

# Heterogeneous Foreign Exchange Market Expectations and Macroeconomic Stability

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## Abstract

In this paper we investigate the macroeconomic interaction of two large economies through trade, price and exchange rate transmission channels within a Keynesian disequilibrium framework. For this purpose we set up a two-country model where each country is specified as an open-economy version of the (D)AS-AD framework discussed in Chen, Chiarella, Flaschel and Semmler (2006). After discussing the core properties of the one-country 5D dynamical subsystem we estimate by means of system GMM and Three-Stage-Least-Squares the two-country model as a whole using time series from the U.S. and the euro area and analyze the response of both economies to monetary policy and labor productivity shocks.

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## 1 Introduction

During the last decade and especially after the prominent contribution by Obstfeld and Rogoff (1995), an important paradigm change concerning the theoretical modeling approach of open-economy issues has taken place. After the long-lasting predominance of Mundell-Fleming-Dornbusch type of models in the academic as well as in the more policy-oriented literature, the so-called “New Open Economy Macroeconomics” approach has become the workhorse for open-economy issues in recent years.

As in their closed-economy DSGE counterparts, such as the ones discussed in Blanchard and Kiyotaki (1987) and Erceg, Henderson and Levin (2000), a central feature in these type of models is the assumption of rational expectations. However, even though theoretically appealing, the notion of fully rational agents is still quite controversial in the academic literature, and especially in the literature on nominal exchange rate dynamics. As pointed out e.g. by De Grauwe and Grimaldi (2005), efficient markets rational expectations models are unable to match empirical data on foreign exchange (hereafter FX) rate fluctuations as well as the occurrence of speculative bubbles, herding behavior and runs. “Non-rational” models, that is models which feature economic agents with heterogenous beliefs, attitudes or trading schemes, seem much more successful in this task. Such models, however, often constrain themselves on the analysis of the FX markets and do not analyze the effects of such non-rational behavior by FX market participants for the dynamics stability at the macroeconomic level.

In this paper an attempt is made to fill in this gap by setting up a two-country semi-structural macroeconomic model with a foreign exchange market consisting of two types of traders with different beliefs concerning the future development of the nominal exchange rate: fundamentalists and chartists. As the model is formulated, it reacts to disequilibrium situations in both goods and labor markets in a sluggish manner primarily due to the gradual adjustment of nominal wages and prices to such situations. This is the first logical step for the understanding of real effects of monetary and fiscal policy in economies which are highly interrelated with each other through a variety of markets and channels, when the general equilibrium requirement is abandoned and instead the alternative view is considered where markets rather adjust to disequilibrium situations in a gradual manner. To do so the theoretical disequilibrium model of AS-AD growth investigated in Chen et al. (2006) is reformulated for the case of two large open economies, first each in isolation and then in

their interaction as two subsystems within a large closed dynamical system.

Our focus is set on the the importance of foreign exchange trading not only for the stability of that single market but for the whole macroeconomic system. We show by means of eigen-value analysis the parameter thresholds by which the steady state of the system loses its stability properties.

The remainder of the paper is organized as following: In section 2 the theoretical two-country semi-structural framework for the case of an open economy is described. Section 3 integrates two open economies and discusses in more detail the linking channels between both economies, as well as the dynamics of the nominal exchange rate (the financial link) and the steady state conditions. In that section the model is also estimated and the resulting dynamic adjustments are analyzed . In section 3.3 the consequences of wage and price flexibility, of monetary policy as well as of the structure in the foreign exchange markets for the stability of the dynamical system are investigated by means of eigen-value analysis. Section 4 draws some concluding remarks from this paper.

## **2 The Baseline Open-Economy Framework**

In this section the closed economy, semi-structural macroeconomic model discussed in Chen et al. (2006) is extended to a macroeconomic two-country framework through the incorporation of trade, price and financial links between two similar economies with imperfectly flexible nominal wages and prices. Hereby it is assumed that both economies have the same macroeconomic structure and are additionally conducted with the same type of monetary policy. Therefore only the structure of the domestic economy will be discussed in this section, denoting with the superscript  $f$  foreign economy variables and assuming equivalent formulations for the foreign economy (with the effect of the log real exchange rate  $\eta = s + \ln(p^f) - \ln(p)$  adequately adjusted).

### **2.1 The Goods and Labor Markets**

Concerning the real part of the economy, a semi-structural approach is pursued assuming that the dynamics of output and employment can be summarized by the

following laws of motion:

$$\hat{u} = -\alpha_{uu}(u - u_o) - \alpha_{ur}(i - \hat{p} - (i_o - \pi_o)) - \alpha_{uv}(v - v_o) + \alpha_{u\eta}\eta + \alpha_{uu^f}\hat{u}^f \quad (1)$$

$$\hat{e} = \alpha_{e\hat{u}}\hat{u} - \alpha_{ev}(v - v_o) \quad (2)$$

The first law of motion is of the type of a dynamic backward-looking open economy IS-equation, here represented by the growth rate of the capacity utilization rate of firms. Concerning the closed economy dimension, it has three important domestic characteristics: (i) it reflects the dependence of output changes on aggregate income and thus on the rate of capacity utilization by assuming a negative, i.e., stable dynamic multiplier relationship in this respect; (ii) it shows the joint dependence of consumption and investment on the domestic income distribution, which in the aggregate in principle allows for positive or negative signs before the parameter  $\alpha_{uv}$ , depending on whether consumption, investment or the next exports are more responsive to relative real wage and wage share changes;<sup>1</sup> and (iii) it incorporates the negative influence of the real rate of interest on the evolution of economic activity. Additionally, in contrast to the closed economy model investigated in Chen et al. (2006), we incorporate (iv) the positive effect of foreign goods demand (proxied by the *growth rate* of capacity utilization in the foreign economy) and (v) the positive influence of the deviation of the log real exchange rate  $\eta = s + \ln(p^f) - \ln(p)$  ( $s$  being the log nominal exchange rate, which law of motion will be defined below) from its PPP consistent steady state level  $\eta_o = 0$ .

In the second law of motion, which describes the growth rate of the rate of employment, we assume that the employment policy of firms follows – in the form of a generalized Okun’s Law – the growth rate of capacity utilization (with a weight  $\alpha_{e\hat{u}}$ ).<sup>2</sup> Moreover, we additionally assume that an increasing wage share has a negative influence on the employment policy of firms. Employment is thus in particular assumed to adjust to the level of current activity since this dependence can be shown to be equivalent to the use of a term  $(u/u_o)^{\alpha_{e\hat{u}}}$  when integrated, i.e., the form of Okun’s law in which this law was originally specified by Okun (1970) himself.

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<sup>1</sup>We will, however, not engage into this debate here but rather adopt the most traditional view according to which  $\partial\hat{u}/\partial v$  is unambiguously negative.

<sup>2</sup>Despite of being largely criticized due to its “lack of microfoundations”, in a large number of microfounded, “rational expectations” models such as Taylor(1994), Okun’s law is used to link production with employment.

## 2.2 The Wage-Price Dynamics

In the modeling of the wage and price inflation dynamics the approach proposed by Chiarella and Flaschel (2000) and Chiarella, Flaschel and Franke (2005) is pursued. This theoretical framework of aggregate demand fluctuations, which allows for under- (or over-)utilized labor *as well as* capital, is based on gradual adjustments to disequilibrium situations of all real variables of the economy. By allowing for disequilibria in both goods and labor markets, the dynamics of wages and prices can be discussed separately from each other in their structural forms, assuming that both react to their own measure of demand pressure, namely  $e - e_o$  and  $u - u_o$ , in the market for labor and for goods, respectively.<sup>3</sup> Hereby  $e$  represents the rate of employment on the labor market and by  $e_o$  the NAIRU-equivalent level of this rate, and similarly by  $u$  the rate of capacity utilization of the capital stock and by  $u_o$  the normal rate of capacity utilization of firms are denoted. Indeed, as for example Barro (1994) points out, perhaps the most important feature that theoretical Keynesian models should comprise is the existence of imperfectly flexible wages as well as prices.

As in Chiarella and Flaschel (2000) and Chiarella et al. (2005), the expectations in both wage and price Phillips curve are modeled in a hybrid way, with crossover myopic perfect foresight (model-consistent) expectations with respect to short-run wage and domestic price inflation on the one hand and an adaptive updating inflation climate expression (symbolized by  $\pi_c$ ) concerning the evolution of the CPI inflation ( $\hat{p}_c$ ), on the other hand. Note that though the specification our model features may not be rational, it nevertheless has model consistent expectations concerning the evolution of the wage and price inflation and also incorporates a similar degree of inertia obtained in New Keynesian models only through also ad-hoc “rules-of-thumb” or price indexation assumptions.<sup>4</sup>

More specifically, concerning the wage Phillips curve it is assumed that the short-run price level considered by workers in their wage negotiations is set by the producer, so that producer price inflation gives the rate of inflation that is perfectly foreseen by workers as their short-run cost-push term. Additionally, in order to incorporate the role of import price inflation in the dynamics of the economy, we assume that the measure that is taken by workers to judge the medium-run evolution of prices in

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<sup>3</sup>As pointed out by Sims (1987), such strategy allows to circumvent the identification problem which arises when both wage and price inflation equations have the same explanatory variables.

<sup>4</sup>See e.g. Galí and Gertler (1999) and Galí, Gertler and López-Salido (2001).

their respective economies is the Consumer Price Index, defined as

$$p_c = p^\xi (Sp^f)^{1-\xi},$$

the geometric average of domestic and import prices – with  $p^f$  being foreign price level and  $S$  the nominal exchange rate.

