Determinacy and Stability Analysis in New Keynesian- and Keynesian (Disequilibrium) Macrodynamics

Peter Flaschel* Christian R. Proaño†

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*Bielefeld University, Bielefeld, Germany. E-mail: pflaschel@wiwi.uni-bielefeld.de

†Macroeconomic Policy Institute (IMK) at the Hans-Böckler Foundation, Düsseldorf, Germany. E-mail: christian.proano@web.de.

1 Introduction

During the last decade Dynamic Stochastic General Equilibrium (DSGE) models along the lines of Erceg, Henderson and Levin (2000), Smets and Wouters (2003) and Christiano, Eichenbaum and Evans (2005) have become the workhorse framework for the study of monetary policy and inflation in the academic literature. However, despite of its popularity, this approach – where the dynamics of the economy is derived from neoclassical microfoundations, the assumption of rational expectations and the condition of general equilibrium – has been starkly questioned from both the theoretical and empirical point of view by numerous researchers like Mankiw (2001), Estrella and Fuhrer (2002) and Solow (2004), among others, primarily due to its highly unrealistic assumptions concerning the alleged "rationality" in the forward-looking behavior of the economic agents, and its failure to explain important empirical stylized facts.

Indeed, besides the "dynamic inconsistencies" concerning, among other things, the interactions between key macroeconomic variables such as the inflation rate and the output gap resulting from the assumed purely forward-looking and "rational" – in the sense of Muth's (1961) mathematical sense – behavior of the economic agents, see e.g. Estrella and Fuhrer (2002), as well as Rudd and Whelan (2005), one of the major shortcomings related with the rational expectations assumption is its lack of economic content, which reduces the determination of the model's solution (and the determinacy conditions of the system), see e.g. Blanchard and Kahn (1980) and Sims (2001), to not much more than a purely mathematical exercise.

We show this by reconsidering the baseline New Keynesian model with staggered wages and prices introduced by Erceg, Henderson and Levin (2000) and discussed in Woodford (2003, ch.4) and Galí (2008, ch.6) concerning the determinacy conditions of this model. As we will show, the role of important feedback channels such as the real wage channel – investigated in Chiarella and Flaschel (2000) and later work – in the shaping of the cyclical adjustment processes and their inflationary consequences is almost inexistent in the New Keynesian framework, since there determinacy is achieved by the specification of a Taylor interest rate rule with parameters values which imply a certain combination of unstable/stable roots for the Jacobian matrix of the dynamics.

By contrast, a closely related reformulation of the 4D New Keynesian baseline model in terms of a wage-price spiral with only model consistent – but not *rational* – expectations enables a thorough theoretical analysis of this and other feedback channels and the related stability issues possible in a world without rational expectations – in the sense of Muth's (1961) theory –. As discussed in section 3, such a framework can be proven to be globally asymptotically stable for conventional types of interest rate policy rules and much more attractive in its deterministic properties than the purely forward-looking 4D baseline New Keynesian approach with its fairly trivial deterministic core (in the case of determinacy), since it integrates different possible scenarios concerning real interest rate effects, real wage effects and a nominal interest rate policy rules.

In this alternative model, we use from the beginning continuous-time as the modeling framework, since that allows for a straightforward stability analysis even in high order dynamical systems (which nevertheless can be simulated adequately with a step length of 1/365). Within this modeling approach, also built on the assumptions of gradually adjusting wages and prices, we can of course consider limit cases where wages, prices or expectations adjust with infinite speed, but these are more a matter of theoretical curiosity than of fundamental importance. As we will show, while the determination of the local stability properties of the (D)AS-AD model is by far no less mathematically demanding than the determinacy analysis of the New Keynesian 4D model, the structure of the former allows us to investigate a large variety of aspects – such as the role of different macroeconomic channels for the dynamic stability of an economy – not analyzed in the New Keynesian framework.

The remainder of this paper is organized as follows. Based on the intuition made by Foley (1975), Sims (1998) and more recently by Flaschel and Proaño (2009) which suggests that period models should feature qualitatively similar dynamics (and thus stability properties) as their continuous time analogues, in section 2 we reformulate the deterministic structure of the discrete-time New Keynesian model with staggered wages and prices in a continuous-time representation and show with it a way how determinacy analysis of this model type can be undertaken, confirming Galí's (2008,ch.6) numerical results in an analytical manner. In contrast, in section 3, we discuss an alternative macroeconomic framework based on gradual adjustments of wages and prices to disequilibrium situations in the real markets, and show how the analysis of the stability properties of different macroeconomic channels can be performed in that framework. In section 4 we compare both approaches and draw some concluding remarks from this study.

2 New Keynesian (Equilibrium) Macrodynamics

As it was previously pointed out, the representation of the dynamics of the economy in New Keynesian DSGE models is derived from first principles (which result from neoclassical microfoundations which imply a rational, forward-looking maximizing behavior by firms and households) and the condition of general equilibrium holding at every moment in time.[1]

In the following we focus on the New Keynesian model with staggered wages and prices developed by Erceg et al. (2000), since it represents in our view – due to the staggered nature of the wage and price setting – the baseline situation to be considered as the natural starting point of a Keynesian version of the New Neoclassical Synthesis (as our own matured approach to be discussed below), as rather than one of its two limit cases (staggered price setting with full wage flexibility or viceversa) – with which it may nevertheless be compared.

2.1 The New Keynesian Model with staggered wages and prices

We begin directly from Galí's (2008, ch.6) presentation of the loglinearly reduced form of the New Keynesian model with staggered wages and prices in order to discuss on this basis analytically the determinacy properties of this model type. The loglinear representation of this New Keynesian model employed in Galí (2008, ch.6) reads:

$$\pi_t^w \overset{\text{WPC}}{=} \beta(h)\pi_{t+h}^w + h\kappa_w\tilde{y}_t - h\lambda_w\tilde{\omega}_t, \quad \pi_t^w = (w_t - w_{t-h})/h \tag{1}$$

$$\pi_t^p \overset{\text{PPC}}{=} \beta(h)\pi_{t+h}^p + h\kappa_p\tilde{y}_t + h\lambda_p\tilde{\omega}_t, \quad \pi_t^p = (p_t - p_{t-h})/h \tag{2}$$

$$\tilde{y}_t \overset{\text{IS}}{=} \tilde{y}_{t+h} - h\sigma^{-1}(i_t - \pi_{t+h}^p - r^n) \tag{3}$$

$$i_t \overset{\text{TR}}{=} r^n + \phi_p\pi_t^p + \phi_w\pi_t^w + \phi_y\tilde{y}_t \tag{4}$$

with

$$\tilde{\omega}_t \equiv \tilde{\omega}_{t-h} + h(\pi_t^w - \pi_t^p) - \Delta\omega_{t+h}^n$$

as the identity relating the changes in the real wage gap $\tilde{\omega}_t = \omega_t - \omega_t^n$ (ω_t^n being the natural real wage) to wage inflation, price inflation, and the change in the natural real wage $\Delta\omega_t^n$. Note here also that $\beta(h) := \frac{1}{1+h\rho}$ is the discount factor that applies to the period length h, and that there holds on this basis $\frac{1-\beta(h)}{\beta(h)} = h\rho$, or; $1/\beta(h) = 1 + h\rho$, when solved for the discount rate ρ of the New Keynesian model, which will be of importance below.

