Public Policy for Economic Growth: Theory and Empirics *

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September 2006

Abstract

This paper responds to the development policy debate involving the World Bank and the IMF on the use of fiscal policy not only for economic stabilization but also to promote growth and the increase of per capita income. A key issue in this debate relates to the effect of the composition of public expenditure, and its financing, on economic growth. The current paper lays out a research strategy to explore the effects of fiscal policy, including the composition of public expenditure, on economic growth, using a time series approach. Based on the modeling strategy by Greiner, Semmler and Gong (2005) we suggest a general model that includes (a) recurrent expenditure on education and health (which influences human capital), as well as expenditure on (b) public infrastructure investment, (c) transfers and consumption of public goods, and (d) public administration. The model can be used to explore the impact of expenditure composition on long run per capita income. Debt and external aid financing are also possible in the general model.

A simplified model is proposed, numerically solved and the out of steady state dynamics studied. The model is then calibrated and the impact of the composition of public expenditure on the long run per capita income explored for low, lower-middle and upper-middle income countries. Policy implications are spelled out, the extension to an estimable model indicated and a debt sustainability test proposed.

*We want to thank Lars Gruene for extensive help in the numerical part of the paper and the audience at the World Bank Workshop on "Growth and Fiscal Policy", June 2006, for useful comments.

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1 Introduction

It has, by now, been recognized that there are many forces that seem to work together to enhance growth and development. Forces that have been considered in recent studies are investment in physical capital, investment in education and human capital, technology and growth of knowledge, infrastructure investment, international trade and so on. Cross-country regressions have explored a large number of forces of growth, but they face methodological perils such as the huge heterogeneity of countries (different technologies and preferences across countries), uncertainty of presumed underlying models and parameters, non-robustness of the outcomes of cross-country regressions, and ambiguous policy implications.

As argued in Greiner, Semmler and Gong (2005), a time series perspective on economic growth maybe more useful to pursue for growth and development strategies. Growth models as advocated in the above book, can (1) allow for a more specific micro behavior of economic agents (though the macro environment is important), (2) allow to pursue time series studies for particular countries or country groups (to study the forces of growth for countries at particular stages of economic growth), (3) permit to be treated by econometric time series methods, and (4) allow to spell out (though mostly in the context of small scale models) important implications for growth and development policies.

Since those models, which lend themselves to some time series tests, are often difficult to treat when a larger number of forces of growth are introduced, Greiner, Semmler and Gong (2005) have suggested to estimate particular countries, in particular stages, where specific forces are important. They mainly consider externalities and learning from others, education and human capital, the creation and accumulation of knowledge and public infrastructure as forces of economics growth in a time series context. Overall, this approach appears to allow for a better study of the integration of public policy and economic growth.

Encouraged by some recent work by Glomm and Rioja (2006) and Agenor et al. (2006)

and country studies undertaken by World Bank research[1], the current paper attempts to build up and to explore a model that includes both education and human capital as well as public investment and their effect on economic growth. A major interest of this paper is to explore, as the Development Committee (2006) suggests, whether there is policy space to promote growth and welfare for low and medium income countries. Given that time series models, studying forces of growth in a historical context, are rather complex and require high quality time series data to be estimated, we pursue here what has been called calibration technique. However, since we only consider this paper as a first step toward a further endeavor to estimate such models, we merely indicate how model variants may be estimated using calibrations.

In section 2 we sketch a rather general model of social planner's type that includes besides the accumulation of physical capital, both education and human capital as well as public infrastructure investment and public capital. The general model allows also for subsistence production and includes a foreign sector (allows for foreign aid received and accounts for foreign debt). In the current version the public budget is presumed to be balanced at all times.[2] The more general model, thus, allows us to keep track of outside debt. This general model is geared toward capturing many important factors of growth, but is too complex for an empirical study.

In section 3, therefore, we leave aside subsistence production, the foreign sector and a number of decision variables (by fixing less relevant decision variables). This simplified model allows us to study the impact of a smaller number of choice variables on economic growth, both theoretically as well as empirically. We solve this simplified model by employing the Hamiltonian and associated first order conditions. We also study the out of steady state dynamics of the simplified model and show how it can be turned into an

[1]See the papers by Arestoff and Hurlin (2006), Balonos (2005), Ferreira and Arajo (2006) and Suescun (2005).

[2]See Greiner, Semmler and Gong (2005, ch. 6) for a model with public deficit and debt. The current model can be extended in this direction. However, due to complexity this extension is left aside in the current model.

estimable model using time series methods.

Section 4 undertakes first a calibration of the model by matching actual differences in per capita income for three groups of countries (for low income, lower-middle-income and upper-middle-income countries). Then the effects of a change in foreign aid and other factors impacting growth are explored. Section 5 undertakes a similar calibration exercise, here, however, by considering the growth and welfare effects arising from a change in the composition of the public expenditures. Section 6 discusses sustainability tests of fiscal policy. Section 7 concludes the paper.

2 A Model of Fiscal Policy and Economic Growth

The model is based on the work by Greiner, Semmler and Gong (2005) but takes into account some of the generalizations that have been put forward by Corsetti and Roubini (1996) and others. In the subsequent model we have included both human capital investment as well public infrastructure investment.

2.1 A General Model

Corresponding to our aim to study a growth model with education and human capital and infrastructure, a first important feature of our model is that we consider four types of public expenditure: (i) public expenditure to enhance education and build up of human capital, (ii) public infrastructure investment to finance general, market and subsistence production (transport system such as roads, bridges, harbors, water supply, sanitation, infrastructure for health care and education), (iii) transfers which may enter into households' preferences and (iv) public consumption representing expenditure with public goods' characteristics (necessary for the functioning of the government). These four expenditure streams allow us to consider the composition effect of public expenditure on growth - as has been suggested in many recent studies.

Since public investment leads to the build up of public capital, we presume that public capital can be used for (i) the production of market goods, (ii) subsistence production and

(iii) facilitating the creation of human capital. Moreover, we also take into account capital inflow (investment from abroad), foreign debt and foreign transfers. The latter we define as foreign aid and consider its effect on growth particular for low income countries. Finally, to keep the analytics simple in this version of our model, we have assumed a balanced budget for domestic spending. To simplify preferences we will suggest a logarithmic welfare function.

When we define the variables, all variables are in per capita form and we define public capital as non-excludable but subject to congestion in the production (of market goods and of goods made by subsistence production) because it is $g = G/L$ which affects per capita output. We want to work with a model of the social optimum. We presume a planner solves

$$\max_{c,e_p,i_p^f,u_1,u_2,v_1,\alpha_1,\alpha_2,\alpha_3} \int_0^\infty e^{-(\rho-n)t} \left(\frac{(c c_h^{\eta_1} (\alpha_2 e_p)^{\eta_2})^{1-\sigma}}{1-\sigma} - 1 \right) dt \tag{1}$$

subject to

$$\begin{aligned}
\dot{k} &= Ak^\alpha (u_1 h)^\beta (v_1 g)^\gamma - c - e_p - (\delta_k + n)k & (2)\\
\dot{h} &= ((1 - u_1 - u_2)h)^\epsilon ((1 - v_1)g)^{1-\epsilon} - \delta_h h & (3)\\
\dot{b} &= (r - n)b + i_p^b - (1 - \alpha_1 - \alpha_2 - \alpha_3)e_p & (4)\\
\dot{g} &= i_p^b + \alpha_1 e_p - (\delta_G + n)g & (5)\\
c_h &= (u_2 h)^\omega (v_1 g)^{1-\omega} & (6)
\end{aligned}$$

All variables are per-capita variables. n : population growth rate, c_h : subsistence production, A : constant, k : physical capital, u_1 and u_2 give that part of human capital h employed in the production of the market goods, eqn. (2) and in the creation of new human capital, eqn. (3). The rest is used for subsistence production c_h (see eqn. (6)).

The amount of resources absorbed by the public sector, e_p, is and used for public infrastructure ($i_p = \alpha_1 e_p$), transfers and public consumption ($c_p = \alpha_2 e_p$), and the functioning of the public sector ($tr = \alpha_3 e_p$). The latter has neither utility nor productive effects, but possesses public goods features and is necessary for the functioning of the state. Finally for the debt service we have $t = (1 - \alpha_1 - \alpha_2 - \alpha_3)e_p$.

When infrastructure investment is turned into public capital, we can think of two uses of the public capital: First, there exists public capital which raises productivity of both market production and subsistence production, such as transport systems (roads, bridges, harbors), water supply, sanitation. In the model, that part is $v_1 g$. (and may be split further in a fraction going to market production and subsistence production). The rest of public capital, $(1-v_1)g$, is used for facilitating the formation of human capital. We may think for example about schooling or, in a broader sense, including health services like hospitals etc. In our view, the formulation with one public capital stock, which is divided between use in production (subsistence and market) and human capital formation, is sufficient.

In a first version, we would consider an exogenous growth model, $\alpha + \beta + \gamma \leq 1$, e.g. $\alpha = \beta = \gamma = 1/3$. Maybe one could introduce here some externalities, as in Greiner, Semmler and Gong (2005, ch.3) to obtain some endogenous growth, but here we have to leave aside such extensions. Note that for simplicity we assume an additively separable utility function such as $U(\cdot) = \ln c + \eta_1 \ln c_h + \eta_2 \ln(\alpha_2 e_p)$.

Our formulation of public debt implies that public debt is foreign public debt. New borrowing from abroad is represented by i_p^b. Hereby interest payments on foreign debt (for example foreign bonds) is rb=interest rate payment (at world interest rate) that does not go to the domestic household sector, but goes to foreigners. The usual transversality condition is assumed to hold (see section 2.2).

Although the above model looks rather high-dimensional with nine decision variables and four state variables, it has the advantage that some control variables such as for example, u_1, u_2, v_1, as well as α_1 and α_2 – maybe also i_p^b – can be fixed as constants and then solutions can be explored in a comparative static analysis and the model calibrated. This is in fact what we do. This allows us to make some inference not only of what public expenditures are most effective and thus should have priority (transfers, public consumption or public investment), but also what type of public capital is most advantageous.

Moreover, the model can also be explored not only by a comparative static analysis, looking at some steady state outcome, but also looking at a dynamic programming solution for situations "out-of-the-steady state". For example one could study with this method, what type of state expenditure, what type of investment, and what type of public capital stock should be given priority for a capital stock (and income) below some steady state of a country or country group. The exploration of the dynamics of such a model is undertaken by a specially developed dynamic programming method (see sect. 3.3).

Moreover, one can collect and construct time series data for those relevant variables and stylized facts on the coefficients in order to estimate and calibrate some model variants. Next we discuss conditions under which the fiscal policy introduced here will be sustainable and the transversality condition will hold.

2.2 Conditions for Debt Sustainability

The general model of section 2.1 permits to borrow from capital markets, in particular to borrow from abroad in order to build up public infrastructure. In this section, we want to explore the effects and implications of government borrowing from capital markets and briefly discuss the sustainability of fiscal policy when borrowing is allowed for.[3]

Let us assume that the government borrows from abroad for investing in public infrastructure. The implication of it is that along the transition path, this raises the growth rate of public infrastructure and leads to a higher infrastructure capital. This higher infrastructure capital brings a distortion into the model by raising the marginal product of private capital. As a consequence, the investment share is increased and the growth rate of consumption rises implying higher welfare after a sufficiently long adjustment period. Note that this also leads to higher growth of physical and human capital. However, in our exogenous growth model these growth effects only hold on the transition path. In the long-run, higher public investment leads to higher levels of output and consumption but

[3] In Greiner, Semmler and Gong (2005, ch. 6) more particular model versions are developed with government borrowing for specific government expenditures.

does not affect the growth rate of endogenous variables.

