On the Mechanisms of Inequality*

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August, 2003

Abstract

Recent research on inequality, in particular as put forward by Brock and Durlauf (2000 a,b) and Durlauf (1999 a,b, 2000), has directed our attention to the fact that the composition and behavior of groups of which a person is a member plays an important role for socioeconomic outcomes. Heterogeneity across groups and group-level effects can either lead to take-offs of individuals or substantial immobility, social lock-in phenomena concerning learning, building up of skills and formation of human capital, and differences in income and status across generations. In the present paper we explore some major mechanism that can lead educational and social lock-ins. Those mechanisms can give rise to persistent inequality. The presence of such mechanisms has important implications for how competition in market economies work and how aggregate inequality is created. If those mechanisms are present the models become highly nonlinear and may give rise to thresholds and poverty traps. The capability of the individual to successfully compete is achieved only after some thresholds are passed, since there are often multiple steady state equilibria and there is path dependency as to the outcome of the dynamics.

*We want to thank Buz Brock and Steven Durlauf for helpful discussions and Lars Gruene for setting up the numerical program to undertake the numerical simulations.

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1 Introduction

This paper intends to describe basic mechanisms of poverty traps and persistent income inequality. One important stream of economic literature in the U.S. sees the causes of inequality in individual-based characteristics which can be overcome by intergenerational mobility. In the work of Becker, see for example, Becker and Thomas (1979), it is competition that erodes excess returns[1] and brings about equality. Obstacles to it, then explain the persistent difference in income across individuals. For example the persistent difference in income status across generations can be explained via the effect of parental income on the childrens' education.

Another approach, more recently developed, for example by Benabou (1993, 1996 a,b), Durlauf (1996 a,b) and Fernandez and Rogerson (1996), consider the persistent effects of residential neighborhood on education and skill formation. In their view group environment and, in particular in the U.S., local school quality, strongly affect the childrens' education and skills.

The membership theory of inequality and poverty (Brock and Durlauf, 2000 a,b, Durlauf, 1996 a,b, 2000) maintains that the composition and behavior of the groups of which a person is a member play an important role of socioeconomic outcomes. Based on this approach one can spell out numerous observable and non observable factors like family, neighborhood, peer group and so on that affect childrens' education and skill potentials (Brock and Durlauf, 2000 a,b). This approach can explain substantial immobility, social lock-in phenomenon concerning the economic status across generations and a lack of education and formation of human skills for certain socioeconomic groups.

Recently, persistent inequality is also seen to arise from the forces of economic growth, for example, inequality, beyond the one already generated by education, can substantially be impacted by the growth of knowledge, technical change and the interaction and spillover effects in production activities. Recent literature on education, skill-biased technical change and skill formation at the workplace, see Acemoglu (2002), Aghion (2002) and Greiner et al. (2002), has stressed the latter approach. [2] This literature demonstrates that there are factors at work, for example spillover effects in production

[1]For an analytical treatment of whether competition in fact can erode excess returns, see Flaschel and Semmler (1987).

[2]For an empirical test as to what extent the latter theory holds across the U.S. and Europe, see Greiner, Rubart and Semmler (2003).

activities, that considerably can add to education-based inequality.

The present paper considers some of the relevant mechanisms that may give rise to persistent inequality. We build on the theory of social interaction as surveyed by Brock and Durlauf (2000 a,b). Yet, we take into account skill-formation through investment into education which Brock and Durlauf (2000 a,b) have not considered. Moreover, group-level effects may also occur in the process of skill formation in production activities. We consider three mechanisms that can produce persistent inequality in market economies. These are (1) locally increasing returns to scale due to interaction effects in skill formation, (2) limited financial resources for the acquisition of skills for individuals (low family income, high cost of loans and credit constraints), and (3) growth of knowledge and group specific spill-over effects in production activities.

The presence of such mechanisms has important implications for how competition in market economies work and how an aggregate inequality is created. The capability of the individual to successfully compete is achieved only after some thresholds are passed, since there are often multiple steady state equilibria and there is path dependency as to the outcome of the dynamics. If those mechanisms are present the models become usually highly nonlinear and may give rise to thresholds and poverty traps.

The remainder of the paper is organized as follows. Section 2 discusses the aforementioned three mechanisms and their origin in the literature. Section 3 presents the different variants in the context of a dynamic model. Section 4 and 5 provide some numerical and quantitative results. Section 6 concludes the paper.

2 The Basic Mechanisms

Next we want to introduce and discuss the major mechanisms that may give rise to more complicated dynamics, thresholds, poverty traps and inequality. These mechanisms could either be relevant for the process of human capital formation or to the activities when human capital is used. We will consider mechanisms that are relevant for the latter but will also affect the former. Overall, however, we keep the process of skill-formation itself rather simple. Our basic model resembles the Uzawa-Lucas human capital model but it disregards physical capital. Here we want to solely give some economic intuition. Analytical and numerical results are presented in sections 3-5.

2.1 Externalities and Locally Increasing Returns

One recent idea that has been employed to study inequality is the idea of externalities and locally increasing returns to scale which has extensively been employed in theory.[3] It has been shown that a variety of positive externalities arising from parental and peer group interrelations, social learning, increasing returns to information and skills can be set in motion if a person enjoys, intentionally, or by historical accident, some better education. Our first variant of a model builds, therefore, on locally increasing returns to scale, arising from externalities. Traditionally locally increasing returns due to local externalities have been approximated by a convex-concave production function[4] which can illustrate those effects.

To present this idea of a convex-concave function for an activity, using solely human capital as input, with externalities and locally increasing returns to scale, we can employ the following production function

$$\begin{aligned} y(h(t)) &= ah(t)^{\alpha_h(t)} \\ \alpha_h(t) &= \begin{cases} \tilde{\alpha}_h & \text{if } h(t) > \tilde{h}(t) \\ \underline{\alpha}_h & \text{otherwise} \end{cases} \end{aligned}$$

with the coefficients $\alpha_h(t)$, varying with the underlying state (h) and the quantity $\tilde{h}(t)$ denoting the threshold for h, a stock of human capital of an agent.[5]

If for $h(t) < \tilde{h}(t)$ one has the coefficient $\underline{\alpha}(t) > 1$, and one has the case of locally increasing returns to scale as Brock and Durlauf (2000 a,b) and Durlauf (2000) refer to. Locally increasing returns may arise due to parental status, peer group effects, social learning etc. All those effects will be summarized in our model by one state variable, namely h. A positive coefficient $\underline{\alpha}_h(t) > 1$ forever would mean that the marginal product $y'(h)$ would rise for ever, not only locally.

On the other hand, if $\tilde{\alpha}_h < 1$ holds forever, the marginal product of $y'(h)$ would fall for ever. In our[6] Figure 1 it would approach the line given by

[3]There is a long tradition that build on this idea. It starts with Marshall (1948), and Sraffa (1926) who study the impact of those forces on competition, and recently has been stressed by Arthur (1989). For a further survey see Deissenberg et al. (2003).

[4]See Brock and Milliaris (1996)

[5]One could interpret h as abroad notion of human capital which may, however, also include other household's assets. This might become relevant if we later consider collaterals for the credit market.

[6]Note that we leave aside here depreciation of human capital, h, which will be introduced in section 3.

the discount rate θ from above if depreciation is allowed for, see case (1) in Figure 1.[7]

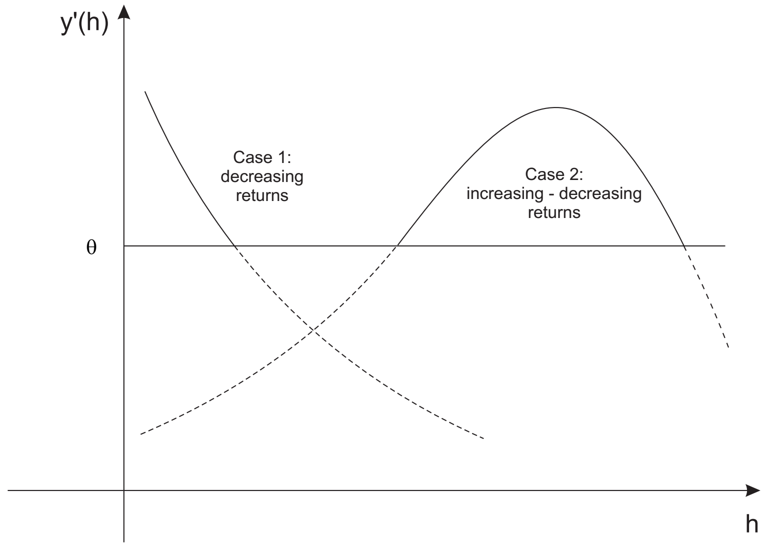

Figure 1: Increasing and decreasing returns

Yet, presuming as in the above convex-concave case, that the parameter α_h is state dependent and approximating the convex-concave production function by a smooth function one would obtain the case 2 in Figure 1. For locally increasing returns to scale, case 2, the marginal product $y'(h)$ will first approach θ from below, then move above this line, θ, and eventually decrease again. In the latter case, because of externalities, too small a human capital will generate a too low return for the agent so that the owner of skills may seek activities other then skill formation activities.