Eq.(1) describes a New Keynesian Wage Phillips Curve (WPC), and eq.(2), analogously, describes a New Keynesian Price Phillips Curve (PPC), all parameters being positive, see Galí (2008) for their derivation. We assume as in Galí (2008, p.128) that the conditions stated there for the existence of a zero steady state solution are fulfilled, namely that a) $\Delta\omega_t^n = 0$ for all t and that b) the intercept in the nominal interest rate rule adjusts always in a one-to-one fashion to variations in the natural rate of interest. The

[1] See also Walsh (2003) for a textbook introduction to the New Keynesian model with staggered wages and prices.

dynamic IS equation (derived by combining the goods markets clearing condition $y_t = c_t$ with the Euler equation of the households) is given by eq.(3), with $\tilde{y}_t \equiv y_t - y_t^n$ as the output gap (y_t^n being the equilibrium level of output attainable in the absence of both wage and price rigidities) and r^n as the natural rate of interest. Finally, eq.(4) describes a generalized type of contemporaneous Taylor interest rate policy rule (TR), whereafter the nominal interest rate is assumed to be a function of the natural rate of interest, of the wage inflation, the price inflation as well as of the output gap, see Galí (2008, 6.2) for details.

Note that we have in this formulation of the model three forward looking variables and one equation that is updating the historically given real wage. For the model to be determinate we thus need the existence of three unstable eigenvalues (three variables that can jump to the 1D stable submanifold) and one eigenvalue that is negative (corresponding to the stable submanifold). In contrast to Galí (2008, fn.6) we use annualized rates, obtained by dividing the corresponding period differences through the period length h (usually 1/4 year in the literature). We show herewith which parameters change with the data frequency or just the iteration step-size h when the model is simulated. We thus use the conventional scaling for the rates here under consideration, but allow for changes in the data collection frequency or iteration frequency.[2] We consequently consider the equations (1) – (4) from an applied perspective, i.e., we take them as starting point for an empirically motivated study of the influence of the data frequency (quarterly, monthly or weekly) on the size of the parameter values to be estimated.

2.2 Determinacy analysis

In principle period analysis and continuous-time modeling should provide qualitatively the same results, which means that the model should not depend in its fundamental qualitative properties on the length of the period h, in particular when frequencies of empirical relevance are considered. In this respect, Foley (1975, p.310) proposes as a methodological precept concerning macroeconomic period models that *No substantive prediction or explanation in a well-defined macroeconomic period model should depend on the real time length of the period.*[3] We therefore expect that it reflects the properties of its continuous-time analogue, abbreviated by $\dot{x} = J_o x$.

[2]For two analyzes of the consequences of such a discrepancy for the resulting dynamics of macroeconomic models see Aadland and Huang (2004) as well as Flaschel and Proaño (2009).

[3]Furthermore, from the view point of economic modeling, Sims (1998, p.318) analyzes a variety of models featuring real and nominal stickiness "formulated in continuous time to avoid the need to use the uninterpretable 'one period' delays that plague the discrete time models in this literature."

In the linear case this can be motivated further by the following type of argument: Consider the mathematically equivalent discrete and continuous-time models (I denoting the identity matrix):

$$x_{t+1} = Ax_t \quad \text{and} \quad \dot{x} = (A - I)x = Jx$$

which follow the literature by assuming an unspecified time unit 1.

Our above arguments suggest that we should generalize such an approach and rewrite it with a variable period length as follows:

$$x_{t+h} - x_t = hJx_t \quad \text{and} \quad \dot{x} = Jx.$$

This gives for their system matrices the relationships

$$A = hJ + I.$$

According to Foley's postulate both J and A should be stable matrices if period- as well as continuous-time analysis is used for macroeconomic analysis in such a linear framework, i.e., all eigenvalues of J should have negative real parts, while the eigenvalues of A should all be within the unit circle. Graphically this implies the situation shown in figure 1 (which shows that, if J's eigenvalues do not yet lie inside the unit circle shown, that they have to be moved into it by a proper choice of the time unit and thus the matrix hJ.)

If the eigenvalues of the matrix J of the continuous time case are such that they lie outside the solid circle shown, but for example within a circle of radius 2, the discrete time matrix $J + I$ would – in contrast to the continuous time case – have unstable roots (on the basis of a period length $h = 1$ that generally is left implicit in such approaches). The system $x_{t+1} = Ax_t, A = J+I$ then has eigenvalues outside the unit circle (which is obtained by shifting the shown solid unit circle by 1 to the right (into the dotted one). Choosing $h = 1/2$ would however then already be sufficient to move all eigenvalues $\lambda(A) = h\lambda(J)+1$ of $A = hJ + I$ into this unit circle, since all eigenvalues of hJ are moved by this change in period length into the solid unit circle shown in Figure 1, since J's eigenvalues have all been assumed to have negative real parts and are thus moved towards the origin of the space of complex numbers when the period length h is reduced.

We also note here already (in view of the New Keynesian approach to be considered next) that matrices J with eigenvalues with only positive real parts will always give rise to totally unstable matrices $A = hJ + I$, since the real parts are augmented by "1" in such a situation. We will however show in the next section that the here considered simple h-dependence of the eigenvalues of the matrix $A: \ \lambda(A) = h\lambda(J) + 1$, – in this linear setup – does not apply to baseline New Keynesian models, since they – though linear –

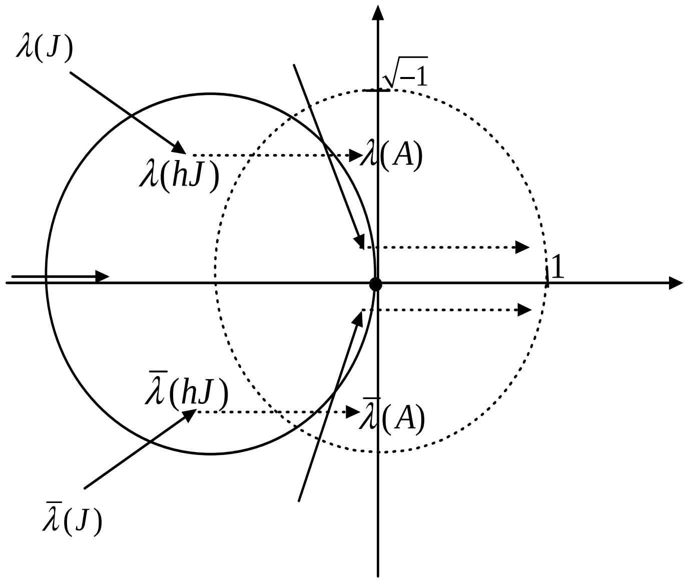

Figure 1: A choice of the period length that guarantees equivalence of continuous and discrete time analysis

depend nonlinearly on their period length h and are only directly comparable to the above in the special case $h = 1$. Comparisons for larger period lengths h are therefore not so easy and demand other means in order to compare determinacy in both continuous- and discrete-time.

The New Keynesian model reformulated in this way represents an implicitly formulated system of difference equations, where all variables with index $t+h$ are expected variables or should be interpreted as representing perfect foresight in the deterministic skeleton of the considered dynamics. Making use again of the TR and the PPC, see equations (1) – (4) and using the above representation of $\tilde{\omega}_t$, it can be made an explicit system of difference equations as follows (with $\eta = \sigma^{-1}$):

$$\pi^w_{t+h} = \frac{\pi^w_t - h\kappa_w\tilde{y}_t + h\lambda_w\tilde{\omega}_t}{\beta(h)} = \pi^w_t + h\rho\pi^w_t - h\frac{\kappa_w\tilde{y}_t - \lambda_w\tilde{\omega}_{t-h} - h\lambda_w(\pi^w_t - \pi^p_t)}{\beta(h)} \quad (5)$$

$$\pi^p_{t+h} = \frac{\pi^p_t - h\kappa_p\tilde{y}_t - h\lambda_p\tilde{\omega}_t}{\beta(h)} = \pi^p_t + h\rho\pi^p_t - h\frac{\kappa_p\tilde{y}_t + \lambda_p\tilde{\omega}_{t-h} + h\lambda_p(\pi^w_t - \pi^p_t)}{\beta(h)} \quad (6)$$

$$\tilde{y}_{t+h} = \tilde{y}_t + h\eta\left[\phi_w\pi^w_t + \left(\phi_p - \frac{1}{\beta(h)}\right)\pi^p_t + \phi_y\tilde{y}_t + h\frac{\kappa_p\tilde{y}_t + \lambda_p\tilde{\omega}_{t-h} + h\lambda_p(\pi^w_t - \pi^p_t)}{\beta(h)}\right] \quad (7)$$

$$\tilde{\omega}_t = \tilde{\omega}_{t-h} + h(\pi^w_t - \pi^p_t) \quad (8)$$

which we can represent in brief through the following matrix equation:

$$x_{t+h} = x_t + h(J_o + hJ_1(h))x_t = x_t + hA(h)x_t = (I + hA(h))x_t.$$

where J_o collects the terms that are linear in h and which therefore will characterize the continuous-time limit case.