Since here borrowing from abroad equals loans or issuing of bonds to foreigners, the government must pay it back plus interest payments. More concretely, the government has to stick to the intertemporal budget constraint which can be written as

$$b(0) = \int_0^\infty e^{-\int_0^\tau (r(\mu)-n)d\mu} s(\tau)d\tau \leftrightarrow \lim_{t\to\infty} e^{-\int_0^t (r(\tau)-n)d\tau} b(t) = 0, \tag{7}$$

where s denotes the primary surplus which, in our model, is given by $(1-\alpha_1-\alpha_2-\alpha_3)e_p - i_p^b$. The first term in (7) states that per-capita government debt at time zero must equal the discounted stream of future primary surpluses. This implies that the government must run primary surpluses if it starts with a positive stock of debt. Equivalent to this formulation is the requirement that discounted debt converges to zero asymptotically. It should be mentioned that in theoretical exogenous growth models the intertemporal budget constraint is met, provided the interest rate exceeds the population growth rate, because per-capita debt converges to a finite value. Nevertheless, it is important for real world economies to test whether fiscal policies of countries are such that they fulfill the inter-temporal budget constraint or not.

One possibility to test for sustainability is to analyze the response of the primary surplus to GDP ratio to variations in the debt-GDP ratio. To see that a positive linear dependence of the primary surplus ratio on the debt ratio can guarantee sustainability, we assume that the primary surplus ratio is given by

$$\frac{s(t)}{y(t)} = \alpha + \beta(t)\,\frac{b(t)}{y(t)}\,, \tag{8}$$

with $y(t)$ per-capita GDP. $\beta(t)$ determines how strong the primary surplus reacts to changes in the public debt-GDP ratio and α, which is assumed to be constant, can be interpreted as a systematic component determining how the level of the primary surplus reacts to a rise in GDP. α can also be seen as representing other constant variables which affect the primary surplus-GDP ratio.

Using equation (8) the differential equation describing the evolution of public debt can be rewritten as

$$\dot{b}(t) = (r(t)-n)b(t) - s(t) = (r(t)-n-\beta(t))\,b(t) - \alpha\,y(t). \tag{9}$$

Solving this differential equation and multiplying both sides by $e^{-\int_0^t (r(\tau)-n)d\tau}$ to get the present value of public debt yields

$$e^{-\int_0^t (r(\tau)-n)d\tau} b(t) = e^{-\int_0^t \beta(\tau)d\tau} \left(b(0) - \alpha\, y(0) \int_0^t e^{\int_0^\tau (\beta(\mu)-r(\mu)+n+\gamma(\mu))d\mu} d\tau \right), \tag{10}$$

with γ the growth rate of GDP. Equation (10) shows that a positive value for β on average, so that $\int_0^t \beta(\tau)d\tau$ converges to plus infinity asymptotically, is necessary for sustainability. It is also sufficient if $r(\mu) - n - \gamma(\mu)$ is positive on average. [4]

It should be noted that a positive value for $r-n-\gamma$ characterizes a dynamically efficient deterministic economy. If the economy is stochastic, however, this need not necessarily hold for the economy to be dynamically efficient. Nevertheless, testing the reaction of the primary surplus to variations in the debt ratio is reasonable because a positive reaction is necessary for sustainability as mentioned above. Further, this test yields insight how governments deal with public debt and, thus, shows how important the goal of stabilizing debt is for the government.

One advantage of the sustainability test just presented is to be seen in the fact that it does not depend on the interest rate, in contrast to other tests where the assumed interest rate may be crucial as to the outcome whether a given policy is sustainable or not. In addition, the proposed test is intuitively plausible from an economic point of view. If a government runs a deficit it has to run a primary deficit in the future to pay back the deficit. Otherwise, sustainability is not given. In our model economy, this implies a withdrawal of resources from the economy. Therefore, a deficit financed increase in public investment is beneficial for the economy if the gain in productivity is sufficiently high to cover the interest payments and the loan. A detailed discussion of different types of sustainability tests and how the above suggested sustainability test can be implemented is undertaken in section 6 of the paper. Next we introduce a simplified version of the model in order to study its dynamic behavior.

[4] This holds because l'Hôpital gives the limit of the second term in (10) multiplied by $e^{-\int_0^t \beta(\tau)d\tau}$ as $e^{-\int_0^t (r(\mu)-n-\gamma(\mu))d\mu}/\beta$.

3 A Simplified Model

We introduce now a simplified model which might already be helpful for growth and policy decisions. This model simplifies matters by reducing the number of choice variables to 3, leaving aside borrowing from abroad and disregarding subsistence production. We thus presume a model without public debt, without subsistence production and where the control variables are c, e_p and u_1 only. The terms $i_p^f, v_1, u_2, \alpha_1, \alpha_2, \alpha_3$ then are taken as parameters. Note that here foreign debt drops out and here i_p^f represents foreign aid which helps to build up domestic infrastructure capital.

3.1 The Model and its Steady State Solution

The model, then, becomes

$$\max_{c,e_p,u_1} \int_0^\infty e^{-(\rho-n)t} \left(\frac{\left(c\left(\alpha_2 e_p\right)^\eta\right)^{1-\sigma}}{1-\sigma} - 1 \right) dt \tag{11}$$

subject to

$$\dot{k} = Ak^\alpha (u_1 h)^\beta (v_1 g)^\gamma - c - e_p - (\delta_k + n)k \tag{12}$$

$$\dot{h} = ((1-u_1)h)^{\epsilon_1} ((1-v_1)g)^{\epsilon_2} - \delta_h h \tag{13}$$

$$\dot{g} = i_p^f + \alpha_1 e_p - (\delta_G + n)g \tag{14}$$

Note that

$$\alpha_1 + \alpha_2 \leq 1$$

must hold. If $\alpha_1 + \alpha_2 = 1$ holds, public spending is used for investment in infrastructure ($i_p = \alpha_1 e_p$) and for public goods which yield utility ($c_p = \alpha_2 e_p$). If $\alpha_1 + \alpha_2 < 1$ holds, a certain part of public spending is used for non-productive public spending which does not yield immediate utility, e.g. spending for the functioning of the public sector or simply waste of resources. A further simplification is obtained by taking u_1 also as a parameter, then c and e_p are the only control variables.

Next we explore the stationary state of the model in its simplified form. We employ the Hamiltonian to sketch a solution of the model. The Hamiltonian is

$$
\begin{aligned}
H(c, e_p, k, h, g) &= ln\ c + \eta ln(\alpha_2 e_p) \\
&+ \lambda_1[Ak^{\alpha}(u_1 h)^{\beta}(v_1 g)^{\gamma} - c - e_p - (\delta_k + n)k] \\
&+ \lambda_2[((1-u_1)h)^{\epsilon_1}((1-v_1)g)^{\epsilon_2} - \delta_h h] \\
&+ \lambda_3[i_p^f + \alpha_1 e_p - (\delta_g + n)g]
\end{aligned} \tag{15}
$$

with $\lambda_1, \lambda_2, \lambda_3$ the co-state variables. The first order conditions for the two choice variables, c and e_p, are:

$$\frac{\partial H}{\partial c} = 0 \quad \Rightarrow \quad c^{-1} = \lambda_1 \tag{16}$$

$$\frac{\partial H}{\partial e_p} = 0 \quad \Rightarrow \quad \eta e_p^{-1} = \lambda_1 - \lambda_3 \alpha_1 \tag{17}$$

For the co-state variables we have

$$\dot{\lambda}_1 = \lambda_1(\rho - n) - \frac{\partial H}{\partial k} = \lambda_1(\rho - n) - \lambda_1[A\alpha k^{\alpha-1}(u_1 h)^{\beta}(v_1 g)^{\gamma} - (\delta_k + n)] \tag{18}$$

$$
\begin{aligned}
\dot{\lambda}_2 = \lambda_2(\rho - n) - \frac{\partial H}{\partial h} = \lambda_2(\rho - n) &- \lambda_1[Ak^{\alpha}\beta(u_1 h)^{\beta-1}u_1(v_1 g)^{\gamma}] \\
&- \lambda_2[\epsilon_1((1-u_1)h)^{\epsilon_1 - 1}(1-u_1)((1-v_1)g)^{\epsilon_2} - \delta_h]
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
\dot{\lambda}_3 = \lambda_3(\rho - n) - \frac{\partial H}{\partial g} = \lambda_3(\rho - n) &- \lambda_1[Ak^{\alpha}(u_1 h)^{\beta}\gamma(v_1 g)^{\gamma-1} \cdot v_1] \\
&- \lambda_2[((1-u_1)h)^{\epsilon_1}((1-v_1)g)^{\epsilon_2-1}\epsilon_2(1-v_1)] \\
&+ \lambda_3(\delta_g + n)
\end{aligned} \tag{20}
$$

Equations (16)-(20), which are derived from the two first order conditions with respect to the choice variables c and e_p and the three equations for the co-state variables λ_1, λ_2, and λ_3, give us, together with the three state variable equations (12)-(14), a system of eight equations in eight variables $\{c, e_p, k, h, g, \lambda_1, \lambda_2, \lambda_3\}$.

Writing eqns. (16) and (17) as

$$0 = \lambda_1 - c^{-1} \tag{16'}$$

$$0 = \lambda_1 - \lambda\alpha_1 - \eta e_p^{-1} \tag{17'}$$

and setting equations (18)-(20) as well as (12)-(14) equal to zero, we can obtain a stationary state $\{c^*, e_p^*, k^*, h^*, g^*, \lambda_1^*, \lambda_2^*, \lambda_3^*\}$. Using then the production function $y = Ak^\alpha(u_1h)^\beta(v_1g)^\gamma$ and k^*, h^* and g^*, we also obtain the per capita income, y^*, at the stationary state.

Furthermore, given our parameter set α_1, α_2 and $\alpha_3 = 1 - \alpha_2 - \alpha_3$, we can explore the impact of the components of public expenditure α_1, α_2 and α_3 on per capita income. We also can explore what impact the reallocation of public capital, for the use in final production, v_1, and the creation of human capital, $(1-v_1)$, has on the long run per capita income. We also can study the impact of v_1 on per capita income. A further effect on the per capita income is predicted to be caused by a change in foreign aid, if $i_p^f > 0$.

Overall, from all of those comparative static studies we expect to obtain information on the per capita income and consumption. We also explore the effects arising from the change of the composition of the public budget.

Using the computer software *Mathematica*, from necessary optimality conditions for the simplified model with a log utility function, i.e. $U = ln\ c + \eta\ ln(\alpha_2 e_p)$, the following stationary solution is obtained:[5]

$k^* = 65.33$, $h^* = 18.16$, $g^* = 11.45$

The parameter values used are: $n = 0.02$, $\rho = 0.03$ $\delta_k = 0.075$, $\delta_h = 0.075$ $\delta_g =$

[5]The *Mathematica* code is available at www.newschool.edu/gf/cem.

$0.05,\ \alpha = \beta = \gamma = 0.33$. Further we have used, $u_1 = v_1 = 0.85, \epsilon_1 = \epsilon_2 = 0.2,\ \alpha_1 = 0.1,\ \alpha_2 = 0.7,\ i_p^f = 0.05,\ A = 1,\ \eta = 0.1$. With this solution technique we can undertake a comparative static analysis and some important calibrations, see sects. 4 and 5.

3.2 Out of Steady State Solutions

Since economies are rarely at their steady states, it is, for practical purposes, of great importance to explore what decisions should be taken out of the steady state. There is, however, no analytical solution for our decision variables out of the steady state. Yet, once the parameters have been set and the stationary state of the variables $\{c, e_p, k, h, g, \lambda_1, \lambda_2, \lambda_3\}$ have been computed, the dynamic programming algorithm developed by Gruene and Semmler (2004) can be used to study the out of steady state dynamics not only of consumption, c, and the use of resources by the public, e_p, but also the state variables: physical capital stock, human capital and public capital stock. Using our parameter set above, next we will compute the out of steady state solutions using a numerical algorithm. This way we can study the following: When, for example, the capital stock of a country is $k < k^*$ (and $h < h^*$, $g < g^*$) the dynamic programming solution allows one to judge whether c should be high or low, how e_p behaves, and how the control variables relate to each other out of the steady state.

