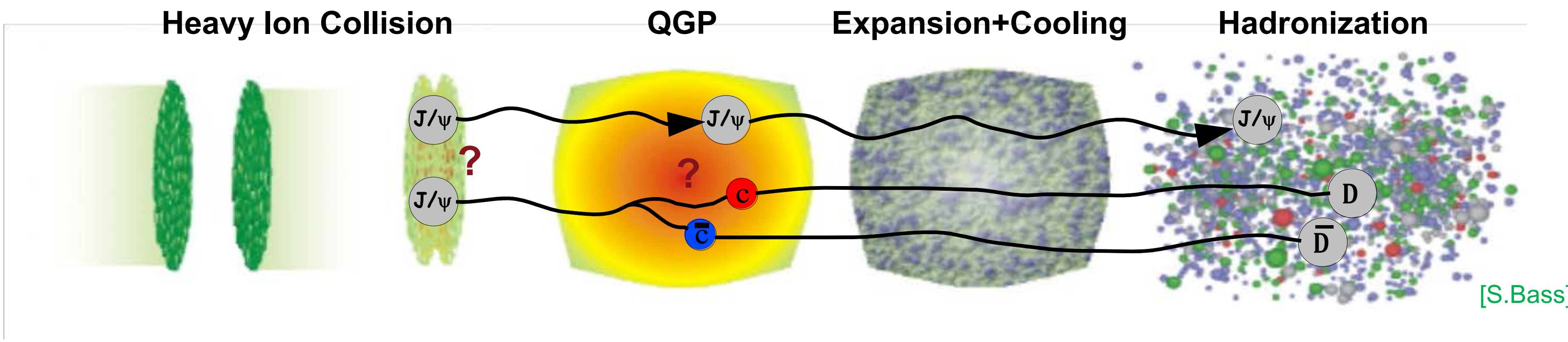
Non-relativistic QCD using a complex heavy quark potential (pNRQCD)

- applicable at least for bottomonium
- shift of bound state masses before the states melt
- thermal broadening of the states due to Im[V]
- sequential dissociation of excited states

[Y. Burnier ,OK, A.Rothkopf, JHEP12(2015)101]



Full relativistic calculations of charmonium and bottomonium difficult, but ongoing…



Light degrees of freedom can rather well be described by hydrodynamics.

How do heavy quarks propagate in the hot and dense medium?

- What is the kinetic equilibration time for heavy quarks?
- Do heavy quarks thermalize and show collective motion?
- What are the transport coefficients of heavy quarks?

Heavy quark diffusion coefficients are crucial ingredients to study these questions

- Can be calculated from current-current (vector meson) correlation functions
- Or in the heavy quark mass limit using EE or BB correlation functions
- Both methods need spectral recontruction methods to obtain spectral functions

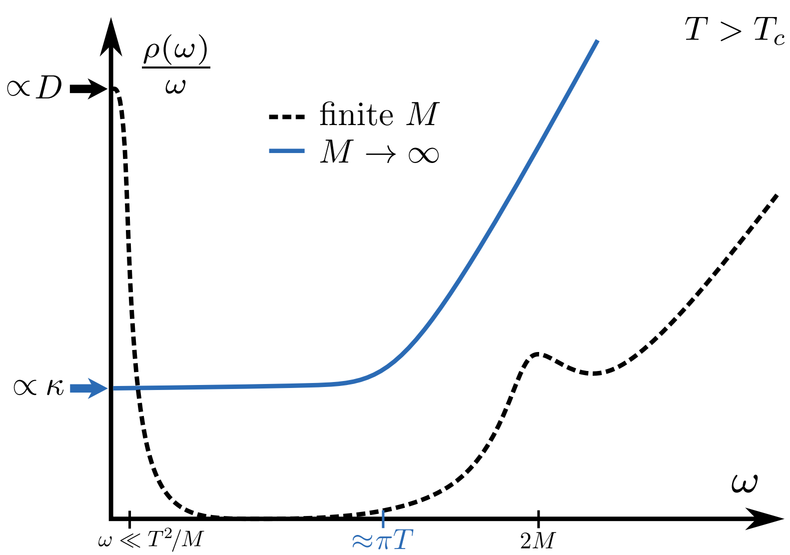
$$G(\tau,\vec{p},T) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\rho(\omega,\vec{p},T)K(\tau,\omega,T)$$

$$K(\tau,\omega,T) = \frac{\cosh\left(\omega(\tau - \frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

Different contributions and scales enter in the spectral function

- continuum at large frequencies
- possible bound states at intermediate frequencies
- transport contributions at small frequencies
- in addition cut-off effects on the lattice

Spectral functions in the QGP



difficult to extract D_s from vector meson correlation fct.

$$G_{\mu\nu}(\tau,\vec{x}) = \langle J_\mu(\tau,\vec{x})J_\nu^\dagger(0,\vec{0})\rangle$$
$$J_\mu(\tau,\vec{x}) = 2\kappa Z_V \bar{\psi}(\tau,\vec{x})\Gamma_\mu\psi(\tau,\vec{x})$$

→ narrow transport peak hard to resolve

→ large lattices and continuum extrapolation needed

→ use perturbation theory to constrain the UV behavior

easier to extract heavy quark momentum diffusion coefficient κ in the heavy quark mass limit

→ smooth $\omega \to 0$ limit expected

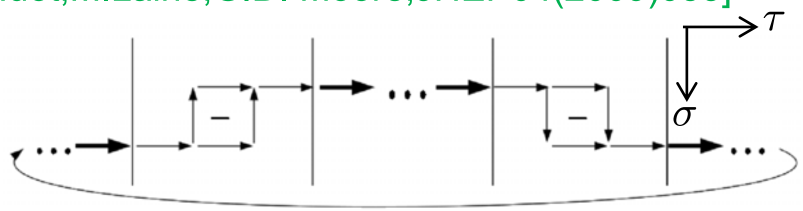
Heavy Quark Effective Theory (HQET) in the large quark mass limit

for a single quark in medium

leads to a (pure gluonic) "color-electric correlator"

[J.Casalderrey-Solana, D.Teaney, PRD74(2006)085012, S.Caron-Huot,M.Laine,G.D. Moore,JHEP04(2009)053]

$$G_E(\tau) \equiv -\frac{1}{3}\sum_{i=1}^{3} \frac{\left\langle \mathrm{Re\,Tr}\left[U(\tfrac{1}{T};\tau)\, g\, E_i(\tau,\mathbf{0})\, U(\tau;0)\, g\, E_i(0,\mathbf{0})\right]\right\rangle}{\left\langle \mathrm{Re\,Tr}[U(\tfrac{1}{T};0)]\right\rangle}$$

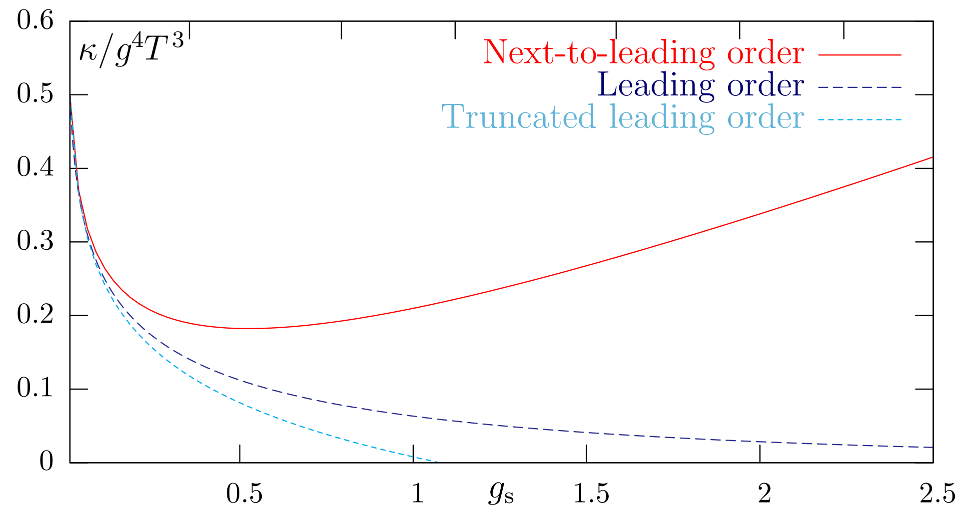


Smooth limit expected from NLO PT
[Caron-Huot, M.Laine, G.Moore, JHEP 0904 (2009) 053]

$$\kappa = \lim_{\omega \to 0} \frac{2T\rho_E(\omega)}{\omega}$$

NLO perturbative calculation:

[Caron-Huot, G. Moore, JHEP 0802 (2008) 081]



→ large correction towards strong interactions

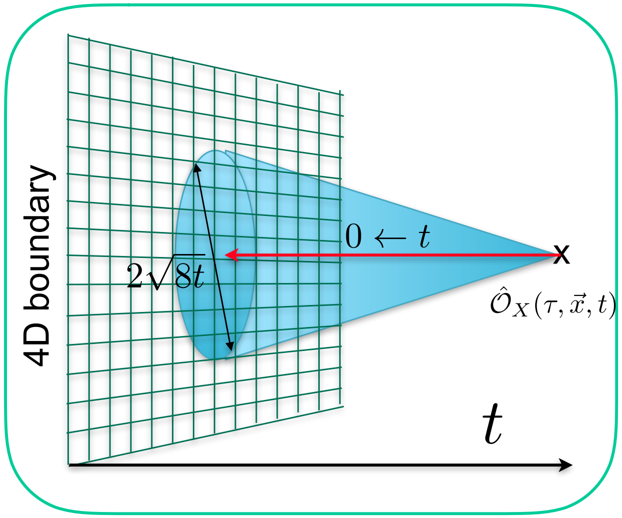
→ non-perturbative lattice methods required

Gradient flow - *diffusion* equation for the gauge fields along extra dimension, *flow-time t*

[M. Lüscher, 2010]

$$\frac{\partial}{\partial t} A_\mu(t,x) = -\frac{\partial S_{\rm YM}}{\partial A_\mu}$$

$$A_\mu(t=0,x) = A_\mu(x)$$



- continuous smearing of the gauge fields, effective smearing radius: $r_{\rm smear} \sim \sqrt{8t}$
- gauge fields become smooth and renormalized
- no UV divergences at finite flow-time *t* → operators of flowed fields are renormalized
- UV fluctuations effectively reduces → noise reduction technique
- applicable in quenched and full QCD
- methods developed in quenched studies now applied in full QCD

What is the flow time dependence of correlation functions?

How to perform the continuum and t→0 limit correctly?

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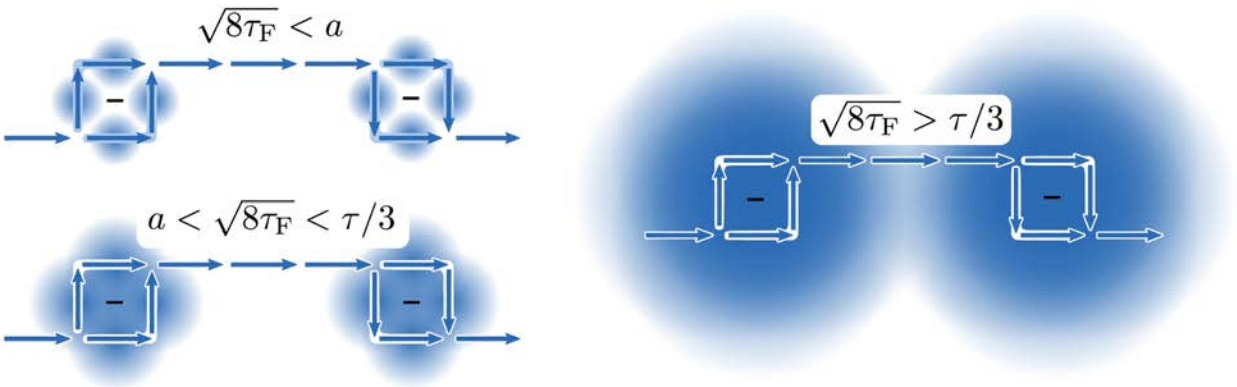