The reformulation of this 4D New Keynesian model in continuous-time can also be justified on the empirical basis that while the actual data generating process (DGP) at the macrolevel, even in the real markets, is by and large of a daily one (concerning averages over the day), the corresponding data collection process (DCP) on the economy-wide goods and labor markets is (due to technological and suitability issues) often of a much lower frequency, namely on a monthly or quarterly basis. In the majority of theoretical New Keynesian models this issue has not been addressed properly, leaving the underlying length of the "one-period delay" unspecified or assuming that the DGP and the DCP are equivalent, with the DGP being set equal to the DCP. However, this modeling strategy leads to the highly questionable implication that all wage and price changes occur in clustered or completely synchronized fashion at the beginning and the end of each considered period (the beginning of the next one). Though in reality micro price and wage changes may be staggered with considerable period lengths in between (at the firms' level), this surely does not hold at the macrolevel, where due to the aggregation of overlapping staggered wage and price decisions the assumption of a quasi continuous-time like behavior is more realistic for the macroeconomic time series.[4]

The New Keynesian baseline model with both staggered wage and price setting, the "Keynesian" version of the New Neoclassical Synthesis, reads thus in its loglinearly approximated form, see Erceg et al. (2000), Woodford (2003, pp.225ff.) and Galí (2008, ch.6):[5]

$$\dot{\pi}^w = \rho\pi^w - \kappa_w\tilde{y} + \lambda_w\tilde{\omega} \quad (9)$$

$$\dot{\pi}^p = \rho\pi^p - \kappa_p\tilde{y} - \lambda_p\tilde{\omega} \quad (10)$$

$$\dot{\tilde{y}} = \eta\phi_w\pi^w + \eta(\phi_p - 1)\pi^p + \eta\phi_y\tilde{y} \quad (11)$$

$$\dot{\tilde{\omega}} = \pi^w - \pi^p \quad (12)$$

With respect to this model type, it is asserted in Galí (2008, p.128) – and illustrated numerically in his Figure 6.1 – that the New Keynesian model is – in the case $\phi_y = 0$ considered below – determinate (exhibits three unstable and one stable root) for all policy parameters ϕ_p, ϕ_w when the following form of the Taylor principle holds: $\phi_w + \phi_p > 1$.

[4]Consequently, in our view the notion that aggregate wage levels and price levels are adjusting only gradually at each moment in time (since they are macro-variables which do not perform noticeable jumps on a daily time scale, which we consider as the relevant time unit for the macro data *generating* process) should be accepted in modern models of the Keynesian variety (but also older ones).

[5]Note that there holds $1/\beta(h) = 1 + h\rho = 1$ in the limit.

To investigate this assertion one has to consider the eigenvalues of the system matrix J_o of our system of differential equations.[6]

$$J_o = \begin{pmatrix} \rho & 0 & -\kappa_w & \lambda_w \\ 0 & \rho & -\kappa_p & -\lambda_p \\ \eta\phi_w & \eta(\phi_p - 1) & \eta\phi_y & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix}$$

Let us start with the case $\rho = 0$. Let γ_j $(j = 1, 2, 3, 4)$ be the roots of the characteristic polynomial $p(\gamma) = \gamma^4 + a_1\gamma^3 + a_2\gamma^2 + a_3\gamma + a_4$ of the matrix J_o. Then, we have[7]

$$\begin{aligned} a_1 &= -\gamma_1 - \gamma_2 - \gamma_3 - \gamma_4 = -\text{trace } J_o = -\phi_y\eta \leq 0 \\ a_2 &= \gamma_1\gamma_2 + \gamma_1\gamma_3 + \gamma_1\gamma_4 + \gamma_2\gamma_3 + \gamma_2\gamma_4 + \gamma_3\gamma_4 \\ &= \text{ sum of the principal second-order minors of } J_o \\ &= -(\lambda_w + \lambda_p) + (\phi_w\kappa_w + (\phi_p - 1)\kappa_p)\eta \\ a_3 &= -\gamma_1\gamma_2\gamma_3 - \gamma_1\gamma_2\gamma_4 - \gamma_1\gamma_3\gamma_4 - \gamma_2\gamma_3\gamma_4 \\ &= \text{ -(sum of the principal third-order minors of)} J_o \\ &= \phi_y(\lambda_w + \lambda_p)\eta \geq 0 \\ a_4 &= \gamma_1\gamma_2\gamma_3\gamma_4 = \det J_o = (1 - \phi_p - \phi_w)(\kappa_w\lambda_p + \lambda_w\kappa_p)\eta \end{aligned}$$

On the basis of these expressions for the four eigenvalues γ_i of the matrix J_o, we can easily prove the following lemma:

Lemma:

Assume $a_1 < 0, a_3 > 0$ for the coefficients of the characteristic polynomial of the matrix J_o. Then: All eigenvalues $\gamma = a + b\sqrt{-1}$ with $a = 0$ also satisfy $b = 0$.

Proof: Assume that there is a pair of eigenvalues $\gamma_1 = b\sqrt{-1}, \gamma_2 = -b\sqrt{-1}$. We then get for the coefficients a_1, a_3 of the characteristic polynomial of the matrix J_o the expressions:

$$a_1 = -\gamma_3 - \gamma_4, \quad a_3 = -\gamma_1\gamma_2\gamma_3 - \gamma_1\gamma_2\gamma_4 = b^2(-\gamma_3 - \gamma_4)$$

[6] We show in this section that this determinacy condition is in fact sufficient and necessary for the 4D New Keynesian model for all positive values of the parameter ϕ_y in front of the output gap in the case of the continuous time version of the model (and thus also for period lengths h that are chosen sufficiently small), *provided that* $\rho = 0$ holds.

[7] The following eigenvalue representation of the coefficients of a characteristic polynomial $p(\gamma)$ is a direct consequence of the fundamental theorem of algebra on the n complex roots of complex polynomials of degree n, since there holds: $p(\gamma) = \prod_{i=1}^{n}(\gamma - \gamma_i)$.

which contradicts the signs we have assumed to apply to these two coefficients if $b \neq 0$ holds. □

On this basis one can derive the following two propositions:

Proposition 1

Assume that $\rho = 0$ and that $\phi_y > 0$. Then: The characteristic equation $|\lambda I - J_o| = 0$ has 3 roots with positive real parts and 1 negative root if and only if the generalized Taylor principle $\phi_p + \phi_w > 1$ holds true.

