Varieties of Capitalism: The Flexicurity Model

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Abstract

We start from the finding that Goodwin's (1967) growth cycle model, i.e., the Marxian reserve army mechanism, does not represent a process of social reproduction that can be considered an adequate socio-economic foundation for a democratic society in the long-run. The paper then derives a basic macrodynamic framework where this distinct form of cyclical growth and social reproduction is overcome by an employer of 'first' resort, added to an economic reproduction process that is highly competitive and thus not of the type of the past Eastern socialism. There is high labor and capital mobility (concerning 'hiring' and 'firing' in particular) where fluctuations of employment in the private sector are made socially acceptable through a second labor market where all remaining workers get occupation and income. The resulting socio-economic system is closely related to the flexicurity model developed for Denmark in particular. We show that this economy exhibits a balanced growth path that is globally attracting. Moreover, pension-fund financed investment can be added to this model without disturbing the prevailing situation of stable full capacity growth The closing section shows however that a paper credit formulation of this process can lead to Keynesian effective demand constraints in this type of economy.

Keywords: Distributive growth cycles, employer of first resort, stable balanced growth, supply-driven business fluctuations.

JEL classifications: E32, E64, H11.

Contents

1 Social reproduction and the reserve army mechanism

This paper starts from the observation that Goodwin's (1967) Classical growth cycle does not represent a process of social reproduction that can be considered as adequate for a social and democratic society in the long-run. The paper therefore derives on this background a basic macrodynamic framework where this form of cyclical growth and social reproduction is overcome by an employer of 'first' resort, added to an economic reproduction process that is highly competitive and flexible and thus not of the type of the past Eastern socialism. Instead, there is high capital and labor mobility (concerning 'hiring' and 'firing' in particular) where fluctuations of employment in the first labor market of the economy (in the private sector) are made socially acceptable through a second labor market where all remaining workers (and even pensioners) find meaningful occupation. The resulting economic system with its detailed transfer payment schemes is in its essence comparable to the flexicurity model developed for the Nordic welfare states and Denmark in particular. We show that this economy exhibits a balanced growth path that is globally attracting. Moreover, credit financed investment can be easily added without disturbing the prevailing situation of full capacity growth. We thus do not get demand-, but only supply-driven business fluctuations in such an environment with both factors of production always full employed. This combines flexible factor adjustment in the private sector of the economy with high employment security for the labor force and thus shows that the flexicurity variety of capitalist reproduction can work in a balanced or at least fairly stable manner.

We start from the (in 1995) still weak empirical evidence for the existence of a long-phase cycle in the state variables e and v, the employment rate and the wage share, that we have presented in Flaschel and Groh (1995) for a number of industrialized market economies. We do this on the basis of fifteen years of further observations and now also partly based on quite modern econometric techniques. Our brief findings will be that the Goodwin growth cycle model of Flaschel and Groh (1995) provides indeed a useful approach to the explanation of the distributive cycle as it was observed in the US, the UK and in other countries after World War II. In Kauermann et al. (2007) we have obtained by specifically tailored econometric techniques the graphical representation of the long-phase wage share v / employment rate e cycle (as centers of the business fluctuations around them) for the U.S. economy over the period 1955 – 2004. Figure 1 shows (bottom-right) a single estimated long-phase core cycle (within the scatter plot of $v, e-$observations) for a period length of approximately 50 years and (bottom-left) the 6-7 cycles of business cycle frequency (approximately 8 years each) that fluctuate around this long-phase cycle. We ignore the shorter cycles in the following and concentrate on the observation that there is evidence for a long-phase overshooting (non-monotonic) interaction between the share of wages v in national income and the employment rate e, the core of which is shown in figure 1, bottom right. This clockwise oriented long-phase cycle appears to be more complex in situations of a high employment rate and is relatively simple structured in the opposite situations. The reader is referred to Kauermann et al. (2007) for details on the applied econometric technique and the results that can be obtained from it.

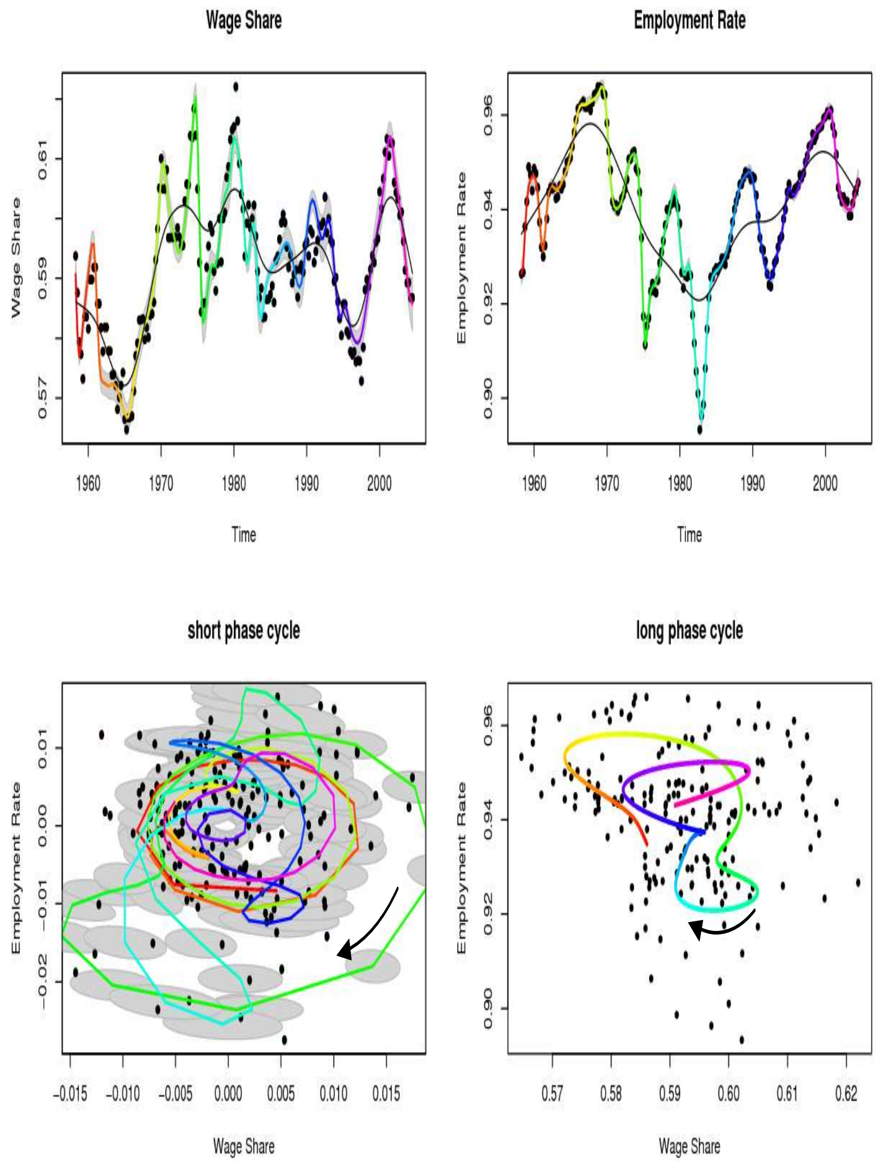

Figure 1: Goodwinian wage share / employment rate dynamics (bottom plots) with estimated long-phase cycle to the right. Top graphs show the data plotted against time.

In order to briefly present a simple model of such a long-phase accumulation cycle in the variables v and e we make use of the seminal growth cycle model of Goodwin (1967). From this perspective, the envisaged cycle-generating feedback structure can be based on the following two laws of motion:

$$\hat{v} = \dot{v}/v = \beta_{ve}(e - \bar{e}) - \beta_{vv}(v - \bar{v}), \quad (1)$$

$$\hat{e} = \dot{e}/e = -\beta_{ev}(v - \bar{v}), \quad (2)$$

where v denotes real unit-wage costs (or the share of wages in GDP) and e the employment rate and the parameters $\beta_{ve} > 0$, $\beta_{ev} > 0$, $\beta_{vv} \geq 0$ determine the speed of adjustment. The coefficients $\bar{e}$ and $\bar{v}$ denote the normal levels of employment and the wage share, respectively, meaning that employment and the wage share are constant at those values. We justify eq. (1) by means of the wage dynamics investigated in Blanchard and Katz (1999), with perfect anticipation of price inflation however (implying a real wage Phillips curve) where in addition to demand pressure we have unit wage costs acting as an error correction mechanism on their own evolution. In the second law of motion we focus on a goods market behavior that is profit-led, i.e., increases in unit wage costs act negatively on aggregate demand and thus negatively on the growth rate of the rate of employment e.

If $\beta_{vv} = 0$ holds, as Blanchard and Katz assert it for the U.S. economy, we have the cross-dual dynamics of the Goodwin (1967) growth cycle model and thus a center type dynamics that is stable, but not asymptotically stable. In the case $\beta_{vv} > 0$ we can apply Olech's Theorem, see Flaschel (1984), and obtain from it global asymptotic stability of the dynamics in the positive orthant of the phase plane with respect to the uniquely determined interior steady state position $\bar{e}, \bar{v}$. For weak Blanchard and Katz (1999) error correction terms we thus get a somewhat damped long-phased cyclical motion in the wage share / employment rate phase space as shown in figure 2. We have a clockwise rotation in the considered phase space with approximately one cycle in 50 years.[1]

We can see that the theoretical 2D dynamics mirrors the empirical phase plot to a certain degree. The Goodwin growth cycle mechanism where employment growth depends negatively on income distribution (is profit-led) and where wage share growth depends positively on the state of the labor market thus not only explains the clockwise orientation observed in the data, but also the long-phased nature of the cycle when adjustment speeds are crudely chosen from an empirical perspective. The unique observation of a single long cycle in income distribution and employment that we have available for the U.S. economy after World War II is thus in fairly close correspondence to the Classical growth cycle model and its suggestion of a long-phase accumulation cycle.[2]

Generating order and economic viability in market economies by large swings in the unemployment rate (mass unemployment with human degradation of part of the families that form the society), as shown above (see also the next figure on the growth cycle structure in the British economy), is one way to make capitalism work, but it must surely be critically reflected with respect to its social consequences. Moreover, it must be contrasted with alternative economic systems that allow to combine the situation of a highly competitive market economy with a human rights bill that includes the right (and the obligation) to work, and to get income from this work that at the least supports basic needs and basic happiness. The Danish flexicurity system provides a typical example for such an alternative. By contrast, a laissez-faire capitalistic society that ruins family structures to a considerable degree (through alienated work, mass unemployment and unlimited media programs) cannot stay a democratic society in the long-run, since it

[1] The parameters underlying this simulation are: $\beta_{ve} = 0.06; \bar{e} = 0.9; \beta_{vv} = 0.01; \beta_{ev} = 0.1; \bar{v} = 0.6$. and are approximately obtained from simple OLS estimates of these dynamics (with no good statistical properties however, but definitely more appropriately chosen compared to the case without any empirical reference).

