Perfect Finance-led World Capitalism in a Nutshell

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Abstract

This paper considers macroeconomic real-financial market interactions in a Turnovsky (1986) two country framework with money, bonds, equities and a foreign exchange market with perfectly flexible exchange rates. It is based on conventional multiplier dynamics in its real part, augmented by either a simple LM curve or a simple Taylor interest rate policy rule, but enriched by a broad spectrum of financial assets, based in their interaction on perfect substitute and perfect foresight assumptions throughout. Contrary to Blanchard's (1981) original contribution, we aim in this paper to analyze and solve a revised form of the model, allowing to avoid the application of the jump-variable technique of the rational expectations school, by the introduction of a Taylor policy rule that makes the dynamics convergent without the imposition of economically poorly motivated jumps of a selected number of the state variables of the model. In a final section inflation dynamics is added as in Turnovsky (1986) in order to also consider Taylor rules that are not solely concentrated on the dynamics of financial markets.

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Contents

1 Introduction

This paper considers macroeconomic real-financial market interactions in a two country framework with money, bonds, equities and a foreign exchange market with a flexible exchange rate. The model uses conventional dynamic multiplier dynamics in its real part as in Blanchard (1981), augmented by either a simple LM curve or later on a simple Taylor interest rate policy rule, but enriched by a broad spectrum of financial assets, based in their dynamic interaction on the assumption of perfect substitute and perfect foresight throughout. Contrary to Blanchard's (1981) original contribution on stock market dynamics, we however aim to analyze and solve a revised form of the model, in order to avoid the heroical assumption of the jump-variable technique (JVT) of the rational expectations school. In approaching such an objective, the model will first be considered in conventional form, starting from the Dornbusch (1976) overshooting exchange rate model, and will then be integrated with the Blanchard (1981) stock market approach in a two-country setup as in Turnovsky (1986). This will lead us to a dynamic model with five laws of motion and a variety of interacting feedback chains and their (in)stability implications.

We do not show however that the saddle-point dynamics (that will again result in such a framework) with their – as demanded by the rational expectations methodology – three non-predetermined and two predetermined variables allows for a proper application of the JVT by proving the existence of in fact three unstable and two stable roots of the Jacobian at their steady state, and by handling therewith the cases of unanticipated as well as anticipated shocks. Instead we suggest the principle that Taylor interest rate policy rules should be designed (independently from what is happening in actual monetary policy) in correspondence to the feedback structure of the dynamics that has been assumed to represent the behavior the private sector.

This principle leads us to a Taylor rule that concentrates on the UIP condition of the Dornbusch model. We assume a behavior of the CB's that is opposite to what policymakers would expect a Central Bank to do in the case of an unwanted depreciation of their currency, i.e., we assume as Taylor rule that they lower the nominal rate of interest in such a case (normally viewed as implying capital outflows and therefore giving further momentum to the on-going depreciation of the exchange rate). In the present ideal modeling of financial markets (where we have perfect substitution and myopic perfect foresight everywhere), this inverted policy reaction is however indeed of help and it implies conventional asymptotic stability towards a uniquely determined steady state (a second implication of our choice of the interest rate policy rule). We thus have that all 5 state variables can be considered as predetermined (but not their rates of growth) and therefore not subject to any explosive tendency. this result no longer enforces the need to apply the JVT, as it would be the case in the model with a conventional Taylor rule we have started from.

Such a modification of a conventional RE model thus shows that it can overcome the conundrums that surround the (nowadays generally purely algorithmic) application of the jump-variable technique. We have achieved this by just searching for Taylor rules that

do their stability-delivering job in the assumed dynamic environment in a conventionally stabilizing way, with all variables being predetermined (so that only their time rates of change are subject to unanticipated shocks).[1]

From the methodological point of view we proceed as in Turnovsky (1986) from a general two-country approach and its uniquely determined interior steady state position to a (partial) linearization of this model around its steady state and to the assumption of symmetry between the two countries, i.e., to assuming identical parameter values for them. This allows us to decompose the dynamics in 2D average dynamics and 3D difference dynamics both of which can be shown to be convergent under standard assumption os the private sector of the economy if the reaction of the interest rate is exchange-rate oriented in a specific way. Moreover, since Turnovsky (1986) also allows for price dynamics as in Dornbusch (1976) we extend the model in a final section towards an integration of phillips curves and can again show convergence if the Taylor rule is adjusted to this new situation in an appropriate way.

The results of this paper question the way rational expectations are exercised in forward looking models, but they also show that the policy that is needed to overcome the saddlepoint instability of perfect foresight models is of a strange type in such a perfect substitute financial world. In the outlook the paper therefore suggests that realistic models of finance-led world capitalism should consider situations of imperfect substitution between financial assets, take account of heterogeneous expectations formation and admit that adjustment processes may be fast, but not infinitely fast, in particular when the limiting situation of rational expectations is a structurally unstable one (i.e., subject to severe discontinuities in the limit).

A side product of the paper moreover is that it shows that one can investigate theoretically situations in continuous time that are out of reach for the neo-Wicksellian period models used for example in Woodford (2003). We consider the use of continuous-time models a necessity in macromodels of the real financial markets interaction, see Asada et al. (2007) for the details of such an argument. It can also be used, if extended appropriately by wage-price dynamics and long-term bonds, for a structural comparison with and a theoretical investigation of the empirical DSGE studies, as they are now the fashion in studies of the working of monetary policy rules, see the paper by Smets and Wouters (2003) for an example.

In the next section we briefly consider the Dornbusch exchange rate dynamics for the small open economy. We then reconsider in section 3 in detail the Turnovsky two-country version of this model type and its rational expectations solutions. Section 4 provides a brief introduction into Blanchard's stock market dynamics for the closed economy. In section 5 we provide an integration of the Dornbusch and Blanchard models on a level that is similar to the Turnovsky approach. We briefly discuss there problems of saddlepoint

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[1]and with a treatment of anticipated shocks that should be event-specific and not just subject to the mechanic use of explosive bubbles that lead us softly in time towards a state where the stable manifold of the new dynamics comes into being.

instability, and then reformulate – in section 6 – the Taylor rule in order to allow for stability in the conventional sense in this model type. 5D Stability analysis is carried out in section 7 under the assumption of symmetry where it can be shown then that both averages over and differences for the two economies converge to their steady state values. In section 9 we show that inflation dynamics can be added to the model without significantly altering its implications significantly. Section 10 concludes.

2 The Dornbusch exchange-rate dynamics

The Dornbusch (1976) exchange rate dynamics reads in the case of the Uncovered Interest Parity condition (UIP) with myopic perfect foresight on the exchange rate dynamic in their simplest form as follows (with e the logarithm of the exchange rate and p the logarithm of the price level, the foreign price level being normalized to 1):

$$
\begin{aligned}
\dot{p} &= \beta_p(Y^d(e-p) - \bar{Y}) \\
\dot{e} &= i(p) - \bar{i}^*
\end{aligned}
$$

We assume that the economy is at its full employment level $\bar{Y}$, but that deviations of aggregate demand Y^d (which only depend on the real exchange rate here) from this level determine with adjustment speed β_p the rate of inflation. The second law of motion is the UIP condition solved for the dynamic of the exchange rate it implies when myopic perfect foresight is assumed. The relationship $i(p)$ is a standard inverted LM curve, i.e., it describes a positive relationship between the price level and the domestic nominal rate of interest. Clearly the steady state is of saddlepoint type (the determinant of the Jacobian is negative) and the implied phase diagram is shown in figure 1.
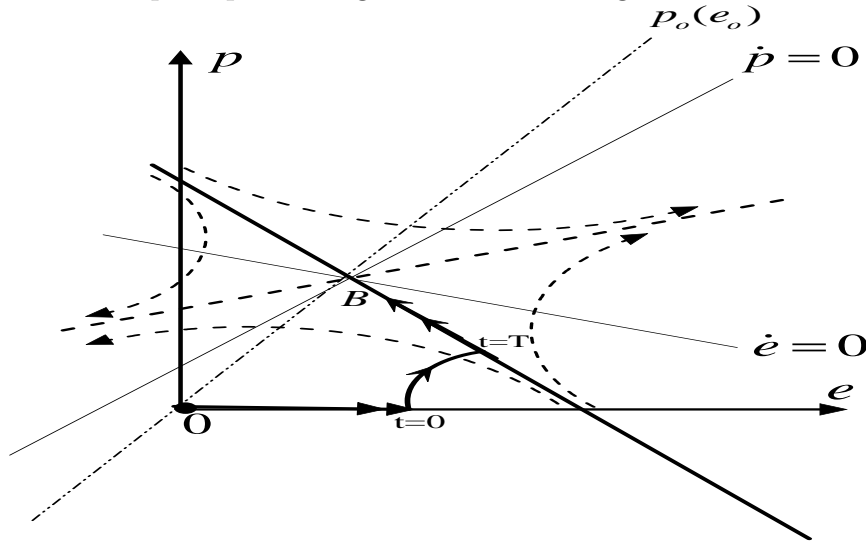

Figure 1: The Dornbusch exchange rate dynamics under myopic perfect foresight

The rational expectations school solves such a dynamical system in the following way. It assumes that the economy is (if no anticipated shocks are occurring) always on the stable manifold of the given saddlepoint, horizontally to the right of the old steady state 0, in the intersection of the stable manifold (a straight line in this simple model) with the horizontal axis, if an unanticipated shock that moves the steady state to point B has occurred, since the price level can only adjust gradually in this model. We therefore get an overshooting exchange rate (with respect to its new steady state values) and thus an increase in goods demand which increases gradually the price level and the nominal interest rate, which appreciates the exchange rate from its excessively high level until all variables reach the new steady state.