Of course, as Figure 1 shows, increasing returns to scale can be assumed to hold, as Greiner, Semmler and Gong (2003, ch. 3) show, only for a certain level of the stock, h. A region of a concave production function may be dominant there after where $y'(h)$ might start falling again, see case 2 above.

2.2 Credit Constraints and Credit Cost

A second strand of literature argues that persons with low human capital, h, will be severly constrained by imperfect capital markets. Poorer individuals

[7] This figure goes back to an idea of Dechert and Nishimura (1983) who capture the essential results of a dynamic optimization model in such a figure.

may face stricter credit conditions than owners of larger human capital, h.[8] Thus, in imperfect capital markets agents who borrow are heterogeneous with respect to their different excess to the capital market.[9] This variant can theoretically be based on credit market theories such as developed by Townsend (1979) and Bernanke, Gertler and Gilchrist (1999), henceforth BGG.[10]

Recently, the credit constraints and the credit cost that an agent faces has been derived from information economics. One here presumes that asymmetric information and agency costs in borrowing and lending relationships. We can draw, as BGG, on the insight of the literature on costly state verification[11] in which lenders must pay a cost in order to observe the borrower's realized returns. This motivates the use of collaterals in credit markets. Uncollateralized borrowing is assumed to pay a larger credit cost than collateralized borrowing. This additional credit cost covers default risk[12] which drives a wedge between the expected return of the borrower and the risk-free interest rate.

Following BGG we measure the inverse relationship between the credit cost and the value of the collateral in a function such as

$$H\left(h(t), B(t)\right) = \frac{\alpha_1}{\left(\alpha_2 + \frac{N(t)}{h(t)}\right)^{\mu}} \theta B(t) \tag{1}$$

with $H\left(h(t), B(t)\right)$ the credit cost depending on the collateral, the net worth, $N(t) = h(t) - B(t)$, with $h((t)$ as asset and $B(t)$ as debt.[13] The parameters are $\alpha_1, \alpha_2, \mu > 0$ and θ is the risk-free interest rate. The shape of this function is shown in figure 2.

[8]For the U.S. there are numerous studies that have shown that poorer families or neighborhoods face more severe credit constraints. Extensive work on this has been undertaken by Dymski, see, for example, Dymski (2000).

[9]More general studies of imperfect capital markets and their impact on physical investment can be found in Kiyotaki and Moore (1997), Bernanke, Gertler and Gilchrist (1999) and Miller and Stiglitz (1999). We here, however, want to study the impact of credit constraints on investment in human capital and the present value of income from human capital.

[10]For a recent extensive survey of the role of imperfect credit market for education, see Aghion, Caroli and Garcia-Penalosa (1999).

[11]This literature originates in the seminal work by Townsend (1979).

[12]The actual cost that arises here may be constituted by auditing, accounting, legal cost and loss of assets.

[13]The above function representing the finance for borrowing can be found in BGG.

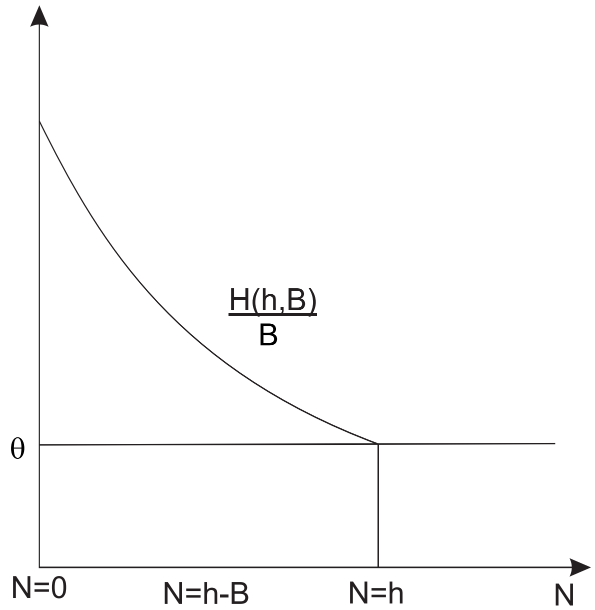

Figure 2: Endogenous Credit Cost

As figure 2 shows a low interest rate, the risk-free interest rate, is only paid by the borrower whose net worth is equal to the value of h.

Another way of how poorer borrowers are disadvantaged on credit markets is that there is credit rationing for them.[14] Thus even if the credit cost depends on individual characteristics of an agent we might want to define credit constraints for a person which, in our model, will be determined by an upper bound of a debt-asset ratio.

2.3 Growth of Knowledge and Spillover Effects

Recent theories on knowledge creation and technical change spells out the consequences of growth of knowledge, technological change, and spillover effects on between-group and within group inequality. Thus, education-based income differences may be exacerbated by group interactions and external effects at the workplace.

In the past, economic theory has maintained that industrialization, mass production and technological change may lead to de-skilling of higher skilled labor force (see Acemoglu, 2002). Yet, in recent times, however, technological change seems to be skill-biased in the sense that it favors higher skills, see

[14]This may, as Aghion et al. (1999) has shown, in particular hold true for groups with low per capita income.

Acemoglu (2002). Moreover, Kremer and Maskin (1996) have shown that the productivity, and thus the wage level of a worker, is determined by his or her coworker's skills which may have, through group specific interaction effects, large influences on the degree of inequality. Less skilled workers may be decoupled more from knowledge-based production activities and thus less spillover effects will occur.[15]

It has been shown that in the last two decades first, the demand for skills has increased faster than the supply of skills. Second, the general growth of knowledge has advantaged the higher skilled workers, third, spillover effects favor high skilled labor and fourth, substitution effects between different skill groups do not sufficiently equalize labor income. Whereas those factors, may have differential effects on labor income, institutions of the labor market, such as labor unions and welfare state measures, may have a wage compressing effect. The above factors are also the likely ones that create different levels of inequality in the U.S. as compared to European countries. A more exact model of growth of knowledge, technical change and spill-over effects on different groups of labor with different skill level is presented in section 5.

3 Basic Dynamic Model Variants

Next we specify different variants of a model that incorporates the above mechanisms. We allow for heterogeneity of agents and spillover effects among them. There will be individual human capital, h_i and aggregate human capital of the group, $\overline{h}$. The latter can be considered as the group level effect on the individual's activity. Before building up the model we want to note that although our model can be nested in utility theory, we use a separation theorem that permits us to separate the present value problem from the consumption problem. In the appendix 2 an analytical treatment is given of why and under what conditions the subsequent dynamic decision problem of the individual can be separated from the consumption decisions.

We may specify a general model that can embody the above mentioned interaction effects as well as the credit market effects. By disregarding the subscript for the agent i, but denoting the group level effect $\overline{h}$, the general decision problem for the formation of human capital of an economic agent can be formulated as follows:

$$V(h) = \underset{j}{Max} \int_0^\infty e^{-\theta t} f\left(h(t), \overline{h}(t), j(t)\right) dt \tag{2}$$

[15]As also Durlauf (2002:9) notes little work has been done concerning this mechanism of inequality. In sect. 6 we will build a model to advance this area of research.

$$\dot{h}(t) = j(t) - \sigma h(t), \quad h(0) = h. \tag{3}$$

.

$$\dot{B}(t) = H\left(h(t), B(t)\right) - \left(f\left(h(t), \overline{h}(t), j(t)\right) - c(t)\right), \; B(0) = B_0 \tag{4}$$

In the general case the agent's net income can be written as

$$f(h, j) = y(h, \overline{h}) - j - j^{\beta} h^{-\gamma} \tag{5}$$

which is generated from human capital, h, through a production function, $y(h, \overline{h})$ with h the individual's human capital and $\overline{h}$ the group-level effect. In our model we presume that human capital investment, j, is undertaken so as to maximize the present value of net income of (5) given $\overline{h}$ and the adjustment costs $j^{\beta} h^{-\gamma}$ in (5). Note that the adjustment cost, pertaining to investment in human capital, could be rather individual specific and might fall with the level of human capital h, already achieved.[16] We assume that $\sigma > 0, \alpha > 0, \beta > 1, \gamma > 0$, are constants and the same across individuals.