[2] Note with respect to figure 2 that it is assumed there that an increasing wage share is accompanied by inflationary pressure as it is suggested by the conflicting income claims approach. Note furthermore that – as is shown in Flaschel, Tavani, Taylor and Teuber (2007) – this cycle can be more complicated in nature if empirically observed nonlinearities in the money wage Phillips curve are taken into account which in fact move the cycle of the theoretical model already fairly close to what is shown in figure 1, bottom right.

produces conflicts that can range from social segmentation to class clashes, racial clashes and more.

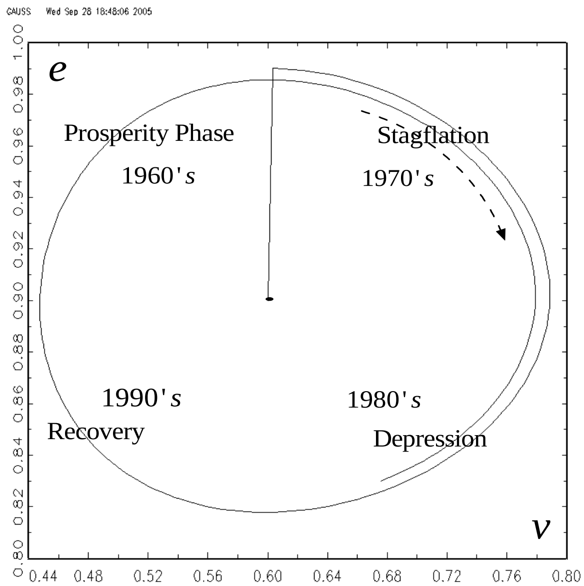

Figure 2: Goodwin-type long-phased wage share / employment dynamics

In addition to what has been shown above, one should, of course also try to take earlier time periods into account than just the 1960's to the present if data – in particular on the wage share – are available. For the United Kingdom we have considered in Flaschel and Groh (1995) such long time–series from 1855 up to 1965[3] which can here be extended to the following phase plot diagram.[4]

The important insight that can be obtained from these diagrams for Great Britain (1855 – 1965) is that the Goodwin cycle – if it really existed – must have been significantly shorter before 1914 (with larger fluctuations in employment during each business cycle), and that there has been a major change in it after 1945. This may be explained by significant differences and changes in the adjustment processes of market economies for these two periods: primarily price adjustment before 1914 and primarily quantity adjustments after 1945. This very tentative judgment must be left for future research here however. Based on Desai's data one could have expected that the growth cycle had become obsolete (and maybe also the business cycle as it was claimed in the 1960's). Yet, extended by the further data from Groth and Madsen (2007), it is now of course obvious that nothing of this sort took place in the UK economy. In fact, we see in figure 3 two periods of excessive employment (in the language of the theory of the NAIRU) which were followed by periods of dramatic unemployment, both started by segments the more or less pronounced occurrence of stagflation.

Such a long-run perspective allows to see to what extent the above shown cycle for the US economy is of a unique nature and therefore possibly representing a specific stage

[3]See Desai(1984) for the sources of these data and for an econometric approach on the basis of these data with respect to the Goodwin growth cycle model.

[4]We have to thank C. Groth and J. Madsen for providing access to these data, they have collected for their paper on 'Medium-term fluctuations and the "Great Ratios" of economic growth" (unpublished).

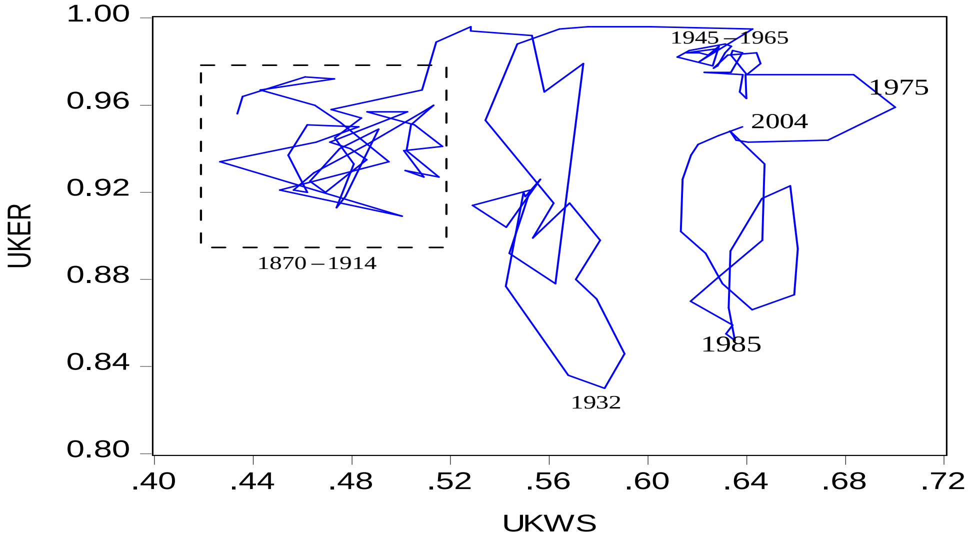

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Figure 3: UK Income Distribution Cycles 1870 –2004: WS = wage share, ER = employment rate

in the social structure of accumulation of capitalist market economies. As the figure 3 shows there are indeed two such phases visible in the case of the United Kingdom which are clearly separated from each other through the period where World War I and II were taking place. In Flaschel and Groh (1995) we had a long time series from 1855 up to 1965 which when taken in isolation could have suggested what was articulated at that time as 'is the business cycle obsolete' (Bronfenbrenner, 1969). When Flaschel and Groh (1995) was published the time-bound illusion in such a statement was of course already obvious, yet it is interesting to see (in figure 3) how radically this illusion was disproved by the development that took place in the UK after 1965. There is a Goodwinian distributive cycle after World War II in the UK and it implies the question of what we will observe in this regard in the next 50 years in the UK and elsewhere.

From the perspective of the flexicurity model that we will formulate and investigate in this paper the phase plot in figure 3 in our view clearly suggests that the depicted evolution in the British economy cannot be considered as an ideal for the next stage of the evolution of capitalism. Instead, we will pursue in this paper the idea that there is a coherent and workable alternative to the depicted distributive cycle in a competitive market environment that mirrors partial as well as macro ideas of the current discussion on the conduct of in particular labor market policies, in particular in the Nordic countries.

In the next section, we augment the Goodwin model by a second labor market where the state acts as the employer of 'first' resort[5] and thus guarantees full employment by specific actions. We show that this extension not only removes the reserve army mechanism from the labor market, despite the possibility of a wage-price spiral mechanism in the first labor market, but also makes the economy convergent to its long-run balanced

[5]and thus not yet of last resort, since this latter approach has been rightly criticized as being too passive and inventory like in nature.

growth path and this the faster the more flexible the labor market is adjusting. Apart from the (important) microeconomic problem of how the second labor market that is here added to the Goodwin growth cycle model can work in an efficient and socially acceptable manner we thus get the result that the macroeconomic performance is not only improved by this reformulation of the Goodwin model, but indeed turned into a state that can be considered as socially superior to the actual working of capitalist market economies like the USA and the UK.

2 Flexicurity growth: A baseline model

We have considered from the theoretical and the empirical perspective a long phase growth cycle that in the theoretical model of Flaschel and Groh (1995) was based – as modification of the simple Goodwin growth cycle approach discussed in the preceding section – on a repelling steady state and behavioral nonlinearities far off the steady state that tame the explosive dynamics and turn it viable and that is confirmed in its qualitative features through econometric measurements for the US economy after World War II. This reserve army mechanism, the distributive cycle as well as the accompanying inflation / unemployment cycle, is obviously a fairly archaic way to provide boundedness and order in a advanced capitalist market economy and its democratic institutions. We are therefore now designing as an alternative to the preceding one a growth model that rests in place of overaccumulation (in the prosperity phase) and mass unemployment (in the stagnant phase) on a second labor market which through its institutional setup guarantees full employment in its interaction with the first labor market, the employment in the industrial sector of the economy that is modelled as highly flexible and competitive. We therefore first reconsider the sector of firms in such an economy:

Firms

Production and Income Account:

Uses	Resources
δK	δK
$\omega_1 L_1^d, \qquad L_1^d = Y^p/z$	$C_1 + C_2 + C_r$
$\omega_2 L_{2f}^w$	G
$\Pi \qquad (= Y^f)$	$I \qquad (= Y^f)$
$\delta_1 R + \dot{R}$	S_1
Y^p	Y^p

This account is a very simple one. Firms use their capital stock (at full capacity utilization as we shall show later on) to employ the amount of labor (in hours): L_1^d in its operation, at the real wage ω_1, the law of motion of which is to be determined in the next section from a model of the wage-price level interaction in the manufacturing sector. They in addition employ labor force $L_{2f}^w = \alpha_f L_1^d, \alpha_f = const$ from the second labor market at the wage ω_2, which is a constant fraction α_ω of the market wage in the first labor market. This labor force L_{2f}^w is working the normal hours of a standard workday, while the workforce L_1^w from the first labor market may be working overtime or undertime depending on the size of the capital stock in comparison to its own size. The rate $u_w = L_1^d/L_1^w$ is therefore the utilization rate of the workforce in the first labor market, the industrial workers of the economy (all other employment comes from the working of households occupied in the second labor market). Note finally, that in line with the model of section, we allow for capital stock depreciation at the rate δ.

Firms produce full capacity output[6] $Y^p + \delta_1 R = C_1 + C_2 + C_r + I + \delta K + G$, that is sold to the two types of consumers (and the retired households), the investing firms and the government. The demand side of the model is formulated here in a way such that indeed this full capacity output can be sold in this way, see the next section on this matter. Deducting from this output and income Y^p of firms their real wage payments to workers from the first and the second labor market (and depreciation)[7] we get the profits of firms which are here assumed to be invested fully into capital stock growth $\dot{K} = I = \Pi$. We thus have Classical (direct) investment habits in this basic approach to a model with an employer of first resort. There is therefore not yet debt or equity financing of investment in this model type.

We assume a fixed proportions technology with $y^p = Y^p/K$ the potential output – capital ratio and with $z = Y^p/L_1^d$ the given value of labor productivity (which determines the employment L_1^d of the workforce L_1^w of firms).

We next consider the households sector of our social growth model which is composed of worker households working in the first labor market and the remaining ones that are all working in the second labor market.

Households I and II (primary and secondary labor market)

Income Account (Households I):

Uses	Resources
$C_1 = c_{h1}(1-\tau_h)\omega_1 L_1^d$	
$\omega_2 L_{2h}^w = c_{h2}(1-\tau_h)\omega_1 L_1^d$	
$T = \tau_h \omega_1 L_1^d$	
$\omega_2(L - (L_1^w + L_{2f}^w + L_{2h}^w + L_{2g}^w))$	
$\omega_2 L^r, L^r = \alpha_r L$	
S_1	$\omega_1 L_1^d$
$Y_1^w = \omega_1 L_1^d$	$Y_1^w = \omega_1 L_1^d$

Income Account (Households II):

Uses	Resources
C_2	$\omega_2 L_2^w, L_2^w = L - L_1^w$
Y_2^w	Y_2^w

Households of type I consume manufacturing goods of amount C_1 and services from the second labor market L_{2h}^w. They pay an (all) income tax T and they pay in addition – via

[6]augmented by company pension payments δR.

