On Poverty Traps, Thresholds and Take-Offs*

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Abstract

Recent studies on economic growth focuses on persistent inequality across countries. In this paper we study mechanisms that may give rise to such a persistent inequality. We consider countries that accumulate capital in order to increase the per capita income in the long run. We show that the long-run growth dynamics of those countries can generate a twin-peak distribution of per capita income. The twin-peak distribution is caused by (1) locally increasing returns to scale and (2) financial market constraints. Those two forces give rise to a twin-peak distribution of per capita income in the long run. In our model investment decisions are separated from consumption decisions and we thus do not have to consider preferences. Empirical evidence in support of a twin-peak distribution of per capita income is provided.

JEL classification: C 61, C 63, L 10, L 11 and L 13

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1 Introduction

For the past twenty years, liberalization of trade, financial deregulation, and privatization of industries occurring in many countries have been viewed as a mean to enhance productivity and growth through more competition. At the same time the criticism was raised that countries and industries are rapidly forced into global competition by premature and fast liberalizations which may enhance the likelihood of countries to fall into poverty traps. Domestic rise of investment rates and a large inflow of capital in less developed countries may not occur. Lasting take-off in growth, improvements in productivity, income and consumption may not be visible in some countries. Globalization of competition could lead to a growing gap of per capita income between countries. This may exacerbate the trend that empirical research has found since long.

From the 1860s to the 1960s, the growth rates of roughly fifteen industrializing nations were only slightly higher than the growth rates of thirty less developed countries. From the 1960s to 1980s the growth rate of the former group was 3.2 percent and of the latter group 2.5 percent. Yet in the period from 1980 to 1995 the growth rate of the former group was 1.5 percent whereas the latter showed only a growth rate of 0.34 percent.[1] It thus seems to have become an empirical regularity that the per capita income and growth rates of per capita GNP has become polarized so that there appear to have arisen convergence clubs and twin peaks in per capita size distribution of income. The proponents of the globalization of competition mislead us to believe that there is a universal way of how a similar level of per capita income can be achieved by all countries.

The polarization of income not only appears to contrast with the above optimistic view but also seems to be in contradiction with the one sector growth models, competitive convex economies, and no capital market constraints, predicting in the long run a convergence to similar per capita income. Recently, the new growth theory has redirected our attention to important long-run forces of economic growth. It is also a great challenge for this new theory to explain the above stylized facts.

One approach of the new growth theory sees persistent economic growth arising from learning from others, externalities in investment and increasing returns to scale. This idea had been formalized by Arrow (1962) and recently rediscovered by Romer (1990), who argues that externalities – arising from

[1] For details, see Azariadis (2001).

learning by doing and knowledge spillover – positively affect the productivity of labor and thus the aggregate level of income of an economy. Lucas (1988), whose model goes back to Uzawa (1965), stresses education and the creation of human capital, Romer (1990) and Grossmann and Helpman (1991) focus on the creation of new technological knowledge as important sources of economic growth.

Another important strand in the development of growth models is the Schumpeterian model, put forward by Aghion and Howitt (1992, 1998). In their work Schumpeter's process of creative destruction is integrated in a formal model where innovations are the major force of sustained economic growth. Another direction argues that persistent economic growth can also be achieved by productive public capital or investment in public infrastructure. [2] A variety of other forces of growth have been added in the literature.[3]

Although what produces persistent growth rates is still controversial, most of the recent growth theories predict empirically that the per capita income of countries will converge to similar high level per capita income. Yet, as the above indicated empirical evidence suggests this does not seem to hold true in the long run. Rather, we can observe an increased gap of per capita income between countries over time. We thus need to explore economic mechanisms that can explain those empirical trends.

In this paper we would like to argue that externalities and increasing returns to scale as well as capital market contraints give rise to such separation of per capita income for countries. Such mechanisms may be able to explain the forces that bring about a twin-peak distribution of per capita income in the long run, namely the convergence of the size distribution to countries with small per capita income and countries with large per capita income.

As many recent growth models do, we start with a capital accumulation model with quadratic adjustment costs as the benchmark model. It represents a basic model of the dynamic decision problem of countries where the capital stock is the state variable and investment is the decision variable. Yet, our model also allows for a capital market. In different variants of the model, we explore mechanisms that may lead to thresholds and the separation of domain of attractions, predicting a twin-peak distribution of per capita income in the long run.[4] We show that only countries that have

[2]This line of research was initiated by Arrow and Kurz (1970), who, however, only considered exogenous growth models. Barro (1990) demonstrated that this approach may also generate sustained per capita growth in the long run. See also Futagami et al. (1993) and Greiner and Semmler (1999).

[3]For a more extensive survey, see Greiner, Semmler and Gong (2005).

[4]An early theoretical study of this problem can be found in Skiba (1978). Further theoretical modeling can be found in Azariadis and Drazen (1990) Aziaridis (2001) and Aziaridis

passed certain thresholds may enjoy a rise of per capita income. The working of the above mechanisms are then empirically explored by applying a kernel estimator and Markov transition matrices to an empirical data set of per capita income across countries.

The remainder of the paper is organized as follows. Section 2 reviews some recent empirical work and describes the economic mechanisms that make such threshold plausible. Sections 3 presents the dynamic model with those properties. Section 4 reports the detailed results from our numerical study on those mechanisms. Section 5 provides empirical evidence for the twin-peak distribution of per capita income for the time period 1960 to 1985. Section 6 concludes the paper. In the appendix we describe the solution methods that allow us to study the different variants of the dynamic model.

2 The Studies on Convergence and Non-Convergence

The above mentioned new growth theory has given rise to numerous empirical studies. The first round of empirical tests, by and large, focused on cross-country studies.[5] We do not exhaustively want to survey the cross-country studies of the new growth theory but their success or failure is reviewed by Sala-i-Martin (1997), Durlauf and Quah (1999) and Greiner, Semmler and Gong (2005). As aforementioned, one of the major issues in recent empirical studies concerns the convergence or non-convergence of per capita income of countries. The large per capita income gap between poor and rich countries has thus become a major issue in the growth literature.

2.1 Convergence and Non-Convergence

Although the above cross-country studies are now numerous, methodological criticism has been raised against those studies. It has been shown that those studies, by lumping together countries at different stages of development, may miss the thresholds of development (Bernard and Durlauf 1995). Moreover, cross-country studies rely on imprecise measures of the economic variables, and the results are amazingly not robust (Sala-i-Martin 1997). In addition, cross-country studies imply that the forces of growth, as well as technology and preference parameters, are the same for all countries in the

and Stachurski (2004). For recent empirical studies see, for example, Durlauf and Johnson (1995), Bernard and Durlauf (1995), Durlauf and Quah (1999), Quah (1996) and Kremer, Onatski and Stock (2001).

[5] See, Mankiw, Romer, and Weil (1992) and Barro and Sala-i-Martin (1995).

sample. When estimating the Solow growth model using a sample consisting of, 100 countries, the obtained parameter values are presumed to be identical for each country. However, if the countries in this sample are highly heterogeneous in their states of development, different parameter values will characterize their technology or preferences.

It is also to be expected that different institutional conditions and social infrastructure in the countries under consideration will affect estimations and will make the countries heterogeneous, with respect to differences in the estimated parameters. Brock and Durlauf (2001) argue that cross-country studies tend to fail because they do not admit institutional differences, uncertainty, and heterogeneity of parameters.

In the spirit of the above view Durlauf and Johnson (1995) allow for different aggregate production functions depending on 1960 per capita incomes and on literacy rates. Durlauf and Johnson use a regression-tree procedure[6] in order to identify threshold levels endogenously. They find that the Mankiw et al. (1992) data set can be divided into four distinct regimes: low-income countries, middle-income countries, and high-income countries, the middle regime divided into two subgroups, one with high, and the other with low, literacy rates. The result of this study is that different groups of countries are characterized by different production possibilities, which implies different parameters on inputs in the aggregate production functions.

On the other hand the assumption of identical preference and production parameters implies that countries in the long run exhibit both identical per capita income growth rates and levels which are known as absolute β-convergence. The absolute β-convergence hypothesis states that poor countries tend to grow faster than rich countries. This indicates a negative relationship between initial per capita income and growth rate. In empirical literature there exist different methodologies to test the hypothesis of absolute β-convergence (see e.g. Bernard and Durlauf (1994)). In general these tests are cross-section regressions and it is accepted that they have not shown a negative and significant relation between initial income and subsequent growth thus rejecting absolute β-convergence hypothesis.

According with these results several authors, Barro (1991) and Mankiw, Romer and Weil (1992) present modified tests on absolute β-convergence which show a negative relationship between initial per capita income and growth exists, after controlling for growth relevant factors such as human capital or political stability that may affect the steady state. This so-called conditional β-convergence is able to explain the differences in per capita income levels. Several tests on the conditional β-convergence hypothesis

[6]For a description see Breiman et al. (1984).

have shown that it generally can not be rejected.

Yet, as mentioned above such growth regressions are subject to several problems. First, numerous growth variables had been researched which lead to approximately 100 different potential variables that significantly explain growth. Second, as above mentioned, cross-country growth regressions assume identical parameters across countries (parameter homogeneity). Landes (1998) and Canova (1999) give evidence for parameter heterogeneity. Third, some of the parameters that explain growth are not exogenous but endogenous. Fourth, cross-country regressions assume that the countries are best stylized by a linear model. Durlauf and Johnson (1995) show that growth behaviour can be determined by initial conditions which serve as thresholds for different regimes of countries. Each regime has specific linear growth behaviour and therefore the model is consistent with multiple steady states.