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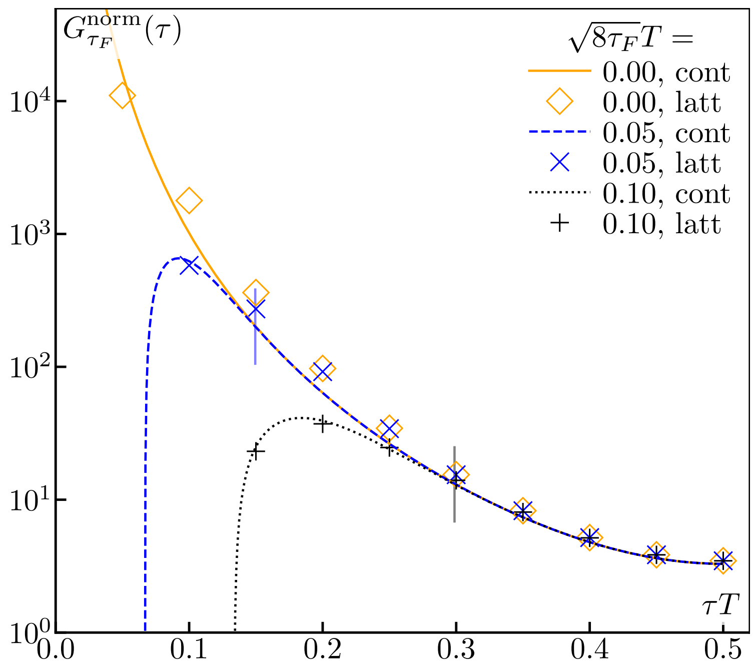
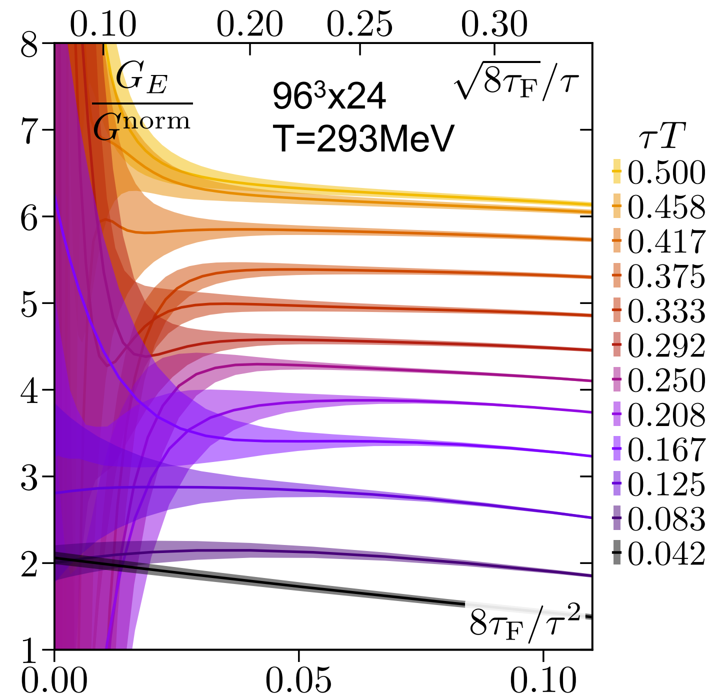
LO perturbative limits for the flow-time dependence:

$$\tilde{\tau}_f < 0.1136(\tau T)^2$$



[A.M Eller, G.D. Moore, PRD97 (2018) 114507]

2+1-flavor lattice QCD results on the flow dependence of the color-electric correlator:



[L. Altenkort, OK, R. Larsen, et al., arXiv:2302.08501]

Effective reduction of UV fluctuations → good noise reduction technique

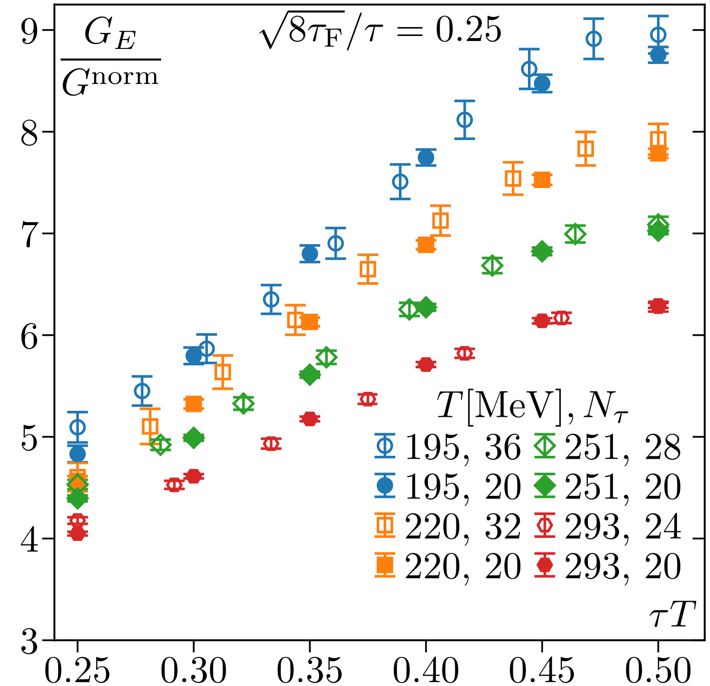
Signal gets destroyed at flow times above the perturbative estimate

Linear behavior at intermediate flow times

2+1-flavor lattice QCD on large and fine isotropic lattices at four temperatures above T_c

- HISQ action with physical strange quark mass and m_s/m_l=5 ($m_\pi \approx 300\ MeV$)
- using gradient flow method to improve the signal

T [MeV]	T/T_c	a [fm]	β	N_σ	N_τ	# conf.
195	1.09	0.0505	7.570	64	20	5899
		0.0421	7.777	64	24	3435
		0.0280	8.249	96	36	2256
220	1.22	0.0449	7.704	64	20	7923
		0.0374	7.913	64	24	2715
		0.0280	8.249	96	32	912
251	1.40	0.0393	7.857	64	20	6786
		0.0327	8.068	64	24	5325
		0.0280	8.249	96	28	1680
293	1.63	0.0336	8.036	64	20	6534
		0.0306	8.147	64	22	9101
		0.0280	8.249	96	24	688



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1) perform the continuum limit, a$\rightarrow$ 0 $\leftrightarrow$ $N_t \rightarrow \infty$

2) perform the flow time to zero limit of the continuum correlators

3) determine κ in the continuum using an Ansatz for the spectral fct. $\rho(\omega)$

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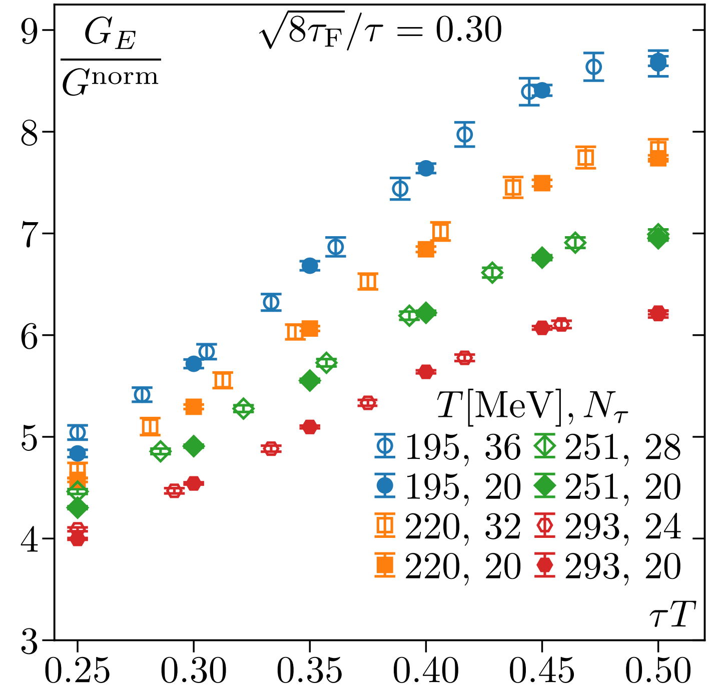

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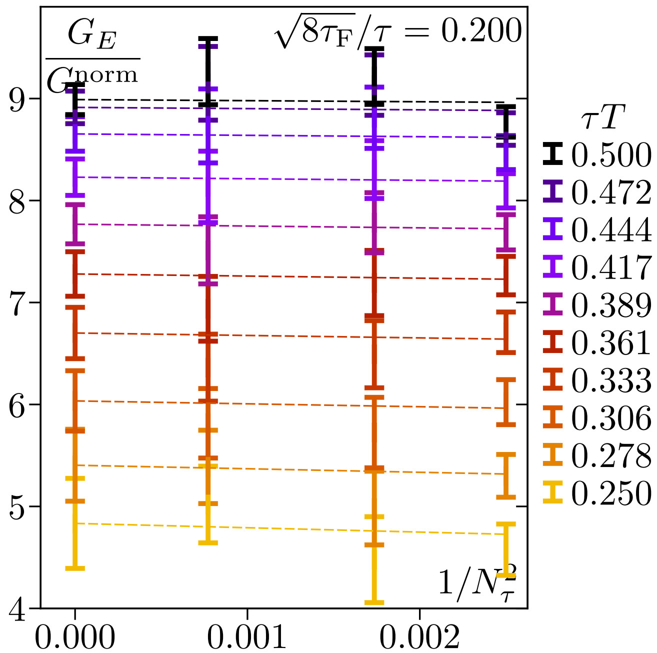
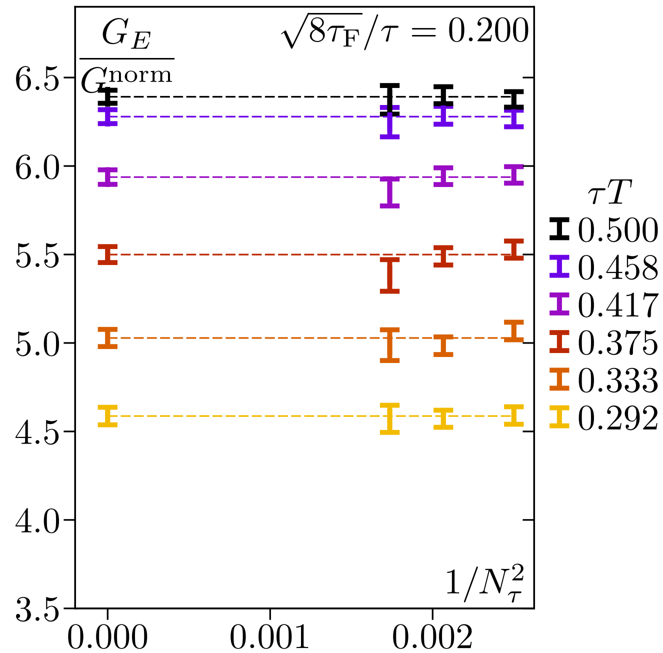
Gradient Flow method – 1) a→0 limit at fixed flow time

- cut-off effects get reduced with increasing flow time
- continuum limit, *a→0 (N_t→ ∞)*, at fixed physical flow time:

T=195MeV N_t=36,24,20



T=293MeV N_t=24,22,20

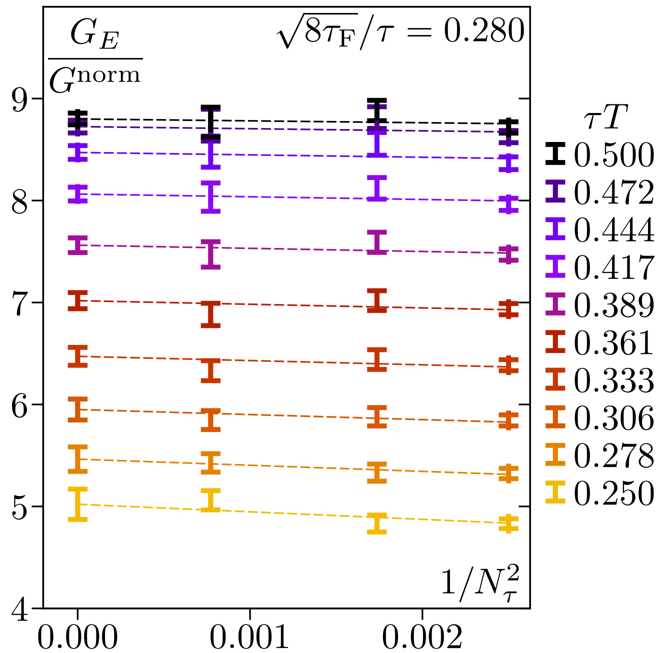


- well defined continuum correlators for different finite flow times

next step: flow time to zero extrapolation of continuum correlators

- cut-off effects get reduced with increasing flow time
- continuum limit, *a→0 (N_t→ ∞),* at fixed physical flow time:

T=195MeV N_t=36,24,20



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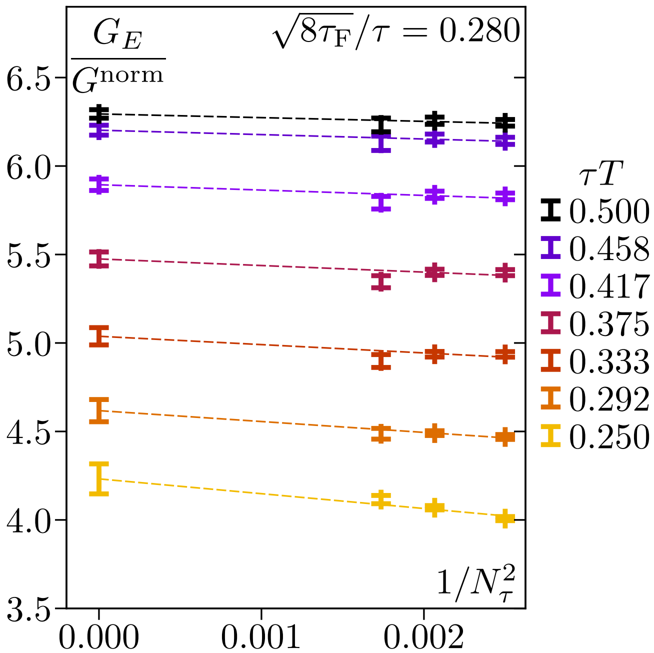