[7]the term S_1 is equal to $\delta_1 R + \dot{R}$.

further tax transfers – all workers' income in the labor market that is not coming from firms, from them and government (which is equivalent to an unemployment insurance). Moreover, they pay the pensions of the retired households ($\omega_2 L^r$) and accumulate their remaining income S_1 in the form a company pension into a fund R that is administrated by firms (with inflow S_1, see the sector of households and outflow $\delta_1 R$).
The transfer $\omega_2(L-(L_1^w+L_{2f}^w+L_{2h}^w+L_{2g}^w)$ can be considered as solidarity payments, since workers from the first labor market that lose their job will automatically be employed in the second labor market where full employment is guaranteed by the government (as employer of first resort). We consider this employment as skill preserving, since it can be viewed as ordinary office or handicraft work (subject only to learning by doing when such workers return to the first labor market, i.e, employment in the production process of firms).
The second sector of households is here modelled in the simplest way that is available: Households employed in the second labor market, i.e, $L_2^w = L_{2f}^w + L_{2h}^w + L_{2g}^w$ pay no taxes and totally consume their income. We have thus Classical saving habits in this household sector, while households of type 1 may have positive or negative savings S_1 as residual from their income and expenditures. We here assume that they can accumulate these savings (or dissave in case of a negative S_1) from the stock of commodities they have accumulated as inventories in the past.
In order to have a consistent distribution of the funds R that are accumulated by households of type I on the basis of their savings S_1, according to the stock-flow relationship $\dot{R} = S_1$ we have to modify this relationship as follows:

$$\dot{R} = S_1 - \delta_1 R$$

where δ_1 is the rate by which these funds are depreciated through company pension payments to the 'officially retired' workers L^r assumed to be a constant fraction of the 'active' workforce $L^r = \alpha_r L$. These worker households are added here as not really inactive, but offer work according to their still existing capabilities that can be considered as an addition to the supply of work organized by the government $L - (L_1^w + L_{2f}^w + L_{2h}^w)$, i.e., the working potential of the officially retired persons remains an active and valuable contribution of the workhours that are supplied by the members of the society. It is obvious that the proper allocation of the work hours under the control of the government needs thorough reflection from the microeconomic and the social point of view, which however cannot be a topic in a paper on the macroeconomics of such an economy.
As the income account of the retired households, shown below, shows they receive pension payments as if they would work in the second labor market and they get in addition individual transfer income (company pensions) from the accumulated funds R in proportion to the time they have been active in the first labor market and as an aggregate household group of the total amount $\delta_1 R$ by which the pension funds R are reduced in each period.

Income Account (Retired Households):

Uses	Resources
C_r	$\omega_2 L^r + \delta_1 R, L^r = \alpha_r L$
Y^r	Y^r

There is finally the government sector which is also formulated in a very basic way:

The Government

Income Account: Fiscal Authority / Employer of First Resort

Uses	Resources
$G = \alpha_g T$	$T = \tau_h \omega_1 L_1^d$
$\omega_2 L_{g2}^w = (1-\alpha_g)T$	
$\omega_2(L - (L_1^w + L_{2f}^w + L_{2h}^w + L_{2g}^w))$	$\omega_2 L_r^w$
$\omega_2 L^r$	$\omega_2 \alpha_r L$
Y^g	Y^g

The government receives income taxes, the solidarity payments (employment benefits) for the second labor market paid from workers in the first labor market and old-age pension payments. It uses the taxes to finance government goods demand G and the surplus of taxes over these government expenditure to actively employ the core workers in the government sector. In addition it employs the workers receiving employment benefits from the households in first labor market and it in fact also employs the 'retired' persons to the extent they can still contribute to the various employment activities. We thus have that the total labor force in the second labor market is employed by firms, by households of type 1 and the remainder through the government as is obvious from the solidarity payments of households working in the first labor market. We thus have that the income payments to workers in the second labor market ($\omega_2 L_2^w$) that are not originating from their services to firms, to households of type I or through an excess of income taxes over government commodity expenditures (base government employment) are paid out of transfers from the household sector that works in industrial production to the government, and that on the basis of these payments the remaining work in the second labor market is organized by government (in the way it does this in the administration of the state in all modern market economies).

In sum we get that workers are employed either in the first labor market and if not there then by doing auxiliary work within firms, services for households of type 1 or services in the government sector concerning public administration, infrastructure services, educational services or other public services (in addition there is potential labor supply $\alpha_r L$ from the retired households, which due to the long-life expectancy in modern societies can remain effective suppliers of specific work over a considerable span of time). In this way the whole workforce is always fully employed in this model of social growth (and the retired persons according to their capabilities) and thus does not suffer from human degradation in particular. Of course, there are a variety of issues concerning state organized work that point to problems in the organization of such work, but all such problems exist also in actual industrialized market economies in one way or another.

We thus have a Classical growth model of the economy where full employment is not assumed, but actively constructed. To motivate the behavioral equations of the social growth model of this paper we derive them as simplification from an advanced Goodwin-Kalecki growth cycle model where indeed the persistent long phase cycle in employment and the wage share we derived and observed in section 1 is augmented by Keynesian goods market dynamics and a Kaleckian reserve army mechanism that concerns the whole social structure of accumulation and in particular an explanation of the rise and the (partial) fall of the welfare state after World War II. We will introduce the behavioral equations of our social growth model by contrasting them – as we go along – with what has been assumed in Flaschel, Franke and Semmler (2007) within the Goodwin-Kaleckian growth cycle model of a distributive reserve army mechanism coupled with Kalecki's (1943) political aspects of full employment.

3 Goodwin-Kalecki dynamics: Progress towards consent economies and beyond?

In this section we go on from the Goodwinian modeling of the Marxian reserve army mechanism to its extension as a Kaleckian model of the evolution of the welfare state after World War II as it was modelled in Flaschel, Franke and Semmler (2007). We progressed in this framework from the case of dissent economies (where in fact a Goodwinian and a Kaleckian type of reserve army mechanism were interacting) to the case of consent economies where we could show the existence of a high and attracting balanced growth path for such an economy. The following table 1 shows the range of possibilities that was considered in Flaschel, Franke and Semmler (2007).

	high steady state	low steady state
stable steady state	Nordic consent economy	Kaleckian market economy type I
unstable steady state	Kaleckian market economy type II	Southern dissent economy

Table 1: Four types of market economies.

As conditions for the existence of a consent economy, we assumed Flaschel, Franke and Semmler (2007), see the behavioral equations below, that demand pressure in the labor market (both inside and outside of the firm) does not influence the rate of wage inflation very much, i.e., the wage level is a fairly stable magnitude. Furthermore, the Kaleckian reserve army mechanism was absent from the model ($i_e = 0$). Moreover, the benchmark values for demand pressures and the employment policy of firms are consistent with each other and all sufficiently high to not imply labor market segmentation and significant disqualification of unemployed workers. This can be coupled with flexible hiring and firing policies then, i.e., the parameter β_{eu} may be chosen as large as it is desirable.

This modified Kaleckian approach to consent economies is contrasted in the following with the dynamics and the balanced growth path of the model of flexicurity capitalism we have introduced in the preceding section. We there compare models of the distributive growth cycle (with more or less conflict between capital and labor) with the flexicurity variant of competitive capitalism. However, the important and difficult topic of the generation of socio-economic progress paths that lead from distributive conflict cycles to consent economies and from there towards the proper functioning of an employer of first resort economy, as the perspective of the flexicurity approach to social growth, must be left for future research here.
We derive the behavioral relationships of our model of flexicurity capitalism by contrasting them with the Kaleckian growth cycle model of Flaschel, Franke and Semmler

(2007). We represent the laws of motion of the latter economy first in a framework of Goodwin-Kalecki type, before we show how these equations simplify in our model of social growth with an employer of first resort.
We consider first the wage-price dynamics in the first labor market, which is the only labor market in the Goodwin-Kalecki approach. For the description of these dynamics we start from a general formulation of a wage-price spiral as shown below, see Flaschel, Franke and Semmler (2007) for a detailed treatment of its structure.[8]

$$\hat{w} = \beta_{we}(e-\bar{e}) + \beta_{wu}(u_w - \tilde{u}_w) - \beta_{w\omega}\ln(\frac{\omega}{\omega^o}) + \kappa_w\hat{p} + (1-\kappa_w)\pi^c$$
$$\hat{p} = \beta_{py}(y-\bar{y}) + \beta_{p\omega}\ln(\frac{\omega}{\omega^o}) + \kappa_p\hat{w} + (1-\kappa_p)\pi^c$$

In these equations, $\hat{w}$, $\hat{p}$ denote the growth rates of nominal wages w and the price level p (their inflation rates) and π^c a medium-term inflation-climate expression which however is of no relevance in the following due to our neglect of real interest rate effects on the demand side of the model. We denote by e the rate of employment on the external labor market and by u_w the ratio of utilization of the workforce within firms. This latter ratio of employment is compared by the workforce in their negotiations with firms with their desired normal ratio of utilization $\tilde{u}_w$. We thus have two employment gaps, an external one: $e - \bar{e}$ and an internal one: $u_w - \tilde{u}_w$, which determine wage inflation rate $\hat{w}$ from the side of demand pressure within or outside of the production process. In the wage PC we in addition employ a real wage error correction term $\ln(\omega/\omega_0)$ as in Blanchard and Katz (1999), see Flaschel and Krolzig (2006) for details, and as cost pressure term a weighted average of short-term (perfectly anticipated) price inflation $\hat{p}$ and the medium-term inflation climate π^c in which the economy is operating.
As the wage PC is constructed it is subject to an interaction between the external labor market and the utilization of the workforce within firms. Higher demand pressure on the external labor market translates itself here into higher workforce wage demand pressure within firms (and demand for a reduced length of the normal working day, etc.), an interaction between two utilization rates of the labor force that has to be and will be taken note of in the formulation of the employment policy of firms. Demand pressure on the labor market thus exhibits two interacting components, where employed workers may make their behavior dependent upon.
We use the output-capital ratio $y = Y/K$ to measure the output gap in the price inflation PC and again the deviation of the real wage $\omega = w/p$ from the steady state real wage ω^o as error correction expression in the price PC. Cost pressure in this price PC is formulated as a weighted average of short-term (perfectly anticipated) wage inflation and again our concept of an inflationary climate π^c. In this price Phillips curve we have three elements of cost pressure interacting with each other, a medium term one (the inflationary climate) and two short term ones, basically the level of real unit-wage labor costs (a Blanchard and Katz (1999) error correction term) and the current rate of wage inflation, which taken by itself would represent a constant markup pricing rule. This basic rule is however modified by these other cost-pressure terms and in particular also

