UNIVERSITÄT BIELEFELD

Charm and beauty spectral and transport properties

From correlators to spectral functions – case studies in the quenched approximation

I) Thermal quarkonium physics in the pseudoscalar channel

[Y. Burnier, H.-T. Ding, OK et al. JHEP11 (2017) 206]

II) Thermal quarkonium physics in the vector channel

[A.-L. Lorenz, H.T. Shu, OK et al. work in progress]

III) Heavy quark momentum diffusion coefficient

[A.Francis, OK, et al., PRD92(2015)116003]

IV) Correlation functions under gradient flow – towards full QCD

[L. Altenkort, A.M. Eller, L. Mazur, OK, G.D Moore, H.T. Shu, work in progress]

CRC-TR 211
Strong-interaction matter under extreme conditions

Information on **bound states and their dissociation** encoded in the spectral function

as well as

Thermal dilepton rate

$$\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3p} = \frac{5\alpha^2}{54\pi^3} \frac{1}{\omega^2(e^{\omega/T}-1)} \rho_{\mathbf{V}}(\omega, \mathbf{T})$$

[H-T.Ding, F.Meyer, OK, PRD94(2016)034504]

Thermal photon rate

$$\omega \frac{\mathrm{d}N_\gamma}{\mathrm{d}^4x \mathrm{d}^3q} = \frac{5\alpha}{6\pi^2} \frac{1}{e^{\omega/T}-1} \rho_V(\omega = |\vec{k}|, T)$$

[J.Ghiglieri, OK, M.Laine, F.Meyer, PRD94(2016)016005]

Transport coefficients are encoded
in the vector meson spectral function
→ Kubo formulae

Diffusion coefficients:

$$DT = \frac{T}{2\chi_q} \lim_{\omega \to 0} \frac{\rho_{\mathrm{ii}}(\omega)}{\omega}$$

On the lattice only correlation functions can be calculated

$$G(\tau, \vec{p}, T) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \rho(\omega, \vec{p}, T) K(\tau, \omega, T)$$

→ spectral reconstruction required

$$K(\tau, \omega, T) = \frac{\cosh\left(\omega(\tau - \frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

In this talk: continuum extrapolated lattice correlation functions compared to perturbation theory

for a comparison of Bayesian and stochastic reconstructions of spectral functions see
[H.-T. Ding, OK, S. Mukherjee, H. Ohno, H.-T. Shu, PRD97(2018)094503]

Vector-meson spectral function – hard to separate different scales

$$G(\tau,\vec{p},T) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\rho(\omega,\vec{p},T)K(\tau,\omega,T)$$

$$K(\tau,\omega,T) = \frac{\cosh\left(\omega(\tau-\frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

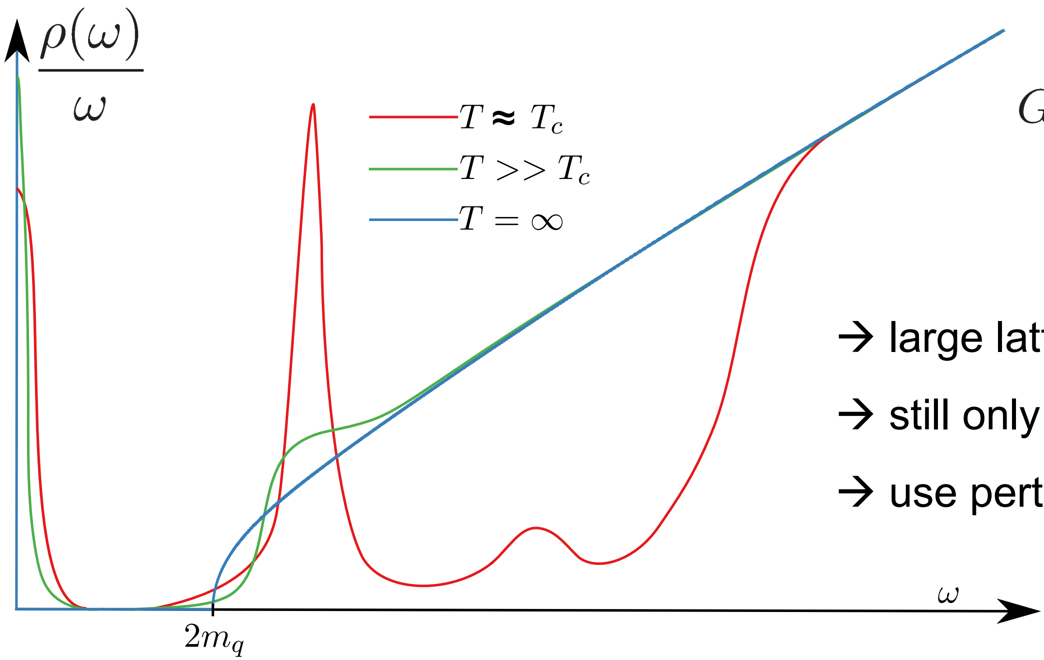
Different contributions and scales enter

in the spectral function

- **continuum at large frequencies**
- **possible bound states at intermediate frequencies**
- **transport contributions at small frequencies**
- in addition cut-off effects on the lattice

notoriously difficult to extract from correlation functions

Spectral functions in the QGP



$$G_{\mu\nu}(\tau,\vec{x}) = \langle J_\mu(\tau,\vec{x})J_\nu^\dagger(0,\vec{0})\rangle$$
$$J_\mu(\tau,\vec{x}) = 2\kappa Z_V\bar{\psi}(\tau,\vec{x})\Gamma_\mu\psi(\tau,\vec{x})$$

→ large lattices and continuum extrapolation needed

→ still only possible in the quenched approximation

→ use perturbation theory to constrain the UV behavior

(narrow) transport peak at small ω: $\rho(\omega \ll T) \simeq 2\chi_{00}\frac{T}{M}\frac{\omega\eta}{\omega^2+\eta^2}\,, \qquad \eta = \frac{T}{MD}$

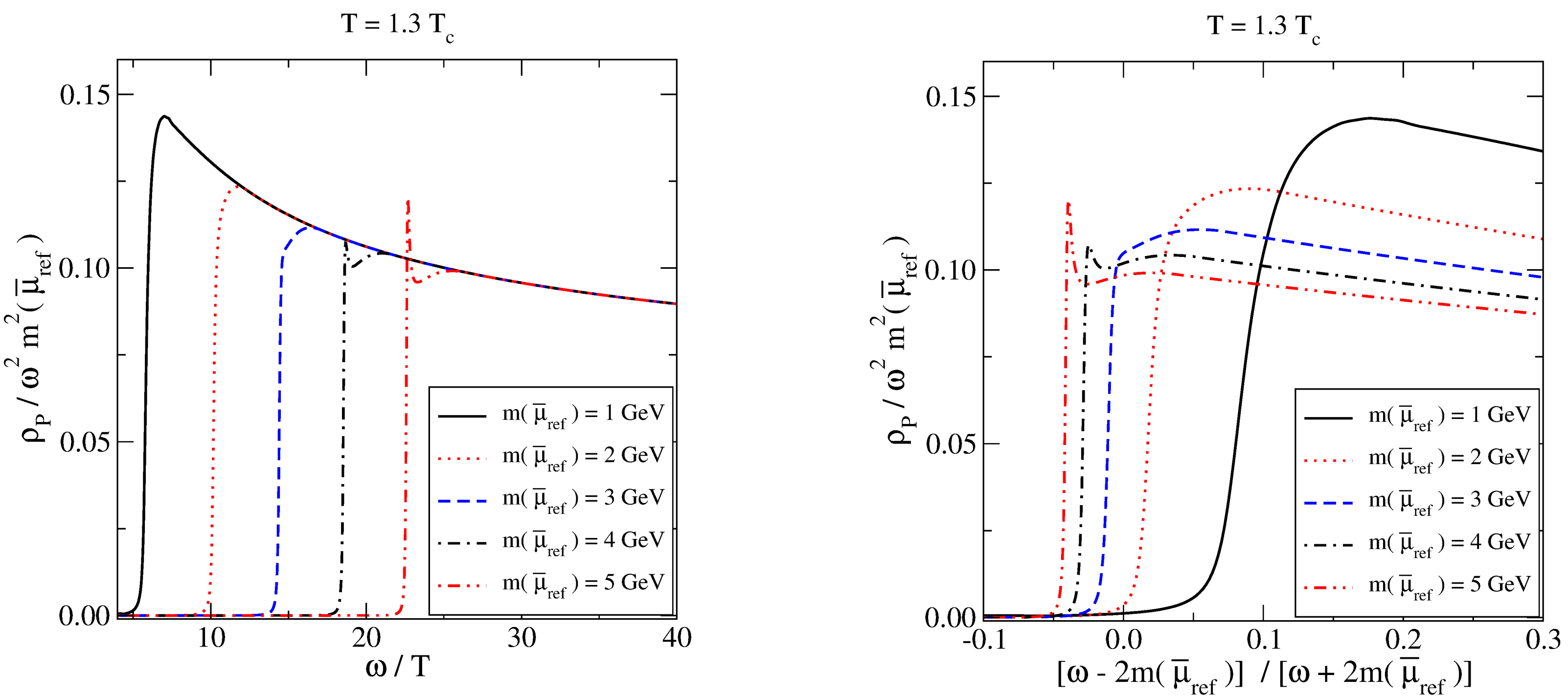
[Y. Burnier, H.-T. Ding, OK et al. JHEP11 (2017) 206]

Using **continuum extrapolated Euclidean correlation functions** from Lattice QCD

$$G_{\mathrm{P}}(\tau) \equiv M_{\mathrm{B}}^2 \int_{\vec{x}} \Big\langle (\bar\psi i\gamma_5\psi)(\tau,\vec{x})\,(\bar\psi i\gamma_5\psi)(0,\vec{0}) \Big\rangle_{\mathrm{c}}\,, \quad 0<\tau<\frac{1}{T}\,,$$

and best knowledge on the spectral function from **perturbation theory and pNRQCD**

interpolated between different regimes



no transport contribution in pseudo-scalar channel channel

Lattice set-up

quenched SU(3) gauge configurations (separated by 500 updates)