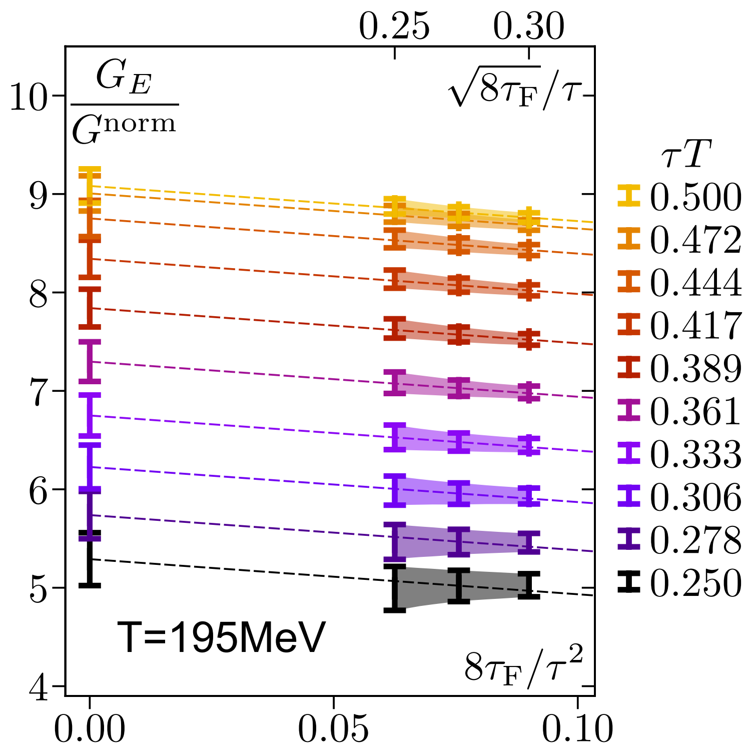
- well defined continuum correlators for different finite flow times

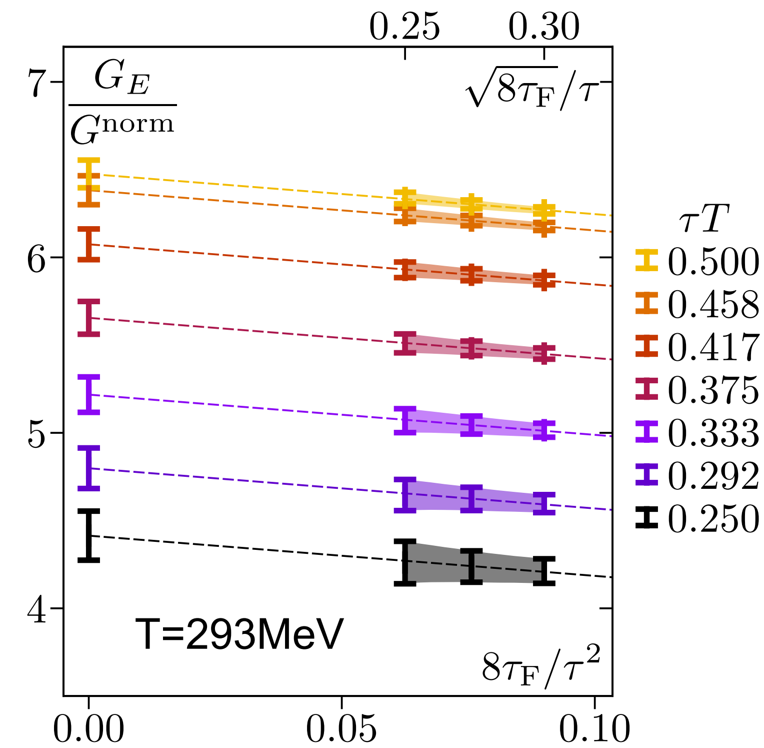
next step: flow time to zero extrapolation of continuum correlators

Continuum limit, *a→0 (N_t→ ∞),* at fixed physical flow time:

followed by

Flow time limit, *t→0,* for each distance:



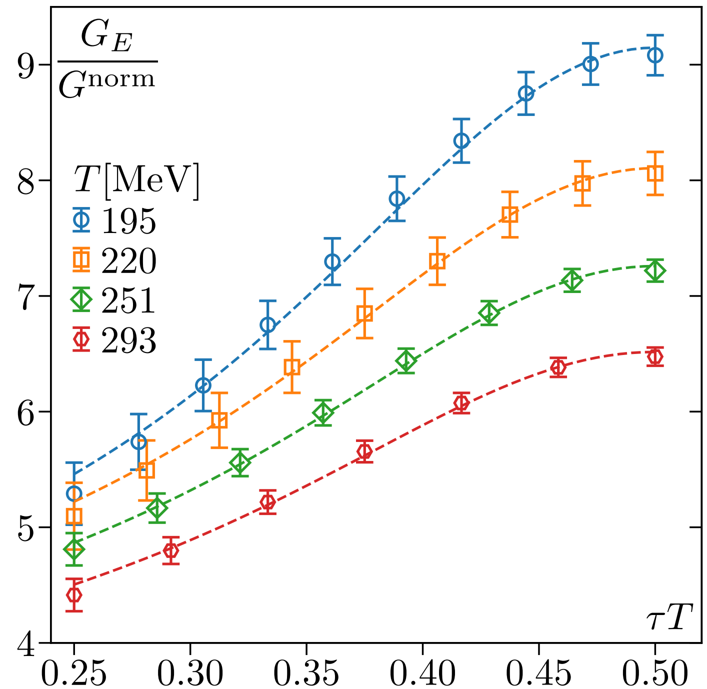


→ well defined continuum and flow time extrapolation

→ well defined renormalized correlation function

Continuum extrapolated color-electric correlation function from

2+1-flavor lattice QCD at four temperatures above T_c



Determine κ in the continuum using various Ansätze for the spectral function $\rho(\omega)$

fitted to the continuum extrapolated correlation functions

$$G(\tau,\vec{p},T) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\rho(\omega,\vec{p},T)K(\tau,\omega,T) \qquad K(\tau,\omega,T) = \frac{\cosh\left(\omega(\tau-\frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

Spectral function models with correct asymptotic behavior

$$\rho_{\rm UV}(\omega) = \frac{g^2(\bar{\mu}_\omega) C_F \omega^3}{6\pi}$$

modeling corrections to ρ_{IR} in various ways

$$\rho_{\rm IR}(\omega) = \frac{\kappa\omega}{2T}$$

Label	$\rho_{\rm model}$	μ	Fit parameters
$\max_{\rm LO}$ $\max_{\rm NLO}$	$\max(\Phi_{\rm IR}, \Phi_{\rm UV})$	$\max(\mu_{\rm eff}, \omega)$ $\max(\mu_{\rm eff}, \mu_{\rm opt})$	$\kappa/T^3, K$
$\mathrm{smax}_{\rm LO}$ $\mathrm{smax}_{\rm NLO}$	$\sqrt{\Phi_{\rm IR}^2 + \Phi_{\rm UV}^2}$	$\max(\mu_{\rm eff}, \omega)$ $\max(\mu_{\rm eff}, \mu_{\rm opt})$	$\kappa/T^3, K$
$\mathrm{plaw}_{\rm LO}$ $\mathrm{plaw}_{\rm NLO}$	$\theta(\omega_{\rm IR}-\omega)\Phi_{\rm IR}+$ $\theta(\omega-\omega_{\rm IR})\theta(\omega_{\rm UV}-\omega)p(\omega)+$ $\theta(\omega-\omega_{\rm UV})\Phi_{\rm UV}$	$\max(\mu_{\rm eff}, \omega)$ $\max(\mu_{\rm eff}, \mu_{\rm opt})$	$\kappa/T^3, K$

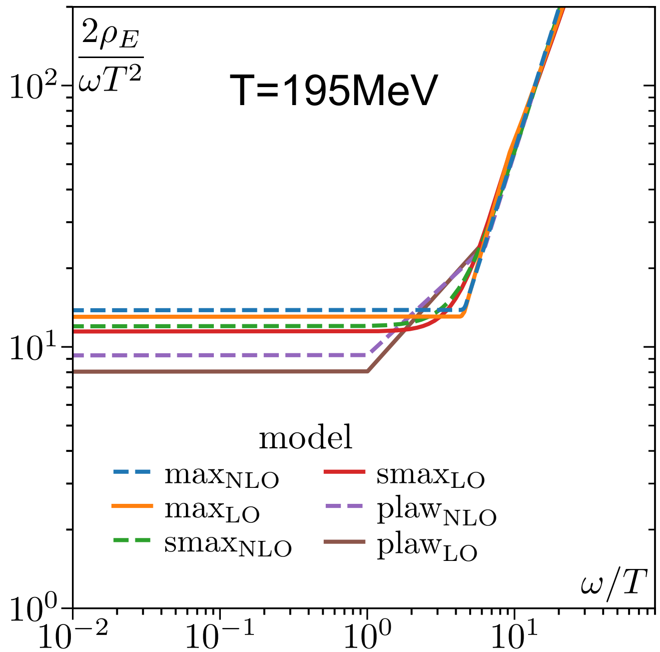
using continuum extrapolated lattice correlators

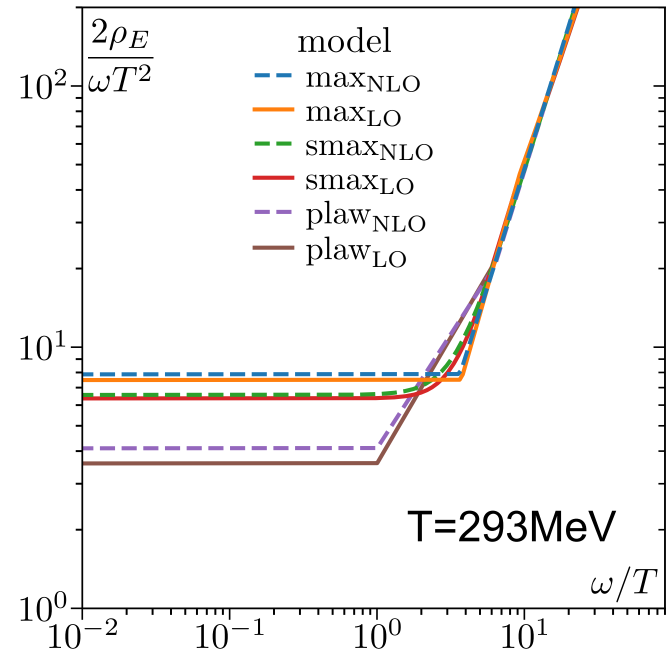
to fit the models and extract κ

$$G_{\rm model}(\tau) \equiv \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \rho_{\rm model}(\omega) \frac{\cosh\left(\frac{1}{2} - \tau T\right)\frac{\omega}{T}}{\sinh\frac{\omega}{2T}}$$