Using the algorithm of Grüne and Semmler (2004) we have solved the model (11)-(14), with u_1 fixed, and thus solely with c and e_p as control variables and k, h and g as state variables. Yet since the value function (11) is a function of the decision variables c and e_p and the state variables k, h and g (the former again depending on k, h and g) one can only obtain proper graphs by appropriate projections and study the value function and behavior of decision variables in a two-dimensional subspace.

We have solved the model (11)-(14) in the vicinity of the steady state $k^* = 65.47$, $h^* = 18.16$, $g^* = 11.47$ which was obtained by solving the equation system arising from the Hamiltonian in sect. 3.1.[6] Next, in the vicinity of the positive steady state we have

[6]There is, however, a second steady state which was neglected because of negative values of some state

taken a cube $[60, 70] \times [16, 20] \times [9, 13]$ for k, u and g, with a subspace for the choice variables $[7, 8] \times [7, 8]$, for c and e_p.

Figure 1 shows four sample trajectories in a three dimensional space, with initial conditions for trajectory 1 (T1): (66.2, 18.9, 11.8) which are above the steady state k^*, h^*, g^*. Trajectory 2 (T2) has initial conditions: (67.5, 18.7, 11.4) and is, thus, far to the right of the steady state k^*, h^*, g^*. Trajectory 3 (T3): (63.2, 16.8, 11.1) starts below the steady state k^*, h^*, g^*. Finally, Trajectory 4 (T4) starts with $k = 68.2$ above, with $h = 16.5$ below and with g at the steady state.

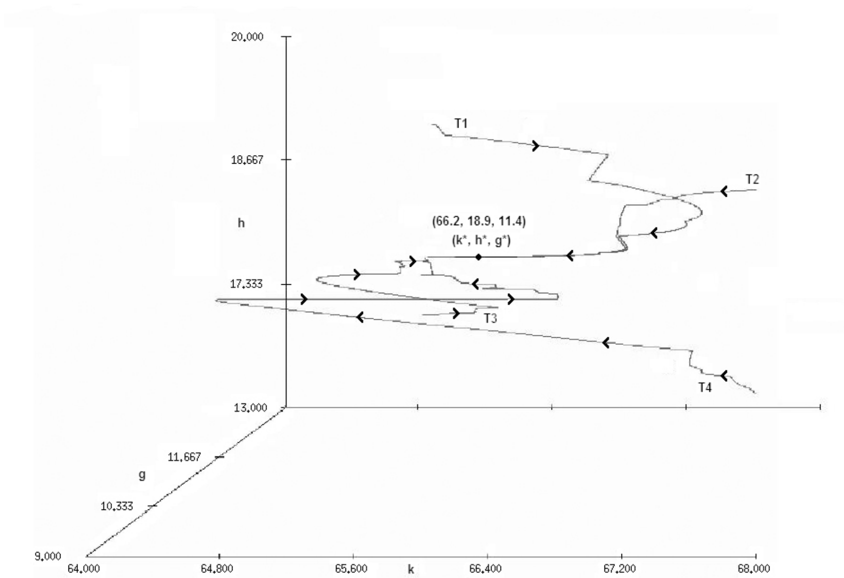

Figure 1: Out of Steady State Dynamics

As one can observe in figure 1, the correctly taken optimal decisions c and e_p make the initial states of state variables converging toward the steady state k^*, h^*, g^* for all four sample trajectories. Note that trajectories (T1), (T2), (T3) and (T4) do not only exhibit some irregular features but also cross each other. This comes from the fact that we follow the trajectories in a 2-dim subspace state variables only, namely in the $k - h$ space. For graphical purposes, the third state variable g is fixed in figure 1.

If one stores the decision variables c and e_p at each grid point of the 3 dim cube of k, h

variables.

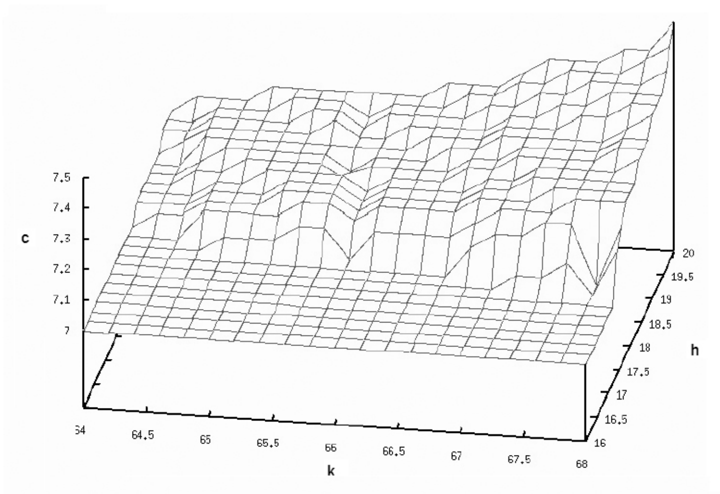

Figure 2: Decision Variable c in the $k-h$ space

and g one can then plot them in a 3-dim space where the height represents the numerical value of the decision variable. In figure 2, for example, the height stands for the value of c, and the other two axes represent k and h.

In figure 2 it is dearly visible that if $k > k^*$ and $h > h^*$ the optimal consumption c is required to be high, above its steady state. The reverse holds for the space $k > k^*$ and $h > h^*$. Here c has to be below its steady state . This behavior of the optimal c is a result that one would also expect from economic intuition.

On the other hand as figure 3 shows the decision variable e_p is not so monotonically dependent on k and h. In some regions e_p is higher than in others. Note that we have neglected here to study the behavior of c depending on government capital, g. This also can be easily done, but this might not be so relevant in the context of our study.

Next, we want to study the dependence of the total use of public resources, e_p, on private capital, k, and public capital, g.

Figure 4 shows how the decision variable e_p behaves in the $k-g$ space. We can observe that e_p will be high above the steady state, for $g > g^*$, and low below the steady state, for $g < g^*$. The decision variable e_p does not seem to depend much on the level of the capital stock, k.

Finally, we want to make a comment on the constraints that we have put on the

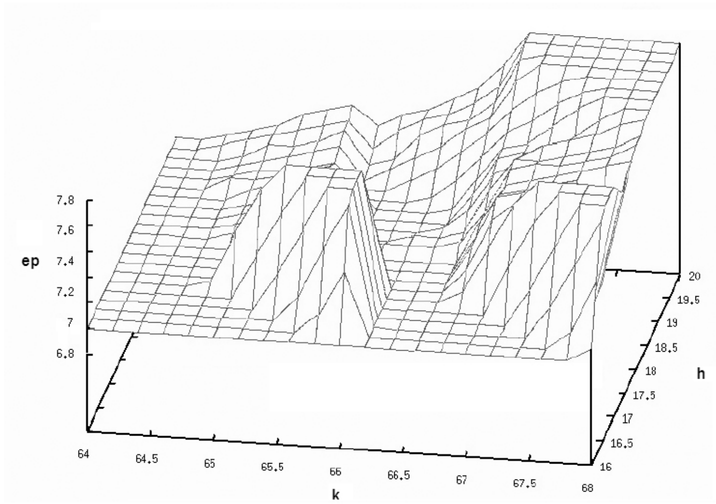

Figure 3: Decision Variable e_p in the $k - h$ space

control variables, c and e_p. For both the lower constraint is 7 and the upper is 8 so that all optimal c and e_p are staying within those constraints. But note that with $c^* = 7.04$ and $e_p^* = 7.53$ the decision variables are above the lower constraints of 7. Putting such constraints on the decision variables just means that the decision variables cannot go fast enough down and thus the state variables that are affected by this are not going up fast enough. This is the reason why the trajectory (T4) in figure 1 first moves far down (to the left) in the state variable k and then rises again moving toward its steady state value.

The above exercise with constraints for the decision variables is nevertheless of great importance for practical economic policy. It basically means that the decision variables do not have to be exactly at their optimal values in order to exhibit convergence dynamics. For actual economies an exact optimal control is indeed hard to achieve, since for actual economies, there is model uncertainty – which model fits the economy – as well as data uncertainty (see Brock, Durlauf and West, 2003). Our computations with constraints for the decision variables then means that the decision variables need to be only above or below their steady state values in the appropriate state space in order to fulfill the requirement to be roughly optimal. This aspect appears to us of great practical importance, since decisions for c and e_p have to be only approximately correct.

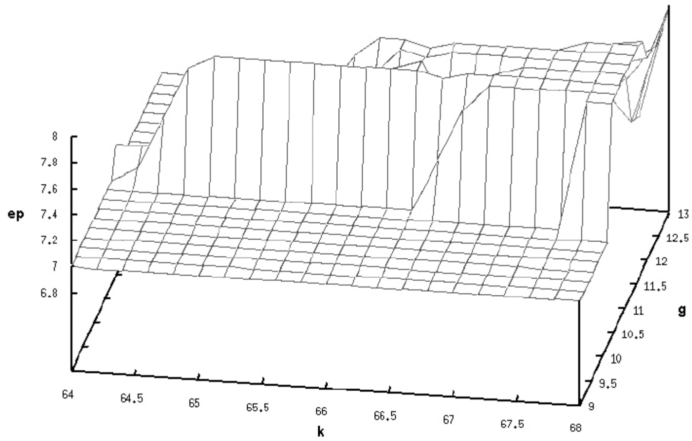

Figure 4: Decision Variable e_p in the $k-g$ space

3.3 The Estimable Form of the Model

Next we want to spell out some implications of how to estimate our model with time series methods. Even for our simplified social planner model of eqns. (11) to (14) it could be too ambitious to estimate the model employing Euler equations, derived from the first order conditions for the two control variables for consumption, c, and total use of resources, e_p, and the decision variable u_1. As the model is written, it gives us the outcome of the social optimum. We propose to reduce the model further, and treat, for the purpose of a time series estimation, only consumption as decision variable. The other choice variables, e_p and u_1 are just treated as historical variables, as time series observations. This is also likely to inject some realistic features into the model.

For a time series study we thus propose to include only one Euler equation in the time series estimation procedure. We suggest to only use the equations (12)-(14) and the equation for $\dot{c}$, which can be derived from the first order condition of the decision variable c as

$$\frac{\dot{c}}{c} = -\frac{\dot{\lambda}_1}{\lambda_1} \rightarrow \dot{c} = c\left(A\alpha k^{\alpha-1}(u_1 h)^{\beta}(v_1 g)^{\gamma} - \rho - \delta_k\right) \tag{21}$$

The model reflects now the fact that it is only optimized with respect to private consumption. The decision making process for all the public variables might be considered too complex to be presumed as the result of some optimization process. Yet, in the normative sense, as we discuss below, we still might consider the other public decision variables as choice variables so as to give us a guidance to welfare improving policies. Below, we will undertake some calibration and comparative static study in order to explore the impact of fiscal policy decisions on per capita income and welfare.