When the human capital is used in production as input, in the production function $y(h, \overline{h})$, we may take a convex-concave production function, as above proposed in section 2.1 which includes the group-level effect $\overline{h}$. To obtain a first variant of the model, for the purpose of simplicity we presume that the group-level effect simply raises the elasticity of output with respect to the agent's human capital input.

Equ. (3) represents the equation for the formation of human capital. Since we want to allow for borrowing to finance education we have introduced equ. (4) which represents the evolution of debt of the agent. Note that in (4), $c(t)$ is a consumption stream arising from the income that is, in the context of our model, treated as exogenous. The consumption stream will be specified further below. Since net income in (5), less the consumption stream $c(t)$, can be negative the temporary budget constraint requires further borrowing from credit markets and if there is positive net income, less consumption, debt can be retired.

Note that in the above general case of adjustment cost in (5), if we take $\beta = 2$ and $\gamma = 0$, we have the standard model with quadratic adjustment cost of investment. When we employ the locally increasing return production function, the convex-concave production function, we will drop the adjustment cost term $j^{\beta} h^{-\gamma}$, as also done in Brock and Milliaris (1996) and assume no extra credit cost.[17]

[16] For adjustment cost concerning physical capital, see Gould (1968) and Lucas (1967).

[17] Note that we use here a general form of adjustment cost which may itself give rise to some interesting dynamics, see Grüne, Semmler and Sieveking (2003).

For the second variant of our model that presumes imperfect capital markets we assume that the credit cost $H(h, B)$ in equ. (4) may be state dependent, depending on the stock, h, and the level of debt B with $H_h > 0$ and $H_B < 0$. As specific function of $H(h, B)$ we will take equ. (1) of section 2.1. The appendix 1 briefly discusses how such a problem with endogenous credit cost can be solved using the Hamilton-Jacobi-Bellman (HJB) equation.

Note, that if we assume that credit cost depends inversely on net worth and the net worth is equal to the value of the stock, h, we get a special case of our model when only the risk-free interest rate determines the credit cost. We then have a constant credit cost and a state equation for the evolution of debt such as

$$\dot{B}(t) = \theta B(t) - f(h, B), \qquad B(0) = B_0 \tag{6}$$

In this case we would only have to consider the transversality condition $\lim_{t\to\infty} e^{-\theta t}B(t) = 0$, as the non-explosiveness condition for debt, to close the model (2)-(3).

In general, however, we define the limit of borrowing, B, equal to $V(h)$ which represents the present value borrowing constraint. This will be particularly relevant when we study the second variant of our model as discussed in section 2.2.

The problem to be solved is then how to compute $V(h)$ and the associated optimal investment j. If the interest rate $\theta = \frac{H(h,B)}{B}$ is constant[18] as in (6), then, as is easy to see, $V(h)$ is in fact the present value of the income from human capital h

$$V(h) = \underset{j}{Max} \int_0^\infty e^{-\theta t} f\left(h(t), j(t)\right) dt \tag{7}$$

$$s.t. \quad \dot{h}(t) = j(t) - \sigma h(t), \quad h(0) = h_0. \tag{8}$$

$$\dot{B}(t) = \theta B(t) - (f(h, B) - c(t)), \quad B(0) = B_0. \tag{9}$$

with $h(0)$ and $B(0)$ the initial value of h and B.

The case with imperfect capital markets, however, when there is an extra credit cost to be paid, thus $H(h, B)$ holds, then the present value itself becomes difficult to treat. Pontryagin's maximum principle is not suitable to solve the problem and we thus need to use a method related to dynamic

[18] As aforementioned in computing the present value of the future net income we do not have to assume a particular fixed interest rate, but the present value, $V(h)$, will, for the optimal investment decision, enter as argument in the credit cost function $H\left(h(t), V((h(t))\right.$.

programming to solve for the present value and optimal investment strategy, see appendix 1.

In the context of the second model variant we can also explore the use of 'ceilings' in debt contracts and their impact on the dynamic investment decision of the agent. Indeed credit restrictions may affect the investment decisions. Suppose the 'ceiling' is of the form $B(t) < \overline{B}$, with $\overline{B}$ a constant, for all t. Either $\overline{B} > V(h)$, then the ceiling is too high because the borrower might be tempted to move close to the ceiling and then is not solvent any more if $\overline{B} > V(h)$. If $\overline{B} < V(h)$, then the borrowing agent may not be able to develop its full potentials, and thus faces a welfare loss.[19] Those conditions obviously are of no practical use if we can not say when $B(t) \leq \overline{B}$. A task of our method will be to compute the investment decision and thus the present value $V(h)$ for the case of an endogenous credit cost and/or credit constraints.

In the two cases – so far locally increasing returns to scale, and imperfect capital markets – the optimal investment strategy may depend on initial conditions of the household or person, on their stock of human capital. Thus, there may be thresholds that separate the optimal solution paths for $V(h)$ to different domains of attraction. For agents with lower stock, h below the threshold it will be optimal for the agents to contract whereas agents with a larger stock, h, may choose an investment strategy to expand. We also will consider the case of a borrowing constrained agent for which holds that $B(t)/h(t) \leq c$ with c a constant and then study the investment strategy.

Our last model variant, that stylizes the spillover effects in production activities will employ a production function with different types of human capital as inputs incorporating externalities of different degrees, see sect. 6 for details. Finally, we want to note that we can admit in our study various paths for the consumption stream, $c(t)$, and their impact on the investment strategy and the present value $V(h)$ for our different model variants, see appendix 2.

Overall we can see that there are thresholds in the competition of agents where only agents above the threshold will be able to successfully take-off and compete. Next, we want to pursue a numerical treatment of the above model variants.

4 Externalities and Increasing Returns

Next, we present numerical results obtained for our model variant that captures externalities and state dependent returns to scale. Throughout this

[19] In Semmler and Sieveking (1996) the welfare gains from borrowing are computed.

section we specify the parameters $\sigma = 0.15$ and $\gamma = 0.3$. The other parameters will be model specific and specified below.[20] Unless otherwise noted we use for the consumption stream $c(t) \equiv 0$ in our study here which will be relaxed in appendix 2..

Let us first start with a numeral example with no - or only weak group level-effects. We thus employ solely a concave production function $y(h) = ah^{\alpha}$, with $0 < \alpha < 1$, as underlying the case 1 in figure 1, and quadratic adjustment cost, bj^{β}. As model parameters we specify $\alpha = 0.5, \beta = 2, b = 0.5, a = 0.29$ and $\theta = 0.1$. This specifies the most simplest variant of a dynamic decision problem with adjustment cost which has often been employed in economics and which can be shown to exhibit solely one positive steady state equilibrium h^* as case 1 in figure 1 predicts. The present value curve is simply given by the present value of the netcome stream of income from human capital, since we here assume a constant credit cost and a debt equation as shown in equ. (9).