error estimates using fully bootstrapped analysis

$$\kappa/T^3 = \lim_{\omega\to 0} \frac{2T\rho_{\rm E}(\omega)}{\omega}$$





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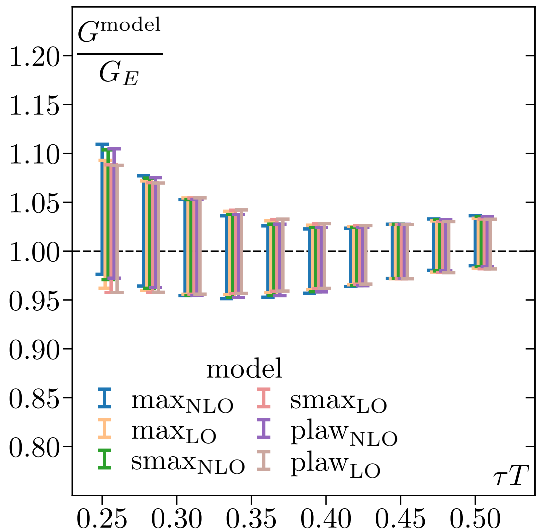
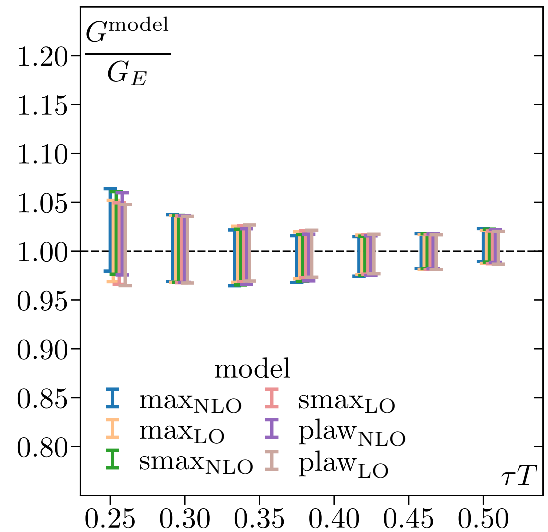
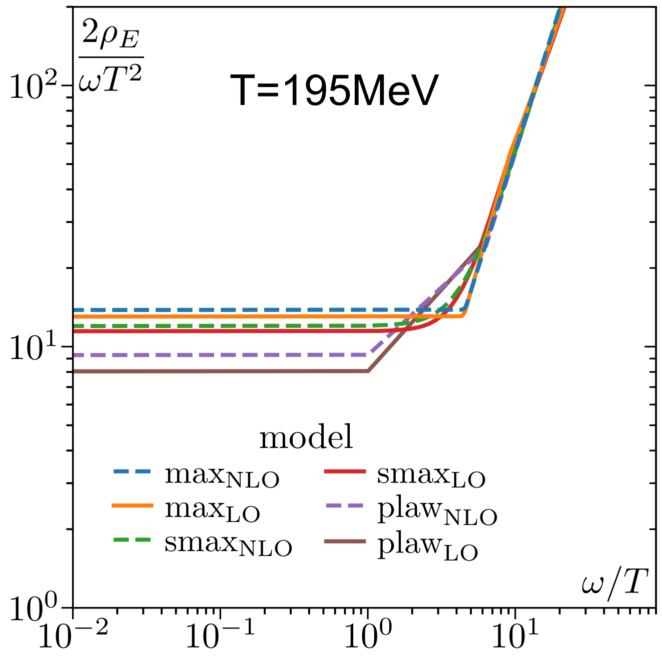
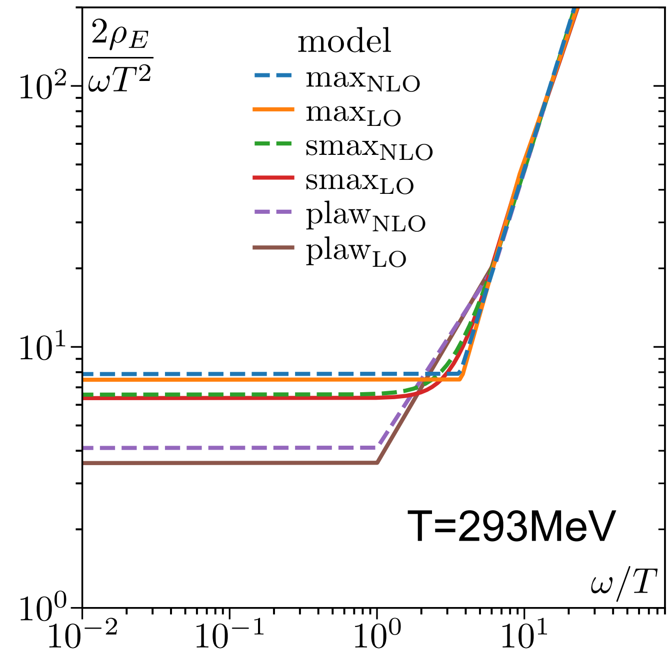
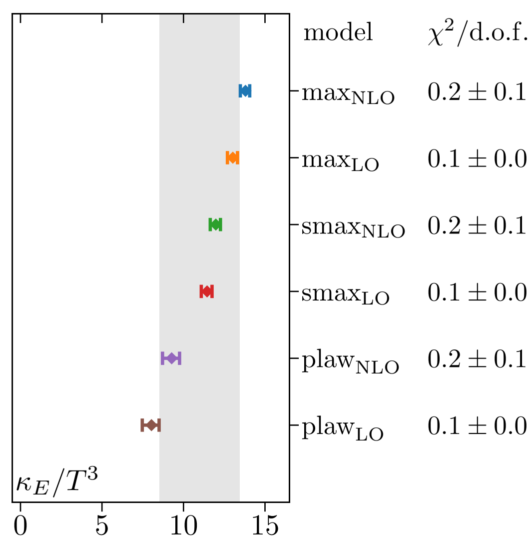


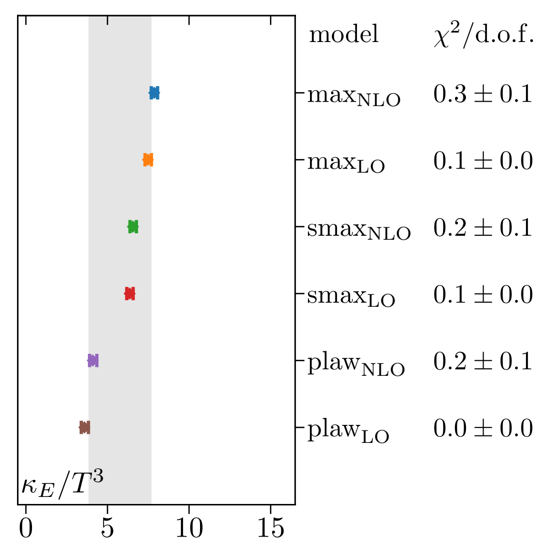






$$\kappa/T^3 = \lim_{\omega\to 0} \frac{2T\rho_E(\omega)}{\omega}$$





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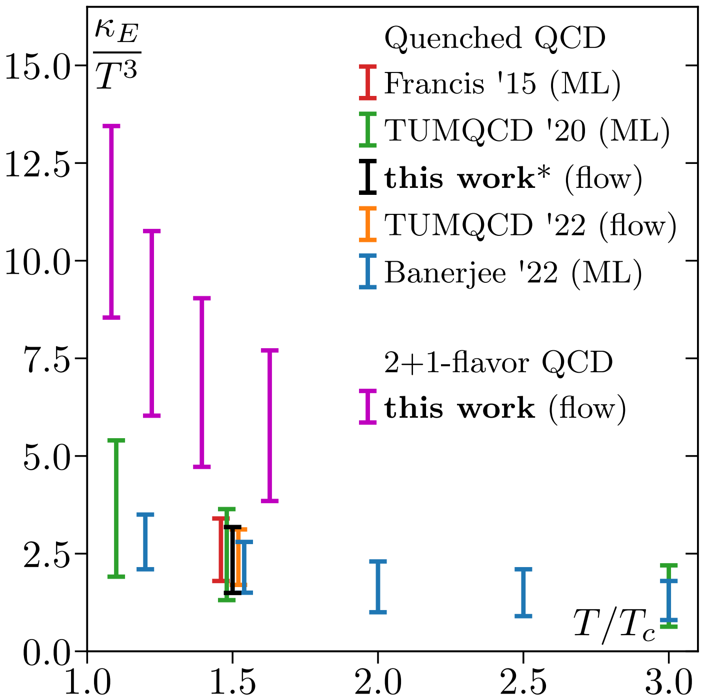
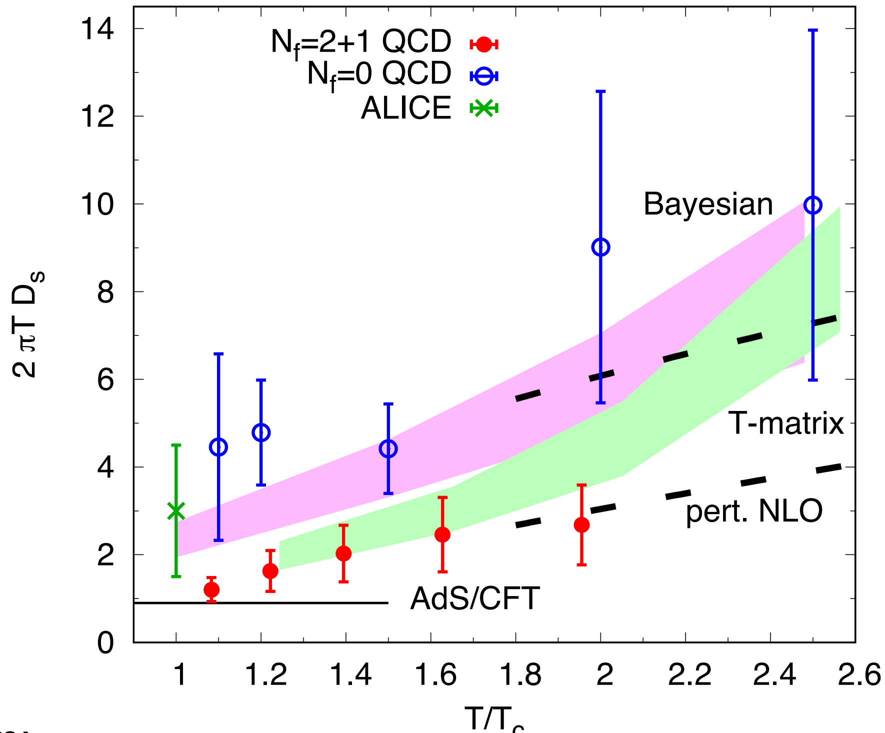
$$\frac{\kappa}{T^3} = \lim_{\omega \to 0} \frac{2T\rho_E(\omega)}{\omega}$$

$$2\pi T D = 4\pi \frac{T^3}{\kappa}$$





kinetic equilibration time for charm and bottom:

$$\tau_{kin}^{-1} = \eta_D = \frac{\kappa}{2M_{kin}T}\left(1 + \mathcal{O}\left(\frac{\alpha_s^{3/2}T}{M_{kin}}\right)\right)$$

T [MeV]	κ_E/T^3	$2\pi TD$	$\tau_{\rm kin}$ [fm/c] ($M = 1.275$ GeV)	$\tau_{\rm kin}$ [fm/c] ($M = 4.18$ GeV)
195	8.5 . . . 13.4	0.9 . . . 1.5	1.0 . . . 1.5	3.2 . . . 5.1
220	6.0 . . . 10.8	1.2 . . . 2.1	1.0 . . . 1.7	3.2 . . . 5.7
251	4.7 . . . 9.0	1.4 . . . 2.7	0.9 . . . 1.7	2.9 . . . 5.5
293	3.9 . . . 7.7	1.6 . . . 3.3	0.8 . . . 1.5	2.5 . . . 5.0

close to T_c charm quark kinetic equilibration appears to be almost as fast as that of light partons.

[L. Altenkort, OK, R. Larsen, S. Mukherjee, P. Petreczky, H.T. Shu, S. Stendebach, Heavy Quark Diffusion from 2+1 Flavor Lattice QCD, PRL 130 (2023) 231902]

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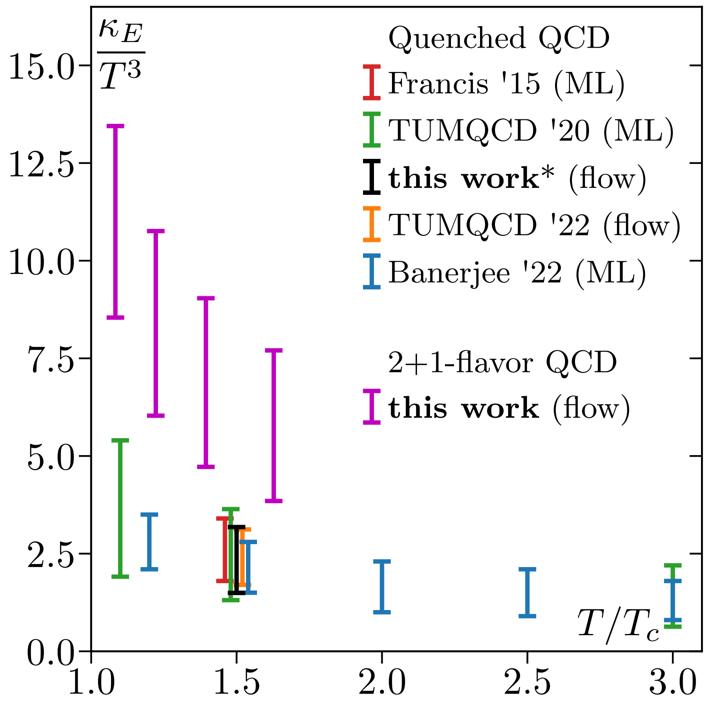
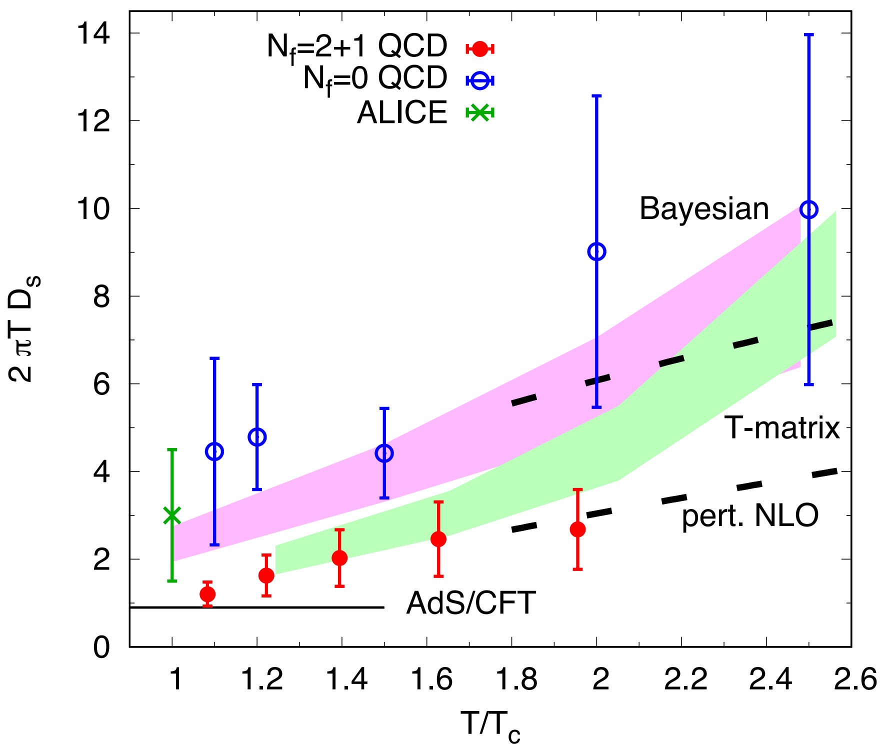


$$2\pi T D = 4\pi \frac{T^3}{\kappa}$$



Next steps:

- determine the quark mass correction: $\kappa \simeq \kappa_E + \frac{2}{3}\langle v^2 \rangle \kappa_B$, $\langle v^2 \rangle \approx \frac{3T}{M_{kin}}\left(1 - \frac{5T}{2M_{kin}}\right)$
- correction may be important for charm
- extend to physical 2+1 flavor QCD
- determine charm and bottom quark diffusion coefficient from vector meson correlators

[A. Bouttefeux, M. Laine, HEP 12 (2020) 150]
[M. Laine, JHEP 06 (2021) 139]

Gauge field configuration generation on GPU supercomputers

previous project: 81 TB gauge field configurations

Generated on supercomputing resources
Perlmutter, JUWELS, Marconi

$96^3 \mathrm{x} N_\tau$ lattice

N_τ	36	32	28	24	20
T [MeV]	195	220	251	293	352
# conf.	2256	912	1680	688	2488

~55.000 gauge field configurations with m_π= 320 *MeV*

$64^3 \mathrm{x} N_\tau$ lattices

T [MeV]	β	am_s	am_l	N_τ	# conf.
195	7.570	0.01973	0.003946	20	5899
	7.777	0.01601	0.003202	24	3435
220	7.704	0.01723	0.003446	20	7923
	7.913	0.01400	0.002800	24	2715
251	7.857	0.01479	0.002958	20	6786
	8.068	0.01204	0.002408	24	5325
293	8.036	0.01241	0.002482	20	6534
	8.147	0.01115	0.002230	22	9101

current project: ~200 TB gauge field configurations

$128^3 \mathrm{x} N_\tau$ and $96^3 \mathrm{x} N_\tau$ lattices with physical pion masses
compute projects on Frontier and LUMI-G
~ 8 Mio GPU-hours for one year

All gauge field configurations will be stored in the International Lattice Data Grid (ILDG)

Measurement of observables on GPU HPC systems

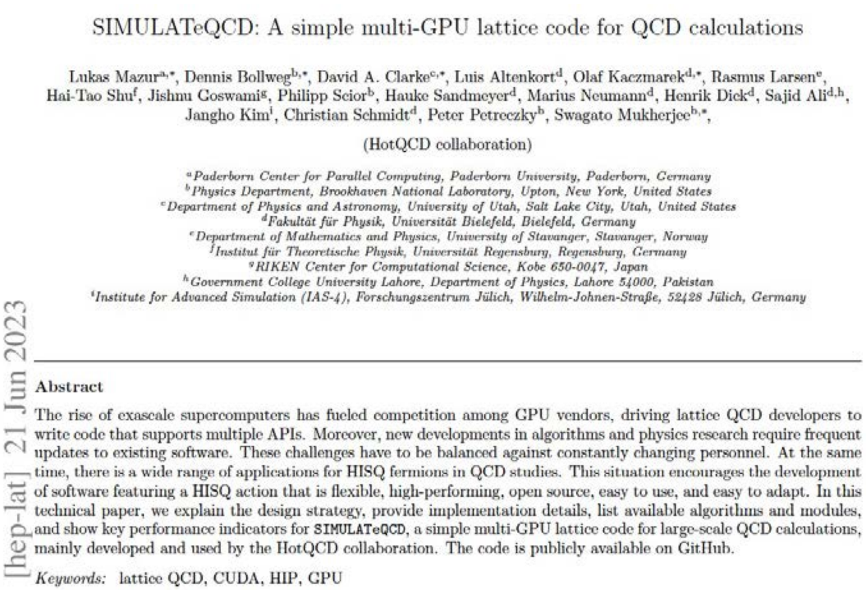
Operators and correlation functions
need to be calculated on each gauge field configuration

Needs optimized multi-GPU code
measurement routines in SIMULATeQCD

Measurement of correlation functions on
Bielefeld GPU Cluster

Measurement of fluctuations and correlations
of charm and conserved charges on
Noctua 2

https://github.com/LatticeQCD/SIMULATeQCD
https://doi.org/10.5281/zenodo.7994982
https://arxiv.org/abs/2306.01098



SIMULATeQCD: A simple multi-GPU lattice code for QCD calculations

Lukas Mazur[a,*], Dennis Bollweg[b,*], David A. Clarke[c,*], Luis Altenkort[d], Olaf Kaczmarek[d,*], Rasmus Larsen[e], Hai-Tao Shu[f], Jishnu Goswami[g], Philipp Scior[b], Hauke Sandmeyer[d], Marius Neumann[d], Henrik Dick[d], Sajid Ali[d,h], Jangho Kim[i], Christian Schmidt[d], Peter Petreczky[b], Swagato Mukherjee[b,*],

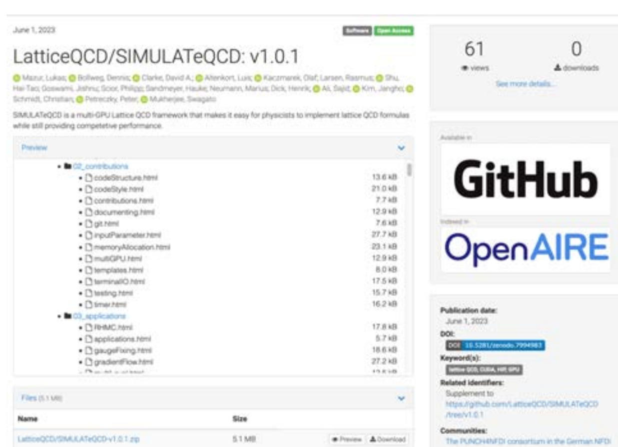
(HotQCD collaboration)

[a]Paderborn Center for Parallel Computing, Paderborn University, Paderborn, Germany
[b]Physics Department, Brookhaven National Laboratory, Upton, New York, United States
[c]Department of Physics and Astronomy, University of Utah, Salt Lake City, Utah, United States
[d]Fakultät für Physik, Universität Bielefeld, Bielefeld, Germany
[e]Department of Mathematics and Physics, University of Stavanger, Stavanger, Norway
[f]Institut für Theoretische Physik, Universität Regensburg, Regensburg, Germany
[g]RIKEN Center for Computational Science, Kobe 650-0047, Japan
[h]Government College University Lahore, Department of Physics, Lahore 54000, Pakistan
[i]Institute for Advanced Simulation (IAS-4), Forschungszentrum Jülich, Wilhelm-Johnen-Straße, 52428 Jülich, Germany

Abstract

The rise of exascale supercomputers has fueled competition among GPU vendors, driving lattice QCD developers to write code that supports multiple APIs. Moreover, new developments in algorithms and physics research require frequent updates to existing software. These challenges have to be balanced against constantly changing personnel. At the same time, there is a wide range of applications for HISQ fermions in QCD studies. This situation encourages the development of software featuring a HISQ action that is flexible, high-performing, open source, easy to use, and easy to adapt. In this technical paper, we explain the design strategy, provide implementation details, list available algorithms and modules, and show key performance indicators for SIMULATeQCD, a simple multi-GPU lattice code for large-scale QCD calculations, mainly developed and used by the HotQCD collaboration. The code is publicly available on GitHub.

Keywords: lattice QCD, CUDA, HIP, GPU



- Developed by HotQCD collaboration (Bielefeld, Brookhaven,…)

- Highly optimized lattice QCD code for multi-GPU (Nvidia and AMD GPUs)

- Optimized for supercomputing resources

- Currently used on Frontier, LUMI-G, Leonardo, Summit, Perlmutter, JUWELS, Noctua2,…

- SIMULATeQCD selected for EuroHPC JU extraordinary support program (ESP) (with AMD and HPE for LUMI-G)

Data analysis and data publication

All analysis performed on Bielefeld compute server

All data and lattice and analysis software as well as a workflow (bash/python) of the project published as open access

https://doi.org/10.4119/unibi/2979080

Dataset for "Heavy Quark Diffusion from 2+1 Flavor Lattice QCD with 320 MeV Pion Mass"

Kaczmarek O, Altenkort L, Larsen R, Mukherjee S, Petreczky P, Shu H-T, Stendebach S (2023)
Bielefeld University.

Datenpublikation

Download
README.sh 12.89 KB
data_pub_altenkort_2023_complete.tar.gz 3.17 GB
figures.tar.gz 15.71 MB
+ **Alle**

DOI **https://doi.org/10.4119/unibi/2979080**

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Volltext(e)

Name	**README.sh** 12.89 KB
Access Level	Open Access
Zuletzt Hochgeladen	2023-05-22T13:40:31Z
MD5 Prüfsumme	f33ce74ad32f2653c49403efd2ff74c1
Name	**data_pub_altenkort_2023_complete.tar.gz** 3.17 GB
Access Level	Open Access

Export
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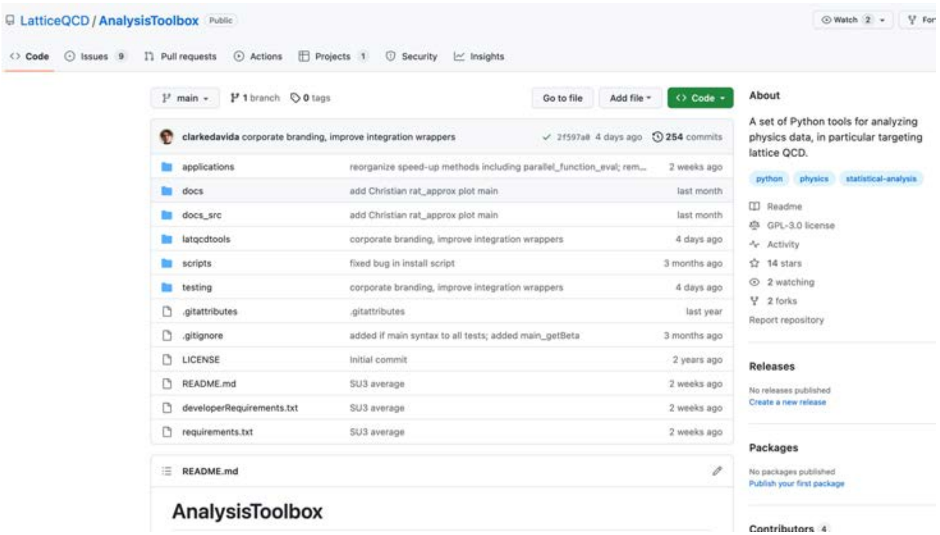
Suchen in
Google Scholar

All raw and derived data is already openly available

gauge field configurations will be published soon on ILDG

All data and analysis software of this project is openly available



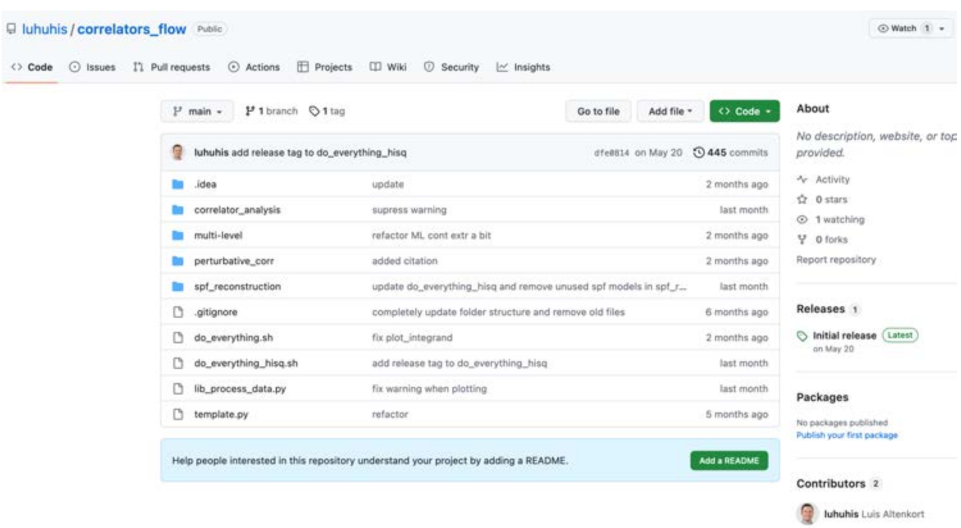
Analysis Software developments

Analysis Toolbox Software development

https://github.com/LatticeQCD/AnalysisToolbox

Heavy quark diffusion analysis based on this

https://github.com/luhuhis/correlators_flow



[A. Bouttefeux, M. Laine, HEP 12 (2020) 150]
[M. Laine, JHEP 06 (2021) 139]

In leading order in 1/M the quark mass dependence of κ depends on another transport coefficient, κ_B,

$$\kappa \simeq \kappa_E + \frac{2}{3}\langle v^2 \rangle \kappa_B$$

$$\langle v^2 \rangle \approx \frac{3T}{M_{kin}}\left(1 - \frac{5T}{2M_{kin}}\right)$$

κ_B can be determined from the color-magnetic correlator

$$G_B(\tau, T) = \sum_{i=1}^{3} \frac{\langle \mathrm{ReTr}\left[U(\beta,\tau)B_i(\mathbf{x},\tau)U(\tau,0)B_i(\mathbf{x},0)\right]\rangle}{3\,\langle \mathrm{ReTr}U(\beta,0)\rangle}$$

and the corresponding spectral function

$$G_B(\tau, T) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\, \rho_B(\omega, T) \frac{\cosh[\omega\tau - \omega/(2T))]}{\sinh[\omega/(2T)]}$$

Problem: In contrast to G_E, G_B has a non-trivial anomalous dimension and the renormalization and continuum extrapolation is more involved.