4 Exploring the Basic Structure of the Model

We want to elaborate on the effect of foreign aid, the productivity factor, the fraction of human capital used in market goods production, and the fraction of public capital used for public infrastructure on the differences in per capita income of the three groups of countries. We follow here the Development Committee (2006) in classifying countries in low-income, lower-middle-income, and upper-middle-income countries. Due to the quality of the data, however, only a reduced list of countries is used for the calibration exercise.[7] We employ only data for the time period 1994 to 2004. First, we will calibrate the effect of foreign aid, i_p^f. By looking at actual data of foreign aid per capita, we are able to determine a range in which our parameters can vary. Before we can evaluate the effect of foreign aid, productivity factor, fraction of human capital used in market goods production, and fraction of public capital used for public infrastructure on per capita income, we need to calibrate our above-mentioned model (11) to (14) such that we can roughly reproduce the differences in per capita income across the three groups of countries. Since, as discussed above, there are many forces of growth, causing the differences in per capita income, we adjust the productivity level of the three groups, the parameter A, such that we roughly obtain the actual differences in per capita income, normalizing the parameter A of the lowest per capita income group. Therefore A is set to 1 for the lowest income group. We hereby take the composition of total public expenditure, α_1, α_2 and α_3, as given for each group as we find them in the data. Now we employ our *Mathematica* program to obtain stationary state solutions for the three groups of countries.[8] This is equivalent

[7]See Appendix for the complete list of countries.

[8]The *Mathematica* code is available at www.newschool.edu/gf/cem.

to a comparative static analysis for the three country groups. Our exercise consists in keeping the parameter A constant for each group of countries and equal to its calibrated value. Then we let foreign aid, i_p^f, vary during that period and see how such a change affects the steady state values k, h, g, c, e_p, y and U. Such an exercise was conducted for the low-, lower-middle-, and upper-middle income groups. In the next step we vary the productivity parameter A for each group of countries, employing for each country group the A that we used to calibrate the model to obtain the actual differences in per capita income, and increasing it gradually. The parameter A is set equal to 1, 1.39, and 1.8 for the low-, lower-middle-, and upper-middle income groups respectively. Third, we vary u_1, the fraction of human capital used in market goods production from 0.1 to 0.8, keeping foreign aid equal to its 10-year average, and the other parameters constant. Finally, we vary v_1, the fraction of public capital used for public infrastructure from 0.1 to 0.8, keeping foreign aid equal to its 10-year average, and the other parameters constant.

4.1 Effect of i_p^f, the Foreign Aid per Capita

The comparative static results for all income groups show that k, h, g, c, e_p, y and U are linear in i_p^f and there is clearly a positive relationship between the level of foreign aid and all the variables, except for public resources per capita, e_p, which shows a negative relationship with respect to foreign aid, i_p^f. As the foreign aid goes up the public resources per capita falls. The results suggest that any increase in foreign aid per capita would increase k, h, g, c and y but would reduce the optimally chosen public resources, e_p. Actual data for the period 1994-2004 shows that the per capita foreign aid went from 2 to 4. To conduct our comparative static exercises, we increased i_p^f by increments of 0.2 from 2 to 4, which is the range provided by the actual data. The results of our comparative static exercises for the low-income group are summarized in Table 2 in appendix A.3. The results for the remaining two income groups can also be found in the appendix. Figure 5 above shows the relationship between i_p^f and y, U, and e_p respectively for the low-income group. Similar results are observed for the lower-middle and upper-middle income groups.

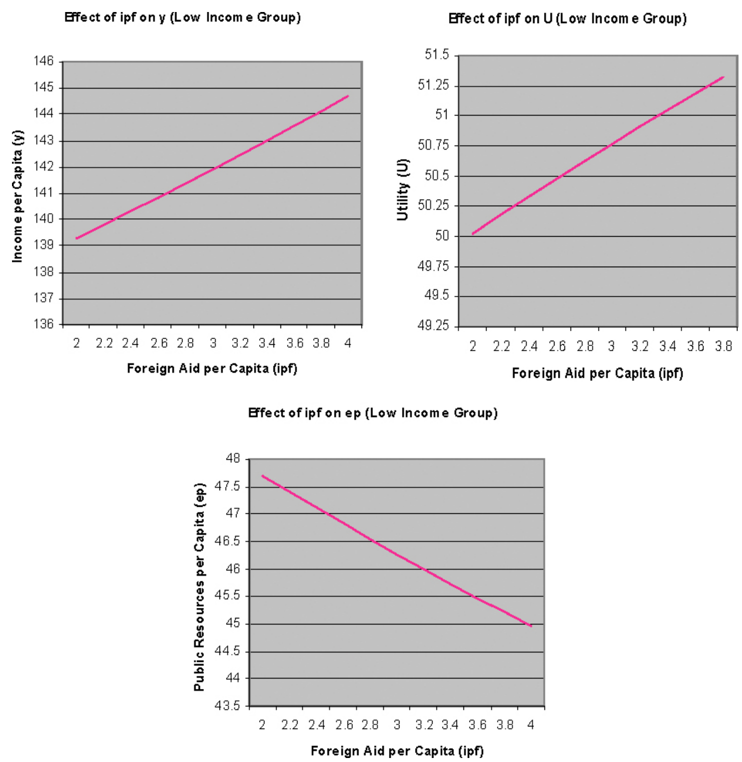

Figure 5: Effects of Foreign Aid i_p^f on the Steady State Variables (Low Income Group)

4.2 Effect of A, the productivity factor

The comparative static results for all income groups show that k, h, g, c, e_p, y and U are linear in A and there is clearly a positive relationship between the level of foreign aid and all the variables. The results suggest that any increase in A would increase k, h, g, c, e_p, y and U. Because the low income group is our reference group, A is increased from 1 to 1.06, and the effect on the steady state variables is observed. As one might predict, results show that there is a positive linear relationship between A and all the steady state values of the variables, with the fastest increase in h and U, relative to the other variables. The results of our comparative static exercises for the low income group are summarized in Table 3 in appendix A.3. The results for the other two income groups can also be found in the appendix. Figure 6 above shows the relationship between A and y, and U

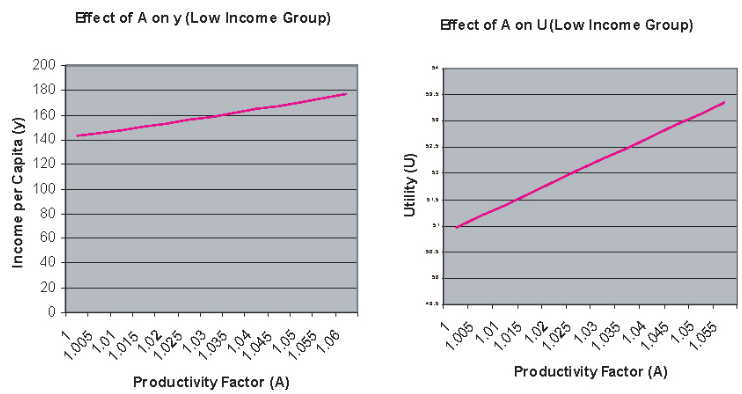

Figure 6: Effects of the Productivity Factor A on the Steady State Variables (Low Income Group)

respectively for the low-income group. Similar results are observed for the lower-middle and upper-middle income groups.

4.3 Effect of u_1, the Fraction of Human Capital Used in Market Goods Production

To analyze the effect of the parameter u_1, it was increased from 0.1 to 0.8 with increments of 0.1, and the steady state values corresponding to the different values of the parameter were recorded. The comparative static results on u_1 for all income groups show a monotonic, positive, non-linear relationship, with decreasing returns with respect to k, g, c, e_p, y, and U. However, with h, we observe a non-monotonic relationship. This hump-shape form of the relationship is characterized by a positive relationship with decreasing returns for $u_1 \leq 0.5$, and a negative relationship with increasing returns for $u_1 \geq 0.5$. It is important to point that such a relationship is common to all income groups. Such a reversal of the direction and type of the relationship could be explained by the interaction or interdependence of u_1 and v_1. It is well possible that in an actual economy, both parameters could correlated, but not necessarily move in the same direction. The results of our comparative static exercise for the low-income group are summarized in Table 4 in appendix A.3. The results for the other two income groups can also be found in the appendix.

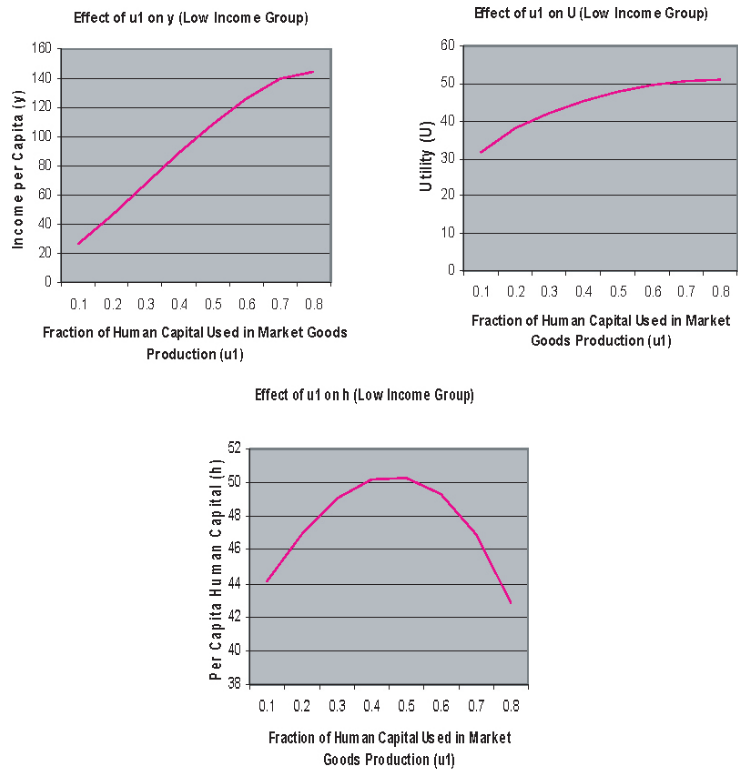

Figure 7: Effect of u_1, the Fraction of Human Capital Used in Market Goods Production Income on the Steady State Variables (Low Income Group)

Figure 7 above shows the relationship between u_1 and y, U, and h respectively for the low-income group. Similar results are observed for the lower-middle and upper-middle income groups.

4.4 Effect of v_1, the Fraction of Public Capital Used in Market Goods Production

As in our comparative static exercises for u_1, to analyze the effect of our parameter v_1, it was increased from 0.1 to 0.8 with increments of 0.1, and the steady state values corresponding to the different values of the parameter were recorded. Comparative static results on v_1 for all income groups, show a monotonic, positive, non-linear, with decreasing returns to v_1 on k, g, c, e_p, y, and U. However, with h, we observe a non-monotonic

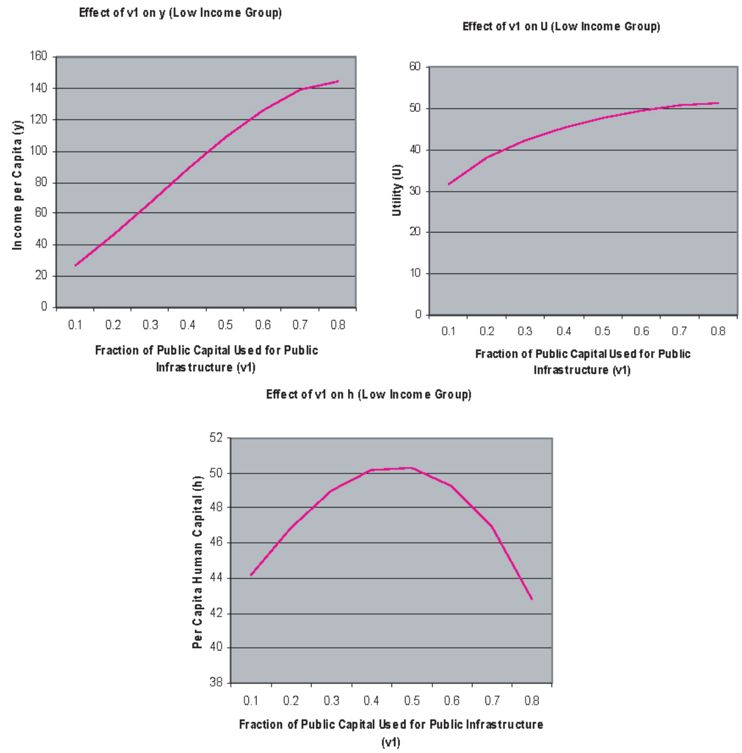

Figure 8: Effects of v_1, the Fraction of Public Capital Used for Public Infrastructure on the Steady State Variables (Low Income Group)

relationship. We observe a positive relationship with decreasing returns for $v_1 \leq 0.5$, and a negative relationship with increasing returns for $v_1 \geq 0.5$. The results of our comparative static results for the low-income group are summarized in Table (5) in appendix A.3. The results for the remaining two income groups can also be found in the appendix. Figure 8 above shows the relationship between v_1 and y, U, and h respectively.