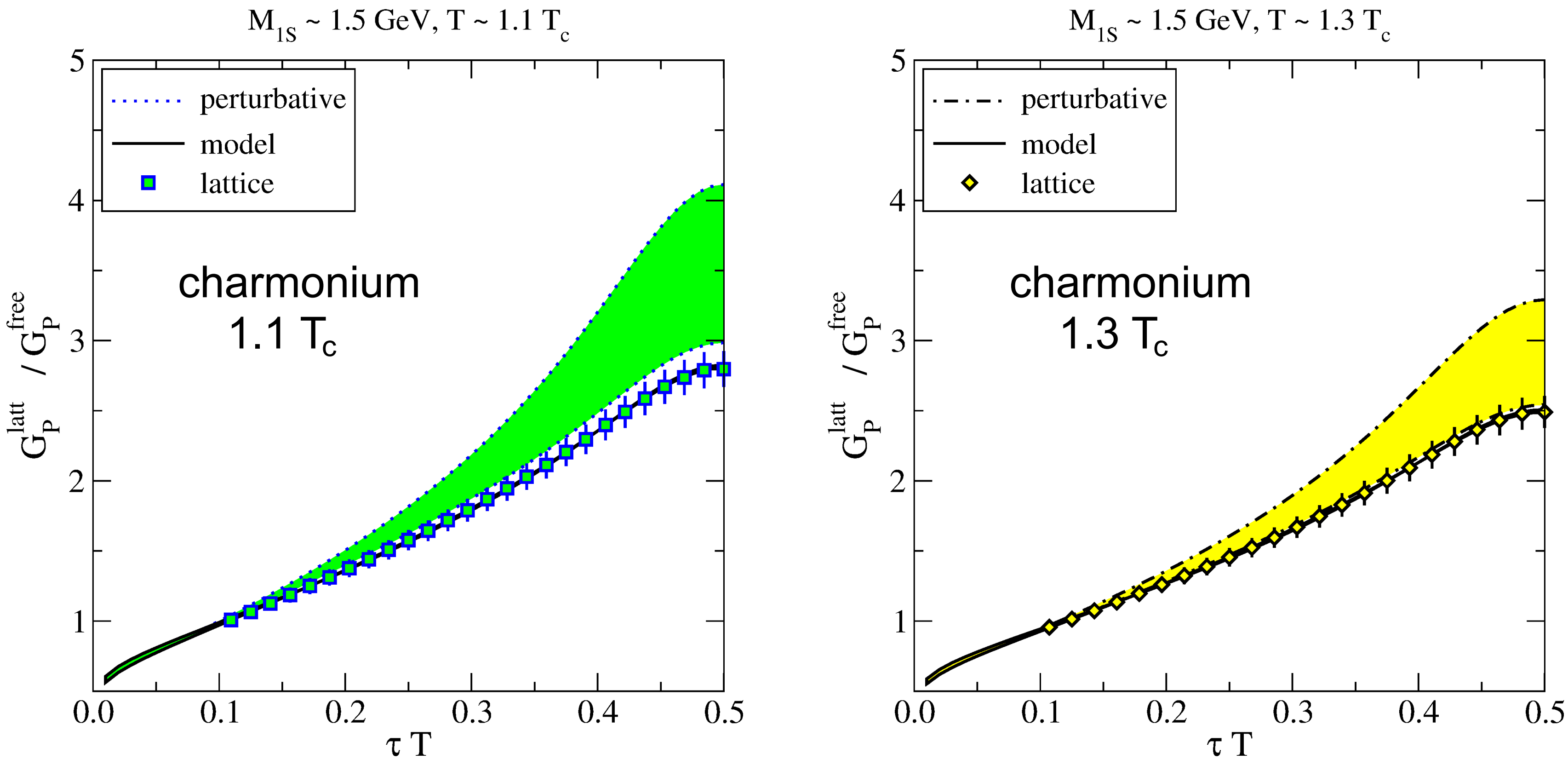
non-perturbatively O(a) clover improved Wilson fermion valence quarks

6 quark masses between charm and bottom → interpolate to physical c and b mass

β	N_s	N_τ	confs	r_0/a	T/T_c	c_{SW}	κ_c	κ	$\frac{m^2(1/a)}{m^2(\bar{\mu}_{ref})}$
7.192	96	48	237	26.6	0.74	1.367261	0.13442	0.12257, 0.12800, 0.13000, 0.13100, 0.13150, 0.13194	0.6442
		32	476		1.12				
		28	336		1.27				
		24	336		1.49				
		16	237		2.23				
7.394	120	60	171	33.8	0.76	1.345109	0.13408	0.124772, 0.12900, 0.13100, 0.13150, 0.132008, 0.132245	0.6172
		40	141		1.13				
		30	247		1.51				
		20	226		2.27				
7.544	144	72	221	40.4	0.75	1.330868	0.13384	0.12641, 0.12950, 0.13100, 0.13180, 0.13220, 0.13236	0.5988
		48	462		1.13				
		42	660		1.29				
		36	288		1.51				
		24	237		2.26				
7.793	192	96	224	54.1	0.76	1.310381	0.13347	0.12798, 0.13019, 0.13125, 0.13181, 0.13209, 0.13221	0.5715
		64	249		1.13				
		56	190		1.30				
		48	210		1.51				
		32	235		2.27				

In this talk: only results based on continuum extrapolated correlation functions

See [Y. Burnier, H.-T. Ding, OK et al. JHEP11 (2017) 206] for more details



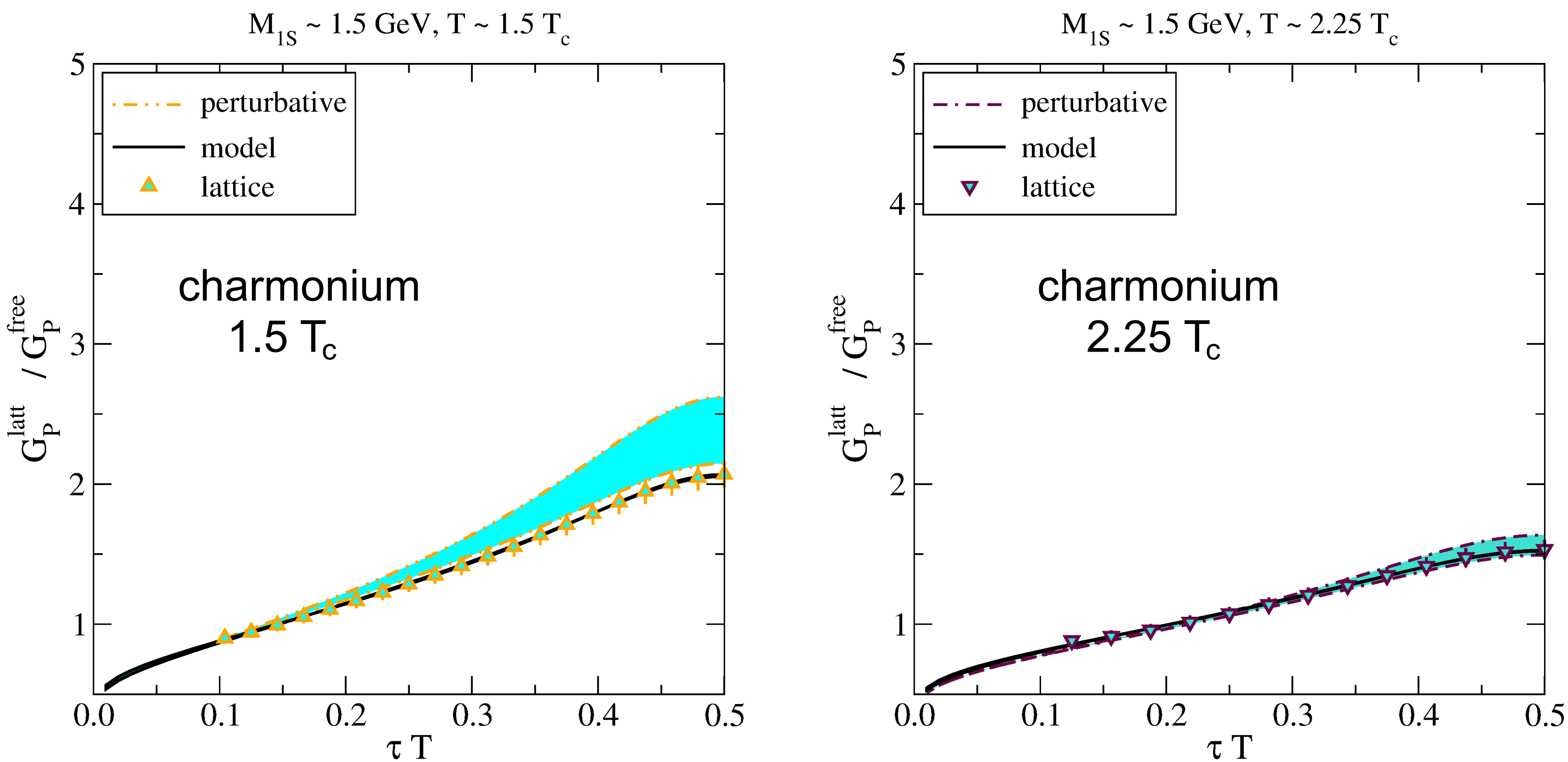
differences between lattice and perturbation theory may have a simple explanation

A: uncertainties related to the perturbative renormalization factors
B: non-perturbative mass shifts

$$\rho_{\mathrm{P}}^{\mathrm{model}}(\omega) \;\equiv\; A\,\rho_{\mathrm{P}}^{\mathrm{pert}}(\omega - B)\;.$$