In the case of anticipated shocks the situation that is assumed by the rational expectations school becomes more complicated, since at the time of the announcement of the policy we are still in the old phase diagram around the point 0. In this case, the exchange rate jumps at $t = 0$ to the right to a uniquely determined level from where it uses the unstable saddlepoint bubble in the old dynamics that starts at this point and from where it then reaches the stable manifold of the new dynamics exactly at the time T when the announced policy shock actually takes place. We thus have – depending on T – a jump in the exchange rate that may still overshoot its new steady state position and that then switches immediately towards a bubble of length T with both rising prices and exchange rates, from which it departs through a soft landing on the new stable manifold at time T.

This is – when appropriately extended – the rational expectations approach to exchange rate dynamics in the frame of a Dornbusch IS-LM model with a price Phillips curve. Its solution techniques looks attractive, since it provides – in addition to the usual treatments of shocks – also a well-defined answer in the case of anticipated events (within certain bounds, depending on the size of the anticipated shock). But it may also be viewed as a rather heroical solution to the treatment of (un)anticipated demand, supply and policy shocks from a descriptive point of view.

3   Symmetric two-country exchange rate dynamics

In this section we introduce and make use of a technique that Turnovsky (1986) has employed to analyze Dornbusch type IS-LM-PC analysis in a symmetric two-country setup by means of (mathematical) average and difference considerations, then used to recover the original dynamics from these two hierarchically ordered subdynamics.

Following Turnovsky (1986) we thus consider in this section the following two–country macroeconomic model. It describes two symmetric economies, characterized by the same parameters, with each specializing in the production of a distinct good and international trading of distinct fix-price bonds. All parameters in the following model are assumed to be positive (with $a_1 < 1$ and $\alpha \in (0.5, 1)$ in addition).

$$
\begin{align}
y &= a_1 y^* - a_2(r - \dot{z}) + a_3(p^* + e - p) + \bar{u}, \tag{1}\\
y^* &= a_1 y - a_2(r^* - \dot{z}^*) - a_3(p^* + e - p) + \bar{u}^*, \tag{2}\\
\bar{m} - z &= b_1 y - b_2 r, \tag{3}\\
\bar{m}^* - z^* &= b_1 y^* - b_2 r^*, \tag{4}\\
r &= r^* + \dot{e}, \tag{5}\\
z &= \alpha p + (1 - \alpha)(p^* + e) = p + (1 - \alpha)(p^* + e - p), \tag{6}\\
z^* &= \alpha p^* + (1 - \alpha)(p - e) = p^* - (1 - \alpha)(p^* + e - p), \tag{7}\\
\dot{p} &= \beta_w y, \tag{8}\\
\dot{p}^* &= \beta_w y^*. \tag{9}
\end{align}
$$

In these equations we make use of the following notation:

y = real output Y (in logarithms) deviation from its natural rate level,

p = price of output, expressed in logarithms,

z = consumer price index, expressed in logarithms,

e = exchange rate (of the domestic economy), measured in logarithms,

r = nominal interest rate,

$\bar{m}$ = nominal money supply, expressed in logarithms,

$\bar{u}$ = real government expenditure, expressed in logarithms.

Domestic variables as usual are unstarred; foreign variables are shown with an asterisk. The equations (1) and (2) describe goods market equilibrium, or the IS curves, in the two economies. Private goods demand depends upon output in the other country, upon the real rate of interest, measured in terms of consumer price inflation, and the real exchange rate. Because of the assumed symmetry, the corresponding effects across the two economies are identical, with the real exchange rate influencing demand in exactly offsetting ways. The money market equilibrium in the economies is of standard textbook type. It is described by the equations (3) and (4). These four equations thus provide a straightforward extension of conventional IS-LM block to the case of two symmetric interacting economies.

The perfect substitutability of the domestic and foreign bonds is described by the uncovered interest parity condition (5). Equations (6) and (7) define the consumer price index (CPI) at home and abroad. The assumption is made that the proportion of consumption α spent on the respective home good is the same in the two economies. We

assume $\alpha > \frac{1}{2}$, so that residents in both countries have a preference for their own good. Finally, equations (8) and (9) define the price adjustment in the two economies in terms of simple Phillips curves (which are not expectations augmented). We note that y is already measured as deviation from its steady state level, which is not true for the other variables of the model. Due to the two-country approach here adopted the world interest rate is not a given magnitude, but will be determined by the equations of the model.

The two-country world described by equations (1) – (9) represents a linear 3D dynamical system in the domestic and the foreign price levels p, p^*, and the exchange rate, e. Following the methodology developed in the preceding section we assume that the prices p, p^* can only adjust continuously, while the exchange rate is free to jump in response to new information and that it will always jump in such a way that the dynamic responses generated remain bounded away from zero and infinity. The jump variable technique therefore now applies to a 3D phase space and can therefore no longer be depicted graphically in the easy way considered in the preceding section.

Fortunately however, due to the symmetry assumption and the linearity of the considered model, the analysis can be simplified considerably by defining the averages and differences for all variables involved, say x for example, as

$$
\begin{aligned}
x^a &\equiv \frac{1}{2}(x + x^*), \\
x^d &\equiv x - x^*.
\end{aligned}
$$

Through elimination of the variables z and z^*, the dynamics can be rewritten in terms of a decoupled system for averages and differences as described below.

The behavior of the 'average economy':

Equations (10) – (12) describe the aggregate world economy. The aggregate IS and LM curves (12) and (13) determine the average output level and average nominal interest rate in terms of the average price level, the evolution of which is described by the Phillips curve (12). Thus

$$
\begin{aligned}
(1 - a_1 - a_2\beta_w)y^a &= -a_2 r^a + \bar{u}^a, & (10) \\
\bar{m}^a - p^a &= b_1 y^a - b_2 r^a, & (11) \\
\dot{p}^a &= \beta_w y^a. & (12)
\end{aligned}
$$

The behavior of this virtual average economy is therefore characterized by virtual IS-LM equilibrium and a virtual Phillips curve, which is not expectations augmented. Note here however that inflation is reflected in aggregate demand in both countries which depends on the actual real rate of interest in both countries where inflation is substituted out by means of the PC's of the model. We assume in this regard

$$
b = 1 - a_1 - \beta_w a_2 > 0,
$$

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i.e., that wages adjust sufficiently sluggishly in order that the resulting IS curve in y^a, r^a space be downward–sloping. We have the stabilizing Keynes-effect present in this formally conventional IS-LM model and no destabilizing Mundell-effect, not however – as in the Dornbusch-Fischer model of section 1.3 – by reducing the real rate of interest to a nominal one, but because of the assumption that the Phillips curves are not yet expectations augmented (exhibit stationary expectations). It can therefore be expected that the linear dynamical model for the averages will converge to its steady state solution. Note that this decoupled part of the original model behaves just like a closed economy.

The dynamics of differences:

The differences in the two economies, together with the exchange rate, are described by

$$(1 + a_1)y^d = a_2(1 - 2\alpha)(\dot{e} - \dot{p}^d) + 2a_3(e - p^d) + \bar{u}^d, \tag{13}$$
$$\bar{m}^d - 2(1 - \alpha)e + (1 - 2\alpha)p^d = b_1 y^d - b_2 \dot{e}, \tag{14}$$
$$\dot{p}^d = \beta_w y^d. \tag{15}$$

It is shown below that the virtual dynamics of the state variable e and p^d is of the saddlepoint type that we considered in the preceding section for the case of a small open economy.

It is convenient to begin with a characterization of the *steady state* equilibrium. Characterizing steady state values by indexation with zeros, we calculate it from the conditions $\dot{p} = \dot{p}^* = \dot{e} = 0$, and first of all get $y_o = 0$ and $r_o = r_o^*$. Thus the steady state equilibrium in the goods and money markets of the two economies is given by

$$a_2 r_o - a_3(p_o^* + e_o - p_o) = \bar{u},$$
$$a_2 r_o + a_3(p_o^* + e_o - p_o) = \bar{u}^*,$$
$$\bar{m} - p_o - (1 - \alpha)(p_o^* + e_o - p_o) = -b_2 r_o,$$
$$\bar{m}^* - p_o^* + (1 - \alpha)(p_o^* + e_o - p_o) = -b_2 r_o.$$

The solutions to these equations are

$$r_o = \frac{1}{2a_2}(\bar{u} + \bar{u}^*) = \bar{u}/a_2 = r_o^*, \tag{16}$$

$$\eta_o \equiv p_o^* + e_o - p_o = \frac{1}{2a_3}(\bar{u} - \bar{u}^*) = \bar{u}^d/(2a_3), \tag{17}$$

$$p_o = \bar{m} + \left\{\frac{b_2}{2a_2} + \frac{(1 - \alpha)}{2a_3}\right\}\bar{u} + \left\{\frac{b_2}{2a_2} - \frac{(1 - \alpha)}{2a_3}\right\}\bar{u}^*, \tag{18}$$

$$p_o^* = \bar{m}^* + \left\{\frac{b_2}{2a_2} - \frac{(1 - \alpha)}{2a_3}\right\}\bar{u} + \left\{\frac{b_2}{2a_2} + \frac{(1 - \alpha)}{2a_3}\right\}\bar{u}^*, \tag{19}$$

$$e_o = \bar{m} - \bar{m}^* + \left\{\frac{1 - 2\alpha}{2a_3}\right\}(\bar{u} - \bar{u}^*). \tag{20}$$

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We obtain that the steady world rate of interest is independent of monetary policy as well as the real exchange rate. With respect to monetary policy we thus have neutrality results as well as a constant real exchange rates as in the Dornbusch (1976) model.