5 Exploring the Composition Effect of Public Expenditure

In this section the simplified model is calibrated for different expenditure structures and the effects of changes in the composition of public expenditure are explored for the three country groups. As above, the classification in low-, lower-middle- and upper-middle-income countries is retained.[9] Here again only the results of the low-income groups are reported. For the other two country groups, see Appendix A.4. The data on public expenditure is obtained from the IMF's Government Finance Statistics. Since these are too detailed for our analysis, the different expenditure categories are summarized to match the following model parameters: public investment, public transfers, public consumption.[10] The three categories are normalized such that their shares can be interpreted as the model's parameters α_1, α_2 and α_3. Note that the values reflect the averages for all countries of one category.

Plugging these parameters into the simplified model, setting $\rho = 0.03$, $\delta_k = 0.075$, $\delta_h = 0.075$, $\delta_g = 0.05$, $\alpha = \beta = \gamma = 0.33$, $u_1 = v_1 = 0.85$, $\epsilon_1 = \epsilon_2 = 0.2$, and $\eta = 0.1$ and using the values for A and i_p^f that were found in the previous section, one can compute the stationary states for all variables of the model.[11]

By computing the stationary states for the different groups, one might ask the question of how changes in the expenditure composition affect the long run values of income, consumption, capital stock, etc. In the following this question is answered for the low income country group.[12] Although all graphs are presented in the 3-dimensional space such that the interaction of all three expenditure parameters can be observed at the same time, the stationary states for shifts from public transfers α_2 to public investment α_1 are of particular interest.[13] Note that the effects of shifts between public transfers α_2 and public consumption α_3 only affect the utility level, since both categories are identical in their effect on other variables, the absorption of resources from the accumulation process.

Figures 9 to 14 depict the effects of changes in the expenditure composition on the various model parameters. The figures show that, except for human capital, all variables

[9]See appendix A.2 for the country lists.

[10]See appendix A.1 for details.

[11]Note that the values for ρ, δ_k, δ_h, δ_g, α, β, γ, u_1, v_1, ϵ_1, ϵ_2, and η remain the same for the remainder of this section.

[12]Similar results hold for the other two groups. Complete sets of graphs for the other groups are listed in Appendix A.4.

[13]Note that $\alpha_1 + \alpha_2 + \alpha_3 = 1$ always has to hold which restricts the plane to a triangular space.

increase with an increase in public investment in a strictly convex manner. However, the curvature for y, co, k and ep increases at a decreasing rate once $\alpha_1 > 0.15$.

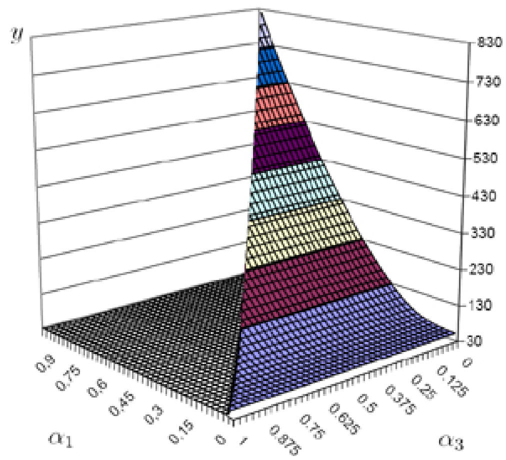

Figure 9: Effects of Expenditure Composition Changes on per capita Income y

Figure 9 depicts the effects of changes of public expenditure on per capita income. The function is clearly strictly increasing in public infrastructure investment. As mentioned above, except for utility shifts between public transfers and public consumption does not affect the long run growth prospects. Hence, one can read one of the graph's axis as an increase in α_3 or a decrease in α_2.

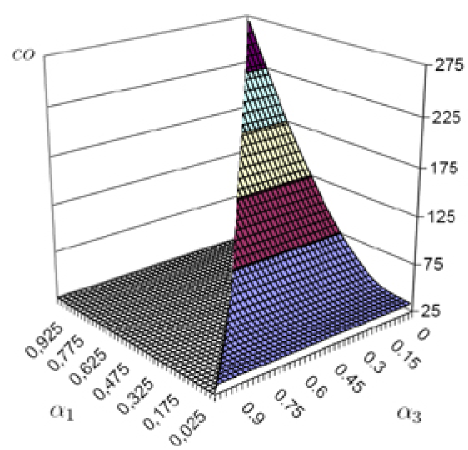

Figure 10: Effects of Expenditure Composition Changes on per capita Consumption c

Figure 10, which shows how consumption changes under different expenditure compositions, is also strictly convex. However, the increase in consumption is not as large as the increase in income, as along the α_1-axis increasing fractions of income need to be invested into infrastructure, which leaves relatively less income to consume.

The figures for capital stock (fig. 11) and public absorption (fig. 12) show qualitatively the same as the previous figures: The functions are strictly convex at an increasing rate up to $\alpha_1 \approx 0.15$ beyond which they are increasing at a decreasing rate.

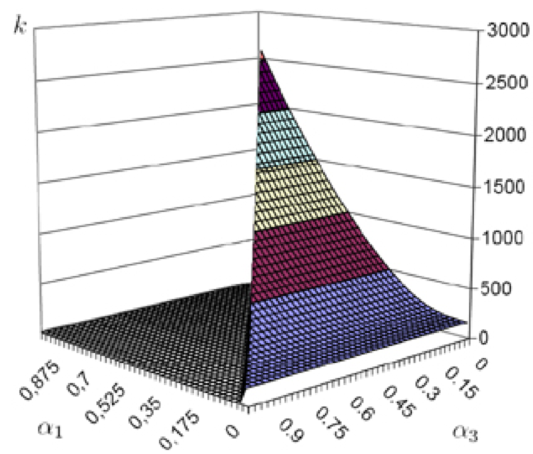

Figure 11: Effects of Expenditure Composition Changes on per capita Capital Stock k

Unlike the previous figures, figure 13 is strictly convex and increases at an increasing rate over the whole parameter range. This can be explained with the fact that overall public absorption, *ep*, increases together with a steady shift to public investment (increases in α_1).

Figure 14 depicts the effects of changes in the public expenditure composition on human capital. The curvature of the graph is not smooth as it is in the previous cases, but possesses both a convex part in the beginning and a concave part at higher levels of α_1. The turning point occurs around $\alpha_1 \approx 0.35$.

In figure 15 the per capita utility for the different public expenditure composition is depicted. It should be noted that for utility, the allocation between public transfer α_2 and public consumption α_3 matters, as only α_2 enters the utility function.

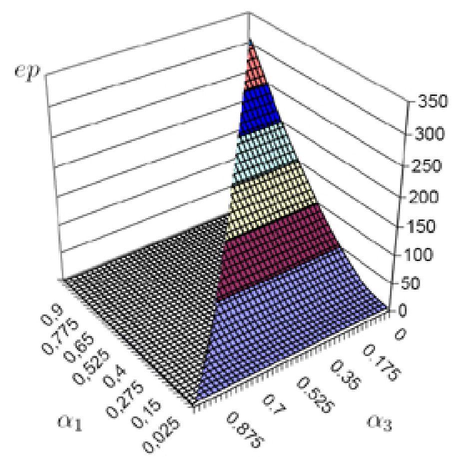

Figure 12: Effects of Expenditure Composition Changes on per capita Public Absorption ep

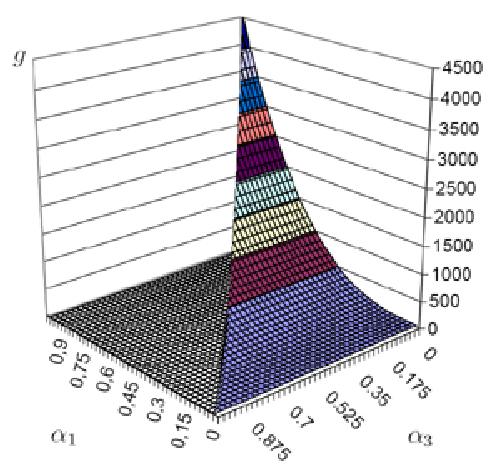

Figure 13: Effects of Expenditure Composition Changes on per capita Public Capital g

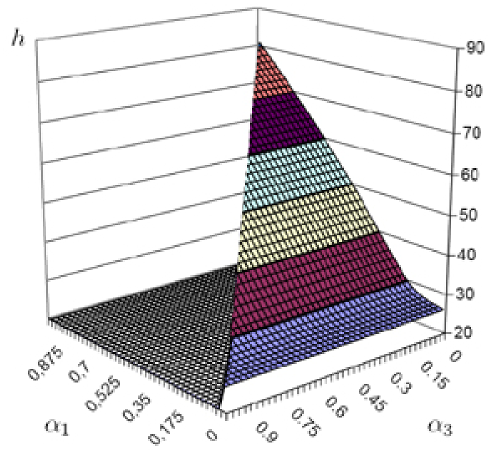

Figure 14: Effects of Expenditure Composition Changes on per capita Human Capital h

The curvature of the graphs is at low levels of public investments α_1 convex and at levels of approx. $\alpha_1 > 0.25$ concave. The reason for this is clearly rooted in the optimizing processes as the dividing line runs along α_1's values. The utility function attains its maximum at the composition of $\alpha_1 = 0.95$, $\alpha_2 = 0.05$ and $\alpha_3 = 0$.

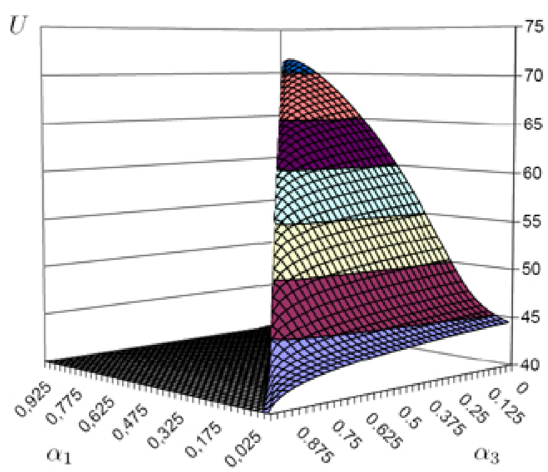

Figure 15: Effects of Expenditure Composition Changes on per capita Utility U

The aim of this section was to conduct a comparative static exercise for changes in the public expenditure composition. The results of which show that in the long run generally higher levels of public investment are more favorable than public transfers and public consumption since the latter do not contribute to the growth process described in this model. This result also holds if one considers the simple utility function assumed for the exercise, which attains its maximum at an investment share of 95%.

6 Testing Debt Sustainability

In our calibration of the three types of countries we did not explore whether the policy is sustainable. In section 2.2 we had briefly modeled of how in the context of our model the intertemporal budget constraint would look like, but we did not pursue this question further in section 4 and 5. Here we want to briefly discuss tests for sustainability of fiscal policies and suggest a version that has been tested for advanced countries. We will then suggest sustainability tests that can be undertaken for specific countries.[14].