Proof: We consider first the case where $\phi_y = 0$ and assume for the time being in addition that $\phi_p + \phi_w = 1$ holds. In this case we have $a_1 = a_3 = a_4 = 0$ and get from this that two roots (γ_1, γ_2) of the matrix J_o must be zero and the other two a) real and of opposite sign or b) purely imaginary. Let us now move away from this special case to a second case and consider $\phi_y > 0$ (assumed however to be sufficiently small). In this case we have $a_1 < 0$, $a_3 > 0$, and $a_4 = 0$. There is then still one zero root (γ_1), but the other zero root must now be positive in case a.) and negative in case b), due to $a_3 = -\gamma_2\gamma_3\gamma_4 > 0$. In the latter case we have in addition that the purely imaginary roots we started from must exhibit a positive real part now, since the trace of J_o would be negative otherwise. The end result is in both cases that there are now two eigenvalues with positive real parts (complex eigenvalues in the case b), and one which is negative.[8]

Assume now moreover that $\phi_p + \phi_w > 1$ holds (sufficiently close to 1). Since $a_4 < 0$ holds in this case we have that the remaining zero eigenvalue must have become positive. The considered case therefore implies for the matrix J_o the existence of 3 unstable roots and 1 stable one, as was illustrated by Galí (2008, figure 6.1), there for the case $\rho > 0$.[9]

In order to show that this result can be extended to arbitrarily large parameter variations (when $\phi_p + \phi_w > 1$ holds) and not only holds for the small variations so far considered, we simply have to note that the assumption $a_4 = \det J_o < 0$ prevents that the real parts of the eigenvalues can change sign, since they also cannot cross the imaginary axis due to what was shown in the lemma.

[8] In the case $a_2 = 0$ we have initially 4 zero eigenvalues, but get here from $\phi_y > 0$ and therefore from $a_1 < 0, a_3 > 0$ the sign distribution $0, -, +, +$ for the real parts of eigenvalues (where the two positive signs may be arising from real or conjugate complex eigenvalues).

[9] In order to show that this result can be extended to arbitrarily large parameter variations (when $\phi_p + \phi_w > 1$ holds) and not only holds for the small variations so far considered, we simply have to note that the assumption $a_4 = \det J_o < 0$ prevents that the real parts of the eigenvalues can change sign, since they also cannot cross the imaginary axis due to what was shown in the lemma.

In the case $\det J_o > 0$, by contrast, we cannot have determinacy since this case only allows for an even number of stable as well as unstable roots. We however can conclude from what was shown above that this case is always characterized by the existence of two unstable and two stable roots. □

The employed proof strategy is summarized in figure 1 by the arrows on its left hand side and the two choices of points A_+, A_- in the (in)determinacy regions of the parameter space.

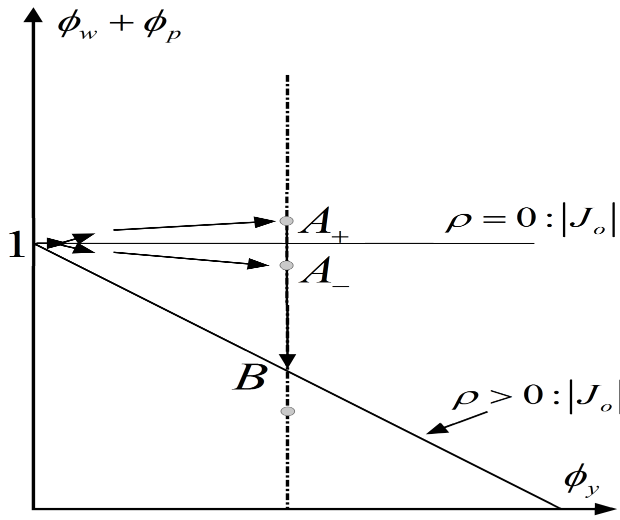

Figure 2: A comparison of $\rho = 0$ with the case $\rho > 0$.

Note that the above proof implies that Galí's (2008) result – shown in his Figure 6.1 – in fact holds for all positive ϕ_y if $\rho = 0$ is assumed. His case $\phi_y = 0$ is however not yet covered by the above proposition 1 and its proof. For the case $\rho > 0$, $\phi_y > 0$ (numerically investigated in Galí's figure 6.2), the reader is referred to Flaschel et al. (2008).

It should be noted, however, that the determinacy analysis undertaken here concerns a loglinear approximation of the true nonlinear model – where rational expectations must be of a global nature – which need not be mirrored through the rational expectations' paths generated by the loglinear approximation. It may therefore well be that the paths that are generated through computer algorithms in the loglinearized version have not much in common with the corresponding ones of the true model.

The above determinacy analysis however opens up the question whether rational expectations models as the New Keynesian model discussed here deliver an adequate representation of the functioning of the economy, and whether such an expectation formation scheme should have such a predominant role in macroeconomics. Indeed, a theory which reduces the complexity of the interaction between economic agents to a purely mathematical exercise – where the convergence of the economy after a shock back towards a uniquely determined steady state is also uniquely determined by purely mathematical arguments – should not be considered as the most adequate representation of a real economy. Additionally, as discussed for example by Fuhrer (2004), models with such a "mathematically rational" and forward-looking behavior need further – ad hoc – assumptions such as habit formation and consumption, investment adjustment costs, "rule-of-thumb" type of behavior in the wage and price setting, etc., and therefore the incorporation of "epicycles" in order to reconcile their theoretical predictions with empirical stylized facts.

There is thus a need for alternative baseline scenarios which can be communicated across scientific approaches, can be investigated in detail with respect to their theoretical properties in their original nonlinear format, and which – when applied to actual economies – remain controllable from the theoretical point of view as far as the basic feedback chains they contain are concerned. As stated by Fuhrer (2004):

In a way, this takes us back to the very old models
— With decent long-run, theory-grounded properties
— But dynamics from a-theoretic sources.

In the following section we provide our alternative to the New Keynesian scenario we have investigated here by means of an extension of the AD-AS model of the Old Neoclassical Synthesis that primarily improves the AS side, the nominal side, of this traditional integrated Keynesian AS-AD approach (and which allows for the impact of wage-price dynamics on the AD side of the model in addition). We call this model type (D)AS-AD where the additional "D" stands for "Disequilibrium". We attempt to show that this matured Keynesian approach can compete with the New Neoclassical Synthesis with respect to an understanding of the basic feedback mechanisms that characterize the working of the macroeconomy, their stability properties and their empirical validity.

3 Keynesian (Disequilibrium)AS-AD Macrodynamics

In this section we discuss a traditionally oriented alternative to the New Keynesian model of the preceding section in the spirit of Chen, Chiarella, Flaschel and Semmler (2006) which, though being based on a quite different philosophy, shares significant similarities with the 4D New Keynesian model previously discussed.[10]

[10] In a plenary lecture at the 'Computing in Economics and Finance' conference in 2007, Volker Wieland compared as two possible approaches simple Traditional Keynesian (KT) models with New Keynesian

In particular it is also our view that in a properly formulated Keynesian model both nominal wage- and price levels should react in a sluggish manner to the state of economic activity. However, we do not found our theoretical formulation on utility/profit maximization under monopolistic competition and Calvo (1983)-like staggered wage and price setting schemes as done in New Keynesian Models, but we assume instead that the gradual adjustment of wages and prices occurs as a reaction to disequilibrium situations in the goods and labor markets, as also done in previous work, see Chiarella and Flaschel (2000), Chiarella, Flaschel and Franke (2005), Chen et al. (2006) and Proaño, Flaschel, Ernst and Semmler (2006). Indeed, since we consider, as already discussed in the previous section, a quasi-continuous time modeling framework as the appropriate one for the study of economic phenomena at the aggregate level – where the assumption of goods and labor markets in equilibrium *at every point in time* is difficult to defend given the assumed sluggishness of wages and prices –, disequilibrium situations in the real markets represent a core feature of our approach, where they, among other things, are the main determinants of wage and price inflation.