Finally, Quah (1996) criticizes a missing distinction in traditional approaches between a growth mechanism that refers to the ability of countries to push back technological and capital constraints and a convergence mechanism that aims to potential different economic process in rich and poor countries. Those mechanisms are related to each other but should be analyzed separately as they can occur isolated. The mechanisms separately can help to understand whether rich countries are more successful in pushing back constraints and whether poor countries adapt technological progress.

Accordingly Quah believes that the concept of β-convergence is irrelevant because it is not significant whether a country converges towards its specific steady state. What is more important is to analyze the development of the entire income distribution of all countries. This idea concerns a concept called σ-convergence which addresses a process of reducing income differences between countries over time. Quah (1997) shows by approximating the distribution of relative per capita income by a kernel density estimation that the distribution of income changed from being unimodal in the 1960s to a bimodal one in the 1980 which is a hint for a widening gap and the formation of convergence clubs. Furthermore he formalizes a bimodal steady state distribution with the help of Markov transition matrices. In section 5 we follow his empirical research strategy but we will refine the methodology considerably.

2.2 Externalities and Increasing Returns

As mentioned above, there is a long tradition in economic theory that has studied the problem of non-convergence of per capita income across countries. More particularly, basic economic mechanisms have been discussed that may lead to divergence of per capita income.

One theory that is often used to explain convergence clubs and poverty traps refers to technological traps. The idea of a technological trap is based on the work by Rosenstein-Rodan (1943, 1961), Singer (1949), Nurske (1953) and others. The starting-point is a modified production function that has both increasing and decreasing returns to scale. The increasing returns can only be realized if a country is capable to build up a capital stock that is above a certain threshold. If this threshold is passed, and enough externalities are generated, the production function exhibits increasing returns. Countries converge to a higher steady state as compared to countries that have fallen short of the threshold. With reference to the technological trap the so called "Big Push Theory"[7] proceeds from the idea that industrial countries had in their past a massive capital inflow and therefore can converge to a steady state with a high income level. In contrast less developed countries have a shortage of such massive capital inflow and accordingly stagnate at a low income level.

A related explanation is given by Myrdal (1957) who points out that a tendency towards automatic stabilization in social systems does not exists and that any process which causes an increase or decrease of interdependent economic factors including income, demand, investment and production will lead to a circular interdependence. Thus this would lead to a cumulative dynamic development that strengthens the effects of up- or downward movement. On this ground poor countries are in a *vicious circle*, becoming poorer. This in contrary to rich countries who will profit by a positive feedback effect, the so-called "Backwash Effects" arising from capital movement and migration to get richer.[8]

As previously mentioned the idea of externalities and increasing returns to scale has been extensively employed in growth theory recently. It is shown that a variety of positive externalities arising from scale economies, learning by using, increasing returns to information and skills are set in motion if a country enjoys, for example, by historical accident, a "big push" and take-off advantages.

Our first variant of a model of dynamic investment decision of countries builds on locally increasing returns to scale arising from externalities. Locally increasing returns due to local externalities may be approximated by a convex-concave production function as proposed by Skiba (1978) and Brock and Milliaris (1996) and Durlauf and Quah (1999) to illustrate those effects.

To present this idea of a convex-concave production function resulting

[7]See Murphy, Shleifer and Vishny (1989)

[8]Scitovsky's work in the 1950s is another example predicting poverty traps, thresholds and take-offs, see Scitovsky (1954).

from externalities and locally increasing returns to scale we use a model by Azariadis and Drazen (1990).[9] We can write a production function such as

$$
\begin{aligned}
y(k(t)) &= ak(t)^{\alpha_k(t)} \\
\alpha_t(t) &= \begin{cases} \overline{\alpha}_k & \text{if } k(t) > \overline{k}(t) \\ \underline{\alpha}_k & \text{otherwise} \end{cases}
\end{aligned}
$$

with the coefficients $\alpha_k(t)$, varying with the underlying state (k) and the quantity $\overline{k}(t)$ denoting the threshold for k, the capital stock.

One can show, from Dechert and Nishimura (1983) that if $\alpha_k < 1$, holds forever, the marginal product of capital, $y'(k)$ would approach the line given by the discount rate ρ plus capital depreciation, δ, from above if depreciation is allowed, see case (1) in Figure 1.

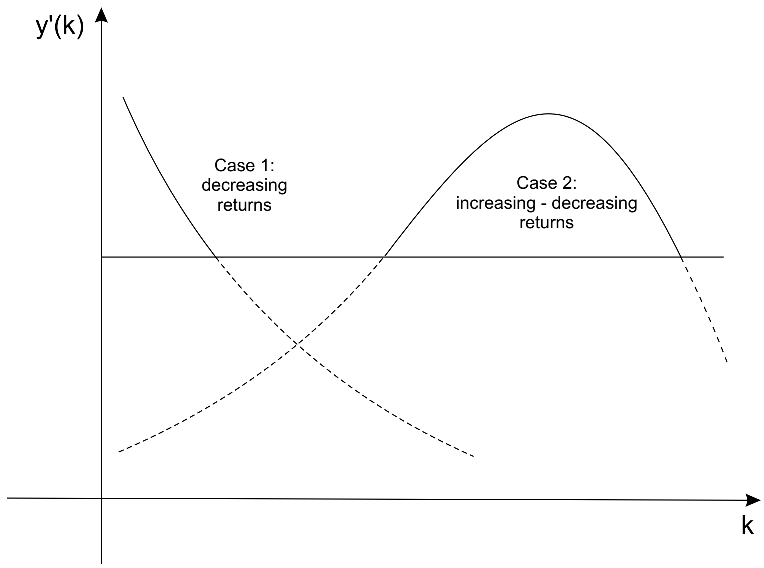

Figure 1: Increasing and decreasing returns

On the other hand, presuming that the parameter α_k is state dependent and approximating the convex-concave production function by a smooth function one obtains the case 2 in Figure 1. For locally increasing returns to scale, case 2, the marginal product of capital $y'(k)$ will first approach

[9]See furthermore Durlauf and Quah (1999), Aziaridis (2001) and Aziaridis and Stachurski (2004).

$\rho + \delta$ from below, then move above this line, $\rho + \delta$, and eventually decrease again. In the latter case, because of externalities, too small a capital stock will generate a too low return in the country so that owners of capital will seek investment somewhere else, perhaps outside the country, where at least $\rho + \delta$ is secured.

As Figure 1 shows, increasing returns can be assumed to hold, as Greiner, Semmler and Gong (2005, ch. 3) show, only up to a certain level of the capital stock. A region of a concave production function may be dominant thereafter where $y'(k)$ might start falling again.

2.3 Capital Market Constraints

A second strand of literature argues that low per capita income countries are severely constrained by less developed capital markets. Poorer countries usually face stricter credit constraint than high per capita income countries. Poor countries also often have to pay a higher risk premium when borrowing from international capital markets. Thus, countries may be heterogeneous with respect to their excess to capital markets.[10] This variant can theoretically be based on studies such as Townsend (1979) and Bernanke, Gertler and Gilchrist (1999), henceforth BGG.

Theoretically the risk premium covering default risk that a country pays has recently been derived from information economics. One presumes that asymmetric information and agency costs in borrowing and lending relationships. One can here draw on the insight of the literature on costly state verification[11] in which lenders must pay a cost in order to observe the borrower's realized returns. This motivates the use of collaterals in credit markets. Uncollateralized borrowing is assumed to pay a larger finance premium than collateralized borrowing. This finance premium represents a risk premium.[12] The finance premium drives a wedge between the expected return of the borrower and the risk-free interest rate.

We thus may measure the inverse relationship between a finance premium and the value of the collateral in a function:

$$H\left(k(t), B(t)\right) = \frac{\alpha_1}{\left(\alpha_2 + \frac{N(t)}{k(t)}\right)^{\mu}} \theta B(t) \tag{1}$$

[10]Studies of capital market constraints can be found in Kiyotaki and Moore (1997), Bernanke, Gertler and Gilchrist (1999) and Miller and Stiglitz (1999), Aghion et al. (1999, 2003, 2004) and Grüne, Semmler and Sieveking (2004).

[11]This literature originates in the seminal work by Townsend (1979).

[12]The actual cost that arises here may be constituted constituted by auditing, accounting, legal cost, loss of assets arising from asset liquidation and reputational domages in credit markets.

with $H\left(k(t), B(t)\right)$ the credit cost depending on the collateral, the net worth, $N(t) = k(t) - B(t)$, with $k((t)$ as capital stock and $B(t)$ as debt.[13] The parameters are $\alpha_1, \alpha_2, \mu > 0$ and θ is the risk-free interest rate. The shape of this function is shown in figure 2.

