Kaleckian Investment and Employment Cycles in Postwar Industrialized Economies

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January 31, 2007

Abstract

The role of the welfare state in the post-war industrialized economies has recently become a major topic. Using a Kaleckian framework we consider an economy where investment depends positively on the rate of return on capital and negatively on the rate of employment. This allows for a possible integration of Kalecki's (1943) analysis of the political aspects of full employment. We use Okun's law to link the goods market with the labor market. We separate laws of motion for wage and price inflation in order to integrate the role of changing income distribution into this framework of effective demand and employment dynamics. There is a balanced growth path solution for this model which however is likely to be locally unstable. From the global perspective, the turning points in long lasting phases of strong economic growth are given by an increasing reaction of investment, and of fiscal and monetary policy, to the consequences resulting from full employment and the evolving welfare state (represented by generous welfare payments, labor market institutions in favor of labor, and co-determination). In subsequent long-phased depressions profit-led goods demand in combination with declining real wages (enforced by mass unemployment and labor market reforms) may account for lower turning points and for a return to normal and subsequently possibly again excessive economic activity. Such nonlinearities in economic behavior far off the balanced growth path imply the global viability of the economic dynamics. On the other hand, in contrast to such a conflict-driven macro economy, as Kalecki (1943) has perceived it, the business leaders and policy makers could pursue a consensus-driven macro economic dynamics stressing a more collaborative and long term approach which will, as we show, take on a more stable path. Pursuing those two social aspects of macro dynamics the paper lays some foundations for an economic analysis of the role of the welfare state in post-war

industrialized economies.
JEL CLASSIFICATION: **E24**, **E31**, **E32**.
KEYWORDS: Long-phase cycles, income distribution, welfare state, fiscal and monetary policy.

1 Introduction

The issue whether the welfare state promotes or retards economic growth has become a major issue in recent macro economic studies. In this paper we investigate some of the aspects of the welfare state in a macro economic model[1] with labor market adjustment process, wage-price spiral, and a Keynesian dynamic multiplier setup. The aggregate demand schedule underlying this dynamic multiplier is derived from Classical saving habits and an investment function which assumes that investment not only reacts to changing profitability in capitalist economies, but also to the conditions characterizing the capital-labor relationship, where persistently high employment rates may give rise to extensive welfare state measures. Those are usually labor market institutions in favor of labor, generous unemployment benefits, labor force participation in the firms' decision making (in the hiring and firing practices of firms) and a considerable reduction of the workday, not all liked by 'industrial leaders' as Kalecki (1943, ch.12) has characterized those achievements.

Our considered model has Kaleckian features with regard to investment and savings behavior and the multiplier approach to the market for goods. It is Goodwinian with respect to labor market dynamics and capital stock growth, though we will now find at least four possible regimes for real wage adjustment, see Rose (1967) in this regard, in place of the single one of the Goodwin (1967) growth cycle approach. There, only a profit-led coupled with a labor-market-led situation was considered, due to the supply side orientation of the Goodwin model, see the table below for details.

In our model, we furthermore employ, in place of Goodwin's (1967) real wage Phillips curve, now a money-wage and a price-level Phillips curve, both of a fairly advanced type, where wage dynamics allows for insider-outsider considerations and Blanchard and Katz (1999) error correction. The price dynamics can be considered as a generalization of Kalecki's theory of markup pricing, with a markup that depends on the state of the business cycle and again a term that represents the Blanchard and Katz (1999) type error correction (a second positive – level – influence) of the real wage on the rate of price inflation. Due to the separate consideration of labor force and workforce utilization rates (the employment rate on the external labor market and the utilization ratio of the workforce within firms) we finally need a two stage Okun-type link between the goods and the labor market. The first stage leads from the utilization of the capital stock (as measured by the output-capital ratio) to the utilization ratio of the employed workforce (as a more or less technological relationship) and the second stage from there to the employment rate of the labor forces by way of the employment policy of firms. Here, firms' employment policies are viewed as dependent on the utilization ratio of their workforce in comparison to a utilization ratio that is desired by them in view of the state of the external labor market.