[L. Altenkort, D. de la Cruz, OK, et al., arXiv:2311:01525]

- Gradient flow serves as a non-perturbative renormalization scheme and the continuum extrapolated correlators are renormalized at the scale $\mu_F = 1/\sqrt{8\tau_F}$
- The renormalization group invariant physical correlators can be obtained via one-loop pQCD matching

$$G_B^{\mathrm{phys.}}(\tau, T) = \lim_{\tau_F \to 0} Z_{\mathrm{match}}(\bar{\mu}_T, \bar{\mu}_{\tau_F}, \mu_F)\, G_B(\tau, T, \tau_F)$$

- This involves three components:
    - matching from gradient flow to $\overline{MS}$ scheme at a scale $\bar{\mu}_{\tau_F}$
    - matching from $\overline{MS}$ to the heavy quark effective theory at a scale $\bar{\mu}_T$
    - running of the anomalous dimension of the operator from $\bar{\mu}_T$ to $\bar{\mu}_{\tau_F}$

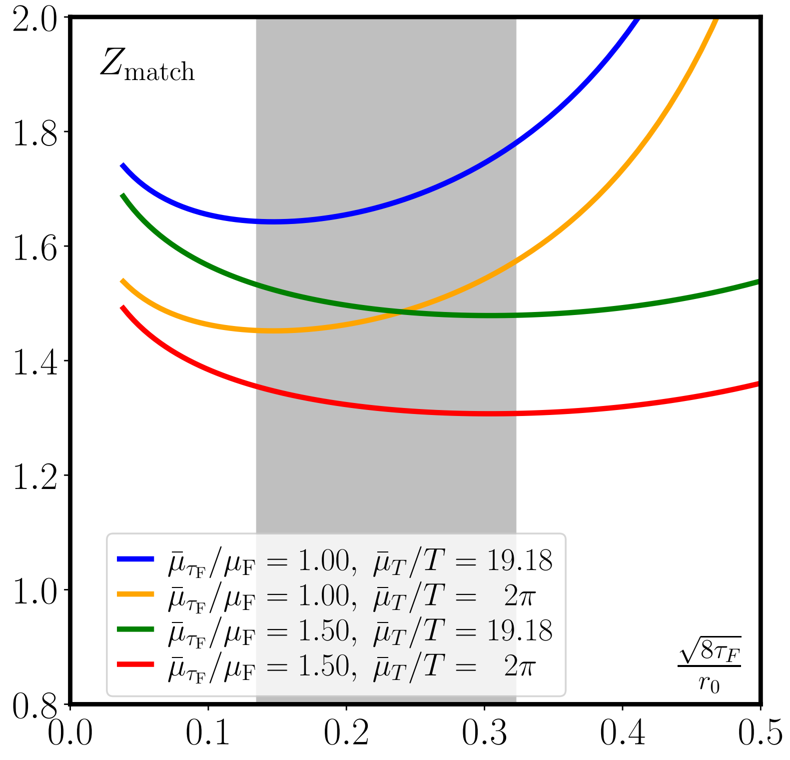
- Estimate uncertainties from unknow higher-order effects by varying the scales

$$\bar{\mu}_T = 2\pi T \ldots 19.18T \quad \text{and} \quad \bar{\mu}_{\tau_F} = \mu_F \ldots 1.4986\mu_F$$

[L. Altenkort, D. de la Cruz, OK, et al., arXiv:2311:01525]

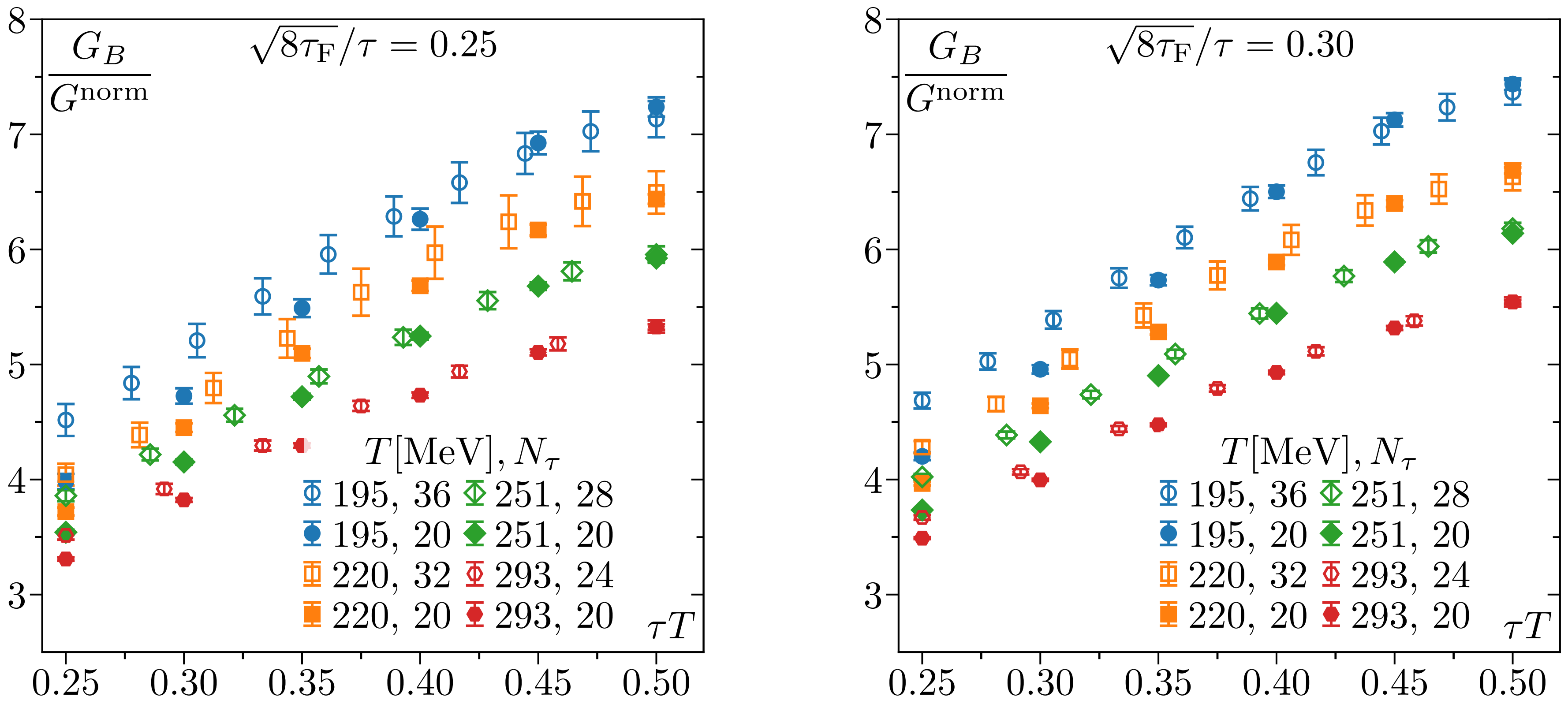
$$G_B^{\mathrm{phys.}}(\tau,T) = \lim_{\tau_\mathrm{F}\to 0} Z_\mathrm{match}(\bar\mu_T, \bar\mu_{\tau_\mathrm{F}}, \mu_\mathrm{F})\, G_B(\tau, T, \tau_\mathrm{F})$$

$$\ln Z_\mathrm{match} = \int_{\bar\mu_T^2}^{\bar\mu_{\tau_\mathrm{F}}^2} \gamma_0 g^2_{\overline{\mathrm{MS}}}(\bar\mu)\frac{d\bar\mu^2}{\bar\mu^2} + \gamma_0 g^2_{\overline{\mathrm{MS}}}(\bar\mu_T)\left[\ln\frac{\bar\mu_T^2}{(4\pi T)^2} - 2 + 2\gamma_\mathrm{E}\right] - \gamma_0 g^2_{\overline{\mathrm{MS}}}(\bar\mu_{\tau_\mathrm{F}})\left[\ln\frac{\bar\mu_{\tau_\mathrm{F}}^2}{4\,\mu_\mathrm{F}^2} + \gamma_\mathrm{E}\right]$$

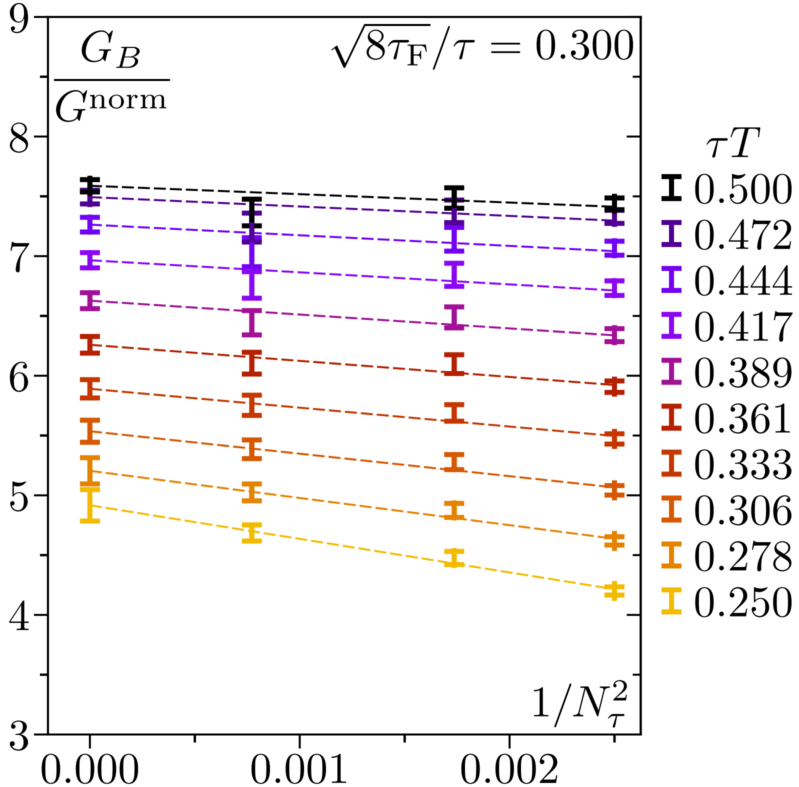


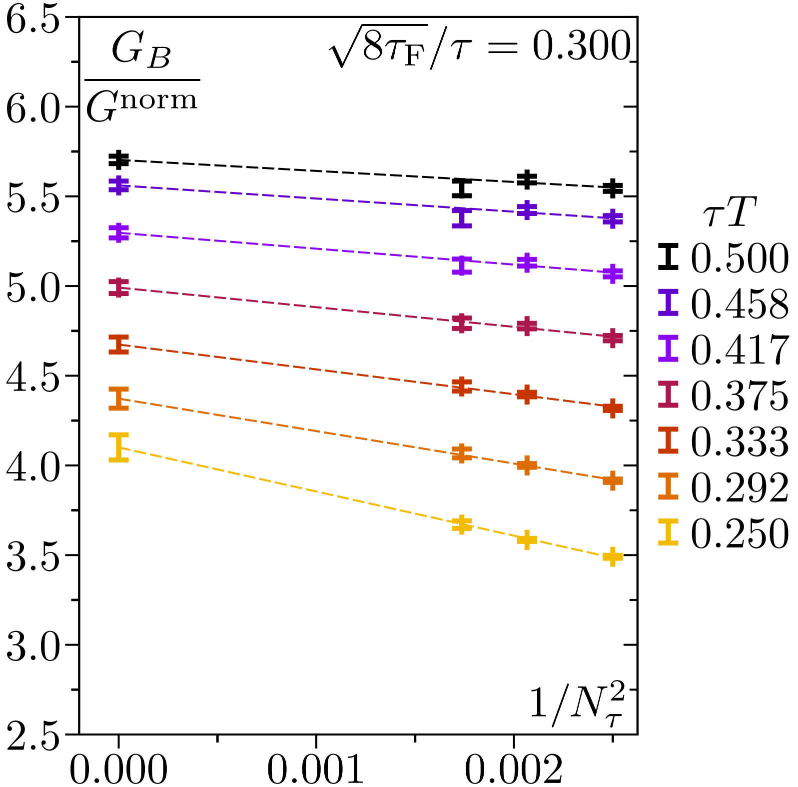
[L. Altenkort, D. de la Cruz, OK, et al., arXiv:2311:01525]

The *B*-field correlators in the gradient flow scheme for different temperatures calculated on the finest (open symbols) and coarsest lattices (filled symbols)