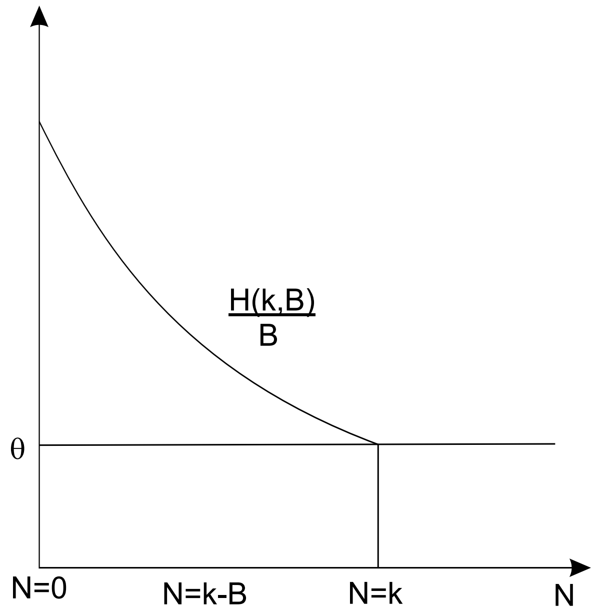

Figure 2: Endogenous Credit Cost

As figure 2 shows a low interest rate, the risk-free interest rate is a limit case which is only paid by the borrowers whose net worth is equal to the value of the capital stock and start borrowing.

Another way of showing how poorer countries are disadvantaged on capital markets is that there is credit rationing for them. This is stressed by Aghion et al. (1999, 2003, 2004) who presumes that this holds true for countries with low per capita income. In Aghion et al. (2003) it is presumed that a low level of financial development and low level of credit protection lead to hard credit constraints for in particular low per capital income countries, In our study we consider both state dependent credit cost as well as credit constraints therefore credit costs depend on individual characteristics of a country. We also define credit constraints for a country which, in our model, will be defined by an upper bound of a debt-capital stock ratio.

[13]Not that for our numerical study as pursuite in section 4.2 parameters will be chosen so that $\alpha_1/(\alpha_2 + \frac{NH}{k(t)}$, will converge to one for $N(t)$ approaching $k(t)$.

3 The Model Variants

Next we specify in a compact way different variants of a model that incorporates the above mechanisms discussed in sections 2.2 and 2.3. We can allow for heterogeneity of countries along dimensions such as capital size, externalities and returns to scale, adjustment costs of moving capital, and capital market constraints. Although our model can be nested in utility theory, a separation theorem that permits us to separate the present value problem from the consumption problem. In Sieveking and Semmler (1998) an analytical treatment is given of why and under what conditions the subsequent dynamic decision problem of a country can be separated from the consumption problem and thus preferences can be neglected.

We may specify a general model that can embody the above mechanisms as well as country specific features. The general decision problem for a country to achieve growth through the accumulation of capital can be formulated as follows:

$$V(k) = \underset{j}{Max} \int_0^\infty e^{-\theta t} f\left(k(t), j(t)\right) dt \tag{2}$$

$$\dot{k}(t) = j(t) - \sigma k(t), \quad k(0) = k. \tag{3}$$

.

$$\dot{B}(t) = H\left(k(t), B(t)\right) - \left(f\left(k(t), j(t)\right) - c(t)\right),\ B(0) = B_0 \tag{4}$$

In the general case the country's net income after accounting for investment and very general adjustment cost of moving capital can be written as

$$f(k, j) = y(k) - j - j^\beta k^{-\gamma} \tag{5}$$

The income is generated from capital stock, through a production function, $y(k)$. Investment, j, is undertaken so as to maximize the present value of net income of (4) given the adjustment cost of capital $j^\beta k^{-\gamma}$ in (4). Note that $\sigma > 0, \alpha > 0, \beta > 1$, $\gamma > 0$, are constants.

As production function, $y(k)$ we may take a convex-concave production function, as introduced in section 2.2 and further specified below, giving us the first variant of our model. By using a Cobb-Douglas production function $y(k) = ak^\alpha$ and stressing the other mechanism discussed above, namely capital market constraints leading to state dependent risk premium and/or credit rationing, will deliver us the second variant of our model.

Equ. (3) represents the equation for capital accumulation and equ. (4) the evolution of debt of the country. We allow for negative investment rates $j < 0$, i.e. reversible investment for simplicity.[14] Note that in (4) $c(t)$ is a consumption stream arising from the income of the country that are, in the context of our model, treated as exogenous. The consumption stream will be specified further below, in sect. 4.3.. Since net income in (5), less the consumption stream $c(t)$, can be negative the temporary budget constraint requires further borrowing from credit markets and if there is positive net income, less consumption, debt can be retired. Our model allows for capital inflows so that the debt accumulated can be reviewed as accumulation of external debt.

In the general case of adjustment cost in (5), if $\beta = 2$ and $\gamma = 0$, we have our benchmark model with quadratic adjustment costs of investment. When we employ the locally increasing return production function as introduced in section 2.2, the convex-concave production function, we will drop the adjustment cost term $j^{\beta}k^{-\gamma}$, and assume no finance premium.[15]

For our second variant we assume that the finance premium $H(k, B)$ in equ. (3) may be state dependent, depending on the capital stock, k, and the level of debt B with $H_k < 0$ and $H_B > 0$. Appendix 1 briefly discusses how the steady states of such a problem with state dependent credit cost can be solved.

If we assume that the borrowing cost is only constituted by a risk free rate, a special case of our model when there is the risk-free interest rate would determine the credit cost. We then have a constant credit cost and a state equation for the evolution of debt such as

$$\dot{B}(t) = \theta B(t) - f(k, j), \qquad B(0) = B_0 \tag{6}$$

In this case we would only have to consider the transversality condition $\lim\limits_{t\to\infty} e^{-\theta t}B(t) = 0$, as the non-explosiveness condition for debt, to close the model(2)-(5).

In general, however, we need to define the limit of borrowing, B, equal to $V(k)$ which represents the present value borrowing constraint. This will be particularly relevant when we study the second variant of our model. The problem to be solved is then how to compute $V(k)$ and the associated optimal

[14]The model can also be interpreted as written in efficiency labor, therefore σ can represent the sum of the capital depreciation rate, and rate of exogenous technical change.

[15]This is also done in Skiba (1978) and Brock and Milliaris (1996). Note that we use here a general form of adjustment cost which may itself give rise to some interesting dynamics, see Grüne, Semmler and Sieveking (2004).

investment j. If the interest rate $\theta = \frac{H(k,B)}{B}$ is constant[16] as in (6), then as is easy to see, $V(k)$ is in fact the present value of k which results from the following decision problem

$$V(k) = \underset{j}{Max} \int_0^\infty e^{-\theta t} f\left(k(t), j(t)\right) dt \tag{7}$$

$$s.t. \quad \dot{k}(t) = j(t) - \sigma k(t), \quad k(0) = k_0. \tag{8}$$

$$\dot{B}(t) = \theta B(t) - (f(k,j) - c(t)), \quad B(0) = B_0. \tag{9}$$

with $k(0)$ and $B(0)$ the initial value of k and B.

The case with capital market constraints, however, when there is a finance premium arising from state dependent credit cost, to be paid, thus $H(k, B)$, then the present value itself becomes difficult to treat. Pontryagin's maximum principle is not suitable to solve the problem and we thus need a method related to dynamic programming to solve for the present value and optimal investment strategy.[17]

From the second model variant we can also explore the use of 'ceilings' in debt contracts and their impact on the dynamic investment decision of the country. Indeed credit restrictions may affect the investment decisions. Suppose the 'ceiling' is the form $B(t) < C$, with C a constant, for all t. Either $C > V(k)$, then the ceiling is too high because the debtor country might be tempted to move close to the ceiling and then goes bankrupt if $B > V(k)$. If $C < V(k)$, then the debtor country may not be able to develop its full potentials, and thus faces a welfare loss.[18] A task of our method will be to compute the present value of the capital stock $V(k)$ for the case of a finance premium and/or credit constraints, so that one obtains information about the ceiling.

In the two cases – locally increasing returns to scale and capital market constraints – the optimal investment strategy may depend on the specific features of the country. As remarked previously those are defined by externalities and increasing returns, adjustment costs of currency capital, and specific capital market constraint. To simplify matters, we presume that the heterogeneity's only defined by the size of the country's capital stock. This

[16]As aforementioned in computing the present value of the future net income we do not have to assume a particular fixed interest rate, but the present value, $V(k)$, will, for the optimal investment decision, enter as argument in the credit cost function $H\left(k(t), V((k(t))\right.$.

[17]See the appendix where the HJB equation is formulated for the above problem, which can be solved through dynamic programming.

[18]In Semmler and Sieveking (1996) the welfare gains from borrowing are computed.

way we can clearly observe that there may be thresholds that separate the optimal solution paths for $V(k)$ to different domains of attraction. For countries with lower capital stock below the threshold, it will be optimal for the country to consume all capital, whereas large countries with a larger capital stock may choose an investment strategy to expand. We also consider the case of a credit constrained country and then study the investment strategy of the country. Moreover, we can admit in our study various paths for the consumption stream, $c(t)$, and their impact on the investment strategy and the present value $V(k)$ for our different model variants.

4 A Numerical Study

Since we cannot obtain analytical solutions to our model variants, we present numerical results obtained for the different specifications of our model variants using a dynamic programming algorithm.[19] This will further motivate our empirical study. Throughout this section we specify the parameter $\sigma = 0.15$. The other parameters will be model specific and specified below.[20] Unless otherwise noted we use for the consumption stream $c(t) \equiv 0$ in our experiments which will be relaxed in the section 4.3..

