Keynesian Macrodynamics and the Phillips Curve. An Estimated Baseline Macromodel for the U.S. Economy

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Abstract:
In this paper we formulate a baseline disequilibrium AS-AD model and empirically estimate it with time series data for the US-economy. The version of the model used here exhibits a Phillips-curve, a dynamic IS curve and a Taylor interest rate rule. It is based on sticky wages and prices, perfect foresight of current inflation rates and adaptive expectations concerning the inflation climate in which the economy operates. A version of Okun's law is used to link capacity utilization to employment. Our proposed nonlinear 5D model of real market dynamics overcomes anomalies of the old Neoclassical synthesis and also the rational expectations methodology of the new Neoclassical Synthesis. It resembles New Keynesian macroeconomics but permits nonclearing of markets. It exhibits typical Keynesian feedback structures with asymptotic stability of its steady state for low adjustment speeds and with loss of stability – generally by way of Hopf bifurcations – when certain adjustment speeds are made sufficiently large. We provide system estimates of our model, for quarterly time series data of the U.S. economy 1965.1-2001.1, and study the stability features of the U.S. economy with respect to its various feedback channels from an empirical perspective. Based on these estimates, which in particular imply that goods market dynamics are profit led, we find that the dynamics are strongly convergent around the steady state, if monetary policy is sufficiently active, but will lose this feature if the inflationary climate variable or the price inflation rate itself adjusts sufficiently fast. We also study to what extent more active interest rate feedback rules or downward wage rigidity can stabilize the dynamics in the large when the steady state is locally repelling. We study the economy's behavior due to faster adjustments. We find that monetary policy should allow for sufficient steady state inflation in order to avoid stability problems in areas of the phase space where wages are not flexible in a downward direction.

Keywords: AS-AD disequilibrium, wage and price Phillips curves, Okun's law, (in-)stability, persistent fluctuations, monetary policy.

JEL CLASSIFICATION SYSTEM: **E24**, **E31**, **E32**.

1 Introduction

This paper formulates and estimates a Keynesian macroeconomic model for the U.S. economy. It builds as recent New Keynesian macrodynamic model, on gradual wage and price adjustments, employs two Phillips-curves, to relate factor utilization rates and wage and price dynamics and includes a dynamic IS curve and a Taylor interest rate rule. It resembles the New Keynesian macromodels in that it includes elements of forward looking behavior, but it permits nonclearing markets, underutilized labor and capital stock and a mix of myopic perfect foresight and adaptively formed medium run expectations concerning an inflation climate of the economy. In order to link output to employment the paper will make use of a dynamic form of Okun's law. Although our model is akin to the traditional AS-AD model the resulting nonlinear 5D model of nonclearing labor and goods market avoids the anomalies of the conventional AS-AD model.

Our approach exhibits similarities but also difference to New Keynesian macroeconomics. We use the same formal structure for the variables that drive wage and price inflation rates (utilizations rates and real wages), but with a microfoundations that are for example based on the Blanchard and Katz (1999) reconciliation of wage Phillips curves and current labor market theories. The basic difference in the wage-price module is that we augment this structure by hybrid expectations formation where the forward-looking part is based on a neoclassical type of dating and where expectations are of cross-over type – we have price inflation expectations in the wage Phillips curve (wage PC) and wage inflation expectations in the price Phillips curve (price PC). Our formulation of the wage-price dynamics permits therefore an interesting comparison to New Keynesian work that allows for both staggered price and wage setting. Concerning the IS-curve we make use of a law of motion for the rate of capacity utilization of firms that depends on the level of capacity utilization (the dynamic multiplier), the real rate of interest and finally on the real wage and thus on income distribution. New Keynesian authors often use a purely forward-looking IS-curve (with only the real rate of interest effect) and a Phillips curve which has been criticized from the empirical point of view; see in particular Fuhrer and Rudebusch (2004) and Eller and Gordon (2003). Since we distinguish between the rate of employment of the labor force and that of the capital stock, namely the rate of capacity utilization, we employ some form of Okun's law to relate capacity utilization to employment.

The present paper intends to provide empirical evidence on a baseline model of a Keynesian disequilibrium AS-AD (DAS-DAD) variety. It presents the feedback structures of this (semi-)reduced form of a macromodel and its stability implications, first on a general level and then on the level of the sign and size restrictions obtained from empirical estimates of the five laws of motion of the dynamics. These estimates, undertaken from the U.S. economy for quarterly data 1965.1-2001.1 also allow us to discuss asymptotic stability for the estimated parameter sizes and to determine stability boundaries.

The remainder of the paper is organized as follows. Section 2 considers, the New Keynesian macrodynamic model with staggered wage and price setting in a deterministic and continuous time framework. Section 3 then presents our formulation of a baseline Keynesian DAS-DAD growth dynamics such that is suitable to empirical estimation. Section 4 considers the feedback chains of the reformulated model and derives cases of local asymptotic stability and of loss of stability by way of Hopf-bifurcations. In section 5 we then estimate the model to find out sign and size restrictions for its behavioral equations and we study which type of feedback mechanisms may apply to the US-economy after World War II. Section 6 investigates in detail the stability properties of the estimated model. Section 7 analyzes on the one hand the stability problems that occur when there is a floor to money wage deflation and the role of monetary policy in such a case. Section 8 discusses related literature and Section 9 concludes. Details of the estimation results are presented in an appendix of the paper.

2 New Keynesian macrodynamics

In this section we consider briefly the modern analog to the old neoclassical synthesis, namely the New Keynesian approach to macrodynamics in its advanced form, where both staggered price setting and staggered wage setting are assumed. We here follow Woodford (2003, p.225) in his formulation of staggered wages and prices, where their joint evolution is coupled with the usual forward-looking output dynamics, and in addition coupled with a derived law of motion for real wages. Here we shall only briefly look at this extended approach and leave to later sections of the paper a consideration of the similarities and differences between these New Keynesian dynamics and our own approach.

Woodford (2003, p.225) basically makes use of the following two loglinear equations for describing the joint evolution of wages and prices (d the backward oriented difference operator[1]).

$$\begin{aligned} d\ln w_t &\overset{WPC}{=} \beta E_t(d\ln w_{t+1}) + \beta_{wy}\ln Y_t - \beta_{w\omega}\ln\omega_t, \\ d\ln p_t &\overset{PPC}{=} \beta E_t(d\ln p_{t+1}) + \beta_{py}\ln Y_t + \beta_{p\omega}\ln\omega_t, \end{aligned}$$

where all parameters are assumed to be positive. Our first aim here is to derive the continuous time analog to these two equations (and the other equations of the full model) and to show on this basis how this extended model is solved in the spirit of the rational expectations school.

In a deterministic setting we obtain from the above

$$\begin{aligned} d\ln w_{t+1} &\overset{WPC}{=} \frac{1}{\beta}[d\ln w_t - \beta_{wy}\ln Y_t + \beta_{w\omega}\ln\omega_t], \\ d\ln p_{t+1} &\overset{PPC}{=} \frac{1}{\beta}[d\ln p_t - \beta_{py}\ln Y_t - \beta_{p\omega}\ln\omega_t]. \end{aligned}$$

If we assume (as we do in all of the following and without much loss in generality) that the parameter β is not only close to one, but in fact set equal to one, then the last two equations can be expressed as

$$\begin{aligned} d\ln w_{t+1} - d\ln w_t &\overset{WPC}{=} -\beta_{wy}\ln Y_t + \beta_{w\omega}\ln\omega_t, \\ d\ln p_{t+1} - d\ln p_t &\overset{PPC}{=} -\beta_{py}\ln Y_t - \beta_{p\omega}\ln\omega_t. \end{aligned}$$

[1]We make use of this convention throughout this paper and thus have to write $r_{t-1} - dp_t$ for the real rate of interest.

Denoting by π^w the rate of wage inflation and by π^p the rate of price inflation (both indexed by the end of the corresponding period) we then obtain the continuous time dynamics (with $\ln Y = y$ and $\theta = \ln \omega$):

$$\boxed{\begin{aligned} \dot{\pi}^w &\stackrel{WPC}{=} -\beta_{wy} y + \beta_{w\omega}\theta, \\ \dot{\pi}^p &\stackrel{PPC}{=} -\beta_{py} y - \beta_{p\omega}\theta. \end{aligned}}$$

From the output dynamics of the New Keynesian approach, namely

$$y_t = y_{t+1} - \alpha_{yi}(i_t - \pi^p_{t+1} - i_0), \quad i.e., \quad y_{t+1} - y_t = \alpha_{yi}(i_t - \pi^p_{t+1} - i_0),$$

we obtain the continuous time reduced form law of motion

$$\boxed{\dot{y} \stackrel{IS}{=} \alpha_{yi}[(\beta_{i\pi} - 1)\pi^p + (\beta_{iy} + \beta_{py})y + \beta_{p\omega}\theta)],}$$

where we have already inserted an interest rate policy rule in order to (hopefully) obtain determinacy as in the New Keynesian baseline model, which is known to be indeterminate for the case of an interest rate peg. Here we have chosen the simple Taylor interest rate policy rule

$$i = i_t = i_o + \beta_{i\pi}\pi + \beta_{iy} y,$$

see Walsh (2003, p.247), which is of a classical Taylor rule type (though without interest rate smoothing yet).

There remains finally the law of motion for real wages to be determined, which setting $\theta = \ln \omega$ simply reads

$$\boxed{\dot{\theta} = \pi^w - \pi^p.}$$

We thus get from this extended New Keynesian model an autonomous linear dynamical system, in the variables π^w, π^p, y and θ. The, in general, uniquely determined steady state of the dynamics is given by $(0.0, 0, i_o)$. From the definition of θ we see that the model exhibits four forward-looking variables, in direct generalization of the baseline New Keynesian model with only staggered price setting. Searching for a zone of determinacy of the dynamics (appropriate parameter values that make the steady state the only bounded solution of the dynamics to which the economy then immediately returns after isolated shocks of any type) thus requires establishing conditions under which all roots of the Jacobian have positive real parts.

The Jacobian of the 4D dynamical system under consideration reads:

$$J = \begin{pmatrix} 0 & 0 & -\beta_{wy} & \beta_{w\omega} \\ 0 & 0 & -\beta_{py} & -\beta_{p\omega} \\ 0 & \alpha_{yi}(\beta_{i\pi}-1) & \alpha_{yi}(\beta_{iy}+\beta_{py}) & \alpha_{yi}\beta_{p\omega} \\ 1 & -1 & 0 & 0 \end{pmatrix}.$$

For the determinant of this Jacobian we calculate

$$-|J| = (\beta_{wy}\beta_{p\omega} + \beta_{py}\beta_{w\omega})\alpha_{yi}(\beta_{i\pi}-1) \gtreqless 0 \quad \text{iff} \quad \beta_{i\pi} \gtreqless 1.$$

We thus get that an active monetary policy of the conventional type (with $\beta_{i\pi} > 1$) is – compared to the baseline New Keynesian model – no longer appropriate to ensure determinacy (for which a positive determinant of J is a necessary condition). One can show in addition, see Chen, Chiarella, Flaschel and Hung (2004), via the minors of order 3 of the Jacobian J, that the same holds true for a passive monetary policy rule, i.e., the model in this form must be blocked out from consideration. There consequently arises the necessity to specify an extended or modified active Taylor interest rate policy rule from which one can then obtain determinacy for the resulting dynamics, i.e., the steady state as the only bounded solution and therefore, according to the logic of the rational expectations approach, the only realized situation in this deterministic set-up. This would then generalize the New Keynesian baseline model with only staggered prices, which is known to be indeterminate in the case of an interest rate peg or a passive monetary policy rule, but which exhibits determinacy for a conventional Taylor rule with $\beta_{i\pi} > 1$.

We briefly observe here that when the discrete time dynamics makes use of a system matrix J the system matrix of the continuous time analog is given by $J - I$, I the identity matrix. The eigenvalues of the discrete time case are thus all shifted to the left by 1 in the continuous time analog. In the case of the considered dynamics this means that determinacy in the continuous time case implies determinacy in the discrete time case, but the same does not at all hold for indeterminacy in the place of determinacy. The discrete time case therefore can be determinate, though the continuous time case has been shown to be indeterminate, for example simply because the stable roots of the continuous time case are situated to the left of -1. In this example, a very stable root in the continuous system may cause strong overshooting divergence in the discrete situation and thus turn stable roots into unstable ones. We would consider the occurrence of such a situation as resulting from over-synchronization in the considered market structure, since theoretical discrete time systems are then allowed only to react in the discrete point in time $t, t{+}1, \ldots$ Depending on the period length that is underlying the model this can mean (in the case of one quarter) that shopping can only be done every three months which – if implemented by law on an actual economy – would make it probably a very unstable one. Discrete time modeling is important in empirical analysis due to data availability, but should not be implemented as a theoretical model, unless it can be checked that it is not in stark contradiction compared to the case where all difference quotients are replaced by differential quotients. There are processes in agriculture and biology where discrete time analysis is reasonable by itself, but this statement does not carry over to the macrolevel of industrialized economies, where staggered price and wage setting is not restricted to four points in time within a year, and where therefore an assumption of this type can give rise to instability results simply due to over-synchronization and a lack of smoothness, aspects that are very questionable from an macroeconomic point of view. We conclude that the lack of determinacy in continuous time is also a problem for the discrete time analogue that should not be overcome by making the period length so large that stable processes are in fact turned into unstable ones.

There are a variety of critical arguments raised in the literature against the New Phillips Curve (NPC) of the baseline model of Keynesian macrodynamics, see in particular Mankiw (2001) and recently Eller and Gordon (2003) for particularly strong statements.[2] These and other criticisms in our view will also apply to the above extended wage and price dynamics. In order to overcome these and other critiques we here propose some modifications to the above presentation of the wage-price dynamics which will remove from it completely the questionable feature of a sign reversal for the role of output and wage gaps. This sign reversal is caused by the fact that future values of the considered state values are used on the right hand side of their determining equations, which implies that the time rates of change of these variables depend on output and wage gaps with a reversed sign in front on them. These sign reversals are at the root of the problem when the empirical relevance of such NPC's is investigated. We instead will make use of the following expectations augmented wage and price Phillips curves:

$$
\begin{aligned}
d\ln w_{t+1} &\overset{WPC}{=} \kappa_w d\ln p_{t+1} + (1-\kappa_w)\pi_t^m + \beta_{wy}\ln Y_t - \beta_{w\omega}\ln \omega_t],\\
d\ln p_{t+1} &\overset{PPC}{=} \kappa_p d\ln w_{t+1} + (1-\kappa_p)\pi_t^m + \beta_{py}\ln Y_t + \beta_{p\omega}\ln \omega_t].
\end{aligned}
$$

We have modified the New Keynesian approach to wage and price dynamics here only with respect to the terms that concern expectations, in order to generate the potential for a wage-price spiral. We first assume that expectations formation is of a crossover type, with perfectly foreseen price inflation in the wage PC of workers and perfectly foreseen wage inflation in the price PC of firms. Furthermore, we make use in this regard of a neoclassical dating in the considered PC's, which means that – as is usually the case in the reduced form PC – we have the same dating for expectations and actual wage and price formation on both sides of the PC's. Finally, following Chiarella and Flaschel (1996), we assume expectations formation to be of a hybrid type, where a certain weight is given to current (perfectly foreseen) inflation rates and the counterweight attached to a concept that we have dubbed the inflationary climate π^m that is surrounding the currently evolving wage-price spiral. We thus assume that workers as well as firms to a certain degree pay attention to whether the current situation is embedded in a high inflation regime or in a low inflation one.

These relatively straightforward modifications of the New Keynesian approach to expectations formation will imply for the dynamics of what we call a matured traditional Keynesian approach – to be completed in the next section – radically different solutions and stability features, with in particular no need to single out the steady state as the only relevant situation for economic analysis in the deterministic set-up here considered. Concerning microfoundations for the assumed wage-price spiral we here only note that the wage PC can be microfounded as in Blanchard and Katz (1999), using standard

[2] With respect to the New Phillips curve it is stated in Mankiw (2001): "Although the new Keynesian Phillips curves has many virtues, it also has one striking vice: It is completely at odd with the facts."

labor market theories, giving rise to nearly exactly the form shown above (with the unemployment gap in the place of the logarithm of the output gap) if hybrid expectations formation is in addition embedded into their approach. Concerning the price PC a similar procedure may be applied based on desired markups of firms. Along these lines one in particular gets an economic motivation for the inclusion of – indeed the logarithm of – the real wage (or wage share) with negative sign into the wage PC and with positive sign into the price PC, without any need for loglinear approximations. We furthermore use the (un-)employment gap and the capacity utilization gap in these two PC's, respectively, in the place of a single measure (the log of the output gap). We conclude that the above wage-price spiral is an interesting alternative to the – theoretically rarely discussed and empirically questionable – New Keynesian form of wage-price dynamics. This wage-price spiral will be embedded into a complete Keynesian approach in the next section, exhibiting a dynamic IS-equation as in Rudebusch and Svensson (1999), but now also including real wage effects and thus a role for income distribution, exhibiting furthermore Okun's law as the link from goods to labor markets, and exhibiting of course the classical type of a Taylor interest rate policy rule in the place of an LM-curve.

3 Keynesian disequilibrium dynamics: Empirical reformulation of a baseline model

In this section we reformulate the theoretical disequilibrium model of AS-AD growth of Asada, Chen, Chiarella and Flaschel (2004) in order to obtain a somewhat simplified version that is more suitable for empirical estimation and for the study of the role of contemporary interest rate policy rules. We dispense with the LM curve of the original approach and replace it here by a Taylor type policy rule. In addition we use dynamic IS as well as employment equations in the place of the originally static ones, where with respect to the former the dependence of consumption and investment on income distribution now only appears in an aggregated format. We use Blanchard and Katz (1999) error correction terms both in the wage and the price Phillips curve and thus give income distribution a role to play in wage as well as in price dynamics. Finally, we will again have inflationary inertia in a world of myopic perfect foresight through the inclusion of a medium-run variable, the inflationary climate in which the economy is operating, and its role for the wage - price dynamics of the considered economy.

We start from the observation that a Keynesian model of aggregate demand fluctuations should (independently of whether justification can be found for this in Keynes' General Theory) allow for under- (or over-)utilized labor *as well as* capital in order to be general enough from the descriptive point of view. As Barro (1994) for example observes, IS-LM is (or should be) based on imperfectly flexible wages and prices and thus on the consideration of wage as well as price Phillips Curves. This is precisely what we will do in the following, augmented by the observation that also medium-run aspects count both in wage and price adjustments, here formulated in simple terms by the introduction of the concept of an inflation climate. We have moreover model-consistent expectations with respect to short-run wage and price inflation. The modification of the traditional AS-AD model that we shall consider thus treats – as already described in the preceding section – expectations in a hybrid way, with crossover myopic perfect foresight of the currently evolving rates of wage and price inflation on the one hand and an adaptive updating of an inflation climate expression with exponential or any other weighting schemes on the other hand.

We consequently assume, see also the preceding section, two Phillips Curves in the place of only one. In this way, we can discuss wage and price dynamics separately from each other, in their structural forms, now indeed both based on their own measure of demand pressure, namely $V^l - \bar{V}^l, V^c - \bar{V}^c$, in the market for labor and for goods, respectively. We here denote by V^l the rate of employment on the labor market and by $\bar{V}^l$ the NAIRU-level of this rate, and similarly by V^c the rate of capacity utilization of the capital stock and $\bar{V}^c$ the normal rate of capacity utilization of firms. These demand pressure influences on wage and price dynamics, or on the formation of wage and price inflation rates, $\hat{w}, \hat{p}$, are both augmented by a weighted average of corresponding cost-pressure terms, based on forward looking myopic perfect foresight $\hat{p}, \hat{w}$, respectively, and a backward looking measure of the prevailing inflationary climate, symbolized by π^m.

We thereby arrive at the following two Phillips Curves for wage and price inflation, which in this core version of Keynesian AS-AD dynamics are – from a qualitative perspective – formulated in a fairly symmetric way.[3] We stress that we include forward-looking behavior here, without the need for an application of the jump variable technique of the rational expectations school in general and the New Keynesian approach in particular as will be shown in the next section.[4]

The structural form of the wage-price dynamics:

$$\hat{w} = \beta_{w_1}(V^l - \bar{V}^l) - \beta_{w_2}(\ln \omega - \ln \omega_o) + \kappa_w \hat{p} + (1 - \kappa_w)\pi^m, \quad (1)$$

$$\hat{p} = \beta_{p_1}(V^c - \bar{V}^c) + \beta_{p_2}(\ln \omega - \ln \omega_o) + \kappa_p \hat{w} + (1 - \kappa_p)\pi^m. \quad (2)$$

Somewhat simplified versions of these two Phillips curves have been estimated for the US-economy in various ways in Flaschel and Krolzig (2004), Flaschel, Kauermann and Semmler (2004) and Chen and Flaschel (2004) and have been found to represent a significant improvement over the conventional single reduced-form Phillips curve. A particular finding was that wage flexibility was greater than price flexibility with respect to their demand pressure measure in the market for goods and for labor,[5] respectively, and workers were more short-sighted than firms with respect to their cost pressure terms. Note that such a finding is not possible in the conventional framework of a single reduced-form Phillips curve. Inflationary expectations over the medium run, π^m, i.e., the inflationary climate in which current inflation is operating, may be adaptively following the actual rate of inflation (by use of some linear or exponential weighting scheme), may be based on a rolling sample (with hump-shaped weighting schemes), or on other

[3]With respect to empirical estimation one could also add the role of labor productivity growth. But this will not be done here in order to concentrate on the cycle component of the model, caused by changing income distribution in a world of stable goods market and interest rate dynamics. With respect to the distinction between real wages and unit wage costs we shall therefore detrend the corresponding time series such that the following types of PC's can still be applied.