→ continuum lattice data well described by this model with χ^2/d.o.f < 1

Modelling the spectral function



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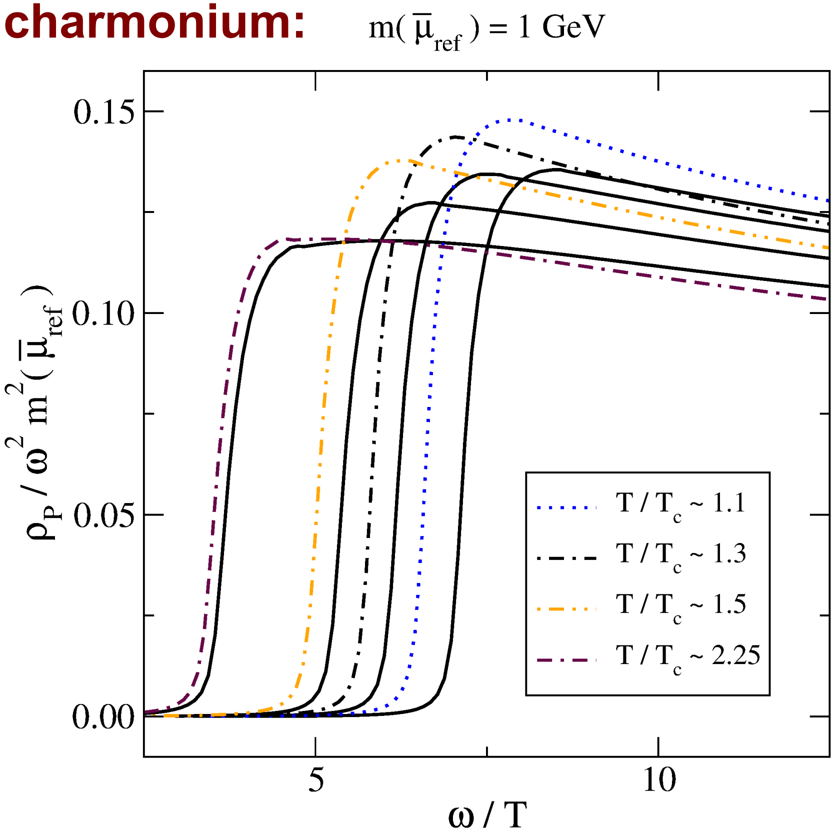
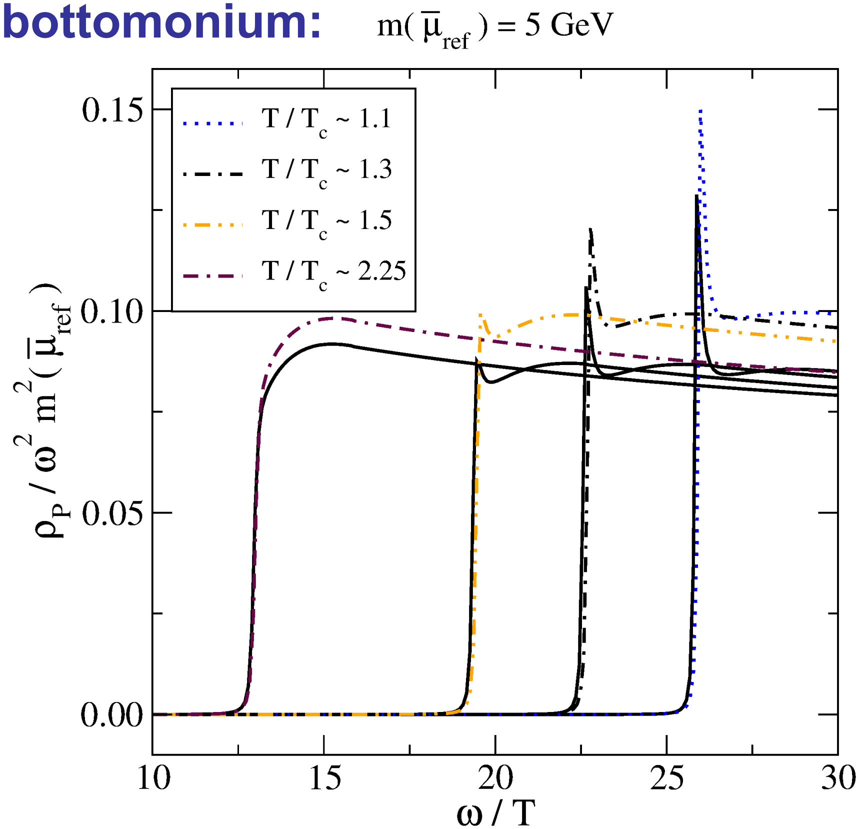
Pseudo-scalar spectral functions

[Y. Burnier, H.-T. Ding, OK et al. JHEP11 (2017) 206]

charmonium:



bottomonium:



charmonium:

no resonance peaks are needed for representing the lattice data even for 1.1 T_c

modest threshold enhancement sufficient in the analyzed temperature region

bottomonium:

thermally broadened resonance peak

present up to temperatures around 1.5 T_c

[H.T. Ding, O. Kaczmarek, A.-L. Lorenz, R. Larsen, Swagato Mukherjee, H. Ohno, H. Sandmeyer, H.-T. Shu, paper in preparation]

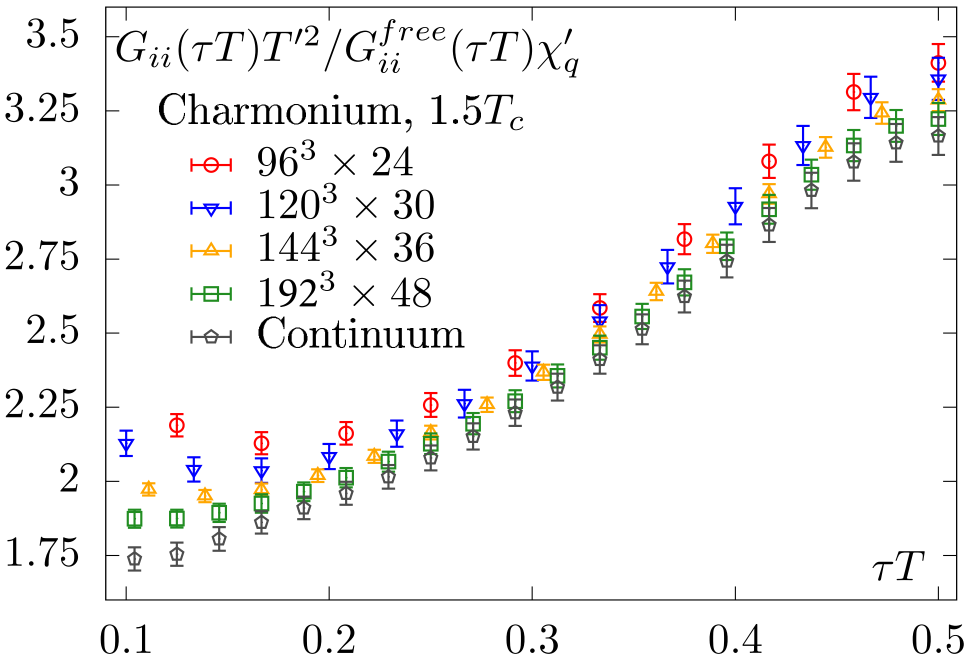
quenched SU(3) gauge configurations (separated by 500 updates)

non-perturbatively O(a) clover improved Wilson fermion valence quarks

6 quark masses between charm and bottom

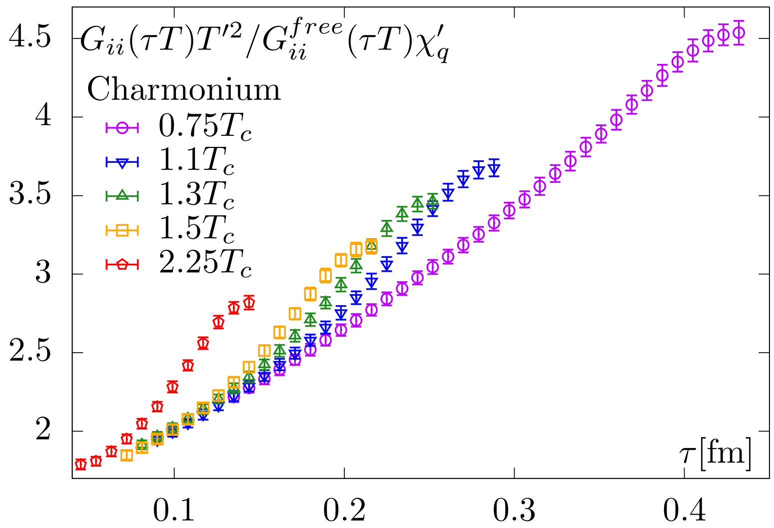
→ interpolate to physical c and b mass

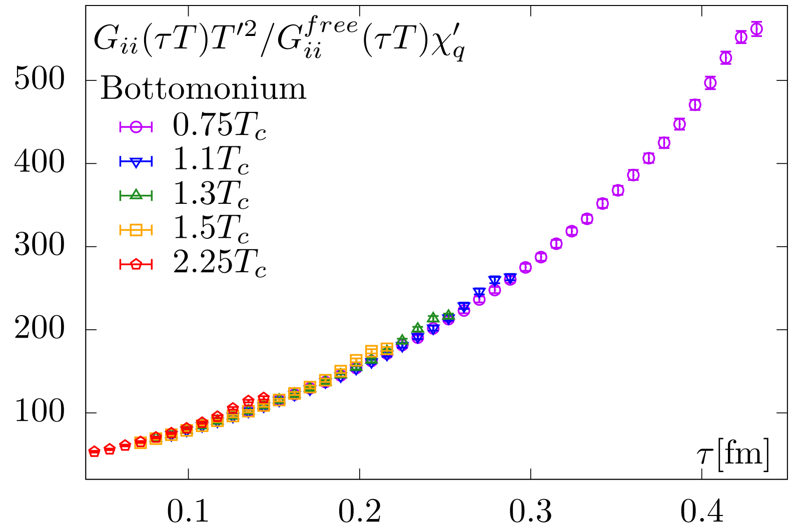
well controlled continuum extrapolation:



β	r_0/a	a[fm](a^{-1}[GeV])	N_σ	N_τ	T/T_c	# confs
7.192	26.6	0.018(11.19)	96	48	0.75	237
				32	1.1	476
				28	1.3	336
				24	1.5	336
				16	2.25	237
7.394	33.8	0.014(14.24)	120	60	0.75	171
				40	1.1	141
				30	1.5	247
				20	2.25	226
7.544	40.4	0.012(17.01)	144	72	0.75	221
				48	1.1	462
				42	1.3	660
				36	1.5	288
				24	2.25	237
7.793	54.1	0.009(22.78)	192	96	0.75	224
				64	1.1	291
				56	1.3	291
				48	1.5	348
				32	2.25	235

[H.T. Ding, O. Kaczmarek, A.-L. Lorenz, R. Larsen, Swagato Mukherjee, H. Ohno, H. Sandmeyer, H.-T. Shu, paper in preparation]





we model the UV part of the spectral function again by a perturbative model

$$\rho_{ii}^{mod}(\omega) = A\rho_V^{pert}(\omega - B)$$

in the vector channel an additional IR contribution appears as a transport peak

$$\rho_{ii}^{trans}(\omega) = 3\chi_q \frac{T}{M}\frac{\omega\eta}{\omega^2+\eta^2} \qquad D = \frac{1}{3\chi_q}\lim_{\omega\to 0}\frac{\rho_{ii}^{trans}(\omega)}{\omega} \qquad \eta = \frac{T}{MD}$$

with an almost constant contribution to the correlator at large distances for small η

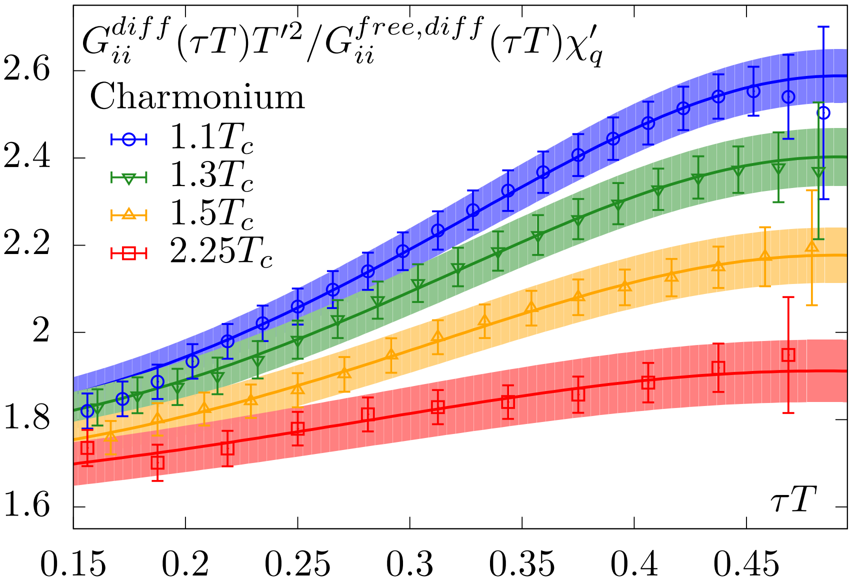
$$\rho_{ii}(\omega) = \rho_{ii}^{trans}(\omega) + \rho_{ii}^{mod}(\omega) \qquad G_{ii}(\tau T) = G_{ii}^{trans}(\tau T) + G_{ii}^{mod}(\tau T)$$