We investigate the *stability properties of the average economy* first. For its steady state position we get from the above that

$$r_o^a = \frac{1}{a_2}\bar{u}^a,$$
$$p_o^a = \bar{m}^a + \frac{b_2}{a_2}\bar{u}^a,$$
$$y_o^a = 0.$$

We can see that the steady state world interest rate $r_o = r_o^* = r_o^a$ only depends on fiscal policy in the two countries and the interest rate elasticity of the aggregate demand function (of the aggregate investment demand function in particular). For the steady state average price level we get the additional influences of the interest rate elasticity of the money demand functions as well as average world money supply. The steady sate of the average economy is therefore of a very simple type.

In order to discuss its stability we have to solve the IS-LM equations of the average economy for y^a and r^a first. Making use of the abbreviation $b = 1 - a_1 - \beta_w a_2$ we get from the IS and LM equation for the average economy the linear system

$$\begin{pmatrix} b & a_2 \\ b_1 & b_2 \end{pmatrix} \begin{pmatrix} y^a \\ r^a \end{pmatrix} = \begin{pmatrix} \bar{u}^a \\ \bar{m}^a - p^a \end{pmatrix}.$$

This in turn gives

$$\begin{pmatrix} y^a \\ r^a \end{pmatrix} = \frac{1}{zb_2 + a_2b_1} \begin{pmatrix} b_2 & a_2 \\ b_1 & -b \end{pmatrix} \begin{pmatrix} \bar{u}^a \\ \bar{m}^a - p^a \end{pmatrix}.$$

Setting $d = 1/(zb_2 + a_2b_1)$ we therefore get

$$\begin{pmatrix} y^a \\ r^a \end{pmatrix} = d \begin{pmatrix} b_2\bar{u}^a + a_2(\bar{m}^a - p^a) \\ b_1\bar{u}^a - b(\bar{m}^a - p^a) \end{pmatrix},$$

which has the expected signs in front of the coefficients that characterize fiscal and monetary policy. Inserting the expression obtained for the output gap y^a into the average PC then leads us to the linear differential equation in the average price level p^a,

$$\dot{p}^a = \beta_w d(b_2\bar{u}^a + a_2\bar{m}^a - a_2 p^a),$$

which shows that the steady state level $p_o^a = \bar{m}^a + \frac{b_2}{a_2}\bar{u}^a$ is obviously a global attractor for the average price level. The average world economy is therefore globally asymptotically stable in a very straightforward way.

A remark: However, the PC that is employed in Turnovsky (1986) is *not expectations augmented* and may still allow for stability results in contrast to what was is known about destabilizing Mundell inflationary expectations effects. We therefore briefly discuss the case when expectations augmented PC's are used in the two-country approach here under consideration. Expectations of workers concern the consumer price indices z and z^* in the present context. Due to the definition of the consumer price index (by way of functions of Cobb-Douglas type) we get for the derivative of its logarithm (and thus for the growth rate of the consumer price index)

$$\dot{z} = \alpha \dot{p} + (1 - \alpha)(\dot{p}^* + \dot{e}), \quad \dot{z}^* = \alpha \dot{p}^* + (1 - \alpha)(\dot{p} - \dot{e}).$$

As expectations augmented Phillips Curves we now define[2]

$$\dot{p} = \beta_w y + \pi, \quad \dot{p}^* = \beta_w y^* + \pi^*,$$

where π and π^* denote the expected growth rates for the consumer price indices of the two countries. For these expected rates we now assume an adaptive expectations mechanism, which in the present context and for the two countries considered must be of the form

$$\dot{\pi} = \beta_\pi(\dot{z} - \pi), \quad \dot{\pi}^* = \beta_\pi(\dot{z}^* - \pi^*),$$

by employing again the symmetry assumption for the considered two-country model.

Note that Turnovsky (1986) and our above presentation of his approach make use of myopic perfect foresight with respect to price inflation as well (in the aggregate demand function), but disregard the fact that this might fix the output level at its NAIRU level. The PC of the above model can therefore be positively sloped, since it has not been augmented by inflationary expectations in the usual way. We now depart from this procedure by inserting expected consumer price inflation into the aggregate demand function in the place of actual consumer price inflation in order to be in line with the conventional IS-LM-PC model type investigated in chapter 2. We therefore now consider a mixed situation with respect to expectations formation: rational ones in the financial markets and adaptive ones with respect to goods markets, labor markets and wage and price inflation. We justify the choice of such a mixed situation with reference to applied models such as the one of McKibbin and Sachs (1991), see also the IMF Multimod Mark III model, where however a more complicated situation is considered , since inflationary expectations are there based on forward and backward looking elements and not included directly in the investment or consumption demand function.

In terms of averages the equations just discussed give rise to

$$\begin{aligned} \dot{\pi}^a &= \beta_\pi(\dot{z}^a - \pi^a) = \beta_\pi(\dot{p}^a - \pi^a), \\ \dot{p}^a &= \beta_w y^a + \pi^a. \end{aligned}$$

There is thus an immediate extension of the model by adaptive inflationary expectations such that an IS-LM-PC analysis is established for the average economy that is of the type

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[2]These equations are based on level representations of the type $\hat{p} = \beta_w \ln(Y/\bar{Y}) + \pi$.

considered in chapter 2 for the case of a closed economy. Note here however again, that the IS-LM part of the model is now given by

$$(1 - a_1)y^a = -a_2(r^a - \pi^a) + \bar{u}^a,$$
$$\bar{m}^a - p^a = b_1 y^a - b_2 r^a,$$

whose solution for the variable y^a, r^a now gives [with $d = 1/((1 - a_1)b_2 + a_2 b_1)$]

$$\begin{pmatrix} y^a \\ r^a \end{pmatrix} = d \begin{pmatrix} b_2 a_2 \pi^a + b_2 \bar{u}^a + a_2(\bar{m}^a - p^a) \\ b_1 a_2 \pi^a + b_1 \bar{u}^a - (1 - a_1)(\bar{m}^a - p^a) \end{pmatrix},$$

again with the expected signs in front of the coefficients that characterize fiscal and monetary policy (and the role of inflationary expectations now). Inserting the expression for output y^a into the revised dynamical system then finally gives

$$\dot{p}^a = \beta_w y^a + \pi^a = \beta_w d(b_2 a_2 \pi^a + b_2 \bar{u}^a + a_2(\bar{m}^a - p^a)) + \pi^a,$$
$$\dot{\pi}^a = \beta_\pi \beta_w y^a = d(b_2 a_2 \pi^a + b_2 \bar{u}^a + a_2(\bar{m}^a - p^a)).$$

These IS-LM-PC dynamics are of the same qualitative type as the one investigated in chapter 2. They therefore now contain the destabilizing Mundell effect (represented by the coefficient $db_2 a_2$) besides the stabilizing Keynes effect (represented by the coefficient $-da_2$) and thus will not be locally asymptotically stable if the Mundell-effect works with sufficient strength. However, the present analysis is strictly local in nature, since aggregate demand $Y^d = C + I + G$ has been approximated by a loglinear expression of the type $a_1 y^a - a_2(r^a - \pi^a) + \bar{u}^a$. The completion of the analysis by means of a kinked PC and the proof of global stability of these average dynamics is therefore not possible here, but demands a level form representation of the whole model which is necessarily nonlinear in nature – to which the averaging method of this section can then no longer be applied. The analysis of this section therefore becomes considerably more complicated when a global IS-LM-PC approach is attempted that generalizes chapter 2 to the case of two (symmetric) interacting open economies.

For the loglinear approximation of this section and the use of positively sloped PC's (static expectations of wage earners) and myopic perfect foresight with respect to price inflation by investors, we have however shown that the average economy is (locally) monotonically and asymptotically stable and thus behaves much simpler than even the traditional monetarist base model and its extension to IS-LM-PC analysis.

Let us now consider *the dynamics of differences* which when slightly reformulated is given by:[3]

$$(1 + a_1)\dot{p}^d/\beta_w = a_2(1 - 2\alpha)(\dot{e} - \dot{p}^d) + 2a_3(e - p^d) + \bar{u}^d,$$
$$\bar{m}^d - 2(1 - \alpha)(e - p^d) - p^d = b_1 \dot{p}^d/\beta_w - b_2 \dot{e}.$$

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[3]Note that the third equation is solved for y^d and inserted into the first two equations of the difference dynamics.