[4]For a detailed comparison with the New Keynesian alternative to our model type see Chiarella, Flaschel and Franke (2004).

[5]For lack of better terms we associate the degree of wage and price flexibility with the size of the parameters β_{w_1}, β_{p_1}, though of course the extent of these flexibilities will also depend on the size of the fluctuations of the excess demands in the market for labor and for goods, respectively.

possibilities for updating expectations. For simplicity of the exposition we shall make use of the conventional adaptive expectations mechanism in the theoretical part of this paper, namely

$$\boxed{\dot{\pi}^m = \beta_{\pi^m}(\hat{p} - \pi^m). \qquad (3)}$$

Note that for our current version of the wage-price spiral, the inflationary climate variable does not matter for the evolution of the real wage $\omega = w/p$, the law of motion of which is given by (with $\kappa = 1/(1 - \kappa_w \kappa_p)$):

$$\hat{\omega} \;=\; \kappa[(1-\kappa_p)(\beta_{w_1}(V^l - \bar{V}^l) - \beta_{w_2}(\ln\omega - \ln\omega_o)) - (1-\kappa_w)(\beta_{p_1}(V^c - \bar{V}^c) + \beta_{p_2}(\ln\omega - \ln\omega_o))],$$

This follows easily from the following obviously equivalent representation of the above two PC's,

$$\begin{aligned} \hat{w} - \pi^m &= \beta_{w_1}(V^l - \bar{V}^l) - \beta_{w_2}(\ln\omega - \ln\omega_o) + \kappa_w(\hat{p} - \pi^m), \\ \hat{p} - \pi^m &= \beta_{p_1}(V^c - \bar{V}^c) + \beta_{p_2}(\ln\omega - \ln\omega_o)) + \kappa_p(\hat{w} - \pi^m), \end{aligned}$$

by solving for the variables $\hat{w} - \pi^m$ and $\hat{p} - \pi^m$. It also implies the following two across-markets or <u>reduced form PC's</u>:

$$\begin{aligned} \hat{p} = &\quad \kappa[\beta_{p_1}(V^c - \bar{V}^c) + \beta_{p_2}(\ln\omega - \ln\omega_o) + \kappa_p(\beta_{w_1}(V^l - \bar{V}^l) - \beta_{w_2}(\ln\omega - \ln\omega_o))] + \pi^m, \\ \hat{w} = &\quad \kappa[\beta_{w_1}(V^l - \bar{V}^l) - \beta_{w_2}(\ln\omega - \ln\omega_o)) + \kappa_w(\beta_{p_1}(V^c - \bar{V}^c) + \beta_{p_2}(\ln\omega - \ln\omega_o))] + \pi^m, \end{aligned}$$

which represent a considerable generalization of the conventional view of a single-market price PC with only one measure of demand pressure, the one in the labor market.

The remaining laws of motion of the private sector of the model are as follows:

$$\boxed{\begin{aligned} \hat{V}^c &= -\alpha_{V^c}(V^c - \bar{V}^c) \pm \alpha_\omega(\ln\omega - \ln\omega_o) - \alpha_r((r - \hat{p}) - (r_o - \bar{\pi})), && (4) \\ \hat{V}^l &= \alpha_{V_1^l}(V^c - \bar{V}^c) + \alpha_{V_2^l}\hat{V}^c. && (5) \end{aligned}}$$

The first law of motion is of the type of a dynamic IS-equation, see also Rudebusch and Svensson (1999) in this regard, here represented by the growth rate of the capacity utilization rate of firms. It has three important characteristics; (i) it reflects the dependence of output changes on aggregate income and thus on the rate of capacity utilization by assuming a negative, i.e., stable dynamic multiplier relationship in this respect, (ii) it shows the joint dependence of consumption and investment on the real wage (which in the aggregate may in principle allows for positive or negative signs before the parameter α_ω, depending on whether consumption or investment is more responsive to real wage changes), and (iii) it shows finally the negative influence of the real rate of interest on the evolution of economic activity. Note here that we have generalized this law of motion in comparison to the one in the original baseline model of Asada, Chen, Chiarella and

Flaschel (2004), since we now allow for the possibility that also consumption, not only investment, depends on income distribution as measured by the real wage. We note that we also use $\ln \omega$ in the dynamic multiplier equation, since this variable will be used later on to estimate this equation.

In the second law of motion, for the rate of employment, we assume that the employment policy of firms follows – in the form of a generalized Okun Law – the rate of capacity utilization (and the thereby implied rate of over- or underemployment of the employed workforce) partly with a lag (measured by $1/\beta_{V_1^l}$), and partly without a lag (through a positive parameter $\alpha_{V_2^l}$). Employment is thus assumed to adjust to the level of current activity in somewhat delayed form which is a reasonable assumption from the empirical point of view. The second term, $\alpha_{V_2^l}\hat{V}^c$, is added to take account of the possibility that Okun's Law may hold in level form rather than in the form of a law of motion, since this latter dependence can be shown to be equivalent to the use of a term $(V^c/\bar{V}^c)^{\alpha_{V_2^l}}$ when integrated, i.e., the form of Okun's law in which this law was originally specified by Okun (1970) himself.

The above two laws of motion therefore reformulate in a dynamic form the static IS-curve (and the employment this curve implies) that was used in Asada, Chen, Chiarella and Flaschel (2004). They only reflect implicitly the there assumed dependence of the rate of capacity utilization on the real wage, due to on smooth factor substitution in production (and the measurement of the potential output this implies in Asada, Chen, Chiarella and Flaschel (2004))), which constitutes another positive influence of the real wage on the rate of capacity utilization and its rate of change. This simplification helps to avoid the estimation of separate equations for consumption and investment C, I and for potential output Y^p.

Finally, we no longer to employ here a law of motion for real balances as was still the case in Asada, Chen, Chiarella and Flaschel (2004). Money supply is now accommodating to the interest rate policy pursued by the central bank and thus does not feedback into the core laws of motion of the model. As interest rate policy we assume the following classical type of Taylor rule:

$$r^* = (r_o - \bar{\pi}) + \hat{p} + \alpha_p(\hat{p} - \bar{\pi}) + \alpha_{V^c}(V^c - \bar{V}^c) \tag{6}$$
$$\dot{r} = \alpha_r(r^* - r) \tag{7}$$

The target rate of the central bank r^* is here made dependent on the steady state real rate of interest augmented by actual inflation back to a nominal rate, and is as usually dependent on the inflation gap and the capacity utilization gap (as measure of the output gap). With respect to this target there is then interest rate smoothing with strength α_r. Inserting r^* and rearranging terms we get from this expression the following from of a Taylor rule

$$\boxed{\dot{r} = -\gamma_r(r - r_o) + \gamma_p(\hat{p} - \bar{\pi}) + \gamma_{V^c}(V^c - \bar{V}^c) \qquad (8)}$$

where we have $\gamma_r = \alpha_r, \gamma_p = \alpha_r(1 + \alpha_p), i.e., \alpha_p = \gamma_p/\alpha_r - 1 \quad \text{and} \quad \gamma_{V^c} = \alpha_r\alpha_{V^c}$.

We thus allow now for interest rate smoothing in this rule in contrast to section 2. Furthermore, the actual (perfectly foreseen) rate of inflation $\hat{p}$ is used to measure the inflation gap with respect to the inflation target $\bar{\pi}$ of the central bank. Note finally that we could have included (but have not done this here yet) a new kind of gap into the above Taylor rule, the real wage gap, since we have in our model a dependence of aggregate demand on income distribution and the real wage. The state of income distribution matters for the dynamics of our model and thus should also play a role in the decisions of the central bank. All of the employed gaps are measured relative to the steady state of the model, in order to allow for an interest rate policy that is consistent with it.

We note that the steady state of the dynamics, due to its specific formulation, can be supplied exogenously. For reasons of notational simplicity we choose: $V_o^c = \bar{V}^c = 1, V_o^l = \bar{V}^l = 1, \omega_o = 1, \pi_o^m = \bar{\pi} = 0.005, r_o = 0.02$ in the later estimation of the model by means of quarterly US-data. As the model is formulated now it exhibits five gaps, to be closed in the steady state and has five laws of motion, which when set equal to zero, exactly imply this result, since the determinant of the Jacobian of the dynamics is shown to be always non-zero in the next section of the paper. Note finally that the model becomes a linear one when utilization gaps are approximated by logs of utilization rates.

The steady state of the dynamics is locally asymptotically stable under certain sluggishness conditions that are reasonable from a Keynesian perspective, loses its asymptotic stability cyclically (by way of so-called Hopf-bifurcations) if the system becomes too flexible, and becomes sooner or later globally unstable if (generally speaking) adjustment speeds become too high, as we shall show below. If the model is subject to explosive forces, it requires extrinsic nonlinearities in economic behavior – like downward money wage rigidity – to manifest themselves at least far off the steady state in order to bound the dynamics to an economically meaningful domain in the considered 5D state space. Chen, Chiarella, Flaschel and Hung (2004) provide a variety of numerical studies for such an approach with extrinsically motivated nonlinearities and thus undertake its detailed numerical investigation. In sum, therefore, our dynamic AS-AD growth model here and there will exhibit a variety of features that are much more in line with a Keynesian understanding of the characteristics of the trade cycle than is the case for the conventional modelling of AS-AD growth dynamics or its radical reformulation by the New Keynesians (where – if non-determinacy can be avoided by the choice of an appropriate Taylor rule – only the steady state position is a meaningful solution in the related setup we considered in the preceding section).

Taken together the model of this section consists of the following five laws of motion (with the derived reduced form expressions as far as the wage-price spiral is concerned and with reduced form expressions by assumption concerning the goods and the labor market dynamics):[6]

[6] As the model is formulated we have no real anchor for the steady state rate of interest (via investment behavior and the rate of profit it implies in the steady state) and thus have to assume here that it is the monetary authority that enforces a certain steady state values for the nominal rate of interest.

$$
\begin{array}{rcll}
\hat{V}^c & \stackrel{Dyn.IS}{=} & -\alpha_{V^c}(V^c - \bar{V}^c) \pm \alpha_\omega(\ln\omega - \ln\omega_o) - \alpha_r((r - \hat{p}) - (r_o - \bar{\pi})), & (9) \\
\hat{V}^l & \stackrel{O.Law}{=} & \beta_{V_1^l}(V^c - \bar{V}^c) + \beta_{V_2^l}\hat{V}^c, & (10) \\
\dot{r} & \stackrel{T.Rule}{=} & -\gamma_r(r - r_o) + \gamma_p(\hat{p} - \bar{\pi}) + \gamma_{V^c}(V^c - \bar{V}^c), & (11) \\
\hat{\omega} & \stackrel{RWPC}{=} & \dot{\theta} = \kappa[(1-\kappa_p)(\beta_{w_1}(V^l - \bar{V}^l) - \beta_{w_2}(\ln\omega - \ln\omega_o)) & \\
 & - & (1-\kappa_w)(\beta_{p_1}(V^c - \bar{V}^c) + \beta_{p_2}(\ln\omega - \ln\omega_o))], \quad \theta = \ln\omega & (12) \\
\dot{\pi}^m & \stackrel{I.Climate}{=} & \beta_{\pi^m}(\hat{p} - \pi^m) \quad \text{or,} & (13) \\
\pi^m(t) & = & \pi^m(t_o)e^{-\beta_{\pi^m}(t-t_o)} + \beta_{\pi^m}\int_{t_o}^{t} e^{\beta_{\pi^m}(t-s)}\hat{p}(s)ds. &
\end{array}
$$

The above equations represent, in comparison to the baseline model of New Keynesian macroeconomics, the IS goods market dynamics (7), here augmented by Okun's Law as link between the goods and the labor market (8), and of course the Taylor Rule (9), and a law of motion (10) for the real wage $\hat{\omega} = \pi^w - \pi^p$ that makes use of the same explaining variables as the New Keynesian one considered in section 2 (but with inflation rates in the place of their time rates of change and with no accompanying sign reversal concerning the influence of output and wage gaps), and finally the law of motion (11) that describes the updating of the inflationary climate expression.[7] We have to make use in addition of the following reduced form expression for the price inflation rate or the price PC, our law of motion for the price level p in the place of the New Keynesian law of motion for the price inflation rate π^p:

$$
\begin{array}{rcll}
\hat{p} & = & \kappa[\beta_{p_1}(V^c - \bar{V}^c) + \beta_{p_2}(\ln\omega - \ln\omega_o) & \\
 & + & \kappa_p(\beta_{w_1}(V^l - \bar{V}^l) - \beta_{w_2}(\ln\omega - \ln\omega_o))] + \pi^m, & (14)
\end{array}
$$

which has to be inserted into the above laws of motion in various places in order to get an autonomous nonlinear system of differential equations in the state variables: capacity utilization V^c, the rate of employment V^l, the nominal rate of interest r, the real wage rate ω, and the inflationary climate expression π^m. We stress that one can consider the eq. (14) as a sixth law of motion of the considered dynamics which however – when added — leads a system determinant which is zero and which therefore allows for zero-root hysteresis for certain variables of the model (in fact in the price level if the target rate of inflation of the Central Bank is zero and if interest rate smoothing is present in the Taylor rule). We have written the laws of motion in an order that first presents the dynamic equations also present in the baseline New Keynesian model of inflation dynamics, and then our formulation of the dynamics of income distribution and of the inflationary climate in which the economy is operating.

With respect to the empirically motivated restructuring of the original theoretical framework, the model is as pragmatic as the approach employed by Rudebusch and Svensson

[7] In correspondence to the Blanchard and Katz error correction terms in our wage and price PC, we here make also use of the log of the real wage in the law of motion which describes goods market dynamics, partly due also to our later estimation of the model.

(1999). By and large we believe that it represents a working alternative to the New Keynesian approach, in particular when the current critique of the latter approach is taken into account. It overcomes the weaknesses and the logical inconsistencies of the old Neoclassical synthesis, see Asada, Chen, Chiarella and Flaschel (2004), and it does so in a minimal way from a mature, but still traditionally oriented Keynesian perspective (and is thus not really 'New'). It preserves the problematic stability features of the real rate of interest channel, where the stabilizing Keynes effect or the interest rate policy of the central bank is interacting with the destabilizing, expectations driven Mundell effect. It preserves the real wage effect of the old Neoclassical synthesis, where – due to an unambiguously negative dependence of aggregate demand on the real wage – we had that price flexibility was destabilizing, while wage flexibility was not. This real wage channel is not really discussed in the New Keynesian approach, due to the specific form of wage-price dynamics there considered, see the preceding section, and it is summarized in the figure 1 for the situation where investment dominates consumption with respect to real wage changes. In the opposite case, the situations considered in this figure will be reversed with respect to their stability implications.

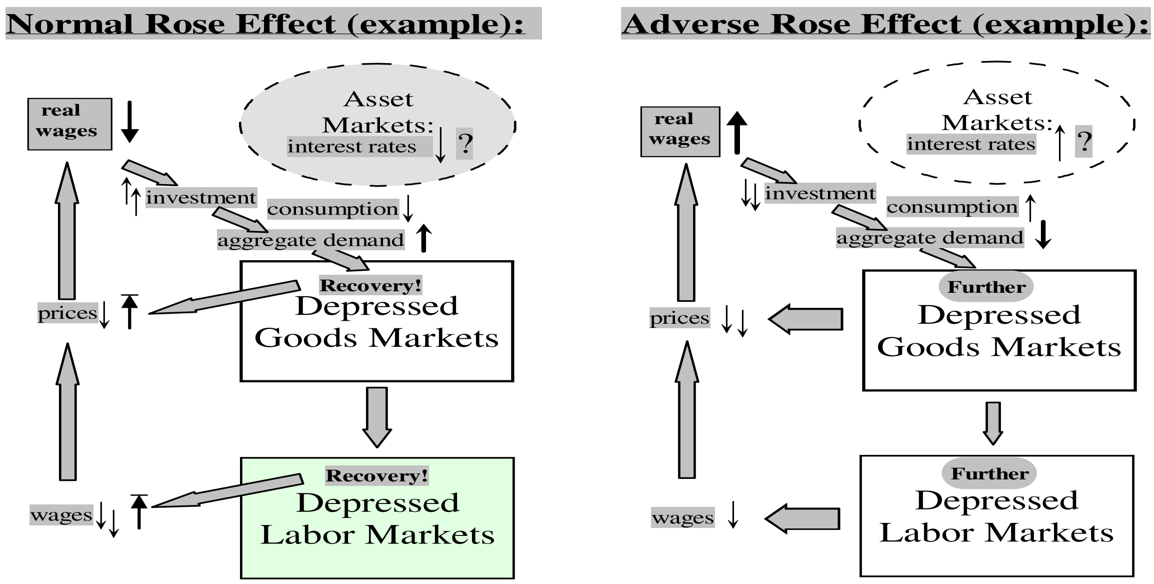

Figure 1: The Rose effects: The real wage channel of Keynesian macrodynamics .

The feedback channels just discussed will be the focus of interest in the now following stability analysis of our D(isequilibrium)AS-D(isequilibrium)AD dynamics. We have employed reduced-form expressions in the above system of differential equations whenever possible. We have thereby obtained a dynamical system in five state variables that is in a natural or intrinsic way nonlinear (due to its reliance on growth rate formulations). We note that there are many items that reappear in various equations, or are similar to each other, implying that stability analysis can exploit a variety of linear dependencies in the calculation of the conditions for local asymptotic stability. This dynamical system will be investigated in the next section in somewhat informal terms with respect to some stability assertions to which it gives rise. A rigorous proof of local asymptotic stability and its loss by way of Hopf bifurcations can be found in Asada, Chen, Chiarella and Flaschel (2004), there for the original baseline model. For the present model variant we supply a more detailed stability proofs in Chen, Chiarella, Flaschel and Hung (2004), where also more detailed numerical simulations of the model are provided.

4 5D Feedback-guided stability analysis

In this section we illustrate an important method to prove local asymptotic stability of the interior steady state of the dynamical system (9) – (13) (with eq. (14) inserted wherever needed) through partial considerations from the feedback chains that characterize this empirically oriented baseline model of Keynesian dynamics. Since the model is an extension of the standard AS-AD growth model, we know from the literature that there is a real rate of interest effect typically involved, first analyzed by formal methods in Tobin (1975), see also Groth (1992). Instead of the stabilizing Keynes-effect, based on activity-reducing nominal interest rate increases following price level increases, we have here however a direct steering of economic activity by the interest rate policy of the central bank. Since the (correctly anticipated) short-run real rate of interest is driving investment and consumption decisions (increases leading to decreased aggregate demand), there is furthermore the activity stimulating (partial) effect of increases in the rate of inflation (as part of the real rate of interest channel) that may lead to accelerating inflation under appropriate conditions. This is the so-called Mundell-effect that normally works in opposition to the Keynes-effect, but through the same real rate of interest channel as this latter effect. Due to our use of a Taylor rule in the place of the conventional LM curve, the Keynes-effect is here implemented in a more direct way towards a stabilization of the economy (coupling nominal interest rates directly with the rate of price inflation) and it is supposed to work more strongly the larger the choice of the parameters γ_p, γ_{V^c}. The Mundell-effect by contrast is stronger the faster the inflationary climate adjusts to the present level of price inflation, since we have a positive influence of this climate variable both on price as well as on wage inflation and from there on rates of employment of both capital and labor.

There is a further important potentially (at least partially) destabilizing feedback mechanism as the model is formulated. Excess profitability depends positively on the rate of return on capital and thus negatively on the real wage ω. We thus get – since consumption may also depend (positively) on the real wage – that real wage increases can depress or stimulate economic activity depending on whether investment or consumption is dominating the outcome of real wage increases (we here neglect the stabilizing role of the additional Blanchard and Katz type error correction mechanisms). In the first case, we get from the reduced-form real wage dynamics:

$$\hat{\omega} \;=\; \kappa[(1-\kappa_p)\beta_{w_1}(V^l - \bar{V}^l) - (1-\kappa_w)\beta_{p_1}(V^c - \bar{V}^c)],$$

that price flexibility should be bad for economic stability, due to the minus sign in front of the parameter β_p, while the opposite should hold true for the parameter that characterizes wage flexibility. This is a situation that was already investigated in Rose (1967). It gives the reason for our statement that wage flexibility gives rise to normal, and price flexibility to adverse, Rose effects as far as real wage adjustments are concerned (if it is assumed – as in our theoretical baseline model – that only investment depends on the real wage). Besides real rate of interest effects, establishing opposing Keynes- and Mundell-effects, we thus have also another real adjustment process in the considered model where now wage and price flexibility are in opposition to each other, see Chiarella and Flaschel (2000) and Chiarella, Flaschel, Groh and Semmler (2000) for further discussion of these as well as of other feedback mechanisms of such Keynesian growth dynamics. We observe again that our theoretical DAS-AD growth dynamics in Asada, Chen, Chiarella and Flaschel (2004) – due to their origin in the baseline model of the Neoclassical Synthesis, stage I – allows for negative influence of real wage changes on aggregate demand solely, and thus only for cases of destabilizing wage level flexibility, but not price level flexibility. In the empirical estimation of the model (9) – (13) we will indeed find that this case seems to be the one that characterizes our empirically and broader oriented dynamics (9) – (13).

