Twin Deficits and Inflation Dynamics in a Mundell-Fleming-Tobin Framework

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Abstract

In this paper we consider a small open economy of the Mundell-Fleming-Tobin type. We study its accumulation laws of government and foreign debt (or surpluses) and couple these dynamics with a standard Phillips curve approach for such an economy. The resulting dynamical system and the equilibrium relationships on which it is based are rather complex and allow for a variety of monetary and exchange rate regimes and stability results. We study in this paper in particular the case of an interest and exchange rate peg and a regime with perfectly flexible exchange rates and given money supply. In both case we get stable adjustment processes of the considered twin deficits or surpluses only under very restrictive assumptions on the behavioral equations of the model and various type of instability results otherwise, to be investigated in more detail in future research.

Contents

1 Introduction

Over the last decade, the unprecedented increase in the internal and external imbalances of two of the largest economies in the world, the U.S. and China, has raised serious concerns about the dangers of such macroeconomic imbalances for the stability of the world economy. Nevertheless, a correction of these imbalances by means of a significant readjustment concerning the U.S. dollar is not very probable in the near future, due to the reluctance to deal with this issue not only of both governments but also because many other countries do not make steps into this direction. In this light, national wage price adjustments as well as foreign asset in- and outflows have become even more important as macroeconomic adjustment mechanisms to these imbalances.

In view of such developments, we consider in this paper in a Mundell-Fleming-Tobin framework the dynamics of government deficits or surpluses as well the direction of foreign assets accumulation, here for a small open economy, when financial assets are assumed to be imperfect substitutes (where therefore the UIP condition does not hold) and where inflation dynamic is driven by an open-economy Phillips curve. We moreover assume, as in Rødseth (2000, that domestic bonds are non-tradables, i.e., the accumulation of foreign assets by the domestic economy can only occur through surpluses in the current account and not via balanced exchanges of domestic against foreign assets on the international capital markets. International capital movements of this sort will be added in Asada, Flaschel, Groh and Proaño (2006) and will then be investigated with respect to the differences this implies compared to the case of non-tradable domestic bonds (where asset reallocations are confined to the stocks held domestically).

Concerning the goods markets in the domestic economy, we introduce a Keynesian demand constraint in place of the assumption of Say's law in Flaschel (2006) where the world market delivered or consumed everything not present or not needed in the domestic economy. This will be done by adding an export function (and now also an investment function) to the description of aggregate goods demand in the domestic economy. Finally, we will now assume regressive exchange rate expectations formation in place of rational expectations which, on the one hand, helps to avoid the questionable jump variable technique of the rational expectations school (in order to get stability by assumption in an otherwise unstable saddlepoint environment) and which, on the other hand, allows for maximum stability of the considered dynamics (since only fundamentalist and thus in principle only converging expectations revisions are allowed for). This simplifying assumption will be helpful for the central objective of the paper which is to focus on the fundamental destabilizing forces contained in the two accumulation equations for internal and external deficits or surpluses, caused by the government budget equation and the evolution of the current account of the considered economy.

We thus allow, compared to Flaschel (2006), for underutilized labor and capital due to insufficient effective demand on the goods market,[1] as they can result from Keynesian consumption and investment demand, augmented by a net export schedule in this open economy, and the goods market equilibrium condition based on these aggregate demand functions. We consider on this basis a Keynesian approach to the business cycle in the real part of the model (based on an IS curve that equates savings to investment). Due

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[1]and of course also overutilized labor and capital depending on the state of aggregate demand.

to our use of a standard open economy money wage Phillips curve, this real cycle is accompanied by labor market driven inflation or deflation dynamics which – in combination with the capital account and government budget dynamics of the type considered in Flaschel (2006) – provides a dynamic model that goes significantly beyond standard Mundell-Fleming type approaches. Finally, since we are now considering exports and imports simultaneously we are of course now using a two commodity (but single small country) approach in contrast to the paper of Flaschel (2006), where primarily only one commodity cases were considered.

Despite the intrinsic government budget and capital account dynamics of the model, we have also a rich set of feedback channels present in it: Hicksian disposable income effects, Pigou price level effects, Keynes price level effects, the Mundell-Tobin effect of inflationary expectations in both the consumption and the investment function, Dornbusch exchange rate effects, portfolio effects, and the stated stock-flow interactions. The interaction of these effects allows for a variety of (in-)stability results, too numerous in order to allow their investigation in a single paper of this model type. We therefore concentrate in this paper on basic studies of a regime with pegged interest as well as exchange rate and contrast this situation with a regime where the exchange rate is perfectly flexible and the money supply a given magnitude, under the control of the Central Bank of the domestic economy.

In the next section we derive the model of this paper and the stock-flow interaction that characterize its intrinsically generated dynamics (which includes also a Phillips curve approach to inflation dynamics), calculate the steady state position of the dynamics and perform some preliminary stability considerations. Section 3 investigates the case of an interest rate peg coupled with an exchange rate peg and derives stability as well as instability results for such a monetary regime. In section 4 we do the same for the regime of perfectly flexible (overshooting) exchange rates and a given money supply. Section 5 concludes.

2   The general framework

As point of departure of our theoretical analysis we employ a standard small open economy Mundell-Fleming-Tobin model, as discussed in detail in Rødseth (2000, Ch.6). On this basis our purpose is to analyze rigorously the stock-flow dynamics that are generated by the capital account in the balance of payments (with respect to the foreign bond accumulation by private households) and by the government budget constraint (with respect to their domestic bond holdings), when these dynamics are linked (and interact) with macroeconomic activity levels through price level adjustments they imply.

2.1   Budget equations and saving/financing decisions

We start with the budget equations of the three relevant sectors, households, the government and the central bank. With respect to firms, we assume that all of their income is transferred to the household sector and that households then provide the credit needed

to allow them to finance their investment expenditures (which is by and large the same as the assumption of a direct investment decision by the household sector). Concerning the Central Bank (CB) we note that it may change its government bond holdings by means of an open market policy $dB_c = dM$, and similarly its foreign bond holdings through $dF_c = dM$, without influencing the flow budget equations to be discussed below, since all interest income from these bond holdings is transferred back to the government sector which therefore only has to pay interest on the bonds B held by the private sector. The domestic bond holdings of the CB can therefore be neglected in the following and an additional indexation of the magnitude that is held by the household sector can thus be avoided.

The budget restrictions for the three sectors of the domestic economy are given by:

$$p(Y - T) + rB + er^*F_p \equiv pC + pI + \dot{M} + \dot{B} + e\dot{F}_p \tag{1}$$

$$pT + er^*F_c + \dot{B} + \dot{M} \equiv pG + rB \tag{2}$$

$$\dot{M} \equiv \dot{B}_c \tag{3}$$

We denote here by B, F_p the domestic and foreign bonds held by the household sector and by F_c the foreign assets held by the central bank (its currency reserves, that can only be changed by open market operations on the domestic market for foreign assets, we assume, as in Flaschel (2006), that domestic bonds are non-tradables). We use r for the nominal rate of interest and e for the nominal exchange rate and index by $*$ foreign set variables, which are obviously not under the control of the domestic economy. All other symbols are fairly standard and thus need no explanation here. We stress again that domestic and foreign interest incomes of the CB are transferred to the government sector so that the central bank can only change its asset position by printing new money. In the normal course of events it will channel this money into the economy by buying of domestic government bonds as shown in the third budget equation, but it can also rearrange its portfolio of domestic and foreign bonds (not considered explicitly here) by open market operations on the domestic financial markets (as already considered above).

These budget equations imply for the evolution of domestic and foreign bonds held by the private sector of the domestic economy:

$$\dot{B} = rB + p(G - T) - er^*F_c - \dot{M} \tag{4}$$

$$e\dot{F}_p = e\dot{F} = p(Y - C - G - I) + er^*F \tag{5}$$

These equations show the nominal evolution of government debt implied by its budget constraint and of the countries foreign debt (or foreign surplus) position as implied by the balance of payments (which – due to the situation assumed to hold for the budget equations of the three sectors of our economy – is here always balanced, independently of the exchange rate and monetary regime that is to be investigated).[2]

Considering the given situation of a balanced balance of payments (without the intervention of the CB) from the viewpoint of savings we can write:

$$pS_p = p(Y - T) + rB + er^*F_p - pC = pI + \dot{M} + \dot{B} + e\dot{F}_p$$

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[2]This is due to assumption that the new issue of money and domestic government bonds is always voluntarily absorbed by private households.

$$pS_g = pT + er^*F_c - rB - pG = -\dot{B} - \dot{M}$$

which gives for the total savings pS of the economy (NX net exports):

$$pS = pY - pC - pG + er^*(F_p + F_c) = pNX + er^*F = pI + e\dot{F}_p = pI + e\dot{F}, \ F = F_p + F_c$$

This is again the formulation of the fact that the balance of payments must be balanced in the considered situation without any further adjustment processes, based on the assumption that the issue of new money $\dot{M}$ and new government bonds $\dot{B}$ is in fact always accepted by the household sector (as it is implicitly made in the above formulation of the three budget equations of our economy).