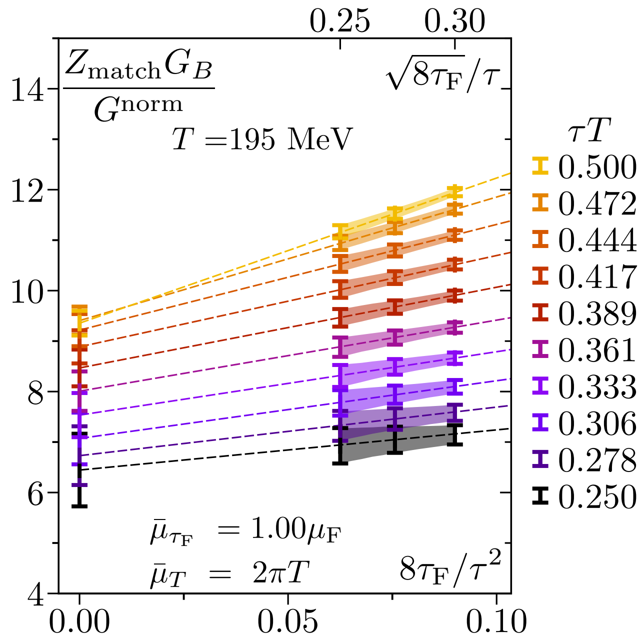


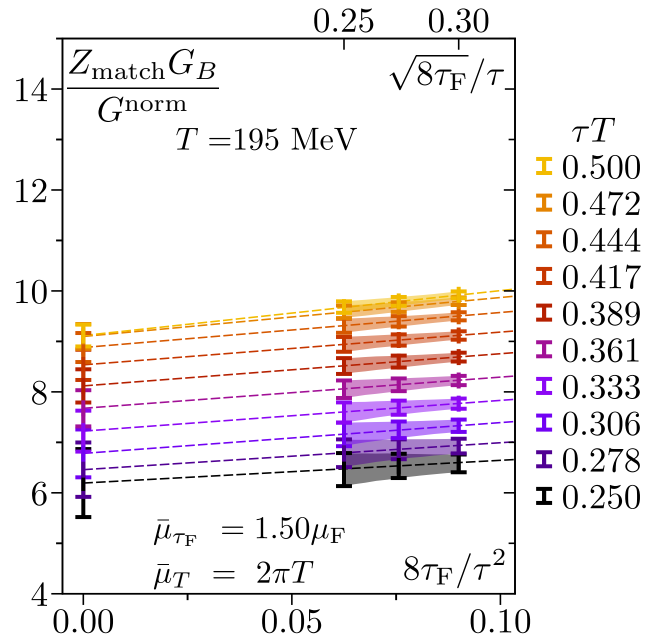
[L. Altenkort, D. de la Cruz, OK, et al., arXiv:2311:01525]

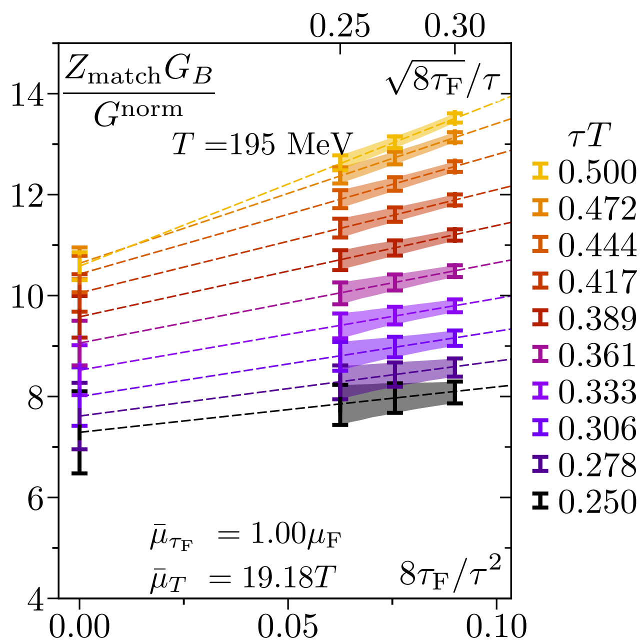


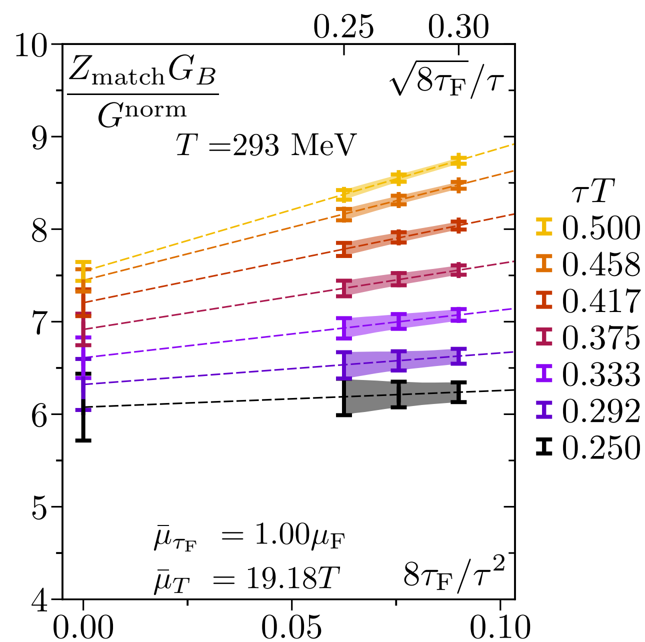


Flow time extrapolation

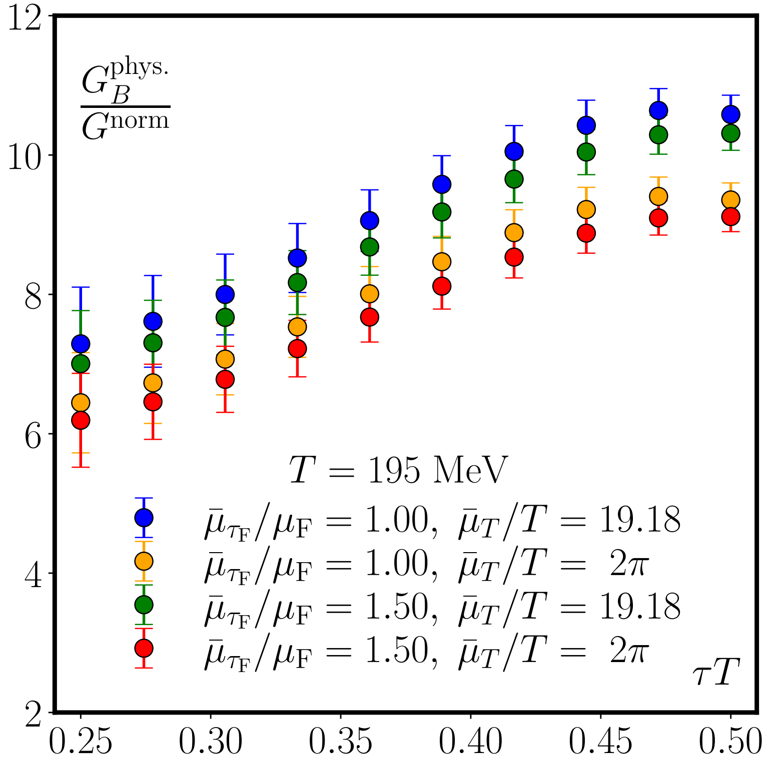




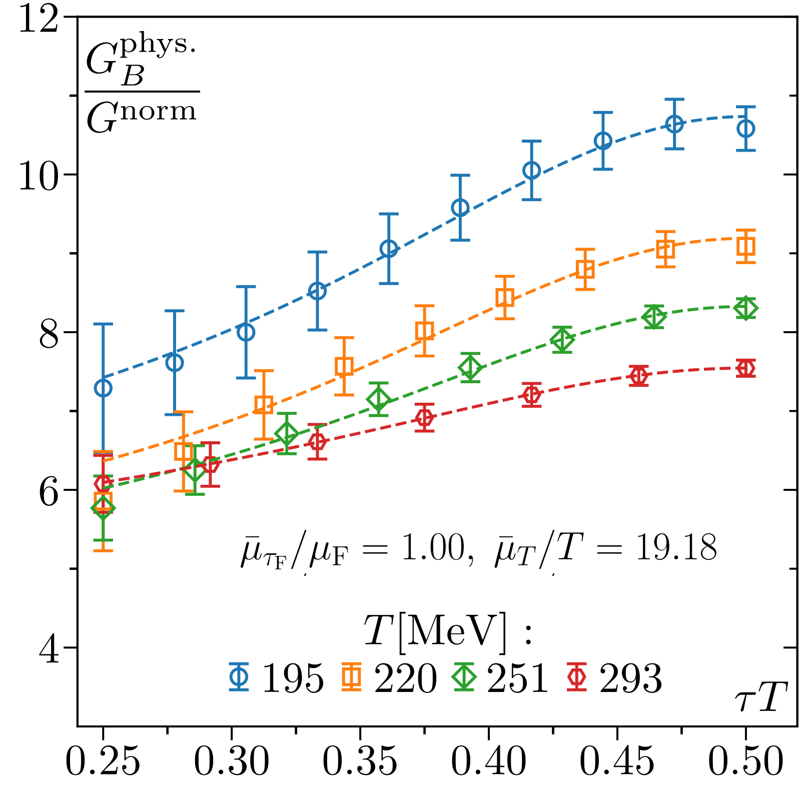




scale dependence of $G_B^{phys.}$



T dependence of $G_B^{phys.}$



Spectral function models with correct asymptotic perturbative behavior

$$\rho_B^{\mathrm{uv,LO}}(\omega,\mu) = \frac{g^2_{\overline{\mathrm{MS}}}(\mu) C_{\mathrm{F}} \omega^3}{6\pi},$$

$$\rho_B^{\mathrm{uv,NLO}}(\omega,\mu) = \frac{g^2_{\overline{\mathrm{MS}}}(\mu) C_{\mathrm{F}} \omega^3}{6\pi} \left\{ 1 + \frac{g^2_{\overline{\mathrm{MS}}}(\mu)}{(4\pi)^2} \left(N_c \left[\frac{5}{3} \ln \frac{\mu^2}{4\omega^2} + \frac{134}{9} - \frac{2\pi^2}{3} \right] - N_f \left[\frac{2}{3} \ln \frac{\mu^2}{4\omega^2} + \frac{26}{9} \right] \right) \right\},$$

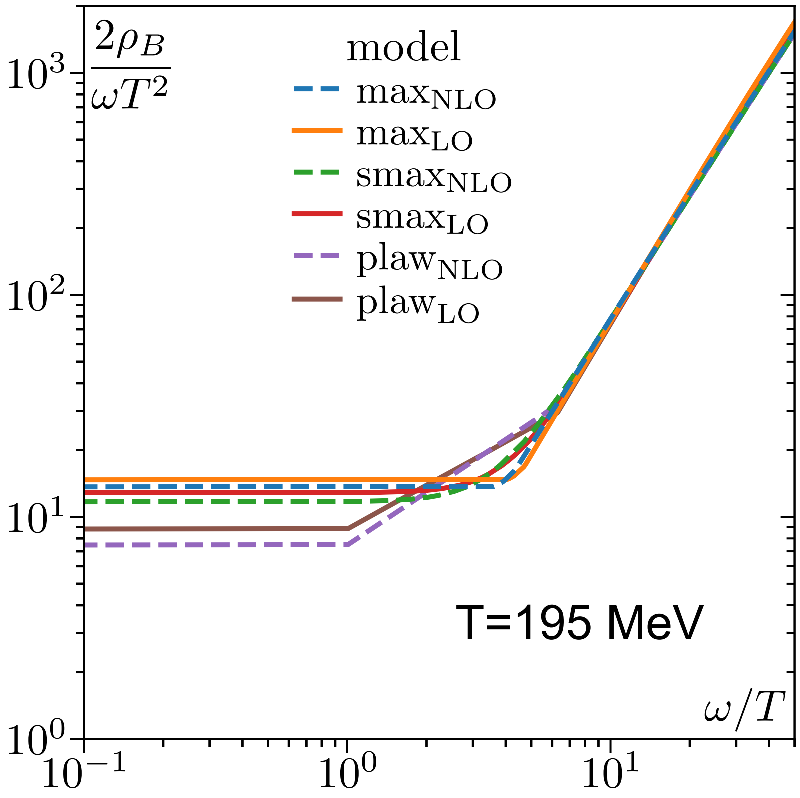
multiplied with $c_B^2(\mu, \bar{\mu}_T) = \exp\left(\int_{\bar{\mu}_T^2}^{\mu^2} \gamma_0 g^2_{\overline{\mathrm{MS}}}(\bar{\mu}) \frac{d\bar{\mu}^2}{\bar{\mu}^2} \right)$ to go from $\overline{MS}$ to physical scheme