The foregoing discussion enhances our understanding of the feedback mechanisms of the dynamical system (9) – (13) whose stability properties will now be investigated by means of varying adjustment speed parameters appropriately. With the feedback scenarios considered above in mind, we first observe that the inflationary climate can be frozen at its steady state value, $\pi_o^m = \bar{\pi}$, if $\beta_{\pi^m} = 0$ is assumed. The system thereby becomes 4D and it can indeed be further reduced to 3D if in addition $\alpha_\omega = 0, \beta_{w_2} = 0, \beta_{p_2} = 0$ is assumed, since this decouples the ω-dynamics from the remaining system dynamics V^c, V^l, r. We will consider the stability of these 3D subdynamics – and its subsequent extensions – in informal terms only here, reserving rigorous calculations to the alternative scenarios provided in Chen, Chiarella, Flaschel and Hung (2004). We nevertheless hope to be able to demonstrate to the reader how one can indeed proceed systematically from low to high dimensional analysis in such stability investigations from the perspective of the partial feedback channels implicitly contained in the considered 5D dynamics. This method has been already applied successfully to various other, often more complicated, dynamical systems; see Asada, Chiarella, Flaschel and Franke (2003) for a variety of typical examples.

Before we start with our stability investigations we establish the fact that for the dynamical system (7)-(11) loss of stability can in general only occur by way of Hopf-bifurcations, since the following proposition can be shown to hold true under mild – empirically plausible – parameter restrictions.

Proposition 1:

> *Assume that the parameter γ_r is chosen sufficiently small and that the parameters $\beta_{w_2}, \beta_{p_2}, \kappa_p$ fulfill $\beta_{p_2} > \beta_{w_2}\kappa_p$. Then: The 5D determinant of the Jacobian of the dynamics at the interior steady state is always negative in sign.*

Sketch of proof: We have for the sign structure in the Jacobian under the given assumptions the following situation to start with (we here assume as limiting situation $\gamma_r = 0$ and have already simplified the law of motion for V^l by means of the one for V^c through row operations that are irrelevant for the size of the determinant to be calculated):

$$J = \begin{pmatrix} \pm & + & - & \pm & + \\ + & 0 & 0 & 0 & 0 \\ + & + & 0 & + & + \\ - & + & 0 & - & 0 \\ + & + & 0 & + & 0 \end{pmatrix}.$$

We note that the ambiguous sign in the entry J_{11} in the above matrix is due to the fact that the real rate of interest is a decreasing function of the inflation rate which in turn depends positively on current rates of capacity utilization.

Using the second row and the last row in their dependence on the partial derivatives of $\hat{p}$ we can reduces this Jacobian to

$$J = \begin{pmatrix} 0 & 0 & - & \pm & + \\ + & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & + \\ 0 & + & 0 & - & 0 \\ 0 & + & 0 & + & 0 \end{pmatrix}$$

without change in the sign of its determinant. In the same way we can now use the third row to get another matrix without any change in the sign of the corresponding determinants

$$J = \begin{pmatrix} 0 & 0 & - & \pm & 0 \\ + & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & + \\ 0 & + & 0 & - & 0 \\ 0 & + & 0 & + & 0 \end{pmatrix}$$

The last two columns can under the observed circumstances be further reduced to

$$J = \begin{pmatrix} 0 & 0 & - & \pm & 0 \\ + & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & + \\ 0 & + & 0 & 0 & 0 \\ 0 & 0 & 0 & + & 0 \end{pmatrix}$$

which finally gives

$$J = \begin{pmatrix} 0 & 0 & - & 0 & 0 \\ + & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & + \\ 0 & + & 0 & 0 & 0 \\ 0 & 0 & 0 & + & 0 \end{pmatrix}.$$

This matrix is easily shown to exhibit a negative determinant which proves the proposition, also for all values of γ_r which are chosen sufficiently small. ■

Proposition 2:

Assume that the parameters $\beta_{w_2}, \beta_{p_2}, \alpha_\omega$ and β_{π^m} are all set equal to zero. This decouples the dynamics of V^c, V^l, r from the rest of the system. Assume furthermore that the partial derivative of the first law of motion (7) depends negatively on V^c, i.e., $\alpha_{V^c} > \alpha_r \kappa \beta_{p_1}$, so that the dynamic multiplier process, characterized by α_{V^c}, dominates this law of motion with respect to the overall impact of the rate of capacity utilization V^c. Then: The interior steady state of the implied 3D dynamical system

$$\begin{aligned} \hat{V}^c &= -\alpha_{V^c}(V^c - \bar{V}^c) - \alpha_r((r - \hat{p}) - (r_o - \bar{\pi})), && (15) \\ \hat{V}^l &= \beta_{V_1^l}(V^c - \bar{V}^c), && (16) \\ \dot{r} &= -\gamma_r(r - r_o) + \gamma_p(\hat{p} - \bar{\pi}) + \gamma_{V^c}(V^c - \bar{V}^c), && (17) \end{aligned}$$

is locally asymptotically stable if the interest rate smoothing parameter γ_r and the employment adjustment parameter β_{V^l} are chosen sufficiently small in addition.

Sketch of proof: In the considered situation we have for the Jacobian of these reduced dynamics at the steady state:

$$J = \begin{pmatrix} - & + & - \\ + & 0 & 0 \\ + & + & - \end{pmatrix}$$

The determinant of this Jacobian is obviously negative if the parameter γ_r is chosen sufficiently small. Similarly, the sum of the minors of order 2: a_2, will be positive if β_{V^l} is chosen sufficiently small. The validity of the full set of Routh-Hurwitz conditions then easily follows, since trace J $= -a_1$ is obviously negative and since det J is part of the expressions that characterize the product $a_1 a_2$. ■

Proposition 3:

Assume now that the parameter α_ω is negative, but chosen sufficiently small, while the error correction parameters β_{w_2}, β_{p_2} are still kept at zero. Then: The interior steady state of the resulting 4D dynamical system (where the state variable ω is now included)

$$\begin{aligned} \hat{V}^c &= -\alpha_{V^c}(V^c - \bar{V}^c) - \alpha_\omega(\ln\omega - \ln\omega_o) - \alpha_r((r - \hat{p}) - (r_o - \bar{\pi})), && (18)\\ \hat{V}^l &= \beta_{V_1^l}(V^c - \bar{V}^c), && (19)\\ \dot{r} &= -\gamma_r(r - r_o) + \gamma_p(\hat{p} - \bar{\pi}) + \gamma_{V^c}(V^c - \bar{V}^c), && (20)\\ \hat{\omega} &= \kappa[(1-\kappa_p)\beta_{w_1}(V^l - \bar{V}^l) - (1-\kappa_w)\beta_{p_1}(V^c - \bar{V}^c), && (21) \end{aligned}$$

is locally asymptotically stable.

Sketch of proof: It suffices to show in the considered situation that the determinant of the resulting Jacobian at the steady state is positive, since small variations of the parameter α_ω must then move the zero eigenvalue of the case $\alpha_\omega = 0$ into the negative domain, while leaving the real parts of the other eigenvalues – shown to be negative in the preceding proposition – negative. The determinant of the Jacobian to be considered here – already slightly simplified – is characterized by

$$J = \begin{pmatrix} 0 & + & - & - \\ + & 0 & 0 & 0 \\ 0 & + & - & 0 \\ 0 & + & 0 & 0 \end{pmatrix}.$$

This can be further simplified to

$$J = \begin{pmatrix} 0 & 0 & 0 & - \\ + & 0 & 0 & 0 \\ 0 & 0 & - & 0 \\ 0 & + & 0 & 0 \end{pmatrix}$$

without change in the sign of the corresponding determinant which proves the proposition. ■

We note that this proposition also holds where $\beta_{p_2} > \beta_{w_2}\kappa_p$ holds true as long as the thereby resulting real wage effect is weaker than the one originating from α_ω. Finally – and in sum – we can also state that the full 5D dynamics must also exhibit a locally stable steady state if β_{π^m} is made positive, but chosen sufficiently small, since we have already shown that the full 5D dynamics exhibits a negative determinant of its Jacobian at the steady state under the stated conditions. Increasing β_{π^m} from zero to a small positive value therefore must mover the corresponding zero eigenvalue into the negative domain of the plane of complex numbers.

Summing up, we can state that a weak Mundell effect, the neglect of Blanchard-Katz error correction terms, a negative dependence of aggregate demand on real wages, coupled with nominal wage and also to some extent price level inertia (in order to allow for dynamic multiplier stability), a sluggish adjustment of the rate of employment towards actual capacity utilization and a Taylor rule that stresses inflation targeting therefore are here (for example) the basic ingredients that allow for the proof of local asymptotic stability of the interior steady state of the dynamics (9) – (13). We expect however that indeed a variety of other and also more general situations of convergent dynamics can be found, but have to leave this here for future research and numerical simulations of the model. Instead we now attempt to estimate the signs and also the sizes of the parameters of the model in order to gain insight into the question to what extent for example the US economy after World War II supports one of the real wage effects considered in figure 1 and also the possibility of overall asymptotic stability for such an economy, despite a destabilizing Mundell effect in the real interest rate channel. Due to proposition 1 we know that the dynamics will generally only loose asymptotic stability in a cyclical fashion (by way of a Hopf-bifurcation) and will indeed do so if the parameter β_{π^m} is chosen sufficiently large. We thus arrive at a radically different outcome for the dynamics implied by our mature traditional Keynesian approach as compared to the New Keynesian dynamics. The question that naturally arises here is whether the economy can be assumed to be in the convergent regime of its alternative dynamical possibilities. This of course can only be decided by an empirical estimation of its various parameters which is the subject of the next section.

5 Estimating the model

We now provide some estimates for the signs and sizes of the parameters of the model of this paper and will do so – with respect to the wage-price spiral – on the level of its structural form (where it has not yet been reduced to the dynamics of real wages, see eq. (14)). The further aim of these estimates is of course to determine whether the equivalent autonomous reduced form 5D dynamics we considered in the preceding section – obtained when equation (14) is inserted into (9) – (13) – exhibits asymptotic stability of (convergence to) its interior steady state position. We proceed in three steps in this matter, first by estimating an unrestricted VAR approach for the core variables of the model, secondly by comparing its results with a linear structural VAR model that is a linear approximation of the (weakly) nonlinear model of the theoretical part of the paper and finally by estimating and evaluating this latter approach in comparison to the result achieved in step 2. The central aim of these estimations is to determine the signs and sizes of the entries in the Jacobian matrix of the considered dynamics, which indeed will allow us later on to formulate certain specific (in)stability propositions on the in fact then 4D dynamics implied by our estimates of Okun's law.

In their theoretical form these dynamics exhibit the following sign structure in their Jacobian, calculated at its interior steady state:

$$J = \begin{pmatrix} \pm & + & - & \pm & + \\ \pm & + & - & \pm & + \\ + & + & - & \pm & + \\ - & + & 0 & - & 0 \\ + & + & 0 & \pm & 0 \end{pmatrix}.$$

There are therefore still a variety of ambiguous effects embedded in the general theoretical form of the dynamics, due to the Mundell-effect and the Rose-effect in the dynamics of the goods-market and the opposing Blanchard-Katz error correction terms in the reduced form price Phillips curve.

In section 4 we have then considered certain special cases of the general model which allowed for the derivation of asymptotic stability of the steady state and its loss of stability by way of Hopf bifurcations if certain speed parameters become sufficiently large. In the present section we now provide empirical estimates for the laws of motion (9) – (13) of our disequilibrium AS-AD model, by means of the structural form of the wage and price Phillips curve, coupled with the dynamic multiplier equation, Okun's law and the interest rate policy rule. These estimates, on the one hand, serve the purpose of confirming the parameter signs we have specified in the initial theory-guided formulation of the model and to determine the sizes of these parameters in addition. On the other hand, we have three different situations where we cannot specify the parameter signs on purely theoretical grounds and where we therefore aim at obtaining these signs from the empirical estimates of the equations whenever this happens.

There is first of all, see eq. (9), the ambiguous influence of real wages on (the dynamics of) the rate of capacity utilization, which should be a negative one if investment is more responsive than consumption to real wage changes and a positive one in the opposite case. There is secondly, with an immediate impact effect if the rates of capacity utilization for capital and labor are perfectly synchronized, the fact that real wages rise with economic activity through money wage changes on the labor market, while they fall with it through price level changes on the goods market, see eq. (11). Finally, we have in the reduced form equation for price inflation a further ambiguous effect of real wage increases, which there lower $\hat{p}$ through their effect on wage inflation, while speeding up $\hat{p}$ through their effect on price inflation, effects which work into opposite directions in the reduced form price PC (14). Mundell-type, Rose-type and Blanchard-Katz error-correction feedback channels therefore make the dynamics indeterminate on the general level.

In all of these three cases empirical analysis will now indeed provide us with definite answers as to which ones of these opposing forces will be the dominant ones. Furthermore, we shall also see that the Blanchard and Katz (1999) error correction terms do play a role in the US-economy, in contrast to what has been found out by these authors for the money wage PC in the U.S. However, we will not attempt to estimate the parameter β_{π^m} that characterizes the evolution of the inflationary climate in our economy. Instead, we will use moving averages with linearly declining weights for its representation, which allows us to bypass the estimation of the law of motion (13). We consider this as the simplest approach to the treatment of our climate expression (comparable with recent New Keynesian treatments of hybrid expectation formation), which should later on be replaced by more sophisticated ones, for example one that makes use of the Livingston index for inflationary expectations as in Laxton et al. (2000) which in our view mirrors some adaptive mechanism in the adjustment of inflationary expectations.

We take an encompassing approach to conduct our estimates. The structural laws of motion of our economy, see section 3, have been formulated in an intrinsically nonlinear way (due to certain growth rate formulations). We note that single equations estimates have suggested the use of only $\alpha_{V^l_2}$ in the equation that describes the dynamics of the employment rate. In the wage-price spiral we use – in line with Blanchard and Katz's theoretical derivation (1999) – the log of unit wage costs, removing their significant downward trend in the employed data (see figure 2) appropriately. Note here again that we use the log of the unit wage costs in the dynamic multiplier equations as well.

We conduct our estimates in conjunction with time-invariant estimates of all the parameters of our model. This in particular implies that Keynes' (1936) explanation of the trade cycle, which employed systematic changes in the propensity to consume, the marginal efficiency of investment and liquidity preference over the course of the cycle, find no application here and that – due the use of detrended measures for income distribution changes and unit-wage costs – also the role of technical change is downplayed to a significant degree, in line with its neglect in the theoretical equations of the model presented in section 3. As a result we expect to obtain from our estimates long-phased economic fluctuations, but not yet long -waves, since important fluctuations in aggregate demand (based on time-varying parameters) are still ignored and since the dynamics is then driven primarily by slowly changing income distribution, indeed a slow process in the overall evolution of the U.S. economy after World War II.

To show that such an understanding of the model is a suitable description of (some of) the dynamics of the observed data, we first fit a corresponding 6D VAR model to the data to uncover the dynamics in the six independent variables there employed. We then identify a linear structural model that parsimoniously encompasses the employed VAR. Finally, we contrast our nonlinear structural model, i.e., the laws of motion (1) to (5) in structural form (and the Taylor rule), with the linear structural VAR model and show through a J test[8] that the nonlinear model is indeed preferred by the data. In this way we show that our (weakly) nonlinear structural model represents a proper description of the data.

The relevant variables for the following investigation are the wage inflation rate, the price inflation rate, the rates of utilization of labor and of capital, the nominal interest rate, the log of average unit wage cost,[9] to be denoted in the following by: $d\ln w_t, d\ln p_t, V^l_t, V^c_t, r_t$ and uc_t, where uc_t is the cycle component of the log of the time series for the unit real wage cost, filtered by the bandpass filter.[10]

[8]See Davidson and Mackinnon (1993) for details.

[9]or alternatively the real wage which does not modify the obtained results in significant ways.

[10]For details on the bandpass filter see Baxter and King (1995, 1999).

5.1 Data Description

The empirical data of the corresponding time series are taken from the Federal Reserve Bank of St. Louis data set (see http:/www.stls.frb.org/fred). The data are quarterly, seasonally adjusted and concern the period from 1965:1 to 2001:2. Except for the unemployment rates of the factors labor, U^l, and capital, U^c (and of course the interest rate and the derived inflation climate) the log of the series are used in table 1 (note however that the intermediate estimation step of a linear structural VAR makes use of the logs of both utilization rates however, see also their representation in figure 2).

We now use $\ln w_t$ and $\ln p_t$, i.e., logarithms, in the place of the original level magnitudes. Their first differences $d\ln w_t, d\ln p_t$ thus give the current rate of wage and price inflation (backwardly dated). We use π_t^{12} in this section to denote specifically a moving average of price inflation rates with linearly decreasing weights over the past 12 quarters, interpreted as a particularly simple measure for the inflationary climate expression of our model, and we denote by $V^l, V^c(U^l, U^c)$ the rates of (under-)utilization of labor and the capital stock.

There is a pronounced downward trend in part of the employment rate series (over the 1970's and part of the 1980's) and in the wage share (normalized to 0 in 1996). The latter trend is not the topic of this chapter which concentrates on the cyclical implications of changed in income distribution. Wage inflation shows three trend reversals, while the inflation climate representation clearly show two periods of low inflation regimes and in between a high inflation regime.

Variable	Transformation	Mnemonic	Description of the untransformed series
$U^l = 1 - V^l$	UNRATE/100	UNRATE	Unemployment Rate
$U^c = 1 - V^c$	1-CUMFG/100	CUMFG	Capacity Utilization: Manufacturing, Percent of Capacity
$\ln w$	ln(COMPNFB)	COMPNFB	Nonfarm Business Sector: Compensation Per Hour, 1992=100
$\ln p$	ln(GDPDEF)	GDPDEF	Gross Domestic Product: Implicit Price Deflator, 1996=100
$\ln yn = \ln y - \ln l^d$	ln(OPHNFB)	OPHNFB	Nonfarm Business Sector; Output Per Hour of All Persons, 1992=100
uc	$\ln\left(\frac{COMPRNFB}{OPHNFB}\right)$	COMPRNFB	Nonfarm Business Sector: Real Compensation Per Output Unit, 1992=100
$d\ln yn$	$d\ln(OPHNFB)$	OPHNFB	Growth Rate of Labor Productivity
r	FEDFUNDS	FEDFUNDS	Federal Funds Rate

Table 1: Data used for the empirical investigation

We expect that the 6 independent time series for wages, prices, capacity utilization rates, the growth rate of labor productivity and the interest rate (federal funds rate) are stationary. The graphs of the series for wage and price inflation, capacity utilization rates, $d\ln w_t, d\ln p_t, \ln V_t^l, \ln V_t^c$ seem to confirm our expectation. In addition we carry out the DF unit root test for each series. The test results are shown in table 2.