4.1 Externalities and Increasing Returns

Let us first start with a numeral example employing a benchmark model with a concave production function $y(k) = ak^{\alpha}$, with $0 < \alpha < 1$ and quadratic adjustment cost, bj^{β}. As model parameters we specify $\alpha = 0.5, \beta = 2, b = 0.5, a = 0.29$ and $\theta = 0.1$. This specifies the most simplest variant of a dynamic decision problem with adjustment costs which is employed in economics and which, because of strict convexities, can be shown to exhibit solely one positive steady state equilibrium k^* toward where all countries should converge. The present value curve is simply given by the present value of the net income stream of the country, since we assume a constant credit cost and a debt equation as shown in equ. (9).

[19] For details of the algorithm see Grüne and Semmler (2004).

[20] Note that we, of course, could choose another source of heterogeneity of countries, namely by assuming different technology parameters for countries. This might be another line of research which we will not pursue here.

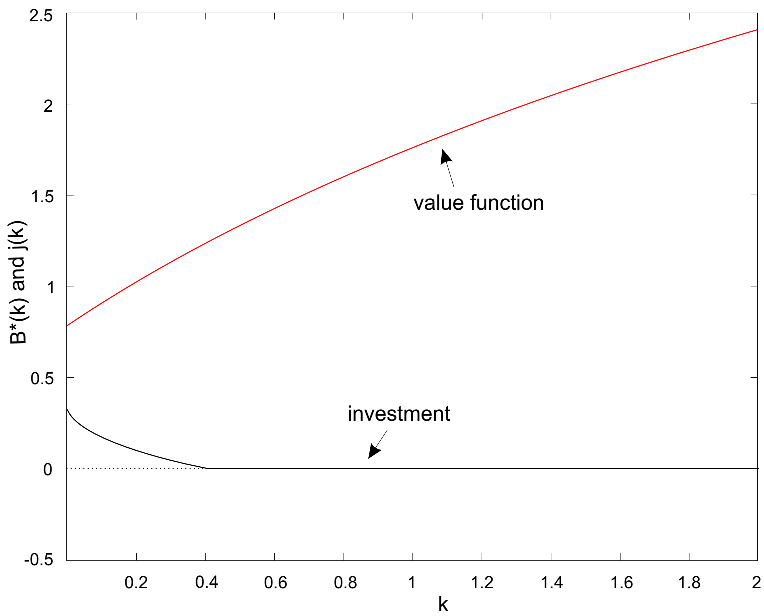

Figure 3: Quadratic adjustment cost of capital

In this case one can use a dynamic programming algorithm of the type suggested in Grüne and Semmler (2004) to solve the model. The value function is given in figure 3 and the solution path of the dynamic decision problem, the investment decision, is given by the optimal control in figure 3. Here the present value is the debt constraint. The debt dynamics holds that all initial levels of debt below the value functions, which can be steered bounded without the investment strategy of the country being affected.

Next we compute the investment strategy for a model variant with a convex-concave production function as suggested in section 2.2. We disregard adjustment costs of capital but again presume a constant borrowing cost, $\theta = 0.1$. The convex-concave production function is for our numerical purpose specified as a logistic function of k

$$y = \frac{a_0 \; exp(a_1 k)}{exp(a_1 k) + a_2} - \frac{a_0}{1 + a_2} \tag{10}$$

with $a_0 = 2500,\; a_1 = 0.0034,\; a_2 = 500$. This convex-concave production function specifies the production function $y(k)$ in equ. (5), yet there is no adjustment cost term $j^\beta k^{-\gamma}$ or j^2. The net income, $f(k, j)$, in equ. (5) is thus linear in the decision variable, j, and one would thus expect a bang-bang solution to exist. In our numerical solution, we restrict the net income such that $f(k, j) \geq 0$. The results are shown in figure 4.

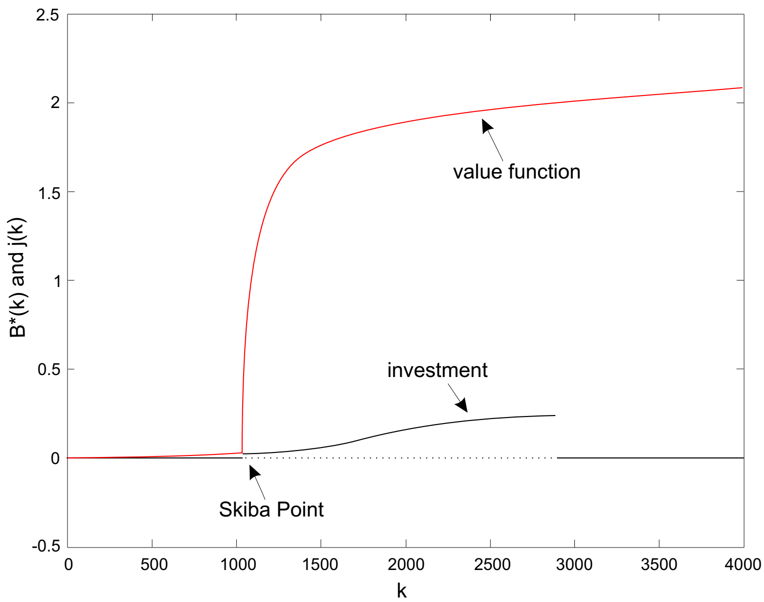

Figure 4: Convex-concave production function

The value function in figure 4 represents the present value curve and the investment curve and the sequence of the optimal investment decision. This variant of our model gives multiple steady states at 0 and 2847 and a threshold, in the literature called a Skiba point, at 1057 in the vicinity of which there is another, but non-optimal steady state. The fact that we can discover a threshold, possibly different from the middle steady state, is due to our solution method. Other work on this topic, for example Aziaridis and Drazen (1990), Aziaridis (2001) and Aziaridis and Stachurski (2004) models, do not provide a method to solve for those thresholds but have only proposed such threshold.

Again any debt, B_0, below the present value curve can be steered bounded but capital stock with initial condition, k_0, to the left of the threshold, the Skiba point, will contract and move to the right of this point and will expand when approaching the high steady state 2847. Thus the threshold is an unstable point but the points 0 and 2847 are attractors. Also clearly visible at the threshold, the control variable is discontinuous and jumps. Note, however, that the jump of the decision variable at the high steady state arises from the fact that, without adjustment cost, we have a decision problem linear in the decision variable.

4.2 Capital Market Constraints

Next we will study our specifications of capital market constraints with finance premium and/or credit constraints. First we will presume that the finance premium, arising from default risk $H(k, B)$, is endogenous, depending on net worth. Second, we presume that there is a debt ceiling as an exogenous credit constraints.

The finance premium that may arise due to costly state verification is positively related to the default cost which is inversely related to the borrowers net worth. Net worth is defined as the country's collateral value of the (illiquid) capital stock less the country's outstanding obligations. As mentioned previously we measure the inverse relationship between the cost of finance and net worth in a function such as equ. (1) where the limit is the risk-free interest rate. In the analytical and numerical study of the model below we presume that the finance premium will be zero in the limit, for $B(t)$ going to zero, the borrowing rate is the risk-free rate. Borrowing at a risk-free rate will be considered here as a benchmark case.

In general, it is not possible to transform the above problem into a standard infinite horizon optimal decision problem for a country. Hence, what we need to use here is an algorithm that computes domains of attraction. We will undertake experiments for different shapes of the credit cost function.

For the credit cost function (1) we specify $\mu = 2$. Taking into account that we want θ to be the risk–free interest rate, we obtain the condition $\alpha_1/(\alpha_2 + 1)^2 = 1$ and thus $\alpha_1 = (\alpha_2 + 1)^2$. Note that for $\alpha_2 \to \infty$ and $0 \leq B \leq k$ one obtains $H(k, B) = \theta B$, i.e., the model from the previous section. In order to compare these two model variants we use the formula $H(k, B) = \frac{\alpha_1}{\alpha_2^2}\theta B$ for $B > k$.[21] We use an adjustment cost of the type $\left(\frac{j}{k}\right)^{\beta}$.

For large α_2 in equ. (1) the model does not necessarily have an unique steady state equilibrium. There can be multiple domains of attraction depending on the initial capital stock size, k. We choose an $\alpha_2 = 100$ and compute the value function and the optimal investment strategy. As can be observed in figure 5 there is a threshold, a Skiba point, at S=0.267 which is clearly visible in the investment curve (lower graph), which is discontinuous at this point. Thus, the dynamic decision problem of the country faces a discontinuity. For countries with initial values of the capital stock $k(0) < S$ the optimal investment strategy is to consume the capital stock and to move to $k^* = 0$. For initial values of the capital stock $k(0) > S$ the optimal investment strategy makes the capital stock growing and tend to the domain

[21] For small values of α_2 it turns out that the present value curve satisfies $V(k) < k$, hence this change of the formula has no effect on $V(k)$.

of attraction $k^{**} = 0.996$.