2.2   Real Disposable Income and Wealth Expressions

Concerning the real disposable incomes of the private and public sectors in the domestic economy, in analogy to the conventional Hicksian definition of private disposable income – which is defined as the level of income which when consumed just preserves the current level of wealth of the considered sector – we first define and rearrange this concept for the aggregate government sector (including the foreign interest income of the central bank) and provide thereafter these concepts as a recapitulation for the sector of private households (cf. Flaschel (2006) in this regard). Since the rate of inflation $\hat{p} = \dot{p}/p$[3] is a variable in the following completion of the model we will use this expression here already in the definition of the real rates of interest to be employed in the calculation of real disposable incomes of the aggregate government sector Y_g^a and of private households Y_p.

By definition, the aggregate real income of the government sector Y_g^a is composed of the tax and interest rate payments and receipts of the government plus the inflation and capital gains on government debt and central bank reserves,

$$Y_g^a \quad := \quad T - \frac{rB}{p} + \frac{er^*F_c}{p} + \hat{p}\frac{M+B}{p} + (\hat{e} - \hat{p})\frac{eF_c}{p}.$$

After some manipulations, we can express the aggregate real income of the government sector as

$$Y_g^a = T + r\frac{M}{p} + \xi\frac{M+B}{p} + \rho^*W_g^a, \tag{6}$$

where real aggregate government wealth is defined as

$$W_g^a := \frac{-(M+B) + eF_c}{p} = -\frac{(M+B)}{p} + \frac{eF_c}{p} = W_g + W_c, \tag{7}$$

where $\xi = r^* + \hat{e} - r$, $\rho^* = r^* + \hat{e} - \hat{p}$, and $\rho^{*e} = r^* + \epsilon(e) - \hat{p}$ represent the actual risk premium on foreign bonds, the actual real rate on return on foreign bonds and finally the expected one (with $\epsilon(e)$ a regressive expectations mechanism for example). Note that the third term in eq. (6) can be ignored if the uncovered interest rate parity condition is assumed to hold. This is however only possible if international trade in domestic and foreign bonds is allowed for as in Asada, Flaschel, Proaño and Semmler (2006) where

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[3]We use in this text $\hat{x}$ to denote the growth rate of a variable x.

this condition is considered as a limit case to imperfect substitutability of internationally traded bonds.

Concerning the Hicksian real disposable income of the private sector, it is defined as

$$Y_p = Y - T + (r^* + \hat{e} - \hat{p})\frac{eF_p}{p} + (r - \hat{p})\frac{B}{p} - \hat{p}\frac{M}{p},$$

or, after some manipulations,

$$Y_p = Y - T + \rho^*(W - W_g^a) - \xi\frac{M + B}{p} - r\frac{M}{p} \tag{8}$$

where the actual wealth of the private sector is defined as the difference between the wealth of the whole economy W and the aggregate wealth of the government sector W_g^a, i.e.

$$W_p = \frac{M + B + eF_p}{p} = W - W_g^a. \tag{9}$$

From the calculations of disposable income of households and the government, we finally in sum get:

$$Y_p = Y - Y_g^a + \rho^*W \quad \text{or} \quad Y_p + Y_g^a = Y + \rho^*W$$

as a relationship between the country's total disposable income, its domestic product and the real interest on domestically held foreign bonds.

2.3 Temporary equilibrium: Output, interest and exchange rate determination

Having described the budget restrictions and disposable income equations of the different sectors in the domestic economy, we follow again Rødseth (2000) in his description of the temporary equilibrium relationships on the goods and the asset markets, which are given by:

$$Y \stackrel{IS}{=} C(Y_p, W - W_g^a, \rho, \rho^{*e}) + I(Y, \rho, \rho^{*e}) + G + NX(\cdot, \eta, Y^*) \tag{10}$$

$$M/p \stackrel{LM}{=} m^d(Y, r), m_Y^d > 0, m_r^d < 0 \tag{11}$$

$$W \stackrel{FF}{=} eF/p = e(F_p^d + F_c^d)/p = f^d(\xi^e, W - W_g^a) + eF_c^d/p, \tag{12}$$
$$f_{\xi^e}^d > 0, f_{W_p}^d \in (0, 1)$$

$$B/p = W_p - f^d(\xi^e, W_p) - m^d(Y, r), \quad \xi^e = r^* + \epsilon(e) - r \tag{13}$$

with $\rho = r - \hat{p}$ and $\rho^{*e} = r^* + \epsilon(e) - \hat{p}$ representing the real domestic interest rate and the expected real return on foreign bonds respectively. While eq. (10) represents an IS-curve of an advanced traditional type,[4] the money market equilibrium, the standard textbook LM relationship, is described by eq. (11), where money demand is as usual assumed to

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[4]The component $\cdot$ in the net export function stands for the variables that determine the domestic absorption in terms of consumption and investment demand.

depend positively on the level of output and negatively on the domestic interest rate. Eq. (12), the FF-curve, which represents the equilibrium on the market for foreign bonds (foreign exchange), is determined primarily by the reaction of private household with respect to the risk premium ξ and the marginal wealth effect $f_{W_p}^d$ in foreign (and domestic) bond demand, as in Rødseth (2000), with $F = F_p^d + F_c^d$. Concerning the market for domestic bonds, it is always cleared, by Walras's Law of Stocks, when the money market and the market for foreign bonds are in equilibrium.

These equations are to be solved (by means of the implicit function theorem) for the variables considered as statically endogenous (depending on the monetary and exchange rate regimes that are assumed) and to be inserted into the laws of motion of p, B and F_p in order to obtain an autonomous system of ordinary differential equations, describing domestic price level dynamic, the dynamic of the government budget constraint, and of the foreign position of the domestic economy. Of course, the definitions of $Y_p, \rho, \rho^{*e}, \eta$ have also to be inserted to achieve this end. Note that the variables M, B, F, r, e may become policy parameters depending on the monetary and exchange regime that is under consideration.[5]

Concerning the interest rate, by applying the implicit function theorem to eq. (11) and assuming that money demand is based on the full employment output level $\bar{Y}$, we get for its level in its reduced form representation the formula:

$$r = r(p, M, \bar{Y}) \quad \text{with} \quad r_1 > 0, r_2 < 0, r_3 > 0.$$

The theory of the nominal rate of interest is thus the standard or even textbook one of Keynesian macroeconomics and closely related to the working of the so-called Keynes-effect whereby money wage decreases stimulate the economy when they imply price level changes in the same direction and on this basis lower interest rate which then work on consumption and investment via the real rate of interest.

Concerning the nominal exchange rate, inserting the above equation into eq. (12) which describes the equilibrium in the market for foreign bonds, gives us an equation in the endogenous variables e and p and thus provides us with a theory of the nominal exchange rate in its dependence on dynamically endogenous stock variables B, F_p and the price level p.

$$F_p = p f^d(r^* + \epsilon(e) - r(p, M, \bar{Y}), \frac{M + B + e F_p}{p})/e \tag{14}$$

We recall that $f_1^d, f_2^d > 0$ is assumed to hold and that we have $\epsilon'(e) < 0$ due to the assumption of a regressive formation of expectations of exchange rate de- or appreciation.

In the case of a perfectly flexible exchange rate e where M, F_c are given magnitudes that are under the control of the CB, and $F = F_p + F_c$ is also assumed to be a given magnitude, we obtain by means of the implicit function theorem the following expressions for the

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[5]In the case of a flexible exchange rate regime and a given money supply we will for example get jumps in the variables e, η, W, W_g^a when there is a shock in money supply occurring.

partial derivatives with respect to the dynamically endogenous variables:

$$\frac{\partial e}{\partial F_p} = -\frac{e(1 - f_2^d)}{F_p(1 - f_2^d) - p f_1^d \epsilon'} < 0$$

$$\frac{\partial e}{\partial B} = \frac{f_2^d}{F_p(1 - f_2^d) - p f_1^d \epsilon'} > 0$$

$$\frac{\partial e}{\partial p} = \frac{f^d - p f_1^d r_1 - f_2^d W_p}{F_p(1 - f_2^d) - p f_1^d \epsilon'} = \frac{e F_p(1 - f_2^d)/p - p f_1^d r_1 - f_2^d \frac{M+B}{p}}{F_p(1 - f_2^d) - p f_1^d \epsilon'} < 0$$

with $0 < f_2^d < 1$ (the portfolio choice condition). We note that the last partial derivative is negative if the degree of capital mobility with respect to the risk premium is chosen sufficiently high (which is what we do in the following). The signs of the partial derivatives shown above will also apply to the real exchange rate in the place of the nominal one, due to its definition $\eta = e(p)/p$. Note finally that the first two partial derivatives will approach zero if capital mobility approaches infinity and that in this case the limit of the partial derivative with respect to p is simply given by r_1/ϵ'. The result $\frac{\partial e}{\partial p} < 0$ reflects the message of the Dornbusch (1976) model, according to which an increase in the domestic interest rate (caused by shrinking real balances) leads to higher depreciation gains expectations which in a regressive expectations environment demands for a decrease, an appreciation of the nominal exchange rate.

Since the dynamic behavior of prices enters in the IS-equilibrium condition through the real interest rate, we already define here the law of motion for the price level p, which is determined, under the assumption of constant markup-pricing, by a standard, expectations augmented, open-economy Phillips curve[6]

$$\hat{p} = \beta_w(Y - \bar{Y}) + \gamma \hat{p} + (1 - \gamma)(\pi^* + \epsilon(e)) = \beta_w(Y - \bar{Y})/(1 - \gamma) + \epsilon(e), \quad \gamma \in (0, 1) \quad (15)$$

where the output gap $Y - \bar{Y}$ measures the demand pressure on the labor market. Note that this form of a Phillips curve derives from an expectations augmented one where the cost pressure item (concerning the consumer price index) is initially given by a weighted average of domestic and import price inflation of the form: $\gamma \hat{p} + (1 - \gamma)(\pi^* + \epsilon(e))$ and where myopic perfect foresight is assumed with regard to the evolution of the domestic inflation rate.