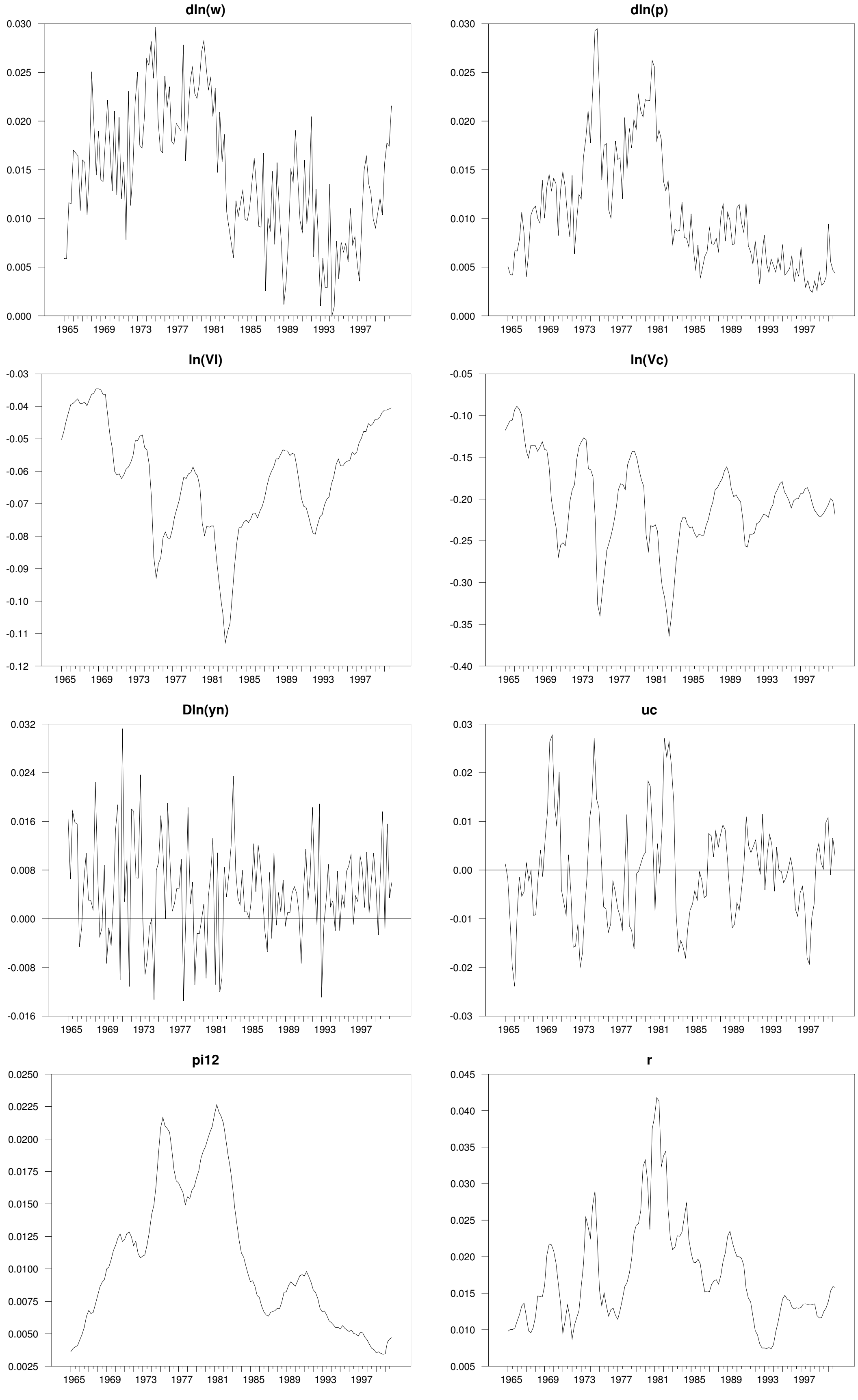

Figure 2: The fundamental data of the model (*uc* bandpass filtered).

Variable	Sample	Critical Value	Test Statistic
$d\ln w$	1965:01 TO 2000:04	-3.41000	-3.74323
$d\ln p$	1965:01 TO 2000:04	-3.41000	-3.52360
$\ln V^l$	1965:01 TO 2000:04	-2.86000	-2.17961
$\ln V^c$	1965:01 TO 2000:04	-3.41000	-3.92688
r	1965:01 TO 2000:04	-2.86000	-2.67530

Table 2: Summary of DF-Test Results

The applied unit root test confirms our expectations with the exception of V^l and r. Although the test cannot reject the null of a unit root, there is no reason to expect the rate of unemployment and the federal funds rate to be unit root processes. Indeed we expect that they are constrained in certain limited ranges, say from zero to 0.3. Due to the lower power of the DF test, this test result should only provide hints that the rate of unemployment and the federal funds rate exhibit strong autocorrelations, respectively.

5.2 Estimation of the unrestricted VAR

Given the assumption of stationarity, we can construct a VAR model for the 6 variables of the structural model to mimic their DGP (data generating process of these 6 variables) by linearizing our given structural model in a straightforward way.

$$\begin{pmatrix} d\ln w_t \\ d\ln p_t \\ \ln V_t^l \\ \ln V_t^c \\ r_t \\ uc_t \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \end{pmatrix} d74 + \sum_{k=1}^{P} \begin{pmatrix} a_{11k} & a_{12k} & a_{13k} & a_{14k} & a_{15k} & a_{16k} \\ a_{21k} & a_{22k} & a_{23k} & a_{24k} & a_{25k} & a_{26k} \\ a_{31k} & a_{32k} & a_{33k} & a_{34k} & a_{35k} & a_{36k} \\ a_{41k} & a_{42k} & a_{43k} & a_{44k} & a_{45k} & a_{46k} \\ a_{51k} & a_{52k} & a_{53k} & a_{54k} & a_{55k} & a_{56k} \\ a_{61k} & a_{62k} & a_{63k} & a_{64k} & a_{65k} & a_{66k} \end{pmatrix} \begin{pmatrix} d\ln w_{t-k} \\ d\ln p_{t-k} \\ \ln V_{t-k}^l \\ \ln V_{t-k}^c \\ r_{t-k} \\ uc_{t-k} \end{pmatrix} + \begin{pmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \\ e_{4t} \\ e_{5t} \end{pmatrix} \tag{22}$$

To determine the lag length of the VAR we apply sequential likelihood tests. We start with a lag length of 24, at which the residuals can be taken as white noise process. The sequence likelihood ratio test procedure gives a lag length of 11. The test results are listed below.

- $H_0 : P = 20$ v.s. $H_1 : P = 24$
  Chi-Squared(144)= 147.13 with Significance Level 0.91

- $H_0 : P = 16$ v.s. $H_1 : P = 20$
  Chi-Squared(144)= 148.92 with Significance Level 0.41

- $H_0 : P = 12$ v.s. $H_1 : P = 16$
  Chi-Squared(36)= 118.13 with Significance Level 0.94

- $H_0 : P = 11$ v.s. $H_1 : P = 12$
  Chi-Squared(36)= 42.94 with Significance Level 0.19

- $H_0 : P = 10$ v.s. $H_1 : P = 11$
  Chi-Squared(36)= 51.30518 with Significance Level 0.04

According to these test results we use a VAR(12) model to represent a general model that should be a good approximation of the DGP. Because the variable uc_t is treated as exogenous in the structural form (1) – (8) of the dynamical system , we factorize the VAR(12) process into a conditional process of $d\ln w_t, d\ln p_t, \ln V_t^l, \ln V_t^c, r_t$ given uc_t and the lagged variables, and the marginal process of uc_t given the lagged variables:

$$\begin{pmatrix} d\ln w_t \\ d\ln p_t \\ \ln V_t^l \\ \ln V_t^c \\ r_t \end{pmatrix} = \begin{pmatrix} c_1^* \\ c_2^* \\ c_3^* \\ c_4^* \\ c_5^* \end{pmatrix} + \begin{pmatrix} b_1^* \\ b_2^* \\ b_3^* \\ b_4^* \\ b_5^* \end{pmatrix} d74 + \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{pmatrix} uc_t \tag{23}$$

$$+ \sum_{k=1}^{P} \begin{pmatrix} a_{11k}^* & a_{12k}^* & a_{13k}^* & a_{14k}^* & a_{15k}^* & a_{16k}^* \\ a_{21k}^* & a_{22k}^* & a_{23k}^* & a_{24k}^* & a_{25k}^* & a_{26k}^* \\ a_{31k}^* & a_{32k}^* & a_{33k}^* & a_{34k}^* & a_{35k}^* & a_{36k}^* \\ a_{41k}^* & a_{42k}^* & a_{43k}^* & a_{44k}^* & a_{45k}^* & a_{46k}^* \\ a_{51k}^* & a_{52k}^* & a_{53k}^* & a_{54k}^* & a_{55k}^* & a_{56k}^* \end{pmatrix} \begin{pmatrix} d\ln w_{t-k} \\ d\ln p_{t-k} \\ \ln V_{t-k}^l \\ \ln V_{t-k}^c \\ r_{t-k} \\ uc_{t-k} \end{pmatrix} + \begin{pmatrix} e_{1t}^* \\ e_{2t}^* \\ e_{3t}^* \\ e_{4t}^* \\ e_{5t}^* \end{pmatrix}$$

$$uc_t = c_6 + \sum_{k=1}^{P} \begin{pmatrix} a_{61k} & a_{62k} & a_{63k} & a_{64k} & a_{65k} & a_{66k} \end{pmatrix} \begin{pmatrix} d\ln w_{t-k} \\ d\ln p_{t-k} \\ \ln V_{t-k}^l \\ \ln V_{t-k}^c \\ r_{t-k} \\ uc_{t-k} \end{pmatrix} + e_{6t} \tag{24}$$

We now examine whether uc_t can be taken as an "exogenous" variable. The partial system (23) is exactly identified. Hence the variable uc_t is weakly exogenous for the parameters in the partial system.[11] For the strong exogeneity of uc_t, we test whether $d\ln w_t, d\ln p_t, \ln V_t^l, \ln V_t^c, r_t$ Granger cause uc_t. The test is carried out by testing the hypothesis: $H_0 : a_{ijk} = 0, (i = 6; j = 1, 2, 3, 4, 5; k = 1, 2, ..., 12)$ in (24) based on the likelihood ratio

- Chi-Squared(60)=57.714092 with Significance Level 0.55972955

The result of the test is uc_t is strongly exogenous with respect to the parameters in (23). Hence we can investigate the partial system (23) taking uc_t as exogenous.

5.3 Estimation of the Structural Model

As discussed in section 3, the law of motion for the real wage rate, eq. (12), represents a reduced form expression of the two structural equations for $d\ln w_t$ and $d\ln p_t$. Noting again that the inflation climate variable is defined in the estimated model as a linearly

[11] For a detailed discussion of this procedure, see Chen (2003).

declining function of past price inflation rates, the dynamics of the system (1) – (8) can be rewritten in linearized form as shown in the following equations:[12]

$$\begin{aligned}
d\ln w_t &= \beta_{w_1}\ln V^l_{t-1} - \beta_{w_2} uc_{t-1} + \kappa_w d\ln p_t + (1-\kappa_w)\pi^{12}_t + c_1 + e_{1t}, && (25)\\
d\ln p_t &= \beta_{p_1}\ln V^c_{t-1} + \beta_{p_2} uc_{t-1} + \kappa_p d\ln w_t + (1-\kappa_p)\pi^{12}_t + c_2 + e_{2t}, && (26)\\
d\ln V^l_t &= \alpha_{V^l_2} d\ln V^l_t + e_{3t}, && (27)\\
d\ln V^c_t &= -\alpha_{V^c}\ln V^c_{t-1} - \alpha_r(r_{t-1} - d\ln p_t) - \alpha_\omega uc_{t-1} + c_4 + e_{4t}, && (28)\\
dr_t &= -\gamma_r r_{t-1} + \gamma_p d\ln p_t + \gamma_{V^c}\ln V^c_{t-1} + c_5 + e_{5t}. && (29)
\end{aligned}$$

Obviously, the model (25) – (29) is nested in the VAR(12) of (23). Therefore we can use (23) to evaluate the empirical relevance of the model (25) – (29). First we test whether the parameter restrictions on (23) implied by (25) – (29) are valid.

The linearized structural model (25) – (29) puts 349 restrictions on the unconstrained VAR(12) of the system (23). Applying likelihood ratio methods we can test the validity of these restrictions. For the period from 1965:1 to 2001:2 we cannot reject the null of these restrictions. The test result is the following:

- Chi-Squared(349)= 361.716689 with Significance Level 0.34902017

Obviously, the specification (25) – (29) is a valid one for the data set from 1965:1 to 2001:2. This result shows strong empirical relevance for the laws of motions as described in (1) – (8) as a model for the U.S. economy from 1965:1 to 2001:2. It is worthwhile to note that altogether 349 restrictions are implied through the structural form of the system (1) – (8) on the VAR(12) model. A p-value of 0.349 thus means that (1) – (8) is a much more parsimonious presentation of the DGP than VAR(12), and henceforth a much more efficient model to describe the economic dynamics for this period.

To get a result that is easier to interpret from the economic perspective, we transform the model (25) – (29) back to its originally nonlinear form (1) – (8):[13]

$$\begin{aligned}
d\ln w_t &= \beta_{w_1} V^l_{t-1} - \beta_{w_2} uc_{t-1} + \kappa_w d\ln p_t + (1-\kappa_w)\pi^{12}_t + c_1 + e_{1t}, && (30)\\
d\ln p_t &= \beta_{p_1} V^c_{t-1} + \beta_{p_2} uc_{t-1} + \kappa_p d\ln w_t + (1-\kappa_p)\pi^{12}_t + c_2 + e_{2t}, && (31)\\
d\ln V^l_t &= \alpha_{V^l_2} d\ln V^c_t + e_{3t}, && (32)\\
d\ln V^c_t &= -\alpha_{V^c} V^c_{t-1} - \alpha_u uc_{t-1} - \alpha_r(r_{t-1} - d\ln p_t) + c_4 + e_{4t}, && (33)\\
dr_t &= -\gamma_r r_{t-1} + \gamma_p d\ln p_t + \gamma_{V^c} V^c_{t-1} + c_5 + e_{5t}. && (34)
\end{aligned}$$

This model therefore differs from the model (25) – (29) by referring now again to the explanatory variables V^c and V^l instead of $\ln V^c$ and $\ln V^l$ which were necessary to construct a linear VAR(12) system. We compare on this basis the model (30) – (34) with the model (25) – (29) in a nonnested testing framework. Applying the J test to such a nonlinear estimation procedure, we get significant evidence that the model (30) – (34) is to be preferred to the model (25) – (29).

[12]Note here that the difference operator d is to be interpreted as backward in orientation and that the nominal rate of interest is dated at the beginning of the relevant period. The linearly declining moving average π^{12}_t in turn concerns the past twelve price inflation rates.

[13]Note that $dr_t = -\gamma_r r_{t-1} + \gamma_p d\ln p_t + \gamma_{V^c} V^c_{t-1} + c_5 + e_{5t}$ can also be represented by $r_t = (1 - \gamma_r)r_{t-1} + ...$ in the equations to be estimated below.

Model	J test
H_1 : Model of (25) – (29) is true	$t_\alpha = 4.611$
H_2 : Model of (30) – (34) is true	$t_\phi = -0.928$

We have already omitted in the following summaries of our model estimates the insignificant parameters in the displayed quantitative representation of the semi-structural model and also the stochastic terms. By putting furthermore the NAIRU expressions and all other expressions that are here still assumed as constant into overall constant terms, we therefore finally obtain the following (approximate) Two Stage Least Squares estimation results (with t-statistics in parenthesis):

$$
\begin{array}{rcllllll}
d\ln w_t & = & \begin{array}{c}0.13V^l_{t-1}\\(3.95)\end{array} & \begin{array}{c}-0.07uc_{t-1}\\(-1.94)\end{array} & \begin{array}{c}+0.49d\ln p_t\\(2.61)\end{array} & \begin{array}{c}+0.51\pi^{12}_t\\(2.61)\end{array} & \begin{array}{c}-0.12,\\(-3.82)\end{array} \\
d\ln p_t & = & \begin{array}{c}0.04V^c_{t-1}\\(2.32)\end{array} & \begin{array}{c}+0.05uc_{t-1}\\(2.52)\end{array} & \begin{array}{c}+0.18d\ln w_t\\(2.32)\end{array} & \begin{array}{c}+0.82\pi^{12}_t\\(2.32)\end{array} & \begin{array}{c}-0.04,\\(-6.34)\end{array} \\
d\ln V^l_t & = & \begin{array}{c}0.18d\ln V^c_t,\\(14.62)\end{array} & & & & \\
d\ln V^c_t & = & \begin{array}{c}-0.14V^c_{t-1}\\(-5.21)\end{array} & \begin{array}{c}-0.94(r_{t-1}\\(-4.72)\end{array} & \begin{array}{c}-d\ln p_t)\\ \ \end{array} & \begin{array}{c}-0.54uc_{t-1}\\(-4.84)\end{array} & \begin{array}{c}+0.12,\\(5.41)\end{array} \\
dr_t & = & \begin{array}{c}-0.08r_{t-1}\\(24.82)\end{array} & \begin{array}{c}+0.06d\ln p_t\\(1.2)\end{array} & \begin{array}{c}+0.01V^c_{t-1}\\(2.46)\end{array} & \begin{array}{c}-0.01.\\(-2.19)\end{array} &
\end{array}
$$

We thus here get that Blanchard and Katz error correction terms matter in particular in the labor market, that the adjustment speed of wages is larger than the one for prices with respect to their corresponding demand pressures and that wage earners are more short-sighted than firms with respect to the influence of the inflationary climate expression. Okun's law which relates the growth rate of employment with the growth rate of capacity utilization is below a 1:5 relationship and thus in fact represents a fairly weak relationship. There is a strong influence of the real rate of interest on the growth rate of capacity utilization in the error correcting dynamic multiplier equation and also a significant role for income distribution in this equation. Since this role is based on a negative sign we have the result that the economy is profit-led, i.e., investment behavior (which is assumed to depend negatively on real unit wage costs) dominates the outcome of a change in income distribution. With respect to the interest rate policy we finally obtain a sluggish form of interest rate smoothing, based on a passive policy rule (with a coefficient 0.06/0.08 in front of the inflation gap).

Next we compare the preceding situation with the case where the climate expression π^m is based on a 24 quarter horizon in the place of the 12 quarter horizon we have employed so far.[14]

[14] Details on the estimation of the subsequently reported results are provided in an appendix of the working paper version of this paper.

$$
\begin{aligned}
d\ln w_t &= 0.12V^l_{t-1} - 0.06uc_{t-1} + 0.71d\ln p_t + 0.29\pi^{24}_t - 0.10,\\
d\ln p_t &= 0.04V^c_{t-1} + 0.09uc_{t-1} + 0.38d\ln w_t + 0.62\pi^{24}_t - 0.03,\\
d\ln V^l_t &= 0.18d\ln V^c_t,\\
d\ln V^c_t &= -0.14V^c_{t-1} - 0.94(r_{t-1} - d\ln p_t) - 0.54uc_{t-1} + 0.12,\\
dr_t &= -0.09r_{t-1} + 0.07d\ln p_t + 0.01V^c_{t-1} - 0.01.
\end{aligned}
$$

We see that the application of a time horizon of 24 quarters for the formation of the inflationary climate variable does not alter the qualitative properties of the dynamics significantly as compared to the case of a moving average with linearly declining weights over 12 quarters only (which approximately corresponds to a value of $\beta_{\pi^m} = 0.15$ in an adaptive expectations mechanism as used for the theoretical version of the model in section 3). Even choosing only a six quarter horizon for our linearly declining weights preserves the qualitative features of our estimated model and also by and large the stability properties of the dynamics as we shall see later on, though inflationary expectations over the medium run are then updated with a speed comparable to the ones used for the price PC in hybrid New Keynesian approaches:

$$
\begin{aligned}
d\ln w_t &= 0.12V^l_{t-1} - 0.08uc_{t-1} + 0.27d\ln p_t + 0.73\pi^{6}_t - 0.11,\\
d\ln p_t &= 0.03V^c_{t-1} + 0.02uc_{t-1} + 0.10d\ln w_t + 0.90\pi^{6}_t - 0.03,\\
d\ln V^l_t &= 0.18d\ln V^c_t,\\
d\ln V^c_t &= -0.14V^c_{t-1} - 0.94(r_{t-1} - d\ln p_t) - 0.54uc_{t-1} + 0.12,\\
dr_t &= -0.08r_{t-1} + 0.06d\ln p_t + 0.01V^c_{t-1} - 0.01.
\end{aligned}
$$

We thereby arrive at the general qualitative result that wages are more flexible than prices with respect to their corresponding measures of demand pressure and that wage earners are more short-sighted than firms with respect to the weight they put on their current (perfectly foreseen) measure of cost pressure as compared to the inflationary climate that surrounds this situation. Blanchard and Katz (1999) type error correction mechanisms play a role both in the wage PC and also in the price PC for the U.S. economy and have the sign that is predicted by theory, in contrast to what is found out by these two authors themselves. We have the validity of Okun's law with an elasticity coefficient of less than 20 percent and have the correct signs for the dynamic multiplier process as well as with respect to the influence of changing real rate of interests on economic activity. Finally, the impact of income distribution on the change in capacity utilization is always a negative one and thus of an orthodox type, meaning that rising average unit wage costs will decrease economic activity, and will therefore imply at least from a partial perspective that increasing wage flexibility is stabilizing, while increasing price flexibility (again with respect to its measure of demand pressure) is not.

We conclude from the above that it should be legitimate to use the system estimate with π^{12} as inflation climate term for the further evaluation of the dynamic properties of our theoretical model of section 3, in order to see what more can be obtained as compared to the theoretical results of section 4 when empirically supported parameter signs and sizes are (approximately) taken into account. As a further support for this parameter approximation we finally also report single equations estimates for our 5D system in order to get a feeling for the intervals in which the parameter values may sensibly assumed to lie:[15]

$$\begin{aligned}
d\ln w_t &= 0.19V^l_{t-1} - 0.07uc_{t-1} + 0.16d\ln p_t + 0.84\pi^{12}_t - 0.17, \\
d\ln p_t &= 0.05V^c_{t-1} + 0.05uc_{t-1} + 0.09d\ln w_t + 0.91\pi^{12}_t - 0.04, \\
d\ln V^l_t &= 0.16d\ln V^c_t, \\
d\ln V^c_t &= -0.14V^c_{t-1} - 0.93(r_{t-1} - d\ln p_t) - 0.54uc_{t-1} + 0.12, \\
dr_t &= -0.10r_{t-1} + 0.10d\ln p_t + 0.01V^c_{t-1} - 0.01.
\end{aligned}$$

Again parameter sizes are changed to a certain degree. We do not expect however that this changes the stability properties of the dynamics in a qualitative sense and we will check this in the following section from the theoretical as well as numerical perspective.