[8]The considered wage-price spiral will imply a law of motion for real wages which in simplified form also appears in the flexicurity model. As these models are formulated their dynamics are however independent of the nominal levels of wages and prices, i.e., everything can be expressed in real terms. For the introduction of the monetary sector see Flaschel, Franke and Semmler (2007).

made dependent on the state of the business cycle by way of the demand pressure term $y - \bar{y}$ in the market for goods.
In our social growth model the above wage-price inflation dynamics simplifies to the following form:

$$\hat{w} = \beta_{wu}(u_w - \tilde{u}_w) - \beta_{w\omega}\ln(\frac{\omega}{\omega^o}) + \kappa_w\hat{p} + (1-\kappa_w)\pi^c \quad (3)$$

$$\hat{p} = \beta_{p\omega}\ln(\frac{\omega}{\omega^o}) + \kappa_p\hat{w} + (1-\kappa_p)\pi^c \quad (4)$$

since we will have – by construction – full employment in this model type (and a NAIRU rate that is zero) and in addition a goods demand that is always equal to the potential output that is produced by firms.
On the demand side of the model the Kaleckian framework used for reasons of simplicity the conventional Keynesian dynamic multiplier process (in place of a full-fledged Metzlerian inventory adjustment mechanism) and extremely classical saving habits together with a Kaleckian type of investment function, i.e.

$$\hat{Y} = \dot{Y}/Y = \beta_y(Y^d/Y - 1) + \bar{a}, \quad Y^d = \omega L^d + I(\cdot) + \delta K + G$$

where Y^d, Y denote aggregate demand and supply and $\bar{a}$ a trend term in the behavior of capitalist firms. Assuming a fixed proportions technology with given output-employment ratio $x = Y/L^d$ and potential output-capital ratio $y^p = Y^p/K$, allows us to determine from the output-capital ratio y the employment u_w of the workforce within firms that corresponds to this activity measure y:

$$u_w = y/(xle), \quad u_w = L^d/L^w, l = L/K, e = L^w/L$$

(with L^d hours worked, L^w the number of workers employed within firms and with L denoting labor supply). This relationship represents by and large a technical relationship (to be calculated by 'engineers') and relates hours worked to goods market activity as measured by y in the way shown above.
In the social growth model we always have the relationship $y^d = y = y^p$ per unit of capital and thus no dynamic on the goods market, and get on this basis then:

$$u_w = L_1^d/L_1^w = \frac{y^p}{zle_1} = \frac{y^p}{zl_1^w}, \quad l = L/K, l_1^w = L_1^w/K, e_1 = L_1^w/L. \quad (5)$$

This technological relationship must be carefully distinguished from the employment (recruitment) policy of firms that reads on the intensive form level:

$$\hat{e} = \beta_{eu}(u_w - \tilde{u}_f) - \beta_{e\omega}(\omega - \omega^o) + \bar{a} - \hat{L}, \ i.e.,$$

$$\dot{e} = \beta_{eu}(y^p/(xl) - \tilde{u}_f e) - \beta_{e\omega}(\omega - \omega^o)e + (\bar{a} - \hat{L})e$$

Basis of this formulation of an employment policy of firms in terms of the employment rate is – by assumption – the following level form representation of this relationship:

$$\begin{aligned}\dot{L}^w &= \beta_{eu}(L^d - \tilde{u}_f L^w) - \beta_{e\omega}(\omega - \omega^o)L^w + \bar{a}L^w, \ i.e. \\ \hat{L}^w &= \beta_{eu}(L^d/L^w - \tilde{u}_f) - \beta_{e\omega}(\omega - \omega^o) + \bar{a}\end{aligned}$$

where $\bar{a}$ again integrates the trend term assumed by firms into now their employment policy and where $\tilde{u}_f$ represents the utilization ratio of the workforce of firms that is desired by them. In order to obtain eq. (6) as the resulting law of motion for the rate of employment one simply has to take note of the definitional relationship $\hat{e} = \hat{L}^w - \hat{L}$, where L denotes the labor supply in each moment in time. We have also included into the above recruitment policy a term that says that intended recruitment will be lowered in case of increasing real wage costs of firms.
In the social growth model the employment policy of firms (on the first labor market) is by and large the same as above. We stress however that the external and the internal labor market and the pressure they are exercising on money wage formation form a capillary system in the Goodwin-Kalecki approach and are handled by firms against this background. Such a situation is no longer present in the social growth model, since there is by construction full employment in this model type and since the second labor market here serves as a buffer for the fluctuations that occur in the employment of workers within firms. Note that the label flexicurity assumes in this regard that firms are completely free in their choice of the hiring and firing parameter β_{eu}.

$$\begin{aligned}\hat{e}_1 &= \beta_{eu}(u_w - \tilde{u}_w) + \rho^o - \hat{L}, \ i.e., && (6)\\ \dot{e}_1 &= \beta_{eu}(y^p/(xl) - \tilde{u}_w e_1) + (\rho^o - \hat{L})e_1, \text{or simpler} && (7)\\ \hat{l}_1^w &= \beta_{eu}(y^p/(zl_1^w) - \tilde{u}_w) + (\rho^o - \rho) && (8)\end{aligned}$$

since investment is equal to profits in this basic version of the social growth model. Note that we now use a common measure $\tilde{u}_w$ in the money wage PC and the recruitment policy of firms and that we assume now ρ^o to be the trend rate of growth of the economy which is used by firms in their trend labor recruitment policy (in place of the $\bar{a}$ used in the Goodwin-Kalecki model).
In the Keynesian Goodwin-Kalecki framework we assumed extremely classical saving habits ($s_w = 0, s_c = 1$) and for the investment behavior of firms:

$$I/K = i_\rho(\rho - \rho_o) - i_e(e - \bar{e}_f) + \bar{a},$$

with $\rho = y(1 - \omega/x)$ the current rate of profit. In this equation, the magnitude $\bar{a}$ denotes again the given trend investment rate (representing investor's 'animal spirits') from which firms depart in a natural way if there is excess profitability (and vice versa). Moreover, firms have a view of what the rate employment should be on the external labor market (Kalecki's (1943, ch.12) analysis of why 'bosses' dislike full employment) and thus reduce their (domestic) investment plans (driven by excess profitability) in situations of a tense labor market. They thus take pressure from the labor market in the future evolution of the economy by their implicit collective understanding that high pressure in the capillary system of internal and external labor markets we have

considered above will lead to conditions in the capital-labor relationship, unwanted by firms, since persistently high employment rates may give rise to significant changes of workforce participation with respect to firms' decision making, in the hiring and firing decision of firms, in reductions of the work-day etc., not at all liked by 'industrial leaders' in the case of a Kaleckian dissent economy.
In the social growth model, the alternative and extension of this paper to / of a Kaleckian consent economy, we have already assumed that workers of type II consume their whole income (they pay no taxes). With respect to the other type of workers we assume as their consumption function

$$
\begin{aligned}
C_1 &= c_{h1}(1-\tau_h)\omega_1 L_1^d, && c_h \text{ propensity to consume}, \tau_h \text{ tax rate} \quad (9)\\
\omega_2 L_{2h}^w &= c_{h2}(1-\tau_h)\omega_1 L_1^d && \text{consumption of household services} \quad (10)
\end{aligned}
$$

Households type I savings is on the basis of our accounting relationships given by

$$S_1 = \omega_1 L_1^d - C_1 - \omega_2 L_{2h}^w - \omega_2(L - (L_1^w + L_{2f}^w + L_{2h}^w + L_{2g}^w) - \omega_2 L^r) \quad (11)$$

due to the assumed solidarity contribution they provide to the second labor market. Investment behavior is in the basic form of the social growth model very simple: all profits of firms are invested and there is no debt or equity financing yet. The growth rate of the capital stock is thus simply given by $\hat{K} = \rho = \Pi/K(\rho^o$ the steady state rate of profit). On the basis of what we have already assumed we thus get:

$$\hat{K} = \rho = y^p[1 - \omega_1(1 + \alpha_\omega \alpha_f)/z] - \delta, \quad \omega_2 = \alpha_\omega \omega_1, L_{2f}^w = \alpha_f L_1^d \quad (12)$$

see below with respect to the parameter α_f which characterizes the employment policy of firms with respect to the second labor market.
For government consumption we finally assume the simple relationship $G = \gamma I$, i.e., government consumption per unit of capital grows at the same rate as the capital stock (which allows to integrate fiscal policy with investment behavior in the intensive form of the model).
Since the government, workers from the second labor market and pensioners do not save and since all tax transfers are turned into consumption and the savings of households of type I into commodity inventories of firms from which company pensions are to be deducted and since finally all profits are invested it can easily be shown from what was presented in accounting form in the preceding section that we must have at all times:

$$Y^p + \delta_1 R = C_1 + C_2 + C_r + I + \delta K + G, \quad C_1 = c_{h1}(1-\tau_h)\omega_1 L_1^d \quad (13)$$

if firms produce at full capacity $Y^p = y^p K, L_1^d = Y^p/z$ (which they can and will do in this case). There is thus no demand problem on the market for goods and thus no need to discuss a dynamic multiplier process as in the Goodwin - Kalecki model with which this model was compared here. Note that moreover we have by construction of the social growth model at all points in time:

$$L = L_1^w + L_{2f}^w + L_{2h}^w + L_{2g}^w + L_r^w = L_1^w + L_2^w \qquad L_r = \alpha_r L \tag{14}$$

We thus assume that households of type 1 must pay as solidarity contribution (employment benefits) those workers of type 2, whose wages are not paid by firms, through households type 2 service to households of type I and through the core employment in the government sector. The government employs in addition as administrative workers and infrastructure workers (public work and education) the remaining workforce in the second labor market (plus the L^r services from pensioners). This completes the discussion of the behavioral equations of the social growth model, the intensive form of which will now be derived in the following section. Compared to the Goodwin-Kalecki model of Flaschel, Franke and Semmler (2007) this model type will be shown to function very easily without need of a discussion of conflict-driven upper and lower turning points in economic activity and income distribution which are necessary to keep the locally centrifugal dynamics of the Goodwin-Kalecki approach bounded and thus viable. There are only mildly conflicting income claims in the social growth model and also only mild conflicts about the role and the extent of the welfare state in such a framework (to be discussed below).

In the Goodwin-Kalecki growth cycle model we have (in the dissent situation) conflict-riddled turning points in economic and social activities than can end prosperity phases in a radical fashion and then lead the society into long-lasting depressions, processes that are harmful and wasteful with respect to human and physical capital and that may not work towards a recovery under all circumstances. The need for an alternative to such a situation is therefore a compelling one from the perspective of a social and democratic society and the potential it may contain for the evolution of mankind. We have already introduced in the previous and this section the economic contours of such an alternative. This alternative model of social reproduction will be analyzed in its macrodynamic features in the next two sections.