[1] A similar model of Steindlian accumulation dynamics is studied in Flaschel and Skott (2006) and Asada, Flaschel and Skott (2006)

We thus distinguish with respect to the labor market the rate of employment that is characterizing this market from the utilization ratio of the workforce of firms and thus pursue an insider-outsider approach as far as wage negotiations and the employment policy of firms are concerned. The utilization ratio of the workforce is always immediately adjusted to the utilization ratio of the capital stock and thus fluctuates as this ratio with the state of aggregate demand on the market for goods, though with a different amplitude (depending on the technology of the firm sector). The change in the external employment rate (on the labor market), by contrast, is determined by the employment policy of the firms which may be subject to many influences (economic and political ones) and thus represents the volatile element in this two stage formulation of Okun's (1970) law, leading from capacity utilization of firms to their labor force utilization and from there to the recruitment (or dismissal) of laborers into (from) the existing work-force of firms. As already stated firms in addition pay attention to the state of the labor market when deciding on their amount of their investment projects. We view the interaction of inside workforce utilization with outside employment rates as a capillary system where pressure as measured by rate of money wage growth is shifted from the inside component to the outside component and back in order to achieve – from the viewpoint of firms – as little combined pressure as possible.

As the model is formulated we can distinguish between wage-led and profit-led situations on the market for goods, depending on whether aggregate demand increases or decreases with increases in the real wage. In addition we can differentiate labor-market led influences from goods-market led influences of economic activity increases on the growth rate of real wages. The first situation is given when the growth rate of real wages essentially depends positively on factor utilization rates. The second situation characterizes the opposite case, in which case the dynamics of the price level dominate in the formation of real wages (and not the dynamics of the money wage). The table shown below summarizes the scenarios that are possible in such an environment and it implies that empirical analysis is needed if definite conclusions are to be drawn with respect to which entry in this table is the relevant one in a particular country at a particular time.

	wage-led goods demand	profit-led goods-demand
labor-market-led real wage adjustment	adverse = divergent	normal = convergent
goods-market-led real wage adjustment	normal = convergent	adverse = divergent

Table 1: Four types of real wage adjustment processes.

If aggregate demand and the real wage dynamics are wage led and labor-market led, respectively, we have increasing activity with increasing real wages and in addition a pro-cyclical real wage dynamics (with a quarter phase displacement) and thus get instability from this two-fold positive feed-back mechanism. The other cases in table 1 are motivated in a similar way, giving in sum rise to two types of adverse and two types of stabilizing real wage reaction patterns.

In the paper we will concentrate on situations where the real wage dynamics are labor market led (for which there is some evidence in the U.S. economy, see Chen et al. (2006) and also the on-going debate on procyclical real wages in this regard). Chen et al. (2006) also provide evidence on profit-led goods market activities, but here the situation is more mixed in the literature, since Postkeynesian authors normally assume wage-led situations in their analysis. In the paper we shall show in this respect that wage-led situations are always coupled with dynamic multiplier stability, but are – as was argued above – plagued by real wage instability if money wages respond strongly to labor market pressure (in particular inside pressure). Profit-led situations by contrast are plagued by dynamic multiplier instability, but imply a stable or convergent partial real wage adjustment mechanism.

We therefore have in both cases instability scenarios – in the first case via the real wage feedback structure and in the second case via a goods market quantity adjustment process that works too fast. In the first situation downward money wage rigidity may be of help in controlling the explosive dynamics of real wages and in the second case a weakening of the multiplier process far off the steady state may allow for bounded dynamics, though not convergent ones. Our model therefore allows for interesting alternative stability scenarios, dependent on the working of the wage-price spiral. This allows also to include Blanchard and Katz type error correction terms, found to be relevant in the case of the U.S. economy in Chen et al. (2006)) and employment policies, pursued by firms, in a Keynesian aggregate demand situation where income distribution and the state of the labor market matters.

Moreover, we also get in this model type interesting steady state relationships, underlying the dynamics of the model. On the one hand we find for the steady state that owners of capital get what they spend, since their profit rate is solely determined by their trend investment behavior. On the other hand, as the model is formulated, we have that the steady state rate of employment on the labor market is determined by the target rate in the investment function solely. Finally, the steady ratio of capacity utilization is generally only equal to the utilization ratio of the capital stock desired by firms. This holds if workers adjust their behavior to the steady state rate of employment set by capitalist firms and if their reaction with respect to the insider utilization ratio of the workforce is the same as in the firms' reaction pattern. This steady ratio of capacity utilization is a fairly complicated function of the parameters of the model and thus dependent on a variety of exogenous magnitudes of our approach to the interaction of goods and labor market dynamics.

There are however processes on the nominal side of the model that may modify such situations significantly. If there is accelerating inflation, the interest rate policy pursued by the central bank may not only enforce upper turning points in economic activity, but may also modify its steady state position back to natural rates of employment and capacity utilization, if these rates are known to the central bank. Such monetary policy thus exercises a strong influence on the working of the economy which may not really function without it. Concerning deflationary episodes the role of the central bank may however be much more limited, since there is a floor to the setting of the nominal rate of interest. In such a situation, downward money wage rigidity and thus behavior of workers may be of decisive importance for the viability of the economy.