Using this alternative framework based on the gradual adjustments of wages and prices to disequilibrium situations in the goods and labor markets, we will be able to study the role of different macroeconomic transmission channels in an economy in a more clear and economics-based fashion – by means of a thorough analysis of the local stability conditions of the steady state of this model – than it was the case in the determinacy analysis of the 4D New Keynesian model discussed in the previous section.

3.1 A Keynesian (D)AS-AD model

Despite our criticism concerning, among other things, the use of the rational expectations assumption in the 4D New Keynesian model of section 2, our alternative framework features many common elements with this model, in particular as far as the formal structure of the Wage- and Price Phillips Curve equations are concerned. Indeed, the output gap and the wage share also enter our wage and price Phillips Curve equations, the latter variable however not being a result of a monopolistic utility/profit maximization of households and firms, respectively, see e.g. Woodford (2003), but rather due to a wage bargaining and price setting situations as they are e.g. discussed in Blanchard and Katz (1999) in their microfoundation of the wage Phillips curve, see also Flaschel and Krolzig (2006) in this regard.

(NK) models. In view of this lecture, the present paper can be considered as an attempt towards the formulation of more advanced models of the TK type.

Concerning the modeling of inflationary expectations and the "rationality" of the agents of our theoretical framework, we assume that the economic agents have a hybrid expectations formation scheme, see Chiarella and Flaschel (1996), based on short-run cross-over and model consistent expectations and the concept of an inflationary climate – within which the short-run is embedded –, which is updated adaptively. We use simultaneous dating and cross-over wage and price expectations in the formulated wage-price spiral, in place of the forward-looking self-reference that characterizes the New Keynesian approach on both the labor and the goods market, and – as stated – in addition hybrid ones that give inertia to our formulation of wage-price dynamics.

Under these modifications, with the inclusion of a conventional IS equation[11] and a standard monetary policy rule, the deterministic part of the model of the preceding section reads (with a neoclassical dating of inflationary expectations now and thus without the need to put an h in front of the terms that drive wage and price inflation):[12]

$$\begin{aligned}
\pi^w_{t+h} &= \tilde{\pi}^p_{t+h} + \beta_{wy} y_t - \beta_{w\omega}\theta_t, \quad \pi^w_{t+h} = (w_{t+h} - w_t)/(w_t h) \\
\pi^p_{t+h} &= \tilde{\pi}^w_{t+h} + \beta_{py} y_t + \beta_{p\omega}\theta_t, \quad \pi^p_{t+h} = (p_{t+h} - p_t)/(p_t h) \\
y_{t+h} &= y_t - h\alpha_{yi}(i_t - \pi^p_{t+h} - i_0) \\
i_t &= i_0 + \beta_{ip}\pi^p_t + \beta_{iy} y_t
\end{aligned}$$

As just discussed, for the impact of price inflation on wage inflation (and v.v) we assume in addition that it is not only of a temporary nature, but subject also to some inertia, here measured by an index for the inflation climate in which the economy is currently operating. It is natural to assume that such a medium-run climate expression π^c is updated in adaptive fashion, i.e. in the simplest approach that it satisfies a law of motion of the following type

$$\pi^c_{t+h} = \pi^c_t + h\beta_{\pi^c}(\pi^p_t - \pi^c_t) \tag{13}$$

We define on this basis the still undefined variables $\tilde{\pi}^p_{t+h}$, $\tilde{\pi}^w_{t+h}$ by the expressions

$$\tilde{\pi}^p_{t+h} = \alpha_p \pi^p_{t+h} + (1-\alpha_p)\pi^c_{t+h}, \quad \tilde{\pi}^w_{t+h} = \alpha_w \pi^w_{t+h} + (1-\alpha_w)\pi^c_{t+h} \tag{14}$$

with α_p,$\alpha_w \in (0,1)$.

[11]For simplicity we abstract from an explicit modeling of the labor market and assume that the employment dynamics can also be represented by the output gap dynamics, see Chen et al. (2006) and Proaño (2007) for alternative modeling approaches of the employment dynamics in the (D)AS-AD framework.

[12]Note that we use as in the New Keynesian models the log of the output level as quantity variable and a zero target rate of inflation of the Central Bank.

In continuous time the system can then be summarized as follows – if $\pi^w$and π^p are used to denote the forward rate of inflation of wages and prices, i.e., the right hand derivatives of $\ln w$ and $\ln p$:

$$\begin{aligned}
\pi^w &= \alpha_w \pi^p + (1-\alpha_w)\pi^c + \beta_{wy} y - \beta_{w\omega}\theta & (15)\\
\pi^p &= \alpha_p \pi^w + (1-\alpha_p)\pi^c + \beta_{py} y + \beta_{p\omega}\theta & (16)\\
\dot{y} &= -\alpha_{yi}\{(\beta_{ip}-1)\pi^p + \beta_{iy} y\} & (17)\\
\dot{\pi}^c &= \beta_{\pi^c}(\pi^p - \pi^c) & (18)\\
\dot{\theta} &= \pi^w - \pi^p & (19)
\end{aligned}$$

where β_{wy}, $\beta_{w\omega}$, β_{py}, $\beta_{p\omega}$, β_{iy}, and α_{yi} are positive parameters and $0 < \alpha_w < 1$, $0 < \alpha_p < 1$, $0 < \beta_{\pi^c} < 1$, $\beta_{ip} > 1$.

We can rewrite equations (15) and (16) as follows:

$$\begin{pmatrix} 1 & -\alpha_w \\ -\alpha_p & 1 \end{pmatrix} \begin{pmatrix} \pi^w \\ \pi^p \end{pmatrix} = \begin{pmatrix} (1-\alpha_w)\pi^c + \beta_{wy} y - \beta_{w\omega}\theta \\ (1-\alpha_p)\pi^c + \beta_{py} y + \beta_{p\omega}\theta \end{pmatrix}$$

Solving this equation, we obtain the following relationships.

$$\begin{aligned}
\pi^w &= \frac{1}{1-\alpha_p\alpha_w} \begin{vmatrix} (1-\alpha_w)\pi^c + \beta_{wy} y - \beta_{w\omega}\theta & -\alpha_w \\ (1-\alpha_p)\pi^c + \beta_{py} y + \beta_{p\omega}\theta & 1 \end{vmatrix} \\
&= \alpha\{(\beta_{wy} + \alpha_w\beta_{py})y + (\alpha_w\beta_{p\omega} - \beta_{w\omega})\theta\} + \pi_c = \pi^w(y,\theta) + \pi^c & (20)\\
\pi^p &= \frac{1}{1-\alpha_p\alpha_w} \begin{vmatrix} 1 & (1-\alpha_w)\pi^c + \beta_{wy} y - \beta_{w\omega}\theta \\ -\alpha_p & (1-\alpha_p)\pi^c + \beta_{py} y + \beta_{p\omega}\theta \end{vmatrix} \\
&= \alpha\{(\beta_{py} + \alpha_p\beta_{wy})y + (\beta_{p\omega} - \alpha_p\beta_{w\omega})\theta\} + \pi^c = \pi^p(y,\theta) + \pi^c & (21)
\end{aligned}$$

where $\alpha = 1/(1-\alpha_p\alpha_w) > 1$. Substituting equations (20) and (21) into equations (17) – (19), we obtain the following three-dimensional linear dynamical system:

$$\begin{aligned}
\dot{y} &= -\alpha_{yi}[(\beta_{ip}-1)(\pi^p(y,\theta) + \pi^c) + \beta_{iy} y] = F_1(y,\pi^c,\theta) & (22)\\
\dot{\pi}^c &= \beta_{\pi^c}\pi^p(y,\theta) = F_2(y,\theta) & (23)\\
\dot{\theta} &= \dot{\theta}(y,\theta) = \pi^w(y,\theta) - \pi^p(y,\theta) = F_3(y,\theta) & (24)
\end{aligned}$$

and it exhibits (as the one in the preceding section) the origin as the steady state.