The above by and large similar representation of the sizes of the parameter values of our dynamics thus reveal various interesting assertions on the relative importance of demand pressure influences as well as cost pressure effects in the wage-price spiral of the U.S. economy. The Blanchard and Katz error correction terms have the correct signs and are of relevance in general. Okun's law holds as a level relationship between the capacity utilization rate and the rate of employment, basically of the form $V^l/\bar{V}^l = (V^c/\bar{V}^c)^b$ with an elasticity parameter b of about 18 percent. The dynamic IS equation shows the from the partial perspective stabilizing role of the multiplier process and a significant dependence of the rate of change of capacity utilization on the current real rate of interest. There is a significant and negative effect of real unit wage costs (we conjecture: since investment dominates consumption) on this growth rate of capacity utilization, which in this aggregated form suggests that the economy is profit-led as far as aggregate goods demand is concerned, i.e., real wage cost increases significantly decrease economy activity.

Finally, for the Taylor interest rate policy rule, we get the result that interest rate smoothing takes place around the ten percent level, and that monetary policy is to be considered as somewhat passive ($\gamma_p/(1-\gamma_r) < 1$) in such an environment as far as the inflation gap is concerned, and that there is only a weak direct influence of the output gap on the rate of change of the nominal rate of interest. It may therefore be expected that instability can be an outcome of the theoretical model when simulated with these estimated parameter values. Finally, we note that it is not really possible to recover the steady state rate of interest from the constant in the above estimated Taylor rules in a statistically significant way, since the expression implied for this rate by our formulation of the Taylor rule would be:

$$r_o = (const + \gamma_p \bar{\pi} + \gamma_{V^c})/\gamma_r,$$

which does not determine this rate with any reliable statistical confidence. This also holds for the other constants that we have assumed as given in our formulation of Keynesian DAS-DAD dynamics.

[15] Details on the t-statistics of the subsequently reported results can be found in the appendix of the working paper version of paper.

In sum the system estimates of this section provide us with a result that confirms theoretical sign restrictions. They moreover provide definite answers with respect to the role of income distribution in the considered disequilibrium AS-AD or DAS-DAD dynamics, confirming in particular the orthodox point of view that economic activity is likely to depend negatively on real unit wage costs. We have also a negative real wage effect in the dynamics of income distribution, in the sense that the growth rate of real wages, see our reduced form real wage dynamics in section 3, depends – through Blanchard and Katz error correction terms – negatively on the real wage. Its dependence on economic activity levels however is somewhat ambiguous, but in any case small. Real wages therefore only weakly increase with increases in the rate of capacity utilization which in turn however depends in an unambiguous way negatively on the real wage, implying in sum that the Rose (1967) real wage channel is present, but may not dominate the dynamic outcomes.

Finally, the estimated adjustment speed of the price level is so small that the dynamic multiplier effect dominates the overall outcome of changes in capacity utilization on the growth rate of this utilization rate, which therefore establishes a further stabilizing mechanism in the reduced form of our multiplier equation. The model and its estimates thus by and large confirm the conventional Keynesian view on the working of the economy and thus provide in sum a result very much in line with the traditional ways of reasonings from a Keynesian perspective. There is one important qualification however, as we will show in the next sections, namely that downward money wage flexibility can be good for economic stability, in line with Rose's (1967) model of the employment cycle, but in opposition to what Keynes (1936) stated on the role of downwardly rigid money wages. Yet, the role of income distribution in aggregate demand and wage vs. price flexibility was not really a topic in the General Theory, which therefore did not comment on the possibility that wage declines may lead the economy out of a depression via a channel different from the conventional Keynes-effect.

6 Stability analysis of the estimated model

In the preceding section we have provided definite answers with respect to the type of real wage effect present in the data of the U.S. economy after World War II, concerning the dependence of aggregate demand on the real wage, the degrees of wage and price flexibilities and the degree of forward-looking behavior in the wage and price PC. The resulting combination of effects and the estimated sizes of the parameters (in particular the relative degree of wage vs. price flexibility) suggest that their particular type of interaction is favorable for stability, at least if monetary policy is sufficiently active.

We start the stability analysis of the semi-structural theoretical model with estimated parameters from the following reference situation (the system estimate where the inflationary climate is measured as by the twelve quarter moving average):

$$\begin{aligned}
d\ln w_t &= 0.13V^l_{t-1} - 0.07uc_{t-1} + 0.49d\ln p_t + 0.51\pi^{12}_t - 0.12,\\
d\ln p_t &= 0.04V^c_{t-1} + 0.05uc_{t-1} + 0.18d\ln w_t + 0.82\pi^{12}_t - 0.04,\\
d\ln V^l_t &= 0.18d\ln V^c_t,\\
d\ln V^c_t &= -0.14V^c_{t-1} - 0.94(r_{t-1} - d\ln p_t) - 0.54uc_{t-1} + 0.12,\\
dr_t &= -0.08r_{t-1} + 0.06d\ln p_t + 0.01V^c_{t-1} - 0.01.
\end{aligned}$$

We consider first the 3D core situation obtained by totally ignoring adjustments in the inflationary climate term, by setting $\pi^m = \bar{\pi}$ in the theoretical model, and by interpreting the estimated law of motion for V^l in level terms, i.e., by moving from the equation $\hat{V}^l = b\hat{V}^c$ to the equation $V^l = \bar{V}^l(V^c/\bar{V}^c)^b$, with $b = 0.18$ (and $\bar{V}^l = \bar{V}^c = 1$ for reasons of simplicity and without much loss of generality). On the basis of our estimated parameter values we furthermore have that the expression $\beta_{p_1} - \kappa_p\beta_{w_1}$ is approximately zero (slightly positive), i.e., the weak influence of the state variable ω in the reduced form price PC will not be of relevance in the following reduced form of the dynamics (which however is not of decisive importance for the following stability analysis). Finally, the critical condition for normal or adverse Rose effects

$$\alpha = (1 - \kappa_p)\beta_{w_1}b - (1 - \kappa_w)\beta_{p_1} \approx 0$$

is also – due to the measured size of the parameter b – close to zero (which is of importance for stability analysis, see the matrix J below). Rose real-wage effects are thus not very strong in the estimated form of the model, at least from this partial point of view, despite a significant negative dependence of capacity utilization on real unit wage costs (the wage share).

Under these assumptions, the laws of motion (9) – (13) – with the reduced form price PC inserted again – can be reduced to the following qualitative form (where the undetermined signs of a_1, b_1, c_1 do not matter for the following stability analysis and where are assumed to be sufficiently close to 0):

$$\hat{V}^c = a_1 - a_2\, V^c - a_3\, r - a_4\, \ln\omega, \tag{35}$$

$$\dot{r} = b_1 + b_2\, V^c - b_3\, r \pm b_4 \ln\omega, \tag{36}$$

$$\hat{\omega} = c_1 \pm c_2\, V^c - c_4\, \ln\omega, \tag{37}$$

since the dependence of $\hat{p}$ on V^c is a weak one, to be multiplied by 0.17 in the comparison with the direct impact of V^c on its rate of growth, and thus does not modify the sign measured for the direct influence of this variable on the growth rate of the capacity utilization rate significantly. Note with respect to this qualitative characterization of the remaining 3D dynamics, that the various influences of the same variable in the same equation have been aggregated always into a single expression, the sign of which has been obtained from the quantitative estimates shown above. We thus have to take note here in particular of the fact that the reduced form expression for the price inflation rate has been inserted into the first two laws of motion for the capacity utilization dynamics and the interest rate dynamics, which have been rearranged on this basis so that the influence of the variables V^c and ω appears at most only once, though both terms appear via two different channels in these laws of motion, one direct channel and one via the price inflation rate.

The result of our estimates of this equation is that the latter channel is not changing the signs of the direct effects of capacity utilization (via the dynamic multiplier) and the real wage (via the aggregate effect of consumption and investment behavior). We note again that the parameter c_2 may be uncertain in sign, but will in any case be close to zero, while the sign of b_4 does not matter in the following. A similar treatment applies to the law of motion for the nominal rate of interest, where price inflation is again broken down into its constituent parts (in its reduced form expression) and where the influence of V^l in this expression is again replaced by V^c through Okun's Law. Finally, the law of motion for real wages themselves is obtained from the two estimated structural laws of motion for wage and price inflation in the way shown in section 3. We have the stated very weak, but possibly positive influence of capacity utilization on the growth rate of real wages, since the wage Phillips curve slightly dominates the outcome here and an unambiguously negative influence of real wages on their rate of growth due to the signs of the Blanchard and Katz error correction terms in the wage and the price dynamics.

On this basis, we arrive – if we set the considered small magnitudes equal to zero – at the following sign structure for the Jacobian at the interior steady state of the above reduced model for the state variables V^c, r, ω:

$$J = \begin{pmatrix} - & - & - \\ + & - & 0 \\ 0 & 0 & - \end{pmatrix}.$$

We therefrom immediately get that the steady state of these 3D dynamics is asymptotically stable, since the trace is negative, the sum a_2 of principal minors of order two is always positive, and since the determinant of the whole matrix is negative. The coefficients $k_i, i = 1, 2, 3$ of the Routh Hurwitz polynomial of this matrix are therefore all positive as demanded by the Routh Hurwitz stability conditions. The remaining stability condition is

$$k_1 k_2 - k_3 = (-traceJ)k_2 + detJ > 0.$$

With respect to this condition we immediately see that the determinant of the Jacobian J, given by:

$$J_{33}(J_{11}J_{22} - J_{12}J_{21})$$

is dominated by the terms that appear in k_1k_2, i.e., this Routh-Hurwitz condition is also of correct sign as far as the establishment of local asymptotic stability is concerned. The weak and maybe ambiguous real wage effect or Rose effect that is included in the working of the dynamics of the private sector thus does not work against the stability of the steady state of the considered dynamics. Ignoring the Mundell effect by assuming $\beta_{\pi^m} = 0$ therefore allows for an unambiguous stability result, basically due to the stable interaction of the dynamic multiplier with the Taylor interest rate policy rule, augmented by real wage dynamics that in itself is stable due to the estimated signs (and sizes) of the Blanchard error correction terms, where the estimated negative dependence of the change in economic activity on the real wage is welcome from an orthodox point of view, but does not really matter for the stability features of the model. The neglectance of the Mundell effect therefore leaves us with a situation that is close in spirit to the standard textbook considerations of Keynesian macrodynamics. Making the β_{π^m} slightly positive does not overthrow the above stability assertion, since the determinant of the 4D case is positive in the considered situation, see also Chen, Chiarella, Flaschel and Hung (2004), where it is also shown in detail, that a significant increase of this parameter must lead to local instability in a cyclical fashion via a so-called Hopf-bifurcation (if $\gamma_r < \gamma_p$).

Figure 3 shows simulations of the estimated dynamics where indeed the parameter β_{π^m} is now no longer zero, but set equal to $0.075, 0.15$ in correspondence to the measures π^{12}, π^6 of the inflationary climate used in our estimates (these values arise approximately when we estimate β_{π^m} by means of these moving averages). We use a large real wage shock (increase by ten percent) to investigate the response of the dynamics (with respect to capacity utilization) to such a shock. The resulting impulse-responses are unstable ones in the case of the estimated policy parameter $\gamma_p = 0.6$ as shown in the two graphs on the left hand side of figure 3. This is due to the fact that the estimated passive Taylor rule allows for a positive real eigenvalue that leads to divergence when the dominant root of the estimated situation has run its course brought the dynamics sufficiently close to the steady state. Increasing the parameter γ_p towards 0.8 (a Taylor rule right at the border towards an active one) or even to 0.12 (where α_p assumes a standard value of 0.5) moves the positive real eigenvalue into the negative half plane of the plane of complex numbers and thus makes the dynamics produce trajectories that converge back to the steady state as shown in figure 3 on its left hand side. On its right hand side we show in the top figure how the positive real eigenvalue (the maximum of the real parts of the eigenvalues of the dynamics) varies with the parameter γ_p. We clearly see that instability is reduced and finally removed as this policy parameter is increased.[16]

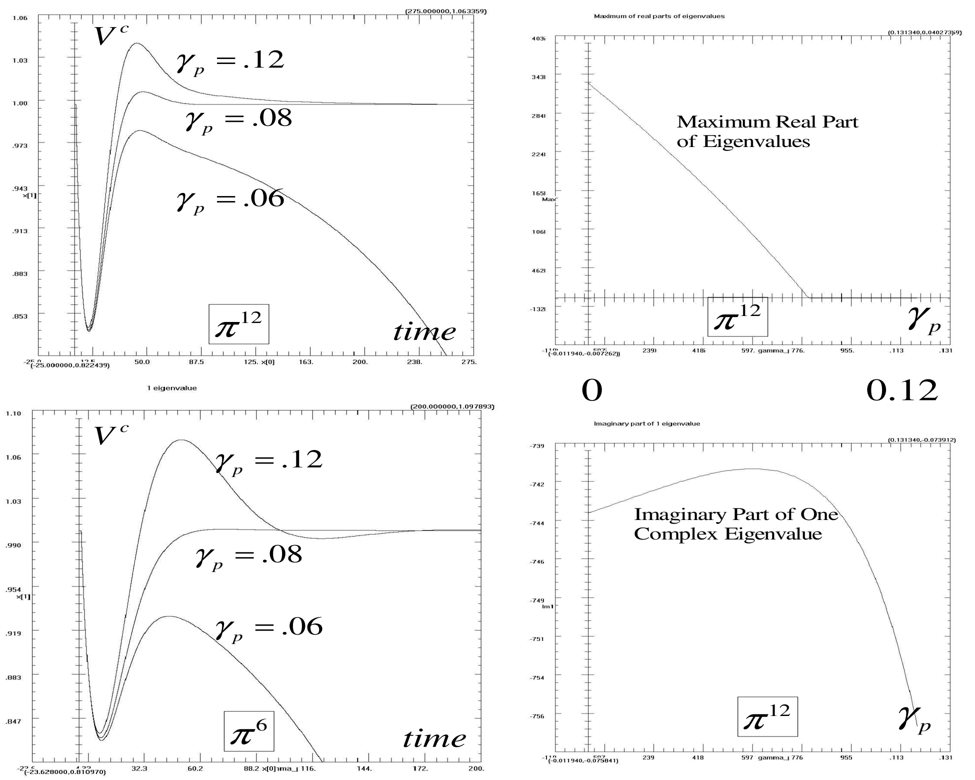

Figure 3: Responses to real wage shocks in the range of estimated parameter values.

More active monetary policy leading to stability is a result that holds for all measures of the inflation climate as shown for the case π^6 in figure 3. Bottom right we finally show in this figure that there are complex roots involved in the considered situation (there for the case π^{12}). Adjustment to the steady state is therefore of a cyclical nature, though only weakly cyclical as shown in this figure). In sum we therefore get that active Taylor rules as estimated for the past to decades (but not for our larger estimation period) will bring stability to the dynamics, since the dominant root then enforces convergence to the steady state without any counteracting force close to the steady state. In the considered range for the parameter β_{π^m} the overall responses of the dynamics are then by and large of the shown type, i.e., the system has strong, though somewhat cyclical stability properties over this whole range, if monetary policy is made somewhat more active than

[16]Note that – due to the estimated form of Okun's law – one eigenvalue of the 5D dynamics must always be zero.

estimated, independently of the particular combination of the speed of adjustment of the inflationary climate and the set of other parameter values we have estimated in the preceding section. The impulse-response situation shown in figure 3 is the expected one. The same holds true for the response of $V^l, \hat{w}, \hat{p}, r$ which all decrease in the contractive phase shown in figure 3.

We again note that the system is subject to zero root hysteresis, since the laws of motion for V^l, V^c are here linearly dependent (since $\alpha_{V_1^l}$, has been estimated as being zero), i.e., it need not converge back to the initially given steady state value of the rate of capacity utilization which was assumed to be 1. Note also that the parameter estimates are based on quarterly data, i.e., the plots in figure 3 correspond to 25 years and thus show a long period of adjustment, due to the fact that all parameters have been assumed as time-invariant, so that only the slow process of changing income distribution and its implications for Keynesian aggregate demand is thus driving the economy here.

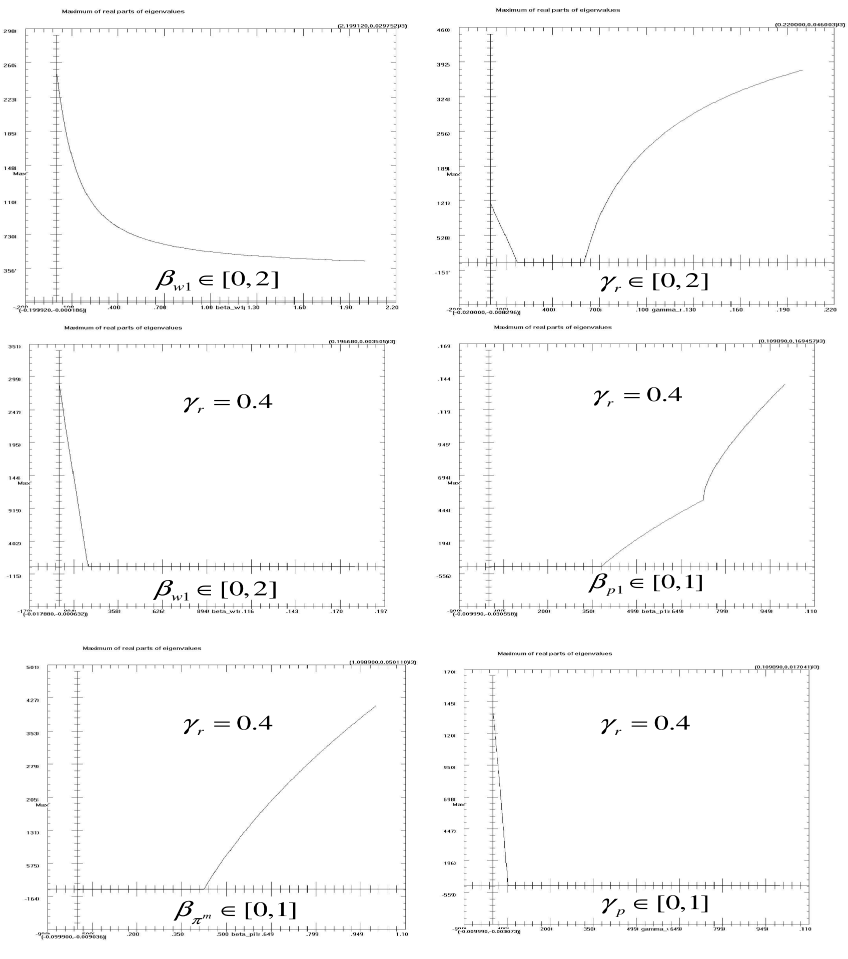

Figure 4: Eigenvalue diagrams for varying parameter sizes.

Next we test in figure 4 the stability properties of the model if one of its parameters is varied in size. We there plot the maximum value of the real parts of the eigenvalues against specific parameter changes shown on the horizontal axis in each case. We by and large find (also for parameter variations that are not shown) that all partial feedback chains (including the working of the Blanchard and Katz error correction terms) translate themselves into corresponding 'normal' eigenvalue reaction patterns for the full 5D dynamics.

With respect to the wage flexibility parameter β_{w_1} we see in figure 4 top-left that its increase helps to reduce the instability of the system with respect to the estimated parameter set, but is not able by itself to enforce convergence. The same holds true for the other adjustment parameters in the wage and the price Phillips curves. Top-right we then determine the range of values for the parameter γ_r where local asymptotic stability is indeed established and find that this is approximately true for a the interval $(0.1, 0.6)$, while large parameters values imply a switch back to local instability. Interest rate smoothing of a certain degree therefore can enforce convergence back to the steady state. We thus now change the estimated parameter set in this respect and assume for the parameter γ_r the value 0.4 in the remainder of figure 4.

The two plots in the middle of figure 4 then show that too low wage flexibility and too high price flexibility will destabilize the dynamics again. this is what we expect from the real wage effect in a profit led economy, due o what has been said on normal and adverse Rose effects. Furthermore, concerning the Mundell effect, we indeed also find that an adjustment of the inflationary climate expression that is too fast induces local asymptotic instability and is therefore destabilizing (see figure 4, bottom-left). Finally, bottom-right we see that a Taylor interest rate policy rule that is too passive with respect to the inflation gap will also endanger the stability we have created by increasing the speed of interest rate smoothing. Such eigenvalue diagrams therefore nicely confirm what is know from partial reasoning on Keynesian macrodynamic feedback chains. Note here that increasing price flexibility is also destabilizing via the Mundell-effect, since the growth rate $\hat{V}^c$ of economic activity can thereby be made to depend positively on its level (via the real rate of interest channel, see eq. (14)), leading to an unstable augmented dynamic multiplier process in the trace of J under such circumstances. Moreover, such increasing price flexibility will give rise to a negative dependence of the growth rate of the real wage on economic activity (whose rate of change in turn depends negatively on the real wage) and thus leads to further sign changes in the Jacobian J. Increasing price flexibility is therefore bad for the stability of the considered dynamics from at least two perspectives.