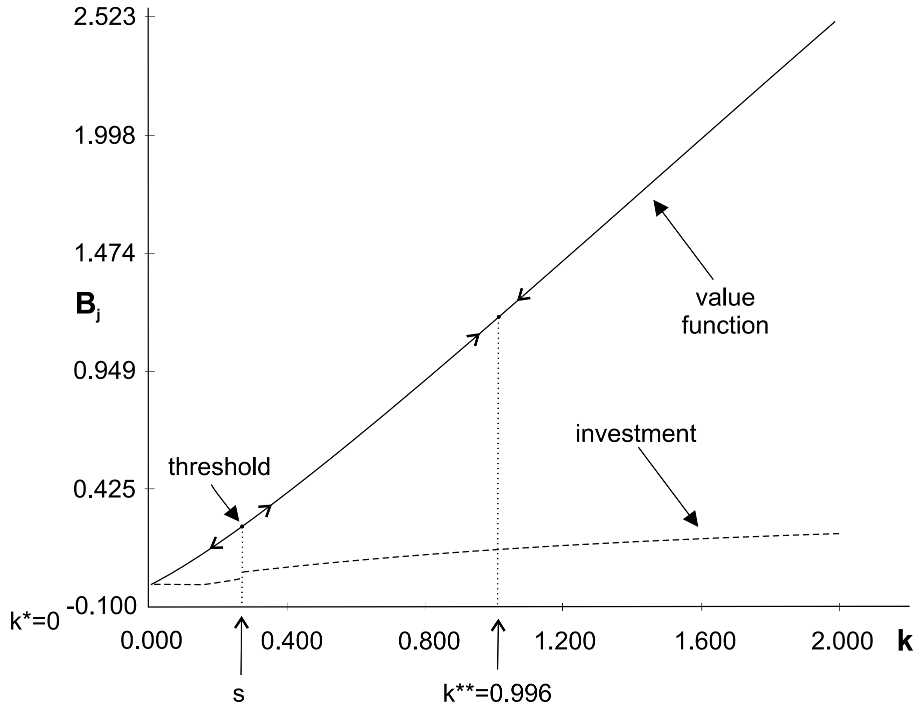

Figure 5: Optimal value function and optimal feedback law

Figure 5 also shows the corresponding optimal value function representing the present value curve, $V(k)$, (upper graph).

In figure 6 we compare the respective present value curves $V(k)$ for $\alpha_2 = 100, 10, 1, \sqrt{2}-1$ (from top to bottom) and the corresponding $\alpha_1 = (\alpha_2+1)^2$.

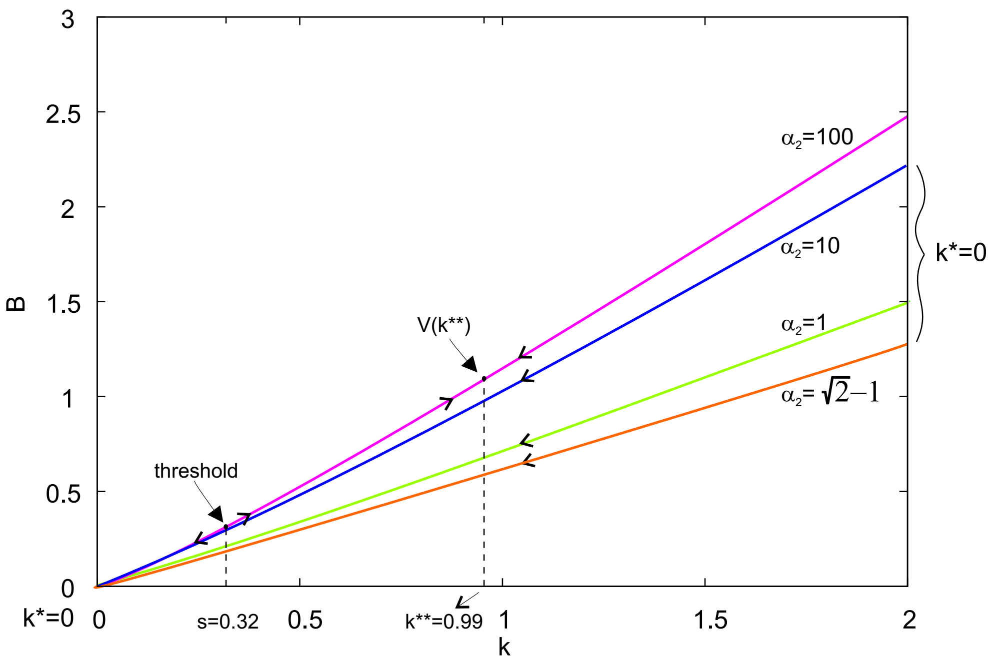

Figure 6: Present value curve $V(k)$ for different α_2

The top trajectory for $\alpha_2 = 100$: There exists a threshold at $S = 0.32$ and two stable domains of attraction at $k^* = 0$ for all capital sizes $k < S$ and $k^{**} = 0.99$ for $k > S$. For the above discrete values somewhere smaller values of α_2 than 100 there is no threshold observable and there exists only one domain of attraction at $k^* = 0$ which is stable. Further simulations have revealed that for decreasing values of $\alpha_2 \leq 100$ the threshold value S increases (i.e., moves to the right) and the stable domain of attraction k^{**} decreases (i.e., moves to the left), until they meet at about $\alpha_2 = 31$. For all smaller values of α_2 there exists just one equilibrium at $k^* = 0$ for all capital stock sizes which is stable.

The reason for this behavior lies in the fact that for decreasing α_2 credit becomes more expensive since the credit market curve in figure 2 becomes very steep. Hence, for small α_2 it is no longer optimal for the country – with any size of the capital stock – to borrow large amounts and to increase the capital stock for a given initial size, instead it is optimal to shrink the capital stock and to reduce the stock of debt $B(t)$ to 0. Thus, with small α_2 and thus large borrowing cost it is for any country, i.e. for any initial capital stock, optimal to shrink the capital stock to zero.

Next we study the decision problem of the country with credit constraints as suggested in the work by Aghion et al. (1999, 2003, 2004). In Aghion et al. (2003) also empirical evidence on credit constraints and its impact on growth is provided. For $H(k, B)$ from equ. (1) with $\alpha_2 = 100$ we test the impact of credit constraints, given by a debt ceiling, on the value function and investment strategy. We impose a credit restriction such as $B(t)/k(t) \leq c$ for some constant c. Thus the constant c represents a maximum debt capacity that the lenders allow for. Figure 7 shows the respective value function curves for the credit constraints $c = 1.2$ and $c = 0.6$ (from top to bottom). In addition, the credit constraints curves $B = ck$ are shown with dots for $c = 1.2$ and $c = 0.6$.

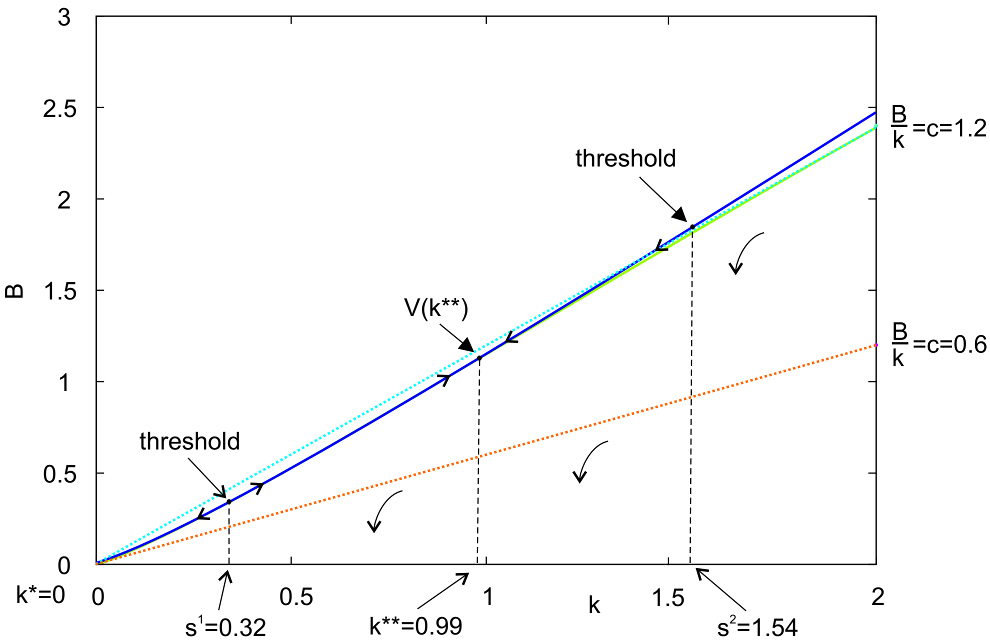

Figure 7: Present value curve $V(k)$ for different debt ceilings, $H(k, B)$ from (1)

In figure 7 for $c = 0.6$ the present value curve $V(k)$ coincides with the credit constraint curve $B(k) = ck$; in this case each trajectory $B(t)$ with $B(t) \leq V(k(t))$ leaves the curve $(k, V(k))$ and eventually $B(t)$ tends to zero. For $c = 1.2$ [22] the curves $B^*(k)$ and $B = ck$ coincide only for capital stock size $k \geq 1.46$. Here one observes the same steady stock equilibria k^* and k^{**} and a threshold S' as for the sup–restriction, in addition to these here a new threshold appears at $S^2 = 1.54$. For initial values of capital stock $(k, V(k))$

[22] This curve is difficult to see because it coincides with the curve for $\sup_{t\geq 0} B(t) < \infty$ for small k and with the credit constraint curve $B = ck$ for large k.

with $S^1 < k < S^2$ the country's capital stock expands or contracts and its capital stock tends to the stable domain of attraction k^{**}. On the other hand, with initial capital stock $k > S^2$ the behavior is the same as for $c = 0.6$, i.e., the corresponding trajectories leave the curve $V(k)$ and eventually $B(t)$ tend to zero.[23] Overall we can observe that credit constraints will reduce the present value and thus the welfare of the country and possibly giving rise to a more complicated dynamics.