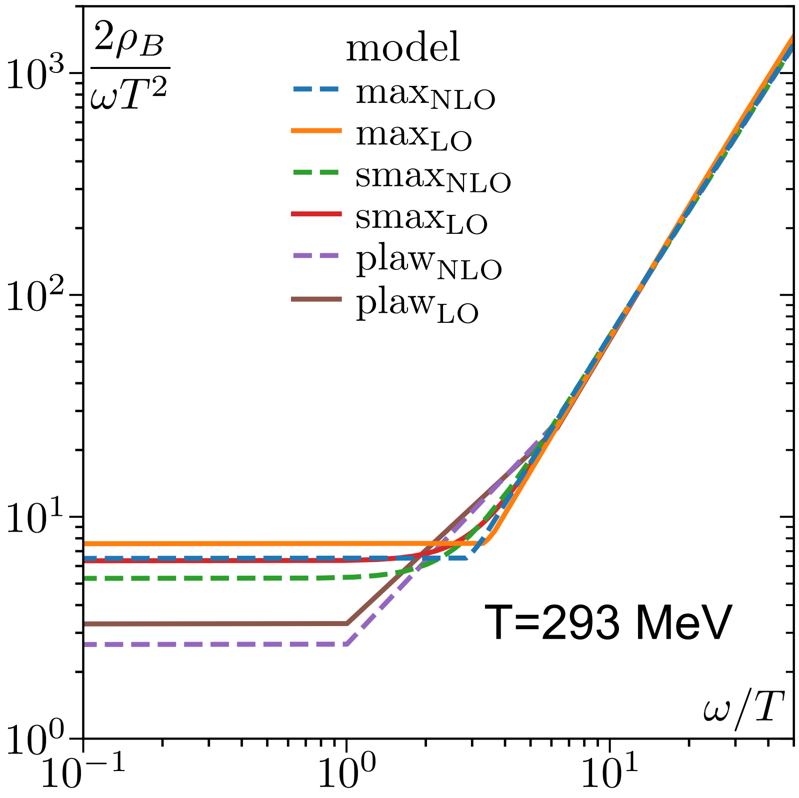
modeling corrections to $\rho_{\mathrm{IR}}(\omega) = \frac{\kappa\omega}{2T}$ in various ways

Label	ρ_{model}	μ	Fit parameters
$\max_{\mathrm{LO}}$ $\max_{\mathrm{NLO}}$	$\max(\Phi_{\mathrm{IR}}, \Phi_{\mathrm{UV}})$	$\max(\mu_{\mathrm{eff}}, \omega)$ $\max(\mu_{\mathrm{eff}}, \mu_{\mathrm{opt}})$	$\kappa/T^3, K$
$\mathrm{smax}_{\mathrm{LO}}$ $\mathrm{smax}_{\mathrm{NLO}}$	$\sqrt{\Phi^2_{\mathrm{IR}} + \Phi^2_{\mathrm{UV}}}$	$\max(\mu_{\mathrm{eff}}, \omega)$ $\max(\mu_{\mathrm{eff}}, \mu_{\mathrm{opt}})$	$\kappa/T^3, K$
$\mathrm{plaw}_{\mathrm{LO}}$ $\mathrm{plaw}_{\mathrm{NLO}}$	$\theta(\omega_{\mathrm{IR}}-\omega)\Phi_{\mathrm{IR}}+$ $\theta(\omega-\omega_{\mathrm{IR}})\theta(\omega_{\mathrm{UV}}-\omega)p(\omega)+$ $\theta(\omega-\omega_{\mathrm{UV}})\Phi_{\mathrm{UV}}$	$\max(\mu_{\mathrm{eff}}, \omega)$ $\max(\mu_{\mathrm{eff}}, \mu_{\mathrm{opt}})$	$\kappa/T^3, K$

Fitting $G_{\mathrm{model}}(\tau) \equiv \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \rho_{\mathrm{model}}(\omega) \frac{\cosh\left(\frac{1}{2} - \tau T\right)\frac{\omega}{T}}{\sinh \frac{\omega}{2T}}$ to obtain $\kappa_B/T^3 = \lim_{\omega\to 0} \frac{2T\rho_B(\omega)}{\omega}$

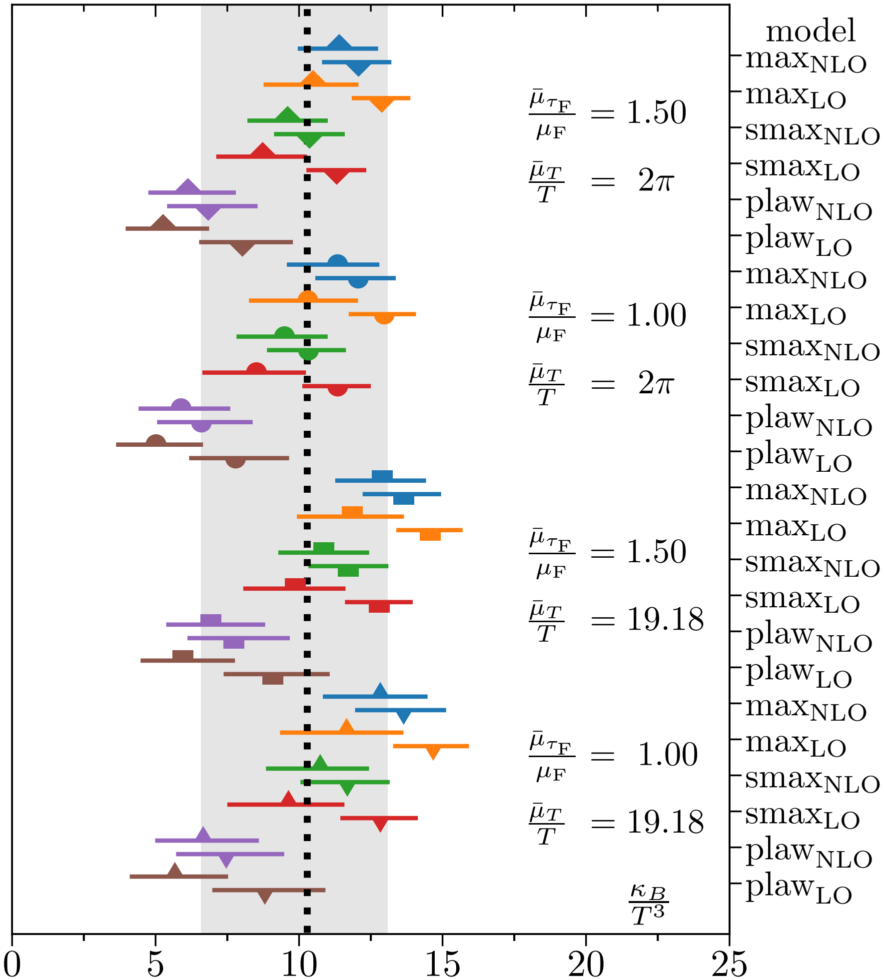
Fit results of the color-magnetic spectral function



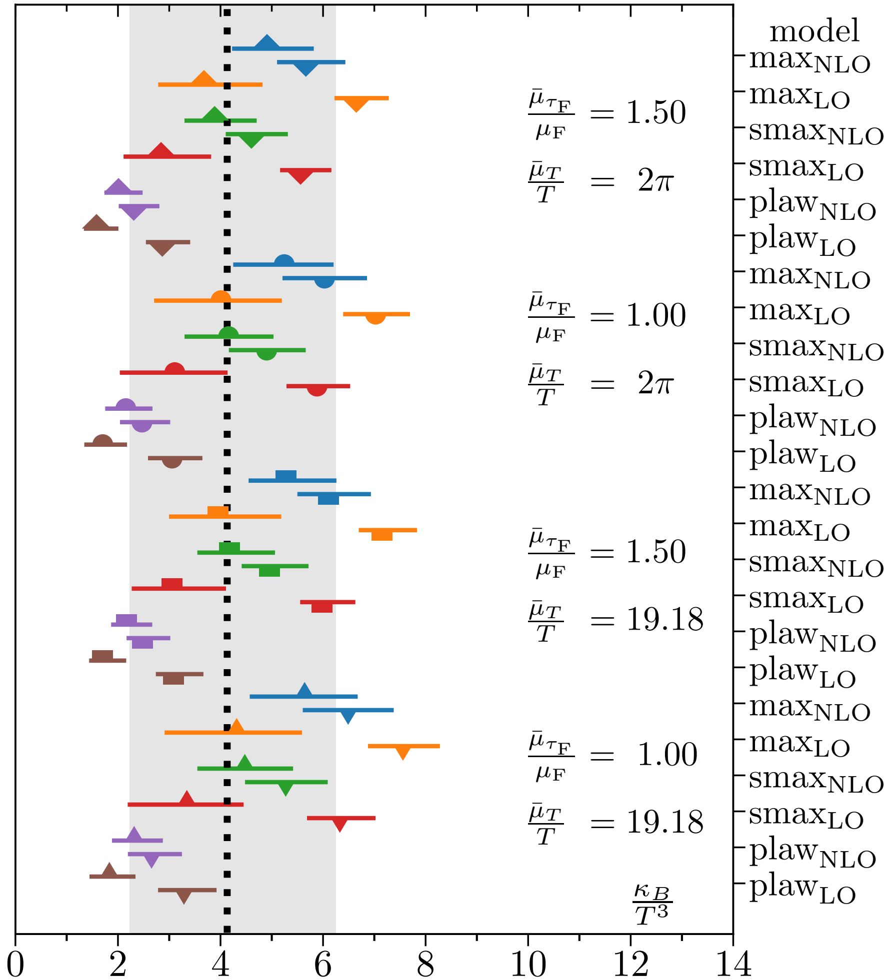


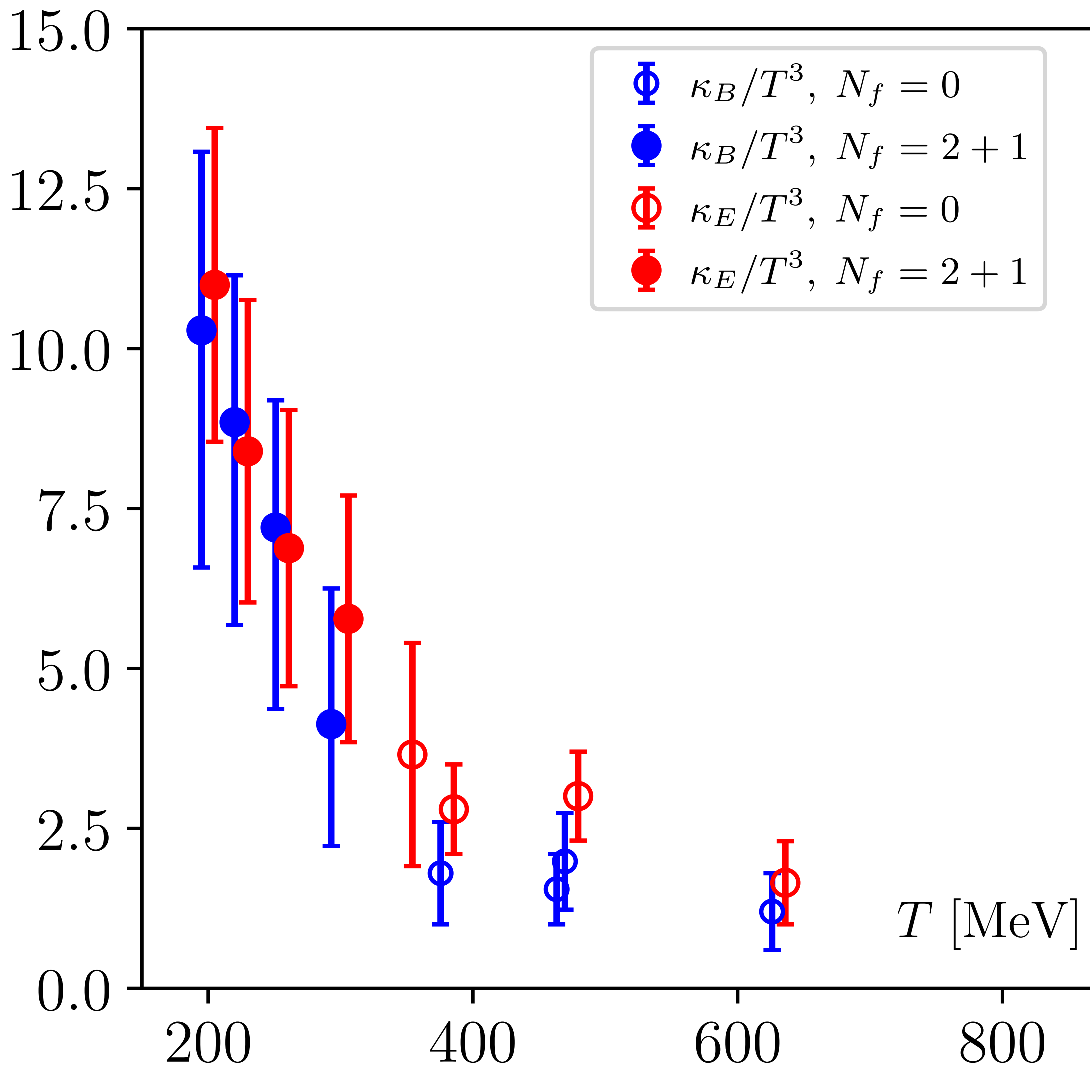
Fit results for κ_B using various models and various scales
to estimate systematic uncertainties

T=195 MeV



T=293 MeV




15.0
12.5
10.0
7.5
5.0
2.5
0.0
200
400
600
800
T [MeV]
$\kappa_B/T^3,\ N_f=0$
$\kappa_B/T^3,\ N_f=2+1$
$\kappa_E/T^3,\ N_f=0$
$\kappa_E/T^3,\ N_f=2+1$

Spatial Diffusion coefficient for charm and bottom quarks

[L. Altenkort, D. de la Cruz, OK, et al., arXiv:2311:01525]

$$D_s = \frac{2T^2}{\kappa} \cdot \frac{\langle p^2 \rangle}{3MT}$$