Consequently, the CPI inflation  $\hat{p}_c$  includes both domestic inflation (with a specific weight  $\xi$ ) and imported goods price inflation (with weight  $1 - \xi$ ), so that

$$\hat{p}_c = \xi \hat{p} + (1 - \xi)(\dot{s} + \hat{p}^f), \quad (3)$$

with  $s = \ln(S)$ . Because of the uncertainty linked with nominal exchange rate movements, we assume for both workers and firms' decision taking processes, that CPI inflation is updated in an adaptive manner according to<sup>5</sup>

$$\dot{\pi}_c = \beta_{\pi_c}(\hat{p}_c - \pi_c) = \beta_{\pi_c}\xi(\hat{p} - \pi_c) + \beta_{\pi_c}(1 - \xi)(\hat{p}^f + \dot{s} - \pi_c) \quad (4)$$

We thereby arrive at the following two Phillips Curves for wage and price inflation, which in this core version of Keynesian AS-AD dynamics are – from a qualitative perspective – formulated in a fairly symmetric way.

The structural form of the wage-price dynamics is:

$$\hat{w} = \beta_{we}(e - e_o) - \beta_{wv} \ln(v/v_o) + \kappa_{wp}\hat{p} + (1 - \kappa_{wp})\pi_c + \kappa_{wz}\hat{z}, \quad (5)$$

$$\hat{p} = \beta_{pu}(u - u_o) + \beta_{pv} \ln(v/v_o) + \kappa_{pw}(\hat{w} - \hat{z}) + (1 - \kappa_{pw})\pi_c, \quad (6)$$

where  $\hat{z}$  denotes the growth rate of labor productivity (which we assume here just to be equal to  $g_z = \hat{z} = \text{const.}$  ( $g_z$  denoting the trend labor productivity growth)).

Note that as the wage-price mechanisms are formulated, the development of the CPI inflation does not matter for the evolution of the domestic wage share  $v = (w/p)/z$ , measured in terms of producer prices, the law of motion of which is given by (with  $\kappa = 1/(1 - \kappa_{wp}\kappa_{pw})$ ):

$$\hat{v} = \kappa [(1 - \kappa_{pw})f_w(e, v) - (1 - \kappa_{wp})f_p(u, v) + (\kappa_{wz} - 1)(1 - \kappa_{pw})g_z]. \quad (7)$$

with

$$f_w(e, v) = \beta_{we}(e - e_o) - \beta_{wv} \ln(v/v_o) \quad \text{and}$$

$$f_p(y, v) = \beta_{pu}(u - u_o) + \beta_{uv} \ln(v/v_o)$$

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<sup>5</sup>In the empirical applications of this adaptive revision of the CPI inflation we will simply use a moving average of the CPI inflation with linearly declining weights.

which follows easily from the following obviously equivalent representation of the above two Phillips Curves:

$$\begin{aligned}\hat{w} - \pi_c &= \beta_{we}(e - e_o) - \beta_{wv} \ln(v/v_o) + \kappa_{wp}(\hat{p} - \pi_c), \\ \hat{p} - \pi_c &= \beta_{pu}(u - u_o) + \beta_{pv} \ln(v/v_o) + \kappa_{pw}(\hat{w} - \pi_c)\end{aligned}$$

by solving for the variables  $\hat{w} - \pi_c$  and  $\hat{p} - \pi_c$ . It also implies the following two across-markets or *reduced form Phillips Curves*:

$$\begin{aligned}\hat{w} &= \kappa [\beta_{we}(e - e_o) - \beta_{wv} \ln(v/v_o) + \kappa_{wp}(\beta_{pu}(u - u_o) + \beta_{pv} \ln(v/v_o)) \\ &\quad + (\kappa_{wz} - \kappa_{wp}\kappa_{pw})g_z] + \pi^c, \\ \hat{p} &= \kappa [\beta_{pu}(u - u_o) + \beta_{pv} \ln(v/v_o) + \kappa_{pw}(\beta_{we}(e - e_o) - \beta_{wv} \ln(v/v_o)) \\ &\quad + \kappa_{pw}(\kappa_{wz} - 1)g_z] + \pi^c,\end{aligned}$$

which represent a considerable generalization of the conventional view of a single-market price PC with only one measure of demand pressure, namely the one in the labor market. Indeed, as it will be discussed below, the incorporation of separate but interdependent dynamics of the nominal wages and prices allows for a much more richer analysis of the macroeconomic interdependencies between these variables and output, employment and income distribution than models with only price rigidities but perfectly flexible nominal wages are able to deliver.

### 2.3 Monetary Policy

As standard in modern macroeconomic models, we assume that money supply accommodates the interest rate policy pursued by the central bank and thus does not feedback into the core laws of motion of the model. As interest rate policy we assume a classical Taylor rule:

$$i_{\tau} = (i_o - \pi_o) + \hat{p} + \phi_{\pi}(\hat{p} - \pi_o) + \phi_y(u - u_o). \quad (8)$$

The target rate of the central bank  $i_{\tau}$  is thus assumed to depend on the steady state real rate of interest – augmented by actual inflation back to a nominal rate –, on the inflation gap and on the capacity utilization gap (as a measure of the output gap). We assume furthermore that the monetary authorities, when pursuing this target rate, do not react automatically but rather adjust to it in a smooth manner according to

$$\dot{i} = \alpha_{ii}(i_{\tau} - i), \quad (9)$$

with  $\alpha_{ii}$  determining the adjustment speed of the nominal interest rate.<sup>6</sup> Inserting  $i_T$  in and rearranging terms we obtain from this expression the following dynamic law of motion for the nominal interest rate

$$\dot{i} = -\gamma_{ii}(i - i_o) + \gamma_{ip}(\hat{p} - \pi_o) + \gamma_{iu}(u - u_o) \quad (10)$$

where we have  $\gamma_{ii} = \alpha_{ii}$ ,  $\gamma_{ip} = \alpha_{ii}(1 + \phi_\pi)$ , i.e.,  $\phi_\pi = \gamma_{ip}/\alpha_{ii} - 1$  and  $\gamma_{iu} = \alpha_{ii}\phi_y$ .

## 2.4 The Real Exchange Rate Dynamics

As stated before, despite of the empirical inability of rational expectations models to explain the dynamics of the nominal exchange rate, the majority of theoretical macroeconomic frameworks (and especially in the nowadays quite popular DSGE models) still assume the rational expectations and a rational behavior of economic agents concerning the dynamics of the nominal exchange rate. A common procedure in these types of models is to assume that the dynamics of the nominal exchange rate are driven by the validity of the purchasing power parity (PPP) postulate, see e.g. Obstfeld and Rogoff (1995). Through a log-linearization around the general equilibrium “rational expectations” steady state of the system, the – correctly – expected depreciation rate of the nominal exchange rate between two economies is simply determined by

$$E_t[s_{t+1} - s_t] = \pi_t - \pi_t^f,$$

with  $s_t$  denoting the log of the nominal exchange rate and  $\pi_t$  and  $\pi_t^f$  the domestic and foreign price inflation rates, respectively. Under the assumption that the price inflation rate is determined by the difference between money- and consumption growth differentials, the actual nominal exchange rate can be expressed (applying the no-bubbles condition) as (see Walsh (2003, p.277))

$$s_t = \frac{1}{1 + \delta} \sum_{i=0}^{\infty} \left( \frac{1}{1 + \delta} \right)^i [(m_{t+i} - m_{t+i}^*) - (c_{t+i} - c_{t+i}^*)]$$

with  $\delta$  as the intertemporal discount rate;  $m$  and  $m^*$  as the money supplies; and  $c$  and  $c^*$  as the consumption levels in the domestic and foreign economies. Thus, in the New Keynesian framework, the actual nominal exchange rate between two

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<sup>6</sup>In the academic literature there is an ongoing and still unsolved debate about whether there is an interest smoothing parameter in the monetary policy reaction rule of the central banks or whether the observed high autocorrelation in the nominal interest rate is simply the result of highly correlated shocks or only slowly available information, see e.g. Rudebusch (2002) and Rudebusch (2006) for a throughout discussion.



countries depends on the current and future paths of the nominal money supply- and consumption differentials between both economies.

Though straightforward in a theoretical rational expectations general equilibrium, this solution implies nevertheless the existence of (solely) purely rationally handling agents in the financial markets, an assumption that has been proven to be unable to explain major stylized facts of the nominal exchange rate dynamics. As shown for example in Ehrmann and Fratzscher (2005), the volatility of fundamentals (modeled in that study through an index of interest rate and output growth differentials and current account deficits) is by far not as large as the dynamics of the corresponding nominal exchange rates.

Due to the empirical failure of rational expectations models, a large literature based on the assumption of heterogenous expectations or beliefs among the traders in the foreign exchange market has arisen in the last decade. The inclusion of such heterogeneity, and therefore of a somewhat “nonrational” behavior by the economic agents has proven quite valuable in providing insights and explanations concerning some of the “puzzles” which arise when “rationality” is assumed.<sup>7</sup>

In the most basic heterogenous expectations framework, see e.g. Frankel and Froot (1990), two basic types of traders with different belief patterns (or expectations) concerning the future behavior of the nominal exchange rate are modeled: the fundamentalists and the chartists. The fundamentalists typically believe that the nominal exchange rate is driven by macroeconomic fundamentals such as interest rate differentials, different developments of production and employment and/or the validity of the PPP postulate and consequently trade conforming to this belief. In contrast, the chartists are assumed to follow the market tendencies, acting thus in principle in a destabilizing manner. The dynamics and stability of the resulting nominal exchange rate, therefore, depend on the relative strength and proportion of these two groups in the foreign exchange market.