The next section presents the model of this paper. In section 3 we derive the laws of motion that characterize the considered economy. Section 4 investigates its steady state and basic stability calculations. In section 5 we consider the essential feedback structures of this model type and compare them with other ones.[2] The feedback structures are then used to construct a case where in fact local stability of the steady state arises. We call this a consensus-driven economy that is considered at the end of this section. Section 6 then considers the case of local instability of the steady state solution characterizing a conflict-driven economy. This provides various scenarios that may nevertheless allow for upper as well as lower turning points from a global point of view, i.e., that make the considered dynamics an economically viable one from the global point of view. In the case of a conflict-driven economy the order is established only through large fluctuations in economic activity or economic policy. Section 7 provides some numerical illustrations of the model. The section 8 points to some problems on the nominal side of the considered economy that do not yet feedback into the real dynamics, but that demand for the extension of the model concerning interest effects on aggregate demand and a Taylor interest rate policy rule in order to tame the explosive dynamics that is existing in the nominal part of the dynamics. Besides monetary policy we there also briefly consider fiscal policy of a Keynesian type which augments the private sector of our Keynes/Kalecki-Goodwin/Rose economy in a straightforward way. Section 9 concludes. The employed stability criteria are considered in an appendix to the paper.

2 The model

We here consider the private sector of the macroeconomy in isolation from the government sector (and fiscal and monetary policies). The role and implications of macroeconomic policy for the modeled investment and employment dynamics will be considered in a later section of this paper. We start from the wage-price dynamics or the aggregate supply side of the model, as it is often called in the literature. For the description of aggregate supply we will employ the following general formulation of a wage-price spiral mechanism

[2]See, for example, Flaschel and Skott (2006), Asada, Flaschel and Skott (2006) for a related model of capital accumulation.

(related to both work of Rose (1967, 1990) and Blanchard and Katz (1999):

$$\hat{w} = \beta_{we}(e-\bar{e}) + \beta_{wu}(u_w - \tilde{u}_w(e)) - \beta_{w\omega}\ln(\frac{\omega}{\omega_0}) + \kappa_w\hat{p} + (1-\kappa_w)\pi^c \quad (1)$$

$$\hat{p} = \beta_{py}(y-\bar{y}) + \beta_{p\omega}\ln(\frac{\omega}{\omega_0}) + \kappa_p\hat{w} + (1-\kappa_p)\pi^c \quad (2)$$

In these equations, $\hat{w}$, $\hat{p}$ denote the growth rates of nominal wages w and the price level p (their inflation rates) and π^c a medium-term inflation-climate expression which however is of no relevance in the following due to our neglect of real interest rate effects on the demand side of the model. We denote by e the rate of employment on the external labor market and by u_w the ratio of utilization of the workforce within firms. This latter ratio of employment is compared by the workforce in their negotiations with firms with their desired normal ratio of utilization $\tilde{u}_w(e), \tilde{u}_w' < 0$ which can and is here assumed to depend negatively on the external rate of employment e, since higher employment on the labor market implies demands for lower worktime within firms. We thus have two employment gaps, an external one: $e-\bar{e}$ and an internal one: $u_w - \tilde{u}_w$, which determine wage inflation rate $\hat{w}$ from the side of demand pressure within or outside of the production process. In the wage PC we in addition employ a real wage error correction term $\ln(\omega/\omega_0)$ as in Blanchard and Katz (1999), see Asada, Chen, Chiarella and Flaschel (2006) for details, and as cost pressure term a weighted average of short-term (perfectly anticipated) price inflation $\hat{p}$ and the medium-term inflation climate π^c in which the economy is operating.

As the wage PC is constructed it is subject to an interaction between the external labor market and the utilization of the workforce within firms. Higher demand pressure on the external labor market translates itself here into higher workforce wage demand pressure within firms (and demand for a reduced length of the normal working day, etc.), an interaction between two utilization rates of the labor force that has to be and will be taken note of in the formulation the employment policy of firms. Demand pressure on the labor market thus exhibits two interacting components, where employed workers may make their behavior dependent upon.