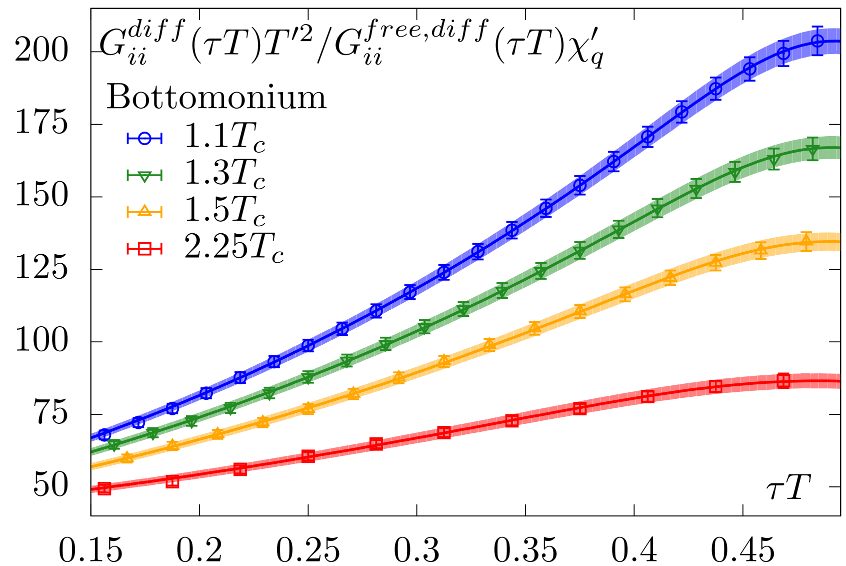
an almost constant contribution of the transport peak is effectively removed in

$$G_{ii}^{diff}(\tau/a) = G_{ii}(\tau/a+1) - G_{ii}(\tau/a)$$

and the UV part of the spectral function well described by the perturbative model

$$\rho_{ii}^{mod}(\omega) = A\rho_V^{pert}(\omega - B)$$





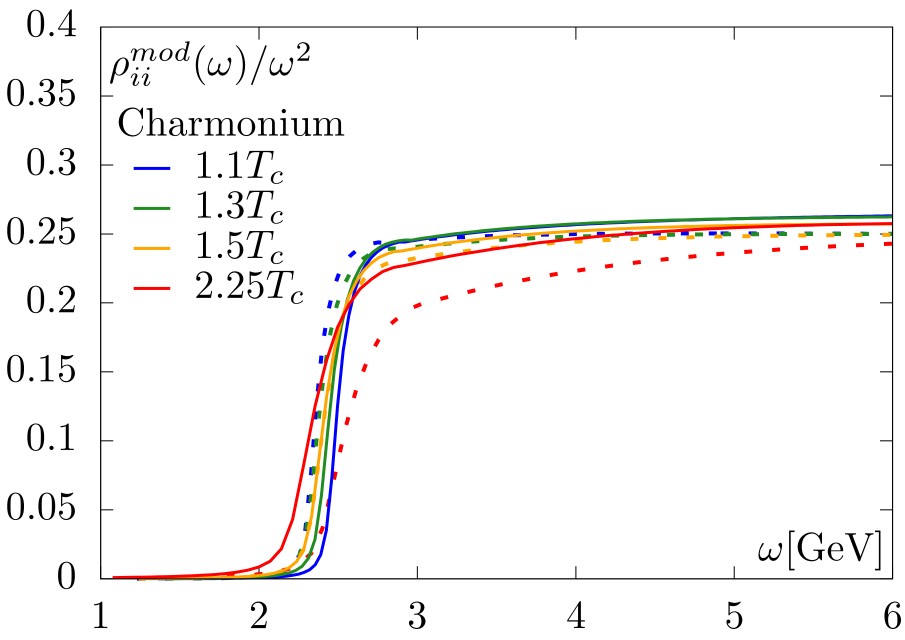
differences between lattice and perturbation theory may have a simple explanation

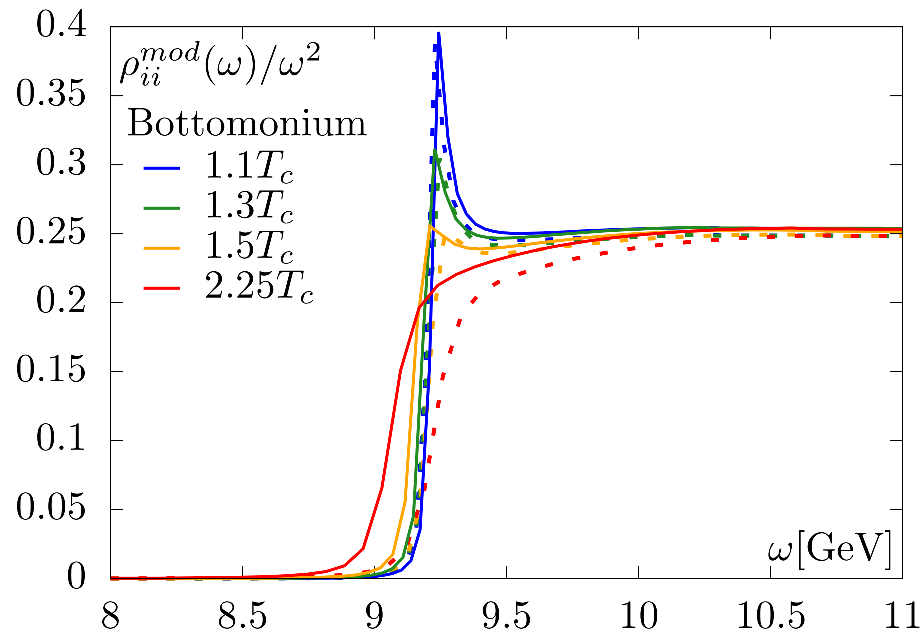
A: uncertainties related to renormalization
B: non-perturbative mass shifts

→ continuum lattice data well described by this model with χ^2/d.o.f < 1

Vector meson spectral functions

UV part well described by the perturbative model: $\rho_{ii}^{mod}(\omega) = A\rho_V^{pert}(\omega - B)$





charmonium:

no resonance peaks are needed for representing the lattice data even for 1.1 T_c

modest threshold enhancement sufficient in the analyzed temperature region

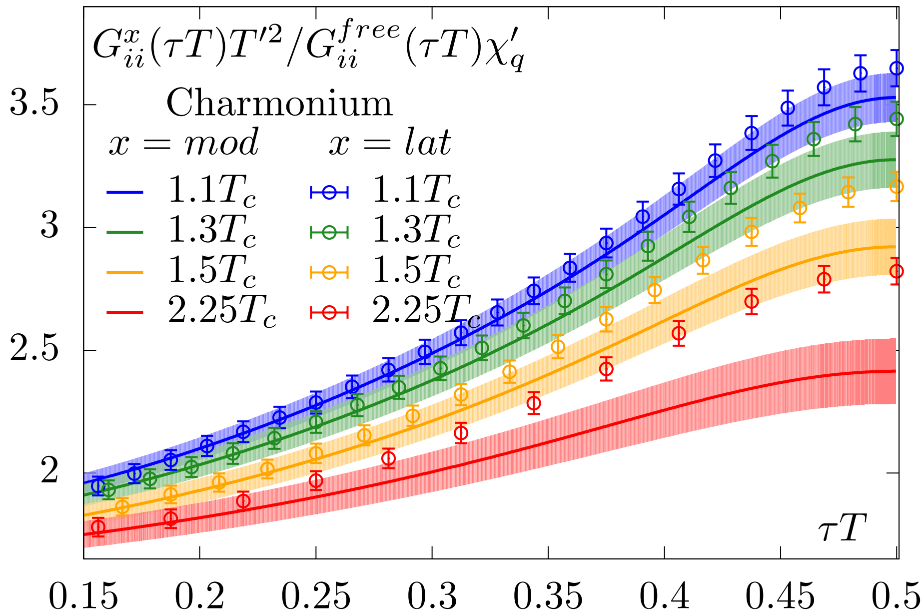
bottomonium:

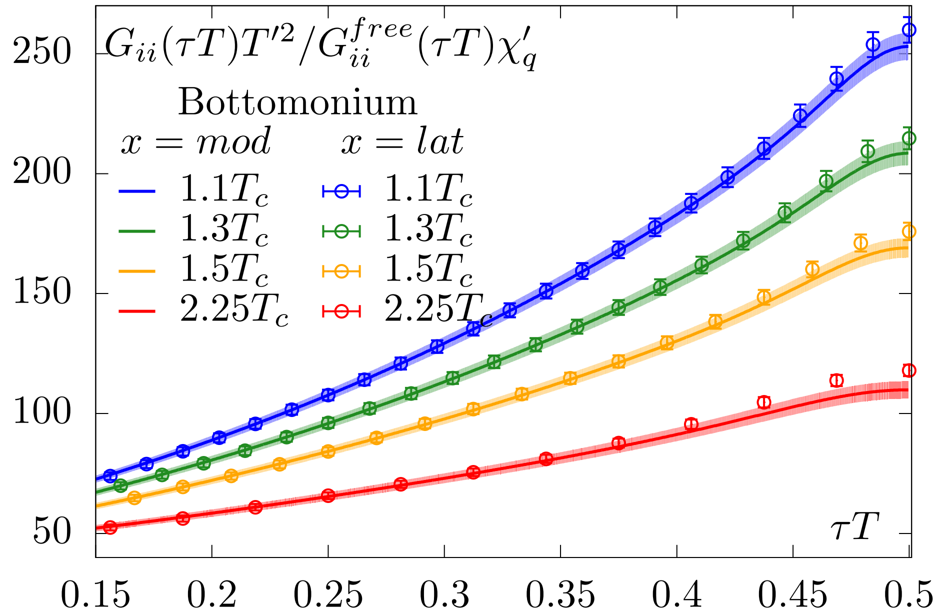
thermally broadened resonance peak

present up to temperatures around 1.5 T_c

comparison of the full correlator to the model

$$\rho_{ii}(\omega) = \rho_{ii}^{trans}(\omega) + \rho_{ii}^{mod}(\omega) \qquad G_{ii}(\tau T) = G_{ii}^{trans}(\tau T) + G_{ii}^{mod}(\tau T)$$