4.3 Growth with Consumption

In the previous model variants we have neglected consumption by setting it equal to zero. Next we investigate the role of consumption. We study the case of a non zero finance premium but in $H(k, B)$ from equ. (1) we use again $\alpha_2 = 100$. This gives rise only to a very small finance premium so that the credit cost is approximately equal to the risk free rate. We now study the case when the country's net income f is reduced by a constant consumption stream $c(t) \equiv \eta$, for example paid out in each period. In this case the present value curve $V(k)$ may become negative at some low level of capital stock. This means that there is a minimum level of capital stock required – the level of capital stock where the present value curve becomes positive – that supports the consumption path $c(t) = \eta$. For all levels of capital stock below this size the consumption path $c(t) = \eta$ is not supported.

Note that for the linear model from section 3 system (7)-(9) subtracting a constant η from f simply results in an optimal value function $V_\eta = V - \frac{\eta}{\theta}$. Since for $\alpha_2 = 100$ the present value curve $V(k)$ for $H(k, B)$ from equ. (1) is very close to the model from section 3 we would expect much the same behavior. Figure 8 shows that this is exactly what happens here.

[23]The simulation are halted at zero, but we would like to report if continued the $B(t)$ curve becomes negative and tends to $-\infty$.

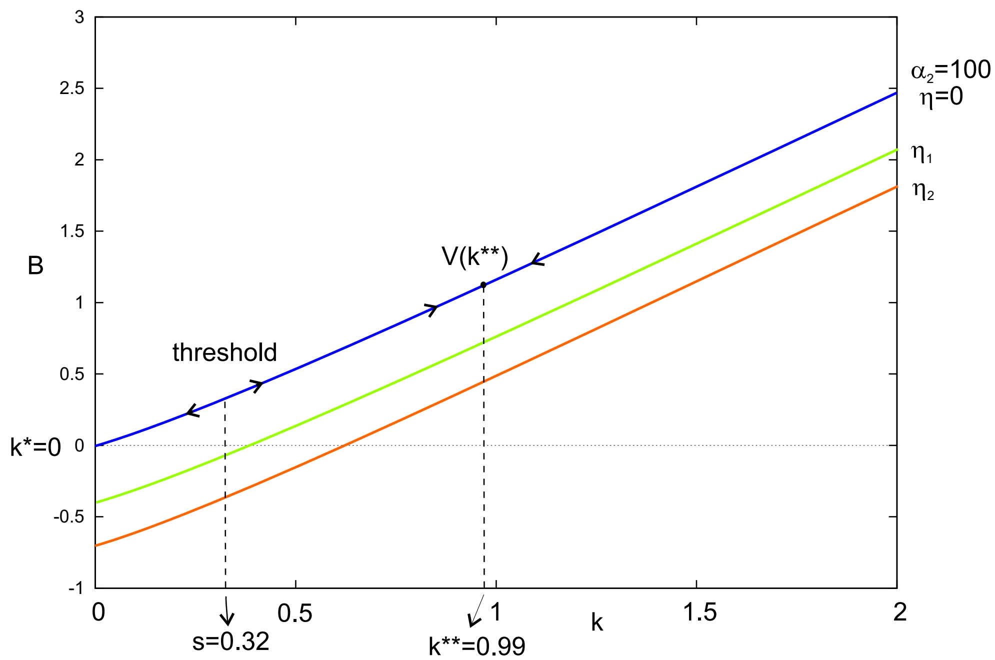

Figure 8: Present value curve $V(k)$ for different η, $H(k, B)$ from (1)

The fact that the curves here are just shifted is also reflected in the shift of the stable equilibria and the threshold, which do not change their positions. The interesting result here is that the dynamic behavior does not depend on the consumption rate. This result holds as long as the dynamic investment decision problem can be separated from consumption decision. [24]

5 Empirical Evidence on Twin-Peaked Distribution

If our above studied economic mechanisms are empirically relevant one would predict that there will be a tendency toward a persistent gap in the per capita income across countries in the long run – a twin-peak distribution of per capita income. This implies that countries will converge to different steady states in the long run.

Our empirical work resembles the one by Quah (1997). We both present the kernel density estimate and the steady state distribution of income in

[24]Under what conditions the separation of the two decisions hold is studied in details in Sieveking and Semmler (1998).

the empirical part of our paper. Yet, Quah (1997), who illustrated with the help of a stochastic kernel the dynamics of income distribution over a 15-year horizon, did not present the steady state of this distribution. We additionally specify the long run steady state distribution in addition to the illustrative ergodic distribution of a Markov transition matrix, as done in Quah (1993). In contrast to Quah (1993) we used data from 1960-1985 but the results are similar: The steady state income distribution is characterized by clusters at the tails and a thinning in the middle, the so-called twin-peaks.

5.1 Kernel Estimators of the Unconditional Density of Per Capita Income

The empirical analysis of countries' convergence properties can be measured by approximating the distribution of the relative per capita income by a kernel density estimation. Relative capita per income is defined as the ratio of a country's per capita income to average per capita world income in the corresponding year. Thus, relative per income of 1/2 indicates that a country has only half of the average per capita world income. For the calculation of relative income we have used data on real GDP per capita (Laspeyres index, 1985 intl. prices) taken from Summer and Heston's Penn Worl Table Mark 5.6 which covers a time period from 1960 to 1985.

The concept of a kernel estimator was developed by Rosenblatt (1956) and Parzen (1962). The basic idea is to construct rectangles of width $2h$ and height $\frac{1}{2nh}$ around every observation and subsequently one sums up the height.

To obtain a smooth curve for the estimated density function one should use a weighting function K with the property that the contribution of an observation to the density decreases with increasing distance to it. In literature it is common to use

$$K = \frac{1}{\sqrt{2\pi}} \cdot e^{\frac{-u^2}{2}}$$

which is the density function of the standard normal distribution and it is denoted Gaussian kernel. Therefore the simplest form of a kernel estimator looks like

$$\overline{f}(x) = \frac{1}{nh} \sum_{i=1}^{n} K\Big(\frac{x_i - x}{h}\Big) \tag{11}$$

where x denotes relative per capita income, h the bandwidth and n the sample size.

Figure 10 and 11 present kernel estimations of relative income in 1960 resp. 1985.[25]

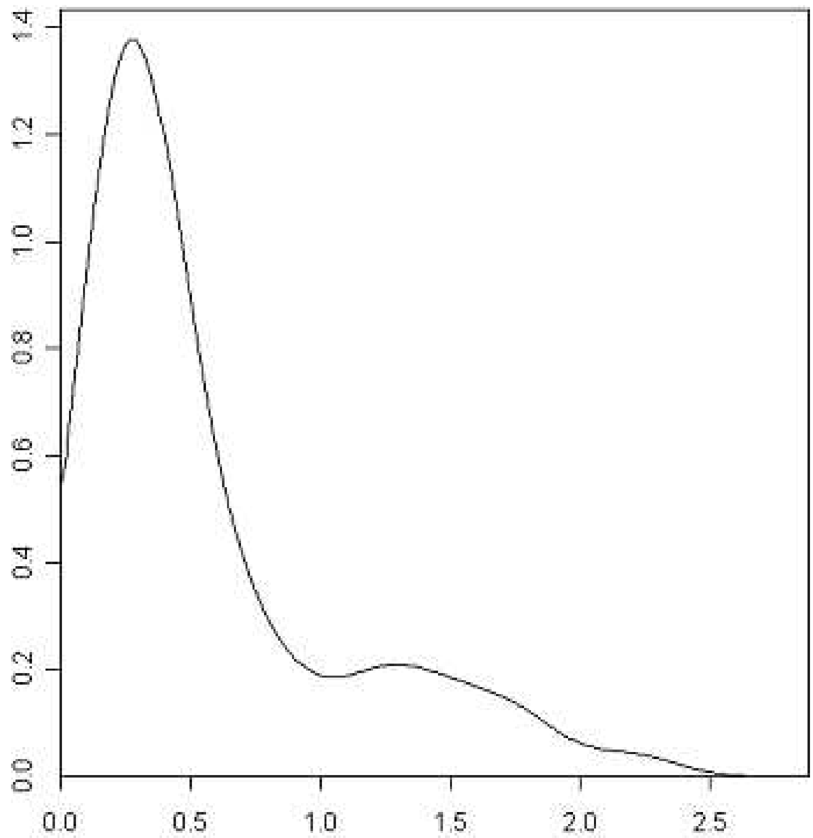

Figure 9: Density of relative income in 1960

[25]With reference to Quah (1996) h was determined using the "optimal bandwidth method" developed in Silverman (1986). Furthermore the data nonnegativity was taken into account.