We use the output-capital ratio $y = Y/K$ to measure the output gap in the price inflation PC and again the deviation of the real wage $\omega = w/p$ from the steady state real wage ω_0 as error correction expression in the price PC. Cost pressure in this price PC is formulated as a weighted average of short-term (perfectly anticipated) wage inflation and again our concept of an inflationary climate π^c. In this price Phillips curve we have three elements of cost pressure interacting with each other, a medium term one (the inflationary climate) and two short terms ones, basically the level of real unit-wage labor costs (a Blanchard and Katz (1999) error correction term) and the current rate of wage inflation, which taken by itself would represent a constant markup pricing rule. This basic rule is however modified by these other cost-pressure terms and in particular also made dependent on the state of the business cycle by way of the demand pressure term $y - \bar{y}$ in the market for goods.

On the demand side of the model we use for reasons of simplicity the conventional dynamic multiplier process (in place of a full-fledged Metzlerian inventory adjustment mechanism) as in Asada, Flaschel and Skott (2006), i.e.

$$\hat{Y} \;=\; \dot{Y}/Y = \beta_y(Y^d/Y - 1) + \bar{a} \tag{3}$$

where Y^d, Y denote aggregate demand and supply and $\bar{a}$ a trend term in the behavior of capitalist firms to be explained below. Assuming a fixed proportions technology with given output-employment ratio $x = Y/L^d$ and potential output-capital ratio $y^p = Y^p/K$, allows us to determine from the output-capital ratio y the employment u_w of the workforce within firms that corresponds to this activity measure y:

$$u_w = y/(xle), \quad u_w = L^d/L^w, l = L/K, e = L^w/L \tag{4}$$

(with L^d hours worked, L^w the number of workers employed within firms and with L denoting labor supply). This relationship represents by and large a technical relationship (to be calculated by 'engineers') and relates hours worked to goods market activity as measured by y in the way shown above.

This technological relationship must be carefully distinguished from the employment (recruitment) policy of firms that reads on the intensive form level:

$$\hat{e} \;=\; \dot{e}/e = \beta_{eu}(u_w - \tilde{u}_f(e)) - \beta_{e\omega}(\omega - \omega_o) + \bar{a} - \hat{L},\ i.e., \tag{5}$$

$$\dot{e} \;=\; \beta_{eu}(y/(xl) - \tilde{u}_f(e)e) - \beta_{e\omega}(\omega - \omega_o)e + (\bar{a} - \hat{L})e, \quad \tilde{u}_f' > 0. \tag{6}$$

Basis of this formulation of an employment policy of firms in terms of the employment rate is – by assumption – the following level form representation of this relationship:

$$\dot{L}^w \;=\; \beta_{eu}(L^d - \tilde{u}_f(e)L^w) - \beta_{e\omega}(\omega - \omega_o)L^w + \bar{a}L^w,\ i.e. \tag{7}$$

$$\hat{L}^w \;=\; \beta_{eu}(L^d/L^w - \tilde{u}_f(e)) - \beta_{e\omega}(\omega - \omega_o) + \bar{a} \tag{8}$$

where $\bar{a}$ again integrates the trend term assumed by firms into now their employment policy and where $\tilde{u}_f(e)$ represents the utilization ratio of the workforce of firms that is desired by them. This ratio is here made dependent on the rate of employment on the external labor market in a positive way, since firms will accept higher demand pressure within their workforce employment relationship (in place of the new recruitment of workers), if the external labor market has become tighter. Firms therefore react to both measures of demand pressure (in the labor market and within the production process) by an attempt to obtain some balance in these two types of wage pressure, just as in a physical capillary system. The growth rate of the workforce of firms is thus a positive function of the utilization gap $u_w - \tilde{u}_f$ of the workforce within firms, where the benchmark utilization ratio $\tilde{u}_f$ desired by firms depends positively on the outside employment ratio, since firms are inclined to allow on an average for larger utilization rates within firms when the outside labor market situation is becoming more tense. In order to obtain eq. (6) as the resulting law of motion for the rate of employment one simply has to take note of the definitional relationship $\hat{e} = \hat{L}^w - \hat{L}$, where L denotes the labor supply in each moment in time. We have also included into the above recruitment policy a term that says that intended recruitment will be lowered in case of increasing real wage costs of firms.