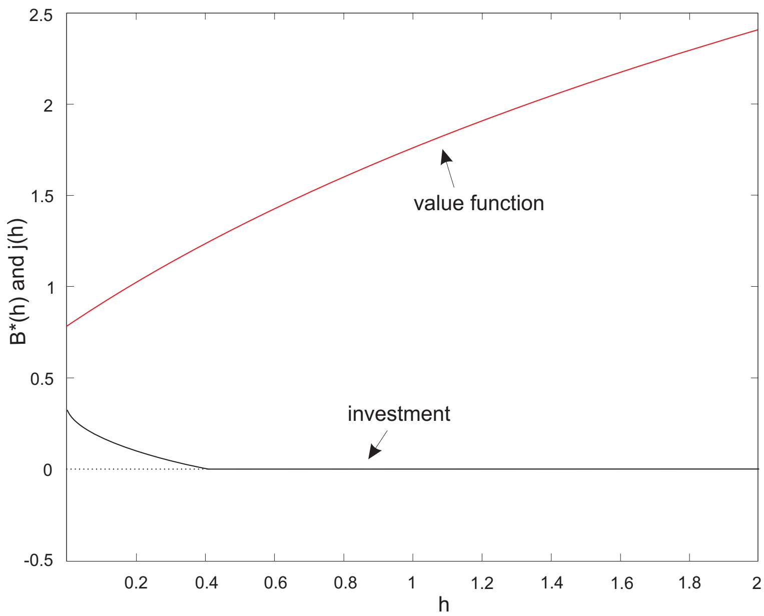

Figure 3: Quadratic adjustment cost of human capital

In this case one can use a dynamic programming algorithm of the type suggested in Grüne and Semmler (2003) to solve the model. The value function is given in figure 3 and the solution path of the dynamic decision problem, the investment decision on building up human capital, is given by the

[20] Note that we, of course, could choose another source of heterogeneity, namely by assuming different technology parameters for the household or individuals. This might be another line of research which we will not pursue here.

curve called investment in figure 3. Here it is assumed that the agent can borrow to build up human capital, up to an amount of debt equal to the present value of its income. Due to the concave production function where the human capital is used as an input the marginal product of human capital would be large, representing case 1, in figure 1, and thus it would be always optimal to invest in human capital if human capital is small. The positive equilibrium h^* would always be the sole attracting point.

Next we compute the investment strategy for skill-formation for a model variant with a convex-concave production function, representing case 2 in figure 1. It embodies strong enough group-level effects so that one obtains, at least locally, increasing returns to scale. We have included the group-level effect in the activity when the human capital or skills are employed. This group-level effect has, of course, also an effect on the formation of skills, i.e. on the investment decision of human capital. We disregard first, as most of the literature, adjustment cost but again presume a constant borrowing cost, for example, $\theta = 0.1$. The convex-concave production function is for our numerical purpose specified as a logistic function of h

$$y = \frac{a_0 \; exp(a_1 h)}{exp(a_1 h) + a_2} - \frac{a_0}{1 + a_2} \tag{10}$$

with $a_0 = 2500,\ a_1 = 0.0034,\ a_2 = 500$. This convex-concave production function specifies the production function $y(h)$ in equ. (5), yet there is no adjustment cost term $j^\beta h^{-\gamma}$ or j^2. The net income, $f(h, j)$, in equ. (5) is thus linear in the decision variable, j, and one would thus expect a bang-bang solution to exist. In our numerical solution, we need to restrict the net income such that $f(h, j) \geq 0$. The results, using this simulation, are shown in figure 4.

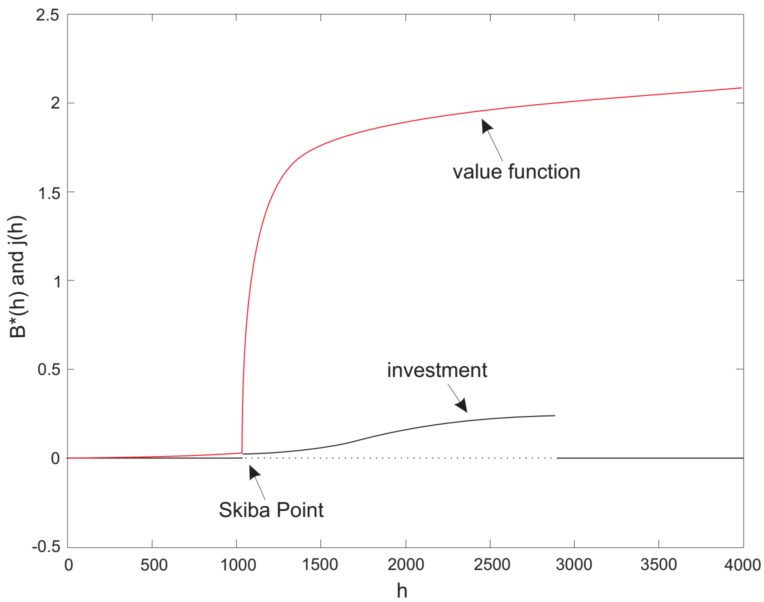

Figure 4: Convex-concave production function

The value function represents the present value curve and the other curve, called investment, the dynamic solutions to the investment decision problem. This variant of our model gives multiple steady states at 0 and 2847 and a threshold, at 1057 in the vicinity of which there is another, but non-optimal steady state.

Again any debt, B_0, below the present value curve can be steered bounded but a stock of skills, h, with initial condition, h_0, to the left of the threshold, will contract and to the right of this point will expand approaching the high steady state 2847. Thus the thresholds, is unstable and 0 and 2847 are attractors. As also clearly visible, at the threshold the investment is discontinuous, it jumps. For the agent with human capital to the left of the threshold it would be optimal to let his or her human capital deteriorate so that it finally shrinks to zero. Note, however, that the jump of investment at the high steady state arises from the fact that, without adjustment cost, we have a decision problem linear in the decision variable. Overall, this latter model variant, with group-level effects on the individual human capital formation (at least if it holds locally, is one that predicts a persistence in inequality in the long-run. In fact, this model more accurately replicates what has been stated for the case 2 in figure 1.

5 Credit Constraints and Credit Cost

Next we will study our specifications of imperfect capital markets with endogenous credit cost and/or credit constraints as stated in sect. 2.2. First we will presume that the credit cost $H(h, B)$ is endogenous in the sense that it depends on net worth. Second, we presume that there is in addition an exogenous borrowing ceiling.

As aforementioned, an extra credit cost may arise due to costly state verification. This cost is inversely related to the borrowers' net worth. Net worth is defined as the agent's collateral value of the stock of assets less the agent's outstanding obligations. As above shown we measure the inverse relationship between the cost of credit and net worth in a function such as

$$H\left(h(t), B(t)\right) = \frac{\alpha_1}{\left(\alpha_2 + \frac{N(t)}{h(t)}\right)^{\mu}} \theta B(t) \tag{11}$$

with $H\left(h(t), B(t)\right)$ the credit cost depending on net worth, $N(t) = h(t) - B(t)$, with $h(t)$ as asset stock and $B(t)$ as debt. The parameters are $\alpha_1, \alpha_2, \mu > 0$ and θ is the risk-free interest rate. In the analytical and numerical study of the model below we presume that the extra credit cost will be zero for $N(t) = h(t)$ and thus, in the limit, for $B(t) = 0$, the borrowing rate is the risk-free rate. Although this could occur for an agent with a small stock a borrowing rate equal to the risk-free rate it is more likely to hold for an agent with large stock, h. Borrowing at a risk-free rate will be considered here as a benchmark case.[21]

Next, we undertake experiments for different shapes of the credit cost function. For the credit cost function (11) we specify $\mu = 2$. Taking into account that we want θ to be the risk–free interest rate, we obtain the condition $\alpha_1/(\alpha_2 + 1)^2 = 1$ and thus $\alpha_1 = (\alpha_2 + 1)^2$. Note that for $\alpha_2 \to \infty$ and $0 \leq B \leq h$ one obtains $H(h, B) = \theta B$, i.e., the model from the previous section. In order to compare these two model variants we use the formula $H(h, B) = \frac{\alpha_1}{\alpha_2^2} \theta B$ for $B > h$.[22] We use an investment cost of the type $\left(\frac{j}{h}\right)^{\beta}$.

For large α_2 in (11) the model does not necessarily have an unique steady state equilibrium. Here again there can be multiple domains of attraction depending on the initial stock, h. There is a threshold, s, at $h^{+} = 0.267$

[21]In general, as above remarked, it is not possible to transform the above problem into a standard infinite horizon optimal control problem. Hence, what we need to use here is an algorithm that computes domains of attraction, see Grüne, Semmler and Sieveking (2003).

[22]For small values of α_2 it turns out that the present value curve satisfies $V(h) < h$, hence this change of the formula has no effect on $V(h)$.

which is clearly visible in the optimal control law, which is discontinuous at this point. Thus, the dynamic decision problem of the agent faces a discontinuity at a threshold. For agents with initial values of the stock $h(0) < h^+$ the optimal trajectories tend to $h^* = 0$, and the human capital will be optimally depleted and tend to zero. For initial values of the stock $h(0) > h^+$ the optimal trajectories tend to the domain of attraction $h^{**} = 0.996$, thus the human capital will rise. Thus, the success in competition is history dependent.