By inserting the open-economy Phillips curve equation in the IS equilibrium equation described by eq. (10), we can calculate the following signs of the partial derivatives, which for the case where the speed of adjustment of money wages β_w is chosen sufficiently low, are:

$$\frac{\partial Y}{\partial F_p} > 0, \qquad \frac{\partial Y}{\partial B} > 0, \qquad \frac{\partial Y}{\partial p} < 0$$

This holds again only in the case where in addition capital mobility is assumed as sufficiently high. We leave the lengthy calculations of the involved partial derivatives here to the reader and only state that increasing wealth of private households with respect to domestic and foreign bond holdings stimulate economic activity, while and increasing price level will reduce economic activity through various channels in the model, in

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[6]See Rødseth (2000, Ch.6) for its motivation.

particular through real wealth effects in consumption demand, but also through interest rate increases and real exchange rate decreases.

In the case where capital mobility is nearly perfect ($f_\xi^d \approx \infty$) and wages nearly rigid with respect to demand pressure ($\beta_w \approx 0$) we may summarize the comparative static properties – as far as the dynamically endogenous variables are concerned – approximately as follows:

$$r = r(p), r' > 0, \quad e = e(p), e' < 0, \quad Y(F_p, B, p), \quad Y_1 > 0, Y_2 > 0, Y_3 < 0$$

2.4 Dynamics and the steady state of the economy

Our dynamical representation of the Mundell-Fleming-Tobin model in its general form (with F_c a given magnitude in the considered exchange rate regime and with $p^* = 1$ for simplicity),[7] expressed in nominal terms, consists of the following differential equations

$$\dot{F}_p = r^*(F_p + F_c) + NX(\cdot, \eta, Y^*)/\eta, \quad \eta = \frac{e}{p}, \; p^* = 1 \tag{16}$$

$$\dot{B} = rB + pG - pT - er^*F_c \tag{17}$$

to be coupled with the law of motion for the price level p:

$$\dot{p} = [\beta_w(Y - \bar{Y})/(1 - \gamma) + \epsilon(e)]p \tag{18}$$

and with the description of the temporary equilibrium given by eq. (10)-(12) and the definitions of private wealth and real disposable income discussed in the previous section.

In order to allow for a sequential determination of the steady state values of this dynamical system, we proceed as follows. We assume (and shall later on prove) that the steady state value r_o of r is given by r^*, i.e., we have $\xi = 0$ in the steady state. Due to the Phillips curve we furthermore know that $Y = Y_o$ must hold true. The given quantity of money M then allows to determine the steady state value of p as $p_o = M/m^d(Y_o, r^*)$. Setting $\dot{B} = 0$ furthermore gives a simple positive relationship between B_o and e_o, which represent the equilibrium in the domestic bond market. The FFB-curve implies on this basis:

$$F_{po} \overset{FFB}{=} p_o f^d(0, \frac{M + B_o(e_o) + e_o F_{po}}{p_o}) = p_o f^d(0, \frac{M}{p_o} + \frac{T - G}{r^*} + \frac{e_o(F_{po} + F_c)}{p_o}),$$

which defines a positive relationship between e_o and F_{po}, as the figure 1 (where we use linear curves for reasons of simplicity).

From $\dot{F} = 0$, the equilibrium in the foreign currency bond market, we furthermore get

$$r^*F_o + p_o NX(\cdot, e_o/p_o, Y^*) = 0,$$

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[7]Such an approach is further justified through the observation that the dynamics of the nominal stock magnitude B is involved in the above considered real dynamics as long as $\xi \neq 0$ holds true which shows that the real dynamics is then in fact a hybrid as far as the distinction between real and nominal variables is concerned.

that is, a negative relationship between e_o and F_{po}, represented in figure 1 by the FB curve. If we assume that $M+B_o$ is nonnegative (compare figure 1), we have a nonnegative demand for foreign bonds at $e_o = 0$ and thus a nonnegative value of F_{po} associated with it. In the case of linearity (which need not hold in the large) we will then get an intersection of the $\dot{F} = 0$ and the FFB curve in the positive half plane of the (F, e) space which gives us positive steady state values of both F_p and e (since one can assume that there is a positive value e where $NX = 0$ holds). Under the stated conditions we thus have that the private agents of the domestic economy will hold a positive amount of foreign bonds in the steady state and that net exports must therefore be negative in the steady state due to a value of the exchange rate that is below the one that balances the trade balance. In the general situation where both curves in figure 1 are nonlinear, one must show that the $\dot{F} = 0$ curve is not to flat in view of the intersection of the $FFB-$curve with the vertical axis and both curves are not approaching $+\infty$ for finite values of the nominal exchange rate.
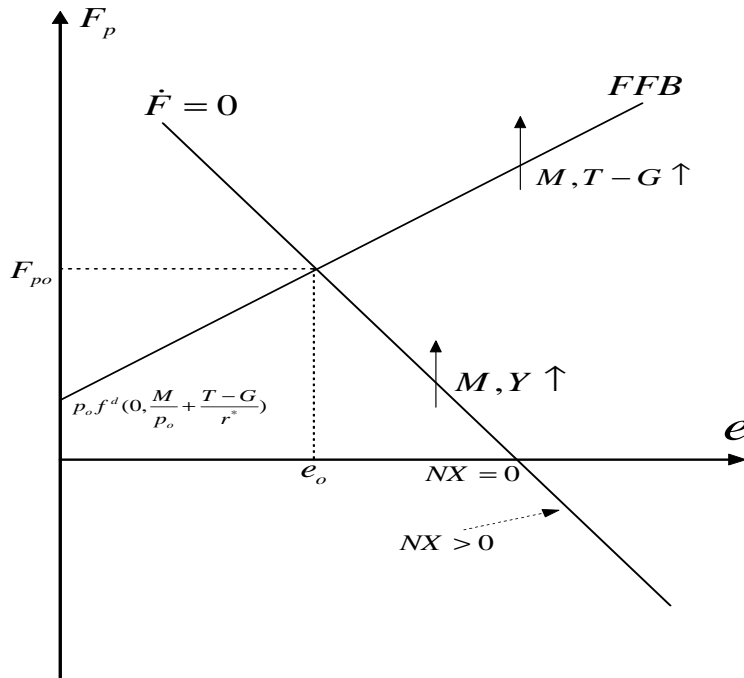

Figure 1: *Steady State Stock Equilibrium ($B_o > 0$).*

Figure 1 shows on the basis of the above the determination of the steady state values of both the nominal exchange rate and the value of foreign debt held domestically, and also indicates how these values can be influenced by fiscal and monetary policy in particular. It shows that fiscal consolidation will make the currency stronger and increase the amount of foreign $-denominated debt held domestically. The latter effect is also produced by a monetary expansion, whose effect on the value of e may however be ambiguous, depending on the competitiveness of the considered economy, i.e., on the elasticity of the net export function with respect to the real exchange rate at the steady state of the economy. Note that at this steady state we have a negative trade account

and a positive interest rate account and of course a balanced capital account in the balance of payments.

Inspecting the IS curve on the basis of what has been determined so far shows however that no endogenous variable is then left that allows for IS-equilibrium at the full employment level Y and the foreign interest rate r^*. We therefore now adjust the values T, G such that $T - G$ remains unchanged (that is without impact on the left hand side of the government budget constraint) until also the IS-curve passes through the determined steady state values. This holds if G, T are – on the basis of a given value for $G - T$ – determined by:

$$Y = C(Y - T + r^* e_o F_{po}/p_o + r^* B_o/p_o, r^*, r^*) + G_o + NX(\cdot, e_o/p_o, Y^*)$$

if $I = 0$ holds in the steady state (see below).

Note here also that B_o/p_o is given by $(T - G)/r^* + \eta_o F_c$ need not be positive and in fact must be negative in the case of a value of $T - G$ that is sufficiently negative. This is due to the fact that interest payments $r B_o$ are positive in the steady state in the case of a positive value of government debt, implying that real government income $T + e_o F_c/p_o$ must be sufficiently high relative to G to allow for the given interest obligation of the government (since there is no issue of new government debt in the steady state). Note also that we assume the behavioral functions and parameters of the model to be such that disposable income Y_p is positive at the steady state. Note finally that our regressive expectation function for exchange rate de- or appreciations is always assumed to fulfill $\epsilon(e_o) = 0$ in the steady state (is assumed to be asymptotically rational) and is therefore shifting with the steady state solution for e_o.

We have determined in advance the steady state values for p, r, Y, ρ, ρ^* and then simultaneously the steady state values for e, η, F_p, B basically from stock equilibrium conditions. We have so far not mentioned the capital stock K here whose growth rate is to be determined by Y/K. The model's dynamics are in fact treated without any consideration of the law of motion of the capital stock. We justify this here by assuming as further consistency condition that $0 = I(Y, r^*, r^*)$ holds in the steady state. The capital stock will then converge to a certain finite value which is of no importance in the model as it has been formulated so far (since we have constant markup pricing by firms, i.e., prices do not react to demand pressure on the market for goods). Measuring this demand pressure by way of Y/K for example would suggest that this term should enter the investment function in the place of just Y. The dynamics of the capital stock would then feed back into the goods market and become interdependent with the rest of the dynamics, even if prices do not react to such a demand pressure term.