In order to close the model we assume extremely classical saving habits ($s_w = 0, s_c = 1$), i.e. the consumption-output ratio is simply given by ω/x. For investment behavior we moreover assume

$$I/K = i(\cdot) + \bar{a} = i_\rho(\rho - \rho_o) - i_e(e - \bar{e}_f) + \bar{a}, \tag{9}$$

with $\rho = y(1 - \omega/x)$ the current rate of profit.[3] In this equation, the magnitude $\bar{a}$ denotes a given trend investment rate, representing investor's 'animal spirits') from which firms depart in a natural way if there is excess profitability (and vice versa). Moreover, firms have a view of what the rate employment should be on the external labor market (Kalecki's (1943, ch.12) analysis of why 'bosses' dislike full employment) and thus reduce their (domestic) investment plans (driven by excess profitability) in situations of a tense labor market. They thus take pressure from the labor market in the future evolution of the economy by their implicit collective understanding that high pressure in the capillary system we have considered above will lead to conditions in the capital-labor relationship, unwanted by firms, since persistently high employment rates may give rise to significant changes of workforce participation with respect to firms' decision making, in the hiring and firing decision of firms, in reductions of the work-day etc., not at all liked by 'industrial leaders'.

We view the rate $\bar{a}$, the trend investment decision of capitalist firms as the (mathematically seen) independent variable of the model, whereas any changes in the growth rate n of the labor force, the so-called natural rate of growth, are following the growth rate of the capital stock more or less passively and have in any case little to do with the biological reproduction rate of a given society. There are of course many possible ways in which the growth rate of the labor force $\hat{L}$ (not the growth rate of the working population) is governed by the accumulation path of the economy. In this paper we make two specific (simplifying) assumptions in this regard: First $\hat{L} = \bar{a}$ and later on even $\hat{L} = \hat{K}$, in order to concentrate on the interaction of income distribution with the goods market dynamics and the demand pressures on the labor market these former dynamics give rise to.

The model neglects on the quantity side unintended inventory changes and on the value side windfall profits or losses caused by the assumed possibility of a discrepancy between savings and investment. It describes through its equations a closed economy where we abstract from all government activities and where we ignore the behavior of central banks

[3] An alternative assumption about investment behavior is given by

$$I/K = i_o(y - \bar{y}) + i_{\rho^m}(\rho^m - \rho_o^m) - i_e(e - \bar{e}_f) + \bar{a}, \quad \rho^m = (1 - \omega/x)\bar{y} \tag{10}$$

which separates short-term business cycle utilization rate effects from normal utilization profitability effects ρ^m caused by changing income distribution. Note also that we do not discuss the financing of investment explicitly. A justification for this is that all profits are paid out as dividends and all investment is financed by the issue of new equities, the value of which is not yet feeding back onto the investment decision of firms.

due to our neglect of real interest effects on the demand side of the model (consisting of workers' consumption and firms' investment demands solely). We also abstract here from the role of financial markets in the financing of investment decisions by assuming that all profits are paid out as dividends and that investment is purely equity-financed with no feedback on the assumed consumption behavior, see Chiarella and Flaschel (2000, Ch.6) for the budget equations that allow for such flow consistency.

As in earlier work, see Asada , Chen, Chiarella and Flaschel (2006) in particular, we can derive as reduced forms from the described wage-price spiral a real wage dynamic $\hat{\omega} = \hat{w} - \hat{p}$, which does not depend on the inflationary climate term π^c and an augmented reduced-form price PC where π^c has a coefficient of unity. However, since the real rate of interest is no issue in our model, this latter reduced-form equation does not yet play a role in the implied dynamical system to be considered below. In this system we only need to consider the dynamics of real wages ω, of the output-capital ratio y and of the rate of employment e (plus full employment labor intensity $l = L/K$ if the natural rate of growth would be given exogenously).

3 The implied dynamics

We now reduce the considered macrodynamic model to a system of four differential equations in the pairs of state variables y, e and ω, l. The first pair can be considered to describe the Keynes-Kalecki goods market and employment dynamics and the second one the Goodwin-Rose growth cycle dynamics, though Kalecki and Rose have also included income distribution effects and goods market effects in their analysis of the capital accumulation process, respectively. Later on in the paper we assume, by way of a special assumption on labor supply L, that the state variable l stays at its steady state level and thus will reduce the dynamics to dimension 3 then.

As the model is formulated we therefore have as laws of motion on its supply side, a law describing the dynamics of real wages and one growth law, describing the capital accumulation induced by the assumed investment behavior. The inflationary tension resulting on the nominal side of the considered dynamics does not yet play a role in their core dynamics, since we do not have any real interest rate effects or wealth effects included in its formulation. On the demand side we have the law of motion for capacity utilization of firms, measured by $y = Y/K$, and the law of motion of the employment rate e on the external labor market implied by the employment policy of firms.