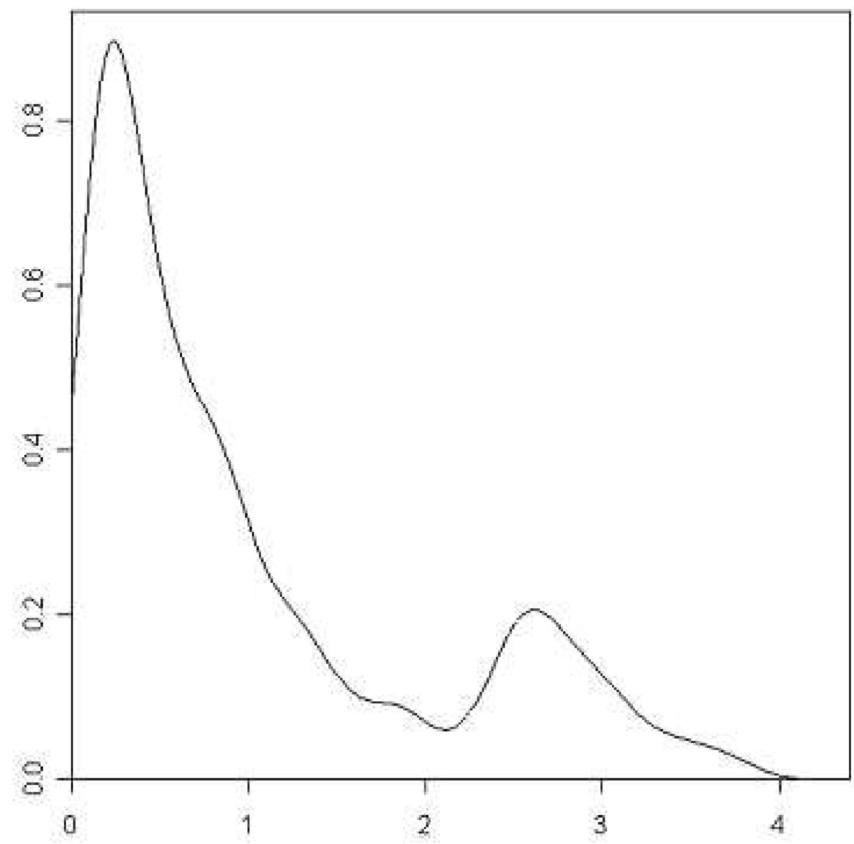

Figure 10: Density of relative income in 1985

Fig. 10 shows a twin-peak distribution in 1960 with a local maximum at about 0.3 of relative income and a second one at 1.3. Thus the distance corresponds exactly to the average world income. Looking at density in 1985, see Figure 11, the difference has increased to 3.2. Furthermore, the maximum at the high levels of relative income has become more pronounced. So there is big group of poor countries and a small group with high income levels. The middle part is nearly empty so that this can be regarded as a hint for a transition towards two different and stable steady states and thus indicating the existence of convergence clubs.

5.2 Transition Matrix and Steady State

To formalize this *visual hint* and to quantify the dynamics in the sequence of distributions using Quah's work (1993) we assume that the evolution of the relative income follows a homogenous first order Markov process.
Let Y_t denote the distribution of relative income at time t, then its evolution is described by the law of motion:

$$Y_{t+1} = M * Y_t \tag{12}$$

M is a one step (annual) Markov chain transition matrix and thus contains probabilities that one country with a relative income corresponding to state i transits to state j in the next year.

To determine unknown transition matrix and its probabilities respectively the first values of the relative income are discretized into five intervals: $Y \leq \frac{1}{4}$, $\frac{1}{4} < Y \leq \frac{1}{2}$, $\frac{1}{2} < Y \leq 1$, $1 < Y \leq 2$, $Y > 2$.
The number of times n, $1 \leq n \leq N$, where $Y_t = i$ and $Y_{t+1} = j$ is the transition, are called transition numbers. We denote them with $F^{(N)}_{ij,\tau t+1}$, which means that the cardinal number, *card* $\phi^{(N)}_{ij} = F^{(N)}_{ij,\tau t+1}$, is attached to the amount $\phi^{(N)}_{ij}$ of transition times. The power of amount $\phi^{(N)}_{ij}$ is called transition number.

If there are no transitions from interval i to j during the period $[t, t+1]$, $\phi^{(N)}_{ij}$ will be empty and the corresponding transition number is zero.
Realizations of $F^{(N)}_{ij,\tau t+1}$ are denoted $f_{ij,\tau t+1}$. The realizations were determined through counting the state sequence of the appropriate distribution Y_t. The elements $f_{ij,\tau t+1}$ form the so-called fluctuation matrix $FM_{\tau t+1} = [f_{ij}]_{i,j \in S}$, $0 \leq f_{ij} \leq N$.

Example:

	$Y \leq \frac{1}{4}$	$\frac{1}{4} < Y \leq \frac{1}{2}$	$\frac{1}{2} < Y \leq 1$	$1 < Y \leq 2$	$Y > 2$
$Y \leq \frac{1}{4}$	17	1	0	0	0
$\frac{1}{4} < Y \leq \frac{1}{2}$	1	29	0	0	0
$\frac{1}{2} < Y \leq 1$	0	1	28	2	0
$1 < Y \leq 2$	0	0	0	14	0
$Y > 2$	0	0	0	0	20

Table 1: Fluctuation matrix $FM_{\tau 63}$ for 1962/1963

Determining fluctuation matrices for all time periods of the sample (1960/61 ($FM_{\tau 61}$) to 1984/85 ($FM_{\tau 85}$)) we obtained 25 of such annual fluctuation matrices. By adding them together they can be transformed into an aggregated fluctuation matrix, that gives the total number of transitions from period t to $t+1$ during the time horizon of 26 years.

Because the amount of transition numbers is completely sufficient for the amount of transition probabilities p_{ij} the probability distribution $F^{(N)}_{ij,\tau t+1} \forall i,j \in S$, as a Likelihood function, can be used for estimating p_{ij}:

The Maximum Likelihood Estimations for the unknown transition probabilities of a irreducible homogenous matrix chain with finite state space are given as:

$$\overline{p}_{ij} = \frac{f_{ij}}{f_{i.}} \quad \forall i,j \in S.$$

and result in the following transition matrix M:

	$Y \leq \frac{1}{4}$	$\frac{1}{4} < Y \leq \frac{1}{2}$	$\frac{1}{2} < Y \leq 1$	$1 < Y \leq 2$	$Y > 2$
$Y \leq \frac{1}{4}$	0.9412	0.0587	0.0000	0.0000	0.0000
$\frac{1}{4} < Y \leq \frac{1}{2}$	0.0693	0.9307	0.0420	0.0000	0.0000
$\frac{1}{2} < Y \leq 1$	0.0000	0.0345	0.9243	0.0411	0.0000
$1 < Y \leq 2$	0.0000	0.0000	0.0369	0.9433	0.0197
$Y > 2$	0.0000	0.0000	0.0000	0.0076	0.9923

Table 2: transition matrix M

The matrix has the following interpretation: For instance on average 94.12% of countries started in the interval $Y \leq \frac{1}{4}$ ended there after one year. 5.87% of them transited to the interval $\frac{1}{4} < Y \leq \frac{1}{2}$ and so on. Probabilities on the main diagonal always exceed 90% and there are only transitions to adjacent intervals (triple diagonal condition).

5.3 Steady State Conditions

To determine the steady state distribution of the transition matrix M it is necessary to analyze under the conditions it exists:
Ergodicity Theorem: A irreducible, aperiodic and positive recurrent Markov chain with a corresponding transition matrix M is always ergodic and has a unique steady state distribution.

- A Markov chain will be called *irreducible* if all states communicate with each other. Accordingly there exists an $n \in \mathbf{N}$ so that $p_{ij} > 0, \ \forall i,j \in S$

- Assume $R_i = n \in \mathbf{N_0} : p_{ii}^n > 0 \ (i \in S)$
  Furthermore
  $$d(i) = \begin{cases} \infty & \text{if } R_i = 0 \\ gcd & \text{else} \end{cases}$$
  Then $d(i)$ is called the period of state i. For $d(i) = 1$ the state is called *aperiodic*.

- The state i of a given Markov chain is called recurrent, if
  $$a_{ii}^* = P(\bigcup_{n=1}^{\infty})\{T_{ii} = n\}) = 1$$
  where
  $$T_{ii} = \begin{cases} min\{n \in \mathbf{N} : X_n = i\}, & \text{if such an } n \text{ exists} \\ \infty & \text{else} \end{cases}$$
  Therefore a Markov chain is recurrent if every state i can be returned to in finite number of steps with probability 1. If the expected return time to every state $E(T_{ii})$ is finite then the chain is denoted *positive recurrent*.

Considering the transition matrix M it is obvious that the corresponding Markov chain is irreducible (because every state communicate with each other) and aperiodic. Furthermore it is positive recurrent because the state space is finite and the chain is irreducible and aperiodic. Thus the Ergodicity Theorem implies a unique steady state distribution (that is independent of the initial distribution).