In more advanced theoretical frameworks about heterogenous beliefs a wide variety of extensions concerning the endogenous determination of the trader groups composition can be found: in Kirman (1993) for example the determination of the two groups is determined by a purely stochastic factor; in Lux (1995) the “contagion” effect, that is, the change in the trading strategy, depends on the overall “mood” of the market and on the observed realized returns. De Grauwe and Grimaldi (2005), in

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<sup>7</sup>See De Grauwe and Grimaldi (2006, ch.1) for an extensive discussion of the advantages of the heterogenous agents-approach with respect to the rational-expectations approach in the explanation of empirical financial market data.

a similar manner, assume the *group change* probability as a function of the relative probability of the forecasting rules of the two groups and the *risk* associated with their use.<sup>8</sup>

However, in the actual theoretical framework, these possible model extensions are left for future research, assuming for simplicity that the composition of the two groups of traders in the FX markets, the chartists and the fundamentalists, is given. The fundamentalists will primarily orient their nominal exchange rate expectations towards levels which would be consistent with the equilibrium real exchange rate. In contrast, the chartists will act in a rather destabilizing manner, paying attention solely to the past patterns of the nominal exchange rate.

Using the notation of Manzan and Westerhoff (2007), we assume that the log of the nominal exchange expected by the fundamentalists is determined by

$$E_t^f(s_{t+1}) = s_t + \beta_s^f(f_t - s_t) \quad (11)$$

where  $f_t$  represents the value of the macroeconomic fundamentals at time  $t$ . By rearranging this equation we obtain for the nominal exchange depreciation rate expected by the fundamentalists

$$E_t^f(s_{t+1} - s_t) = \beta_s^f(f_t - s_t). \quad (12)$$

Now, as usually done in the literature, we assume that the macroeconomic variables which serve as reference for the fundamentalists is the PPP postulate, that is

$$f_t = \ln(p_t) - \ln(p_t^f) \quad (13)$$

with  $p_t$  and  $p_t^f$  denoting the price levels in the domestic and foreign economies, respectively. Inserting this expression in eq. (12) delivers

$$E_t^f(s_{t+1} - s_t) = \beta_s^f(\ln(p_t) - \ln(p_t^f) - s_t) \quad (14)$$

$$= \beta_s^f(-\eta_t) \quad (15)$$

with  $\eta_t$  as the log of the real exchange rate at time  $t$ .

Concerning the second group of traders, the chartists, I assume that their expected nominal exchange rate depreciation for  $t + 1$  is simply determined by

$$E_t^c(s_{t+1} - s_t) = \beta_s^c(s_t - s_{t-1}). \quad (16)$$

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<sup>8</sup>See Samanidou, Zschischang, Stauffer and Lux (2007) for a comprehensive survey article on this strain of research.

Assuming that the factual nominal exchange rate depreciation rate is determined by

$$s_{t+1} - s_t = i^f - i + \lambda(\beta_s^f(-\eta_t)) + (1 - \lambda)\beta_s^c(s_t - s_{t-1}) \quad (17)$$

where the relative influence of both groups is represented by the factor  $\lambda$ , and taking the continuous time approximation for  $s_{t+1} - s_t$ , we obtain

$$\dot{s} = i^f - i - \lambda(\beta_s^f\eta) + (1 - \lambda)\beta_s^c\dot{s}_t, \quad (18)$$

which, after some manipulation, delivers

$$\dot{s} = \frac{i^f - i - \lambda\beta_s^f\eta}{1 - (1 - \lambda)\beta_s^c}. \quad (19)$$

Eq.(19) clearly shows that for

$$\beta_s^c > \frac{1}{1 - \lambda}$$

we have explosive nominal exchange rate dynamics, while otherwise they are intrinsically stable as a reverting process towards the steady state, PPP consistent nominal exchange rate level.

This law of motion, together with the price inflation adjustment equations for the domestic and the foreign economies deliver

$$\begin{aligned} \dot{\eta} &= \dot{s} + \hat{p}^f - \hat{p} \\ &= \frac{i^f - i - \lambda\beta_s^f\eta}{1 - (1 - \lambda)\beta_s^c} + \hat{p}^f - \hat{p}. \end{aligned} \quad (20)$$

Note that in this formulation the real exchange rate dynamics are determined by a wide variety of macroeconomic factors, as well as by the composition of fundamentalists and traders in the foreign exchange markets. Note that through the effect of  $\eta$  on  $u$ , the real exchange rate acts intrinsically in a stabilizing manner. This nevertheless might, at the end, not hold if the influence of the chartists in the foreign exchange markets is predominant.

Altogether the model of this section consists of the following six laws of motion (with the derived reduced form expressions as far as the wage-price spiral is concerned):<sup>9</sup>

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<sup>9</sup>As the model is formulated we have no real anchor for the steady state rate of interest and thus have to assume here that it is the monetary authority that enforces a certain steady state values for the nominal rate of interest.

### The One-Country Sub-Module

$$\begin{aligned} \hat{u} \quad \text{Dynamic IS} & \quad -\alpha_{uu}(u - u_o) - \alpha_{ur}(i - \hat{p} - (i_o - \pi_o)) \\ & \quad -\alpha_{uv}(v - v_o) + \alpha_{u\eta}\eta + \alpha_{uuf}\hat{u}^f \end{aligned} \quad (21)$$

$$\hat{e} \quad \text{Okun's Law} \quad \alpha_{e\hat{u}}\hat{u} - \alpha_{ev}(v - v_o) \quad (22)$$

$$\begin{aligned} \hat{v} \quad \text{Wage Share} & \quad \kappa[(1 - \kappa_{pw})(\beta_{we}(e - e_o) - \beta_{wv} \ln(v/v_o)) \\ & \quad - (1 - \kappa_{wp})(\beta_{pu}(u - u_o) + \beta_{pv} \ln(v/v_o)) + \delta g_z], \end{aligned} \quad (23)$$

with  $\delta = (\kappa_{wz} - 1)(1 - \kappa_{pw})$

$$\dot{\pi}_c \quad \text{CPI Climate} \quad \beta_{\pi_c}(\hat{p}_c - \pi_c), \quad \hat{p}_c = \gamma\hat{p} + (1 - \gamma)(\dot{s} + \hat{p}^f) \quad (24)$$

$$\dot{i} \quad \text{Taylor Rule} \quad -\gamma_{ii}(i - i_o) + \gamma_{ip}(\hat{p} - \pi_o) + \gamma_{iu}(u - u_o) \quad (25)$$

$$\dot{\eta} \quad \text{Real Exchange} \quad \frac{i^f - i - \lambda\beta_s^f\eta}{1 - (1 - \lambda)\beta_s^c} + \hat{p}^f - \hat{p}. \quad (26)$$

The above equations represent, in comparison to the baseline model of New Keynesian macroeconomics, the IS goods market dynamics, here augmented by Okun's Law as link between the goods and the labor market, the Taylor Rule, a law of motion for the wage share  $v$  that makes use of the same explaining variables as in the New Keynesian model with both staggered prices and wages,<sup>10</sup> and the law of motion that describes the updating of the inflationary climate expression. In addition, we have to make use of the reduced form expression for the price inflation rate or the price PC,

$$\hat{p} = \kappa[\beta_{pu}(u - u_o) + \beta_{pv} \ln(v/v_o) + \kappa_{pw}(\beta_{we}(e - e_o) - \beta_{wv} \ln(v/v_o))] + \pi_c \quad (27)$$

which has to be inserted into the above laws of motion in various places in order to get an autonomous nonlinear system of differential equations in the state variables: capacity utilization  $u$ , the rate of employment  $e$ , the nominal rate of interest  $i$ , the wage share  $v$ , and the inflationary climate expression  $\pi_c$ . Eq.(27) could be considered as a sixth law of motion of the considered dynamics which however – when added – leads a system determinant which is zero, allowing therefore for zero-root hysteresis for certain variables of the model. The laws of motion have been written in an order that first presents the dynamic equations also present in the baseline New Keynesian model of inflation dynamics, and then our formulation of the dynamics of income distribution and of the inflationary climate in which the economy is operating.

<sup>10</sup>but with inflation rates  $\hat{p}, \hat{w}$  in place of their time rates of change and with no accompanying sign reversal concerning the influence of output and wage gaps

In sum, therefore, our dynamic AS-AD growth model exhibits a variety of features that are much more in line with a Keynesian understanding of the characteristics of the trade cycle than is the case for the conventional modeling of AS-AD growth dynamics or its radical reformulation by the New Keynesians.

## 2.5 Local Stability Analysis: The Small-Open Economy Case

We start our analysis of the stability properties of the system with the small-open economy case, assuming that the foreign economy is and remains at its steady state level ( $u^f = u_o^f$ ,  $e^f = e_o^f$ ,  $v^f = v_o^f$ ). Since we assume the same structure for both economies, the local stability of one subsystem would imply the same for the other subsystem, assuming that similar parameter dimensions. Due to its specific formulation, the steady state of the 5D subdynamics can be supplied exogenously. It exhibits five gaps, to be closed in the steady state and has five laws of motion, which when set equal to zero, exactly imply this result.

In order to investigate the role of heterogenous expectations in the foreign exchange market and the main international transmission channels for the stability of the whole macroeconomic system analytically, the dimensions of our theoretical framework are reduced through the following simplifying assumptions:

- The monetary authorities do not pursue an interest rate smoothing strategy, so that  $i = i_T$  always holds. This is the case when  $\alpha_{ii} \rightarrow \infty$ .
- $\beta_{\pi_c} = 0$ . In this case the inflationary climate is constant ( $\pi_c = 0$  is assumed).
- We can replace  $e$  through  $\alpha_{eu}u$  in the wage and price inflation adjustment equations without loss of generality.