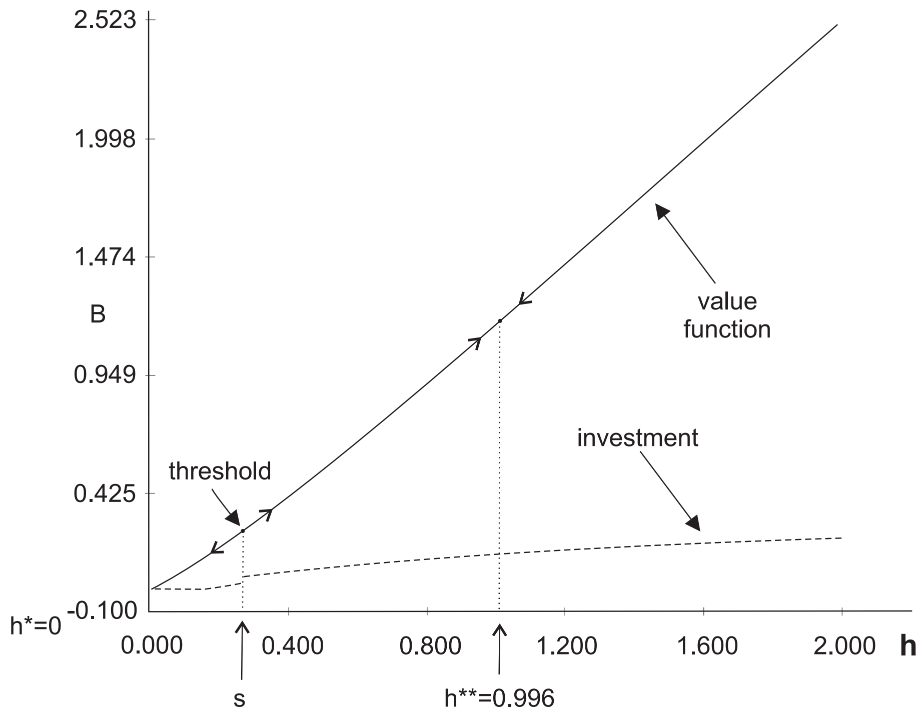

Figure 5: Optimal value function and optimal feedback law

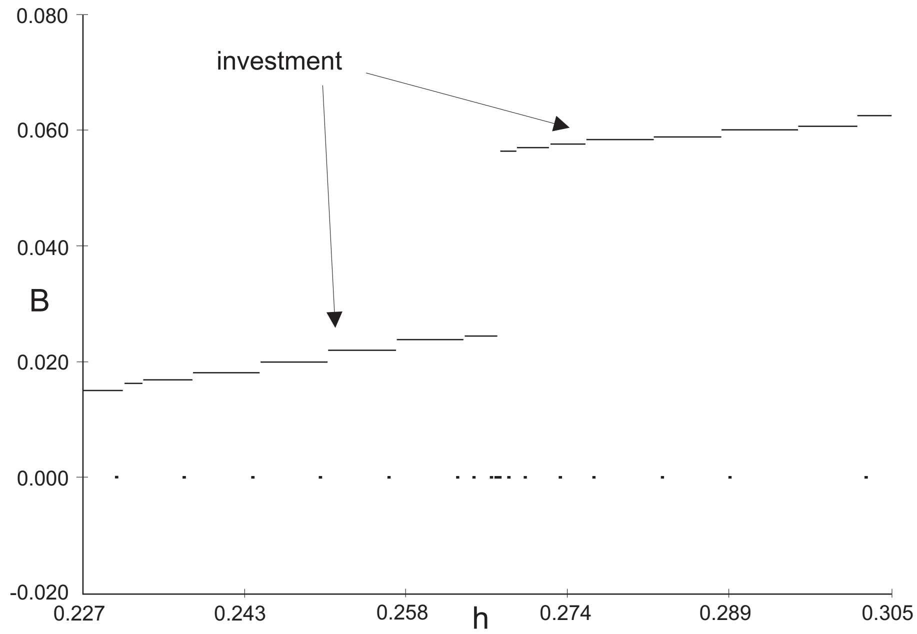

Figure 6: The jump in investment and distribution of grid points at the threshold

Figure 5 shows for an $\alpha_2 = 100$ the corresponding value function representing the present value curve, $V(h)$, (upper graph) and the related, the investment decision. Figure 6 shows the optimal feedback control, the investment decision, in a neighborhood of the threshold for the size of the human capital. The discontinuity in the control variable, and thus in the investment strategy, is clearly observable. Investment for an agent to the left of h^+ is lower than σh and makes the human capital shrinking whereas investment for an agent to the right of h^+ is larger than σh and let the human capital increase. At h^+ investment in education for the agent then jumps.[23]

Figure 7 shows the respective present value curves $V(h)$ for $\alpha_2 = 100$, 10, 1 , $\sqrt{2} - 1$ (from top to bottom) and the corresponding $\alpha_1 = (\alpha_2 + 1)^2$.

[23] In addition, in figure 6 the adaptively distributed grid points are shown. As mentioned, the grid is in particular refined around the threshold, the reason for this is the (barely visible) kink in the optimal value function at this point, resulting in a non–differentiable value function and hence in large local errors.

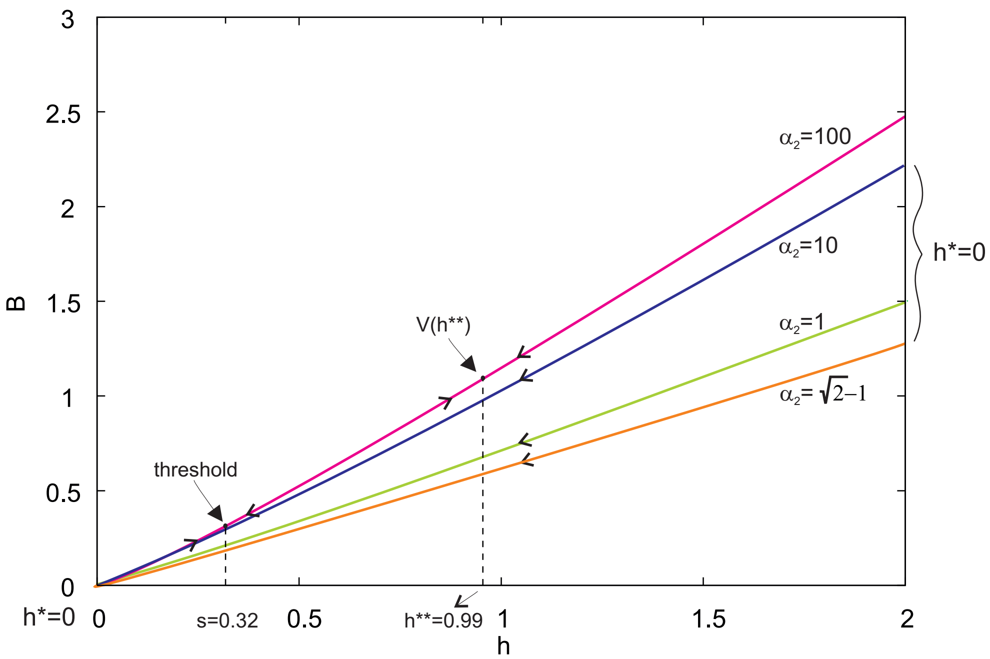

Figure 7: Present value curve $V(h)$ for different α_2

The top trajectory for $\alpha_2 = 100$: There exists a threshold at $h^+ = 0.32$ and two stable domains of attraction at $h^* = 0$ for all human capital sizes and $h^{**} = 0.99$.

Further numerical studies have revealed that for decreasing values of $\alpha_2 \leq 100$ the threshold value h^+ increases (i.e., moves to the right) and the stable domain of attraction h^{**} decreases (i.e., moves to the left), until they meet at about $\alpha_2 = 31$. For all smaller values of α_2 there exists just one equilibrium at $h^* = 0$ for all human capital stock sizes which is stable. The reason for this behavior lies in the fact that for decreasing α_2 credit becomes more expensive, hence for small α_2 it is no longer optimal for the agent – with any size of the human stock – to borrow large amounts and to increase the human capital for a given initial size, instead it is optimal to shrink the human capital to zero. Thus, with small α_2 and thus large borrowing cost it is for any agent, i.e. for any initial value, optimal to shrink the human capital to zero.