5.4 Steady State Distribution

It is possible to estimate future distributions of relative income by the following iteration:

$$Y_{t+s} = M^s * Y_t \tag{13}$$

Let s go to infinity resulting in the limiting distribution:

$$\lim_{s \to \infty} Y_{t+s} = M^s * Y_t \tag{14}$$

To reduce matrix multiplication the following remark is helpful:
The steady state long run distribution solves $\pi^T(I - M) = 0$ *where* I *the identity matrix and* π *a column vector.* Furthermore considering the triple diagonal simplifies calculations due to the following relations between ergodic and transition probabilities:
$\frac{\pi_1}{\pi_2} = \frac{p_{21}}{p_{12}}, \frac{\pi_2}{\pi_3} = \frac{p_{32}}{p_{23}}, \frac{\pi_3}{\pi_4} = \frac{p_{43}}{p_{34}}, \frac{\pi_4}{\pi_5} = \frac{p_{54}}{p_{45}}$ *and* $\sum_{i=1}^{5} \pi_i = 1$
The ergodic distribution, the steady state distribution respectively is given in table 3

	$Y \leq \frac{1}{4}$	$\frac{1}{4} < Y \leq \frac{1}{2}$	$\frac{1}{2} < Y \leq 1$	$1 < Y \leq 2$	$Y > 2$
ergodic	0.14	0.12	0.10	0.26	0.38

Table 3: Ergodic Distribution

The ergodic distribution is *twin-peaked* because probabilities for low and high relative income levels are higher than for the middle income. 38% of all countries will converge to a steady state with a relative income level twice or more than the average of all countries. In contrast about 14% of countries will converge to a steady state with an income only a quarter of the average.[26]

6 Conclusions

Many academics and economic policy makers have argued that globalization and increased international competition should lead, in the long run, to productivity increase, faster capital accumulation, increased per capita income and thus to a take off for less developed countries. On the other, there is a critical view of those suggested welfare improvements of globalization by maintaining that globalization of competition may just lead to a poverty trap of some countries and thus to a growing gap of per capita income between countries, exacerbating a trend that empirical research has found since long.

In studying this issue of how countries may respond to such a globalization of competition, we presume that countries are exposed to externalities, increasing returns to scale and financial market constraints. As economic theory has taught us since long, externalities and increasing returns to scale

[26] Kremer, Onatski and Stock (2001) slightly modified this approach by using five year transition intervals with the argument that one year transition intervals may lead to the violation of a homogenous first order Markov process. Their results also shows a bimodal steady state distribution but with 72 % of countries in the highest income level and 12 % in the lowest.

require a certain level of economic activity to allow a country to enjoy those effects. On the hand, underdeveloped financial markets are likely to lead to either severe credit constraints or to the payment of high default premia for a country. By incorporating such ideas in a model of capital accumulation and growth we presume that countries are at different stages of development. As our study then shows, the path of capital accumulation can be expansionary or contractionary: The long run per capita income depends on a threshold that acts as tipping point where small shocks in the vicinity of those tipping points may lead to drastically different outcomes in the development of per capita capital stock and income. Our model thus predicts a twin-peak distribution of per capita income in the long run.

Those theoretical results motivate us to pursue an empirical study on the long run distribution per capita income across countries. For the empirical study we take the per capita income as measure of progress, since in many studies this has been used as standard measure to account for the differences of the level of welfare across countries. Our kernel estimator of the unconditional density of relative per capita income as well as the ergodic distribution, obtained from aggregate (annual) transition matrices, show that indeed a tendency toward a twin-peak distribution of per capita income across countries can be predicted. If our above mentioned forces are at work they are likely to produce a polarization of per capita income distribution in the long run.

Of course, recent research[27] has also studied other important forces of economic growth, such education and formation of human capital, knowledge creation through deliberate research efforts, public infrastructure investment, openness, well organized financial sector, rule of law, economic and political stability, attitude toward work, and so on. A broader study might have to examine those forces of growth as well. Yet, here too externalities and increasing returns maybe at work. In our study here we have mainly focused on spillover effects and externalities, increasing returns and the lack of developed capital markets as possible candidates to create poverty traps. Future studies may have to include other forces of growth as well.[28]

[27]See, for example, Greiner, Semmler and Gong (2005).

[28]Recent studies of the World Bank stress in particular the lack of education and infrastructure –and the externalities arising from them– as playing an important role for poverty traps.

7 Appendix: The Solution of the Basic and Extended Model

The Hamilton-Jacobi-Bellman (HJB) equation for our problem (7) - (9) reads

$$\theta V = \max_{j} \left[k^{\alpha} - j - j^2 k^{-\gamma} + V'(k)(j - \sigma k) \right] \quad \text{(A1)}$$

We can compute the steady state equilibria and the rough shape of the value function and thresholds in three steps. These three steps provide some intuition of how to compute multiple equilibria and thresholds for a dynamic decision problem such as (7) - (9). The actual computation of the value function and thresholds is, however, undertaken with dynamic programming, for details, see Grüne, Semmler and Sieveking (2004).

Step 1: Compute the steady state candidates.

For the steady state candidates, for which $0 = j - \sigma k$ holds, we obtain:

$$V(k) = \frac{f(k, j)}{\theta} \quad \text{(A2)}$$

$$V'(k) = \frac{f'(k, j)}{\theta} = \frac{\frac{\partial}{\partial k}(k^{\alpha} - \sigma k - \sigma^2 k^{2-\gamma})}{\theta} \quad \text{(A3)}$$

Using the information of (A2)-(A3) in (A1) gives, after taking the derivatives of (A1) with respect to j, the steady states for the stationary HJB equation:

$$-1 - 2jk^{-\gamma} + \frac{a^{\alpha} k^{\alpha-1} - \sigma - \sigma^2 (2 - \gamma) k^{1-\gamma}}{\theta} = 0 \quad \text{(A4)}$$

Note that hereby $j = \sigma k$. Given our parameters the equation admits three steady states.

Step 2: Derive the differential equation V'.
Next, we derive the differential equation V$'$ by taking

$$\frac{\partial \theta V}{\partial j} = 0;$$

We obtain

$$-1 - 2jk^{-\gamma} + V'(k) = 0$$

Solving for the optimal j and using the optimal j in (A1) we get

$$V' = 1 + 2\sigma k^{1-\alpha} \pm \sqrt{\left(1 + 2\sigma k^{1-\alpha}\right)^2 + 4\delta k^{-\alpha} V + k^{\gamma-\alpha} - 6} \tag{A5}$$

To solve (A5) we could start the iteration with steady states as initial conditions. For e, a steady state, we get as initial value for the solution of the differential equation (A5):

$$\begin{aligned} V_0 &= \int_0^\infty e^{-\delta t} g(e,j)dt \\ V_0 &= \frac{1}{\delta} g(e,j) \end{aligned}$$

Step 3: Compute the global value function by taking

$$V(k) = \max_i V_i$$

where $V(k)$ is the outer envelope of the piece-wise value function obtained through Step 2.

The more general case has the credit cost as endogenous. If we have $H(k,B)$, as in equ. (1) and thus equs. (2)-(4) hold then, the present value itself becomes difficult to treat. Pontryagin's maximum principle is not suitable to solve the problem with endogenous credit cost and we thus need to use a variant of a dynamic programming to solve for the present value and investment strategy of for our problem (2) - (4).

In the general case of equ. (2)-(4) with default risk and a finance premium as stated in equ. (1), and shown in Figure 1, we have the following HJB-equation

$$H(k, B^*(k)) = \max_j \left[f(k,j) + \frac{dB^*(k)}{dk}(j - \sigma k) \right] \tag{A6}$$

Note that in the limit case, where there is no borrowing and $N = k$, and thus the constant discount rate θ holds we obtain the HJB-equation (A1). Note also that in either case B^* the credit worthiness, the maximum amount the country can borrow, is equal to the asset price $V(k)$. The HJB-equation (A6) can be written as

$$B^*(k) = \max_j H^{-1} \left[f(k,j) + \frac{dB^*(k)}{dk}(j - \sigma k) \right] \tag{A7}$$

which is a standard dynamic form of a HJB-equation. Next, for the purpose of an example, let us specify $H(k,B) = \theta B^\kappa$ where, with $\kappa > 1$, the interest payment is solely convex in B. We then have

$$B^*(k) = \max_j \left[f(k,j) + \frac{dB^*}{dk}(j - \sigma k) \right]^{\frac{1}{\kappa}} \theta^{-\frac{1}{\kappa}} \tag{A8}$$

The equilibria of the HJB-equation (A8), with $\kappa > 1$, are shown below. The algorithm to study the more general problem of equ. (A6) is summarized in Grüne, Semmler and Sieveking (2004).

If the HJB-equation (A6) holds with $H(B) = \theta B^\kappa$, the finance premium, depends on the debt of the country. This extension is presented in Semmler and Sieveking (2000) and Grüne, Semmler and Sieveking (2004). For $H(B) = \theta B^\kappa$ for $\kappa \geq 1$ it leads to the following equation for candidates of equilibrium steady states

$$1 + 2jk^{-\gamma} = \frac{\alpha k^{\alpha-1} - \sigma - \sigma^2(2-\gamma)k^{1-\gamma}}{\theta\kappa(k^\alpha - \sigma k - \sigma^2 k^{2-\gamma})^{(\kappa-1)/\kappa}} \tag{A9}$$

Note that the steady state candidates are the same as in (A1) if in (A6) and (A8), $\kappa = 1$ holds. For details of the solution, for the problems (A1) and (A6), and for numerical methods to solve them, see Grüne, Semmler and Sieveking (2004).

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