Under the simplifying assumptions, the initial 5D dynamical system can be reduced to the following 3D subsystem

$$\begin{aligned} \hat{u} &= -\alpha_{uu}(u - u_o) - \alpha_{ur}((i_o - \pi_o) + \phi_\pi(\hat{p} - \pi_o) + \phi_y(u - u_o)) \\ &\quad - \alpha_{uv}(v - v_o) + \alpha_{u\eta}\eta \end{aligned} \quad (28)$$

$$\begin{aligned} \hat{v} &= \kappa[(1 - \kappa_{pw})(\beta_{we}(\alpha_{eu}u - e_o) - \beta_{wv} \ln(v/v_o)) \\ &\quad - (1 - \kappa_{wp})(\beta_{pu}(u - u_o) + \beta_{pv} \ln(v/v_o)) + \delta g_z], \end{aligned} \quad (29)$$

$$\dot{\eta} = i_o^f - (i_o + (1 + \phi_\pi)(\hat{p} - \pi_o) + \phi_y(u - u_o)) - \frac{\lambda\beta_s^f\eta}{1 - (1 - \lambda)\beta_s^c} - \hat{p}. \quad (30)$$

with

$$\hat{p} = \kappa[\beta_{pu}(u - u_o) + \beta_{pv} \ln(v/v_o) + \kappa_{pw}(\beta_{we}(\alpha_{eu}u - u_o) - \beta_{wv} \ln(v/v_o))] \quad (31)$$

to be inserted in several places.

The corresponding Jacobian of this reduced 3D subsystem

$$J_{3D} = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix}.$$

with

$$J_{11} = \frac{\partial \hat{u}}{\partial u} = -\alpha_{uu} - \alpha_{ur}(\phi_y + (1 + \phi_\pi)\kappa(\beta_{pu} + \kappa_{pw}\beta_{we}\alpha_{eu})) < 0 \quad (32)$$

$$J_{12} = \frac{\partial \hat{u}}{\partial v} = -\alpha_{uv} - \alpha_{ur}\phi_\pi\kappa\left(\frac{\beta_{pv} - \kappa_{pw}\beta_{wv}}{v_o}\right) < 0 \quad (33)$$

$$J_{13} = \frac{\partial \hat{u}}{\partial \eta} = \alpha_{u\eta} > 0 \quad (34)$$

$$J_{21} = \frac{\partial \hat{v}}{\partial u} = \kappa((1 - \kappa_{pw})\beta_{we}\alpha_{eu} - (1 - \kappa_{wp})\beta_{pu}) \quad (35)$$

$$J_{22} = \frac{\partial \hat{v}}{\partial v} = -\kappa\left(\frac{(1 - \kappa_{pw})\beta_{wv} + (1 - \kappa_{wp})\beta_{pv}}{v_o}\right) < 0 \quad (36)$$

$$J_{23} = \frac{\partial \hat{v}}{\partial \eta} = 0 \quad (37)$$

$$J_{31} = \frac{\partial \hat{\eta}}{\partial u} = \frac{\lambda\beta_f^s(\alpha_{ur} + (1 + \phi_\pi)\kappa(\beta_{pu} + \kappa_{pw}\beta_{we}\alpha_{eu}))}{1 - \beta_c^s(1 - \lambda)} \quad (38)$$

$$J_{32} = \frac{\partial \hat{\eta}}{\partial v} = \frac{\beta_f^s(1 + \phi_\pi)\kappa\lambda\left(\frac{\beta_{pv} - \beta_{wv}\kappa_{pw}}{v_o}\right)}{1 - \beta_c^s(1 - \lambda)} - \kappa\left(\frac{\beta_{pv} - \beta_{wv}\kappa_{pw}}{v_o}\right) \quad (39)$$

$$J_{33} = \frac{\partial \hat{\eta}}{\partial \eta} = -\frac{\beta_f^s\lambda}{1 - \beta_c^s(1 - \lambda)} < 0 \quad (40)$$

has the following sign structure

$$J_{3D} = \begin{bmatrix} - & ? & + \\ ? & - & 0 \\ ? & ? & ? \end{bmatrix}.$$

According to the Routh-Hurwitz stability conditions for a 3D dynamical system, asymptotic local stability of a steady state is fulfilled when

$$a_i > 0, \quad i = 1, 2, 3 \quad \text{and} \quad a_1a_2 - a_3 > 0,$$

where  $a_1 = -\text{trace}(J)$ ,  $a_2 = \sum_{k=1}^3 J_k$  with

$$J_1 = \begin{vmatrix} J_{22} & J_{23} \\ J_{32} & J_{33} \end{vmatrix}, J_2 = \begin{vmatrix} J_{11} & J_{13} \\ J_{31} & J_{33} \end{vmatrix}, J_3 = \begin{vmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{vmatrix}.$$

and  $a_3 = -\det(J)$ .

Our reduced 3D dynamical system is stable around its interior steady state, if the following proposition is fulfilled:

**Proposition 1:**

*Assume that the influence of the chartists in the foreign exchange market is not predominant, i.e., that (i)  $\beta_c^s(1 - \lambda) < 1$  holds. Additionally, assume that (ii)  $(\lambda\beta_s^f(1 + \phi_\pi))/(1 - \beta_c^s(1 - \lambda)) < 1$ . Furthermore, assume that (iii)  $\beta_{we}\alpha_{eu} > \beta_{pu}$ , that is, that wage inflation reacts more strongly to changes in capacity utilization than price inflation. And finally, assume that (iv)  $\kappa_{pw}$  is of a sufficiently small dimension, so that  $\partial\hat{u}/\partial v < 0$  and  $\partial\dot{\eta}/\partial v < 0$  hold.*

*Then: The Routh-Hurwitz conditions are fulfilled and the unique steady state of the reduced 3D dynamical system is locally asymptotic stable.*

**Sketch of Proof:**

As already state, as long as the influence of the chartists in the foreign exchange market is not predominant, the dynamics of the nominal exchange rate are asymptotically stable. In this case  $\partial\dot{\eta}/\partial\eta < 0$ , and the trace of  $J$  is unambiguously negative (and  $a_1 > 0$  holds), since

$$\text{tr}(J) = J_{11} + J_{22} + J_{33} < 0. \tag{41}$$

if condition (i) in Proposition 1 is fulfilled.

Condition (ii) ensures the partial derivative of  $\dot{\eta}$  with respect to  $u$  to be negative and, together with condition (iv), that  $\partial\dot{\eta}/\partial v < 0$ . Condition (iii) ensures that

$\partial \hat{v} / \partial u$ . In this case,  $J_1$ ,  $J_2$  and  $J_3$ , the second-order minors of  $J$  are given by

$$\begin{aligned} J_1 &= J_{22} \cdot J_{33} - J_{32} \cdot J_{23} \\ &= \frac{\beta_f^s \lambda \kappa (\beta_{wv}(1 - \kappa_{pw}) + \beta_{pv}(1 - \kappa_{wp}))}{v(1 - \beta_c^s(1 - \lambda))} > 0 \end{aligned} \quad (42)$$

$$\begin{aligned} J_2 &= J_{11} \cdot J_{33} - J_{31} \cdot J_{13} \\ &= \frac{\lambda \beta_f^s [\alpha_{uu} + (\alpha_{ur} + \alpha_{u\eta})(\phi_y + \phi_\pi \kappa (\beta_{pu} + \kappa_{pw} \beta_{we} \alpha_{eu}))]}{(1 - \beta_c^s(1 - \lambda))} \\ &\quad + \alpha_{u\eta} \kappa (\beta_{pu} + \alpha_{eu} \beta_{we} \kappa_{pw}) \end{aligned} \quad (43)$$

$$\begin{aligned} J_3 &= J_{11} \cdot J_{22} - J_{21} \cdot J_{12} \\ &= [\alpha_{uv} + \alpha_{ur}(\phi_\pi \kappa (\beta_{pv} - \beta_{wv} \kappa_{pw}) + \phi_y)] \kappa \left( \frac{\beta_{wv}(1 - \kappa_{pw}) + \beta_{pv}(1 - \kappa_{wp})}{v_o} \right) \\ &\quad + \kappa (\alpha_{eu} \beta_{we} (1 - \kappa_{pw}) - \beta_{pu} (1 - \kappa_{wp})) \left[ \alpha_{uu} + \alpha_{ur} \phi_\pi \kappa \left( \frac{\beta_{pu} + \alpha_{eu} \beta_{we} \kappa_{pw}}{v_o} \right) \right] \\ &> 0 \end{aligned} \quad (44)$$

If conditions (i)-(iv) hold, the sign structure of the Jacobian matrix is given by

$$J_{3D} = \begin{bmatrix} - & - & + \\ + & - & 0 \\ - & - & - \end{bmatrix}.$$

It can be easily confirmed that under such a sign structure,  $a_2 = \sum_{k=1}^3 J_k > 0$  and  $a_3 = -\det(J) > 0$ , as well as the critical condition  $a_1 a_2 - a_3 > 0$  for local asymptotic stability of the steady state of the system hold under the assumed parameter dimensions.

Concerning the determinant of  $J$ , from the sign structure of the 3D Jacobian it can be easily seen that it is negative, so that  $a_3 = -\det(J) > 0$ .