Rearranging these equations appropriately and using the auxiliary variable $k = e - p^d$ then gives

$$2a_3 k + \bar{u}^d = (1 + a_1)/\beta_w \dot{p}^d - a_2(1 - 2\alpha)\dot{k},$$
$$2(1 - \alpha)k + p^d - \bar{m}^d = (b_2 - b_1/\beta_w)\dot{p}^d + b_2\dot{k}.$$

In matrix notation this in turn gives with respect to the signs involved in these two equations[4]

$$\begin{pmatrix} + & + \\ - & + \end{pmatrix} \begin{pmatrix} \dot{p}^d \\ \dot{k} \end{pmatrix} = \begin{pmatrix} 0 & + \\ + & + \end{pmatrix} \begin{pmatrix} p^d \\ k \end{pmatrix}.$$

Since the determinant of the matrix on the left hand side of this matrix equation is positive (and thus also the determinant of the inverse of this matrix) and the determinant of the matrix on the right hand side is negative, we get that this implicit differential equation system gives rise to a negative system determinant when solved explicitly (by multiplication of the right hand side with the inverse of the matrix on the left-hand side). Not surprisingly we therefore get (for adjustment speeds of wages chosen sufficiently small) that the difference dynamics is of saddlepoint type with respect to its unique steady state solution which is given by

$$p_o^d = \bar{m}^d + (1 - \alpha)/a_3 \bar{u}^d, \quad s_o = e_o - p_o^d = -\bar{u}^d/(2a_3) \quad \text{(and } y_o^d = 0\text{)}.$$

Of course, reformulating the dynamics in terms of p^d and e provides us with the same result, see Turnovsky (1986, p.143) in this regard). We thus have now the situation that the jump-variable technique of the rational expectations school must be applied to the differences between the two countries and be translated back to the individual countries thereafter in order to discuss the consequences of monetary or fiscal shocks in such a two-country framework.

In this way Turnovsky (1986) obtains, for example, the following two results on unanticipated and anticipated monetary shocks, specifically an increase in the domestic money supply by one unit, with foreign money supply held constant, which raises the steady state values of p and e by one unit and leaves the steady state value of p^* unaltered. For both situations we quote directly from Turnovsky (1986) in order to describe the results of this analysis in the spirit of the originally chosen presentation and explanation.[5]

Unanticipated monetary expansion

> "The formal solution to the model in the case of an unanticipated monetary
> disturbance is illustrated in figure 2 In this case, the exchange rate overshoots

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[4]If β_w is again assumed to be sufficiently small and considering that $\alpha \in (0.5, 1)$.
[5]Note we have changed references to figures and their notation to conform with the numbering and notation of this chapter.

its long-run response, on impact, thereafter appreciating toward its new steady state level. The price of domestic output gradually increases, while domestic output initially increases, thereafter falling monotonically towards its natural rate level. The monetary expansion causes an immediate fall in the domestic interest rate, which thereafter rises monotonically toward its equilibrium. All these effects are familiar from the Dornbusch model or its immediate variants. The effects of the domestic monetary expansion on the foreign economy are less clear cut. The rate of inflation p^* of foreign goods and the level of output abroad will rise or fall on impact, depending on an eigenvalue relationship. The monetary expansion in the domestic economy increases both the average world output, y^a and the difference y^d. While this obviously implies a rise in the domestic level of output, foreign output will rise if and only if the increase in average output exceeds half the increase in the difference.
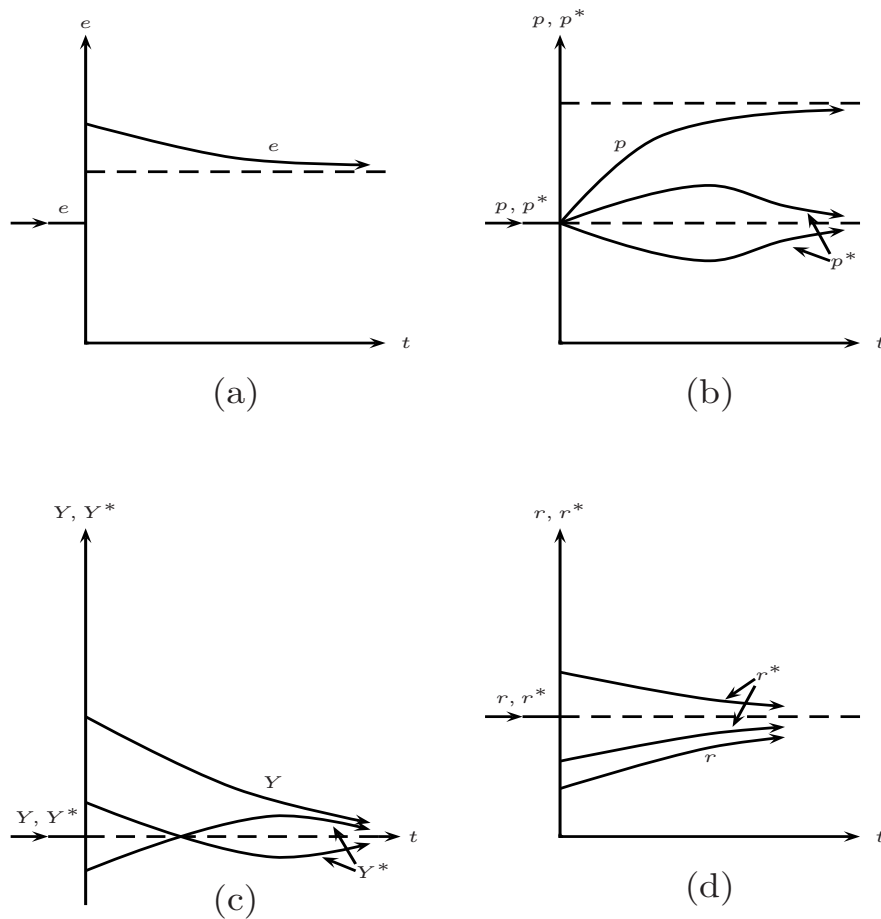

Figure 2: Unanticipated monetary expansion: (a) exchange rate; (b) prices; (c) outputs; (d) interest rates.

More intuitively, the appreciation of the foreign currency vis-a-vis the domestic currency, following the domestic monetary expansion, leads to an increase in the relative price of foreign goods, leading to a fall in demand, and hence in

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output and in inflation abroad. However, the fact that the foreign currency immediately begins to depreciate following its initial (discrete) appreciation increases the rate of inflation abroad, thereby reducing the foreign real rate of interest. This leads to a positive effect on foreign demand, output, and inflation. In addition, the increase in domestic output leads to an increase in imports by the domestic economy, and this also leads to an increase in output and inflation abroad. The net effect on output and inflation abroad depends upon which of these two opposing effects dominates. However, if the foreign level of output does initially rise, it will immediately begin to fall, and vice versa. This is because, if foreign output does increase initially, the accompanying rise in the foreign inflation rate will begin to increase the foreign price level. Thus the level of the real foreign money supply will begin to fall, thereby inducing a contraction in foreign output.
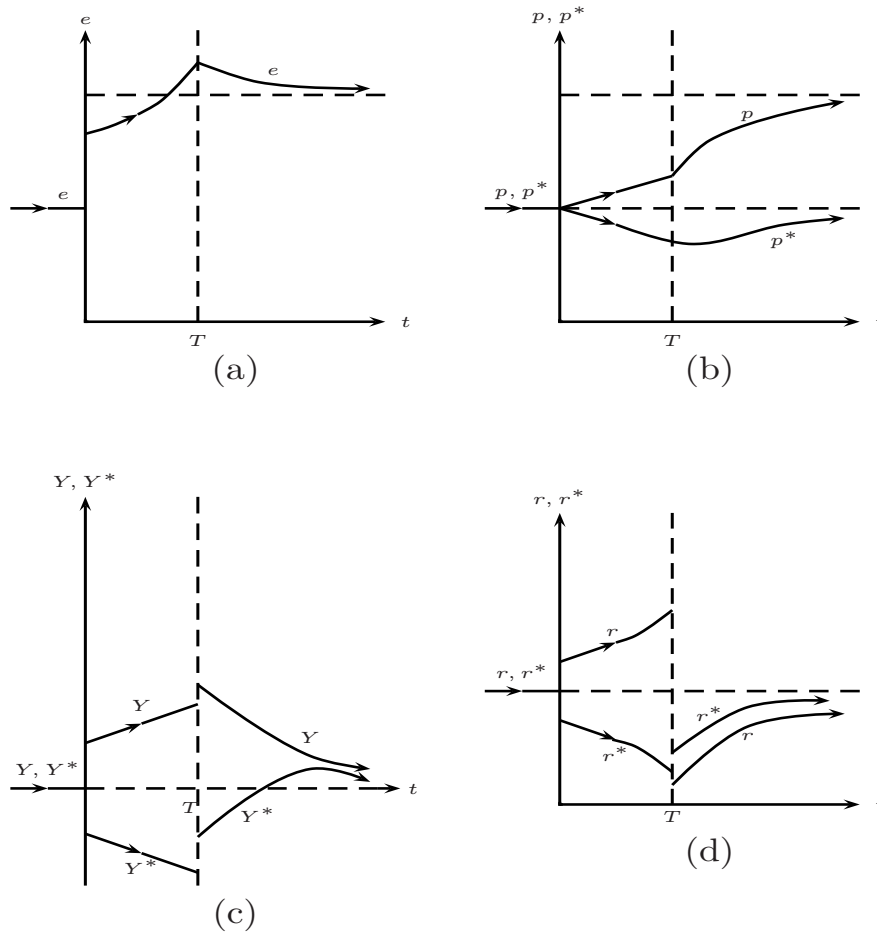

Figure 3: Announced monetary expansion: (a) exchange rate; (b) prices;(c) outputs; (d) interest rates.