Next we study the decision problem of an agent with borrowing constraints. For $H(h, B)$ from (11) with $\alpha_2 = 100$ we test a different criterion for borrowing as before and its impact on the value function: we impose the restriction $B(t)/h(t) \leq c$ for some constant c. Again we use the algorithm as indicated in appendix 1. Figure 8 shows the respective curves for the restriction $\sup_{t \geq 0} B(t) < \infty$ and for the ratio–restriction with $c = 1.2$ and

$c = 0.6$ (from top to bottom). In addition, the restriction curves $B = ch$ are shown with dots for $c = 1.2$ and $c = 0.6$.

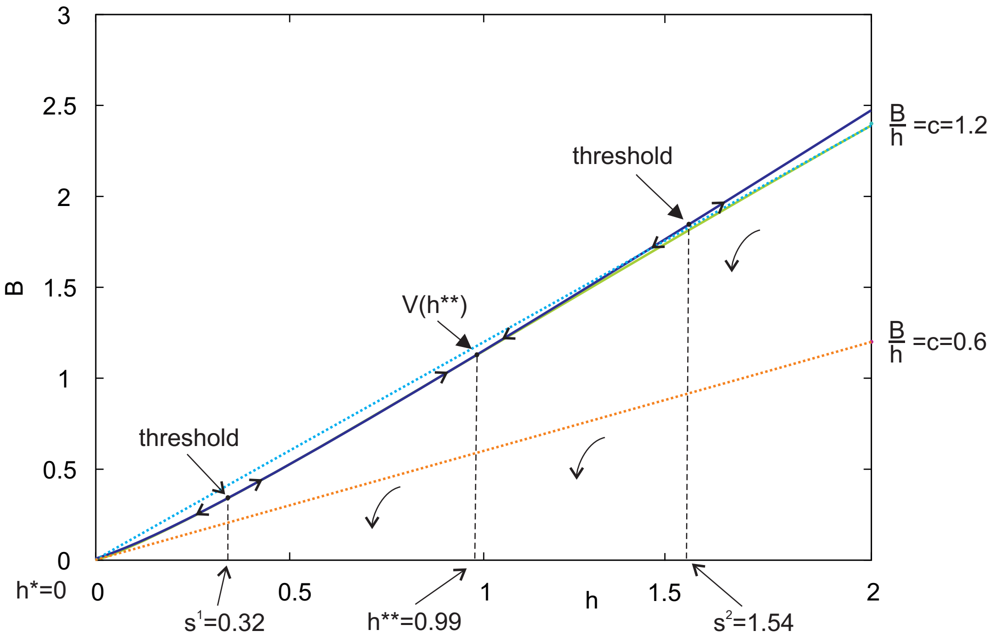

Figure 8: Present value curve $V(h)$ for different debt ceilings, $H(h, B)$ from (11)

For $c = 0.6$ the present value curve $V(h)$ coincides with the "restriction curve" $B(h) = ch$; in this case the curve $(h, V(h))$ is no longer invariant for the dynamics, i.e., each trajectory $B(t)$ with $B(t) \leq V(h(t))$ leaves the curve $(h, V(h))$ and eventually $B(t)$ tends to $-\infty$. For $c = 1.2$ [24] the curves $B^*(h)$ and $B = ch$ coincide only for human capital of size $h \geq 1.46$. Here one observes the same steady equilibria h^* and h^{**} and threshold h^+ as for the sup–restriction, however, in addition to these here a new threshold, s, appears at $h^{++} = 1.54$. For initial values of the stock $(h, V(h))$ with $h^+ < h < h^{++}$ the stock expands and tends to the stable domain of attraction h^{**}, while for agents with initial human capital $h > h^{++}$ the behavior is the same as for $c = 0.6$, i.e., the corresponding trajectories leave the curve $V(h)$ and eventually $B(t)$ tends to zero.[25]

[24] This curve is difficult to see because it coincides with the curve for $\sup_{t\geq 0} B(t) < \infty$ for small h and with the restriction curve $B = ch$ for large h.

[25] The simulation are halted at zero, but we would like to report if continued the $B(t)$ curve becomes negative and tends to $-\infty$.

6 Growth of Knowledge and Spillover Effects

The formation of human capital takes also place through general growth of knowledge and spillover effects in productive activities. As Kremer and Maskin (1996) show the productivity of an employee depends on the general knowledge in the workplace and the knowledge and productivity of the co-workers. To explore those spillover effects we use a model with growth of technological knowledge, A, and with two types of human capital. The model is based on Greiner et al. (2003). The framework goes back to Romer (1990) and, furthermore, to Murphy et al. (1998). It presumes, that two types of human capital, skilled and unskilled labor, are needed in production activities as well as technological knowledge, A.[26]

To derive the income inequality between The two types of human capital, we may presume the following extended production function:[27]

$$y(A, h_h, h_l) = A^{\alpha}\overline{\eta}\Big\{\gamma_1[A^{\xi}h_h]^{\frac{\sigma_p-1}{\sigma_p}} + (1-\gamma_1[A^{\epsilon}h_l]^{\frac{\sigma_p-1}{\sigma_p}}\Big\}^{\frac{\alpha\sigma_p}{\sigma_p-1}}, \tag{12}$$

where h_h and h_l denote high and low skilled human capital. The stock of technological knowledge is denoted by A which is taken here as exogenous[28] and $\alpha \in (0,1)$. $\sigma_p > 0$ gives the elasticity of substitution between h_h and h_l. As in Acemoglu (2002) we say that skilled and unskilled workers are gross substitutes for $\sigma_p > 1$ and gross complements when $\sigma_p < 1$. ξ and ϵ measure the impact of the external effect, i.e. the impact of the knowledge at the workplace, A, on h_h and h_l.

One important difference of eqn. (12) to the work, for example, of Acemoglu (2002) lies in the fact that we introduce here the external effects of knowledge and technology. Generally it is assumed that two kinds of technologies exist which are either complementary to high skilled workers or low skilled workers respectively. Furthermore, $\overline{\eta}$ gives the units of foregone output which are needed to produce one unit of a good.

Defining $X = \gamma_1[A^{\xi}h_h]^{\frac{\sigma_p-1}{\sigma_p}} + (1-\gamma_1)[A^{\epsilon}h_l]^{\frac{\sigma_p-1}{\sigma_p}}$ and assume that there is sufficient competition on the labor markets then, the wages of high and low skilled human capitals will tendentially become equal to the marginal products of high and low skilled human capital in production activities. This gives

$$w_H = \alpha\gamma_1\overline{\eta}A^{\alpha}X^{\frac{\alpha\sigma_p}{\sigma_p-1}-1}A^{\frac{\xi(\sigma_p-1)}{\sigma_p}}h_h^{-\frac{1}{\sigma_p}}, \tag{13}$$

[26]See Greiner et al. (2002) for the complete derivation of the model.

[27]The subsequent equation represents a simplified model of Greiner et al. (2002).

[28]For a further developed model making the creation of A endogenous, see Greiner et al. (2002).

$$w_L = \overline{\eta}\alpha(1-\gamma_1)A^{\alpha}X^{\frac{\alpha\sigma_p}{\sigma_p-1}-1}A^{\frac{\epsilon(\sigma_p-1)}{\sigma_p}}h_l^{-\frac{1}{\sigma_p}}. \tag{14}$$

The wage inequality, w_p, is given by the ratio of the marginal products:

$$w_p \equiv \frac{w_h}{w_l} = \frac{\gamma_1}{1-\gamma_1}\left[\frac{A^{\xi}}{A^{\epsilon}}\right]^{\frac{\sigma_p-1}{\sigma_p}}\left[\frac{h_h}{h_l}\right]^{-\frac{1}{\sigma_p}} \tag{15}$$

As regard inequality, equ. (15) spells out the four main factors determining inequality.

First, the factor of the productivity parameters $\gamma_1/(1-\gamma_1)$ is relevant for inequality. If γ_1 is very small and close to zero the wage premium will have a small value, too. A small value for γ_1 means that the productivity of the high-skilled workers relative to the low-skilled workers is small, i.e. low-skilled workers contribute more to the output than high-skilled workers. Consequently, the wage of the low skilled workers is relatively high and the wage premium is relatively low. If γ_1 is large, say near to one, the reverse holds. That is the productivity of the high-skilled workers is relatively high and, as a consequence, their wage rate and the wage premium are high, too.