Concerning the local asymptotic stability properties of the 6D subsystem, we can infer without an analytical proof that it will lose stability if a) the conditions (i)-(iv) in Proposition 1 are no longer fulfilled, b) the adjustment speed of the inflationary climate  $\beta_{\pi c}$  approaches infinity or c) the nominal interest rate does not adjust sufficiently fast to the target rate pursued by the monetary authorities, that is, when the interest rate smoothing parameter  $\alpha_{ii}$  is insufficiently low.

### 3 The Two-Country Framework: Estimation and Analysis

After having set up the basic structure of an open-economy framework of Keynesian nature, in this section we integrate two economies (and therefore, two small-open-



economy dynamic models if considered separately) with similar characteristics (as the U.S. and the euro area) into a consistent two country framework.

Considering both economies as a single macroeconomic framework, the resulting 11D dynamical system comprises 11 dynamic variables with the gaps

$$u - u_o, \quad e - e_o, \quad v - v_o, \quad i - i_o, \quad \hat{p} - \pi_o, \quad \eta - \eta_o,$$

plus the five ones for the foreign economy that correspond to the first (domestic) five of the list shown above.

For the unique determination of the steady state position we set  $\hat{u}, \hat{e}, \hat{v}, \dot{i}$  equal to zero (and of course have the same situation for the foreign economy). This holds only when all gaps are zero simultaneously, what additionally delivers (for  $\eta = \eta_o = 0$ )  $\dot{s} = 0$ .

Assuming a constant steady state log nominal exchange rate  $s$ , we obtain from the reduced form price Phillips curves

$$\begin{aligned} \hat{p}_o &= \pi_{co} = \xi \pi_{co} + (1 - \xi) \pi_{co}^f \\ \hat{p}_o^f &= \pi_{co}^f = \xi^f \pi_{co}^f + (1 - \xi^f) \pi_{co} \iff \\ \pi_c &= \pi_c^f. \end{aligned}$$

By inserting again eq.(3) and its foreign economy counterpart, we obtain

$$\xi \hat{p}_o + (1 - \xi) \hat{p}_o^f = \xi^f \hat{p}_o + (1 - \xi^f) \hat{p}_o^f$$

which only holds true for  $\hat{p} = \hat{p}^f$ . At the steady state, thus, both countries share the same inflationary climate and equilibrium inflation rate, independently of the actual composition of the CPI index in both economies. Under this condition, the nominal exchange rate equation (19) delivers indeed a constant nominal exchange rate at the steady state, and therefore also a constant real exchange rate, since  $\eta = \eta_o$ .

The structure of the model is summarized in Figure 1. This figure shows at its top the interaction of the foreign exchange market with the two economies and towards the bottom the interaction of both economies through their goods markets. As this diagrammatic exposition of quantity and price trade channels linking the two economies shows, the macroeconomic interaction between them seems intrinsically stable with the sole obvious source of instability (or even chaos) is from the foreign exchange markets. Indeed, in the absence of predominant unstable nominal exchange rate dynamics (which would occur if the FX market would be governed

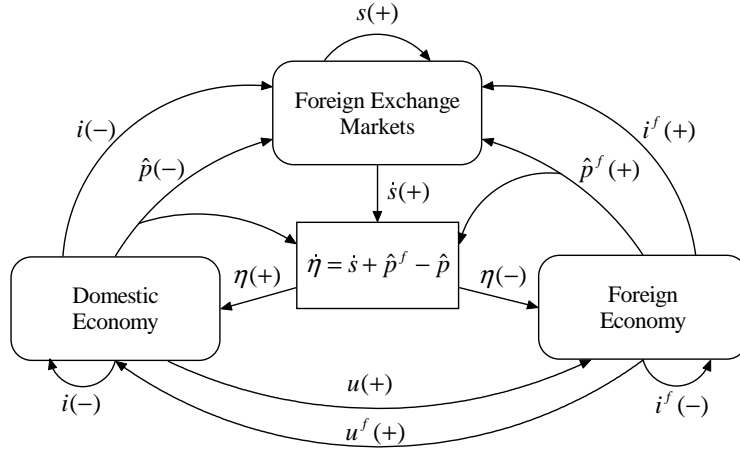


Figure 1: The real and financial links of the two country model

by the chartists), the dynamics of the two-country framework seem to be of a self-regulating nature through the interaction of quantity and price trade linkages. This, however, is not necessarily the case: So for example leads an exogenous increase in the foreign demand ( $u^f \uparrow$ ) on the one hand to an increase of price and (through the related increase in foreign employment) wage inflation abroad, which in turn leads to a loss of competitiveness ( $\eta \uparrow$ ) and to a cooling down of the foreign economy. On the other hand, an increase in  $u^f$  leads (through the “locomotive” effect) to an increase in the domestic level of economic activity, to an increase in domestic wage and price inflation and subsequently to a fall of  $\eta$ , which, in turn, is likely to boost furthermore the economic activity abroad. The net effect of these two opposite effects – and therefore the stability of the system – depends thus to an important degree on the degree of wage and price flexibility in both economies. However, since a throughout analytical calculation of the Routh-Hurwitz local stability conditions for a 11D system would be an extremely complicated and, more importantly, nontransparent task, the stability of the 11D system concerning variations of the structural parameters will be investigated in a numerical manner by means of eigen-value analysis in section 3.3.

### 3.1 Structural Estimation Results

The empirical data of the corresponding time series stem from the Federal Reserve Bank of St. Louis data set (see <http://www.stls.frb.org/fred>) and the OECD database for the U.S. and the euro area, respectively. The data is quarterly, seasonally adjusted

and concern the period from 1980:1 to 2004:4.

Table 1: Data Set

Variable	Description of the original series
$e$	U.S. : Employment rate Euro area : Employment Rate (HP cyclical component, $\lambda = 640000$ )
$u$	U.S. : Capacity utilization: Manufacturing, percent of capacity Euro area : Output Gap
$w$	U.S. : Nonfarm Business Sector: Compensation per hour, 1992=100 Euro area : Business sector: Wage Rate Per Hour,
$p$	U.S. : Gross Domestic Product: Implicit Price Deflator, 1996=100 Euro area : Gross Domestic Product: Implicit Price Deflator, 2000=100
$z$	U.S. : Nonfarm Business Sector; Output per hour of all persons, 1992=100 Euro area : Labor Productivity of the business economy,
$v$	U.S. : Nonfarm Business Sector: Real compensation per output unit, 1992=100 Euro area : Business Sector: Real compensation per output unit (HP cyclical component, $\lambda = 640000$ )
$i$	U.S. : Federal Funds rate Euro Area : Short-term interest rate
$s$	EUR/USD Nominal exchange rate

The logarithms of wages and prices are denoted  $\ln(w_t)$  and  $\ln(p_t)$ , respectively. Their first differences (backwardly dated), i.e. the current rate of wage and price inflation, are denoted  $\hat{w}_t$  and  $\hat{p}_t$  as in the theoretical framework. The inflationary climate  $\pi_c$  of the theoretical part of this paper is approximated here in a very simple way, by a linearly declining moving average of price inflation rates with linearly decreasing weights over the past twelve quarters, denoted  $\pi_t^{12}$ .<sup>11</sup> Figure 2 shows the time series of both the U.S. and the Euro Area described in Table 1. As it can be observed there, the U.S. and the Euro Area have featured in the last two decades a remarkable similarity in their respective wage and price inflation developments, as well as – to a somewhat lesser extent – in the dynamics of the capacity utilization and the output gap, respectively.

This, however, does not hold for the dynamics of the employment rate and the wage share of both economies. As it can be observed in Figure 2, while the U.S. unemployment rate has fluctuated, roughly speaking, around a constant level over the last two decades, the European employment (unemployment) rate displayed a

<sup>11</sup>We also estimated the structural model shown in Table 3 with other proxies for the inflationary climate which also covered the four, six and eighteen last quarters. These estimates could be rejected even at the 10% significance level.

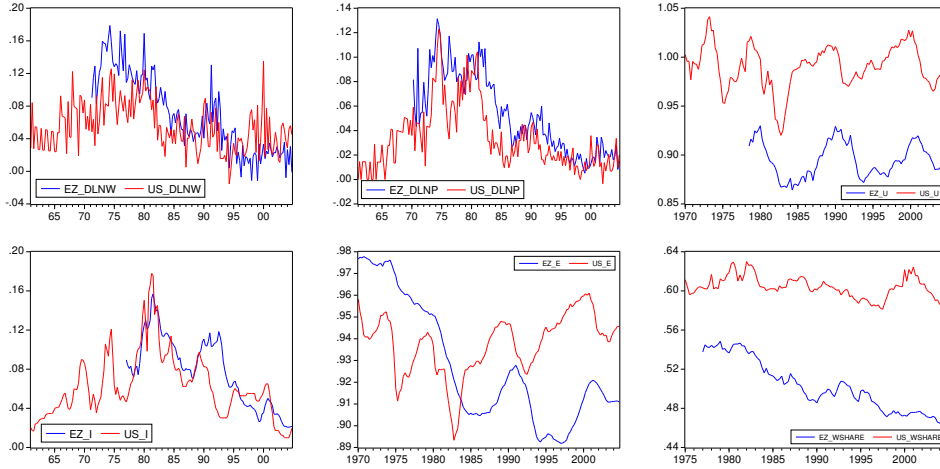


Figure 2: U.S. and Euro Area Aggregate Time Series

persistent downwards (upwards) trend over the same time period. This particular European development has been explained by Layard, Nickell and Jackman (1991) and Ljungqvist and Sargent (1998) by an over-proportional increase in the number of long-term unemployed (i.e. workers with an unemployment duration over 12 months) with respect to short term unemployed (workers with an unemployment duration of less than 12 months) and the phenomenon of hysteresis especially in the first group. Because the long-term unemployed become less relevant in the determination of nominal wages (since primarily the short-term unemployed are taken into account), the potential downward pressure on wages resulting from the unemployment of the former diminishes, with the result of a higher NAIRU level, see Blanchard and Wolfers (2000). When long-term unemployment is high, the aggregate unemployment rate of an economy thus, “becomes a poor indicator of effective labor supply, and the macroeconomic adjustment mechanisms – such as downward pressure on wages and inflation when unemployment is high – will then not operate effectively” (OECD (2002, p.189)).