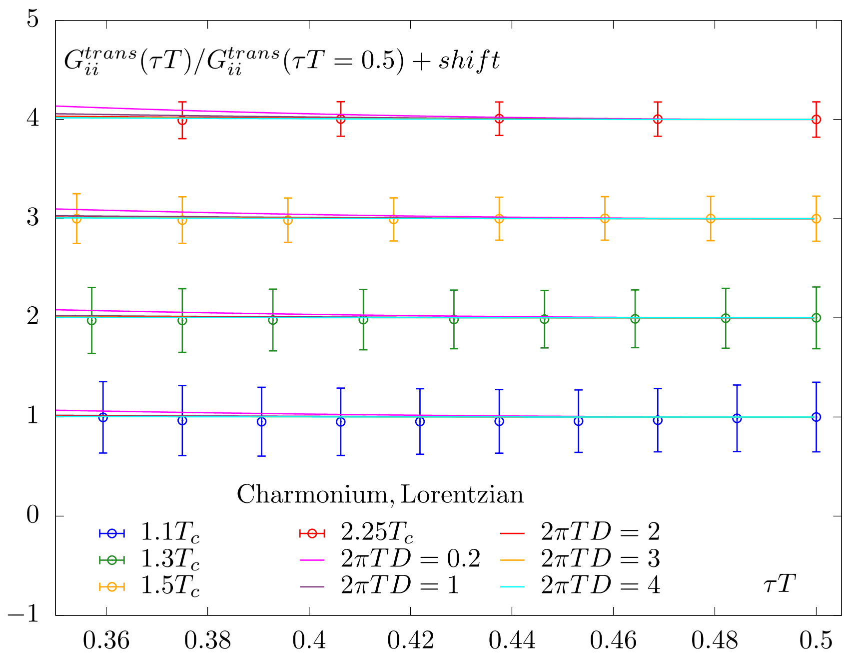


comparison of the full correlator to the model → difference due to transport contribution

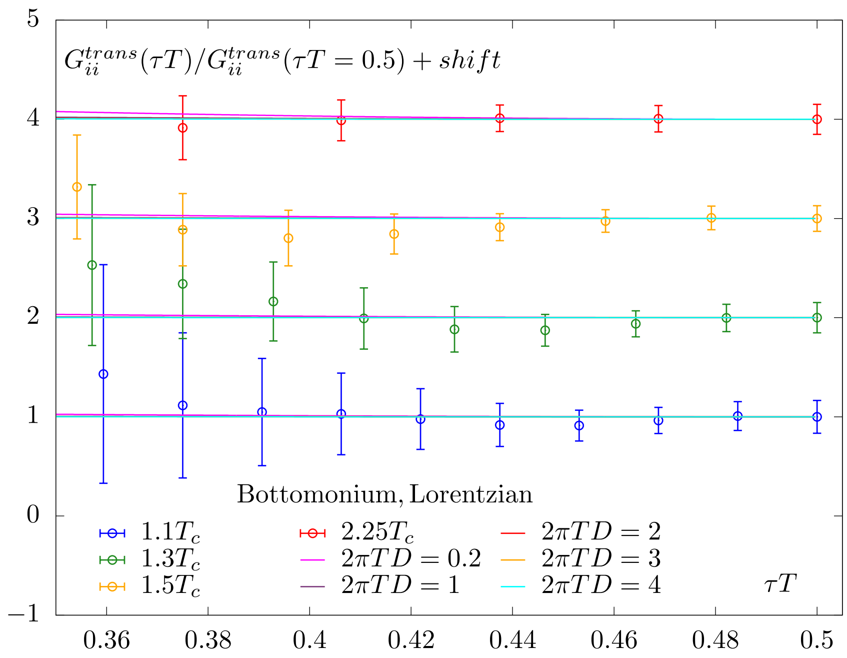
$$\rho_{ii}^{trans}(\omega) = 3\chi_q \frac{T}{M}\frac{\omega\eta}{\omega^2+\eta^2} \qquad D = \frac{1}{3\chi_q}\lim_{\omega\to 0}\frac{\rho_{ii}^{trans}(\omega)}{\omega} \qquad \eta = \frac{T}{MD}$$

transport contribution in the vector channel: $\rho(\omega \ll T) \simeq 2\chi_{00} \frac{T}{M} \frac{\omega\eta}{\omega^2+\eta^2}\,, \qquad \eta = \frac{T}{MD}$

charmonium:



bottomonium:



varying *2πDT* between 0.2 and 4

→ only small curvature due to transport contribution within our errors

→ hard to determine transport coefficients with present data

→ easier in the heavy quark mass limit

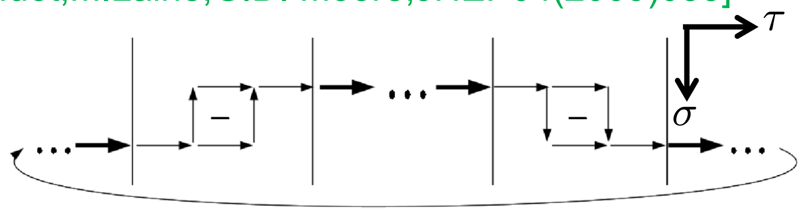
Heavy Quark Effective Theory (HQET) in the large quark mass limit

for a single quark in medium

leads to a (pure gluonic) "color-electric correlator"

[J.Casalderrey-Solana, D.Teaney, PRD74(2006)085012, S.Caron-Huot,M.Laine,G.D. Moore,JHEP04(2009)053]

$$G_E(\tau) \equiv -\frac{1}{3}\sum_{i=1}^{3} \frac{\left\langle \mathrm{Re\,Tr}\left[U(\tfrac{1}{T};\tau)\, g\, E_i(\tau,\mathbf{0})\, U(\tau;0)\, g\, E_i(0,\mathbf{0})\right]\right\rangle}{\left\langle \mathrm{Re\,Tr}[U(\tfrac{1}{T};0)]\right\rangle}$$



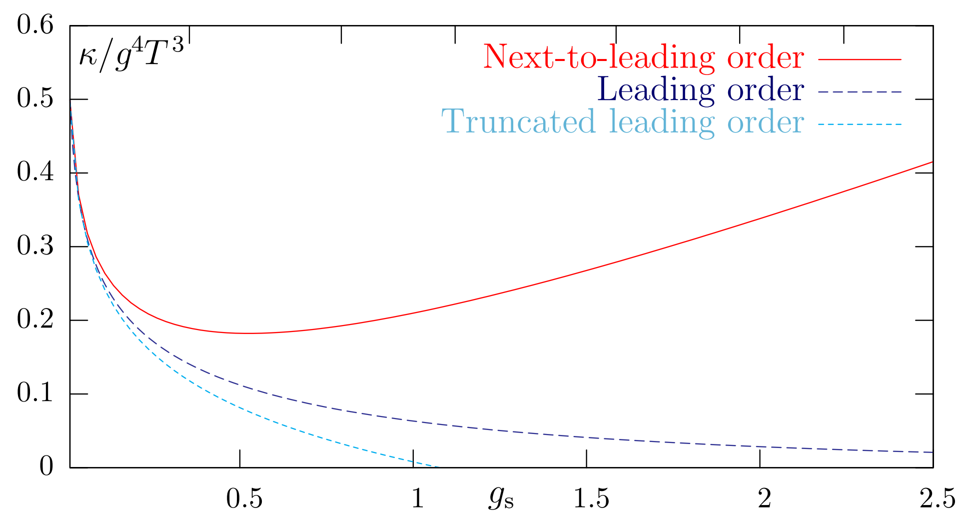
Smooth limit expected from NLO PT

[Caron-Huot, M.Laine, G.Moore, JHEP 0904 (2009) 053]

$$\kappa = \lim_{\omega \to 0} \frac{2T\rho_E(\omega)}{\omega}$$

NLO perturbative calculation:

[Caron-Huot, G. Moore, JHEP 0802 (2008) 081]



→ large correction towards strong interactions

→ non-perturbative lattice methods required

[A.Francis, OK, M.Laine, T.Neuhaus, H.Ohno, PRD92(2015)116003]

Quenched Lattice QCD on large and fine isotropic lattices at T$\simeq$ 1.5 T_c

- standard Wilson gauge action
- algorithmic improvements to enhance signal/noise ratio
- fixed aspect ration N_s/N_t = 4, i.e. fixed physical volume $(2\text{fm})^3$
- perform the continuum limit, a$\to$ 0 $\leftrightarrow$ $N_t \to \infty$
- determine κ in the continuum using an Ansatz for the spectral fct. $\rho(\omega)$
- scale setting using r_0 and t_0 scale [A.Francis,OK,M.Laine, T.Neuhaus, H.Ohno, PRD91(2015)096002]
- multilevel combined with link-integration techniques to improve the signal

β_0	$N_s^3 \times N_\tau$	confs	$T\sqrt{t_0}^{(\mathrm{imp})}$	$T/T_c\|_{t_0}^{(\mathrm{imp})}$	$T\sqrt{t_0}^{(\mathrm{clov})}$	$T/T_c\|_{t_0}^{(\mathrm{clov})}$	Tr_0	$T/T_c\|_{r_0}$
6.872	$64^3 \times 16$	172	0.3770	1.52	0.3805	1.53	1.116	1.50
7.035	$80^3 \times 20$	180	0.3693	1.48	0.3739	1.50	1.086	1.46
7.192	$96^3 \times 24$	160	0.3728	1.50	0.3790	1.52	1.089	1.46
7.544	$144^3 \times 36$	693	0.3791	1.52	0.3896	1.57	1.089	1.46
7.793	$192^3 \times 48$	223	0.3816	1.53	0.3955	1.59	1.084	1.45

similar studies by [Banerjee,Datta,Gavai,Majumdar, PRD85(2012)014510]
[H.B.Meyer, New J.Phys.13(2011)035008]
[Brambilla, Laino, Petreczky, Vairo, arXiv:2007.10078]

[A.Francis, OK et al., PRD92(2015)116003]

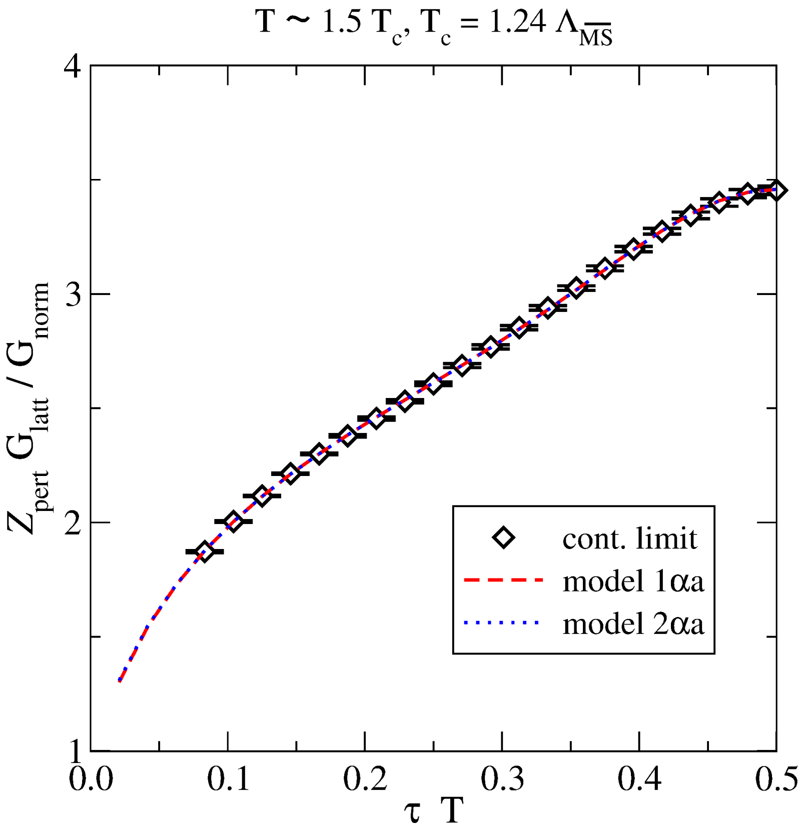
Using continuum extrapolated lattice correlators