4 Flexicurity growth: Full capacity convergence towards balanced reproduction

4.1 The dynamics and their balanced growth path

Inserting the equations of the social growth model appropriately into each other gives rise to the following 3D dynamics in the state variables $\omega_1 = w/p, l_1^w = L_1^w/K$ and

$l = L/K$ where the last variable does however not feedback into the first two laws of motion due to the construction of the labor markets of the model.[9]

$$\hat{\omega}_1 = \kappa[(1-\kappa_p)(\beta_{wu}(\frac{y^p}{zl_1^w} - \tilde{u}_w) - \beta_{w\omega}\ln(\frac{\omega_1}{\omega_1^o})) - (1-\kappa_w)\beta_{p\omega}\ln(\frac{\omega_1}{\omega_1^o})] \quad (15)$$

$$\hat{l}_1^w = \beta_{eu}(\frac{y^p}{zl_1^w} - \tilde{u}_w) + n - \rho, \quad \rho = y^p[1-\omega_1(1+\alpha_\omega\alpha_f)/z] - \delta \quad (16)$$

$$\hat{l} = n - (y^p[1-(1+\alpha_\omega\alpha_f)\omega_1/z] - \delta) = y^p[(1+\alpha_\omega\alpha_f)(\omega_1 - \omega_1^o)/z] \quad (17)$$

However, in order to get a stationary value of l in the long-run we must assume a special value for ω_1^o in the first two equations (as the steady state reference real wage in the first labor market), which is determined by:

$$\hat{l} = 0, \quad i.e., \quad y^p[1-(1+\alpha_\omega\alpha_f)\omega_1/z] = \delta + n. \quad (18)$$

The reference wage used in the first two laws of motion must therefore be chosen such that the capital stock grows with the natural rate n in the steady state which is one of the conditions needed for steady growth in the Harrod (1939) growth model. Since our model is based on Say's law the other conditions of the Harrod model do not apply here. Based on this assumption we get for the interior steady state or balanced growth path of the social growth economy the equations ($\tilde{u}_w = 1$ in the following for reasons of simplicity):

$$l_1^{wo} = \frac{y^p}{\tilde{u}_w z} = \frac{l_1^{do}}{\tilde{u}_w} = y^p/z \quad (19)$$

$$\omega_1^o = \frac{1 - \frac{n+\delta}{y^p}}{1+\alpha_f\alpha_\omega} z < z \quad (20)$$

$$l^o = \text{arbitrary} \quad (21)$$

Since we have a zero determinant for the 3D Jacobian of the above dynamics (since the third law of motion only depends on the first state variable) we have zero root hysteresis in the 3D system which in the given form allows to treat and solve the first two equations independently of the third one which when appended can converge to any value of l, depending on shocks to labor supply, capital formation and the like. Note however that this only applies if there is social consensus with respect to the steady state real wage ω_1^o as the benchmark for real wage negotiations in the first labor market. Choosing in addition (and for example) as parameter values[10] $\alpha_w = 0.5, n = 0.05, \delta = 0.1, y^p = 0.5$ gives for the ratio $v_1 = \omega_1/z$, the wage share in the first labor market, the approximate

[9]The steady state value, see below, is here assumed to underlie Blanchard and Katz (1999) type error correction in the first labor market.

[10]The value of n must be chosen that high since technical change is still ignored in this baseline social growth model.

value $v_1 = 0.64$ and for the profit share Π^o/Y^p the value 0.1 which in sum implies for the shares of wages and government expenditures the value 90%. Note finally that the living standards in this society, as measured by real wages, depend of course on the value of the labor productivity of workers in the first labor market.

4.2 Monotonic convergence towards balanced growth

For the Jacobian of the 3D dynamics evaluated at the steady state we get from the laws of motion:

$$J^o = \begin{pmatrix} -\kappa[(1-\kappa_p)\beta_{w\omega} + (1-\kappa_w)\beta_{p\omega}] & -\kappa(1-\kappa_p)\beta_{wu}\frac{y^p}{z}(l_1^{wo})^{-2}\omega_1^o & 0 \\ \frac{y^p(1+\alpha_\omega\alpha_f)}{z}l_1^{wo} & -\beta_{eu}\frac{y^p}{zl_1^{wo}} & 0 \\ \frac{y^p(1+\alpha_\omega\alpha_f)}{z}l & 0 & 0 \end{pmatrix}$$

Since we only have to investigate the first two laws of motion, it suffices to consider the following matrix with respect to its eigenvalues:

$$\begin{aligned} J^o &= \begin{pmatrix} J_{11}^o & J_{12}^o \\ J_{21}^o & J_{22}^o \end{pmatrix} = \begin{pmatrix} -\kappa[(1-\kappa_p)\beta_{w\omega} + (1-\kappa_w)\beta_{p\omega}] & -\kappa(1-\kappa_p)\beta_{wu}\frac{y^p}{z}(l_1^{wo})^{-2}\omega_1^o \\ \frac{y^p(1+\alpha_\omega\alpha_f)}{z}l_1^{wo} & -\beta_{eu}\frac{y^p}{zl_1^{wo}} \end{pmatrix} \\ &= \begin{pmatrix} - & - \\ + & - \end{pmatrix} \end{aligned}$$

From the sign structure in this matrix it is obvious that we always have locally asymptotically stable dynamics (i.e., trace $J^o < 0, \det J^o > 0$). Furthermore, the condition trace $J^o = 4\det J^o$, i.e.,

$$(J_{11}^o + J_{22}^o)^2 = 4(J_{11}^o J_{22}^o + J_{21}^o J_{12}^o)$$

separates monotonic convergence (for parameters β_{eu} sufficiently large) from cyclical convergence (parameters β_{eu} sufficiently small). Reformulated, this condition reads:

$$|J_{22}^o| = |J_{11}^o| + 2\sqrt{|J_{21}^o J_{12}^o|}, \quad i.e., \quad \beta_{eu}^H = \frac{zl_1^{wo}}{y^p}[|J_{11}^o| + 2\sqrt{|J_{21}^o J_{12}^o|}]$$

We thus get for the bifurcation value β_{eu}^H that separates monotonic from cyclical convergence:

$$\beta_{eu}^H = \kappa[(1-\kappa_p)\beta_{w\omega} + (1-\kappa_w)\beta_{p\omega}] + 2\sqrt{(1+\alpha_\omega\alpha_f)\kappa(1-\kappa_p)\beta_{wu}\omega_1^o l_1^{wo}} \qquad (22)$$

This critical parameter for the hiring and firing speed parameter in our social growth economy is therefore in particular the larger, the larger the reaction of money wage inflation with respect to workforce utilization, i.e., the larger the parameter β_{wu} becomes. We thus get that economic fluctuations can be avoided in this type of economy if wages in the first labor market respond relatively sluggishly to demand pressure in this market (as measured by the utilization rate of the insiders) and if hiring and firing is a sufficiently flexible process as envisaged by the concept of flexicurity capitalism.

4.3 Global viability

For the investigation of global asymptotic stability we will now analyze the core dynamical system by means of so-called Liapunov functions. For this purpose we represent the 2D dynamics of the preceding section as follows.

$$\hat{\omega}_1 = G^1(\omega_1) + G^2(l_1^w), \quad G^{1'} < 0, G^{2'} < 0 \tag{23}$$
$$\hat{l}_1^w = H^1(\omega_1) + H^2(l_1^w), \quad H^{1'} > 0, H^{2'} < 0 \tag{24}$$

The Liapunov function to be used in the stability proof then reads as follows:

$$V(\omega_1, l_1^w) = \int_{\omega_1^o}^{\omega_1} H^1(\tilde{\omega}_1)/\tilde{\omega}_1 d\tilde{\omega}_1 + \int_{l_1^{wo}}^{l_1^w} -G^2(\tilde{l}_1^w)/\tilde{l}_1^w d\tilde{l}_1^w$$

This function describes by its graph a 3D sink with the steady state of the economy as its lowest point, since the above integrates two functions that are negative to the left of the steady state values and positive to their right. For the first derivative of the Liapunov function along the trajectories of the considered dynamical system we moreover get:

$$\begin{aligned}
\dot{V} &= dV(\omega_1(t), l_1^w(t))/dt = \left(H^1(\omega_1)/\omega_1\right)\dot{\omega}_1 - \left(G^2(l_1^w)/l_1^w\right)\dot{l}_1^w && (25)\\
&= H^1(\omega_1)\hat{\omega}_1 - G^2(l_1^w)\hat{l}_1^w && (26)\\
&= H^1(\omega_1)(G^1(\omega_1) + G^2(l_1^w)) - G^2(l_1^w)(H^1(\omega_1) + H^2(l_1^w)) && (27)\\
&= H^1(\omega_1)G^1(\omega_1) - G^2(l_1^w)H^2(l_1^w) && (28)\\
&= -H^1(\omega_1)(-G^1(\omega_1)) - (-G^2(l_1^w))(-H^2(l_1^w)) && (29)\\
&\leq 0 \qquad [= 0 \quad \text{if and only if} \quad \omega_1 = \omega_1^o, l_1^w = l_1^{wo}] && (30)
\end{aligned}$$

since the multiplied functions have the same sign to the right and to the left of their steady state values and thus lead to positive products with a minus sign in front of them (up to the situation where the economy is already sitting in the steady state). We thus have proved that there holds:

Proposition 1:

The interior steady state of the dynamics

$$\hat{\omega}_1 = \kappa[(1-\kappa_p)(\beta_{wu}(\frac{y^p}{zl_1^w} - \tilde{u}_w) - \beta_{w\omega}\ln(\frac{\omega_1}{\omega_1^o})) - (1-\kappa_w)\beta_{p\omega}\ln(\frac{\omega_1}{\omega_1^o})] \tag{31}$$
$$\hat{l}_1^w = \beta_{eu}(\frac{y^p}{zl_1^w} - \tilde{u}_w) + \rho^o + (y^p[(\omega_1 - \omega_1^o)(1+\alpha_\omega\alpha_f)/z] - \delta) \tag{32}$$

is a global sink of the function V, defined on the positive orthant of the phase space, and is attracting in this domain, since the function V is strictly decreasing along the trajectories of the dynamics in the positive orthant of the phase space.

From the global perspective there may however be supply bottlenecks in the second labor market. Here we assume that the economy is working always in a corridor around the steady state where the government as the employer of first resort has still a sufficient amount of workforce working in the range of activities that is organized by it. Due to the stability results obtained in the present and the preceding section this is not a very restrictive assumption under the normal working of the economy.