If one studies the effect of fiscal policy on economic growth, one needs to check, as indicated in section 2.2, whether the government fulfills the intertemporal budget constraint. As shown in section 2.2, in economic terms, this constraint states that public (net) debt at time zero must equal the expected value of future present-value primary surpluses. This requirement is also often referred to as the no-Ponzi game condition. In general form we can write this as follows. Neglecting stochastic effects and assuming that the interest rate is constant the intertemporal budget constraint can be written as

$$B(0) = \int_0^\infty e^{-r\tau}\, S^p(\tau)\, d\tau, \tag{22}$$

with r the constant interest rate, $B(0)$ public debt at time zero and S^p the primary surplus.[15] Equivalent to equation (22) is the following equation

$$\lim_{t\to\infty} e^{-rt} B(t) = 0, \tag{23}$$

with $B(t)$ public debt at time t, stating that the present value of public debt converges to zero for $t \to \infty$.

[14]For a detailed discussion of sustainability tests, see Greiner, Koeller and Semmler (2005).

[15]For a derivation see e.g. Blanchard and Fischer (1989), ch. 2.

In the economic literature numerous studies exist which explore whether (22) and (23) hold in real economies.[16] For example, Hamilton and Flavin (1986) suggest to test for the presence of a bubble term in the time series of public net debt which would indicate that a given fiscal policy is not sustainable. Trehan and Walsh (1991) proposed to test whether the budget deficit is stationary or to test whether the primary budget deficit and the public debt series are cointegrated and $(1-\lambda L)S_t^p$ is stationary, with $0 \leq \lambda < 1+r_t$. Another test, proposed by Wilcox (1989), is to test whether the series of undiscounted debt displays an unconditional mean of zero. If this holds the intertemporal government constraint will be fulfilled, because the intertemporal budget constraint requires the discounted debt to converge to zero.[17] This implies that all government debt will be repaid sometimes.

Moreover, another aspect of these tests which has given rise to criticism is that those test need strong assumptions, because the transversality condition involves an expectation about states in the future that are difficult to obtain from a single set of time series data and, because assumptions about the discount rate have to be made.[18]

Alternative tests check solely if in the long run some debt to income ratio remains stationary. A test procedure which circumvents the problems associated with the above first type of test focuses on the time series of the debt ratio, i.e. on the ratio of public debt to GDP. If this series is constant the intertemporal budget constraint is fulfilled for dynamically efficient economies. To see this let $B/Y = c_1$ be the constant debt ratio, with Y the GDP and c_1 a positive constant. Inserting $B/Y = c_1$ in (23) yields

$$\lim_{t\to\infty} c_1\, Y_0\, e^{(\gamma-r)t} = 0, \tag{24}$$

for $\gamma < r$, with $\gamma > 0$ the constant growth rate of GDP. The condition $\gamma < r$ characterizes a dynamically efficient economy and is likely to hold in real economies. For example, in EU countries this seems to be obvious if one compares interest rates with GDP growth rates. But even the US, where growth rates have exceeded interest rates on safe government

[16]See e.g. Hamilton and Flavin (1986), Kremers (1988), Wilcox (1989) and Trehan and Walsh (1991).

[17]As to these tests applied to Germany see Greiner and Semmler (1999).

[18]See e.g. Bohn (1995, 1998).

bonds since the 1990s, is a dynamically efficient economy.[19] Therefore, for advanced countries we may limit our considerations to the case of dynamic efficient economies and assume that the discount rate of government debt exceeds the GDP growth rate.[20] For developing countries, like the low- and lower-middle income countries of this study, this would have to be explored.

However, testing for stationarity of the debt to GDP ratio is characterized by some shortcomings, too. It is difficult to distinguish between a time series which is stationary about a positive intercept and one that shows a trend. This holds because standard unit root regressions have low power against autoregressive alternatives if the AR coefficient is close to one. As a consequence, the hypothesis that a given fiscal policy is sustainable has been rejected too easily.

Therefore, Bohn (1998) suggests to test whether the primary deficit to GDP ratio is a positive linear function of the debt to GDP ratio. If this holds, a given fiscal policy will be sustainable. As discussed in section 2.2 the intuitive reasoning behind this argument was that if a government raises the primary surplus as the public debt ratio rises, it takes a corrective action which stabilizes the debt ratio and makes public debt sustainable. Before one can undertake empirical tests which apply this concept we need to advance some theoretical considerations about the relevance of this test. We limit our considerations to deterministic economies.

Assuming, as in section 2.2, that the primary surplus to GDP ratio depends on a constant and linearly on the debt to GDP ratio this variable can be written as

$$\frac{S^p(t)}{Y(t)} = \frac{T(t)-G(t)}{Y(t)} = \alpha + \beta\left(\frac{B(t)}{Y(t)}\right), \tag{25}$$

with $T(t)$ tax revenue at t, $G(t)$ public spending at t, α and β are constants which can be negative or positive. α is a systematic component which determines how the level of the primary deficit reacts to variations in GDP. α can also be interpreted as other (constant)

[19]For details see Abel et al. (1989).

[20]In dynamically inefficient economies the government budget constraint is irrelevant because in that case the government can play a Ponzi game.

economic variables which affect the surplus ratio.

The coefficient β can be called a reaction coefficient since it gives the response of the primary surplus ratio to an increase in the debt ratio. Inserting (25) in the differential equation, giving the evolution of public debt, the latter equation is then given by

$$\dot{B}(t) = r\, B(t) + G(t) - T(t) = (r - \beta)\, B(t) - \alpha\, Y(t). \tag{26}$$

Solving this equation we get

$$B(t) = \left(\frac{\alpha}{r - \gamma - \beta}\right) Y(0)\, e^{\gamma\, t} + e^{(r-\beta)\, t}\, C_1, \tag{27}$$

with $B(0) > 0$ debt at time $t = 0$ which is assumed to be strictly positive and with C_1 a constant given by $C_1 = B(0) - Y(0)\, \alpha/(r - \gamma - \beta)$. Multiplying both sides of (27) by e^{-rt} leads to

$$e^{-rt} B(t) = \left(\frac{\alpha}{r - \gamma - \beta}\right) Y(0)\, e^{(\gamma - r)t} + e^{-\beta\, t}\, C_1. \tag{28}$$

The first term on the right hand side in (28) converges to zero in dynamically efficient economies and the second term converges for $\beta > 0$ and diverges for $\beta < 0$. These considerations show that $\beta > 0$ guarantees that the intertemporal budget constraint of the government holds.

Further, with equation (25) the debt to GDP ratio evolves according to

$$\dot{b} = b \left(\frac{\dot{B}}{B} - \frac{\dot{Y}}{Y}\right) = b\,(r - \beta - \gamma) - \alpha, \tag{29}$$

Solving this differential equation we get the debt to GDP ratio b as

$$b(t) = \frac{\alpha}{(r - \beta - \gamma)} + e^{(r-\gamma-\beta)t}\, C_2, \tag{30}$$

where C_2 is a constant given by $C_2 = b(0) - \alpha/(r - \beta - \gamma)$, with $b(0) \equiv B(0)/Y(0)$ the debt-GDP ratio at time $t = 0$.

Equation (30) shows that the debt to GDP ratio remains bounded if $r - \gamma - \beta < 0$ holds. This shows that a positive β does not assure boundedness of the debt-GDP ratio although the intertemporal budget constraint of the government is fulfilled in this case.

Only if β is larger than the difference between the interest rate and the GDP growth rate the debt ratio remains bounded. These considerations demonstrate that sustainability of debt may be given even if the debt-GDP ratio rises over time. A situation which seems to hold for some countries.

But it must also be pointed out that for too high a level of public debt the government will probably not be able to raise the primary surplus any further. Then, our rule formulated in equation (25) will brake down. This holds because, the present value of future surpluses must equal public debt at any finite point in time. So, if the government is not able to raise the primary surplus as public debt rises any longer from a certain point of time, say t_1, onwards, β is zero or even negative. Then, a fiscal policy is sustainable only if the government has succeeded to reduce public debt to zero up to that point of time t_1. This implies that in the long-run the debt/GDP ratio must be constant although it may well rise transitorily. However, lastly it is an empirical question what the country specific β will be.[21]

In this section therefore we suggest some method related to the Bohn method (Bohn (1998)) which allows one to estimate sustainability, but at the same time works, as section 2.2 has indicated, with a time varying reaction of governments to the debt to GNP ratio. This allows one to get some insight using empirical tests about the sustainability of policies. As we have pointed out in section 2.2 our strategy for testing the sustainability of debt has the advantage that the test does not depend on the interest rate which is used to discount public debt as needed in the first type of the above discussed tests.

Following our setup in section 2.2, the equation suggested to be tested is as follows:

$$s_t^y = \beta_t b_t^y + \alpha^\top \mathbf{Z_t} + \epsilon_t, \tag{31}$$

where s_t^y and b_t^y are the primary surplus to GDP and the debt to GDP ratio, respectively. $\mathbf{Z_t}$ is a vector which consists of the number 1 and of other factors related to the primary surplus and ϵ_t is an error term which is i.i.d. $N(0, \sigma^2)$.

The idea behind estimating (31) is the tax smoothing hypothesis according to which

[21] This will be estimated separately for each country in country studies.

public deficits should be used to finance transitory government spending, for example higher public spending during recessions. Further, the variables contained in Z_t may differ depending on which country is analyzed. In the US, for example, military spending is a variable which exerts a strong influence on the primary surplus. In European countries, social spending plays an important role affecting the primary surplus. For other countries, for example, low income countries, other spending categories may be relevant.

We want to note, as concerns the primary surplus one has to distinguish between the primary surplus including the social surplus and the primary surplus exclusive of the social surplus, where social surplus means the surplus of social insurance system. This holds because in some countries deficits of the social insurance system raise the stock of public debt, because the government sector and the social insurance system are not separated whereas this does not hold for other countries. So, in some European countries the social insurance systems are autonomous and do not borrow in capital markets. For example, if the social insurance system runs deficits, these deficits are either transitory because they must be paid back in the next period or the deficits are covered by reserves from earlier years.

If, on the other hand, the social insurance system has a surplus this does not raise the surplus of the government, but is used as reserve. However, it happens that the government subsidizes the social insurance which leads to a decline in the surplus of the government. This amount, however, is included in the regular surplus or deficit of the government so that taking it into account in the surplus or deficit of the government and adding that of the social insurance would lead to double counting.

These considerations demonstrate that institutional regulations determine which concept for the primary surplus should be used. As we noted above, social security and transfer arrangements in countries of different income groups seem to be different. Our suggested test should therefore be adjusted to each type of income group - or country if country specific fiscal policy studies are undertaken.

7 Conclusion

The World Bank has in its recent work raised the issue of whether fiscal policy should not only serve as stabilization policy, but also promote growth and the long run welfare of a country. In this paper we suggest a general model on growth and fiscal policy that includes both education and human capital as well as infrastructure investment and explores their impact on the long run per capita income. We also demonstrate the conditions under which fiscal policy is sustainable. A simplified model is proposed, its out of steady state dynamics is solved and the impact of foreign aid, the allocation of human and public capital and fiscal expenditure on per capita income explored.

The calibration for the three income groups showed that foreign aid per capita, i_p^f, and the productivity factor, A, have a positive and linear effect on per capita GDP and welfare. Such a result is clearly what would have been predicted by the theory as inflows of foreign aid are assumed to be used for investment in roads, schools, hospitals, or any other infrastructure that plays an important role in raising productivity, or leading to real effects. Our results show a negative and linear effect of the foreign aid, i_p^f, on the stock of resources absorbed by the public sector, e_p. Such a result could be due to some crowding in effect. It could be due to the fact that as foreign aid flows in, more investment and production opportunities open up and resources are used more privately and less publicly. Second, it was also observed that the fraction of human capital used in market goods production, u_1, and the fraction of public capital used for public infrastructure, v_1, on growth have a positive, but non linear effect on per capita GDP and welfare. The decreasing return effect indicates that any increase in the fraction of resources takes away resources from other areas that may indeed contribute to growth. Furthermore, the hump-shape form of the effect of u_1 and v_1 on per capita human capital, h, implies that any increase in the parameters would first increase h, but beyond the 50% threshold, the effect becomes negative. Such a reversal of the relationship could signal the fact that as too much human capital is devoted to market goods production, there is less and less of it available in the accumulation process of new human capital.