In order to take into account the lower influence of the long-term unemployed in the determination of wage and price inflation, we proceed as in Proaño (2008) and use, for the euro area, the adjusted cyclical component of the unemployment rate as a proxy for the short-term unemployment, since time series data for long-term unemployment in the euro area are not available for the analyzed sample period.

In order to check the stationarity of the analyzed time series, Phillips-Perron unit root tests were carried out in order to account for residual autocorrelation (as done

Table 2: Phillips-Perron Unit Root Test Results. Sample: 1980:1-2004:4

Country	Variable	Lag Length	Determ.	Adj. Test Stat.	Prob.*
U.S.	$\hat{w}$	-	const.	-6.7769	0.0000
	$\hat{p}$	-	const.	-2.7647	0.0671
	$\hat{u}$	-	-	-7.0655	0.0000
	$\hat{e}$	-	-	-4.8206	0.0000
	$i$	-	-	-1.8553	0.0608
euro area	$\hat{w}$	-	const.	-3.4982	0.0100
	$\hat{p}$	-	none	-2.3617	0.0183
	$\hat{u}$	-	const.	-8.0891	0.0000
	$\hat{e}$	-	-	-3.1516	0.0019
	$i$	-	-	-1.4810	0.1290

\*McKinnon (1996) one-sided p-values.

by the standard ADF Tests), and also for possible residual heteroskedasticity. The Phillips-Perron test specifications and results are shown in Table 2.

The applied unit root tests reject the hypothesis of a unit root for all series with exception of the euro area nominal interest rate  $i$ . Although the test cannot reject the null of a unit root, there is no reason to expect this time series to possess a unit root. We reasonably expect these rates to be constrained to certain limited ranges in the euro area. Due to the general low power of the unit root tests, these results can be interpreted as only providing a hint that the nominal interest exhibit a strong autocorrelation.

As discussed in the previous section, the law of motion for the real wage rate, given by eq.(7), represents a reduced form expression of the two structural equations for  $\hat{w}_t$  and  $\hat{p}_t$ . Noting again that the inflation climate variable is defined in the estimated model as a linearly declining function of the past twelve price inflation rates, the dynamics of the system (21)–(24) can be reformulated as

$$\begin{aligned}
 \hat{w}_t^j &= \beta_{we}(e_{t-1}^j - e_o^j) - \beta_{wv} \ln(v_{t-1}^j/v_o^j) + \kappa_{wp}\hat{p}_t^j + \kappa_{w\pi^{12}}\pi_t^{12,j} + \kappa_{wz}\hat{z}_t^j + \epsilon_{wt} \\
 \hat{p}_t^j &= \beta_{pu}(u_{t-1}^j - u_o^j) + \beta_{pv} \ln(v_{t-1}^j/v_o^j) + \kappa_{pw}(\hat{w}_t^j - \hat{z}_t^j) + \kappa_{p\pi^{12}}\pi_t^{12,j} + \epsilon_{pt} \\
 \ln u_t^j &= \ln u_{t-1}^j + \gamma_u(u_{t-1}^j - u_o^j) - \alpha_{ur}(i_{t-1}^j - \hat{p}_t^j) \pm \alpha_{uv}(v_t^j - v_o^j) + \alpha_{u\eta}\eta_{t-4} + \epsilon_{ut} \\
 \hat{e}_t^j &= \alpha_{eu-1}\hat{u}_{t-1}^j + \alpha_{eu-2}\hat{u}_{t-2}^j + \alpha_{eu-3}\hat{u}_{t-3}^j + \epsilon_{et} \\
 i_t^j &= \phi_i i_{t-1}^j + (1 - \phi_i)\phi_\pi \hat{p}_t^j + (1 - \phi_i)\phi_y u_{t-1}^j + \epsilon_{it}, \quad \text{with } j = us, ez. \\
 s_t &= i_{t-1}^{us} - i_{t-1}^{ez} + \alpha_{ss}s_{t-1} - \lambda\beta_s^f \eta_t + (1 - \lambda)\beta_s^c \hat{s}_{t-1}.
 \end{aligned}$$

with  $\gamma_{uu} = 1 - \alpha_u$  and sample means denoted by a subscript  $o$ .

In order to account for a possible regressor endogeneity and heteroskedasticity, we estimate the two-country system comprising the U.S. and the euro area by means of Three-Stage-Least-Squares (3SLS).

Table 3: 3SLS Parameter Estimates: Two-Country System

Estimation Sample: 1980 : 1-2004 : 4							
$\hat{w}_t$	$\beta_{we}$	$\beta_{wv}$	$\kappa_{wp}$	$\kappa_{w\pi^{12}}$	$\kappa_{wz}$	$\bar{R}^2$	DW
Euro Area	0.475	-0.437	0.862	0.270	0.239	0.706	1.603
	[2.664]	[-3.794]	[3.560]	[1.173]	[2.896]		
U.S.	0.689	-0.399	0.490	0.574	0.348	0.341	1.815
	[4.341]	[-3.033]	[1.827]	[3.072]	[5.059]		
$\hat{p}_t$	$\beta_{pu}$	$\beta_{pv}$	$\kappa_{pw}$	$\kappa_{p\pi^{12}}$		$\bar{R}^2$	DW
Euro Area	0.252	0.114	0.073	0.877		0.898	1.520
	[4.391]	[2.138]	[1.897]	[24.356]			
U.S.	0.130	0.144	0.109	0.579		0.788	1.389
	[2.748]	[2.550]	[3.243]	[19.274]			
$\ln u_t$	$\gamma_{uu-1}$	$\alpha_{ur}$	$\alpha_{uv}$	$\alpha_{u\eta}$	$\alpha_{uuf}$	$\bar{R}^2$	DW
Euro Area	-0.109	-0.062	-0.163	0.012	0.101	0.927	1.753
	[-3.416]	[-3.184]	[-2.724]	[2.353]	[1.531]		
U.S.	-0.091	-0.038	-0.112	-0.008	0.152	0.904	1.520
	[-3.387]	[-1.829]	[-2.088]	[-1.308]	[1.545]		
$\hat{e}$	$\alpha_{eu-1}$	$\alpha_{eu-2}$	$\alpha_{eu-3}$			$\bar{R}^2$	DW
Euro Area	0.135	0.128	0.074			0.615	1.114
	[7.176]	[6.864]	[4.110]				
U.S.	0.151	0.101	0.045			0.369	1.436
	[5.250]	[3.439]	[1.575]				
$i$	$\phi_i$	$\phi_{ip}$	$\phi_{iu}$			$\bar{R}^2$	DW
Euro Area	0.931	1.562	1.925			0.981	1.366
	[49.645]	[10.653]	[2.915]				
U.S.	0.823	2.168	0.404			0.928	1.892
	[31.659]	[15.328]	[1.892]				
$s$	$\alpha_{ss}$	$\lambda$	$\alpha_{s\hat{s}}$			$\bar{R}^2$	DW
	0.834	0.115	0.232			0.917	1.608
	[19.110]	[1.978]	[2.771]				

As it can be observed in Table 3, we find a wide support for the theoretical formulation discussed in the previous section. In the first place we find similar and statistically significant coefficients for  $\ln(v/v_o)$ , the Blanchard-Katz error correction terms, in both the wage and price adjustment equations of both the U.S. and the euro area. In the second place, our cross-over formulation of the inflationary expectations cannot be statistically rejected in the wage inflation equation of both economies and

the U.S. price inflation equation. For the euro area, the cross over term of actual wage inflation determining the actual price inflation seems not to be significant.<sup>12</sup> Furthermore, the influence of the market specific demand pressure terms (the capacity utilization in the price- and the employment rate in the wage Phillips curve equations) is also corroborated by our estimations, as well as the fact that wage flexibility is higher than price flexibility (concerning their respective demand pressure measures) in both the U.S. and the Euro Area, a result in line with the findings of Chen and Flaschel (2006) and Flaschel, Kauermann and Semmler (2007).

Concerning the estimated open economy IS equation, our 3SLS estimations show, as expected, the negative influence of the expected real interest rate on the dynamics of capacity utilization in both economies. The same holds true for the effect of  $v - v_o$ , the deviation of the labor share from its steady state level, in both U.S. and Euro Area, showing that a relatively high labor share (or real average unit labor costs) has a negative impact on the domestic rate of capacity utilization, something that holds for a profit led economy. The coefficient  $\alpha_{uuf}$ , which represents the effect of foreign goods demand on the dynamics of the domestic capacity utilization rate, are both positive and significant (with the U.S. coefficient of an unexpectedly high value, if one takes into account that the U.S. influence is probably higher for the euro area in this respect than otherwise) for both economies.<sup>13</sup>

The parameter estimates in the dynamic Okun's law and Taylor rule equations of both economies are positive, statistically significant and of reasonable dimension, with nevertheless a higher reaction coefficient to the inflation in the U.S. than in the euro area for the analyzed sample period. Concerning the law of motion of the log nominal exchange rate, both the log real exchange rate and the interest rate differential influence the level of the log nominal exchange rate, the former in a negative and the latter in a positive manner.