5 Company pension funds: Dynamics and steady state levels

There is a further law of motion in the background of the model that needs to be considered in order to provide an additional statement on the viability of the considered model of flexicurity capitalism. This law of motion describes the evolution of the pension fund per unit of the capital stock $\eta = \frac{R}{K}$ and is obtained from the defining equation $\dot{R} = S_1 - \delta_1 R$ as follows:

$$\begin{aligned}\hat{\eta} &= \hat{R} - \hat{K} = \frac{\dot{R}}{K}\frac{K}{R} - \rho = \frac{S_1 - \delta_1 R}{K}/\eta - \rho, \quad i.e.: \\ \dot{\eta} &= \frac{S_1}{K} - (\delta_1 + \rho)\eta = s_1 - (\delta_1 + \rho)\eta\end{aligned}$$

with savings of households of type I and profits of firms per unit of capital being given by:

$$\begin{aligned}s_1 &= (1 - (c_{h1} + c_{h2})(1 - \tau_h) - \tau_h)\omega_1 y^p/z - \alpha_\omega \omega_1 (l_x^w + l^r) \\ & \quad l_x^w = l - (l_1^w + l_{2f}^w + l_{2h}^w + l_{2g}^w) \\ & \quad l^r = \alpha_r l, \quad i.e., \text{ due to the financing of the employment terms } \quad l_{2h}^w + l_{2g}^w \\ s_1 &= (1 - c_{h1}(1 - \tau_h) - \alpha_g \tau_h)\omega_1 y^p/z - ((1 + \alpha_r)l - (l_1^w + l_{2f}^w))\alpha_\omega \omega_1, \quad l_{2f}^w = \alpha_f y^p/z \\ \rho &= y^p[1 - (1 + \alpha_\omega \alpha_f)\omega_1/z] - \delta\end{aligned}$$

For reasons of analytical simplicity we now assume that the government pursues an immigration policy that ensures for the total growth rate of the labor force the condition $n = \hat{K}, i.e.$, the total labor supply grows by this migration policy with the same rate as the capital stock. This keeps the ratio $l = L/K$ constant, a simplifying assumption that must be accompanied later on by the assumption that the actual $\bar{l}$ must be chosen in a certain neighborhood of a base value $\bar{l}_o$ that is determined later on. Since we are now no longer able to determine the steady state value of the real wage ω_1 from the law of motion for l, we have to supply it from the outside now: $\omega_1^o = \bar{\omega}_1 =$ given. This also provides us with the steady state value of the rate of profit $\rho^o = \bar{\rho} = y^p[1 - (1 + \alpha_\omega \alpha_f)\bar{\omega}_1/z] - \delta$ which also determines the steady value of natural growth $n_o = \bar{\rho}$. Moreover we also assume for simplicity $\delta_1 = \delta$ for the depreciation rates of the capital stock and the stock of pension funds.

This gives for the law of motion of the pension fund to capital ratio the differential equation:

$$\dot{\eta} = (1 - c_{h1}(1-\tau_h) - \alpha_g\tau_h)\omega_1 y^p/z - ((1+\alpha_r)\bar{l} - (l_1^w + \alpha_f y^p/z))\alpha_\omega\omega_1 \\ -[y^p - (1+\alpha_\omega\alpha_f)\omega_1 y^p/z]\eta$$

We thus get that the trajectory of the pension fund ratio η is driven by the autonomous evolution of the state variables ω_1, l_1^w that characterize the dynamics of the private sector of the economy and that has been shown to be convergent to the steady state values $\bar{\omega}_1, l_1^{wo} = y^p/z$ as usual). Assuming that these variables have reached their steady state positions then gives

$$\dot{\eta} = (1 - c_{h1}(1-\tau_h) - \alpha_g\tau_h)\bar{\omega}_1 y^p/z - ((1+\alpha_r)\bar{l} - (l_1^{wo} + \alpha_f y^p/z))\alpha_\omega\bar{\omega}_1 - (\delta_1 + \bar{\rho})\eta$$

which gives a single linear differential equation for the ratio η. This dynamic is globally asymptotically stable around its steady state position ($l_1^{wo} = y^p/z$):

$$\eta_o = \frac{(1 - c_{h1}(1-\tau_h) - \alpha_g\tau_h)\bar{\omega}_1 y^p/z - ((1+\alpha_r)\bar{l} - (1+\alpha_f)y^p/z)\alpha_\omega\bar{\omega}_1}{\delta_1 + \bar{\rho}}$$

In this simple case we thus have monotonic adjustment of the pension-fund capital ratio to its steady state position, while in general we have a non-autonomous adjustment of this ratio that is driven by the real wage and employment dynamics of the first labor market. the steady state level of η is positive iff there holds for the full employment labor intensity ratio:

$$\bar{l} < \frac{(1 - c_{h1}(1-\tau_h) - \alpha_g\tau_h)\bar{\omega}_1 y^p/z + ((1+\alpha_f)y^p/z)\alpha_\omega\bar{\omega}_1}{(\delta_1 + \bar{\rho})(1+\alpha_r)\alpha_\omega\bar{\omega}_1}$$

We now assume moreover that the additional company pension payments to pensioners should add the percentage $100\alpha_c$ to their base pension $\omega_2\alpha_r\bar{l}$ per unit of capital. We thus have as further restriction on the steady state position of the economy, if there is an α_c target given, namely:

$$\delta_1\eta_o = \alpha_c\omega_2^o\alpha_r\bar{l}, \quad \omega_2^o = \alpha_\omega\bar{\omega}_1$$

Inserting the value for η_o then gives

$$\alpha_c = \delta_1\frac{(1 - c_{h1}(1-\tau_h) - \alpha_g\tau_h)\bar{\omega}_1 y^p/z - ((1+\alpha_r)\bar{l} - (1+\alpha_f)y^p/z)\alpha_\omega\bar{\omega}_1}{(\delta_1 + \bar{\rho})\omega_2^o\alpha_r\bar{l}}$$

We thus get that a target value for α_c demands a certain labor intensity ratio $\bar{l}$ and vice versa. For a given total labor intensity ratio there is a given percentage by which company pensions compare to base pension payments. This percentage is the larger the smaller the ratio $l_1^{wo}/\bar{l}$ due to the following reformulation of the α_c formula:

$$\alpha_c = \delta_1\frac{[(1 - c_{h1}(1-\tau_h) - \alpha_g\tau_h)\bar{\omega}_1 + (1+\alpha_f)\alpha_\omega\bar{\omega}_1]l_1^{wo}/\bar{l} - (1+\alpha_r)\alpha_\omega\bar{\omega}_1}{(\delta_1 + \bar{\rho})\alpha_r\alpha_\omega\bar{\omega}_1} \tag{33}$$

If this value of the total employment labor intensity ratio prevails in the considered economy (where it is of course as usually assumed that $c_{h1}(1-\tau_h)+\alpha_g\tau_h < 1$ holds) we have that core pension payments to pensioners are augmented by company pension payments by a percentage that is given by the parameter α_c and that these extra pension payments are distributed to pensioners in proportion to their contribution to the time that they have worked in the private sector of the economy. There is thus a negative trade-off between the ratios $\bar{l}, \alpha_c$, as expressed by the relationship (33). It also shows that the total working population must have a certain ratio to the capital stock in order to allow for a given percentage of extra company pension payments.: Due to $\delta_1\eta_o = \alpha_c\omega_2^o\alpha_r\bar{l}$ and $s_1^o = (\delta_1+\bar{\rho})\eta_o$ we also have the equivalence between positive savings per unit of capital of households of type I and positive values for α_c, η_o. Moreover, these values are in fact positive if there holds:[11]

$$\bar{l} \;<\; \frac{(1-c_{h1}(1-\tau_h)-\alpha_g\tau_h)\bar{\omega}_1 y^p/z+(1+\alpha_f)y^p/z)\alpha_\omega\bar{\omega}_1}{(\delta_1+\bar{\rho})(1+\alpha_r)\alpha_\omega\bar{\omega}_1}$$

This inequality set limits to the total labor-supply capital-stock ratio $\bar{l}$ which allows for positive savings of households of type I in the steady state and thus for extra pension payments to them later on. Households of type I are by and large financing the second labor market through taxes and employment benefits (besides their contribution to the base income of the retired people). Since firms have a positive rate of profit in the steady state, since the government budget is always balanced and since only households of type I save in this economy, we have thus now established the condition under which such an economy accumulates not only capital, but also pensions funds – under appropriate restrictions on labor supply – to a sufficient degree.

6 Pension funds and credit

In this section we will investigate the implications of the situation where pension funds are used for real capital formation instead of remaining idle except of being used for company pension payments (of amount $\delta_1 R$ at each point in time). The productive use of part of the pension fund R is here assumed to be rewarded at the constant interest rate r applied to the debt level D accumulated by the firms in the private sector of the economy.

6.1 Accounting relationships

Pension funds as quasi commercial banks who give credit to firms out of their funds and thus allow firms to invest in good times much beyond their retained earnings, i.e., profits net of interest payments on loans.

[11] Note that the numerator is easily shown to be not only positive, but even larger than 1 under standard Keynesian assumptions on expenditure and taxation rates.

Firms

Production and Income Account:

Uses	Resources
δK	δK
$\omega_1 L_1^d = \omega_1 Y^p/z$	$C_1 + C_2 + C_r$
$\omega_2 L_{2f}^w = \alpha_\omega \omega_1 \alpha_f Y^p/z$	G
rD	
Π	$I = (i_\rho(\rho - \rho_o) - i_d(d - d_o) + \bar{a})K$
Y^p	Y^p

We denote by ρ^n the profit rate that would prevail at normal rates of capacity utilization, while the capacity utilization effect on investment is captured by the first term in the investment function. Animal spirits are here represented by the exogenously given term $\bar{a}$ which may be subject to sudden shifts, representing golden periods or leaden ages. respectively. We assume that enough credit is generally available to support sudden upward shifts in trend investment. The financing of gross investment is shown in the next account.

Investment and Credit:

Uses	Resources
δK	δK
$I = (i_\rho(\rho - \rho_o) - i_d(d - d_o) + \bar{a})K$	Π
	$\dot{D} = I - \Pi$
I^g	I^g

We assume as investment behavior of firms the functional relationship:

$$I/K = i_\rho(\rho - \rho_o) - i_d(d - d_o) + \bar{a}.$$

This investment schedule states that investment plans depend positively on the deviation of the profit rate from its steady state level and negatively on the deviation of the debt to capital ratio from its steady state value. The exogenous trend term in investment is $\bar{a}$ and it is again assumed that it represents the influence of investing firms 'animal spirits' on their investment activities.

Firms Net Worth:

Assets	Liabilities
K	D
	Real Net Worth
K	K

In the management of pension funds we assume that a portion sR of them is held as minimum reserves and that a larger portion of them has been given as credit D to firms. The remaining amount are idle reserves D^s, not yet allocated to any interest bearing activity.

Pension Funds

Pension Funds and Credit (stocks):

Assets	Liabilities
R	sR
	D
	X excess reserves
R	R

Pension funds receive the Savings of households of type 1 (the other households do not save) and they receiver the interest payments of firms. They allocate this into required reserve increases, payments to pensioners, new credit demand of firms and the rest as an addition or substraction to their idle reserves.