Let us return now to our analytical stability considerations again. The destabilizing role of price flexibility is enhanced if we add to the above stability analysis for the 3D Jacobian the law of motion for the inflationary climate surrounding the current evolution of price inflation. Under this extension we go back to a 4D dynamical system, the Jacobian J of which is obtained by augmenting the previous one in its sign structure in the following way (see again eq. (14)):

$$J = \begin{pmatrix} - & - & - & + \\ + & - & 0 & + \\ 0 & 0 & - & 0 \\ + & 0 & 0 & 0 \end{pmatrix}.$$

As the positive entries J_{14}, J_{41} show, there is now a new destabilizing feedback chain included, leading from increases in economic activity to increases in inflation and climate inflation and from there back to increases in economic activity, again through the real rate of interest channel (where the inflationary climate is involved due to the expression that characterizes our reduced form price PC). This destabilizing, augmented Mundell effect must become dominant sooner or later (even under the estimated simplified feedback structure) as the adjustment speed of the climate expression β_{π^m} is increased. This is obvious from the fact that the only term in the Routh-Hurwitz coefficient a_2 that depends on the parameter β_{π^m} exhibits a negative sign, which implies that a sufficiently high β_{π^m} will make the coefficient a_2 negative eventually. The Blanchard and Katz error correction terms in the fourth row of J, obtained from the reduced form price Phillips curve, that are (only as further terms) associated with the speed parameter β_{π^m}, are of no help here, since they do not appear in combination with the parameter β_{π^m} in the sum of principal minors of order 2. In this sum the parameter β_{π^m} thus only enters once and with a negative sign implying that this sum can be made negative (leading to instability) if this parameter is chosen sufficiently large. This stands in some contrast to the estimation results where the there defined inflation climate term has been varied significantly without finding a considerable degree of instability.

Assuming – as a mild additional assumption – that interest rate smoothing is sufficiently weak furthermore allows for the conclusion that the 4D determinant of the above Jacobian exhibits a positive sign throughout. We thus in sum get that the local asymptotic stability of the steady state of the 3D case extends to the 4D case for sufficiently small parameters $\beta_{\pi^m} > 0$, since the eigenvalue that was zero in the case $\beta_{\pi^m} = 0$ must become negative due to the positive sign of the 4D determinant (since the other three eigenvalues must have negative real parts for small β_{π^m}). Loss of stability can only occur through a change in the sign of the Routh-Hurwitz coefficient k_2, which can occur only once by way of a Hopf-bifurcation where the system looses its local stability through the local death of an unstable limit cycle or the local birth of a stable limit cycle. This result is due to the destabilizing Mundell-effect of a faster adjustment of the inflationary climate into which the economy is embedded, which in the present dynamical system works through the elements J_{14}, J_{41} in the Jacobian J of the dynamics and thus through the positive dependence of economic activity on the inflationary climate expression and the positive dependence of this climate expression on the level of economic activity.

To sum up we have established that the 4D dynamics will be convergent for sufficiently small speeds of adjustments β_{π^m}, and for a monetary policy that is sufficiently active, while they will be divergent for parameters β_{π^m} chosen sufficiently large. The Mundell effect thus works as expected from a partial perspective. There will be a unique Hopf bifurcation point $\beta^H_{\pi^m}$ in between (for γ_r sufficiently small), where the system loses asymptotic stability in a cyclical fashion. Yet sooner or later purely explosive behavior will be indeed be established (as can be checked by numerical simulations), where there is no longer room for persistent economic fluctuations in the real and the nominal magnitudes of the economy.

In such a situation global behavioral nonlinearities must be taken into account in order to limit the dynamics to domains in the mathematical phase space that are of economic relevance. Compared to the New Keynesian approach briefly considered in section 2 of this paper we thus have that – despite many similarities in the wage-price block of our dynamics – we have completely different implications for the resulting dynamics which – for active interest rate policy rules – are convergent (and thus determined from the historical perspective) when estimated empirically (with structural Phillips curves that are not all at odds with the facts) and which – should loss of stability occur via a faster adjustments of the inflationary climate expression – must be bounded by appropriate changes in economic behavior far off the steady state and not just by mathematical assumption as in the New Keynesian case. Furthermore, we have employed in our model type a dynamic IS-relationship in the spirit of Rudebusch and Svensson's (1999) approach, also confirmed in its backward orientation by a recent article of Fuhrer and

Rudebusch (2004). One may therefore state that the results achieved in this and the preceding section can provide an alternative of mature, but traditional Keynesian type that does not lead to the radical – and not very Keynesian – New Keynesian conclusion that the deterministic part of the model is completely trivial and the dynamics but a consequence of the addition of appropriate exogenous stochastic processes.

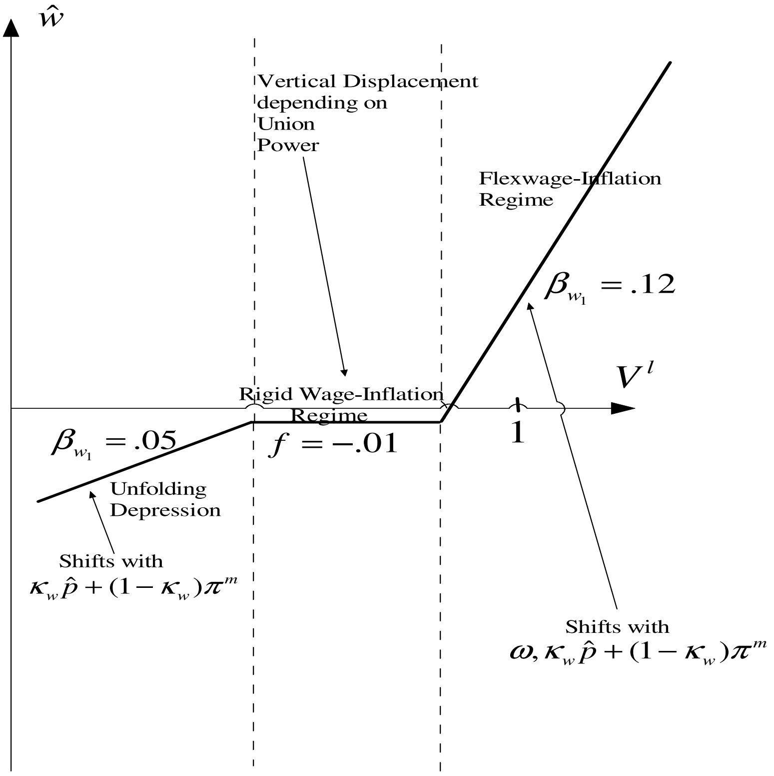

Figure 5: Three possible regimes for wage inflation.

7 Instability, global boundedness and monetary policy

Next we will express some conjectures concerning other scenarios. Based on the estimated range of parameter values, and for an active monetary policy rule, the preceding section has shown that the model then exhibits strong convergence properties, with only mild fluctuations around the steady state in the case of small shocks, but with a long downturn and a long-lasting adjustment in the case of strong shocks (as in the case of figure 3, where a 10 percent increase in real wages shocks the economy). Nevertheless, the economy is reacting in a fairly stable way to such a large shock and thus seems to have the characteristics of a strong shock absorber. Figure 3 however is based on estimated linear Phillips curves, i.e., in particular, on wage adjustment that is as flexible in an upward as well as in a downward direction. It is however much more plausible that wages behave differently in a high and in a low inflation regime, see Chen and Flaschel (2004) for a study of the wage PC along these lines which confirms this common sense statement. Following Filardo (1998) we here go even one step further and indeed assume a three regime scenario as shown in figure 5 where we make use of his figure 4 and for illustrative purposes of the parameter sizes there shown[17] (though they there refer to output gaps on the horizontal axis, inflation surprises on the vertical axis and a standard reduced form Phillips curve relating these two magnitudes):

The figure 5 suggests that the wage PC of the present model is only in effect if there holds simultaneously that wage inflation is above a certain floor f – here (following Filardo) shown to be negative[18] – and the employment rate is still above a certain floor $\underline{V}^l$, where wage inflation starts to become (downwardly) flexible again. In this latter area (where wage inflation according to the original linear curve is below f and the employment rate below $\underline{V}^l$,) we assume as form for the resulting wage-inflation curve the following simplification and modification of the original one:

$$\hat{w} \;=\; \beta_{w_1}(V^l - \underline{V}^l) + \kappa_w \hat{p} + (1-\kappa_w)\pi^m,$$

i.e., we do not consider the Blanchard and Katz error correction term to be in operation then any more. In sum, we therefore assume a normal operation of the economy if both lower floors are not yet reached, constant wage inflation if only the floor f has been reached and further falling wage inflation or deflation rates (as far as demand pressure is concerned) if both floors have been passed. Downward wage inflation or wage deflation rigidity thus does not exist for all states of a depressed economy, but can give way to its further downward adjustment in severe states of depression in actual economies.[19]

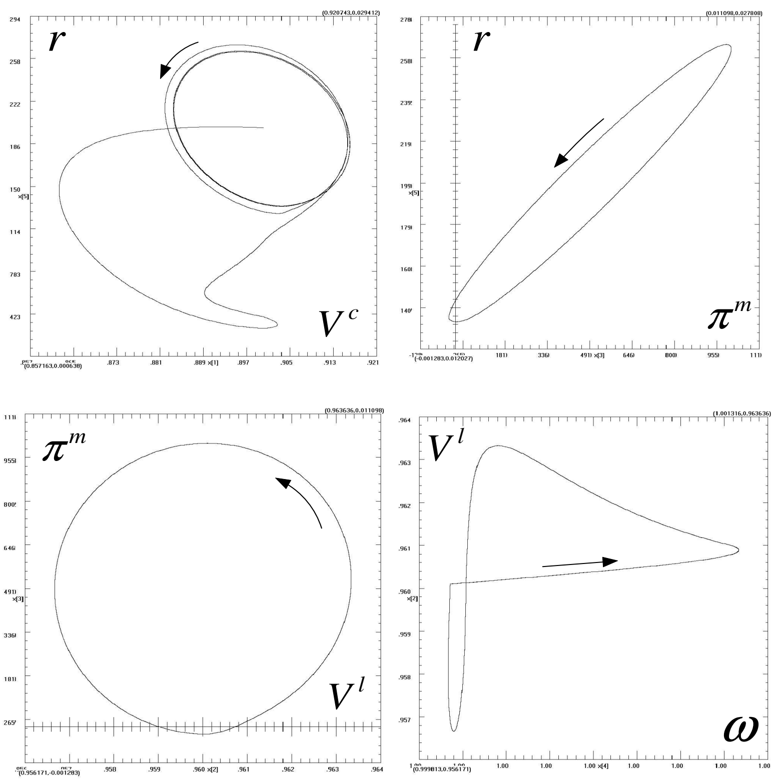

Figure 6: Rose-Filardo type wage Phillips curves and the emergence of persistent business fluctuations.

[17]here adjusted to quarterly data.

[18] by contrast, this floor is claimed and measured to be positive for six European countries in Hoogenveen and Kuipers (2000).

[19]An example for this situation is given by the German economy, at least since 2003.

In figure 6 we consider a situation as depicted in figure 3, i.e., the working of a wage Phillips curve as it was already formulated in Rose (1967) and again contractive real wage shocks.[20] Top-left we plot the rate of capacity utilization against the nominal rate of interest and obtain that the economy now adjusts to a fairly simple persistent fluctuation in this projection of the 5D phase space with an overshooting interest rate adjustment, since for example the interest rate keeps on rising though economic activity has started to fall already for quite a while. There is a strict positive correlation between the rates of utilization of capital and labor, i.e., all assertions made with respect to one utilization rate also hold for the other one. The reason for this overshooting reaction of interest rate policy is that this policy closely follows the inflation gap and not the utilization gap, here represented with respect to the inflation climate term in the figure 6, top- right. This figure also shows that deflation is indeed occurring in the course of one cycle, though only weakly in a brief subperiod of it.

As already indicated there is also overshooting involved in the phase plot between the rate of employment and the inflation climate (figure 6, bottom-left), i.e., the model clearly generates periods of stagflation and also periods where disinflation is coupled with a rising employment rate. This pattern is well-known from empirical investigations. Less close to such investigations, see Flaschel and Groh (1995) for example, is the pattern that is shown bottom-right in figure 6, i.e., a phase plot between the real wage and the rate of employment which according to the Goodwin (1967) model of a growth cycle should be also an overshooting one with a clockwise orientation which in figure 6 is only partly visible in fact. Taken together, we however have the general result that a locally unstable steady state can be tamed towards the generation of a persistent fluctuation around it if wages become sufficiently flexible far off the steady state (both in an upward as well as in a downward direction).

We next show that the corridor $(\underline{V}^l, \bar{V}^l)$ where the second regime in figure 5 applies may be of decisive importance for the resulting dynamics. Small changes in the size of this interval can have significant effects on the observed volatility of the resulting trajectories as the figure 7 exemplifies. In figure 7 we lower the value of $\underline{V}^l$ from 0.96 to $0.959, 0.585, 0.958, 0.9575$ and see that the limit cycle is becoming larger and is approached in more and more complicated ways. In the case 0.575 we finally get a quite different limit cycle with lower r, V^c on an average and only a small amplitude which is shown in enlarged form in figure 7 bottom-right.

Finally, increasing the opposing forces β_{π^m} and γ_p even further to 1.4 and 0.6 respectively, and assuming now $\underline{V}^l = 0.94 < \bar{V}^l = 1$, ie.e, a large range where there is a floor to money wages (and adjusting the interest rate such that it does not become negative along the trajectories that are shown) provides as with an (somewhat extreme) scenario where even complex dynamics are generated from the mathematical point of view (not directly from the economic point of view) as is shown in figure 8.

[20]The parameters of the plot are the estimated one with the exception of $\beta_{\pi^m} = 0.4, \gamma_p = 0.12$. The first parameter has therefore been increased in order to get local instability of the steady state and the policy parameter has been increased in order to tame the resulting instability to a certain degree. Moreover, we have assumed in this plot that the steady state value of V^l coincides with the value where wages become downwardly flexible again, i.e., we here only switch of the Blanchard and Katz error correction term in the downward direction, but have added a floor $f = 0.0004$ to wage inflation for employment rates above th steady state level ($\bar{V}^c = 0.9$ now). This combination of wage regimes indeed tames the explosive dynamics and gives rise to a limit cycle attractor instead as is shown below.

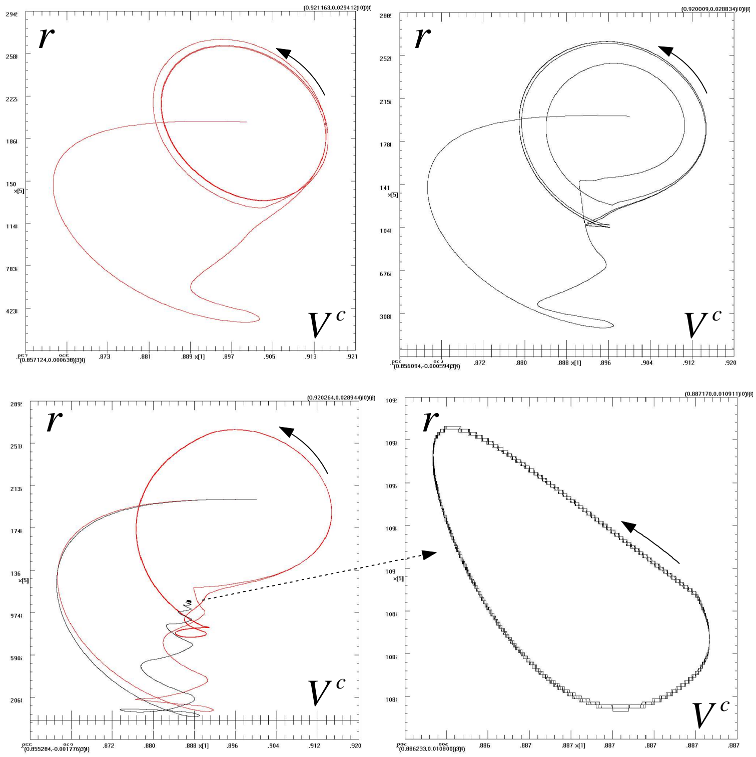

Figure 7: Changes in the regime where rigid wage inflation or deflation prevails.

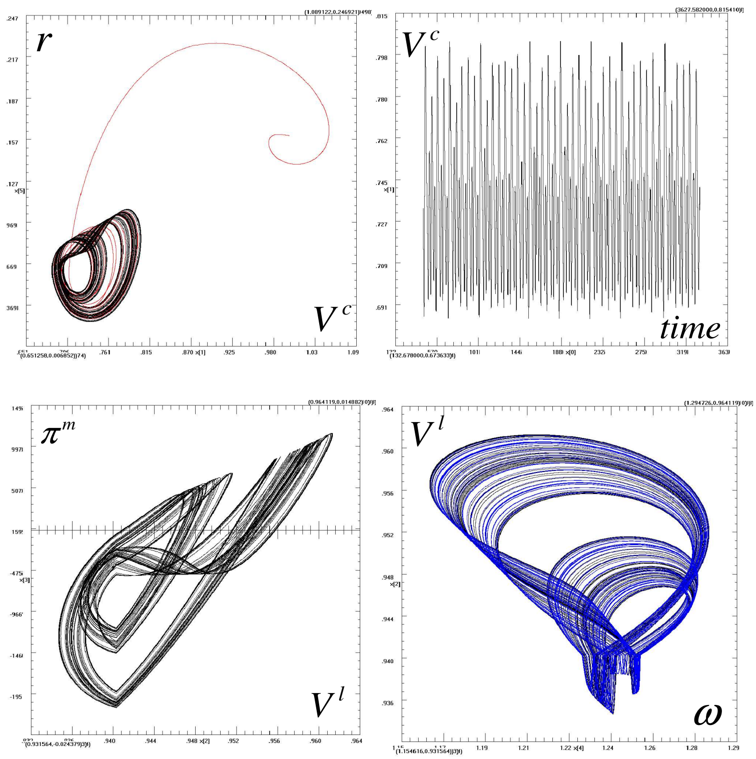

Figure 8: Depressed complex dynamics with long deflationary episodes

Turning now to the effects of monetary policy we first show in figure 9 the situation where the economy is strongly convergent to the steady state in the case of the active monetary policy underlying figure 3. Adding a global floor to this situation radically changes the situation and implies economic collapse once this floor is reached, since real wages are then rising due to falling prices. This situation is again shown to be prevented if wages become flexible again (in a downward direction) at 92 percent of employment. Monetary policy that is then assumed more active either with respect to the utilization gap or the inflation gap can however prevent both situations from occurring, when it implies – as shown – that the floor to money wages can be avoided to come into operation thereby.

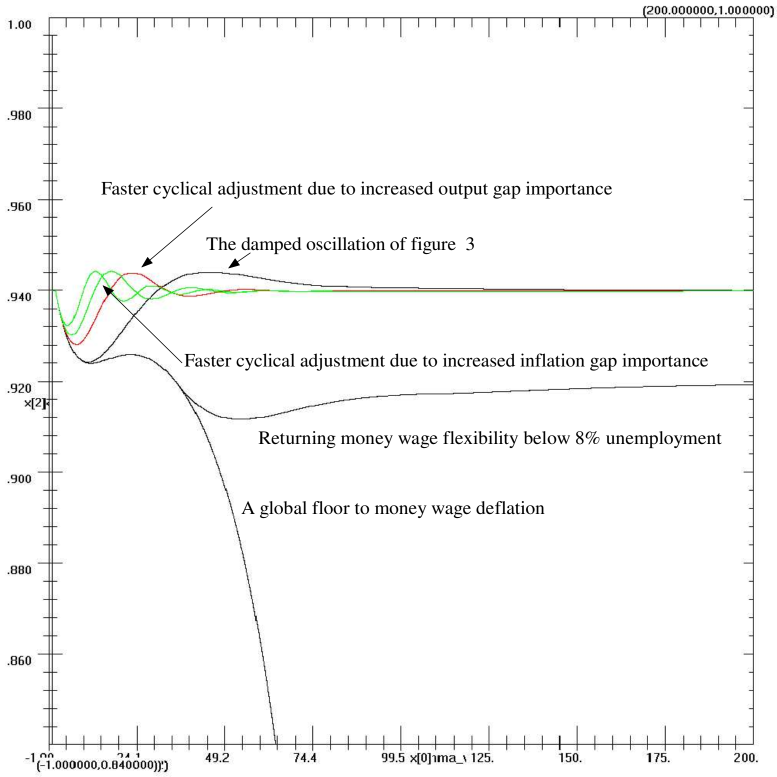

Figure 9: The existence of floors and more active monetary policy rules

The question arises whether a monetary policy is working the better the stronger its reaction is with respect to the inflation gap, i.e., the larger the parameter γ_p becomes. From an applied perspectives it is not to be expected that this is to be the case in reality, since active monetary policy is surely limited by some cautiousness bounds from above. In this 5D dynamical system with its still simple trajectories in may however be a theoretical possibility, though we have already presented a counterexample to a general proposition of this kind in figure 4. Figure 10 now exemplifies this again by means of phase plots, i.e., of projections of the full dynamics into the V^c, r plane. We see in this figure top-left with respect to our estimated parameter set (but with $\beta_{\pi^m} = 0/4$ now again) that a more active monetary policy ($0.12 \rightarrow 0.15$) enlarges the generated limit cycle (see figure 6) and thus makes the economy more volatile. By contrast, see figure 10, top-right, a lower value of $\gamma_p = 0.10$ as compared to figure 6, makes the dynamics in fact convergent with smaller cycles when the transient behavior is excluded, but with a long transient than in the case of figure 10, top-left. This longer transient behavior can be made of an extreme type – with severely underutilized capital along the depressed transient cycles – when the policy reaction to the inflation gap is further reduced (to 0.092), see figure 10, bottom-right and -left. The degree of activeness of monetary policy matters therefore a lot for the business fluctuations that are generated and this in a way with clear benchmark for the appropriate choice of the parameter γ_p.