We now consider some features of the 3D MFT dynamics of this paper close to the steady state as determined in the preceding subsection by means of the Jacobian matrix of the dynamics calculated at the steady state position of the economy.

2.5 Local stability analysis

In the presently considered policy regime we will here first of all derive a situation where the steady state of the model is surrounded by centrifugal forces (is repelling). This

case will then be contrasted with a situation where an attracting steady state is given. The calculations needed to show these two results will show that there exists indeed a multiplicity of situations where either stability or instability may prevail around the steady state. The conclusions will be that empirical and numerical methods are needed in order to get a more complete picture of the stock-flow dynamics of the considered open MFT economy and that it is likely that policy must be more active than it is currently assumed (in particular with respect to the government budget) to enforce convergent behavior around the steady state of such a small open economy.

Inserting the functions obtained from our short-term comparative static analysis into the laws of motion for the variables F_p, B, p gives rise to the following eigen-feedbacks of the considered state variables (if $\frac{\partial Y}{\partial F_p}$ is used in explicit form in the first partial derivative):

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\begin{aligned}
\frac{\partial \dot{F}_p}{\partial F_p} &= r^*(1 - C_Y)\frac{-NX_Y}{1 - C_Y - I_Y - NX_Y}) - C_{W_p}\frac{-NX_Y}{1 - C_Y - I_Y - NX_Y} > 0, \\
&\quad \text{if} \quad 1 - C_Y - I_Y > 0 \quad \text{and} \quad C_{W_p} \quad \text{sufficiently small} \\
\frac{\partial \dot{B}}{\partial B} &= r^* > 0 \\
\frac{\partial \hat{p}}{\partial p} &= \epsilon'(e)e'(p) > 0
\end{aligned}
$$

We thus have in such a situation that the trace of the Jacobian of the dynamics at the steady state is positive (as is the determinant). The three state variables taken in isolation are therefore subject to repelling forces as far as their own steady state position is concerned:

$$
J = \begin{pmatrix} + & - & \pm \\ 0 & + & \pm \\ 0 & 0 & + \end{pmatrix}
$$

with in fact all eigenvalues of this Jacobian being real and positive and thus destabilizing.

Even if one assumes that the first of the above partial derivatives is negative, by allowing a high wealth effect on consumption C_{W_p}, and furthermore that the second partial derivative (in the diagonal of J) is sufficiently low (i.e., if the accumulation of foreign and domestic bonds is not by itself subject to strong destabilizing forces), we get for the considered Jacobian approximately still the following sign structure:

$$
J = \begin{pmatrix} - & - & \pm \\ 0 & 0 & \pm \\ 0 & 0 & + \end{pmatrix}
$$

due to the assumed nearly perfect capital mobility. High capital mobility is therefore problematic for the stability of the balance of payments adjustment process, the evolution of the government debt and for the dynamic price level of the considered small open MFT type economy. There are some forces, by contrast that appear to be stabilizing, but more detailed analysis is in fact needed in order to get clear-cut results on local asymptotic stability and thus local convergence towards the steady state. Such stability issues will be approached in the next sections by means of two basic scenarios in the case of fixed as well as flexible exchange rate regimes.

2.6 Real twin deficit accumulation and inflation dynamics

In order to highlight the role of price level adjustments, as well as the complications they imply for the theoretical modelling of the dynamics of the economy, we now reformulate the model in real terms, with the evolution of the real aggregate wealth of the economy and the government sector instead of the law of motions for F_p and B. On this basis we show in particular that the aggregate wealth of the government sector W_g^a is in its time rate of change – as in the case of the private sector – determined by deducting from its real disposable income the real consumption of this sector. This then provides us with a simple law of motion for real aggregate wealth of the government sector, besides the one we have determined for the total (foreign) wealth of the domestic economy in Flaschel (2006), to be reconsidered below. These two laws describe on the one hand the evolution of surpluses or deficits in the government sector and the evolution of current account surpluses or deficits, and thus in particular allow the joint treatment of the issue of twin deficits in an open economy with a government sector, but not yet with real capital accumulation and economic growth.

For the time rate of change of the aggregate wealth of the government, we get from eq. (7):

$$
\begin{aligned}
\dot{W}_g^a &= \frac{-(\dot{M}+\dot{B})+\dot{e}F_c}{p} - \frac{\dot{p}}{p}\frac{-(M+B)+eF_c}{p} \\
&= \frac{-(pG+rB-pT-r^*eF_c)+\hat{e}eF_c}{p} - \hat{p}\frac{-(M+B)+eF_c}{p} \\
&= T - r\frac{B}{p} + \hat{p}\frac{M+B}{p} + (r^*+\hat{e}-\hat{p})\frac{eF_c}{p} - G \quad \text{which finally gives} \\
\dot{W}_g^a &= Y_g^a - G = \rho^*W_g^a + r\frac{M}{p} + \xi\frac{M+B}{p} + T - G
\end{aligned}
\tag{19}
$$

as the law of motion for the real aggregate wealth or debt that characterizes the government sector as a whole.

Since this debt position is no longer constant, as in Flaschel (2006), we adjust next from this text, but in a self-contained way, the equation for the evolution of total wealth W of the economy to this case and then consider again private disposable income Y_p and private wealth W_p in its interaction with the evolution of aggregate government debt and the foreign position of the economy. Note that we assume goods market equilibrium $Y - C - I - G = NX$ in the following derivations.

Concerning the aggregate (foreign) wealth of the domestic economy, we start defining it as

$$
W := W_p + W_g + W_c = \frac{eF}{p}, \quad \text{with} \quad F = F_p + F_c
$$

or in growth rates

$$
\hat{W} = \hat{e} + \hat{F} - \hat{p} \quad \text{or} \quad \dot{W} = \hat{e}W + \frac{e\dot{F}}{p} - \hat{p}W
\tag{20}
$$

Making use of eq. (5), we obtain

$$
\begin{aligned}
\dot{W} &= \hat{e}W + \frac{p(Y - C - G - I) + er^*F}{p} - \hat{p}W \\
&= (\hat{e} - \hat{p})W + r^*\frac{eF}{p} + Y - C - G - I,
\end{aligned}
$$

or alternatively,

$$
\dot{W} = (r^* + \hat{e} - \hat{p})W + Y - C - G - I = \rho^*W + Y - C - G - I, \quad \rho^* = r^* + \hat{e} - \hat{p}
$$

In the following we set foreign inflation equal to zero ($\pi^* = 0$),[8] and assume given policy parameters (M, T, G). Inserting the behavioral equations given by eqs. (10) - (12), we obtain, together with the law of motion for the price level described by eq. (15), the following 3D dynamical system[9]

$$
\begin{aligned}
\dot{W} &= \rho^*W + Y - C(Y_p, W - W_g^a, \rho, \rho^{*e}) - I(Y, \rho, \rho^{*e}) - G & (21) \\
&= \rho^*W + NX(\cdot, \eta, Y^*) & (22) \\
\dot{W}_g^a &= \rho^*W_g^a + \xi\frac{M + B}{p} + r\frac{M}{p} + T - G & (23) \\
\hat{p} &= \beta_w(Y - \bar{Y})/(1 - \gamma) + \epsilon(e), \quad \gamma \in (0, 1). & (24)
\end{aligned}
$$

Note that we assume myopic perfect foresight with respect to inflation on the market for goods, but allow for errors in exchange rate expectations here. Note also that actual laws of motions are to be based on actual rates of changes in the exchange rate e, but that we have to use the expected depreciation rate inside behavioral relationships. We have assumed above – following Rødseth (2000, Ch.6) – that total private consumption of domestic and foreign goods depends (positively) on disposable income, private wealth and (negatively) on the real rate of returns expected for the two types of bonds considered in this paper, but not on the real exchange rate $\eta = ep^*/p$ that characterizes this small open economy, and similarly that total net investment depends (positively) on domestic economic activity and (negatively) on the same real rates of return of bonds. The real exchange rate $\eta = ep^*/p$ only enters the goods market equilibrium condition only via exports by way of an export function of the type $X = X(\eta, Y^*)$.

Note again that the above laws of motion for W, W_g^a are actual laws of motion based on actual rates of return and thus actual changes in the exchange rate, while some arguments in the consumption function and the investment function are expected ones (relying on our use of a regressive expectations scheme later on) and have thus to be characterized by an index e for 'expected'. Furthermore, we have assumed myopic perfect foresight with respect to the inflation dynamics and thus do distinguish there between expected and actual inflation rates, with the former to be used in the behavioral relationships later on while the latter apply to the actual laws of motion for the considered wealth variables. The distinction between actual and perceived rates of return will become important when exchange rate dynamics is considered later on (in our representation of

---

[8]and consider of course all foreign variables as given for the small open economy.

[9]Note that – due to the assumed goods market equilibrium – $Y - C - I - G$ can always be replaced by NX if this is convenient for certain calculations of the model's implications.

the Dornbusch model in a MFT approach to financial markets). Note finally that we may extend the consumption function of Rødseth (2000, Ch.6) later on, if we want to distinguish between the consumption of the domestic and the foreign commodity and thus have to include then the real exchange rate into the consumption function explicitly. Note also that nothing has been said yet about which of the standard regimes od the MFT open economy is actually prevailing in the considered economy. the choice of which will determine later on which variable of the model have to be considered as endogenous and which ones as exogenous.