Spectral function models with correct asymptotic behavior

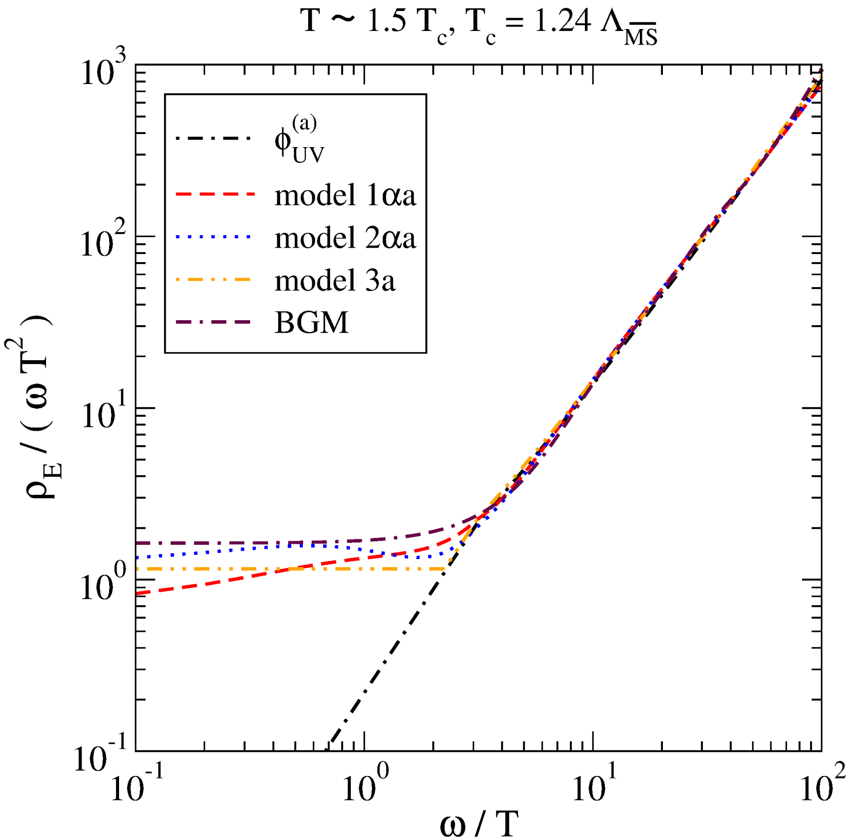
$$\rho_{\mathrm{UV}}(\omega) = \frac{g^2(\bar{\mu}_\omega) C_F \omega^3}{6\pi}$$

modeling corrections to ρ_{IR} by a power series in ω

$$\rho_{\mathrm{IR}}(\omega) = \frac{\kappa\omega}{2T}$$

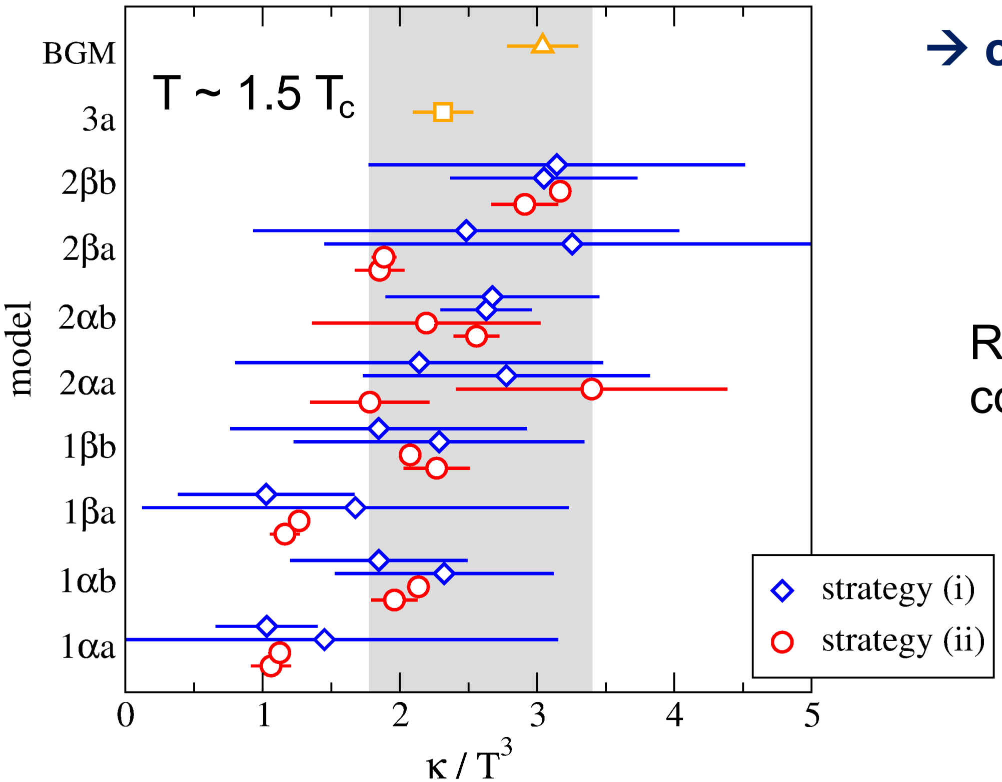


$$G_{\mathrm{model}}(\tau) \equiv \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \rho_{\mathrm{model}}(\omega) \frac{\cosh\left(\frac{1}{2} - \tau T\right)\frac{\omega}{T}}{\sinh \frac{\omega}{2T}}$$



$$\kappa / T^3 = \lim_{\omega \to 0} \frac{2T \rho_{\mathrm{E}}(\omega)}{\omega}$$

[A.Francis, OK et al., PRD92(2015)116003]



Detailed analysis of systematic uncertainties

→ continuum estimate of κ :

$$\kappa/T^3 = \lim_{\omega \to 0} \frac{2T\rho_E(\omega)}{\omega} = 1.8 \ldots 3.4$$

Related to diffusion coefficient D and drag coefficient η_D (in the non-relativistic limit)

$$2\pi T D = 4\pi \frac{T^3}{\kappa} = 3.7 \ldots 7.0$$

$$\eta_D = \frac{\kappa}{2M_{kin}T}\left(1 + O\left(\frac{\alpha_s^{3/2}T}{M_{kin}}\right)\right)$$

time scale associated with the kinetic equilibration of heavy quarks:

$$\tau_{\rm kin} = \frac{1}{\eta_D} = (1.8 \ldots 3.4)\left(\frac{T_c}{T}\right)^2\left(\frac{M}{1.5\ \mathrm{GeV}}\right)\ \mathrm{fm/c}$$

→ close to T$_c$, $\tau_{kin} \simeq$ 1fm/c and therefore charm quark kinetic equilibration appears to be almost as fast as that of light partons.

Gradient Flow as a noise reduction method

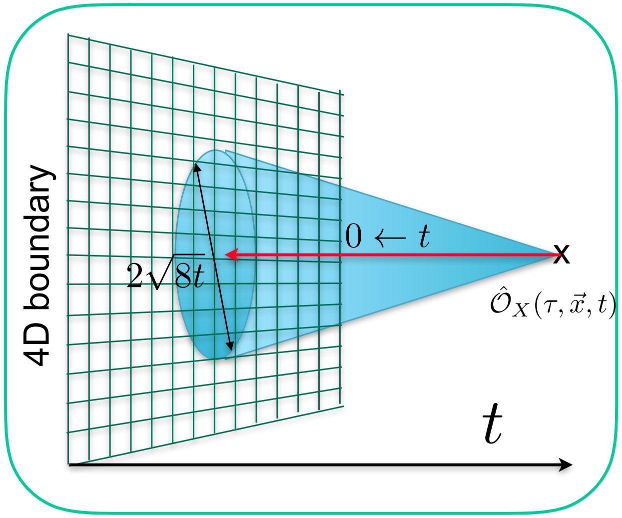
Noise reduction methods used in the quenched approximation not applicable in full QCD

Gradient flow - *diffusion* equation for the gauge fields along extra dimension, *flow-time t*

[M. Lüscher, 2010]

$$\frac{\partial}{\partial t} A_\mu(t,x) = -\frac{\partial S_{\rm YM}}{\partial A_\mu}$$

$$A_\mu(t=0,x) = A_\mu(x)$$



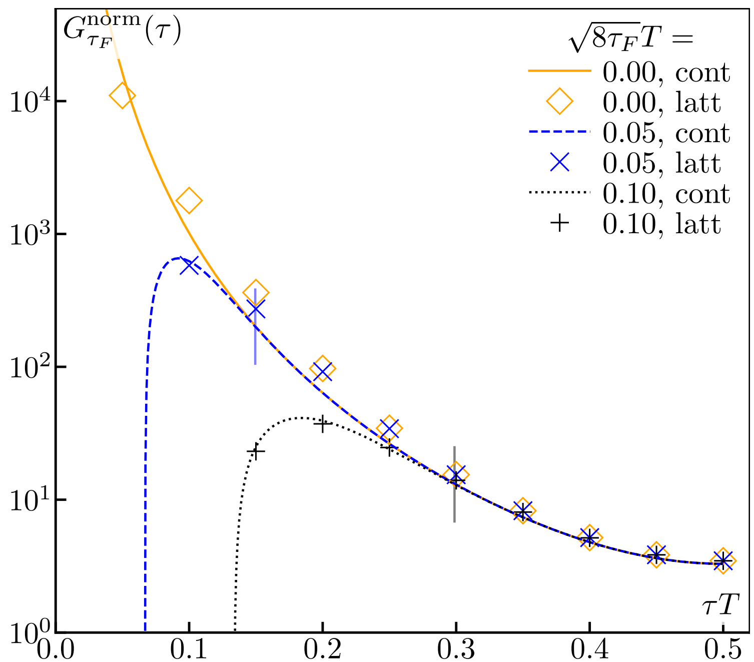
- continuous smearing of the gauge fields, effective smearing radius: $r_{\rm smear} \sim \sqrt{8t}$
- gauge fields become smooth and renormalized
- no UV divergences at finite flow-time *t* → operators of flowed fields are renormalized
- UV fluctuations effectively reduces → noise reduction technique
- Applicable in quenched and full QCD → first case study in quenched

What is the flow time dependence of correlation functions?

How to perform the continuum and t→0 limit correctly?