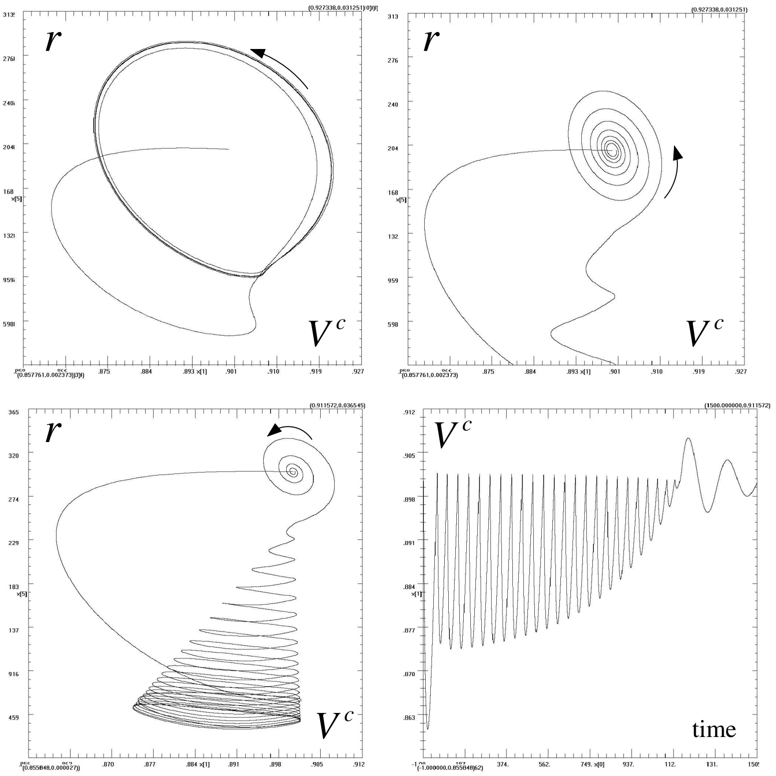

Figure 10: Corridor problems for active monetary policy rules

In figure 11, finally, we vary the parameter $\bar{\pi}$ that characterizes the inflation target of the Central Bank. A first implications of such variations is shown in the plot top-left where we show the limit cycle of figure 6 once again, now together with two trajectories that are based on the assumptions $f = \bar{\pi} = 0.0004$ and $f = \bar{=}0.0004 > \pi = 0.0002$ concerning the temporary floor in money wages and the inflation target of the central bank. In the first case the limit cycle disappears completely and we get instead convergence to the steady state, though with a long transient again. In the second case of an even more restrictive monetary policy we no longer get complete convergence back to th steady state, but instead convergence to a small limit cycle below this steady state, shown in enlarge form in figure 11, top-right. Lowering $\bar{\pi}$ even further (to zero) gives the same result, but with a slightly more depressed limit cycle now, see figure 11, bottom-left. In figure 11, bottom-right, finally, we compare an inflation target 0f 0.05 with an in fact deflation target of the Central bank of -0.002 (both not topical themes in monetary policy). In the first case the persistent fluctuation of the initial situation gets lost and is changed into a business fluctuation with increasing amplitude, that is the economy becomes an unstable one. In the second case we now get in a pronounced way a stable, but depressed limit cycle below the NAIRU levels for the rates of capacity utilization.

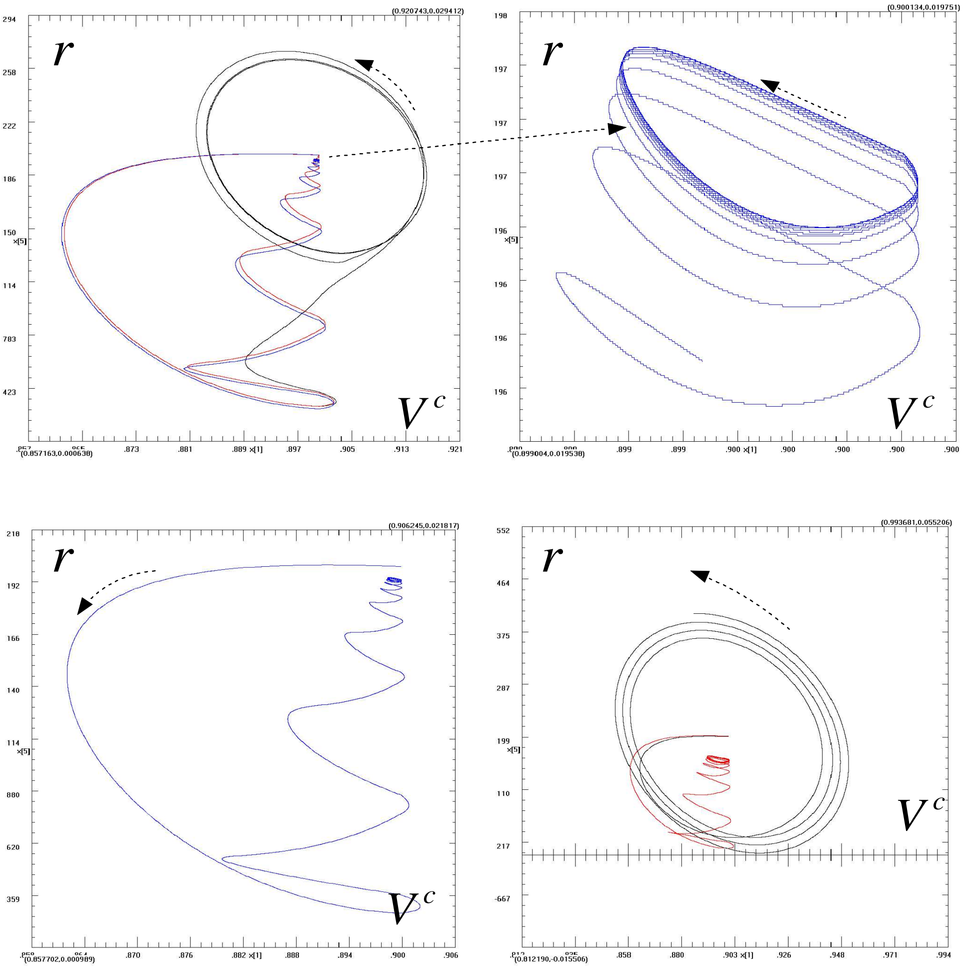

Figure 11: Tight and lose inflation targets

We conclude from these few simulation exercises on other scenarios in the neighborhood of our estimated Keynesian disequilibrium dynamics, that one might observe a variety of interesting scenarios when certain kinks in money wage behavior and changes in the adjustment speed of our inflationary climate expression are taken into account. These changes furthermore show that further investigations of such behavioral nonlinearities are needed, see Chen and Flaschel (2004) and Flaschel, Kauermann and Semmler (2004) for some attempts into this direction for the U.S. economy.

8 Related literature

In this section we want to compare some technical aspects of various recent macrodynamic models. An important technical aspect of the model of this paper is that it exhibits Keynesian feedback dynamics with in particular asymptotic stability of its unique interior steady state solution for low adjustment speeds of wages, prices, and expectations among others. The loss of stability occurs cyclically, generally by way of Hopf bifurcations, when some of these adjustment speeds are made sufficiently large, even leading eventually to purely explosive dynamics sooner or later. This latter fact – if it occurs – implies the need to look for appropriate extrinsic (behavioral) nonlinearities that can bound the dynamics in an economically meaningful domain, such as (some) downward rigidity of wages and prices and the like, if the economy departs too much from its steady state position. Our procedure for making an explosive dynamics bounded and thus viable stands in stark contrast to the New Keynesian approach to macrodynamics where on a similar level of formalization total instability is desirable and achieved by the choice of an appropriate Taylor policy rule. The economy is then made a bounded one simply by assumption (and is thus always sitting in the steady state if exogenously given stochastic processes are removed from the dynamics). The model of this paper can usefully be compared (on the formal level) to the New Keynesian one with staggered wage and price dynamics. Yet, it is radically different from this approach with respect to implications, since we are not forced into a framework with four forward-looking variables where we would have to look for four unstable roots in order to get the conclusion that the variables are at their steady state values (assuming boundedness as part of the solution procedure) as long as only isolated shocks occur and is thus driven as far as business cycle implications are concerned solely by the exogenous stochastic processes that are added to its deterministic core. We have forward-looking behavior (with neoclassical dating) and will find asymptotic stability in the traditional sense of the word over certain ranges in the parameter space. In the case of local instability we look for behavioral nonlinearities that allow the dynamics to remain bounded in an economically meaningful range, in the place of an imposition of such boundedness on admissible solution curves and a quest for determinacy.

We, by contrast, therefore obtain and can prove – from the purely theoretical perspective – based on empirically still unrestricted sizes of the considered adjustment speeds of wages, prices, and quantities, the existence of damped, persistent or explosive fluctuations in the real and the nominal part of the dynamics, in the rates of capacity utilization of both labor and capital, and of wage and price inflation rates. These effects here induce interest rate adjustments by the monetary authority through the attempt to stabilize the observed output and price level fluctuations. We thus obtain a Keynesian theory of an income distribution driven cycle, including a modern approach to monetary policy in such a context. This even holds in the case of myopic perfect foresight, where the structure of the old Neoclassical synthesis radically dichotomizes into independent supply-side real-wage and growth dynamics – that cannot be influenced by monetary policy – and a subsequently determined inflation dynamics, that are purely explosive if the price level is taken as a predetermined variable, a situation that forces conventional approaches to these dichotomizing dynamics to assume convergence by an inconsistent application of the jump-variable technique;[21] see Asada, Chen, Chiarella and Flaschel (2004) for details. In our new matured type of Keynesian labor and goods market dynamics we however can treat myopic perfect foresight of both firms and wage earners without the need to adopt this 'rational expectations' solution methodology in the context of an unstable saddlepoint dynamics, be it of old or new Keynesian type.

From the global perspective, if our theoretical model loses asymptotic stability for higher adjustment speeds, in the present framework specifically of prices and our inflationary climate expression, purely explosive behavior is the generally observed outcome, as it can be demonstrated by means of numerical simulations. The considered, so far only intrinsically nonlinear, model type therefore cannot be considered as being completely

[21]since the nominal wage is transformed into a non-predetermined variable there, despite the initial assumption of only gradually adjusting money wages.

specified under such circumstances, since some mechanism is then required to bound the fluctuations to economically viable regions. Downward money wage rigidity was the mechanism we have often used for this purpose and which we will tried here, however with limited success, in contrast to its successful application in the numerical investigations of the underlying theoretical model in Asada, Chen, Chiarella and Flaschel (2004).

The here estimated somewhat simplified feedback structure of their theoretical model, now indeed no longer (in general) supports the view (of Keynes and others) that downward money wage rigidity will stabilize the economy (as was shown in the structurally more elaborated theoretical paper). Instead, this downward rigidity may now even cause economic breakdown when applied to situations that were strongly stable (convergent to the steady state) without it. This is due to our estimation of the dynamics of capacity utilization rates where we find, on the one hand, besides the usual negative dependence on the real rate of interest, a strong negative dependence on the real wage and thus on income distribution. On the other hand, we find in the wage-price block of the model the sign restrictions of New Keynesian wage and price inflation equations (but do not have their sign reversals in their reduced-form expressions later on, due to our different handling of forward-looking expectations and the inclusion of backward-looking ones). As far as the money wage Phillips curve is concerned we also confirm the form specified in Blanchard and Katz (1999) and find a similar general form to hold for the price inflation Phillips curve. These estimated curves then by and large suggest that real wage changes depend at best slightly positively on economic activity, and that this effect becomes more pronounced the more strongly nominal wages react to the employment gap on the market for labor. In sum we therefore find that the growth rate of real wages depends negatively on its level, with this stabilizing feedback chain to being the stronger the more flexibly nominal wages react to labor market imbalances in the upward as well as in the downward direction. Complete downward wage rigidity may therefore become a problem, and this already in situations where the economy is producing fairly damped cycles (due to an interest rate policy rule that is sufficiently active), if – for instance – the monetary authority is using too low an inflation target.

In the numerical simulations of the estimated model we indeed then found that the reestablishment of money wage flexibility in severely depressed regions of the phase space (however coupled with some midrange downward wage rigidity) can avoid this breakdown, however at the costs of persistent economic fluctuations, to some extent below the normal operating level of the economy. In the present framework (of a profit-led goods demand regime, where real wage increases lead to a decrease in economic activity) downward money wage flexibility is therefore good for economic stability. The opposite conclusion however holds with respect to price flexibility.

The dynamic outcomes of this baseline disequilibrium AS-AD or DAS-DAD model can be usefully contrasted with those of the currently fashionable New Keynesian alternative (the new Neoclassical Synthesis) that in our view is more limited in scope, at least as far as the treatment of interacting Keynesian feedback mechanisms and the thereby implied dynamic possibilities are concerned. A detailed comparison with this New Keynesian approach is provided in Chiarella, Flaschel and Franke (2004, Ch.1). This comparison reveals in particular that one does not really need the typical (in our view strange) dynamics of rational expectation models, based on the specification of certain forward looking variables, if such forward-looking behavior is coupled with backward-looking behavior for the medium-run evolution of the economy (and neoclassical dating in the forward-looking part) and if certain non-linearities in economic behavioral make the obtained dynamics bounded far off the steady state. In our approach, standard Keynesian feedback mechanisms are coupled with a wage-price spiral having – besides partial forward-looking behavior – a considerable degree of inertia, with the result that these feedback mechanisms by and large work as expected (as known from partial analysis), in their interaction with the added wage and price level dynamics.

9 Conclusions and Outlook

We have considered in this paper a significant extension and modification of the traditional approach to AS-AD growth dynamics, primarily by way of an appropriate reformulation of the wage-price block of the model, that allows us to avoid the dynamical inconsistencies of the traditional Neoclassical Synthesis. It also allows us to overcome the empirical weaknesses and theoretical indeterminacy problems of the New Keynesian approach that arise from the existence of only purely forward looking behavior in baseline models of staggered price and wage setting. Conventional wisdom, based on the rational expectations approach, however is here used to avoid the latter indeterminacy problems by appropriate extensions of the baseline model that enforce its total instability (the existence of only unstable roots), implying that the steady state represents the only bounded trajectory in the deterministic core of the model (to which the economy then immediately returns when hit by a demand, supply or policy shock).

By contrast, our alternative approach – which allows for sluggish wage as well as price adjustment and also for certain economic climate variables, representing the medium-run evolution of inflation – completely bypasses the purely formal imposition of such boundedness assumptions. Instead it allows us to demonstrate in a detailed way, guided by the intuition behind important macroeconomic feedback channels, local asymptotic stability under certain plausible assumptions (indeed very plausible from the perspective of Keynesian feedback channels), cyclical loss of stability when these assumptions are violated (if speeds of adjustment become sufficiently high), and even explosive fluctuations in the case of further increases of the crucial speeds of adjustment of the model. In the latter case further behavioral nonlinearities have to be introduced in order to tame the explosive dynamics, for example as in Chiarella and Flaschel (2000, Ch.6,7) where a kinked Phillips curve (downward wage rigidity) is employed to achieve global boundedness.

The stability features of these – in our view properly reformulated – Keynesian dynamics are based on specific interactions of traditional Keynes- and Mundell-effects or real rate of interest effects with real-wage effects. In the present framework – for our estimated parameters – these effects simply imply that increasing price flexibility will be destabilizing but increasing wage flexibility might be stabilizing. On the other hand, of monetary policy responds sufficiently strong to the output or inflation gap there can be less wage flexibility to obtain boundedness of fluctuations. This is based on the estimated fact that aggregate demand here depends negatively on the real wage and on the extended types of Phillips curves we have employed in our new approach to traditional Keynesian growth dynamics. The interaction of these three effects is what explains the obtained stability results under the (in this case not very important) assumption of myopic perfect foresight, on wage as well as price inflation, and thus gives rise to a traditional type of

Keynesian business cycle theory, not at all plagued by the anomalies of the textbook AS-AD dynamics.[22]

Our model provides an array of stability results, which however are narrowed down to damped oscillations when the model is estimated with data for the U.S. economy after World War II (and monetary policy is made somewhat more active in the theoretical model than estimated). Yet, also in the strongly convergent case, there can arise stability problems if the linear wage PC is modified to allow for some downward money wage rigidities. In such a case, prices may fall faster than wages in a depression, leading to real wage increases and thus to further reductions in economic activity. We have shown in this regard how the reestablishment of downward movement may be avoided leading then to a persistent business fluctuations of more or less irregular type and thus back to a further array of interesting stability scenarios. As we have shown monetary policy can avoid such downward movements and preserve damped oscillatory behavior, primarily through the adoption of a target rate of inflation that is chosen appropriately, and in case of the establishment of persistent fluctuations of the above type, reduce such fluctuations by a controlled activation of its response to output gaps or a choice of its response to inflation gaps in a certain corridor, as was shown by way of numerical examples. Therefore, not a simple answer can be given into which direction monetary policy should be changed in order to make the economy less and not indeed more volatile.[23]

Taking all this together, our general conclusion here is that the here proposed framework does not only overcome the anomalies of the Neoclassical Synthesis, Stage I, but also provides a coherent alternative to its second stage, the New Keynesian theory of the business cycle, as for example sketched in Gali (2000). Our alternative to this approach to macrodynamics is based on disequilibrium in the market for goods and labor, on sluggish adjustment of prices as well as wages and on myopic perfect foresight interacting with certain economic climate expression. The rich array of dynamic outcomes of our model provide great potential for further generalizations. Some of these generalizations have already been considered in Chiarella, Flaschel, Groh and Semmler (2000) and Chiarella, Flaschel and Franke (2004). Our overall approach, which may be called a disequilibrium approach to business cycle modelling of mature Keynesian type, thus provides a theoretical framework within which to consider the contributions of authors such as Zarnowitz (1999) who also stresses the dynamic interaction of many traditional macroeconomic building blocks.

[22]See Chiarella, Flaschel and Franke (2004) for a detailed treatment and critique of this textbook approach.

[23]The model of the current paper is numerically further explored in a companion paper, see Chen, Chiarella, Flaschel and Hung (2004), in order to analyze in greater depth, the interaction of the various feedback channels present in the considered dynamics. There is made use of LM curves as well as Taylor interest rate policy rules, kinked Phillips curves and Blanchard and Katz error correction mechanisms in order to investigate in detail the various ways by which locally unstable dynamics can be made bounded and thus viable. The question then is which assumption on private behavior and fiscal and monetary policy – once viability is achieved – can reduce the volatility of the resulting persistent fluctuations. Our work on related models suggests that interest rate policy rule may not be sufficient to tame the explosive dynamics in all conceivable cases, or even make them convergent. But when viability is achieved – for example by downward wage rigidity – we can then investigate parameter corridors where monetary policy can indeed reduce the endogenously generated fluctuations of this approach to Keynesian business fluctuations.

References

Asada, T., Chiarella, C., Flaschel, P. and R. Franke (2003): *Open Economy Macrodynamics. An Integrated Disequilibrium Approach.* Heidelberg: Springer Verlag.

Asada, T., Chen, P., Chiarella, C. and P. Flaschel (2004): Keynesian dynamics and the wage-price spiral. A baseline disequilibrium approach. UTS Sydney: School of Finance and Economics.

Barro, R. (1994): The aggregate supply / aggregate demand model. *Eastern Economic Journal,* 20, 1 – 6.

Baxter, M. and King, R. (1995): Measuring business cycles: Approximate band-pass filters for economic time series. *NBER working paper*, No. 5022.

Baxter, M. and King, R. (1995). Measuring business cycles: Approximate band-pass filters for economic time series. *Eastern Economic Journal,* 81, 575-5.

Blanchard, O.J. and L. Katz (1999): Wage dynamics: Reconciling theory and evidence. American Economic Review. Papers and Proceedings, 69 – 74.

Chen, P. (2003): Weak Exogeneity in Simultaneous Equations Systems . University of Bielefeld: Discussion Paper No. 502.

Chen, P., Chiarella, C., Flaschel, P. and H. Hung (2004): Keynesian disequilibrium dynamics. Estimated convergence, roads to instability and the emergence of complex business fluctuations. UTS Sydney: School of Finance and Economics.