3 Capital account and inflation dynamics under interest and exchange rate pegs

We are now going to consider the dynamic implications of a particular regime among the ones that are economically possible in the considered MFT framework. We here follow Rødseth (2000, ch. 6.6) and choose a case where in fact the asset markets are sent into the background of the model, a case which therefore solely studies the interactions of the IS-curve with a conventional type of Phillips curve and the dynamics of the capital account. The conventional type of IS-PC analysis (without an LM-curve) is therefore here augmented by the change in foreign assets resulting from the excess of domestic savings over domestic investment (or v.v.). The case considered in this section may be applicable – after some modifications – to an economic situation as represented by the Chinese economy (at least for certain time periods of this economy).

3.1 Assumptions

The assumptions we employ in order to derive this special case from our general framework are the following ones:

1.) Given Y^*, p^*, r^* : The small open economy assumptions

2.) $r = r^*$: An interest rate peg by the central bank (via an accommodating monetary policy)

3.) $\bar{e} = \text{const} \ (= 1)$: A fixed exchange rate via an endogenous supply of dollar denominated bonds by the central bank (which is never exhausted)

4.) W_g^a : A tax policy of the government that keeps the aggregate wealth of the government fixed

5.) consumption goods are import commodities and never rationed in this respect

6.) ω : The real wage is fixed by a conventional type of markup pricing

7.) ρ_f^n : The normal rate of return of firms is fixed (since the real wage is a given magnitude) and set equal to r^* for simplicity

8.) $Y^p = y^p K, L^d = Y/x$: Fixed proportions in production (y^p, x capital and labor productivity, respectively)

9.) $K = const$: The capacity effect of investment is ignored. Potential output $Y^p(= 1)$ is therefore a given magnitude

10.) $\bar{Y} = x\bar{L}(= 1)$: A given level of the full employment output

On the basis of these assumptions we get that the real rates of interest are equalized for the domestic economy: $\rho^* = r^* + \hat{e} - \hat{p} = r - \hat{p} = \rho$. Furthermore, the risk premium ξ is zero in the considered situation. Finally, due to the assumed tax policy we have for the disposable income in the household sector: $Y_p = Y + \rho^* W - G$. Private wealth W_p is given by $W - W_g^a$ in the considered situation.

3.2 The model

We define again the real exchange rate by $\eta = (ep^*)/p, i.e.$, the amount of domestic goods that are exchanged for one unit of the foreign good. This rate reduces to $1/p$ due to the above normalization assumptions. Households directly buy investment goods for their firms and use only the normal rate of profit in order to judge their performance, which is a given magnitude here due to the above assumptions (normal output times the profit share). We moreover need only consider one real rate, the rate ρ, in the following formulation of the consumption decisions (for domestic and foreign goods) and the investment decisions of the household sector now. Note that we now distinguish imported from domestic goods in domestic consumption demand and include an of the real exchange rate on this substructure of total private consumption now.

$$
\begin{aligned}
C_1 &= C_1(Y_p, W_p, \rho, \eta) : \quad \text{consumption demand for the domestic good} \\
C_2 &= C_2(Y_p, W_p, \rho, \eta) : \quad \text{consumption demand for the foreign good} \\
C &= C_1(Y_p, W_p, \rho, \eta) + C_2(Y_p, W_p, \rho, \eta)/\eta : \quad \text{total consumption} \\
I &= I(Y, \rho) : \text{investment demand, for domestic goods solely}
\end{aligned}
$$

On this basis the goods market equilibrium or the IS-curve of the model is given by:

$$
C(Y_p, W_p, \rho, \eta) + I(Y, \rho) + G + NX(\cdot) = Y
$$

where net exports are based on a standard export function and import demand as determined by C_2. Imports can be suppressed in this equation by reformulating it as follows

$$
Y = C_1(Y + \rho W - G, W - W_g^a, \rho, \eta) + I(Y, \rho) + G + X(Y^*, \eta), \quad \eta = 1/p, \rho = r^* - \hat{p}.
$$

We have here assumed that exports X depend as usually on foreign output and the real exchange rate.

We note that we have to add the law of motion for K : $\dot{K} = I(Y, r^* - \frac{1}{1-\gamma}\beta_w(Y - 1))$ for a complete treatment of the dynamics of this example of a small open economy. Since however the analysis without such capacity effects of investment is already complicated

enough we do not go into such an extended dynamic analysis here. Furthermore, we may also return to an endogenous treatment of the variable W_g^a which would increase the complexity of the analysis further, despite the simple framework that is here chosen (where portfolio choice does not matter for the analysis of the dynamics of the real part of the model, T, G given magnitudes):

$$\hat{p} = \beta_w(Y-1)/(1-\gamma)$$
$$\dot{W}_g^a = (r^*-\hat{p})W_g^a + r^*m^d(Y,r^*) + T - G,$$
$$\dot{W} = (r^*-\hat{p})W + X(Y^*,1/p) - C_2\left(Y + (r^*-\hat{p})(W-W_g^a) - r^*m^d(Y,r^*) - T,\right.$$
$$\left. W - W_g^a, r^*-\hat{p}, 1/p\right)/p.$$

This extension would again allow for the discussion of the occurrence of twin deficits and other situations of domestic and foreign debt / surpluses.

3.3 Steady state determination

For reasons of simplicity we here return however to the situation where aggregate government wealth (basically the government deficit) stays constant in time (by choosing T appropriately) and thus investigate now the steady state solution and the dynamics of the following system:

$$\hat{p} = \frac{1}{1-\gamma}\beta_w(Y(p,W)-1)$$
$$\dot{W} = (r^*-\hat{p})W + X(Y^*,1/p) - C_2(Y(p,W) + (r^*-\hat{p})W - G, W - W_g^a, (r^*-\hat{p}), 1/p)/p$$

where the properties of the IS-equilibrium are characterized by the standard partial derivatives we have discussed above.

With respect to the steady state solution of this dynamical system we assume that the government pursues – in addition to its tax policy – a constant government expenditure policy that is aimed at fixing the steady state value of exports at the level $\bar{X}$. This implies that the steady state value of the price level, p_o, is to be determined from $\bar{X} = X(Y^*, 1/p_o)$. Assuming that this equation has a (uniquely determined) positive solution for p_o we then can obtain the steady state value of W from the labor market equilibrium equation $1 = \bar{Y} = Y(p_o, W_o)$. The solution of this equation may be positive or negative and is again uniquely determined, since the right hand side of this equation is strictly increasing in W_o. The level of government expenditure G that allows for this solution procedure, finally, is then given by

$$0 = r^*W_o + \bar{X} - C_2(\bar{Y} + r^*W_o - G, W_o - W_g^a, r^*, 1/p_o)/p_o$$

In this equation, this expenditure level is adjusted such that net imports are equal to the foreign interest income of domestic residents, i.e. an excess of imports over exports is needed in the case of a positive foreign bond holdings in the domestic economy in the steady state. Note that the above is also based on the assumption that $I(\bar{Y}, r^*) = 0$ holds in the steady state, since there must be a stationary capital stock in the steady state of the model.

18

3.4 Stability analysis

Due to our simplifying assumption on the goods-market equilibrium equation $Y = Y(p, W)$ which guarantee that price level increases reduce economic activity and thus provide a check to further inflationary tendencies and which induce economic activity to increase with dollar denominated wealth due to its effects on consumption we have a straightforward sign structure in the partial derivatives of the first law of motion. The second law of motion is however much more difficult to handle. Its partial derivative with respect to W is much more involved than the one in Flaschel (2006) and given by (due to $\hat{p}_o = 0$):

$$\frac{\partial \dot{W}}{\partial W} = r^*(1 - \eta_o C_{2Y_p}) - \eta_o C_{2Y_p} \frac{\partial Y}{\partial W} - \eta_o C_{2W_p} + \hat{p}_W[(\eta_o C_{2Y_p} - 1)W_o + \eta_o C_{2\rho}]$$

where we have denoted by $\hat{p}_W$ the partial derivative of the first law of motion with respect to W. We have now considerably more terms in the feedback of foreign debt on its time rate of change than was the case in Flaschel (2006) for the there considered perfectly open economy. These additional terms seem to provide more support for a negative feedback chain compared to Flaschel (2006) (if the assumptions on $Y(p, W)$ hold true) where we simply had that this partial feedback mechanism became positive (destabilizing) when wealth effects in consumption are sufficiently weak.