Pension Funds and Credit (flows):

Resources	Uses
S_1	$s\dot{R}$
rD	$\delta_1 R + rD$
	$\dot{D} = I - \Pi$
	$\dot{X}$
$S_1 + rD$	$S_1 + rD$

The above representation of the flows of funds in the pension funds system implies for the time derivative of accumulated funds R the relationship

$$\dot{R} = S_1 - \delta_1 R - (I - \Pi) = S_1 + \Pi - \delta_1 R - I, \quad i.e.,$$

it is given by the excess of savings of households of type I over current company pension funds payments to retired households and the new credit that is given to firms to finance the excess of investment over retained profits.

Households I and II (primary and secondary labor market)

Income Account (Households I):

Uses	Resources
$C_1 = c_{h1}(1-\tau_h)Y_1^w$	
$\omega_2 L_{2h}^w = c_{h2}(1-\tau_h)Y_1^w$	
$T = \tau_h Y_1^w$	
$\omega_2(L - (L_1^w + L_{2f}^w + L_{2h}^w + L_{2g}^w))$	
$\omega_2 L^r$	
S_1	$\omega_1 L_1^d$
Y_1^w	Y_1^w

Households in the first labor market consume with a constant marginal propensity out of the income after primary taxes and they employ households services in constant proportions to the consumption habits. They pay the wages of the workers in the second labor market that are not employed by firms, by them and the government as a quasi unemployment benefit insurance (a generational solidarity contribution) and they pay the common base rent of all pensioners (as intergenerational contribution). The remainder represents their contribution the pension scheme of the economy, from which they will receive $\delta_1 R + rD$ when retired. We consider this as a possible scheme of funding the excess employment and the pensioners, not necessarily a just one however.

Income Account Households II

Uses	Resources
C_2	$\omega_2 L_2^w$
Y_2^w	Y_2^w

Income Account (Retired Households):

Uses	Resources
C_r	$\omega_2 L^r + \delta_1 R + rD$
Y^r	Y^r

<u>The Government</u>

Income Account – Fiscal Authority / Employer of First Resort:

Uses	Resources
$G = \alpha_g \tau_h Y_1^w$	$T = \tau_h Y_1^w$
$\omega_2 L_{2g}^w = (1-\alpha_g)\tau_h Y_1^w$	
$\omega_2 L_x^w$	$\omega_2(L - (L_1^w + L_{2f}^w + L_{2h}^w + L_{2g}^w))$
$\omega_2 L^r$	$\omega_2 L^r$
Y^g	Y^g

Government gets primary taxes and spends them on goods as well as services in the government sector (which are here determined residually). It administrates the common base rent payments as well as the payments of those not yet employed in the sectors of the economy. Its workforce consists of all workers that are not employed by firms of households of type 1 and also of all pensioners that are still capable to work. The model therefore assumes not only that there is a work guarantee for all, but also a work obligation for all members in the workforce, with the addition of those that are retired but still able and willing to work.

6.2 Investment and credit dynamics in flexicurity growth

For simplicity we here again assume that the government pursues an immigration policy that ensures for the growth rate of the labor force the condition $n = \hat{K}$, *i.e.*, the total labor supply grows by this migration policy with the same rate as the capital stock.

This again keeps the ratio $l = L/K = \bar{l} =$ constant. Since we are again no longer able to determine the steady state value of the real wage ω_1 from the law of motion for l, we have to supply it again from the outside: $\omega_1^o = \bar{\omega}_1 =$ given. This however no longer also provides us with the steady state value of the rate of profit, since profits are now to be determined net of interest payments: $\rho = y^p[1-(1+\alpha_\omega\alpha_f)\bar{\omega}_1/z] - \delta_1 - rd$, where $d = D/K$ denotes the indebtedness of firms per unit of capital. We assume again as trend term in Okun's law the growth rate of the capital stock (i.e., this part of the new hiring is just determined by the installation of new machines or whole plants (under the assumption of fixed proportions in production). The normal level of the rate of employment of the workforce employed by firms is again set equal to '1' for simplicity.
On the basis of these assumptions we get from what was formulated in the preceding subsection (where investment was assumed to be given now by $I/K = i_\rho(\rho-\rho_o) - i_d(d-d_o) + \bar{a}$):

$$\begin{aligned}
\hat{l}_1^w &= \beta_{eu}(\frac{y^p}{zl_1^w} - 1) \\
\hat{\omega}_1 &= \kappa[(1-\kappa_p)(\beta_{wu}(\frac{y^p}{zl_1^w} - 1) - \beta_{w\omega}\ln(\frac{\omega_1}{\bar{\omega}_1})) - (1-\kappa_w)\beta_{p\omega}\ln(\frac{\omega_1}{\bar{\omega}_1})] \\
\dot{d} &= [i_\rho(\rho-\rho_o) - i_d(d-d_o) + \bar{a}](1-d) - \rho \\
\hat{\eta} &= s_1 + \rho - (\delta_1\eta + (1+\eta)[i_\rho(\rho-\rho_o) - i_d(d-d_o) + \bar{a}]) \\
&= (1 - c_{h1}(1-\tau_h) - \alpha_g\tau_h)\omega_1 y^p/z - ((1+\alpha_r)\bar{l} - (l_1^w + \alpha_f y^p/z))\alpha_\omega\omega_1 \\
&+ [y^p[1-(1+\alpha_\omega\alpha_f)\bar{\omega}_1/z] - \delta_1 - rd] - (\delta_1\eta + (1+\eta)[i_\rho(\rho-\rho_o) - i_d(d-d_o) + \bar{a}])
\end{aligned}$$

The introduction of debt financing of firms thus makes the model considerably more advanced in its economic structure, but not so much from the mathematical point of view, due to the recursive structure that characterizes the dynamical system at this level of generality. We note that there is not yet an interest rate policy rule involved in these dynamics, but the assumption of an interest rate peg: $r = const.$
We make use in the following of the following abbreviations:

$$s_1^o = (1 - c_{h1}(1-\tau_h) - \alpha_g\tau_h)\bar{\omega}_1 y^p/z - ((1+\alpha_r)\bar{l} - y^p/z(1++\alpha_f))\alpha_\omega\bar{\omega}_1$$

and

$$\rho_{max} = y^p[1-(1+\alpha_\omega\alpha_f)\bar{\omega}_1/z] - \delta_1.$$

On the basis of such steady state expressions we then have:

Proposition 2:

The interior steady state of the considered dynamics is given by:[12]

$$l_1^{wo} = \frac{y^p}{z},\ \omega_1^o = \bar{\omega}_1,\ \eta_o = \frac{s_1^o + \rho_o - \bar{a}}{\delta_1 + \bar{a}},$$

where d_o, ρ_o have to be determined by solving the two equations

$$\rho_o = \rho_{max} - rd_o, \quad \rho_o = \bar{a}(1-d_o)$$

[12]The steady state value of s_1^o is the same as in the preceding section.

which gives for the steady state values of d, ρ, η the expressions:

$$d_o = \frac{\bar{a} - \rho_{max}}{\bar{a} - r}, \quad \rho_o = \bar{a}\frac{\rho_{max} - r}{\bar{a} - r}, \quad \eta_o = \frac{s_1^o + \bar{a}\frac{\rho_{max}-r}{\bar{a}-r}}{\delta_1 + \bar{a}} = \frac{s_1^o(\bar{a} - r) - \bar{a}(\bar{a} - \rho_{max})}{(\delta_1 + \bar{a})(\bar{a} - r)}.$$

We assume that both the numerator and the denominator of the fraction that defines d_o are positive, i.e., the trend term in investment is sufficiently strong (larger than the rate of profit before interest rate payments ρ_{max} and larger than the rate of interest r). Moreover, it is also assumed that $\rho_{max} > r$ holds so that all fractions shown above are in fact positive. In the case where $\bar{a} = \rho_{max} = y^p[1 - (1 + \alpha_\omega\alpha_f)\bar{\omega}_1/z] - \delta_1$ holds we have $d_o = 0$ and $\rho_o = \bar{a}$ in which case the value of η_o is the same as in the sections on investment without debt financing. Nevertheless the dynamics around the steady state remain debt financed and are therefore different from the one of the preceding section. We thus can have a 'balanced budget' of firms in the steady state while investment remains driven by $I/K = i_\rho(\rho - \rho_o) - i_d(d - d_o) + \bar{a}$ outside the steady state position.
For the fraction of company pension funds divided by base pension payments we now get as relationship in the steady state

$$\alpha_c = \frac{\delta_1\eta_o + r d_o}{\alpha_\omega\alpha_r\bar{\omega}_1\bar{l}}$$

an expression that in general does not give rise to unambiguous results concerning comparative dynamics. In the special case $d_o = 0$ we however can state that this fraction depends positively on s_o^1 (also in general) and negatively on $\bar{a}, \delta_1, \bar{l}$.
The Jacobian at the interior steady state of the here considered 4D dynamics reads

$$J^o = \begin{pmatrix} -\beta_{eu}/l_1^{wo} & 0 & 0 & 0 \\ ? & -\kappa[(1-\kappa_p)\beta_{w\omega} + (1-\kappa_w)\beta_{p\omega}] & 0 & 0 \\ ? & ? & -(i_\rho + i_d)(1 - d_o)) - (\bar{a} - r) & 0 \\ ? & ? & ? & -\bar{a}(1 + \delta_1) \end{pmatrix}$$

This lower triangular form of the Jacobian immediately implies that the elements on the diagonal of the matrix J^o are just equal to the 4 eigenvalues of this matrix which are therefore all real and negative. This gives:

Proposition 3:

> *The interior steady state of the considered dynamics is locally asymptotically stable and is characterized by a strict hierarchy in the state variables of the dynamics.*

Due to the specific form of the considered laws of motion we conjecture that the steady state is also a global attractor in the economically relevant part of the 4D phase space. We then would get again monotonically convergent trajectories from any starting point of this part of the phase space and thus fairly simple adjustment processes also in the case where investment is jointly financed by profits (retained earnings) and credit.
The stability of the steady state is increased (i.e., the eigenvalues of its Jacobian matrix become more negative) if the speed parameter characterizing hiring and firing is

increased, if Blanchard / Katz type error correction becomes more pronounced and if the parameters $i_\rho, i_d, \bar{a}$ in the investment function are increased.
Summing up, we thus can state that the adjustment processes and their stability properties remain very supportive for the working of our model of flexicurity type which is generally monotonically convergent with full capacity utilization of both capital and labor to a steady state position with a sustainable distribution of income between firms, our three types of households and the government. We conclude that flexicurity capitalism may be a workable alternative to current forms of capitalism and can avoid in particular the severe social deformations and the human degradation caused by the reserve army mechanism and the mass unemployment it implies for certain stages in a long-phase distributive and welfare state cycle, in the US and the UK more of a neoclassical cold turkey type and in Germany and in France more gradualistic in nature.[13]

7 Outlook: Keynesian demand problems

As an outlook we here briefly sketch a situation where capacity utilization problems as well as stability problems may arise within the flexicurity variant of a capitalistic economy. We modify the baseline model of section 2 in a minimal way in order to obtain such results. In place of pension funds as well as the credits they give to firms we now consider the situation where firms finance their investment plans through their profits and through the issuing of corporate bonds. We assume these bonds to be of the fixprice variety and we also keep the rate of interest that is paid on these bonds fixed for simplicity.
The amount of such bonds that firms have issued in the past is denoted by B and their price is 1 in nominal units. Firms thus have to pay rB as interest at the current point in time and they intend to use their real profits net of interest rate payments and in addition the issue $\dot{B}^s/p$ to finance their rate of investment $I/K = i_\rho(\rho - \rho_o) - i_b(\frac{B}{pK} - (\frac{B}{pK})_o) + \bar{a}$. This rate of investment is assumed to depend positively on excess profitability compared to the steady state rate of profit and negatively the deviation of their debt from its steady state level.