The Jacobian matrix J of this simple 2-D dynamical system at the interior steady state is characterized by:

$$J = \begin{pmatrix} \partial\dot{y}/\partial y & \partial\dot{y}/\omega \\ \partial\dot{\omega}/\partial y & \partial\dot{\omega}/\omega \end{pmatrix} = \begin{pmatrix} - & \pm \\ \pm & 0 \end{pmatrix}$$

Table 1: Four Baseline Real Wage Adjustment Scenarios

	wage-led goods market	profit-led goods market
labor market-led real wage adjustment	$\begin{pmatrix} - & + \\ + & 0 \end{pmatrix}$ – divergent or convergent –	$\begin{pmatrix} - & - \\ + & 0 \end{pmatrix}$ – convergent –
goods market-led real wage adjustment	$\begin{pmatrix} - & + \\ - & 0 \end{pmatrix}$ – convergent –	$\begin{pmatrix} - & - \\ - & 0 \end{pmatrix}$ – divergent of convergent –

As it can be easily observed, the above Jacobian matrix allows for four different scenarios which can be jointly summarized as in Table 1.

As illustrated there, there exist two cases where the Rose (1967) real wage channel operates in a stabilizing manner: In the first case, the goods markets (represented in our analysis by *the output gap in the Price Phillips Curve* equation) depend negatively on the real wage – a situation usually referred to as "a profit-led goods market" – and the dynamics of the real wage are determined primarily by the nominal wage adjustments and therefore by the developments in the labor market (represented here by *the output gap in the Wage Phillips Curve* equation). In this case labor market-led real wage increases *receive a check* through the implied negative effect on goods markets activity levels. In the second case, the goods markets depend positively on the real wage (a wage-led goods market), and the price level dynamics, and therefore the goods markets, primarily determine the behavior of the real wages.[13]

It should be clear that an identification of an economic by means of these four cases cannot be done a priori, since the concerned partial effects depend directly on the model parameters (which are additionally likely to be state- and/or time-varying), see Chen et al (2006) and Proaño (2009) for an empirical analysis of the (D)AS-AD model.

Our traditional Keynesian model therefore exhibits an interesting feedback structure – the Rose (1967) real wage channel – that is rarely considered in the literature from the theoretical or the empirical point of view. Furthermore, our alternative – traditional – Keynesian dynamics also overcomes the trivial explanation of turning points in economic activity of the monetarist baseline models (with its narrow quantity theory driven inflation ceiling, see Flaschel et al. (2008, ch.1)) and remains – just as these simpler models – under certain mild assumptions globally asymptotically stable in a setup which integrates real interest rate effects and a nominal interest rate policy rule with the real wage feed-

[13]Note here that also the cost-pressure parameters play a role here and may influence the critical stability condition that characterizes the real wage channel, see Flaschel and Krolzig (2006) for details.

back channel of our Keynesian approach to the wage-price spiral, allowing us moreover to address modern issues of monetary policy, as they are typical for the New Keynesian approaches, as well as other types of issues which are more related with the distributive cycle, see Proaño, Diallo, Flaschel and Teuber (2009).

3.2 Local stability analysis

In the following we discuss – in contrast to the determinacy analysis of the New Keynesian model discuss in the previous section – the local stability conditions of the steady state of the Disequilibrium AS-AD model of this section. As it will be shown, the local stability analysis of this second system relies on much more economic grounds than the determinacy analysis required by rational expectations models, and delivers therefore a much deeper economic insight on the workings of the economy.

The equilibrium solution of this system such that $\dot{y} = \dot{\pi}^c = \dot{\theta} = 0$ is determined by

$$\tilde{J}\begin{pmatrix} y \\ \pi^c \\ \theta \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{25}$$

where

$$\tilde{J} = \begin{pmatrix} -\alpha_{yi}G_{11} & -\alpha_{yi}G_{12} & -\alpha_{yi}G_{13} \\ \beta_{\pi^c}\alpha G_{21} & 0 & \beta_{\pi^c}\alpha G_{23} \\ \alpha G_{31} & 0 & \alpha G_{33} \end{pmatrix} \tag{26}$$

is the Jacobian matrix of this system such that

$$\begin{aligned}
G_{11} &= (\beta_{ip}-1)\alpha(\beta_{py}+\alpha_p\beta_{wy})+\beta_{iy} > 0, \\
G_{12} &= \beta_{ip}-1 > 0, \\
G_{13} &= (\beta_{ip}-1)\alpha(\beta_{p\omega}-\alpha_p\beta_{w\omega}), \\
G_{21} &= (\beta_{py}+\alpha_p\beta_{wy}) > 0, \\
G_{23} &= \beta_{p\omega}-\alpha_p\beta_{w\omega}, \quad G_{31} = (1-\alpha_p)\beta_{wy}-(1-\alpha_w)\beta_{py}, \\
G_{33} &= -\{(1-\alpha_p)\beta_{w\omega}+(1-\alpha_w)\beta_{p\omega}\} < 0.
\end{aligned}$$

Since

$$\begin{aligned}
\det\tilde{J} &= -\alpha^2\alpha_{yi}\beta_{\pi^c}\begin{vmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & 0 & G_{23} \\ G_{31} & 0 & G_{33} \end{vmatrix} = \alpha^2\alpha_{yi}\beta_{\pi^c}\underset{(+)}{G_{12}}(\underset{(+)}{G_{21}}\,\underset{(-)}{G_{33}}-\underset{(?)}{G_{23}}\,\underset{(?)}{G_{31}}) \\
&= -\alpha^2\alpha_{yi}\beta_{\pi^c}\underset{(+)}{G_{12}}\{(1-\alpha_p)(\beta_{py}\beta_{w\omega}+\beta_{p\omega}\beta_{wy})+(1-\alpha_w)\alpha_p(\beta_{wy}\beta_{p\omega} \\
&\quad +\beta_{w\omega}\beta_{py})\} < 0,
\end{aligned} \tag{27}$$

we have the unique equilibrium solution $y* = \theta* = \pi^c* = \pi^w* = \pi^p* = 0$.

The characteristic equation of this system becomes as follows.

$$\left|\lambda I - \tilde{J}\right| = \lambda^3 + b_1\lambda^2 + b_2\lambda + b_3 = 0 \tag{28}$$

Let $\lambda_j (j = 1, 2, 3)$ be the characteristic roots of eq.(28). Then, the Routh-Hurwitz conditions for local stability of the steady state (see Hirsch and Smale 1974), are

$$b_1 = -\lambda_1 - \lambda_2 - \lambda_3 = -\text{trace } \tilde{J} = \alpha_{yi} \underset{(+)}{G_{11}} - \alpha \underset{(-)}{G_{33}} > 0 \tag{29}$$

$$\begin{aligned} b_2 &= \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3 = \text{sum of all principal second-order minors of } \tilde{J} \\ &= \alpha \begin{vmatrix} 0 & \beta_{\pi^c} G_{23} \\ 0 & G_{33} \end{vmatrix} - \alpha\alpha_{yi} \begin{vmatrix} G_{11} & G_{13} \\ G_{31} & G_{33} \end{vmatrix} - \alpha\alpha_{yi}\beta_{\pi^c} \begin{vmatrix} G_{11} & G_{12} \\ G_{31} & 0 \end{vmatrix} = C + D\beta_{\pi^c}; \end{aligned} \tag{30}$$

$$b_3 = -\lambda_1\lambda_2\lambda_3 = -\det \tilde{J} > 0 \tag{31}$$

with

$$\begin{aligned} C &= \alpha\alpha_{yi}(-G_{11}G_{33} + G_{13}G_{31}) = \alpha^2\alpha_{yi}(\beta_{ip} - 1)[(1 - \alpha_p)\{(\beta_{py} + \beta_{iy})\beta_{w\omega} + \beta_{p\omega}\beta_{wy}\} \\ &\quad +(1 - \alpha_w)\{\alpha_p(\beta_{wy}\beta_{p\omega} + \beta_{w\omega}\beta_{py}) + \beta_{iy}\beta_{p\omega}\}] > 0, \\ D &= \alpha\alpha_{yi}G_{12}G_{31} = \alpha\alpha_{yi}(\beta_{ip} - 1)\{(1 - \alpha_p)\beta_{wy} - (1 - \alpha_w)\beta_{py}\} \end{aligned}$$