Indeed, as long as foreign output is above its equilibrium level and the foreign price level continues to rise, the contractionary force on foreign output will continue. It can be shown that y* will always fall below its original level during

the transition, giving rise to the overshooting pattern shown in figure 3 (c). The time path is reversed in the case of an initial fall in foreign output. In some case, with the domestic currency appreciating following the initial monetary expansion, it follows that the fall in the domestic nominal interest rate exceeds that of the foreign rate, which in fact may either rise or fall. On the one hand, the initial appreciation of the foreign currency causes the foreign real stock of money to rise, leading to a fall in the foreign interest rate. On the other hand, the rise or fall in the foreign level of income may generate an increase or decrease in the demand for money abroad, thereby rendering the overall effect on the foreign interest rate indeterminate. Both time paths for y^*, p^* and r^* are illustrated."

Announced monetary expansion

"Consider now the behavior of the economy in response to a monetary expansion which the authorities announce at time zero to take place at some future time $T > 0$. The time paths for the relevant domestic and foreign variables are illustrated in figure 3.

At the time of announcement ($t = 0$) the domestic currency immediately depreciates in anticipation of the future monetary expansion. Whether the initial jump involves overshooting of the exchange rate depends upon the lead time T. Following the announcement, the domestic currency continues to depreciate until time T, when it reaches a point above the new long-run equilibrium. Thereafter, it appreciates steadily until the new steady state equilibrium is reached. This behavior is identical with that in the Gray and Turnovsky (1979) model. The anticipation of the future monetary expansion causes the domestic price level to begin rising at time zero. The inflation rate increases during the period 0 to T, when the monetary expansion occurs. This expansion causes a further increase in the inflation rate, which thereafter begins to slow down as the new equilibrium price level is approached. The behavior of the inflation rate is mirrored in the level of output. The positive inflation rate generated by the announcement is accompanied by an immediate increase in output, which increases continuously until the monetary expansion occurs at time T. At that time a further discrete increase in output occurs. Thereafter, as a domestic currency appreciates, the relative price of domestic goods rises and demand and domestic output gradually decline to the equilibrium level. The initial depreciation of the domestic currency causes an immediate jump in the domestic CPI, which, with the domestic nominal money supply fixed (prior to time T), creates an initial fall in the real money supply. At the same time, the initial increase in domestic real output, stimulated by the depreciation of the domestic currency as a result of the announcement, increases the demand for real money balances. In order for domestic money market equilibrium to be maintained, the domestic nominal interest rate must rise. As

the price of domestic output increases during the period prior to the monetary expansion, the real domestic money supply contracts further, while the increasing real income causes the real money demand to continue rising. In order for money market equilibrium to be maintained, the domestic nominal interest must therefore continue to rise. At time T, when the anticipated monetary expansion takes place, the domestic interest rate drops, falling to a level below its long-run equilibrium. Thereafter, it rises steadily back towards its (unchanged) long-run equilibrium. As we have seen, the average world economy remains unchanged by the anticipation of the impending monetary expansion until the moment that it is actually implemented. Thus, during the period 0 – t < T, the averages of the domestic and foreign variables all remain fixed at their initial equilibria. Since all adjustments during this phase stem entirely from the initial announcement and the jump in the exchange rate this generates, it follows that, given the symmetry of the two economies, the adjustment in the foreign economy is an exact mirror image of that in the domestic economy. Thus, during the period prior to the monetary expansion, the rising price of domestic output is matched by a falling price of foreign output, which arises from the appreciating foreign currency.

At time T, following the domestic monetary expansion, the domestic currency begins to appreciate, as we have noted. This depreciation of the foreign currency puts upward pressure on the price of foreign output. The downward trend is therefore gradually reversed, and eventually the price of foreign output increases back to its original exchange level.

Similarly, the behavior of foreign output mirrors that of domestic output during the initial phases. The initial appreciation of the foreign currency at the announcement date causes an immediate fall in foreign output, which continues to fall further in response to the continuing appreciation of the foreign currency. The increase in the foreign inflation rate occurring at time T causes the real interest rate abroad to decline, thereby stimulating foreign demand and foreign output at that time. During the subsequent transition, the depreciating foreign currency stimulates foreign output sufficiently to cause it to rise above its national rate level, after which it declines monotonically. The appreciation of the foreign currency at the time of announcement causes the real money supply abroad to increase, while the fall in foreign output leads to a decline in the foreign demand for money. These two effects together ensure an immediate reduction in the foreign interest rate. This continues to be the case with the appreciating foreign currency, the declining foreign output, and its price level during this initial phase. At time T, when the domestic monetary expansion occurs, the foreign interest rate increases to a level above the domestic rate but below its long-run equilibrium. Thereafter, r^* increases steadily toward its equilibrium. The reason is that the increase in foreign output at time T, stimulated by the domestic monetary expansion, increases the demand-for-money balances abroad. Since the foreign nominal money balances remain fixed, and since prices and the exchange rate move continuously at time T, the real stock

of foreign money balances remains fixed at time T. Thus, in order for foreign money market equilibrium to hold, the foreign nominal interest rate must rise in order to offset the increased money demand resulting from the higher level of income. With the continuous appreciation of the domestic currency following the monetary expansion, the fact that r^* must lie above r during the subsequent transition back to equilibrium is an immediate consequence of the interest rate parity condition. Finally, we should note that it is possible to analyze the effects of the monetary disturbance upon other variables such as the real exchange rate $e+p^*-p$, the domestic and foreign CPIs z and z^* and the domestic and foreign real interest rates $r - \dot{z}$ and $r^* - \dot{z}^*$. Their responses are essentially composites of those that we have been discussing. Taking an overview of figure 3, it is seen that the anticipation of a future domestic monetary expansion has markedly different effects on the two economics, particularly during the initial phases. Domestically, it generates an increase in output, together with rising prices, although the boom is reversed after the expansion occurs. Abroad, it initially generates a recession with falling prices, although this is also reversed after the expansion."

This only partial discussion of the very detailed and numerous results obtained in Turnovsky (1986) shows that interesting conclusions can be drawn from the application of the jump variable technique to unanticipated and even more to anticipated policy shocks. The reader is referred to this very rigorous article for further details on the formal and the verbal explanation of these and other policy studies.

4   The Blanchard stock-market dynamics

The Blanchard (1981) output and stock market dynamics reads in its basic form as follows:

$$\dot{Y} = \beta_y(Y^d(Y, q) - Y) \tag{21}$$
$$i(Y) = r(Y)/q + \hat{q}: \quad \Rightarrow \quad \dot{q} = i(Y)q - r(Y) \tag{22}$$

We here combine a standard dynamic multiplier story for the dynamics of the output level Y (where however aggregate demand Y^d depends positively on Tobin's q in place of a negative dependence on the real rate of interest) with the situation that bonds and equities are perfect substitutes, including myopic perfect foresight concerning the capital gains on equities. We here identify Tobin's average $q = p_e E/pK$ with the share price p_e by assuming that the number of equities E to the value of the capital stock pK is constant and set equal to 1. We denote by $i(Y)$ the inverted LM curve and by $r(Y)$ the profit rate function of the economy (profits per unit of capital) with all profits paid out as dividends to equity owners. The ratio $r(Y)/q$ is then the dividend rate of return on equities (since pK cancels) and $\hat{q} = \dot{q}/q$ are the capital gains per unit of equity, i.e., $r(Y)/q + \hat{q}$ is the

(total) rate of return on equities, set equal to the interest rate here due to the perfect substitute assumption.

The two laws of motion of the Blanchard output and stock-market dynamics give rise to two isoclines, an example of which is shown in figure 4. Moreover, they typically imply saddlepoint dynamics for their intersections (for the exceptions, see below). Their phase diagram and the stable arm that is directed towards the saddlepoint steady state can therefore be used as usually to apply the JVT of the rational expectations school. This is extensively done in Blanchard's (1981) original paper and need not be repeated here.
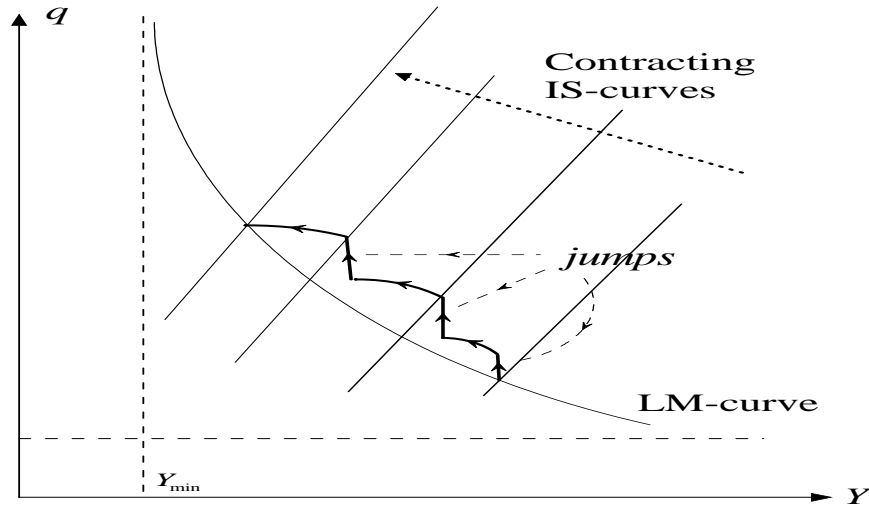

Figure 4: The Blanchard stock-market dynamics in the bad news case

There are however some problems with this approach to a real-financial market interaction. Even in the case where all behavior is assumed as linear we have obtained here at a 2D non-linear dynamical system as indicated in figure 4 for Blanchard's bad news case (with a decreasing LM curve). In Blanchard's good news case (where the interest rate is more sensitive to output changes than the profit rate) the LM curve of figure 4 is positively sloped and it allows for situations of 2, 1 or no steady state, with one steady state being a stable node or focus in the first case. The Jump Variable Technique (JVT) of the Rational Expectations School is therefore then not always applicable which again shows one of its limitations.