The calibration exercise was also undertaken for the different compositions of public expenditure for the three country groups. The results of this exercise show that the composition of public expenditure matters. Generally, higher investment shares in public expenditure increase per capita income and the other main variables. Only at very high investment shares ($\alpha_1 > 0.95$), welfare decreases with further increases in investment. While this is outcome is not surprising, as investment is the only public expenditure category that promotes the accumulation process, the spending categories are still very rough. In the model the remaining two public expenditure categories are defined as mere transfers and public consumption, α_2, as well as public expenditure for the proper function of the state, α_3, of which the latter does not feed back into the economy, and the former enters in the households' welfare function.

Although not undertaken here, we spell also out how the model, instead of calibrating it, can be estimated through time series methods. In order to do so, long time series of high quality data are needed which are so far unavailable. We also point out how the model can be extended to include subsistence production, a foreign sector and public deficit and debt. Further extensions that seem to be important pertain to the revenue side of fiscal expenditure (various forms of taxes and their effect on per capita income) and issues related to the sustainability of fiscal policy. A method of how to conduct a debt sustainability test is proposed, but not undertaken, in section 6 of the paper. This type of test will become relevant when the model is applied to the study of specific countries.

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A Appendix

Appendix A.1 explains how the data used in the paper was constructed. In Appendix A.2 the different country groups are listed. Appendix A.3 lists the data used to compute the graphs shown in section 4 and additional data on the two other country groups which have not been discussed in the paper. Appendix A.4 graphs on the long run effects of public expenditure composition are depicted for the two country groups that have been omitted in the main part of the paper.

A.1 Data

Two different sources were used for gathering the necessary data. First, information on GDP, population size and foreign aid flows was taken from the World Bank data base (WDI). Second, information on the public expenditure composition was taken from the Government Finance Statistics (GFS). The classification of countries in low, lower-middle and upper-middle income countries follows the report of the IMF Development Committee (2006). In order to make the data compatible with the model, only category 7 of the GFS was used. 'Recreation, culture, and religion' (708) and 'Social protection' (710) was subsumed under public transfers. Public investment consists of 'Economic affairs' (704), 'Environmental protection' (705), 'Housing and community amenities' (706) and 'Health' (707). Public consumption was defined as the sum of 'General public services' (701), 'Defense' (702) and 'Public order and safety' (703). Education was taken directly from the GFS, 'Education' (709). Due to the bad quality of the data in the GFS, however, the list of countries was narrowed to the ones listed in the reduced list in appendix A.2.

A.2 Country Groups

Table 1: Reduced List of Countries

Table 1: List of Countries

Low Income	Lower-Middle Income	Upper-Middle Income
Bhutan	Belarus	Argentina
Burundi	Bolivia	Croatia
Ethiopia	Bulgaria	Czech Republic
India	Dominican Republic	Estonia
Moldova	Egypt	Hungary
Myanmar	El Salvador	Lithuania
Pakistan	Iran	Mauritius
Yemen	Jamaica	Mongolia
Zambia	Maldives	Panama
	Morocco	Poland
	Philippines	Uruguay
	Romania	
	Sri Lanka	
	Thailand	
	Tunisia	

A.3 Appendix to Section 4

i_p^f	k	h	g	c	e_p	y	U
2	437.75	39.33	252.29	50.35	47.7	139.28	50.02
2.2	439.35	39.39	253.8	51.01	47.4	139.79	50.17
2.4	440.98	39.45	255.32	51.66	47.11	140.31	50.33
2.6	442.62	39.5	256.87	52.32	46.82	140.83	50.47
2.8	444.29	39.57	258.45	52.97	46.54	141.37	50.62
3	445.98	39.63	260.04	53.62	46.27	141.9	50.76
3.2	447.68	39.69	261.66	54.28	46	142.45	50.91
3.4	449.41	39.75	263.3	54.93	45.73	142.99	51.05
3.6	451.16	39.81	264.97	55.58	45.47	143.55	51.19
3.8	452.92	39.88	266.65	56.24	45.21	144.11	51.32
4	454.71	39.94	268.36	56.89	44.96	144.68	51.46

Table 2: Effects of Foreign Aid, i_p^f, on the Steady State Variables (Low Income Group)

A	k	h	g	c	e_p	y	U
1	448.37	39.71	262.32	54.54	45.89	142.66	50.96
1.005	456.59	39.89	266.92	55.4	46.87	145.28	51.18
1.01	464.93	40.06	271.6	56.27	47.87	147.93	51.4
1.015	473.39	40.23	276.34	57.15	48.88	150.62	51.62
1.02	481.97	40.41	281.15	58.04	49.91	153.35	51.84
1.025	490.68	40.58	286.04	58.95	50.95	156.12	52.06
1.03	499.51	40.76	290.99	59.87	52.01	158.93	52.27
1.035	508.46	40.93	296.02	60.8	53.08	161.78	52.49
1.04	517.55	41.11	301.12	61.75	54.17	164.67	52.71
1.045	526.76	41.28	306.29	62.71	55.28	167.61	52.92
1.05	536.11	41.46	311.54	63.68	56.4	170.58	53.14
1.055	545.58	41.63	316.86	64.67	57.53	173.59	53.35
1.06	555.19	41.81	322.26	65.67	58.69	176.65	53.57

Table 3: Effect of the Productivity Factor, A, on the Steady State Variables (Low Income Group)

u_1	k	h	g	c	e_p	y	U
0.1	83.99	44.16	66.87	14.65	4.16	26.72	31.73
0.2	144.94	46.87	95.4	22.22	10.25	46.12	38
0.3	211.26	49.03	130.63	29.55	17.77	67.22	42.21
0.4	278.29	50.2	167.44	36.7	25.63	88.55	45.34
0.5	341.51	50.3	202.56	43.36	33.13	108.66	47.72
0.6	396.42	49.28	233.23	49.11	39.68	126.14	49.49
0.7	437.29	46.94	256.1	53.38	44.56	139.14	50.66
0.8	454.5	42.81	265.75	55.18	46.62	144.61	51.13

Table 4: Effect of u_1, the Fraction of Human Capital Used in Market Goods Production Income on the Steady State Variables (Low Income Group)

v_1	k	h	g	c	e_p	y	U
0.1	83.99	44.16	66.87	14.65	4.16	26.72	31.73
0.2	144.94	46.87	95.4	22.22	10.25	46.12	38
0.3	211.26	49.03	130.63	29.55	17.77	67.22	42.21
0.4	278.29	50.2	167.44	36.7	25.63	88.55	45.34
0.5	341.51	50.3	202.56	43.36	33.13	108.66	47.72
0.6	396.42	49.28	233.23	49.11	39.68	126.14	49.49
0.7	437.29	46.94	256.1	53.38	44.56	139.14	50.66
0.8	454.5	42.81	265.75	55.18	46.62	144.61	51.13
0.9	425.24	35.43	249.36	52.13	43.12	135.3	50.33

Table 5: Effects of v_1, the Fraction of Public Capital Used for Public Infrastructure on the Steady State Variables (Low Income Group)

i_p^f	k	h	g	c	e_p	y	U
3	1832.53	57.61	1161.99	227.47	195.08	583.08	66.1
3.2	1834.15	57.63	1163.66	228.11	194.81	583.59	66.14
3.4	1835.78	57.65	1165.34	228.75	194.55	584.11	66.17
3.6	1837.42	57.68	1167.03	229.39	194.28	584.63	66.2
3.8	1839.05	57.7	1168.72	230.03	194.02	585.15	66.23
4	1840.7	57.72	1170.41	230.68	193.76	585.68	66.26
4.2	1842.35	57.74	1172.12	231.32	193.5	586.2	66.29
4.4	1844	57.76	1173.83	231.96	193.24	586.73	66.32
4.6	1845.66	57.78	1175.54	232.6	192.98	587.26	66.35
4.8	1847.32	57.8	1177.26	233.24	192.72	587.78	66.38
5	1848.99	57.82	1178.99	233.88	192.46	588.31	66.41

Table 6: Effects of Foreign Aid, i_p^f, on Steady State Variables (Lower-Middle-Income Group)

A	k	h	g	c	e_p	y	U
1.39	1841.52	57.73	1171.27	231	193.63	585.94	66.28
1.395	1866.91	57.92	1187.22	234.05	196.42	594.02	66.45
1.4	1892.55	58.12	1203.35	237.15	199.24	602.18	66.61
1.405	1918.47	58.31	1219.64	240.27	202.1	610.42	66.78
1.41	1944.65	58.51	1236.1	243.42	204.98	618.75	66.94
1.415	1971.1	58.71	1252.72	246.61	207.89	627.17	67.11
1.42	1997.83	58.9	1269.52	249.83	210.83	635.67	67.27
1.425	2024.82	59.1	1286.49	253.08	213.8	644.26	67.44
1.43	2052.09	59.29	1303.64	256.37	216.8	652.94	67.6
1.435	2079.64	59.49	1320.96	259.69	219.84	661.7	67.76
1.44	2107.46	59.69	1338.45	263.04	222.9	670.56	67.93
1.445	2135.57	59.88	1356.12	266.43	225.99	679.5	68.09
1.45	2163.96	60.08	1373.97	269.85	229.12	688.53	68.25

Table 7: Effects of the Productivity Factor, A, on the Steady State Variables (Lower-Middle-Income Group)

u_1	k	h	g	c	e_p	y	U
0.1	246.54	56.01	173.04	38.02	18.83	78.45	42.96
0.2	519.65	64.47	341.59	71.48	48.34	165.34	51.26
0.3	814.07	69.45	525.87	107.1	80.61	259.02	56.48
0.4	1106.83	72.03	709.6	142.43	112.79	352.17	60.12
0.5	1380.75	72.66	881.67	175.46	142.92	439.33	62.78
0.6	1617.8	71.45	1030.64	204.03	169	514.75	64.7
0.7	1793.82	68.21	1141.28	225.25	188.37	570.76	65.95
0.8	1867.88	62.25	1187.84	234.17	196.53	594.33	66.45

Table 8: Effect of u_1, the Fraction of Human Capital Used in Market Goods Production Income on the Steady State Variables (Lower-Middle-Income Group)

v_1	k	h	g	c	e_p	y	U
0.1	246.54	56.01	173.04	38.02	18.83	78.45	42.96
0.2	519.65	64.47	341.59	71.48	48.34	165.34	51.26
0.3	814.07	69.45	525.87	107.1	80.61	259.02	56.48
0.4	1106.83	72.03	709.6	142.43	112.79	352.17	60.12
0.5	1380.75	72.66	881.67	175.46	142.92	439.33	62.78
0.6	1617.8	71.45	1030.64	204.03	169	514.75	64.7
0.7	1793.82	68.21	1141.28	225.25	188.37	570.76	65.95
0.8	1867.88	62.25	1187.84	234.17	196.53	594.33	66.45

Table 9: Effects of v_1,the Fraction of Public Capital Used for Public Infrastructure on the Steady State Variables (Lower-Middle-Income Group)

i_p^f	k	h	g	c	e_p	y	U
3	3639.45	65.09	1892.61	519.6	349.07	1158.01	70.59
3.2	3642.04	65.1	1894.8	520.5	348.79	1158.83	70.61
3.4	3644.64	65.12	1897	521.4	348.51	1159.66	70.63
3.6	3647.24	65.14	1899.19	522.3	348.23	1160.48	70.65
3.8	3649.84	65.16	1901.4	523.2	347.95	1161.31	70.66
4	3652.45	65.18	1903.61	524.1	347.67	1162.14	70.68
4.2	3655.07	65.2	1905.82	525	347.39	1162.98	70.7
4.4	3657.68	65.22	1908.04	525.9	347.12	1163.81	70.71
4.6	3660.31	65.24	1910.26	526.81	346.84	1164.64	70.73
4.8	3662.94	65.26	1912.49	527.71	346.57	1165.48	70.75
5	3665.57	65.27	1914.73	528.61	346.3	1166.32	70.77