Finally, for the last Routh-Hurwitz condition we have

$$b_1b_2 - b_3 = -(\lambda_1 + \lambda_2)(\lambda_1 + \lambda_3)(\lambda_2 + \lambda_3) = E - H\beta_{\pi^c} \tag{32}$$

with

$$\begin{aligned} E &= b_1C = \alpha^2\alpha_{yi}(\beta_{ip} - 1)[\alpha_{yi}\{(\beta_{ip} - 1)\alpha(\beta_{py} + \alpha_p\beta_{wy}) + \beta_{iy}\} + \alpha\{(1 - \alpha_p)\beta_{w\omega} \\ &\quad +(1 - \alpha_w)\beta_{p\omega}\}][(1 - \alpha_p)\{(\beta_{py} + \beta_{iy})\beta_{w\omega} + \beta_{p\omega}\beta_{wy}\} + (1 - \alpha_w)\{\alpha_p(\beta_{wy}\beta_{p\omega} \\ &\quad +\beta_{w\omega}\beta_{py}) + \beta_{iy}\beta_{p\omega}] > 0, \\ H &= b_3 - b_1D = \alpha^2\alpha_{yi}(\beta_{ip} - 1)[(1 - \alpha_p)(\beta_{py}\beta_{w\omega} + \beta_{p\omega}\beta_{wy}) + \alpha_p(1 - \alpha_w)(\beta_{wy}\beta_{p\omega} \\ &\quad +\beta_{w\omega}\beta_{py}) + [\alpha_{yi}\{(\beta_{ip} - 1)(\beta_{py} + \alpha_p\beta_{wy}) + \beta_{iy}\} + \{(1 - \alpha_p)\beta_{w\omega} + (1 - \alpha_w)\beta_{p\omega}\}] \\ &\quad [(1 - \alpha_w)\beta_{py} - (1 - \alpha_p)\beta_{wy}]] \end{aligned}$$

For the Jacobian J of these dynamics we get – under the assumptions of an active monetary policy rule ($\beta_{ip} > 1$) and $\partial\dot{\theta}_y/\partial y > 0$ (which implies a procyclicity of real wages with respect to economic activity) –:

$$J = \begin{pmatrix} - & - & ? \\ + & 0 & ? \\ + & 0 & - \end{pmatrix} \tag{33}$$

It can be shown that the steady state of this alternative dynamical system is locally stable if the following Proposition holds:[14]

Proposition 2

> *The interior steady state of the dynamical system (22) – (24) is locally asymptotically stable if the growth rate of real wages depends positively on economic activity, if monetary policy is active with respect to the inflation gap (which overcomes the destabilizing Mundell effect in this model type) and if the state of the business cycle operates on the interest rate setting policy of the Central Bank with sufficient strength.*

Sketch of Proof: Exploiting the linear dependencies within the considered dynamics and its Jacobian, one can show for the characteristic polynomial of the matrix J:

$$\lambda^3 + b_1\lambda^2 + b_2\lambda + b_3, \quad \text{the conditions} \quad b_1, b_3 > 0.$$

Furthermore the parameter β_{iy} only appears in the entry J_{11} of the matrix J. Making it sufficiently large (assuming thus an active monetary policy) therefore will obviously ensure that b_2 and $b_1 b_2 - b_3 > 0$ hold true in addition. This stability result even holds for all choices of the parameter β_{iy}, i.e., we have – in the case of a law of motion of real wages that is labor market led – always global stability of the considered dynamics if the interest rate is reacting to the inflation gap with a strength that is larger than one. □

But what if the growth rate of real wages depends negatively on economic activity and where the dynamics of real wages is therefore goods market led? In order to investigate this case, assume to begin $H > 0$. A sufficient (but not necessary) condition for $H > 0$ to be satisfied is $(1 - \alpha_w)\beta_{py} \geq (1 - \alpha_p)\beta_{wy}$, which describes the case of a goods market led real wage dynamics (the opposite case of the first condition of Proposition 3). Let us now define the value $\beta^0_{\pi^c}$ as $\beta^0_{\pi^c} = E/H > 0$. Then, under $H > 0$, we have the following proposition

Proposition 3

(1) Suppose that $\beta^0_{\pi^c} < 1$.Then, the characteristic equation (28) has

(i) three roots with negative real parts for all $\beta_{\pi^c} \in (0, \beta^0_{\pi^c})$,

(ii) a set of pure imaginary roots and a negative real root at $\beta_{\pi^c} = \beta^0_{\pi^c}$, and

[14]The proofs of the proposition 3 can be obtained on request from the authors.

(iii) two roots with positive real parts and a negative real root for all $\beta_{\pi^c} \in (\beta^0_{\pi^c}, 1)$.

(2) Suppose that $\beta^0_{\pi^c} \geqq 1$. Then, the characteristic equation (28) has three roots with negative real parts for all $\beta_{\pi c} \in (0, 1)$.

Proof:

(1) (i) Suppose that the parameter β_{π^c} is fixed at $\beta_{\pi^c} \in (0, \beta^0_{\pi^c})$. Then, we have a set of inequalities $b_1 > 0, \quad b_3 > 0$, and $b_1 b_2 - b_3 > 0$, which means that all of the Routh-Hurwitz conditions for stable roots are satisfied (cf. Gandolfo 1996, p.221 and Asada, Chiarella, Flaschel and Franke 2003, p.519).

(ii) Suppose that β_{π^c} is fixed at $\beta_{\pi^c} = \beta^0_{\pi^c}$. Then, we have $b_1 b_2 - b_3 = 0$ and $b_2 = b_3/b_1 > 0$. In this case, three roots of eq.(28) become $\lambda_1 = i\sqrt{b_2}$, $\lambda_2 = -i\sqrt{b_2}$, and $\lambda_3 = -b_1 < 0$, where $i = \sqrt{-1}$(cf. Asada 1995, p.248 and Asada, Chiarella, Flaschel and Franke 2003, p.522).

(iii) Suppose that β_{π^c} is fixed at $\beta_{\pi^c} \in (\beta^0_{\pi^c}, 1)$. Then, we have a set of inequalities $b_1 > 0, \quad b_3 > 0$, and $b_1 b_2 - b_3 < 0$. These inequalities imply that $\lambda_1 + \lambda_2 + \lambda_3 < 0, \quad \lambda_1\lambda_2\lambda_3 < 0$, and $(\lambda_1 + \lambda_2)(\lambda_1 + \lambda_3)(\lambda_2 + \lambda_3) > 0$ (cf. equations (29), (31), and (33)). This proves the assertion (iii).