We here however concentrate on the case shown in figure 4 where the relevant stable manifold of the then uniquely determined steady state of saddlepoint type is an upward sloping curve. We assume that the economy is hit by three severe and unanticipated contractive demand shock which shift the IS curve of this model significantly to the left as shown in figure 4. Equity-financed firms thus experience severe underutilization problems with respect to their productive capacity, but the stock market shows a strong stock-price rally until the final steady state is reached with low rates of capacity utilization of firms,

low dividend payments, but high stock prices. This is a situation which surely needs some explanation from the proponents of the RE approach, in the justification of the economic meaningfulness of the JVT.

5   Interacting Blanchard stock-market and Dornbusch exchange-rate dynamics: A two-country framework

Since the preceding model types are well documented in the literature on real - financial markets interaction, we do not go into a further investigation of the Blanchard and Dornbusch model here, see however Chiarella et al. (2007) in this respect. Instead we ask the question what the outcome will be if we synthesize the Dornbusch and Blanchard model into a two country model of the world economy. This is a big step forward in the dynamics to be considered and therefore simplified here somewhat by considering for the time being prices (and wages) as given, and normalized to 1. Phillips-curve driven price dynamics is however the next step that has to be considered (see the section 8 in this paper).

As Keynesian aggregate demand functions we postulate now the relationship (all prices = 1, but the nominal exchange rate $s[\frac{€}{\$}]$ is perfectly flexible):

$$
\begin{aligned}
Y^d &= C(Y,s) + I(q,s) + G(s) + X(Y^*,s) \\
Y^{d*} &= C^*(Y^*,-s) + I^*(q^*,-s) + G^*(-s) + X^*(Y,-s)
\end{aligned}
$$

concerning the two goods markets that exist in this two - country two - commodity world.[6] These functions represent the demand side of the two considered economies and they are to be inserted again into the dynamic multiplier process of the Blanchard model:

$$
\dot{Y} = \beta_y(Y^d(Y,Y^*,q,s) - Y) \tag{23}
$$
$$
\dot{Y}^* = \beta_{y^*}(Y^{d*}(Y^*,Y,q^*,s) - Y^*) \tag{24}
$$

in order to provide a full description of the dynamics of the real part of the economy. We note that domestic demands, as well as exports, depend of course on the exchange rate s (positively in the domestic economy and negatively in the foreign one). We have made this latter negative dependency an explicit one, since we can then simply say, that the case of symmetric economies is characterized by identical demand functions and adjustment speeds in the two economies. In the local stability analysis we shall later on work with their linearizations and thus with identical corresponding parameter values in order to allow to calculate average as well as difference economies from the model of this section.

Besides these two laws of motion we have now six financial assets in our world economy (2 types of money, 2 types of bonds and two types of equities) which under the assumptions

---

[6]C consumption, I investment, G government expenditure and X exports.

of perfect substitution and myopic perfect foresight give rise to three independent laws of motion now:[7]

$$
\begin{aligned}
i(Y) &= r(Y)/q + \hat{q}: &\Rightarrow\quad \hat{q} &= i(Y) - r(Y)/q & (25)\\
i^*(Y^*) &= r^*/q^* + \hat{q}^*: &\Rightarrow\quad \hat{q}^* &= i^*(Y^*) - r^*(Y^*)/q^* & (26)\\
i(Y) &= i^*(Y^*) + \hat{s}: &\Rightarrow\quad \hat{s} &= i(Y) - i^*(Y^*) & (27)
\end{aligned}
$$

We interpret the functions $i(Y), i^*(Y^*)$ now as interest rate policy rules which here only exhibit the output gap as argument, since inflation is still excluded from the considered dynamics.

Concerning steady state calculations we here refer to the later on considered situation of symmetric countries which implies that averages behave exactly as the closed economy case considered in Blanchard (1981). All difficulties of the original Blanchard model with respect to its steady state determination are therefore also present in the two-country case. The Jacobian of the considered 5D dynamical system in the state variables Y, Y^*, q, q^*, s reads in its sign structure as follows:

$$
J_0 = \begin{pmatrix}
- & + & + & 0 & + \\
+ & - & 0 & + & - \\
\pm & 0 & + & 0 & 0 \\
0 & \pm & 0 & + & 0 \\
+ & - & 0 & 0 & 0
\end{pmatrix}
$$

The Dornbusch and Blanchard type feedback chains (between Y, s and Y, q) are clearly visible, but this does not easily imply that this matrix will have three unstable and two stable roots as it would be needed for a successful application of the JVT. The task for the rational expectations school is moreover, since the model has in correspondence to the characteristics of its eigenvalues, three forward looking, non-predetermined variables q, q^*, s and two predetermined ones, Y, Y^*, that are only gradually adjusting, to determine a 2D stable manifold (and a 3D unstable manifold) in the 5D phase space such that the non-predetermined variables can always jump to a unique point of the stable manifold with predetermined (temporarily given) output levels of the two countries. This must hold in the case of unanticipated shocks, while in the case of anticipated ones one has to find in addition the single bubble of length T in the old 5D dynamics through a jump to a position in the 5D phase space in the old dynamics such that this bubble has a soft landing on the new stable manifold exactly at time T (where the anticipated shock would occur).

The solution to these problems (with significant calculation costs for the theorist as well as the assumed type of economic agents) assumes calculation capabilities that are much beyond anything that can be characterized as 'rational expectations'. We view such a procedure as a wrong axiomatization of what is actually happening in the factual world,

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[7]The following equations imply that all rates of return on the internationally traded asset must be the same (and therefore equal to $r^*(Y^*) + \hat{q}^* + \hat{s} = r(Y) + \hat{q} - \hat{s}$ in addition).

with an inherent tendency to use more and more complicated constructions or eventually purely mathematical (economically seen: black-box) iteration mechanisms in order to get by assumption the result that the economic dynamics of such models – in their deterministic core – are always of the type of a shock absorber (or occupied with finding the correct bubble that leads them safely to the new shock absorber (coming into existence at time T). It is obvious that we do not regard this as a promising route for further macrodynamic investigations. We therefore will now reconsider the structure of the private sector in order to find an interest rate policy rule that can stabilize it in the conventional sense of this word.

6 A model-oriented reformulation of the Taylor interest rate rule

Instead of pursuing the JVT methodology here any further, we raise the question whether the interest rate policy rule of the Central Banks should not consider the given structure of the economy first before one decides on the economic signals that should guide their choice of an interest rate in such a world. We thus stress that the Taylor rule should be tailored in view of the structure of the economy in which it is supposed to work. Since output dynamics are of stable multiplier type, and since repelling forces only concern the financial sector of the economy (tamed by assumption through the JVT of the RE school), it appears plausible to look at the evolution of either Tobin's q or the UIP exchange rate dynamics to find the variables that should steer the interest rate setting strategy of the CB's. After some experimentation with these possibilities we propose here to use and test the rules:

$$i = i_o + a_i(s_o - s) \tag{28}$$
$$i^* = i_o^* - a_i^*(s_o - s) \tag{29}$$

When these rules are applied to the UIP condition of the private sector

$$i \overset{UIP}{=} i^* + \hat{s}$$

they deliver the astonishingly simple result:

$$\hat{s} = i_o + a_i(s_o - s) - (i_o^* - a_i^*(s_o - s)) = g(s) \quad \text{with} \quad g(s_o) = i_o - i_o^*, g'(s) < 0$$

We thus get that the exchange rate dynamics become independent of the rest of the economy under these choices of policy rules in the two countries and give rise to monotonic convergence to the steady state value s_o jointly set by the two CB's if policy coordination implements the condition $i_o = i_o^*$.

Policy formulation of the described kind (that exhibits private sector structure awareness) therefore removes all repelling forces from the exchange rate dynamic as considered in

figure 1. The only troubling aspect may here be that the above implies that the CB's should do just the opposite of what they might be induced to do in the real world, since – for example the ECB – should lower here the interest rate in the case of an exogenous upward jump in the nominal exchange rate s, a depreciation of the Euro. Leaning against a depreciation in actual economies may however mean that one should attract capital inflows and thus raise the domestic rate of interest, but in the model we are considering this would imply instability and not convergence to the steady state value s_o.

In order to investigate the full dynamics of the model one has to consider of course the remaining four laws of motion in addition. These laws of motion are fully interacting and thus from the perspective of the Routh-Hurwitz stability conditions somewhat demanding. In the next section we will therefore consider the case of symmetric economies, as in Turnovsky (1986), and will linearize the model around its (indeed now uniquely determined) steady state position to prove the local asymptotic stability of the steady state of such a two-country real and financial markets interaction.