Table 10: Effects of Foreign Aid, i_p^f, on Steady State Variables (Upper-Middle-Income Group)

A	k	h	g	c	e_p	y	U
1.8	3648.41	65.15	1900.19	522.71	348.1	1160.86	70.65
1.805	3687.4	65.32	1920.33	528.19	351.93	1173.26	70.77
1.81	3726.71	65.49	1940.63	533.71	355.78	1185.77	70.9
1.815	3766.33	65.67	1961.09	539.28	359.67	1198.38	71.02
1.82	3806.27	65.84	1981.72	544.9	363.59	1211.09	71.13
1.825	3846.53	66.01	2002.51	550.56	367.54	1223.89	71.25
1.83	3887.1	66.18	2023.47	556.26	371.52	1236.81	71.37
1.835	3928	66.35	2044.59	562.01	375.53	1249.82	71.49
1.84	3969.22	66.53	2065.88	567.8	379.58	1262.93	71.61
1.845	4010.76	66.7	2087.34	573.64	383.65	1276.15	71.73
1.85	4052.63	66.87	2108.97	579.53	387.76	1289.47	71.85
1.855	4094.83	67.04	2130.76	585.46	391.9	1302.9	71.97
1.86	4137.36	67.22	2152.73	591.44	396.07	1316.43	72.08

Table 11: Effects of the Productivity Factor, A, on the Steady State Variables (Upper-Middle-Income Group)

u_1	k	h	g	c	e_p	y	U
0.1	464.64	61.95	258.84	74.59	36.31	147.84	47.9
0.2	1013.34	72.29	540.09	152.13	89.73	322.43	56.31
0.3	1601.2	78.15	843.16	234.87	147.31	509.47	61.38
0.4	2184.82	81.17	1144.38	316.95	204.53	695.17	64.86
0.5	2730.61	81.94	1426.19	393.69	258.06	868.83	67.37
0.6	3202.8	80.61	1670.05	460.07	304.38	1019.07	69.18
0.7	3553.39	76.97	1851.11	509.35	338.78	1130.63	70.35
0.8	3700.91	70.26	1927.3	530.09	353.25	1177.56	70.82

Table 12: Effect of u_1, the Fraction of Human Capital Used in Market Goods Production Income on the Steady State Variables (Upper-Middle-Income Group)

v_1	k	h	g	c	e_p	y	U
0.1	464.6	62.0	258.8	74.6	36.3	147.8	47.9
0.2	1013.3	72.3	540.1	152.1	89.7	322.4	56.3
0.3	1601.2	78.2	843.2	234.9	147.3	509.5	61.4
0.4	2184.8	81.2	1144.4	317.0	204.5	695.2	64.9
0.5	2730.6	81.9	1426.2	393.7	258.1	868.8	67.4
0.6	3202.8	80.6	1670.1	460.1	304.4	1019.1	69.2
0.7	3553.4	77.0	1851.1	509.4	338.8	1130.6	70.4
0.8	3700.9	70.3	1927.3	530.1	353.3	1177.6	70.8

Table 13: Effects of v_1, the Fraction of Public Capital Used for Public Infrastructure on the Steady State Variables (Upper-Middle-Income Group)

A.4 Appendix to Section 5

Since the calibration exercise for the composition of public expenditure involves too many parameters to list all data in tables, the depiction of the lower-middle- and upper-middle-income cases is limited to graphs. As one can see, these graphs show the same qualitative effects as the low-income group's that were discussed in the main part of the paper.

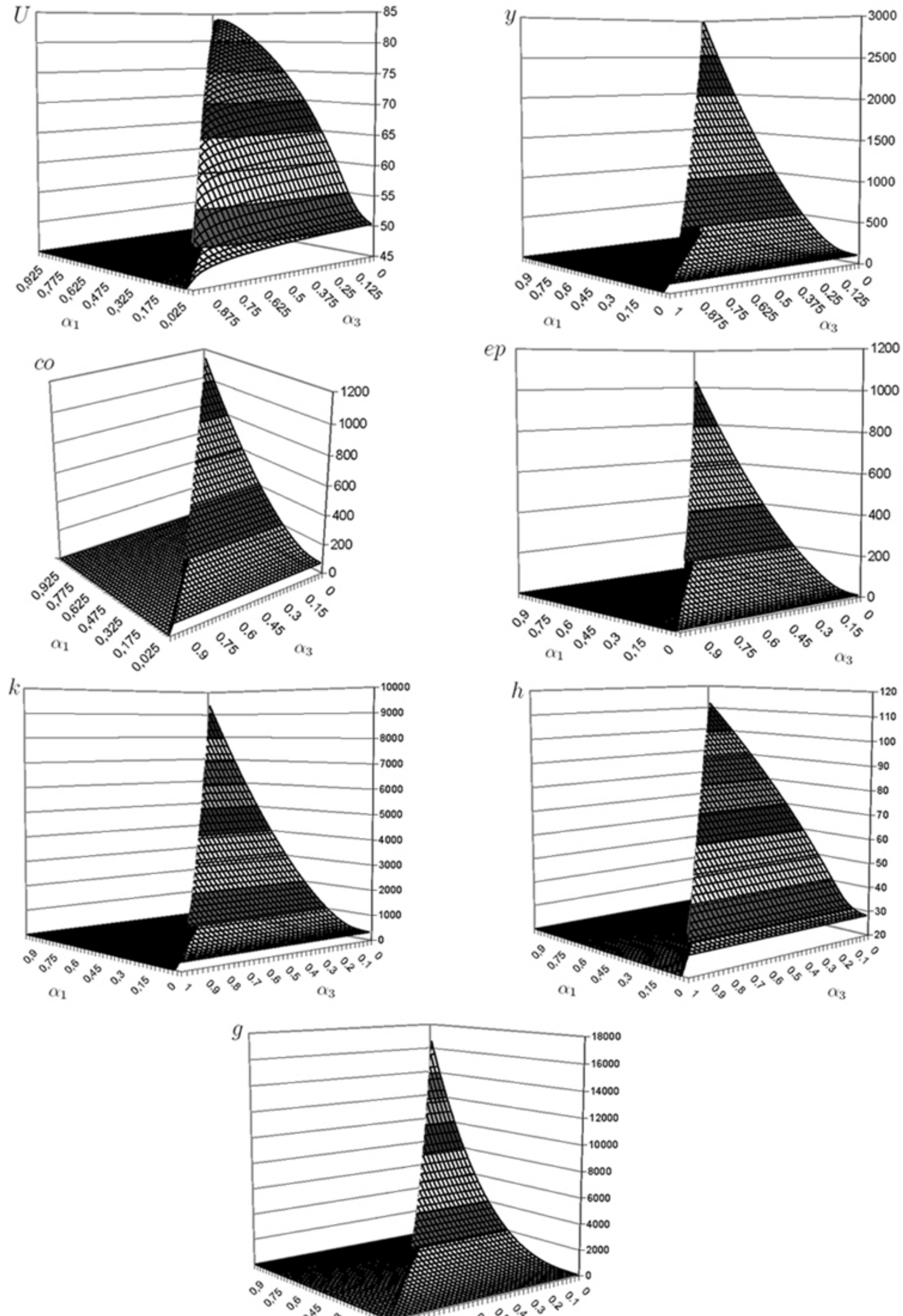

Figure 16: Effects of Expenditure Composition Changes on the per capita Parameters in the Lower-Middle-Income Case

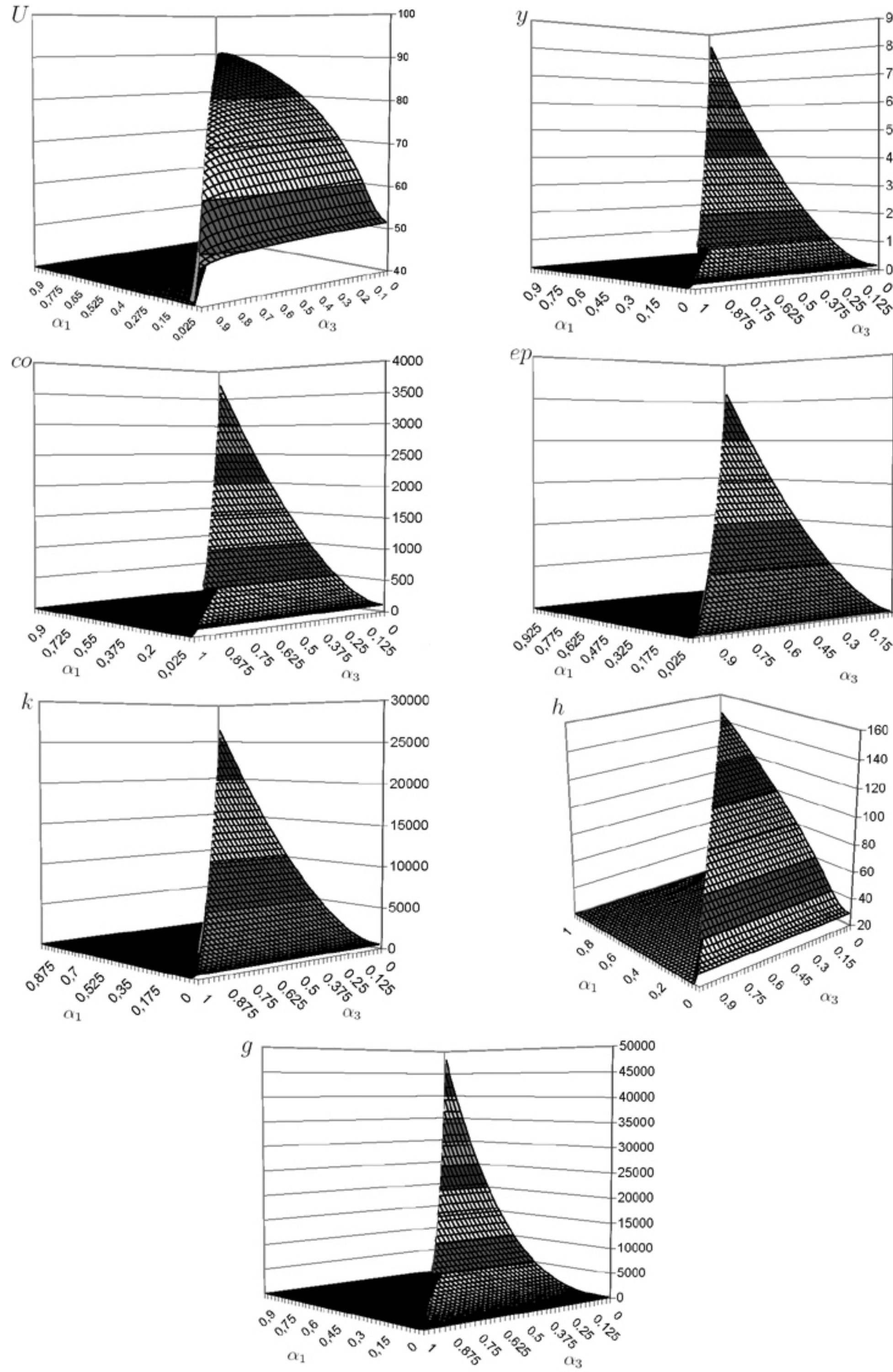

Figure 17: Effects of Expenditure Composition Changes on the per capita Parameters in the Upper-Middle-Income Case