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<sup>12</sup>In Proaño (2008), where the closed economy dimension of this theoretical model was investigated, almost identical results concerning both economies are reported. The only exception was the cross-over term (specified there not as  $\hat{w}_t - \hat{z}_t$  - with  $\hat{z}_t$  as the actual labor productivity growth rate, but only as  $\hat{w}$ ) in the price inflation equation for the Euro Area, which there, turned out to be statistically significant.

<sup>13</sup>In the dynamic adjustments simulations of the next section we will calibrate this coefficient to be if not *lower*, at least *equal* to that of the euro area.

### 3.2 Dynamic Adjustments

In order to evaluate the empirical plausibility of our theoretical framework, we simulate an approximate discrete time version of the semi-structural model discussed in section 2 based on the estimated 3SLS structural model parameters discussed in the last section.<sup>14</sup> Additionally, the parameters concerning the theoretical CPI inflationary climate for both countries were calibrated with the following values:

$$\beta_{\pi_c} = 0.5 \quad \kappa_{\pi_c} = 0.5 \quad \xi = 0.85.$$

Both countries have thus the same degree of inflation climate inertia (represented by  $\beta_{\pi_c}$ , the adjustment coefficient of the CPI inflationary climate), whereafter each new (monthly) CPI inflation rate observation updates with only a 0.5 weight the inflationary climate. Both countries have also the same degree of credibility in the monetary policy target ( $\kappa_{\pi_c}$ ) as well as the same composition of domestic and foreign goods in the CPI index.<sup>15</sup>

In Figure 3 the dynamic adjustments of two artificial economies based on the structural parameters estimates of both the U.S. and the euro area (depicted in table 3) to a hypothetic one percent (100 basis points) U.S. monetary policy shock are shown.

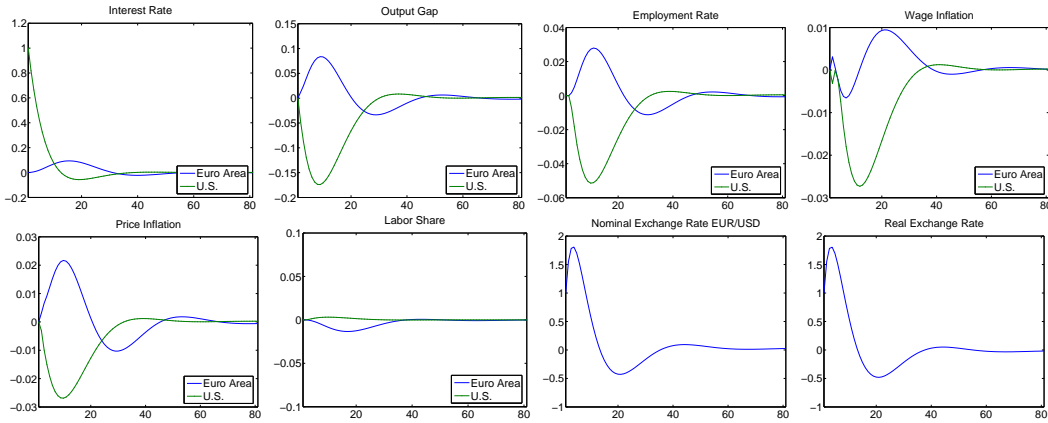


Figure 3: Simulated Impulse-Responses to a One Std. Dev. U.S. Monetary Policy Shock

As Figure 3 shows, the numerical simulations of the calibrated theoretical discrete time model deliver, to a large extent, the stylized facts on monetary policy shocks

<sup>14</sup>The numerical simulations of this section were performed using MATLAB. The simulation code is available upon request.

<sup>15</sup>This specific value is taken from Rabanal and Tuesta (2006).



discussed in the last section (the time axis shows the *months* after the shock). As expected, a positive monetary policy shock in the U.S. leads to an increase in the EUR/USD nominal exchange rate primarily via the uncovered interest rate parity (UIP) condition comprised in the law of motion of the log nominal exchange rate. This nominal appreciation of the US Dollar, together with the effect of the interest rate increase, leads to an economic slowdown, observable in the decrease in the capacity utilization rate (the output gap). Following the downturn of this variable, employment also falls, followed by wage and price inflation. These variables return back to baseline in approximately 40 months, or nearly four years. Concerning the reaction of the euro area variables to a U.S. monetary policy shocks, Figure 3 shows that the euro area is affected by the contractionary U.S. monetary policy shock through two macroeconomic channels: Firstly, by the drop in foreign aggregate demand and secondly, by the gain of relative competitiveness resulting from the nominal (and real, due to the sluggishness in the wage and price adjustment) depreciation of the euro. As figure 3 shows, in the calibrated two-country model underlying these simulations the increase in relative competitiveness of the euro area clearly dominates the fall in the U.S. foreign goods demand, with economic activity in the euro area increasing after a contractionary monetary shock in the U.S. economy.

Concerning the dynamic reaction of the nominal exchange rate, in the model discussed here, there is indeed a somewhat delayed reaction of this variable, with the maximum effect taking place not instantaneously but after one or two periods.

Next we focus on the stability properties of this two-country macrodynamic system concerning variations in the parameter values.

### 3.3 Eigen-Value-Based Stability Analysis

As previously mentioned, if the stability of a macrodynamic system is not simply imposed through the rational expectations assumption, the relative strength of the different macroeconomic channels interacting in an economy (and in this case, between two economies) become central for the local and global stability properties of the system analyzed.

The main purpose of this section is to highlight this issue within the semi-structural two-country macro-framework discussed and estimated in the previous section. For this an eigen-value stability analysis is used taking as the benchmark parameters the estimated values presented in the previous section. After calibrating the 11D continuous time system, the eigen-values of the system are calculated ceteris

paribus for different parameter of the models (mostly in the 0-1 interval) using the SND package discussed by Chiarella, Flaschel, Khomin and Zhu (2002).

In Figure 4 the maximal eigen-values of the system for varying values in the closed-economy- (the one-country submodule under  $\alpha_{uuf}$ ,  $\alpha_\eta = 0$ ,  $\xi = 1$ ) calculated with the U.S. parameter estimates of Proaño (2008) – shown in Table 4 – and in the open-economy cases are sketched.

Table 4: Closed-Economy Model – Calibration Parameters (Proaño (2008))

<b>Goods Markets</b>	$\gamma_{uu}$	$\alpha_{ur}$	$\alpha_{yv}$	
	0.077	0.042	-0.173	
<b>Labor Markets</b>	$\alpha_{eu1}$	$\alpha_{eu2}$	$\alpha_{eu3}$	$\alpha_{ev}$
	0.201	0.113	0.039	0.100
<b>Wage Phillips Curve</b>	$\beta_{we}$	$\beta_{wr}$	$\kappa_{wp}$	$1 - \kappa_{wp}$
	0.679	0.208	0.420	0.580
<b>Price Phillips Curve</b>	$\beta_{pe}$	$\beta_{pr}$	$\kappa_{pp}$	$1 - \kappa_{pp}$
	0.294	0.113	0.044	0.956
<b>Monetary Policy Rule</b>	$\alpha_{ii}$	$\phi_{ip}$	$\phi_{iu}$	
	0.830	2.17	0.423	

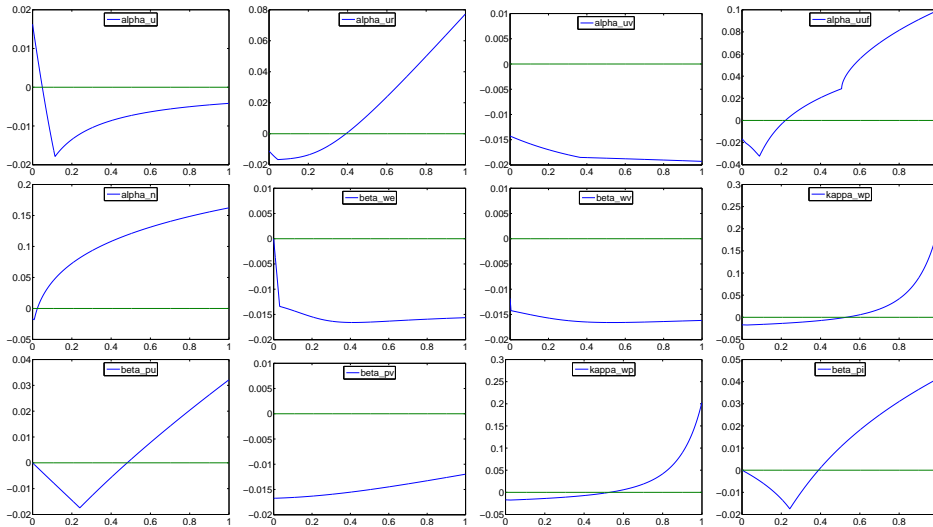


Figure 4: Eigen-Value-based Stability Analysis: The Real Economy

The comparison between Figures 4 and 5 reveals by and large the same qualitative implications of a variation of the analyzed coefficients for the stability of the system (in the two- and the one-country case): So, while the stability properties of the respective systems seem to be invariant for different parameters of  $\alpha_{uv}$  (the