The chosen modified form of the Taylor rule does not only simplify the dynamics that is implied by the model, but it also implies that there is in general a unique steady state solution, in contrast to the Blanchard model type considered beforehand. We here use, as in Blanchard (1981), linear aggregate demand functions and ignore the already given steady state position s_o in their formulation.[8] On this background they read:

$$
\begin{aligned}
Y^d &= a_y Y + b_y q + G + c_y Y^* + const. \\
Y^{d*} &= a_{y^*} Y^* + b_{y^*} q^* + G^* + c_{y^*} Y + const.
\end{aligned}
$$

with the propensities to consume, invest and export all given parameters. For Tobin's q we in similar way get (assuming a linear profit rate function as in Blanchard (1981)):

$$
q = (a_r Y + b_r)/i_o, \qquad q^* = (a_{r^*} Y^* + b_{r^*})/i_o^* \quad (i_o = i_o^*).
$$

Inserting these two equations, which are now linear ones, into the two goods-market equilibrium conditions, provides us with two linear equations for the state variables Y, Y^* which in general have a uniquely determined solution. Of course, the parameters of the model have to be chosen in addition such that the steady state levels of the outputs of the two countries are positive and imply positive profit rates for both of them. We therefore in sum get that coordinated monetary policy provides us with the steady state values of both i and s via the UIP condition, while stationary financial market equilibrium allows us to remove Tobin's q from the goods market equilibrium conditions which then imply the steady state values for both Y and Y^*, on the basis of which the steady state values of the $q's$ can then be determined.

Linearizing also the above 5D dynamics (with the exception of the intrinsically nonlinear q-dynamics) in this way (around the just determined steady state position) gives the

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[8]Note however that we will apply this linearity assumption also in the next section (where s is a variable). This means that we have to linearize there the goods market demand function (with respect to the s-influence and thus will get local results there only.

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dynamical system:

$$
\begin{aligned}
\dot{Y} &= \beta_y[-(1-a_y)Y + b_y q + c_y Y^* + d_y s + e_y] \\
\dot{Y}^* &= \beta_{y^*}[-(1-a_{y^*})Y^* + b_{y^*}q^* + c_{y^*}Y - d_{y^*}s + e_{y^*}] \\
\dot{q} &= (i_o + a_i(s_o - s))q - r, \quad r = a_r Y + b_r \\
\dot{q}^* &= (i_o^* - a_i^*(s_o - s))q^* - r^*, \quad r^* = a_r^* Y^* + b_r^* \\
\hat{s} &= i_o + a_i(s_o - s) - (i_o^* - a_i^*(s_o - s))
\end{aligned}
$$

The Jacobian matrix of this linearized dynamics exhibits the following sign structure:

$$
J_0 = \begin{pmatrix}
- & + & + & 0 & + \\
+ & - & 0 & + & - \\
- & 0 & i_o & 0 & -0 \\
0 & - & 0 & i_o & + \\
0 & 0 & 0 & 0 & -
\end{pmatrix}
$$

The stability of its unique steady state solution will be investigated in the following section by means of an important simplifying device that can be applied to study the interaction of large economies that are sufficiently similar to each other in their real as well as their financial parts.

7 Symmetric countries: Stability analysis

We consider now the artificial variables $Y^a = (Y + Y^*)/2, q^a = (q + q^*)/2$, the averages of the GDP's and Tobin's q's as well as $Y^\delta = Y - Y^*, q^\delta = q - q^*$, the differences of the GDP's and Tobin's q's and will use in the following the above (partial) linear representation, which includes the export functions $X(Y^*, s), X^*(Y, -s)$. Moreover we now use the pair of Taylor rules $i = i_o + a_i(s_o - s), i^* = i_o - a_i(s_o - s)$ for our stability investigations of the symmetric two-country case. It is obvious from the preceding section that the steady values of the case of a difference economy are zero as far as outputs and Tobin's q's are concerned, i.e., the steady state average values share symmetry with the parameters of the model and are given just by the unique values Y_o, q_o of the case of the average economy.

7.1 The average economy

In order to show the stability of the full 5D dynamics we assume the case of two symmetric large open economies, in which case all corresponding parameters of the two countries are of the same size, with opposite signs if the exchange rate is involved (also in the policy

rules). Taking averages $Y^a = (Y + Y^*)/2, q^a = (q + q^*)/2$ therefore allows to combine the goods market equilibrium conditions into a single world market equation, where the exchange rate effect has been canceled. The stock market dynamics can in the same way be aggregated into a single equation. This in sum gives a 2 dimensional dynamical system as it is shown below. This average economy represents a hypothetical economy that is of the Blanchard (1981) closed economy type (without the intrinsic non-linearities in the equity dynamic that have complicated the Blanchard stock market model, since the product iq is now simply given by $i_o q$).

$$
\begin{aligned}
\dot{Y}^a &= \beta_y[(a_y + c_y - 1)Y^a + b_y q^a + const.] \\
\dot{q}^a &= i_o q^a - a_r Y^a + const.
\end{aligned}
$$

For the Jacobian matrix of this economy we immediately get the result:

$$
J = \begin{pmatrix} \beta_y(a_y + c_y - 1) & \beta_y b_y \\ -a_r & i_o \end{pmatrix} = \begin{pmatrix} - & + \\ - & i_o \end{pmatrix}
$$

if we have multiplier stability $(a_y + c_y < 1)$ and if the output adjustment speed is sufficiently large $(\beta_y[1 - (a_y + c_y)] > i_o$ is solely needed). These conditions are of conventional type in the first case and not at all restrictive in the second case, implying that the average economy is generally asymptotically stable and thus always convergent to its steady state position. Since the interest rate i_o is indeed a small number, setting it 0 in addition exemplifies, that adjustment to the steady state is cyclical for an intermediate range of the output adjustment speed.

7.2 The difference economy

Taking differences $Y^\delta = Y - Y^*, q^\delta = q - q^*$ implies a three dimensional dynamics (with these state variables and the exchange rate s.) These dynamics are, formally seen, of the type of the small open economy as considered in Dornbusch (1976). Note that we have a $d_y s$ term in the aggregate demand equation now and that the domestic Taylor rule is given by: $i = a_i(s_o - s) + i_o$, while the one of the foreign economy has a negative sign in front of the adjustment speed a_i.

$$
\begin{aligned}
\dot{Y}^\delta &= \beta_y[(a_y - c_y - 1)Y^\delta + b_y q^\delta + 2d_y s] \\
\dot{q}^\delta &= [-2a_i(s - s_o) + i_o]q^\delta - a_r Y^\delta \\
\hat{s} &= -2a_i(s - s_o)
\end{aligned}
$$

$$
J_0 = \begin{pmatrix} \beta_y[(a_y - c_y - 1) & \beta_y b_y & \beta_y 2d_y \\ -a_r & i_o & -2a_i- \\ 0 & 0 & -2a_i s_o \end{pmatrix} = \begin{pmatrix} - & + & + \\ - & i_o & - \\ 0 & 0 & - \end{pmatrix}
$$

It is obvious from the structure of this Jacobian that asymptotic stability and convergence here holds under the same (and even weaker) conditions as in the case of averages.

7.3 Summary

Summing up, we have that differences must converge to 0 (and s to s_o) and that averages must converge to their common steady state values implying that the full 5D dynamics is also characterized by convergence towards their steady state position. This is a convenient short-cut to the analysis of the full 5D system when the parameters of the two countries differ from each other. We thus have an economy now that is stable in the conventional sense of this word. These should be of interest to policy makers if they could accept that leaning against the wind means in this setup just the opposite of what they might be inclined to do intuitively.

8 Dornbusch inflation dynamics

As in Turnovsky (1986) we now add inflation dynamics in the two countries in the form of the following two Phillips curves (assuming again symmetry between the two countries):

$$\hat{p} = \beta_w(Y - \bar{Y}), \quad \hat{p}^* = \beta_w(Y^* - \bar{Y})$$

We thus assume that the current output gaps drive current inflation rate, but do not yet consider acceleration terms in these two Phillips curves.

Linearizing these equations around the steady state position (for local stability analysis) gives rise to:[9]

$$\dot{p} = \beta_w p_o(Y - \bar{Y}), \quad \dot{p}^* = \beta_w p_o^*(Y^* - \bar{Y})$$

Since the price levels in the two country are now moving in time we have to use the real exchange rate $\sigma = sp^*/p$ in the goods markets dynamics behind the parameter d_y now which when linearized by first order Taylor approximation gives rise to the following modification of the model considered in section 6 (with symmetry now assumed in

---

[9]Note here and in the following that we do not use logarithms, but only use first order Taylor approximations for the terms we want to linearize.

addition):

$$\dot{Y} = \beta_y[-(1-a_y)Y + b_yq + c_yY^* + d_y\sigma_o(\frac{s}{s_o} + \frac{p^*}{p_o^*} - \frac{p}{p_o}) + e_y]$$

$$\dot{Y}^* = \beta_y[-(1-a_y)Y^* + b_yq^* + c_yY - d_y\sigma_o(\frac{s}{s_o} + \frac{p^*}{p_o^*} - \frac{p}{p_o}) + e_y]$$

$$\dot{q} = (i_o + a_i(s_o - s))q - r, \quad r = a_rY + b_r$$

$$\dot{q}^* = (i_o - a_i(s_o - s))q^* - r^*, \quad r^* = a_rY^* + b_r$$

$$\hat{s} = 2a_i(s_o - s)$$

$$\dot{p} = \beta_wp_o(Y - \bar{Y})$$

$$\dot{p}^* = \beta_wp_o^*(Y^* - \bar{Y})$$

Considering the averages of these 5D dynamical system gives rise to the same 2D dynamics as already investigated in section 7. Convergence to the steady state averages is therefore ensured in this extension of the model of section6. With respect to differences we now however get the following 4D dynamics (under the assumption that $p_o = p_o^*$ holds, see below):

$$\dot{Y}^\delta = \beta_y[-(1-a_y-c_y)Y^\delta + b_yq^\delta + 2d_y\sigma_o\frac{s}{s_o} - 2d_y\sigma_o/p_o \cdot p^\delta]$$

$$\dot{q}^\delta = [-2a_i(s-s_o) + i_o]q^\delta - a_rY^\delta$$

$$\hat{s} = -2a_i(s-s_o)$$

$$\dot{p}^\delta = \beta_wp_oY^\delta$$

We assume again that the autonomous dynamic of the nominal exchange rate has already settled down at its steady state position s_o and need therefore only investigate the stability of the remaining 3D dynamics in the state variables $Y^\delta, q^\delta, p^\delta$. The Jacobian matrix of this reduced dynamics exhibits the following sign structure:

$$J_0 = \begin{pmatrix} - & + & - \\ - & i_o & 0 \\ + & 0 & 0 \end{pmatrix}$$

It is easy to show that the Routh-Hurwitz stability conditions are all fulfilled (since i_o is small) with the exception of the determinant of J_o which is positive in this case (but small, due to its multiplicative dependence on i_o). The system is therefore slightly explosive and needs further stabilizing effort from the side of monetary policy in order to allow full convergence to its steady state position.

