Continuous-Discrete Filtering using the Duncan-Mortensen-Zakai (DMZ) Equation: Smooth Likelihood Surface

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The continuous time state space model consists of a dynamical equation for the unobserved state (stochastic differential equation SDE) and a measurement equation, also described by an SDE. We attempt to estimate the unknown parameters (drift, diffusion, factor loadings, measurement error etc.) by maximum likelihood.

The unnormalized filter density (state probability density at present time, given the measurements up to this time), fulfils a stochastic partial differential equation, the DMZ equation. It is a Fokker-Planck equation with additional potential term containing the measurements. The likelihood function is determined by integration over the unnormalized filter density. In the proposed algorithm, numerical quadrature will be utilized. For continuous-discrete filtering, only integrals at discrete times of measurement must be considered.

As it is well known, standard particle filter algorithms suffer from the problem (due to resampling), that the estimated likelihood is not a smooth function of the model parameters. One can improve the resampling procedure (Malik and Pitt, 2011), but this approach is involved in the multivariate case.

The DMZ equation is transformed to a backward Kolmogorov equation which is solved by Monte Carlo integration using importance sampling (similar to option pricing in finance). Since no resampling is required, we obtain a smooth likelihood surface. A drawback is the possibly large number of grid points where the unnormalized filter density must be evaluated. The results are compared with approximations based on a matrix representation of the Fokker-Planck operator and with particle filtering.