Second, the ratio A^{ξ}/A^{ϵ}affects the wage premium. A high (low) value for ξ relative to ϵ means that the positive external effect of technical change affects high-skilled workers to a greater (lower) degree compared to low-skilled workers. That is, due a growth of knowledge, an increase in A, leads to a stronger (smaller) increase in the productivity of high-skilled workers compared to low-skilled workers.

Yet, the spillover effect interacts with a third effect, the substitution effect. The larger the positive difference $\xi - \epsilon$ the higher the inequality, provided that skilled and unskilled labor are gross substitutes, i.e. for $\sigma_p >$ 1. Further, the case of increase of knowledge, i.e. an increase in A, may raise the income inequality. Yet, if skilled and unskilled labor are gross complements ($\sigma_p < 1$) technical change, i.e. an increase in A, leads to a decline in the income inequality. This holds because in this case skilled and unskilled labor are gross complements and, therefore, the relative increase in the labor productivity of skilled labor also raises the demand for unskilled labor, where the latter increase exceeds the increase in demand for skilled labor.

Fourth, as our last term in equ. (15) shows the number of high-skilled workers relative to the number of low-skilled workers also determines the inequality. If this ratio is high the supply of high-skilled workers is relatively large. As a consequence, the inequality will take on a low value. The reverse will hold if the ratio of high to low skilled labor is low.

Referring to equations (15) the main parameters of interest are, the growth of knowledge, the relative quantities of the two types of human capital, the externalities and elasticity of substitution. Empirical knowledge about the sign and values of these parameters allows for a better understanding of the forces driving the different patterns of wage inequality. Taking logs of equation (15) and differentiating with respect to time we obtain the growth rate of the inequality:

$$\hat{w}_p = \frac{\dot{w}_p}{w_p} = \Big(\frac{\sigma_p - 1}{\sigma_p}\Big)(\xi - \epsilon)g_A - \frac{1}{\sigma_p}(g_H - g_L), \tag{16}$$

where $g_A = \frac{\dot{A}}{A}$, $g_H = \frac{\dot{h}_h}{h_h}$ and $g_L = \frac{\dot{h}_l}{h_l}$. The above model has analytically been solved in Greiner, Semmler and Gong (2002, ch. 7). We thus bypass the analytical study of the model but rather want to spell out the model's empirical implications.

Indeed, equation (16) allows for a closer examination of the technology effect and of the value of the elasticity of substitution. Rewriting equation (16) we get:[29]

$$\hat{w}_p = \beta_0 + \beta_1 g_A + \beta_2 g_{HL} + \varepsilon \tag{17}$$

With equ. (17) we obtain an empirically testable equation. Note that β_1 and β_2 describe the influence of technological change and the elasticity of substitution. Furthermore, β_0 accounts for other factors determining the trend of the wage differentials over time. These might be institutional factors such as labor unions or welfare state measures.

Overall, however, we can see that in such a model of growth of knowledge and the interaction and spillover effects in productive activities can generate inequality across skill-groups whenever $(\xi - \epsilon) > 0$. Greiner, Rubart and Semmler (2002) present empirical evidence that in fact for both the U.S. as well Europe such externalities of the growth of knowledge exist, which are stronger for high-skilled than low-skilled labor and thus one obtains $(\xi - \epsilon) > 0$.

Although because of the complexity of this extended model, the dynamic behavior of this model concerning thresholds and multiple equilibria cannot be completely studied, yet the model and the empirical evidence in Greiner,

[29]Note that β_1 and β_2 represent:

$$\beta_1 = \frac{\sigma_p - 1}{\sigma_p}(\xi - \epsilon) \Rightarrow (\xi - \epsilon) \approx \frac{\beta_1}{(\beta_1 - 1)}\beta_1 \text{ and } \beta_2 = \frac{1}{\sigma_p}.$$

Rubart and Semmler (2002) clearly implies that education-based inequality will be exacerbated by spillover effects at the workplace.

7 Conclusions

Recent research on inequality, in particular as put forward by Brock and Durlauf (2000 a,b) and Durlauf (1999 a,b, 2000), has directed our attention to the fact that the composition and behavior of groups of which a person is a member plays an important role for socio-economic outcomes. Heterogeneity across groups and group-level effects can either lead to take-offs of individuals or substantial immobility, social lock-in phenomena concerning learning, building up of skills and formation of human capital, and differences in status across generations. In the present paper three major mechanisms were explored that can give rise to such a creation of persistent inequality over time. We see such mechanisms arising from externalities and locally increasing returns to scale in the formation of human skills, credit constraints and credit cost in financing the formation of skills and from the growth of knowledge, and knowledge spillover in production activities. There are surely other factors affecting inequality beyond our discussed factors, for example, public financing of education, welfare state measures, union organization at the workplace, international trade and competition etc. Yet the above mechanisms may be viewed as very essential. Those above mechanisms that can give rise to multiple equilibria, thresholds and poverty traps imply a threshold theory of competition, meaning that individuals will be able to compete successfully only if they have passed certain thresholds. Thus the aggregate outcome of inter- or intra-group dynamics can be rather resilient leading to educational and social lock-ins. Therefore Brock and Durlauf rightly discuss numerous policies that suitably should be applied when such thresholds exist.

8 Appendix 1: The Solution of the Basic Model Variants

The Hamilton-Jacobi-Bellman (HJB) equation for our problem (7) - (9) reads

$$\theta V = \max_j \left[h^\alpha - j - j^2 h^{-\gamma} + V'(h)(j - \sigma h)\right] \tag{18}$$

We can compute the steady state equilibria and the rough shape of the value function and thresholds in three steps. These three steps provide some intuition of how to compute multiple equilibria and thresholds for a dynamic decision problem such as (7) - (9). The actual computation of the value function and thresholds is, however, undertaken with dynamic programming, as suggested in Grüne, Semmler and Sieveking (2003).

Step 1: Compute the steady state candidates

For the steady state candidates, for which $0 = j - \sigma h$ holds, we obtain:

$$V(h) = \frac{f(h,j)}{\theta} \tag{19}$$

$$V'(h) = \frac{f'(h,j)}{\theta} = \frac{\frac{\partial}{\partial h}(h^\alpha - \sigma h - \sigma^2 h^{2-\gamma})}{\theta} \tag{20}$$

The steady state candidates for the stationary HJB equation are

$$-1 - 2jh^{-\gamma} + \frac{\alpha h^{\alpha-1} - \sigma - \sigma^2(2-\gamma)h^{1-\gamma}}{\theta} = 0 \tag{21}$$

Note that hereby $j = \sigma h$. Given our parameters the equation admits three steady states.

Step 2: Derive the differential equation $V^{'}$

Next, we derive the differential equation V$^{'}$ by taking

$$\frac{\partial \theta V}{\partial j} = 0;$$

We obtain

$$-1 - 2jh^{-\gamma} + V'(h) = 0$$

Solving for the optimal j and using the optimal j we get

$$V' = 1 + 2\sigma h^{1-\alpha} \pm \sqrt{(1 + 2\sigma h^{1-\alpha})^2 + 4\theta h^{-\alpha} V + h^{\gamma-\alpha} - 6} \tag{22}$$

To solve (22) we could start the iteration with steady states as initial conditions. For e, a steady state candidate, we get as initial value for the solution of the differential equation:

$$\begin{aligned} V_0 &= \int_0^\infty e^{-\theta t} g(e,j)dt \\ V_0 &= \frac{1}{\delta} g(e,j) \end{aligned}$$

Step 3: Compute the global value function by taking

$$V(h) = \max_i V_i$$

where $V(h)$ is the outer envelop of the piece-wise value function obtained through Step 2.

The more general case is, however, when the credit cost is endogenous. If we have $H(h,B)$, as in equ. (1) and thus equs. (2)-(4) hold, then the present value itself becomes difficult to treat. Pontryagin's maximum principle is not suitable to solve the problem with endogenous credit cost and we thus need to use a variant of a dynamic programming to solve for the present value and investment strategy of our problem (2) - (4).