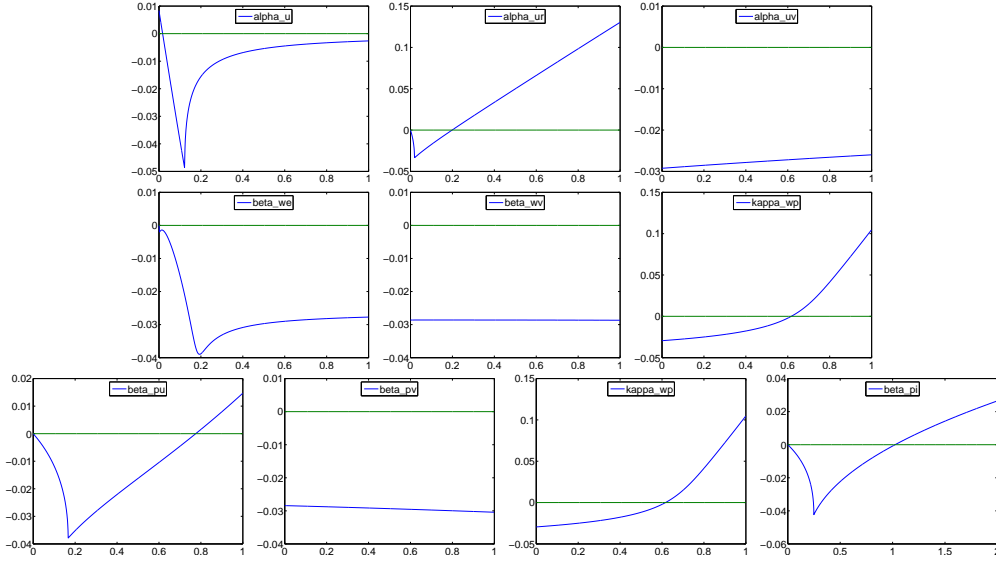


Figure 5: Eigen-Value-based Stability Analysis: The Real Economy

reaction strength of capacity utilization to an increase in the wage share),  $\beta_{we}$  (the wage inflation reactiveness parameter with respect to labor market disequilibrium situations), as well as  $\beta_{wv}$  and  $\beta_{pv}$  (the Blanchard-Katz error correction terms in both wage and price inflation adjustment equations), the same does not hold for the remaining real economy parameters. Indeed, high coefficients of  $\alpha_{ur}$  (the real interest rate reactiveness of the capacity utilization), both  $\kappa_{wp}$  and  $\kappa_{pw}$  (the cross-over inflation terms in the wage and price Phillips Curve equations), a high price flexibility with respect to goods market disequilibrium situations (represented by the parameter  $\beta_{pu}$ ) as well as as a high adjustment of the inflationary climate  $\pi_c$ , determined by  $\beta_{\pi_c}$  seem to induce instability in the system. Concerning the open-economy dimension of the model, Figure 4 shows that both a high reactiveness of capacity utilization towards the real exchange rate *and* the dynamics of the foreign economy (determined by  $\alpha_{\eta}$  and  $\alpha_{uuf}$ , respectively), are likely to induce instability of the system due to an eventual over-synchronization of both economies which might feature reinforcing properties.

Figure 6 shows the eigen-value diagrams resulting from variations in the monetary policy parameters. As expected, while an increase in the  $\alpha_{ii}$  (the adjustment speed of the actual nominal interest rate with respect to  $i_T$ ) induces stability into both the closed and the open economy systems, the steady state stability properties seem to be invariant to changes in  $\phi_y$  (the reaction coefficient of the monetary policy

instrument with respect to the output gap).

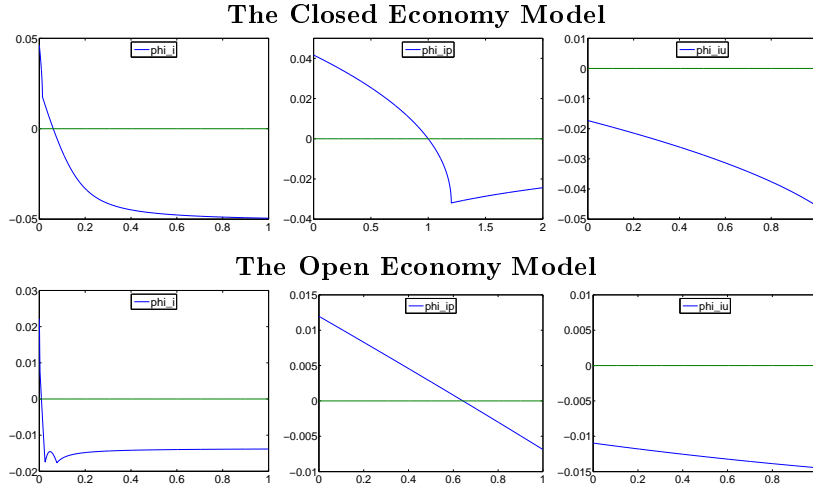


Figure 6: Eigen-Value-based Stability Analysis: Monetary Policy

This, however, does not hold for  $\phi_\pi$ , the reaction coefficient with respect to the inflation gap. Indeed, in line with the academic literature on monetary policy, we find in the closed economy case that the steady state of the economic system is stable only if  $\phi_\pi > 1$ , that is, only if monetary policy reacts in a sufficiently active manner with respect to inflationary developments. The eigen-value diagram of  $\phi_\pi$ , however, shows that the threshold value for stability in the open economy case lies much lower than in the closed-economy case, relativizing up to a certain extent the validity of the prominent Taylor Principle, at least for large economies such as the U.S. and the euro area.

This result, though somewhat surprising at first sight, is actually quite reasonable: In contrast to the closed economy case, in an open economy the monetary policy transmission mechanism is enriched by other transmission channels such as the nominal exchange rate and the competitiveness channels. So for example an interest rate increase leads not only to higher borrowing costs and therefore to a lower consumption and investment demand, but also, in an open economy, to a nominal (and real) appreciation of the domestic currency. This in turn leads to a decrease in the net exports. In an open economy, thus, monetary policy can rely on the activation of more transmission channels and therefore needs not to be as aggressive as in the closed-economy case in order to stabilize the economy.

In Figure 7 we can observe the eigen-value diagrams concerning the parameters

of the nominal exchange rate dynamics. As it can be observed, an increase in  $\beta_s^f$ , the “fundamentalists” parameter (which was restricted to one in the estimations of the previous section) induces stability into the system. This is also the case for  $\lambda$ , the relative share of fundamentalists in the foreign exchange market, which *ceteris paribus* seems not needing to be particularly high in order to stabilize the system.

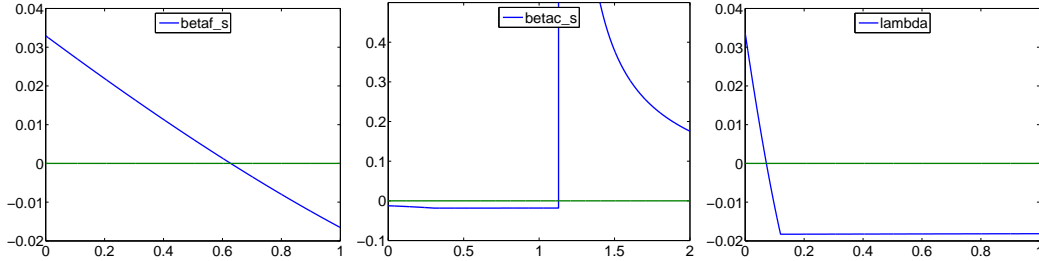


Figure 7: Eigen-Value-based Stability Analysis: FX Dynamics

Last but not least, the middle graph in Figure 7 shows the eigen-value diagram for  $\beta_s^c$ , the parameter denoting the “trend-chasing” degree by the chartists. As expected, an increase in  $\beta_s^c$  brings about instability into the system. The switch from stability to instability, however, does not happen in a smooth manner but is rather of a quite nonlinear nature. This is due to the fact that an increase in  $\beta_s^c$  does not affect the stability of the system as long as the inequality  $1 > (1 - \lambda)\beta_s^c$  (the denominator in the nominal FX dynamics given by eq. (19)) is fulfilled. As it can be observed, there exists a threshold value of  $\beta_s^c$  for which this inequality is not fulfilled anymore and which causes a sign reversal in the nominal FX dynamics which leads to a total instability of the system.

## 4 Concluding Remarks

In this paper a basic two-country theoretical framework in the line of the disequilibrium approach by Chiarella and Flaschel (2000) and Chiarella et al. (2005) was modeled where two large open economies interacted with each other and indeed influenced each other through trade, price and financial channels. The discrete time version of this theoretical framework (which was calibrated with estimated parameters of the U.S. and the euro area), while still quite basic in its structure, was nevertheless able to deliver dynamic responses in line with other empirical studies on the dynamics of open economies such as Eichenbaum and Evans (1995), Kim (1999) and Kim and Roubini (2000).

Through the calculation of the eigen-values of the 11D system, the effects of variations in the parameter values were investigated. This eigen-value analysis confirmed by and large the findings of previous studies such as Chen et al. (2006) concerning the role of wage and price flexibility for the stability of the economy. Furthermore, the maximum-eigenvalue-diagram concerning the inflation gap coefficient  $\phi_\pi$  in the monetary policy rule showed that in an open economy the threshold value for the effectiveness of monetary policy is not one as discussed for example in Woodford (2003), but rather less than one, due to the functioning of other stabilizing macroeconomic channels such as the real exchange rate channel in an open economy.

Particularly with regard to the modeling of nominal exchange rate dynamics, this eigen-value analysis corroborated the standard notion concerning the destabilizing influence of the chartists not only for the FX dynamics, but also for the macroeconomic system as a whole. The incorporation of a more elaborated specification of the chartists/fundamentalists behavior and their interaction into a macroeconomic model as the one discussed here seems to represent a promising research direction for the understanding of the link between nominal exchange rate dynamics and macroeconomic fundamentals. We intend to do this in further research.

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