Note that we assume in this section that $\hat{L}$ is given by $\bar{a}$, i.e., this latter trend term also applies to the conditions of labor supply, and thus not only to investment and the dynamic multiplier story that is based on it (and the employment policy of firms). Note furthermore that aggregate demand per unit of capital, the Keynesian heart of our accumulation dynamics, is always given by the expression:

$$y^d = (\omega/x)y + i(\cdot) + \bar{a}, \quad i(\cdot) = i_\rho(\rho - \rho_o) - i_e(e - \bar{e}_f), \quad \rho = (1 - \omega/x)y$$

in the following.

3.1 DAD: multiplier and employment dynamics

The Disequilibrium AD or DAD part of the model is given by:

$$\begin{aligned} \dot{y} &= -\beta_y(1 - \frac{\omega}{x})y + (\beta_y - y)i(\cdot) + \beta_y \bar{a}, \quad y = Y/K \quad \text{with} \qquad (11) \\ \dot{y}_y &= -\beta_y(1 - \frac{\omega}{x}) + (\beta_y - y)(1 - \frac{\omega}{x})i_\rho = -\beta_y(1 - i_\rho)(1 - \frac{\omega}{x}) - y(1 - \frac{\omega}{x})i_\rho \end{aligned}$$

We assume that $\beta_y > y_o$ holds in all of the following and thus get that the sign of $1 - i_\rho$ is decisive for the sign of $\dot{y}_y$, which is negative if the propensity to invest into capital stock growth from the obtained profit rate is less than one at the steady state.

This law of motion is obtained from

$$\begin{aligned} &\frac{\dot{Y}}{Y} = \beta_y(\frac{Y^d}{Y} - 1) + \bar{a}, \quad C = \omega L^d = \frac{\omega}{x}Y \quad (s_w = 0, s_c = 1) \quad \text{and} \\ &\frac{I}{K} = i(\cdot) + \bar{a} = i_\rho(y(1 - \frac{\omega}{x}) - \rho_o) - i_e(e - \bar{e}_f) + \bar{a}. \end{aligned}$$

We furthermore have:

$$u_w = \frac{1}{xl}\frac{y}{e} \quad \text{via fixed proportions in production}$$

which represents a static (technological) relationship between the ratio of capital stock utilization $y = Y/K$ and the ratio of workforce utilization u_w within firms.

Finally:

$$\dot{e} = \beta_{eu}(y/(xl) - \tilde{u}_f(e)e) - \beta_{e\omega}(\omega - \omega_o)e - i(\cdot)e, \quad \tilde{u}_f' > 0. \qquad (12)$$

gives again the law of motion for the employment policy of firms.

The reduced-form dynamic IS-relationship is depending on income distribution (due to the assumed savings behavior) and on the investment decision of firms in a specific way, translated into the time rate of change of the output-capital ratio by means of the adjustment speed in the dynamic multiplier process that is driving the goods market. The utilization ratio of the employed labor force is a linear function of the ratio between the utilization ratio of the capital stock y, and the utilization rate of the labor force e on the external labor with a multiplier that is given by the product of the reciprocal values of both labor productivity and the full employment labor-capital ratio (this expression has to be inserted into the laws of motion in various places in order to get an autonomous system of four differential equations describing the evolution of the considered macroeconomy). The expression u_w can be altered by firms through changes in labor productivity (technical change), by manipulating labor supply or the external employment rate or – politically – by enforced changes in income distribution that lead to appropriate changes of Keynesian aggregate demand.

3.2 DAS: real wage dynamic and capital accumulation

The Disequilibrium AS or briefly DAS part of the model is given by:[4]

$$
\begin{aligned}
\frac{\dot{\omega}}{\omega} &= \kappa[(1-\kappa_p)(\beta_{we}(e-\bar{e})+\beta_{wu}(u_w-\tilde{u}_w(e))-(1-\kappa_w)\beta_{py}(y-\bar{y})] \\
&- \kappa[(1-\kappa_p)\beta_{w\omega}\ln(\frac{\omega}{\omega_o})+(1-\kappa_w)\beta_{p\omega}\ln(\frac{\omega}{\omega_o})], \quad u_w=\frac{1}{xl}\frac{y}{e} \qquad (13) \\
\hat{l} &= \hat{L}-\hat{K}=-i_\rho(\rho-\rho_o)+i_e(e-\bar{e}_f)=-i(\cdot) \qquad (14)
\end{aligned}
$$

if it is assumed that labor force growth is governed by $n=\hat{L}=\bar{a}$ which is an assumption of at least the same ideal type as the assumption of constant labor force growth.