(2) In case of $\beta^0_{\pi^c} > 1$, all of the Routh-Hurwitz conditions for stable roots ($b_1 > 0, \quad b_3 > 0$, and $b_1 b_2 - b_3 > 0$) are satisfied for all $\beta_{\pi^c} \in (0, 1)$. □

Remark: The point $\beta_{\pi^c} = \beta^0_{\pi^c}$ is a degenerated "Hopf Bifurcation point" in a system of linear differential equations (S_2).

Corollary of Proposition 3

1 Suppose that $\beta^0_{\pi^c} < 1$. Then, we have the following properties.

(i) The equilibrium point of the system (S_2) is asymptotically stable for all $\beta_{\pi^c} \in (0, \beta^0_{\pi^c})$, and it is unstable for all $\beta_{\pi^c} \in (\beta^0_{\pi^c}, 1)$.

(ii) Even if the equilibrium point of the system (S_2) is unstable, it does not become totally unstable, but it becomes a saddle point.

(iii) Cyclical fluctuations occur in the system (S_2) at some range of the parameter value β_{π^c} which is sufficiently close to $\beta^0_{\pi^c}$. In particular, a family of closed orbits exists at $\beta_{\pi^c} = \beta^0_{\pi^c}$.

2 Suppose that $\beta^0_{\pi^c} > 1$. Then, the equilibrium point of the system (S_2) is asymptotically stable for all $\beta_{\pi^c} \in (0,1)$.

Proof: These results directly follow from Proposition 3. For instance, let us consider the following numerical example.

$$\beta_{wy} = \beta_{v\omega} = \beta_{py} = \beta_{p\omega} = \beta_{iy} = \alpha_{yi} = 1, \quad \alpha_w = \alpha_p = 0.5. \tag{34}$$

Then, we have $\beta^0_{\pi^c} \cong 2.2 + 5(\beta_{ip} - 1) > 1$ for all $\beta_{ip} > 1$.

In this case, the equilibrium point of the system (S_2) is asymptotically stable for all $\beta_{\pi^c} \in (0,1)$. □

It should be clear that in this conceivable, but limited situation of $\beta_{\pi^c} > \beta^0_{\pi^c}$ values strong monetary policy reactions with respect to the parameter β_{iy} or meaningful behavioral nonlinearities off the steady state may be needed in addition in order to make the dynamics bounded or viable if it departs by too much from the steady state, see for example Chen et al. (2006).[15]

The above stability investigations imply that we will always get asymptotic stability if $(1-\alpha_p)\beta_{wy} - (1-\alpha_w)\beta_{py} > 0$ holds true, i.e., in the case of a labor market led real wage dynamics, since we then have $D > 0$, $H < 0$. The labor market led case thus is completely unambiguous as far as stability results are concerned.

4 Concluding Remarks

In this paper we compared two alternative theoretical approaches to macroeconomics, focusing on their determinacy/stability conditions and the implications of such analysis for the understanding of the functioning of an economic system.

The approach to determinacy analysis of the 4D New Keynesian model pursued in section 2 made use of the notion that the intrinsic dynamics and determinacy properties of a dynamic model should be invariant to the assumed frequency of the decision making of the economic agents in the discrete time version of the model, and therefore, should not depend on whether such model is formulated in continuous- or discrete time.[16] On this basis the approach pursued there made determinacy analysis of New Keynesian models

[15]The reader is referred to this and other earlier works for more details on such dynamical systems and further empirical investigations of this model prototype.

[16]For counterfactual examples where the determinacy properties of the rational expectations equilibrium in an economy do depend on the decision frequency assumed, see Hintermaier (2005).

with staggered wages and prices as studied for example in Woodford (2003) much more tractable, because it allowed us to circumvent the calculation of the significantly more complicated conditions which hold for the corresponding discrete time case, see for example the mathematical appendices in Woodford (2003) for the difficulties that exist already in the 3D case.

However, respecting this New Keynesian approach to macroeconomic modeling, we also intended to highlight the fact that the solution method implied by the rational expectations assumption in this type of models lacks to a significant extent of economic insight. As we showed, the analysis of the determinacy conditions even of a simple rational expectations models such as the 4D New Keynesian model discussed here, resembles much more a mathematical exercise than an economic analysis.

Furthermore, there are additional issues related with the appropriateness of the New Keynesian approach as the workhorse framework in macroeconomics. On the one hand there is the validity of its use of the word Keynesian as a label: There is in fact no IS-curve, representing Keynesian demand rationing on the market for goods, as the model is formulated, but simply a Walrasian type of notional goods demand and on this basis the assumption of goods market equilibrium. The theory of rational expectations has also very little to do with Keynes' (1936) views on the difficulties of expectations formation, in particular for the evaluation of long-term investment projects. Finally, Keynes' liquidity preference theory is no longer a subject that is paid attention to here, due to the disappearance (irrelevance) of the LM schedule, which is at best present in the background of a simple to handle Taylor interest rate policy rule. Therefore, when compared with Keynes' (1936, ch. 22) "Notes on the Trade Cycle" and its important constituent parts – the marginal propensity to consume out of rationed income, the marginal efficiency of investment (and the expected cash flow that is underlying it) and the parameters that shape liquidity preference –, not much of this is left in the New Keynesian approach to macrodynamics, in particular concerning the systematic forces within the business cycle and its turning points as they are discussed in Keynes' (1936, ch.22). Moreover, as previously discussed, in the New Keynesian framework further important feedback channels such as the real wage channel – investigated in Chiarella and Flaschel (2000) and later work – is almost inexistent, since there determinacy is achieved by the specification of a Taylor rule with parameters values which imply a certain combination of unstable/stable roots for the Jacobian matrix of the dynamics.

Furthermore, while the *microfoundation* of economic behavior is per se an important desideratum to be reflected also by behaviorally oriented macrodynamics, the use of "representative" consumers and firms for the explanation of macroeconomic phenomena is too simplistic and also too narrow to allow a proper treatment of what is *really* interesting on the behavior of economic agents – the interaction of heterogenous agents –, and it is also not detailed enough to discuss the various feedback channels present in the real world. *Market Clearing*, the next ingredient of such approaches, may however be a questionable device to study the macroeconomy in particular on its real side. The data generating process is too fast in order to allow for period models with a *uniform* period length of a quarter or more. In continuous time however it is much too heroic to assume market clearing at all moments in time, but real markets are then only adjusting towards moving equilibria in such an framework.

Yet, neither microfoundations per se nor market clearing assumptions are the true dividing line between the approaches we are advocating and the ones considered in this section. It is the ad hoc, i.e., not behaviorally microfounded assumption of *Rational Expectations* that by the chosen analytical method makes the world in general loglinear (by construction) and the generated dynamics convergent (by assumption) to its unique steady state which is the root of the discontent that our paper tries to make explicit. Indeed, agents are heterogeneous, form heterogeneous expectations along other lines than suggested by the rational expectations theory, and have differentiated short- and long-term views about the economy.

We conclude that the New Keynesian approach to macrodynamics creates more theoretical and empirical problems than it helps to solve, therefore not (yet) representing a theoretically and empirically convincing strategy for the study of the fluctuating growth that we observe in capitalist economies. The alternative theoretical framework discussed in section 3, in contrast, features a number of advantages which, in our opinion, facilitate to a significant extent the analysis and understanding of the role of the different macroeconomic channels working in an economy, such as the disequilibrium specification of the dynamics of the economy – which seems to us to proper approach to follow given that fluctuations of macroeconomic aggregates occur on a daily basis and not on a quarterly basis as implicitly assumed in many macroeconomic models – and the alternative (and maybe more realistic, but on all accounts more tractable) specification of the expectations formation, that allows already in its deterministic setup for a meaningful theory of the business cycle with monotonic convergence or damped fluctuations in economic activity towards its steady state.

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