The remaining partial derivative for local stability analysis is given by

$$\frac{\partial \dot{W}}{\partial p} = ((\eta_o C_{2Y_p} - 1)W_o + \eta_o C_{2\rho})\hat{p}_p - \eta_o C_{2Y_p} \frac{\partial Y}{\partial p} + (C_2 + \eta_o C_{2\eta} - X_\eta)/p^2).$$

The first two expressions in this equation are positive in sign while the last term in brackets – the quantitative reaction of net imports to price level changes via the real exchange rate channel – is generally assumed as being negative ($\eta = (ep^*)/p = 1/p$). For the Jacobian J we thus in sum get as sign structure:

$$J = \begin{pmatrix} \frac{\partial \hat{p}}{\partial p} p & \frac{\partial \hat{p}}{\partial W} p \\ \frac{\partial \dot{W}}{\partial p} & \frac{\partial \dot{W}}{\partial W} \end{pmatrix} = \begin{pmatrix} - & + \\ \pm & \pm \end{pmatrix}.$$

The easiest case for a stability result is

$$J = \begin{pmatrix} \frac{\partial \hat{p}}{\partial p} p & \frac{\partial \hat{p}}{\partial W} p \\ \frac{\partial \dot{W}}{\partial p} & \frac{\partial \dot{W}}{\partial W} \end{pmatrix} = \begin{pmatrix} - & + \\ - & - \end{pmatrix},$$

i.e., the case where interest effects do not dominate the capital account adjustment process and where the normal reaction of the trade balance (based on the so-called Marshall-Learner conditions) dominates the income, wealth and interest rate effects generated by the general form of a consumption function used in this paper and Flaschel (2006). The steady state is in this case obviously locally asymptotically stable (since trace $J < 0$, det $J > 0$). Graphically, we get in this situation the phase diagram shown in the following figure:
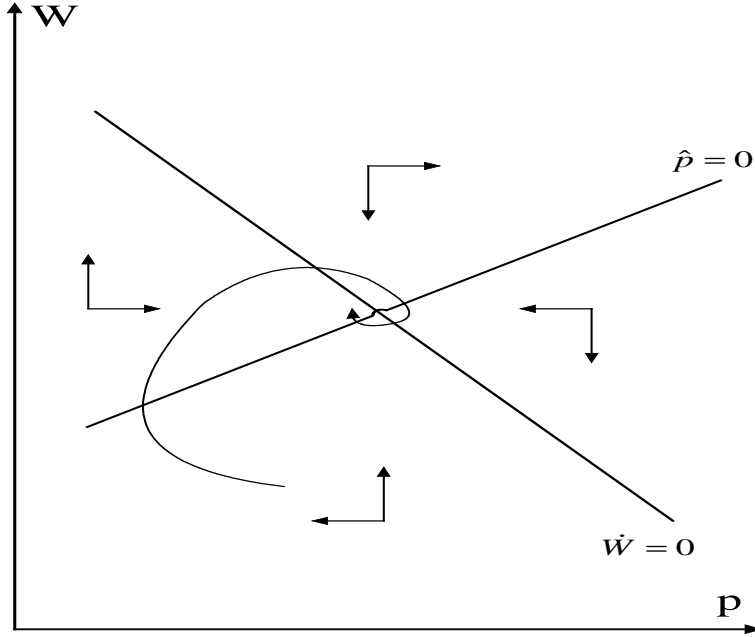
Figure 2: the special example of stable inflation and capital account dynamics.

In view of this figure we must however keep in mind the very restrictive assumptions we have made with respect to the IS-curve and its replacement by the evolution of the state variables of the dynamics, the reaction of the balance of payments and the dynamics of the capital account and also on the reaction of foreign bond accumulation with respect to price level changes. It is therefore by no means clear how dominant the case of stable price level and capital account dynamics are in the set of all possible stability scenarios even in the special case of a MFT economy here under consideration. In the case of divergence the question is of course what mechanisms in the private sector may then keep the dynamics bounded or what policy actions are needed to ensure this.

The depicted dynamical implications are by and large of the type considered in Rødseth (2000, Ch.6.6), though our consumption and investment functions differ to some extent from the ones used by Rødseth. Note that Rødseth (2000, Ch.6.6) is using the negative of W in order to characterize the international credit or debt position of the domestic economy. Rødseth (2000, Ch.6.6) also considers a variety of further issues for this case of a stable steady state of the Mundell-Fleming regime with an interest and ab exchange rate peg. The reader is referenced to this analysis for further aspects of this stable monetary and exchange rate regime.

By contrast, the worst case scenario (for instability) is given by the situation

$$
J = \begin{pmatrix} \frac{\partial \hat{p}}{\partial p} p & \frac{\partial \hat{p}}{\partial W} p \\ \frac{\partial \dot{W}}{\partial p} & \frac{\partial \dot{W}}{\partial W} \end{pmatrix} = \begin{pmatrix} - & + \\ + & + \end{pmatrix},
$$

in which case the steady state clearly is of saddlepoint type. This situation is depicted in figure 3.
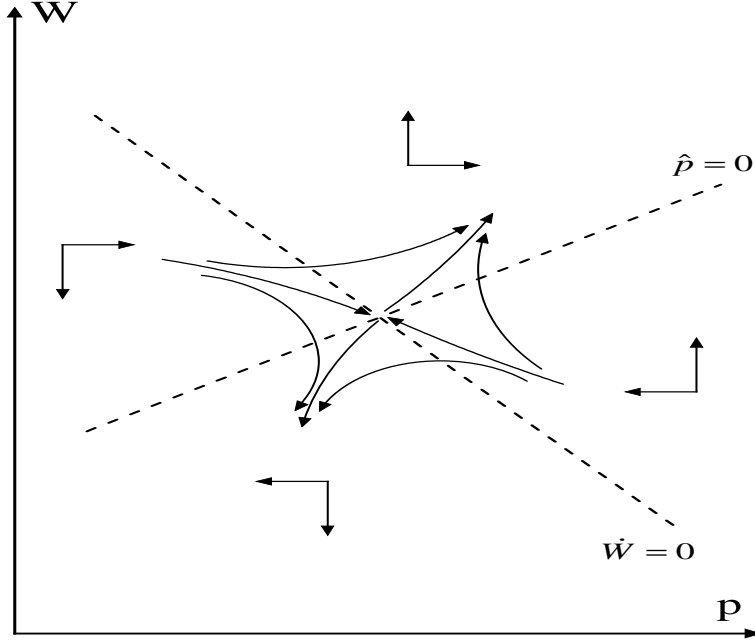
Figure 3: the case of centrifugal inflation and capital account dynamics.

Advocates of the jump variable technique will not be able to apply this technique here under consideration, since both the price level p and the foreign position W of the economy are predetermined variables here (despite myopic perfect foresight as far as domestic inflation rates are concerned). The solution to the instability shown in figure 3 can thus not be found in an ad hoc imposition of appropriate jumps in the price level, but must be found through the consideration of private sector or public policy behavioral changes when the economy has departed too much from its steady state position. We consider this a descriptively relevant approach to observed instabilities in the adjustment of the balance of payments in particular.

3.5 Twin deficit or surplus accumulation

In order to highlight the important role of the price dynamics for the stability of the system, as discussed above, we now set foreign inflation equal to zero ($\pi^* = 0$) and assume also constant price levels for the domestic economy ($p = p^* = 1$ for notational simplicity). We assume given policy parameters (M, T, G). Inserting the behavioral equations given by eqs. (10) - (12), we obtain in the presently considered case the 2D dynamical system[10]

$$\dot{W} = r^*W + Y - C(Y_p, W - W_g^a, r^*) - I(Y, r^*) - G = r^*W + NX(\cdot) \quad (25)$$

$$\dot{W}_g^a = r^*W_g^a + r^*\frac{M}{p} + T - G, \quad (26)$$

---

[10]Note that – due to the assumed goods market equilibrium – $Y - C - I - G$ can always be replaced by $NX(Y, Y^*, e)$ if this is convenient for certain calculations of the model's implications.

The corresponding Jacobian at the steady state of the dynamics is characterized by:

$$J = \begin{pmatrix} \frac{\partial \dot{W}}{\partial W} & \frac{\partial \dot{W}}{\partial W_g^a} \\ \frac{\partial \dot{W}_g^a}{\partial W} & \frac{\partial \dot{W}_g^a}{\partial W_g^a} \end{pmatrix} = \begin{pmatrix} r^* - C_W & -C_{W_g^a} \\ 0 & r^* \end{pmatrix} = \begin{pmatrix} \pm & + \\ 0 & + \end{pmatrix}$$

with $\det J < 0$ if $r^* < C_W$ and $\text{tr } J > 0$ else. In the absence of price adjustments, therefore, the dynamics of the system become intrinsically unstable in any case (with an unstable equilibrium in the first and a saddlepoint in the second situation), even if the wealth effect on consumption influences in a larger extent the dynamics of the economy's wealth position than the interest payments. We conclude that the dynamics if twin deficits (or surpluses) is far from being self-correcting.

4 Overshooting exchange rates and inflation dynamics for perfectly flexible exchange rate regimes

As a second case study we now consider a regime of perfectly flexible exchange rates and given money supply. This implies that supply of foreign bonds at each moment in time is just given by the stock of these bonds held in the private sector, since domestic bonds are assumed to be non-tradables in this paper. The supply of financial assets is thus fixed in each moment of time and can only change in time via the flows induced in the capital account. The central bank may use open market operations in domestic bonds to change the composition of these bonds and money in the households portfolio, but does not issue money otherwise (to buy foreign bonds from domestic residents in particular). The case considered in this section may be applicable – after some modifications – to an economic situation as represented by the Australian economy (at least for certain time periods of this economy). We stress that we reconsider here the Dornbusch (1976) overshooting exchange rate dynamics for imperfect asset substitutability in the case of the empirically questionable case of the uncovered interest rate parity condition.