In the general case of equ. (2)-(4) endogenous credit cost as stated in equ. (1), and shown in Figure 1, we have the following HJB-equation

$$H(h, B^*(h)) = \max_j \left[f(h,j) + \frac{dB^*(h)}{dh}(j - \sigma h) \right] \tag{23}$$

Note that in the limit case, where there is no borrowing and $N = h$, and thus the constant discount rate θ holds we obtain the HJB-equation (18). Note also that in either case B^*, the creditworthiness, the maximum amount the agent can borrow, is equal to the value of human capital $V(h)$. The HJB-equation (23) can be written as

$$B^*(h) = \max_j H^{-1} \left[f(h,j) + \frac{dB^*(h)}{dh}(j - \sigma h) \right] \tag{24}$$

which is a standard dynamic form of a HJB-equation. Next, for the purpose of an example, let us specify $H(h,B) = \theta B^\kappa$ where, with $\kappa > 1$, the interest payment is solely convex in B. We then have

$$B^*(h) = \max_j \left[f(h,j) + \frac{dB^*}{dh}(j - \sigma h) \right]^{\frac{1}{\kappa}} \theta^{-\frac{1}{\kappa}} \tag{25}$$

The equilibria of the HJB-equation (25), with $\kappa > 1$, are shown below. The algorithm to study the more general problem of equ. (25) is summarized in Grüne, Semmler and Sieveking (2003). We can also observe that the HJB equation (18) is obtained if we set $\kappa = 1$ in the HJB equation.

If the HJB-equation (25) holds with $H(B) = \theta B^{\kappa}$, the finance premium, depends on the debt of the agent. For $H(B) = \theta B^{\kappa}$ for $\kappa \geq 1$ it leads to the following equation for candidates of equilibrium steady states

$$1 + 2jh^{-\gamma} = \frac{\alpha h^{\alpha-1} - \sigma - \sigma^2(2-\gamma)h^{1-\gamma}}{\theta\kappa(h^{\alpha} - \sigma h - \sigma^2 h^{2-\gamma})^{(\kappa-1)/\kappa}} \tag{26}$$

Note that the steady state candidates are the same as in (18) if in (25), $\kappa = 1$ holds. For details of the solution, for the problem (18), and for the more complicated case (25), see Grüne, Semmler and Sieveking (2003).

8.1 Appendix 2: Dynamics with Consumption

In this appendix we briefly want to demonstrate of how our model of section 3 is nested in a more general model with utility functional. The essential feature of the more general model is that the study of the problem of creditworthiness and the present value borrowing constraints can be separated from the consumption problem.

We start with the more general problem where both c and j are control variables. In order to optimize the utility functional

$$\hat{U}(c) := \int_0^{\infty} e^{-\theta t} U(c(t))dt$$

we have to solve

$$\begin{cases} \max_{c,j} \int_0^{\infty} e^{-\theta t} U(c(t))dt, & \\ \dot{h} = i(h,j); & h(0) = k_0 \\ \dot{B} = \theta B + c - f(h,j); & B(0) = B_0, \\ \lim\limits_{t\to\infty} e^{-\theta t} B(t) = 0 & \end{cases} \qquad (P_F(h_0, B_0))$$

where j and c are control variables and where U is a strictly monotone increasing instantaneous utility function.

This problem can be separated into two optimization problems.

1) Solve the investment problem for $h_0 \in R_+$

$$\begin{cases} \max_j \int_0^{\infty} e^{-\theta s} f(h(s), j(s))ds, & \\ \dot{h} = i(h,j); & h(0) = h_0. \end{cases} \qquad (P_I(h_0))$$

By using an optimal solution $(h^*(t), j^*(t))$ of $(P_I(h_0))$ we define the wealth of the economy at time $t = 0$ by $\omega^* := \int_0^\infty e^{\theta s} f(h^*(s), j^*(s))ds - B_0$.
2) Solve the problem of optimal consumption for given (h, j, B_0), and $\omega \in R_+$

$$\begin{cases} \max_{c, \bar{c} \leq \omega} \int_0^\infty U(c(s)) e^{-\theta s} ds, \\ \dot{B} = \theta B + c - f(h, j), \qquad\qquad B(0) = B_0, \\ \lim\limits_{t \to \infty} e^{-\theta t} B(t) = 0 \end{cases} \qquad\qquad (P_C(\omega, h, j, B_0))$$

where $\bar{c} := \int_0^\infty e^{-\theta s} c(s) ds$. We denote a solution of $(P_C(\omega, h, j, B_0))$ by $(B^*(t), c^*(t))$.

In Sieveking and Semmler (1998) it is shown that $(\widetilde{h}, \widetilde{B}, \widetilde{j}, \widetilde{c})$ is an optimal solution of $(P_F(h_0, B_0))$ if and only if $(\hat{h}, \overline{j})$ is an optimal solution of $(P_I(h_0))$ and $(\widetilde{B}, \overline{c})$ is an optimal solution of $(P_C((\widetilde{\omega}, h, j, B_0))$, where $\overline{\omega} := \int_0^\infty e^{-\theta s} f((\widetilde{h}(s), (\widetilde{j}(s))ds - b_0$.

A further analytical treatment why and under what conditions such problems can be separated as well as an example for the case of a utility function of CRRA type are given in Sieveking and Semmler (1998).

We next numerically investigate the role of consumption. We study for $H(h, B)$ from (11), with $\alpha_2 = 100$ the case when the agent's net income f is reduced by a constant consumption $c(t) \equiv \eta$, for example paid out in each period. In this case the present value curve $V(h)$ may become negative at some low level of the stock. This means that there is an initial level of the stock, required – the level of the stock where the present value curve becomes positive – that supports the consumption path $c(t) = \eta$. For all levels of capital stock below this size the consumption path $c(t) = \eta$ is not supported.

Note that for the linear model from sect. 3 system (7)-(9) subtracting a constant η from f simply results in an optimal value function $V_\eta = V - \frac{\eta}{\theta}$. Since for $\alpha_2 = 100$ the present value curve $V(h)$ for $H(h, B)$ from (11) is very close to the model from sect. 3 we would expect much the same behavior. Figure 9 shows that this is exactly what happens here.

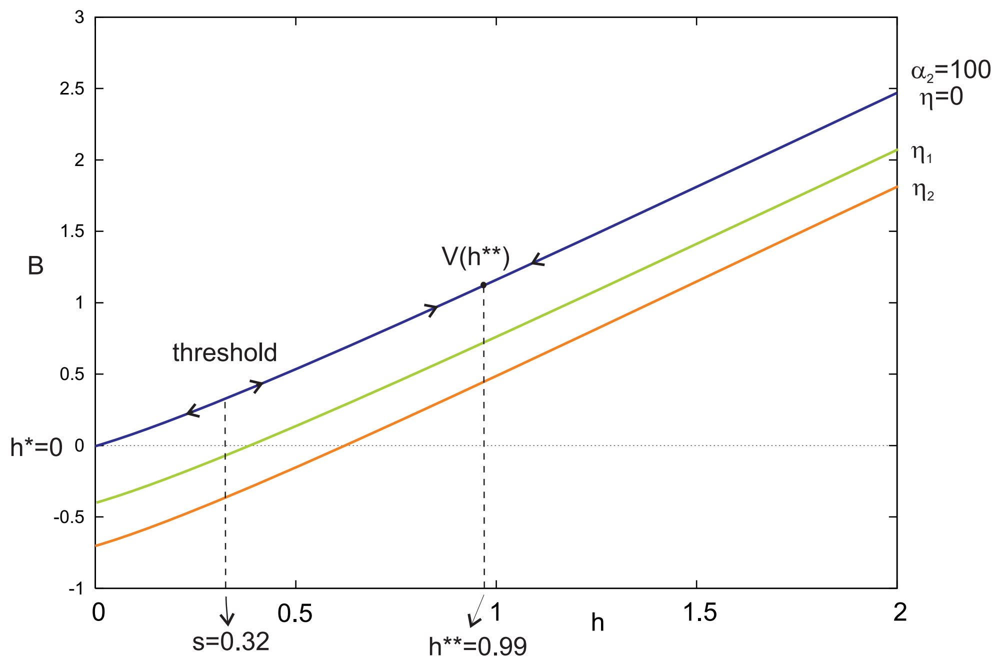

Figure 9: Present value curve $V(h)$ for different η, $H(h, B)$ from (11)

The fact that the curves here are just shifted is also reflected in the stable equilibria and the threshold, which do not change their positions. In particular, the dynamic behavior does not depend on the consumption rate. This result holds as long as the dynamic decision problem of maximizing the present value of the income flow of the agent can be separated from consumption decision as stated above.[30]

[30]For more details, see Sieveking and Semmler (1998).

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