Chen, P. and Flaschel, P. (2004): Testing the dynamics of wages and prices for the US economy. Bielefeld: Center for Empirical Macroeconomics: Working paper.

Chiarella, C. and P. Flaschel (1996): Real and monetary cycles in models of Keynes-Wicksell type. *Journal of Economic Behavior and Organization*, 30, 327 – 351.

Chiarella, C. and P. Flaschel (2000): *The Dynamics of Keynesian Monetary Growth: Macro Foundations*. Cambridge, UK: Cambridge University Press.

Chiarella, C., P. Flaschel, G. Groh and W. Semmler (2000): Disequilibrium, Growth and Labor Market Dynamics. Heidelberg: Springer Verlag.

Chiarella, C., Flaschel, P. and R. Franke (2004): *Foundations for a Disequilibrium Theory of the Business Cycle. Qualitative Analysis and Quantitative Assessment*. Cambridge, UK: Cambridge University Press, to appear.

Davidson, R. and J. Mackinnon (1993): *Estimation and Inference in Econometrics*. New York: Oxford University Press.

Eller, J.W. and R.J. Gordon (2003): Nesting the New Keynesian Phillips Curve within the Mainstream Model of U.S. Inflation Dynamics. Paper presented at the CEPR Conference: The Phillips curve revisited. Berlin: June 2003

Filardo, A. (1998): New Evidence on the output cost of fighting inflation. *Economic Review.* Federal Bank of Kansas City,, 33 –61.

Flaschel, P. and G. Groh (1995): The Classical growth cycle: Reformulation, simulation and some facts. *Economic Notes*, 24, 293 – 326.

Flaschel, P. and H.-M. Krolzig (2004): Wage and price Phillips curves. An empirical analysis of destabilizing wage-price spirals. Oxford: Oxford University. Discussion paper.

Flaschel, P., Kauermann, G. and W. Semmler (2004): Testing wage and price Phillips curves for the United States. Bielefeld University: Center for Empirical Macroeconomics, Discussion Paper.

Fuhrer, J.C. and G.D. Rudebusch (2004): Estimating the Euler equation for output. *Journal of Monetary Economics*, 51, 1133 ff.

Gali, J. (2000): The return of the Phillips curve and other recent developments in business cycle theory. *Spanish Economic Review,* 2, 1 – 10.

Groth, C. (1992): Some unfamiliar dynamics of a familiar macromodel. *Journal of Economics,* 58, 293 – 305.

Hoogenveen, V.C. and S.K. Kuipers (2000): The long-run effects of low inflation rates. *Banca Nazionale del Lavoro Quarterly Review,* 53, 267 – 286.

Keynes, J.M. (1936): *The General Theory of Employment, Interest and Money.* New York: Macmillan.

Laxton, D., Rose, D. and D. Tambakis (2000): The U.S. Phillips-curve: The case for asymmetry. *Journal of Economic Dynamics and Control*, 23, 1459–1485.

Mankiw, G. (2001): The inexorable and mysterious tradeoff between inflation and unemployment. *Economic Journal,* 111, 45 – 61.

Okun, A.M. (1970): *The Political Economy of Prosperity.* Washington, D.C.: The Brookings Institution.

Rose, H. (1967): On the non-linear theory of the employment cycle. *Review of Economic Studies*, 34, 153 – 173.

Rudebusch, G.D. and L.E.O. Svensson (1999): Policy rules for inflation targeting. In: J.B.Talor (ed.): *Monetary Policy Rules.* Chicago: Chicago University Press.

Tobin, J. (1975): Keynesian models of recession and depression. *American Economic Review,* 65, 195 – 202.

Walsh, C.E. (2003): *Monetary Theory and Policy.* Cambridge, MA: The MIT press.

Woodford, M. (2003): *Interest and Prices.* Princeton: Princeton University Press.

Zarnowitz, V. (1999): *Theory and History Behind Business Cycles: Are the 1990s the onset of a Golden Age*? NBER Working paper 7010, http://www.nber.org/papers/w7010.

10 Appendix: Estimation Results

```
System-Estimate: Inflation climate expression PI24
---------------------------------------------------------------------------


DW    = C(11)*DP + (1-C(11))*PI24(-1) + C(12)*VL(-1) + C(13)*UCBP(-1) + C(14)
DP    = C(21)*DW + (1-C(21))*PI24(-1) + C(22)*VC(-1) + C(23)*UCBP(-1) + C(24)
        + C(25)*D74
DLVL  = C(31)* DLVC  = C(41)*VC(-1) + C(42)*RRATE(-1) +
C(43)*UCBP(-1) + C(44)*D75 + C(45) RATE  = C(51)*DP + C(52)*VC(-1)
+ C(53)*RATE(-1) + C(54)

Estimation Method: Two-Stage Least Squares

Sample: 1965Q1 2000Q3
Included observations: 143
Total system (balanced) observations 715
```

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	0.710220	0.162715	4.364818	0.0000
C(12)	0.115200	0.036651	3.143111	0.0017
C(13)	-0.059254	0.037881	-1.564192	0.1182
C(14)	-0.104465	0.034438	-3.033454	0.0025
C(21)	0.377586	0.072645	5.197683	0.0000
C(22)	0.038070	0.007830	4.862081	0.0000
C(23)	0.092927	0.024459	3.799222	0.0002
C(24)	-0.032589	0.006211	-5.247158	0.0000
C(25)	0.008626	0.002118	4.072382	0.0001
C(31)	0.182985	0.012594	14.52998	0.0000
C(41)	-0.143177	0.027502	-5.206075	0.0000
C(42)	-0.935544	0.198089	-4.722845	0.0000
C(43)	-0.543269	0.112208	-4.841622	0.0000
C(44)	-0.108770	0.013845	-7.856175	0.0000
C(45)	0.123288	0.022759	5.417063	0.0000
C(51)	0.068661	0.045923	1.495118	0.1353
C(52)	0.011430	0.004742	2.410350	0.0162
C(53)	0.913851	0.036922	24.75112	0.0000
C(54)	-0.008538	0.003981	-2.144603	0.0323

```
Determinant residual covariance 2.25E-25


Equation: DW = C(11)*DP + (1-C(11))*PI24(-1) + C(12)*VL(-1) + C(13)*UCBP(-1)
             + C(14)
Instruments: D74 D75 PI24(-1) UCBP(-1) UCBP(-2) UCBP(-3) UCBP(
        -4) UCBP(-5) UCBP(-6) UCBP(-7) UCBP(-8) UCBP(-9) UCBP(-10)
        UCBP(-11) UCBP(-12) VL(-1) VC(-1) RATE(-1) RRATE(-1) C
R-squared   0.575093        Mean dependent var  0.014427
Adjusted R-squared  0.565923        S.D. dependent var  0.006773
S.E. of regression  0.004462        Sum squared resid   0.002768
Durbin-Watson stat  1.680537


Equation: DP = C(21)*DW + (1-C(21))*PI24(-1) + C(22)*VC(-1) + C(23)*UCBP(-1)
             + C(24) + C(25)*D74
Instruments: D74 D75 PI24(-1) UCBP(-1) UCBP(-2) UCBP(-3) UCBP(
        -4) UCBP(-5) UCBP(-6) UCBP(-7) UCBP(-8) UCBP(-9) UCBP(-10)
        UCBP(-11) UCBP(-12) VL(-1) VC(-1) RATE(-1) RRATE(-1) C
R-squared   0.792474        Mean dependent var  0.010623
Adjusted R-squared  0.786459        S.D. dependent var  0.006097
S.E. of regression  0.002817        Sum squared resid   0.001095
Durbin-Watson stat  1.622519


Equation: DLVL  =  C(31)*DLVC
Instruments: D74 D75 PI24(-1) UCBP(-1) UCBP(-2) UCBP(-3) UCBP(
        -4) UCBP(-5) UCBP(-6) UCBP(-7) UCBP(-8) UCBP(-9) UCBP(-10)
        UCBP(-11) UCBP(-12) VL(-1) VC(-1) RATE(-1) RRATE(-1) C
R-squared   0.661433        Mean dependent var  7.08E-05
Adjusted R-squared  0.661433        S.D. dependent var  0.003563
S.E. of regression  0.002073        Sum squared resid   0.000610
Durbin-Watson stat  1.339455


Equation: DLVC  = C(41)*VC(-1) + C(42)*RRATE(-1) + C(43)*UCBP(-1)
                + C(44)*D75 + C(45)
Instruments: D74 D75 PI24(-1) UCBP(-1) UCBP(-2) UCBP(-3) UCBP(
        -4) UCBP(-5) UCBP(-6) UCBP(-7) UCBP(-8) UCBP(-9) UCBP(-10)
        UCBP(-11) UCBP(-12) VL(-1) VC(-1) RATE(-1) RRATE(-1) C
R-squared   0.450896        Mean dependent var  -0.000399
Adjusted R-squared  0.434980        S.D. dependent var  0.017845
S.E. of regression  0.013414        Sum squared resid   0.024830
Durbin-Watson stat  1.494882
```

```
Equation: RATE  = C(51)*DP + C(52)*VC(-1) + C(53)*RATE(-1) + C(54)
Instruments: D74 D75 PI24(-1) UCBP(-1) UCBP(-2) UCBP(-3) UCBP(
        -4) UCBP(-5) UCBP(-6) UCBP(-7) UCBP(-8) UCBP(-9) UCBP(-10)
        UCBP(-11) UCBP(-12) VL(-1) VC(-1) RATE(-1) RRATE(-1) C
R-squared   0.891711        Mean dependent var  0.017217
Adjusted R-squared  0.889374        S.D. dependent var  0.007143
S.E. of regression  0.002376        Sum squared resid   0.000785
Durbin-Watson stat  1.706658
```

```
System-Estimate Inflation climate expression PI12
---------------------------------------------------------------------------

DW    = C(11)*DP + (1-C(11))*PI12(-1) + C(12)*VL(-1) + C(13)*UCBP(-1) + C(14)
DP    = C(21)*DW + (1-C(21))*PI12(-1) + C(22)*VC(-1) + C(23)*UCBP(-1) + C(24)
      + C(25)*D74
DLVL  = C(31)*DLVC
DLVC  = C(41)*VC(-1) + C(42)*RRATE(-1) + C(43)*UCBP(-1) + C(44)*D75 + C(45)
RATE  = C(51)*DP + C(52)*VC(-1) + C(53)*RATE(-1) + C(54)

Estimation Method: Two-Stage Least Squares

Sample: 1965Q1 2000Q3
Included observations: 143
Total system (balanced) observations 715
```

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	0.490226	0.187713	2.611574	0.0092
C(12)	0.131183	0.033209	3.950200	0.0001
C(13)	-0.066962	0.034465	-1.942892	0.0524
C(14)	-0.119487	0.031206	-3.828928	0.0001
C(21)	0.184763	0.079613	2.320760	0.0206
C(22)	0.042185	0.007050	5.983426	0.0000
C(23)	0.054794	0.021707	2.524261	0.0118
C(24)	-0.035235	0.005556	-6.342266	0.0000
C(25)	0.008689	0.001883	4.613379	0.0000

C(31)	0.184250	0.012602	14.62041	0.0000
C(41)	-0.143177	0.027502	-5.206075	0.0000
C(42)	-0.935544	0.198089	-4.722845	0.0000
C(43)	-0.543269	0.112208	-4.841622	0.0000
C(44)	-0.108770	0.013845	-7.856175	0.0000
C(45)	0.123288	0.022759	5.417063	0.0000
C(51)	0.055179	0.046312	1.191477	0.2339
C(52)	0.011726	0.004754	2.466403	0.0139
C(53)	0.920816	0.037100	24.81997	0.0000
C(54)	-0.008756	0.003991	-2.193869	0.0286

```
Determinant residual covariance 2.73E-25

Equation: DW = C(11)*DP + (1-C(11))*PI12(-1) + C(12)*VL(-1) + C(13)*UCBP(-1)
            + C(14)
Instruments: C D74 D75 PI12(-1) UCBP(-1) UCBP(-2) UCBP(-3) UCBP(
        -4) UCBP(-5) UCBP(-6) UCBP(-7) UCBP(-8) UCBP(-9) UCBP(-10)
        UCBP(-11) UCBP(-12) VL(-1) VC(-1) RATE(-1) RRATE(-1)
R-squared   0.611075        Mean dependent var  0.014427
Adjusted R-squared  0.602681        S.D. dependent var  0.006773
S.E. of regression  0.004269        Sum squared resid   0.002533
Durbin-Watson stat  1.687612


Equation: DP = C(21)*DW + (1-C(21))*PI12(-1) + C(22)*VC(-1) + C(23)*UCBP(-1)
            + C(24) + C(25)*D74
Instruments: C D74 D75 PI12(-1) UCBP(-1) UCBP(-2) UCBP(-3) UCBP(
        -4) UCBP(-5) UCBP(-6) UCBP(-7) UCBP(-8) UCBP(-9) UCBP(-10)
        UCBP(-11) UCBP(-12) VL(-1) VC(-1) RATE(-1) RRATE(-1)
R-squared   0.835578        Mean dependent var  0.010623
Adjusted R-squared  0.830812        S.D. dependent var  0.006097
S.E. of regression  0.002508        Sum squared resid   0.000868
Durbin-Watson stat  1.641296


Equation: DLVL  =  C(31)*DLVC

Instruments: C D74 D75 PI12(-1) UCBP(-1) UCBP(-2) UCBP(-3) UCBP(
        -4) UCBP(-5) UCBP(-6) UCBP(-7) UCBP(-8) UCBP(-9) UCBP(-10)
        UCBP(-11) UCBP(-12) VL(-1) VC(-1) RATE(-1) RRATE(-1)
R-squared   0.660159        Mean dependent var  7.08E-05
Adjusted R-squared  0.660159        S.D. dependent var  0.003563
S.E. of regression  0.002077        Sum squared resid   0.000613
Durbin-Watson stat  1.345388
```

```
Equation: DLVC  = C(41)*VC(-1) + C(42)*RRATE(-1) + C(43)*UCBP(-1) + C(44)*D75
                + C(45)

Instruments: C D74 D75 PI12(-1) UCBP(-1) UCBP(-2) UCBP(-3) UCBP(
        -4) UCBP(-5) UCBP(-6) UCBP(-7) UCBP(-8) UCBP(-9) UCBP(-10)
        UCBP(-11) UCBP(-12) VL(-1) VC(-1) RATE(-1) RRATE(-1)
R-squared   0.450896        Mean dependent var  -0.000399
Adjusted R-squared  0.434980        S.D. dependent var  0.017845
S.E. of regression  0.013414        Sum squared resid   0.024830
Durbin-Watson stat  1.494882


Equation: RATE  = C(51)*DP + C(52)*VC(-1) + C(53)*RATE(-1) + C(54)

Instruments: C D74 D75 PI12(-1) UCBP(-1) UCBP(-2) UCBP(-3) UCBP(
        -4) UCBP(-5) UCBP(-6) UCBP(-7) UCBP(-8) UCBP(-9) UCBP(-10)
        UCBP(-11) UCBP(-12) VL(-1) VC(-1) RATE(-1) RRATE(-1)
R-squared   0.891221        Mean dependent var  0.017217
Adjusted R-squared  0.888873        S.D. dependent var  0.007143
S.E. of regression  0.002381        Sum squared resid   0.000788
Durbin-Watson stat  1.708475




System-Estimate: Inflation climate expression PI6
---------------------------------------------------------------------------


DW    = C(11)*DP + (1-C(11))*PI6(-1) + C(12)*VL(-1) + C(13)*UCBP(-1) + C(14)
DP    = C(21)*DW + (1-C(21))*PI6(-1) + C(22)*VC(-1) + C(23)*UCBP(-1) + C(24)
      + C(25)*D74
DLVL  = C(31)*DLVC
DLVC  = C(41)*VC(-1) + C(42)*RRATE(-1) + C(43)*UCBP(-1) + C(44)*D75 + C(45)
RATE  = C(51)*DP + C(52)*VC(-1) + C(53)*RATE(-1) + C(54)



Estimation Method: Two-Stage Least Squares

Sample: 1965Q1 2000Q3
Included observations: 143
Total system (balanced) observations 715


       Coefficient  Std. Error  t-Statistic Prob.
```

C(11)	0.274457	0.235362	1.166108	0.2440
C(12)	0.123058	0.029410	4.184269	0.0000
C(13)	-0.081909	0.033900	-2.416156	0.0159
C(14)	-0.111857	0.027638	-4.047154	0.0001
C(21)	0.100938	0.091078	1.108260	0.2681
C(22)	0.030465	0.006397	4.762441	0.0000
C(23)	0.018408	0.022273	0.826471	0.4088
C(24)	-0.025338	0.005025	-5.042319	0.0000
C(25)	0.006947	0.001863	3.728085	0.0002
C(31)	0.184470	0.012573	14.67239	0.0000
C(41)	-0.143177	0.027502	-5.206075	0.0000
C(42)	-0.935544	0.198089	-4.722845	0.0000
C(43)	-0.543269	0.112208	-4.841622	0.0000
C(44)	-0.108770	0.013845	-7.856175	0.0000
C(45)	0.123288	0.022759	5.417063	0.0000
C(51)	0.057076	0.046682	1.222635	0.2219
C(52)	0.011684	0.004754	2.457612	0.0142
C(53)	0.919836	0.037216	24.71633	0.0000
C(54)	-0.008725	0.003991	-2.186345	0.0291

Determinant residual covariance 3.15E-25

```
Equation: DW = C(11)*DP + (1-C(11))*PI6(-1) + C(12)*VL(-1) + C(13)*UCBP(-1)
             + C(14)
Instruments: D74 D75 PI6(-1) UCBP(-1) UCBP(-2) UCBP(-3) UCBP(
        -4) UCBP(-5) UCBP(-6) UCBP(-7) UCBP(-8) UCBP(-9) UCBP(-10)
        UCBP(-11) UCBP(-12) VL(-1) VC(-1) RATE(-1) RRATE(-1) C
R-squared   0.612699        Mean dependent var  0.014427
Adjusted R-squared  0.604340        S.D. dependent var  0.006773
S.E. of regression  0.004260        Sum squared resid   0.002523
Durbin-Watson stat  1.608989
```

```
Equation: DP = C(21)*DW + (1-C(21))*PI6(-1) + C(22)*VC(-1) + C(23)*UCBP(-1)
             + C(24) + C(25)*D74
Instruments: D74 D75 PI6(-1) UCBP(-1) UCBP(-2) UCBP(-3) UCBP(
        -4) UCBP(-5) UCBP(-6) UCBP(-7) UCBP(-8) UCBP(-9) UCBP(-10)
        UCBP(-11) UCBP(-12) VL(-1) VC(-1) RATE(-1) RRATE(-1) C
R-squared   0.835884        Mean dependent var  0.010623
Adjusted R-squared  0.831127        S.D. dependent var  0.006097
S.E. of regression  0.002506        Sum squared resid   0.000866
Durbin-Watson stat  1.722328
```

```
Equation: DLVL  =  C(31)*DLVC
Instruments: D74 D75 PI6(-1) UCBP(-1) UCBP(-2) UCBP(-3) UCBP(
        -4) UCBP(-5) UCBP(-6) UCBP(-7) UCBP(-8) UCBP(-9) UCBP(-10)
        UCBP(-11) UCBP(-12) VL(-1) VC(-1) RATE(-1) RRATE(-1) C
R-squared   0.659930        Mean dependent var  7.08E-05
Adjusted R-squared  0.659930        S.D. dependent var  0.003563
S.E. of regression  0.002078        Sum squared resid   0.000613
Durbin-Watson stat  1.346403


Equation: DLVC  = C(41)*VC(-1) + C(42)*RRATE(-1) + C(43)*UCBP(-1) + C(44)*D75
                + C(45)
Instruments: D74 D75 PI6(-1) UCBP(-1) UCBP(-2) UCBP(-3) UCBP(
        -4) UCBP(-5) UCBP(-6) UCBP(-7) UCBP(-8) UCBP(-9) UCBP(-10)
        UCBP(-11) UCBP(-12) VL(-1) VC(-1) RATE(-1) RRATE(-1) C
R-squared   0.450896        Mean dependent var  -0.000399
Adjusted R-squared  0.434980        S.D. dependent var  0.017845
S.E. of regression  0.013414        Sum squared resid   0.024830
Durbin-Watson stat  1.494882


Equation: RATE  = C(51)*DP + C(52)*VC(-1) + C(53)*RATE(-1) + C(54)
Instruments: D74 D75 PI6(-1) UCBP(-1) UCBP(-2) UCBP(-3) UCBP(
        -4) UCBP(-5) UCBP(-6) UCBP(-7) UCBP(-8) UCBP(-9) UCBP(-10)
        UCBP(-11) UCBP(-12) VL(-1) VC(-1) RATE(-1) RRATE(-1) C
R-squared   0.891300        Mean dependent var  0.017217
Adjusted R-squared  0.888954        S.D. dependent var  0.007143
S.E. of regression  0.002380        Sum squared resid   0.000788
Durbin-Watson stat  1.708345
```