4.1 Equilibrium conditions

In our reformulation of the Dornbusch overshooting exchange rate dynamics[11] within the framework of a MFT model we assume for simplicity also – just as in the original Dornbusch (1976) approach – that transactions demand in the money demand function is based on full employment output $\bar{Y} = 1$, i.e., we get from the LM curve of our Tobinian portfolio approach the result that the domestic rate of interest is solely dependent on the price level (positively) and on money supply (negatively), i.e.:

$$r = \frac{\ln p - \ln M}{\alpha} + const$$

---

[11]See Asada, Chiarella, Flaschel and Franke (2003, ch.5) for a detailed presentation of the Dornbusch model and its various extensions.

in the case of the Cagan money demand function considered in Flaschel (2006). For full asset markets equilibrium we need only consider the market for foreign bonds in addition which in the considered exchange rate regime reads:

$$\frac{eF_p}{p} = f^d(r^* + \epsilon(e) - r, \frac{M + B + eF_p}{p}) = f^d(\rho^{*e} - \rho, \frac{M + B + eF_p}{p}),$$

$$\rho^{*e} = r^* + \epsilon(e) - \hat{p}, \quad \rho = r - \hat{p}$$

with $F_p, M + B, p$ given magnitudes in each moment of time. Since $0 < f_2^d < 1$ holds true in a Tobinian portfolio model, we get from this equilibrium condition that the exchange rate e depends negatively on r, F_p and positively on p (when the effect of the price level on the nominal interest rate is ignored). The theory of the exchange rate of the considered Mundell-Fleming regime thus can be represented as follows:

$$e = e(r(p, M), p, F_p), \quad e_1 < 0, e_2 > 0, e_3 < 0, \quad r_1 > 0, r_2 < 0.$$

Note that the overall effect of price level changes on the exchange rate may be an ambiguous one.

Next we consider the IS-equilibrium curve of the presently considered situation:

$$Y = C_1(Y + \rho^{*e}W - G, W - W_g^a, \rho, \rho^{*e}, \eta) + I(Y, \rho, \rho^{*e}) + G + X(Y^*, \eta)$$

with $\eta = e/p, p^* = 1$ and G, W_g^a again given magnitudes. One has to use our regressive expectations regime, the dependence of the nominal rate of interest and the real exchange rate on the price level and the functional dependence of the nominal exchange rate on r, p derived above in order to derive conclusions on how the equilibrium output level depends on the price level p. The outcome is however ambiguous, but pointing to a certain degree to a (conventional) negative overall dependence of Y on p. We shall assume that this holds true in our following discussion of overshooting exchange rates, since the opposite case would imply a destabilizing feedback of the price level on its rate of change via the Phillips curve mechanism. The dependence of Y on W is obviously a positive one, though we will have ambiguity in the movement of W later on.

4.2 Dynamics and steady state determination

We have by now determined the statically endogenous variables of the considered MFT regime r, e, Y by the three equilibrium relationships that now characterize the model. The state variables of the model are again p, W (while the movement of the capital stock is still neglected). The laws of motion for these variables are now given by:

$$\dot{W} = \rho^* W + X(Y^*, \eta) - \eta C_2(Y + \rho^{*e}W - G, W - W_g^a, \rho, \rho^{*e}, \eta)$$

$$\hat{p} = \beta_w(Y - 1)/(1 - \gamma),$$

where $\rho^* = r^* + \hat{e} - \hat{p}$ and $\rho^{*e} = r^* + \epsilon(e) - \hat{p}$ and $\rho = r - \hat{p}$. The law of motion for W is now a very complicate one, since the static relationships have to be inserted into it in various places. We will therefore not discuss its (in)stability implications in the following, but just assume for the time being that this variable is placed into its new

long-run equilibrium position after a shock and kept constant there. We therefore only study the adjustment of the price level p after in particular an open market operation of the central bank (which leaves $M + B$ unchanged). We note however that this implies a jump in the variable $W = (eF)/p$ that is neglected in our following analysis of such shocks.

In the construction of a steady state reference solution we proceed as follows. We again assume that the government pursue an export target $\bar{X}$ by means of its expenditure policy G, besides the tax policy that keeps W_g^a at a constant level $\bar{W}_g^a$. The steady state real exchange rate η_o is then uniquely determined by the equation $\bar{X} = X(Y^*, \eta)$. On this basis we can use the equilibrium condition on the market for foreign bonds to determine the steady state level of W, since this equilibrium condition can be rewritten as

$$f^d(\xi, W - W_g^a) = W - \eta_o F_c \quad \text{or} \quad 0 = W - \eta_o F_c - f^d(\xi, W - W_g^a) = g(\xi, W)$$

since money supply and thus F_c is held constant by the central bank. Since $\epsilon(e_o)$ and ξ_o should be zero in the steady state (to be further justified below) the above equation can be assumed to have a uniquely determined positive solution if the function f^d is chosen appropriately. We note that we only consider positive values of W_o here, since we assume that households and the central bank hold such assets and since firms do not finance there investment expenditures abroad.

Due to the Phillips curve we get next that $Y_o = \bar{Y}$ must hold in the steady state ($\hat{p}_o = 0$). Furthermore, the regressive expectations scheme is built such that $\epsilon(e_o) = 0$ holds for the steady state value e_o of the nominal exchange rate (that remains to be determined still). We thus have $\rho_o = r_o, \rho_o^* = \rho^{*e} = r^*$ in the steady state and postulate that $I(Y, r^*, r^*) = 0$ holds for the investment function used in this MFT model. We assume on this basis in addition that the government chooses the level of G and thus $\bar{X}$ such that $r_o = r_o^*$ is enforced by the IS-equation in the steady state:[12]

$$\bar{Y} = C_1(\bar{Y} + rW_o - G, W_o - \bar{W}_g^a, r_o, r^*, \eta_o) + I(\bar{Y}, r_o, r^*) + G + \bar{X}.$$

On this basis we can then get the steady state value of the price level p_o from the LM-curve $p_o = M/m^d(\bar{Y}, r^*)$ and thus also the steady state value of the exchange rate $e_o = p_o \eta_o$.

We thus have quite a different 'causality' now in the determination of the steady state, where fiscal policy has to provide the necessary anchor for a meaningful steady state solution, whereas in the short-run we have that IS-LM-FF curves determine the variables Y, r, e in principle in this order, while the Phillips curve and the balance of payments determine the dynamics of the price level and of domestically held foreign bonds (in real terms). Note again that the dynamics of the capital stock still remains excluded from consideration here. Note also that we will simplify in the following even further, since we

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[12]In principle, the government here enforces two things in the steady state, namely that the domestic interest rate must be at the international level and that the level of exports is such that goods market equilibrium is then assured, with a zero level of net investment by the assumption on the investment function. The real exchange rate is then a consequence of the level of $\bar{X}$ needed for goods market equilibrium.

shall also exclude the complicated adjustment process for $W = eF/p$ from consideration and instead assume that this magnitude will immediately jump to its new steady state value after any shock and will be kept frozen there. The aim of the following simplified presentation instead will be to reconsider the Dornbusch (1976) model of overshooting exchange rates in the context of a Tobinian portfolio model of the financial sector and somewhat advanced formulations of consumption and investment behavior.

4.3   Dornbusch (1976) exchange rate dynamics

Let us now consider an open market operation of the central bank $dM = -dB$, that increases the money holdings of private agents by reducing their holdings of domestic bonds (which therefore keeps $M + B$ and F_c constant). Our way of constructing a steady state for the considered dynamics immediately implies then that all real magnitudes remain the same (in particular W_o) and that we have as sole steady state changes the following ones:

$$dM/M = dp_o/p_o = de_o/e_o \quad (\eta_o = const, r_o = r^*)$$

The long-run reaction of the dynamics is therefore as in the original Dornbusch (1976) model a very straightforward one, strict neutrality of money and the relative form of the PPP, i.e., there is no change in the real exchange rate caused by the monetary expansion that is undertaken.

In the short-run, prices are fixed and the burden of adjustment in the money market falls entirely on the nominal rate of interest r which is decreased below r^* through the monetary expansion. Since the new steady state value e'_o of the nominal exchange rate is above the old level now and since the assumed regressive expectations mechanism is completely rational in this respect, we would have that $\epsilon(e'_o/e), \epsilon' < 0$ would become positive (generate the expectation of a depreciation) if the short-run exchange rate would remain unchanged. Since we ignore – as described above – adjustments in the value of W we however must have that the current exchange rate depreciates beyond e'_o in order to imply the expectation of an appreciation of the currency such that $\xi = \hat{r}^* + \epsilon(e'_o/e) - r = 0$ can remain unchanged (due to the unchanged value of W_o,[13] since the adjustment process of W is here ignored).[14] We thus get that r decreases and e increases under the assumed monetary expansion and expect that goods market equilibrium output increases through these two influences, since $r^* + \epsilon = r$ has decreased and since η has been increased (the price level still being fixed at p_o). Again there may be an ambiguous reaction possible, but we assume here – as before – that goods markets behave normally in this respect.

The nominal exchange rate therefore overshoots in the short-run its new long-run level as in the original Dornbusch model. Due to the increase in the output level beyond its normal level we have now for the medium-run that the price level starts rising according to

$$\hat{p} = \beta_w(Y - 1)/(1 - \gamma) \quad \text{with} \quad Y = Y(p, W_o), \ \partial Y \partial p < 0$$

---

[13]and also $\bar{W}_g^a, \eta_o F_c$.