The law of motion for the real wage depends on a variety of demand pressure items, since all cost-pressure terms (relating to the κ-coefficients can be reduced to their underlying demand pressure terms when the two linear equations of nominal wage and price dynamics are solved for the two unknowns $\hat{w}-\pi^c, \hat{w}-\pi^c$, from which $\hat{\omega}=\hat{w}-\hat{p},\ \omega=w/p$ can then be obtained. Of course, demand pressure terms on the labor market influence real wages in a positive fashion and those on the goods market in a negative way. The Blanchard and Katz real wage error correction terms in the WPC and PPC both act on real wages in a negative fashion as suggested by the label that is used for their working.

We assume in a later stability analysis for the law of motion of real wages that the insider term $\beta_{wu}(u_w-\tilde{u}_w(e))$ is of a size that does not alter the positive dependence of real wage growth on the rate of employment e and its negative dependence on the ratio of capacity utilization y. Depending on the choice of parameter values it may however easily happen that the term $u_w=\frac{1}{xl}\frac{y}{e}$ overthrows the negative influence of y on the growth rate of real wages, in which case the dynamics of real wages is completely dominated by the labor market in- and outside of firms and thus completely labor-market led. Since u_w depends negatively on the rate of employment e it may furthermore also happen that an overall negative dependence of $\hat{\omega}$ on the rate of employment is established. The movement of real

[4] $\kappa=1/(1-\kappa_w\kappa_p)$.

wages is thus subject to a variety of influences which may imply that it is not moving in a clear or even in a strictly procyclical fashion.

4 4D steady state configuration and reduced-form 3D dynamics

In this section we will calculate the steady state of the dynamics (12) – (14),. This system is an autonomous one, once the definitions of u_w and ρ have been inserted into it. As steady state solutions we then get:

4.1 Steady state solution of the 4D dynamics

From the conditions $\dot{y} = 0, \hat{l} = -i(\cdot) = 0$ we get as our first steady state result that there must hold for the rate of profit ρ:

$$\rho_o \quad = \quad y_o(1 - \omega_o/x) = \bar{a}, \quad i.e., \tag{15}$$

capitalists get what they spend in the steady state. Note that the values of y_o, ω_o are restricted by this steady state condition, but remain to be determined still.

Next we find, by way of $i_\rho(\cdot) = i(\cdot) = 0$, that there holds for the steady state rate of employment on the external labor market:

$$e_o \quad = \quad \bar{e}_f \quad \neq \bar{e} \quad \text{in general,} \tag{16}$$

This states that the steady state rate of employment (not at all to be interpreted as an inflation oriented NAIRU rate of employment)[5]) is solely determined by the investment behavior of firms, i.e., by the benchmark level they set for this rate, beyond which firms believe that the social structure of accumulation (workforce participation in firms' decision making and the like) will change significantly to their disadvantage.

As an intermediate step we furthermore get for the value of $(y/l)_o$ by way of the condition:

$$\tilde{u}_f(e_o) = (y/l)_o/(xe_o), \quad \text{i.e., the steady state value} \quad (y/l)_o = \tilde{u}_f(e_o)xe_o$$

and from this for the utilization ratio of the workforce of firms in the steady state

$$u_{wo} = \tilde{u}_f(e_o), \quad i.e.,$$

the steady state utilization rate of the labor force is determined by the benchmark rate in the employment policy of firms which in turn depends on the external rate of employment

[5] see also the concluding section of the paper.

that firms desire in order to keep workforce relationships within firms under sufficient control.

Making use of the condition $\hat{\omega}(\cdot) = 0$ in the reduced form expression for real wage dynamics then gives ($\kappa_w < 1$):

$$y_o = \bar{y} + \frac{(1-\kappa_p)(\beta_{we}(e_o - \bar{e}) + \beta_{wu}(\tilde{u}_f(e_o) - \tilde{u}_w(e_o))}{(1-\kappa_w)\beta_{py}} \neq \bar{y} \tag{17}$$

$$\tag{18}$$

since the Blanchard / Katz error correction terms are zero in the steady state (by assumption). We assume that the adjustment parameters in the numerator of this expression are sufficiently small such that – in the case where $e_o - \bar{e}$ or $u_{wo} - \tilde{u}_w(e_o)$ become negative – the steady state output capital ratio y_o stays positive. We note also that increasing labor market flexibility in whatever respect may in this approach have positive or negative effects on steady state capacity utilization $y_o = (Y/K)_o$ depending on the benchmark choices of both workers and firms. We note however also that the model is 'ill-defined' (indeterminate) in the case where price inflation does not depend on the demand pressure in the market for goods.