Moreover, also this steady state position needs some further discussion, since output values are now equal to the NAIRU value $\bar{Y}$ in the steady state and therefore no longer determined through goods market equilibrium (but through labor market equilibrium instead). From the (nonlinearized) equations of the model we can first of all conclude that s must be equal to the value s_o (not yet determined) via the UIP condition. This

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in turn implies by means of the postulated Taylor rules $i = i^* = i_o$, i.e., interest rate equality with the steady state rate of interest rate set by the Central Banks. We then get for the values of Tobin's q given expressions of the type $r(\bar{Y})/i_o$ which when inserted into the goods market equilibrium equations (which are identical for the two countries) determine the steady state value of the real exchange rate σ, since output must be equal to its natural level. We now assume that the Central Banks know this natural level of σ and base their interest rate policy on the (so far undetermined) condition $s_o = \sigma_o$. This then finally implies that $p_o = p_o^*$ must hold true in the steady state, a condition that is needed for the application of the symmetric country assumption and the mathematical methodology based on it (concerning $\dot{p}^\delta here$).

In order to get full convergence in the above two country model with Dornbusch inflation dynamics we now augment the coordinated Taylor rules of the two countries as follows:

$$i = i_o + a_i(s_o - s) + b_i\hat{p} \tag{30}$$
$$i^* = i_o - a_i(s_o - s) + b_i\hat{p}^* \tag{31}$$

This gives for the law of motion for s now the equation:

$$\hat{s} = i - i^* = 2a_i(s_o - s) + b_i\dot{p} - b_i\dot{p}^* = 2a_i(s_o - s) + b_i\dot{p}^\delta = 2a_i(s_o - s) + b_i\beta_w p_o Y^\delta$$

This extended Taylor rule modifies the Jacobian of the difference economy for the now 4 interacting state variables $Y^\delta, q^\delta, s, p^\delta$ as follows:

$$J_0 = \begin{pmatrix} - & + & + & - \\ - & i_o & 0 & 0 \\ 0 & 0 & - & + \\ + & 0 & 0 & 0 \end{pmatrix}$$

Since i_o is small, the trace of the matrix J_o is surely negative. Using the same argument again, the sum of principal minors of order two is easily shown to be positive. It is also easily shown that the determinant of the 4D Jacobian is positive (and small). For the sum of principal minors of order three we finally get a negative value if the value of the new parameter b_i in the Taylor rule is chosen sufficiently small. We thus have that the coefficients a_i of the characteristic polynomial of the matrix J_o are all positive as required by the Routh-Hurwitz stability conditions. According to Asada et al. (2003, Theorem A.6) we have to show however in addition for the validity of asymptotic stability that there holds: $a_1 a_2 a_3 - a_1^2 a_4 - a_3^2 > 0$. Since the determinant of J_o is small we have that a_4 is close to zero. There thus remains to be shown that $a_1 a_2 - a_3 > 0$ holds true for the trace and the principal minors of order 2 and 3. Using again the conditions that i_o is small and b_i chosen with care then implies also this result, since the expressions in $a_1 a_2$ are then dominating the principal minors of order three of the Jacobian matrix J_o.

We thus get that a cautious anti-inflationary interest policy rule can stabilize the economy where Phillips curve dynamics has been added to the goods markets and asset markets

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dynamics we have considered in the preceding sections. This makes our model comparable to the Turnovsky (1986) IS-LM two country model, but now with Tobin's q in the aggregate demand function in place of the real rate of interest of conventional textbook analysis. We here however arrive at the result that the Central Banks should use conventional gaps as well as an unconventionally signed nominal exchange rate gap in order to stabilize the economy in the presence of perfect substitution between bonds and equities and perfect foresight on capital gains through stock price and exchange rate dynamics.

9   Outlook: Imperfect capital markets

This paper has shown – if one accepts the perfectness assumptions made with respect to asset substitution and expected capital gains – that a better solution to the then implied instability problems (the centrifugal exchange rate as well as stock price dynamics) may be to choose the Taylor interest rate policy rule appropriately in the light of the structure of the private sector, rather than to try to enforce three unstable roots and two stable ones on the dynamics by a conventional type of Taylor rule in order to allow for the application of the JVT of the RE school. Yet, even if one can generate stability in this way, this solution procedure nevertheless shows that the extremely perfect structure of the financial sector implies the need for a – from an applied point of view – fairly strange interest rate reaction function (as far as financial markets are concerned), where it has to be assumed that a monetary policy intended to counteract the country's exchange rate depreciation should lower the interest in this country, but not increase it as conventional wisdom might imply.

The reason for this strange result can be easily detected if exchange rate dynamics are formulated with some sluggishness in their reaction to interest rate differentials. A simple illustration is provided by the following example, where we assume given exchange rate expectations $\hat{s}^e$ for the time being:

$$\hat{s} = \beta_s(i^*(s) + \hat{s}^e - i(s))$$

Clearly, a positive reaction of the domestic interest rate and a negative reaction of the foreign to the exchange rate, the opposite of what we have used in the preceding section, would contribute now to exchange rate stability, if it is (of course) assumed that exchange rates are increasing if expected returns on foreign bonds are higher than the ones on domestic bonds. This postulated exchange rate reaction is compatible with the direction of capital flows behind the assumed adjustment equation. Yet, going from such fast to infinitely fast exchange rate reactions implies $i(s) = i^*(s) + \hat{s}^e$ which together with the assumption of myopic perfect foresight gives

$$\hat{s} = \hat{s}^e = i(s) - i^*(s),$$

i.e., a sign reversal with respect to the role played by the interest rate differential. This

is the reasons why also policy must accept a sign reversal in its orientation in order to be successful in this limit case. It also suggests that approaching the limit case is producing a discontinuity in the behavior of the economy. More generally speaking, we claim here that the limit case is structurally unstable with respect to any model that allows for imperfect substitution and imperfect foresight, i.e., it cannot be approached by considering degrees of imperfections that are shrinking to zero. But if the limit is of strictly isolated importance only, it may be questioned whether it is useful for policy advice. This also holds for the models of sections 3 and 5 which are close in spirit to the currently fashionable DSGE model type which is in fact used for applied policy analysis, see Smets and Wouters (2003) for an example.

The conclusion we draw from this is that model's of portfolio choice with only imperfect asset substitution augmented be heterogeneous (necessarily imperfect) expectation formation and somewhat delayed adjustment processes are the better choice if one wants to model the world in a (mathematically seen) robust and descriptively seen relevant way, in place of the extremely idealized stock market (long-term bond markets) and exchange rate dynamics of today's DSGE models, where determinacy and convergence is enforced, but not proved. Such modifications of the perfectness assumptions of the present chapter will be the topics we will investigate in the remaining chapters of this part of the book.

References

ASADA, T., C. CHIARELLA, P. FLASCHEL and R. FRANKE (2003): *Open Economy Macrodynamics : An Integrated Disequilibrium Approach.* Springer: Berlin.

ASADA, T., FLASCHEL, P. GROH, G. and C. PROAÑO (2008): Period analysis, continuous time and chaos from an empirical perspective. In: Flaschel, P., Groh, G., Proaño, C. and W. Semmler (2008): *Topics in Applied Macrodynamics.* Heidelberg, Springer, forthcoming.

BLANCHARD, O.J. (1981): Output, the stock market, and interest rates. *American Economic Review*, 71, 132 – 143.

CHIARELLA, C., FLASCHEL, P., FRANKE, R. and W. SEMMLER *Financial Markets and the Macroeconomy. A Keynesian Perspective.* London: Routledge, forthcoming.

DORNBUSCH, R. (1976): Expectations and exchange rate dynamics. *Journal of Political Economy*, 84, 1161 – 1175.

GRAY, M. and S. TURNOVSKY (1979): The stability of exchange rate dynamics under myopic perfect foresight. *International Economic Review*, 20, 641 – 660.

MCKIBBIN, W. and J. SACHS (1991): *Global Linkages. Macroeconomic Interdependence and Cooperation in the World Economy.* Washington, D.C.: The Brookings Institution.

SMETS, F. and R. WOUTERS (2003): An estimated stochastic dynamic general equilibrium model of the Euro area. *Journal of the Economic Association*, 5, 1123-1175

TURNOVSKY, S.J. (1986): Monetary and fiscal policy under perfect foresight: a symmetric two country analysis. *Economica*, 53, 139 – 157.

WOODFORD, M. (2003): *Interest and Prices. Foundations of a Theory of Monetary Policy.* Princeton: Princeton University Press.