[14]the argument must be more detailed if the short run reaction in the value of W is taken into account, but would then of course demand a thorough discussion of the conditions under which dynamics drives this variable back to its steady state level.
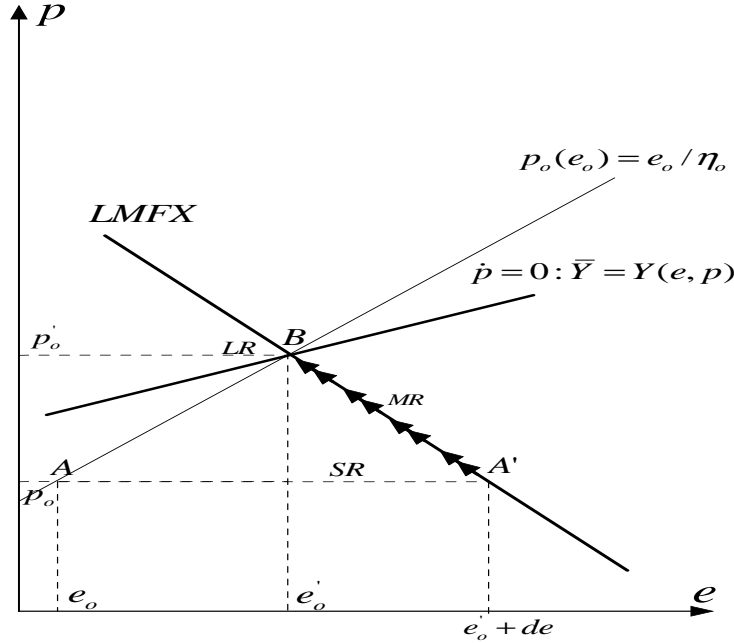
Figure 4: the Dornbusch overshooting exchange rate dynamics.

according to what has been shown above. The dynamics therefore converges back to its steady state position with output levels falling back to their normal levels, prices rising to their new steady state value p'_o, the exchange rate appreciating back to its risen steady state value e'_o and the nominal rate of interest rising again to the unchanged international level r^*. All this takes place in a somewhat simplified portfolio approach to financial markets (in place of the UIP condition under rational expectations) and without any complications arising from possibly adverse adjustments in the balance of payments. It is therefore to be expected that a treatment of the full model along the lines of Flaschel (2006), there for the extremely open economy, will reveal a variety of more complicated situations and in the worst case an unstable dynamics for which then actions of households or the government have again to be found that bound their trajectories to economically meaningful domains. The present discussion is therefore only the beginning of a detailed discussion of the Dornbusch mechanism in a full-fledged MFT model with inflation and balance of payments adjustment dynamics.

The dynamic adjustment processes just discussed are summarized in figure 4. We have the simple LMFF curve describing full portfolio equilibrium by way of

$$f^d(r^* + \epsilon(e) - r(p), W_o - \bar{W}_g^a) = W_o - \eta_o F_c$$

where all revaluation effects of assets have been ignored, where transactions balances are based on normal output still and where most importantly the dynamics of the state variable W is set aside. The true LMFF curve is of course shifting with the changes in W, η and the output level Y. We have furthermore the curve along which the price level is stationary and which also in general is not as simple as shown in this graph (where $\partial Y/\partial e, \partial Y/\partial p < 0$ is assumed), but may be quite complicated due to the assumed consumption and investment behavior.

If the old steady state A is disturbed by an expansionary monetary shock that shifts the old LMFF curve (not shown) into the new position (shown) we have as immediate response (SR) that the exchange rate adjusts (to A') such that full portfolio equilibrium is restored. In the medium run (MR) we then have rising price levels, rising interest rates and falling exchange rates until the new steady state position (LR) is reached at point B. We note that the variables W, W_g^a may be subject to jumps in a regime with flexible exchange rates, but are then following their laws of motion if no further shocks occur (in the case of W), respectively remain then fixed at their new level (in the case of W_g^a).

4.4 Capital account and budget deficit dynamics

Furthermore, we may also return to an endogenous treatment of the variable W_g^a (by making use again of given lump sum taxation T again). This would increase the complexity of the analysis even further and lead us to the consideration the following 3D dynamics:

$$
\begin{aligned}
\hat{p} &= \beta_w(Y-1)/(1-\gamma), \\
\dot{W}_g^a &= (r^* + \hat{e} - \hat{p})W_g^a + (r^* + \hat{e} - r)\frac{M+B}{p} + rm^d(Y, r) + T - G, \\
\dot{W} &= (r^* + \hat{e} - \hat{p})W + X(Y^*, ep^*/p) - (ep^*/p)C_2(Y + (r^* + \epsilon(e) - \hat{p})(W - W_g^a) \\
&\quad - rm^d(Y, r) - (r^* + \epsilon(e) - r)\frac{M+B}{p} - T, W - W_g^a, r - \hat{p}, r^* + \epsilon(e) - \hat{p}, ep^*/p).
\end{aligned}
$$

This extension would again allow for the discussion of the occurrence of twin deficits and other situations of domestic and foreign debt/surplus accumulation. Note however that the system has become a very complicated in this case of a perfectly flexible exchange rate, where the statically endogenous variables r, e have to be obtained from the portfolio part of the model, in interaction with the IS-curve of the model, and where the growth rate of e has then to be calculated on this basis in order to insert it into the 3D laws of motion where necessary. This is not at all an easy task and makes an analysis of the model nearly untractable. In view of this, ways have to be found that model the dynamics of the exchange rate directly, so that a differentiation of the portfolio equilibrium equations for this purpose can be avoided.

5 Conclusions

In this paper, we have considered a small open economy of the Mundell-Fleming-Tobin type, exhibiting on the market for goods a Keynesian demand constraint. Moreover, we have also assumed imperfect substitutability of financial assets in place of an UIP condition. This imperfectness was coupled with the assumption that domestic bonds are non-tradables, i.e., the amount of foreign bonds held domestically was only changed to the extent that there is a surplus or a deficit in the current account. Finally, we also assumed regressive exchange rate expectations. This simplifying assumption was

helpful for the central objective of the paper which was to isolate the fundamental destabilizing forces contained from the two accumulation equations of the model, concerning internal and external deficits or surpluses caused by the government budget equation and the evolution of the current account of the considered economy. Due to our use of a standard open economy money wage Phillips curve, the Keynesian business fluctuations approach was accompanied by labor market driven inflation or deflation dynamics which – in combination with the capital account and government budget dynamics – provided a dynamic model that goes significantly beyond standard Mundell-Fleming type approaches.

Despite the intrinsic government budget and capital account dynamics of the model, we had also a rich set of feedback channels present in it: Hicksian disposable income effects, Pigou price level effects, Keynes price level effects, the Mundell-Tobin effect of inflationary expectations in both the consumption and the investment function, Dornbusch exchange rate effects, portfolio effects, and the stated stock-flow interactions. The interaction of these effects allowed for a variety of (in-)stability results, too numerous to allow their investigation in a single paper of this model type. We therefore concentrated in this paper on a regime with pegged interest rate as well as exchange rate and contrasted this situation with a regime where the exchange rate is perfectly flexible and the money supply a given magnitude under the control of the Central Bank of the domestic economy. The (in-)stability results that were obtained suggested that this type of approach is rich in implications, but unfortunately poor in providing simple and unambiguous answers to those who prefer simple economic conclusions and on this basis also simple advices for policy interventions.

References

ASADA, T., CHIARELLA, C., FLASCHEL, P. and R. FRANKE (2003): *Open Economy Macrodynamics. An Integrated Disequilibrium Approach.* Heidelberg: Springer Verlag.

ASADA, T., FLASCHEL, P., GROH, G. and C.R. PROAÑO (2006): International capital flows. The MFT approach revisited. Bielefeld: Mimeo.

DORNBUSCH, R. (1976): Expectations and exchange rate dynamics. *Journal of Political Economy*, 84, 1161 – 1175.

Flaschel, P. (2006): Instability Problems and Policy Issues in Perfectly Open Economies. In: T. Asada and T. Ishikawa (eds.): The Economics of Time and Space. Tokyo: Springer, forthcoming

Rødseth, A. (2000): Open Economy Macroeconomics. Cambridge, UK: Cambridge University Press.

6 Appendix: Notation

The following list of symbols contains only domestic variables and parameters. Magnitudes referring to the foreign country are defined analogously and are indicated by an asterisk ($*$). Superscript d characterizes demand expressions, while the corresponding supply expressions do not have any index (in order to save notation). A subscript p stands for private households and g, c for government and the central bank. A 'dot' is used to characterize time derivatives and a 'hat' for corresponding rates of growth. We furthermore use an subscript o to denote steady state expressions. Finally, we characterize exogenous variables by means of a bar over the considered variable if this is not stated otherwise.

A. Statically or dynamically endogenous variables:

p	price level
w	wage level
r	nominal interest rate
ρ	real interest rate (ρ^{*e} the expected one)
ξ	risk premium (ξ^e the expected one)
e	exchange rate
Y	output
T	lump sum taxes
G	government expenditures
Y_p	private disposable income
Y_p^a	government disposable income
$f(N)$	production function
N	labor supply
C	private consumption
S_p	private savings
S	total savings
M	money supply
B	domestic bonds
F	foreign bonds
W_p	real private wealth
W_g^a	real wealth of the state
W	real foreign bonds

B. Mathematical notation

$\dot{x}$	Time derivative of a variable x
$\hat{x}$	Growth rate of x
$r_o, etc.$	Steady state values