<u>Firms</u>

Production and Income Account:

Uses	Resources
δK	δK
$\omega_1 L_1^d, L_1^d = Y/z$	$C_1 + C_2 + C_r$
$\omega_2 L_{2f}^w, \omega_2 = \alpha_\omega \omega_1, L_{2f}^w = \alpha_f L_1^d$	G
rB/p	$I = i_\rho(\rho - \rho_o)K - i_b(\frac{B}{p} - (\frac{B}{p})_o) + \bar{a}K$
$\Pi (= Y^f)$	$[I = \Pi + \dot{B}^s/p]$
Y	Y

[13]We refer the reader back to what is shown in figure 3 where the postwar period up into the 1960's seemed to suggest that the working of the reserve army mechanism had been overcome, a suggestion that was disproved in the subsequent years in a striking way.

Households of type I behave as was assumed so far, but attempt to channel their real savings now into corporate bond holdings as shown below. They will be able to exactly satisfy their demand for new bonds when there is goods market equilibrium prevailing ($I = S$), since only firms and these households act on this market, while all other economic units just spend what they get (with balanced transfer payments organized by the government). The real return from savings in corporate bonds rB/p, at each moment in time, will be added below to the base rent payments of retired households, who receive these benefits in proportion to the bonds they have allocated during their worklife in the private sector of the economy. The bonds allocated in this way thus only generate a return when their holders are retired and then – as in the pension fund scheme of section 2 – at the then prevailing market rate of interest (which is here a given rate still). The pension fund model is therefore here only reformulated in terms of nominal paper holdings (coupons) and thus no longer based on the storage of physical magnitudes. Hence, corporate bonds are here not only of a fix-price variety, but also provide their return only after retirement. This is shown in the income account of retired persons below. The income account of the workers in the second labor market is unchanged and therefore not shown here again.

Households I (primary labor market) and Retired Households

Income Account (Households I):

Uses	Resources
$C_1 = c_{h1}(1-\tau_h)\omega_1 L_1^d$	
$\omega_2 L_{2h}^w = c_{h2}(1-\tau_h)\omega_1 L_1^d$	
$T = \tau_h \omega_1 L_1^d$	
$\omega_2(L - (L_1^w + L_{2f}^w + L_{2h}^w + L_{2g}^w))$	
$\omega_2 L^r, L^r = \alpha_r L$	
$S_1[= \dot{B}^d/p]$	$\omega_1 L_1^d$
$Y_1^w = \omega_1 L_1^d$	$Y_1^w = \omega_1 L_1^d$

Income Account (Retired Households):

Uses	Resources
C_r	$\omega_2 L^r + rB/p, L^r = \alpha_r L$
Y^r	Y^r

The government income account (not shown) is also kept unchanged and in particular balanced in the way used in the preceding model types. The modifications of the model of section 2 are therefore of a minimal kind, largely concerning a different type of investment behavior of firms and a new type of organizing the formerly assumed company pension funds. However, the assumed flexicurity system becomes now of real importance, since we here will get demand determined (Keynesian) business cycle fluctuations in the dynamics implied by the model, whereas firms did not face capacity under- or over-utilization problems in the earlier model types. Keynesian IS-equilibrium determination has to be considered now and gives rise to the following equation for the effective output per unit of capital (characterizing goods market equilibrium):[14]

$$
\begin{aligned}
Y/K = y &= C_1/K + C_2/K + C_r/K + \delta + I/K + G/K \\
&= c_h(1-\tau_h)\omega_1 \frac{y}{z} + \alpha_\omega \omega_1(\bar{l} - l_1^w) + \alpha_\omega \alpha_r \omega_1 \bar{l} + rb \\
&\quad + \delta + i_\rho(\rho - \rho_o) - i_b(b - b_o) + \bar{a} + \alpha_g \tau_h \omega_1 y/z \\
\rho &= y - (1 + \alpha_f \alpha_\omega)\omega_1 y/z - \delta - rb, \qquad b = B/(pK) \\
&\quad \text{which taken together gives:} \\
y &= \frac{\alpha_\omega \omega_1(\bar{l} - l_1^w) + \alpha_\omega \alpha_r \omega_1 \bar{l} + (rb + \delta)(1 - i_\rho) - i_\rho \rho_o - i_b(b - b_o) + \bar{a}}{1 - [c_h(1-\tau_h) + \alpha_g \tau_h - i_\rho(1 + \alpha_f \alpha_\omega)]\omega_1/z - i_\rho} \\
&= y(l_1^w, \omega_1, b, \dots)
\end{aligned}
$$

Note that we have modified the investment function in this section to $i(\cdot) = i_\rho(\rho - \rho_o) - i_b(b - b_o) + \bar{a}$. Note also that we have again assumed that natural growth n is always adjusted to the growth rate of the capital stock $\hat{K}$. We also assume that the denominator in the above fraction is positive and now get the important result that output per unit of capital is no longer equal to its potential value, but now depending on the marginal propensity to spend as well as on other parameters of the model. This is due to the new situation that firms use corporate bonds to finance their excess investment (exceeding their profits) or buy back such bonds in the opposite case and that households of type I buy such bonds from their savings (and thus do not buy goods in this amount anymore to increase the pension fund). We thus have independent real investment and real savings decisions which – when coordinated by the achievement of goods market equilibrium as shown above – lead to a supply of new corporate bonds that is exactly equal to the demand for such bonds at this level of output and income. This simply follows from the fact that only firms and households of type I are saving, while all other budgets are balanced. Households of type I thus just have to accept the amount of the fixed price bonds offered by firms and are thereby accumulating these bonds (whose interest rate payments are paid out to retired people according to the percentage they have achieved when retiring).

Assuming the accumulation of corporate bonds in the place of real commodities and an investment function that is independent from these savings conditions thus implies that the economy is subject to Keynesian demand rationing processes (at least close to its steady state). These demand problems are here derived on the assumption of IS-equilibrium and thus represented in static terms in place of a dynamic multiplier approach that can also be augmented further by means of Metzlerian inventory adjustment processes. We stress once again that the possibility for full capacity output is here prevented through the Keynesian type of underconsumption assumed as characterizing the household type I sector and the fact that there is then only one income level that allows savings in bonds to become equal to bond financed investment in this simple credit market that is characterizing this modification of the flexicurity model, due to the now existing effective demand schedule $y(l_1^w, \omega_1, b, \dots)$. We assume that the parameters are chosen such that we get for the partial derivatives of the effective demand function

[14]Standard Keynesian assumptions will again ensure that $y^o > 0$ holds true.

y :

$$y_{l_1^w}(l_1^w,\omega_1,b,\dots)<0,\quad y_{\omega_1}(l_1^w,\omega_1,b,\dots)>0,\quad y_b(l_1^w,\omega_1,b,\dots)<0$$

holds true. This is fulfilled for example if the expression in the denominator of the effective demand function is negative and if the parameter i_b is chosen sufficiently large. Effective demand is then wage led and flexible wages therefore dangerous for the considered economy.
As now significantly interacting laws of motion we have in the consider case:

$$\begin{aligned}
\hat{l}_1^w &= \beta_{eu}(\frac{y}{zl_1^w}-1)\\
\hat{\omega}_1 &= \kappa[(1-\kappa_p)(\beta_{wu}(\frac{y}{zl_1^w}-1)-\beta_{w\omega}\ln(\frac{\omega_1}{\omega_1^o}))-(1-\kappa_w)(\beta_{pu}(\frac{y}{y^p}-\bar{u}_c)+\beta_{p\omega}\ln(\frac{\omega_1}{\omega_1^o}))]\\
\dot{b} &= (1-b)(i_\rho(\rho-\rho_o)-i_b(b-b_o)+\bar{a})-\rho-\hat{p}b\\
&\quad \hat{p}=\kappa[\beta_{py}(\frac{y}{y^p}-\bar{u}_c)+\beta_{p\omega}\ln(\frac{\omega_1}{\omega_1^o}+\kappa_p(\beta_{wu}(\frac{y}{zl_1^w}-1)-\beta_{w\omega}\ln(\frac{\omega_1}{\omega_1^o})]+\pi^c\\
&\quad \dot{\pi}^c=\beta_{\pi^c p}(0-\pi^c)
\end{aligned}$$

where $\hat{p}$ has to be inserted into the other equation (where necessary) in order to arrive at an autonomous system of 4 ordinary differential equations. This particular formulation of the debt financing of firms thus makes the model considerably more advanced from the mathematical as well as from an economic point of view. We note that there is not yet an interest rate policy rule involved in these dynamics, but that the assumption of an interest rate peg is maintained still: $r = const.$
The laws of motion describe again (in this order) our formulation of Okun's law, the real wage dynamics as it applies in a Keynesian environment (see section 3), the debt dynamics of firms and a simple regressive expectations scheme concerning the inflationary climate surrounding the wage-price spiral where it is assumed (and in fact also taking place) that inflation converges back to a constant price level. There is therefore not yet an inflation accelerator present in the formulation of the dynamics of the four state variables of the model. Nevertheless, price level inflation is now explicitly taken account of, indeed for the first time in this paper.
Steady state and stability analysis is no longer straightforward in this Keynesian variant of flexicurity capitalism. With respect to steady state positions we have to solve now a simultaneous equation system in the variables ω_1, ρ, b. Due to the structure of the effective demand function we have moreover no longer zero entries in the Jacobian of the dynamics at the steady state of the first three state variables (the last law of motion is a completely trivial one). As economic mechanism we can identify a real wage channel as in the Kaleckian dynamics (working here in a wage led environment by assumption). There is furthermore the dynamic of the debt to capital ratio of firms. these feedback channels can be tamed through appropriate assumptions, but are even then working in an environment that gives no straightforward economically plausible stability assertions, due to the strong interactions present in the dynamics. We therefore have to leave the stability analysis for future research here.
The conclusion of this section therefore is that effective demand problems can make flexicurity capitalism significantly more difficult to analyze and therefore demand a treatment of much more depth – including inflation and interest rate policy rules, government

deficits and fiscal policy rules, etc. – than was possible in this concluding section. Moreover, credit relationships may be looked for that avoid the increase in complexity of the dynamics of this concluding section.

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