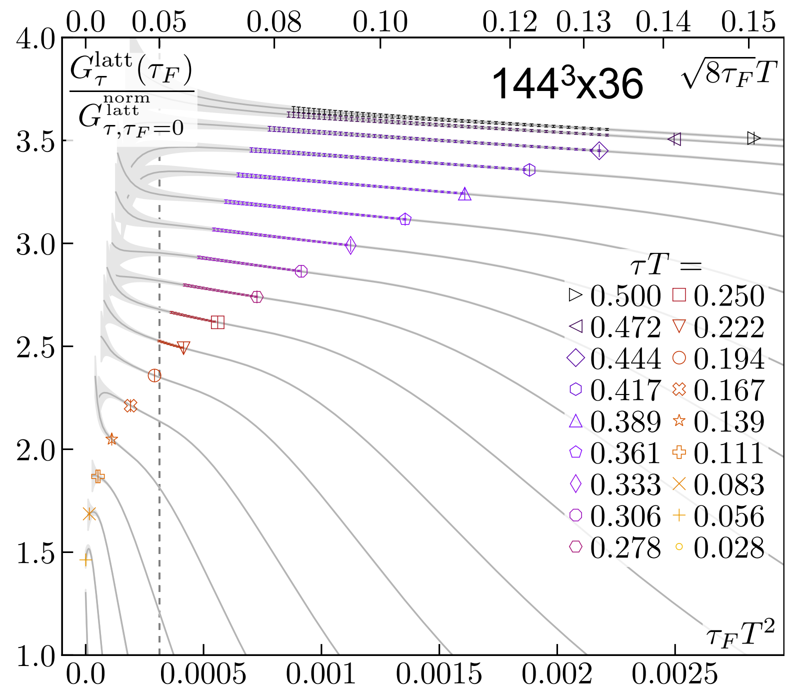
LO perturbative limits for the flow-time dependence:

$$\tilde{\tau}_f < 0.1136(\tau T)^2$$



[A.M Eller, G.D. Moore, PRD97 (2018) 114507]

First lattice QCD results on the flow dependence of the color-electric correlator:

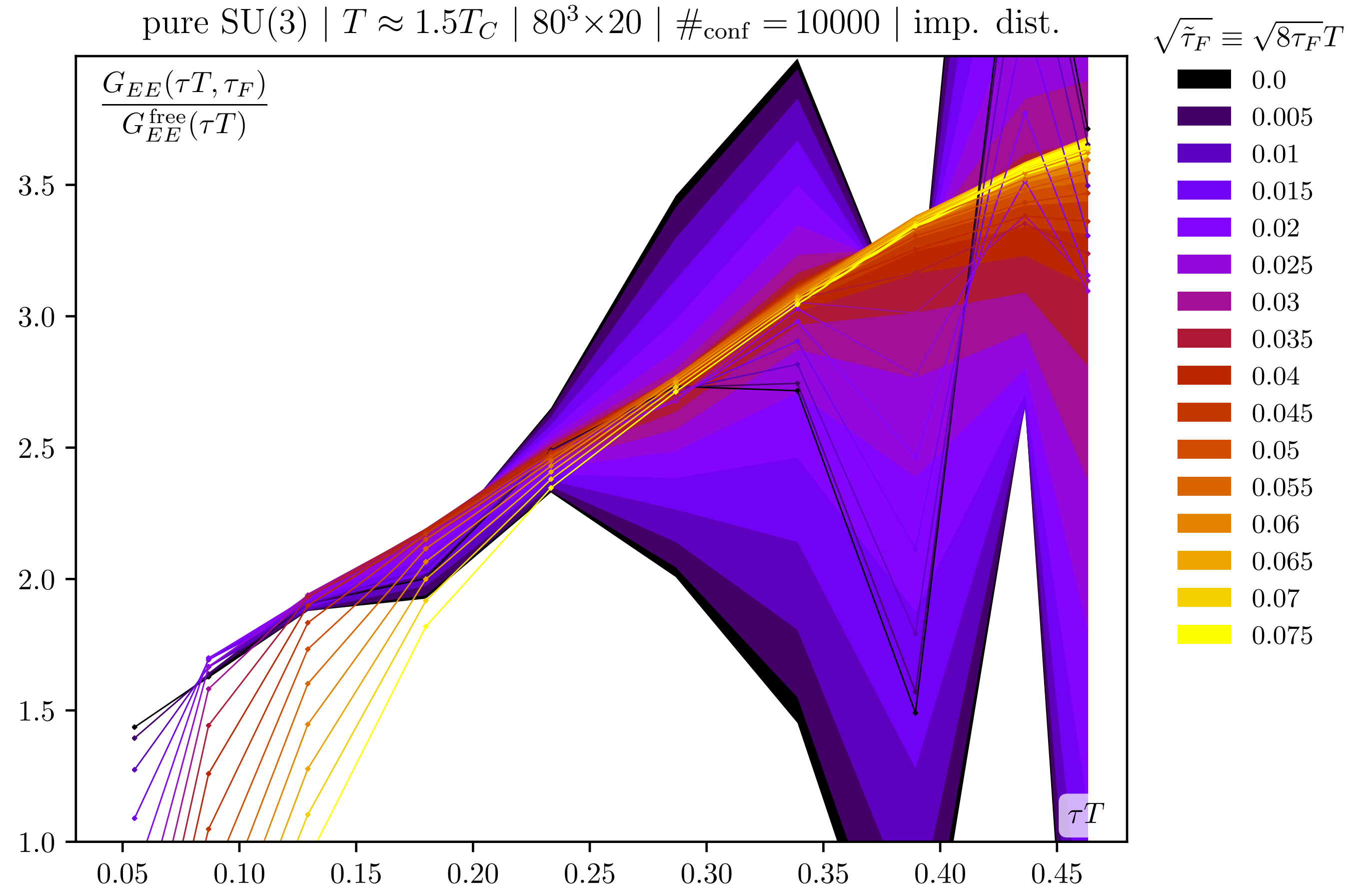


[L. Altenkort, A.M. Eller, L. Mazur, OK, G.D Moore, H.T. Shu, work in progress]

Effective reduction of UV fluctuations → good noise reduction technique

Signal gets destroyed at flow times above the perturbative estimate

Linear behavior at intermediate flow times

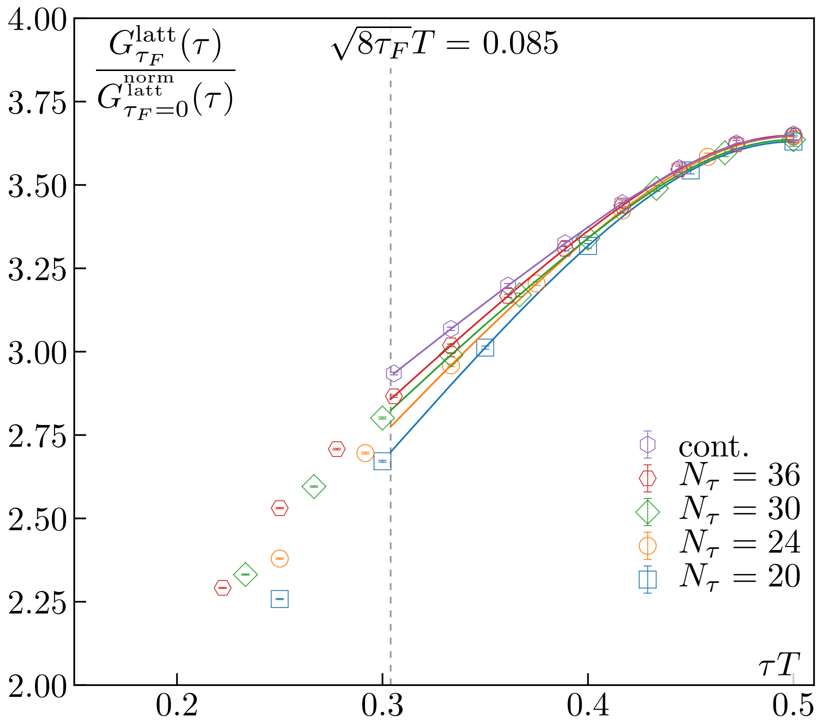


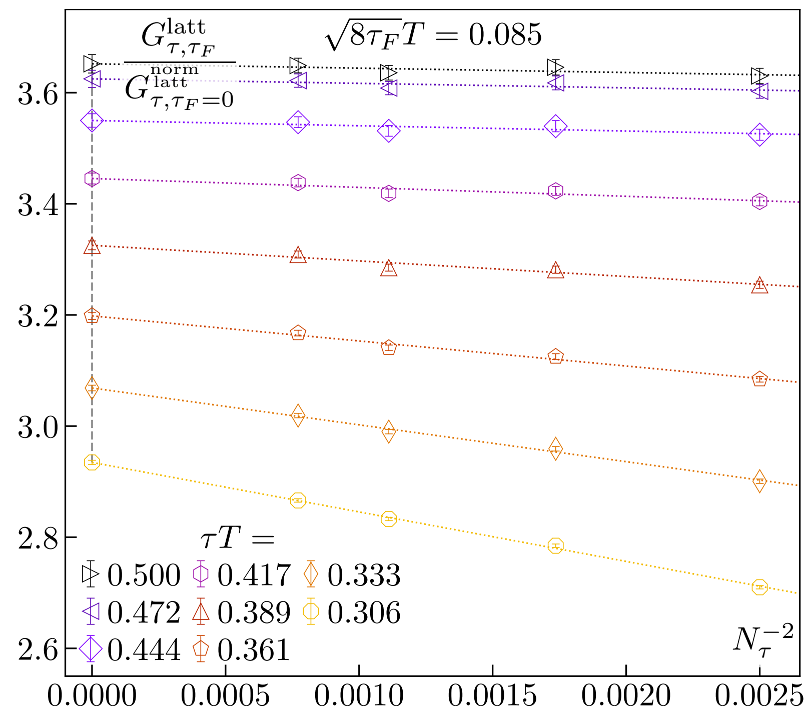
Effective reduction of UV fluctuations → good noise reduction technique

Signal gets destroyed at small distances → large-ω part of the spectral function modified

Strategy: 1) continuum limit at fixed physical flow times 2) flow time limit t→0

- signal to noise ratio strongly improves with flow
- operators become renormalized after some amount of flow
- cut-off effects get reduced with increasing flow time
- continuum limit, *a→0 (N_t→ ∞)*, at fixed physical flow time:



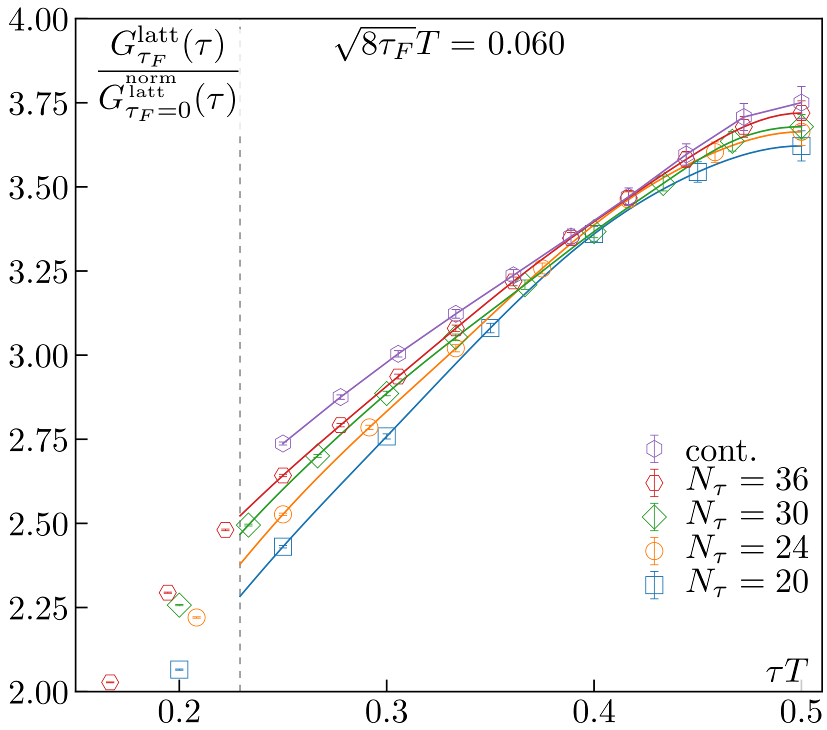


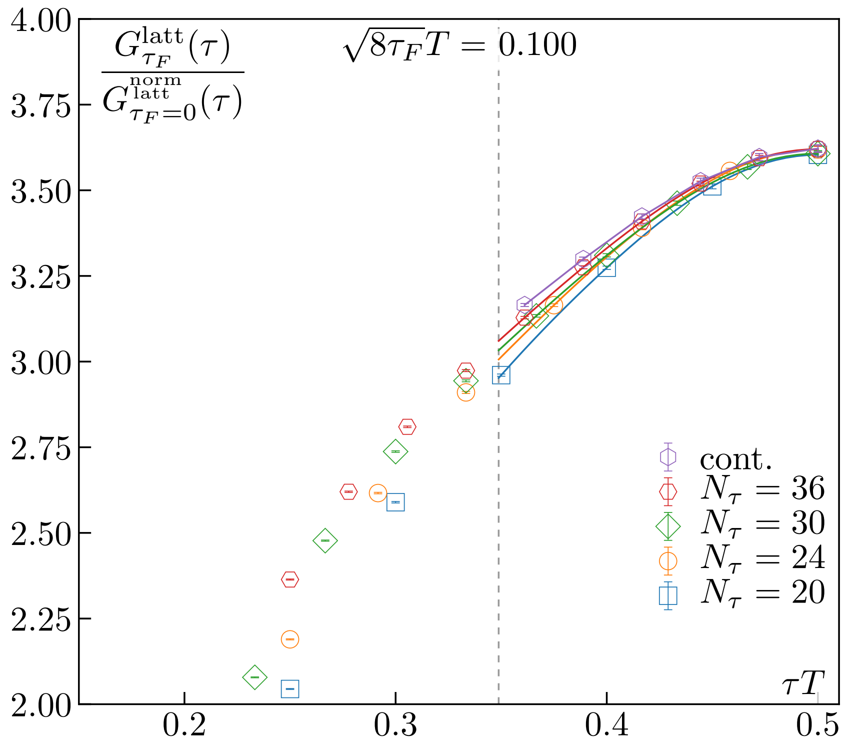
- well defined continuum correlators for different finite flow times

next step: flow time to zero extrapolation of continuum correlators

Gradient Flow method – a→0 limit

- signal to noise ratio strongly improves with flow
- operators become renormalized after some amount of flow
- cut-off effects get reduced with increasing flow time
- continuum limit, *a→0 ($N_t \to \infty$)*, at fixed physical flow time:





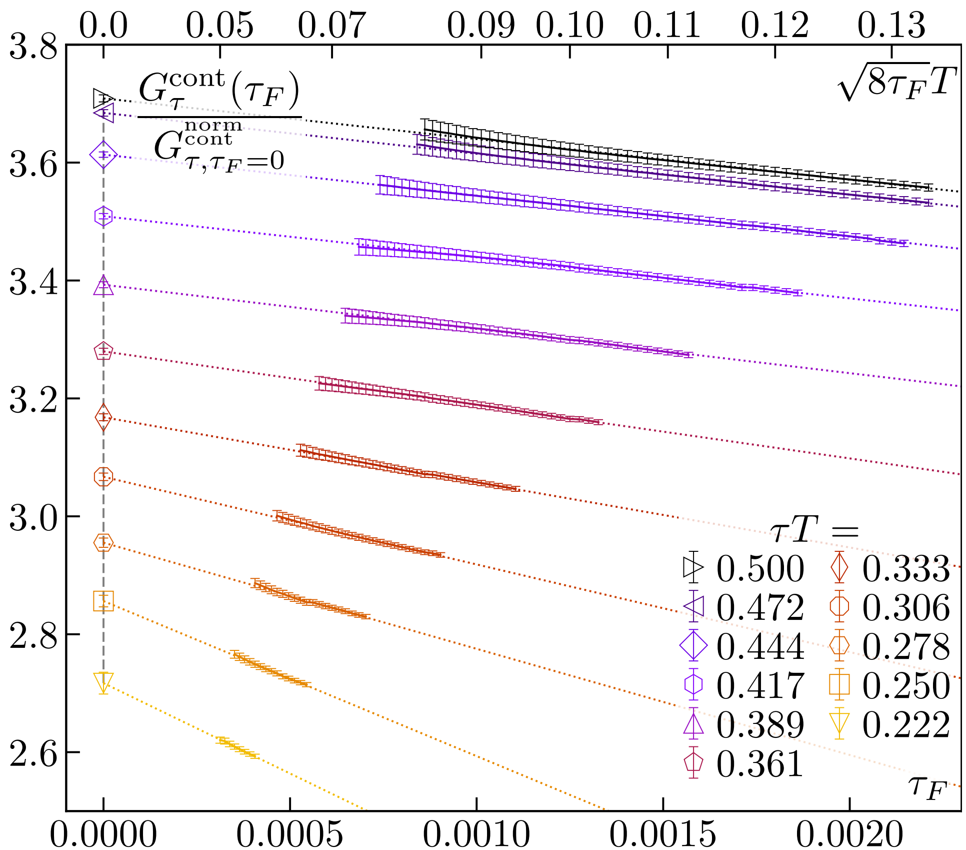
- well defined continuum correlators for different finite flow times

next step: flow time to zero extrapolation of continuum correlators

Continuum limit, *a→0 (N_t→ ∞)*, at fixed physical flow time:

followed by

Flow time limit, *t→0*, for each distance:

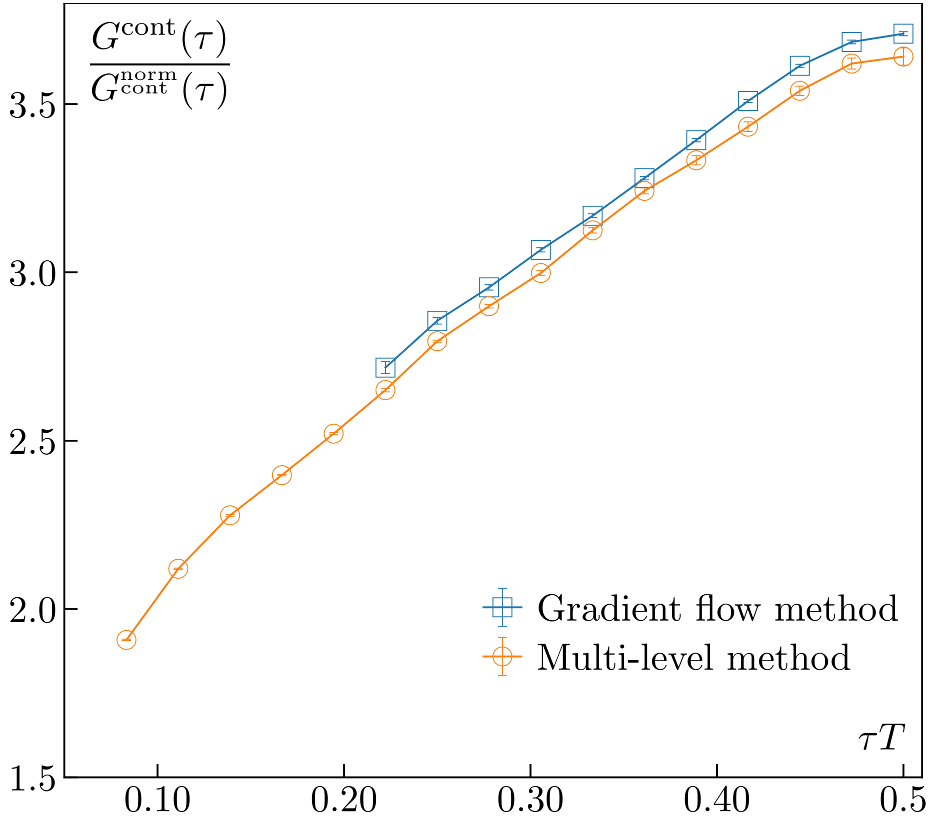


→ well defined continuum and flow time extrapolation

→ well defined renormalized correlation function

Continuum extrapolated EE-correlation function

Comparison of gradient flow and multi-level method:



gradient flow: N_t=16,20,24,30,36

[L. Altenkort, A.M. Eller, L. Mazur, OK, G.D Moore, H.T. Shu, work in progress]

multi-level: N_t=20,24,36,48

with an update in the analysis and perturbative renormalization of data from [A.Francis, OK et al., PRD92(2015)116003]

Comparable τT dependence at large distances

Uncertainty in the renormalization resolved by gradient flow

Heavy quark momentum diffusion coefficient expected to be slightly larger

Extension to full QCD possible using gradient flow method

Well defined methodology to extract spectral and transport properties from lattice QCD

Continuum extrapolated correlators from quenched lattice QCD are

well described by perturbative model spectral functions down to T $\approx$ T_c

for observable with an external scale (mass, momentum) $\gtrsim \pi$T

All results in this talk were obtained in the quenched approximation

What may change when going to full QCD?

$$\Lambda_{\overline{\mathrm{MS}}}|_{N_f=0} \approx 255\mathrm{MeV} \qquad \Lambda_{\overline{\mathrm{MS}}}|_{N_f=3} \approx 340\mathrm{MeV}$$

$$T_c|_{N_f=0} \approx 1.24\Lambda_{\overline{\mathrm{MS}}}|_{N_f=0} \qquad T_c|_{N_f=3} \approx 0.45\Lambda_{\overline{\mathrm{MS}}}|_{N_f=3}$$

$$\alpha_s^{EQCD}|_{T\simeq T_c} \simeq 0.2 \qquad \alpha_s^{EQCD}|_{T\simeq T_c} > 0.3$$

1st order deconfinement transition **chiral crossover transition**

Physics may become more non-perturbative, more interesting, more complicated…

Quenched theory is a nice playground but full QCD studies crucial!

Stay tuned in the next years for the first results in full QCD...