Common inside and outside employment benchmark levels of firms and workers give rise to $y_o = \bar{y}$ (also for $\beta_p y = 0$) and to specific deviations from this simple benchmark case otherwise. We note however in addition that there need not exist any pressure around the balanced growth path (and even less far away from it) that tends to harmonize the various benchmarks on the labor and the goods market with each other. Instead, the departure of steady state values from their corresponding benchmark values, in the wage and price Phillips curves in particular, simply means that the steady state inherits some of the problems that govern the course of the economy in general such as upward or downward pressure on wage and price inflation that is neutralized through opposite deviations from the benchmark levels.

On the basis of the steady state value for the output capital ratio y_o we finally get:

$$\omega_o = (1 - \bar{a}/y_o)x \tag{19}$$

$$l_o = y_o/(y/l)_o \tag{20}$$

where ω_o is the value used in the dynamic error correction mechanism (and the corresponding ρ_o in the investment equation of firms) as the relevant benchmark values.

All steady state values are positive as long as this holds for the parameter $\bar{a}$. Here it must however also be assumed that $\bar{a} < y_o$ holds true. If this parameter is increased, the steady rate of profit increases while the steady state value of the real wage declines (since output per unit of capital remains constant in such a case). There are no further changes implied by such an increase in trend investment, up to the fact that capitalist firms here always exactly get what they spend. Note furthermore that the steady state value of y depends on various adjustment speeds in the wage-price module and is in particular not defined if β_{py} becomes zero (unless equal to $\bar{y}$). This problem on the real side of the considered dynamics will disappear, when nominal adjustment problems and a Taylor interest rate policy rule are taken into account in a later section of the paper.

If – by contrast – the employment rate target of firms is decreased (because they want to exercise further pressure on the behavior of their workforce) we have that the steady state rate of employment on the external labor market is fully adjusting to this new target of firms implying in addition a fall in the steady state values of y_o, ω_o, while the steady rate of profit does not change in such a case. We thus primarily get in such a situation a lower steady state employment rate, a lower steady rate of capacity utilization and a lower steady state share of wages in national income.

4.2 Reduced-form 3D dynamics

We now assume for the following stability analysis that even $n := \hat{K}$ holds (for analytical simplicity) and have of course the same steady state solution as before (with l now frozen – by assumption – at its predetermined steady state value l_o at all points in time). The implication of this assumption is a modified law of motion for the employment rate which now integrates the capital accumulation relationship (as represented by the law of motion for full employment labor intensity ratio $l = L/K$) into the law of motion for e. The implied DAS- DAD dynamics now are:

$$\dot{y} = f(y,e,\omega) = -\beta_y(1-\frac{\omega}{x})y + (\beta_y - y)i(\cdot) + \beta_y \bar{a}, \tag{21}$$

$$\dot{e} = g(y,e,\omega) = \beta_{eu}(\frac{y}{xl_o} - \tilde{u}_f(e)e) - \beta_{e\omega}(\omega-\omega_o)e - i(\cdot)e, \tag{22}$$

$$\hat{\omega} = h(y,e,\omega), \quad h_y < 0,\ h_e > 0,\ h_\omega < 0, \quad \text{see eq. (13)} \tag{23}$$

with $i(\cdot) = i_\rho(\rho - \rho_o) - i_e(e - \bar{e}_f), \quad \rho = (1 - \omega/x)y$. Note again that we have assumed in the third law of motion, see (13), that real wage growth depends positively on the rate of employment e, i.e., we take on a Goodwin (1967) perspective in this matter. However, the influences of changing capacity utilization y are here separated from employment rate changes and act negatively on real wage growth, dominating the positive effect on this growth rate that is caused by the accompanying parallel effect on workforce utilization ratios. Note also that the first equation allows for two cases (and a borderline case), namely

$$\text{Case 1:} \quad f_y < 0,\ f_e < 0,\ f_\omega > 0 \qquad \text{or} \quad \text{Case 2:} \quad f_y > 0,\ f_e < 0,\ f_\omega < 0,$$

respectively. This is implied by the following partial derivatives

$$\dot{y}_y = c(1 - \frac{\omega}{x}), \quad \dot{y}_\omega = -c\frac{y}{x}, \quad c = -\beta_y(1 - i_\rho) - yi_\rho = (\beta_y - y)i_\rho - \beta_y.$$

We thus have that a partially stable integrated multiplier process (i_ρ sufficiently small) is necessarily coupled with a wage-led regime in the aggregate demand function and an unstable multiplier process with a profit-led regime in the aggregate demand function. These two cases thereby give rise to two typical stability scenarios in our model of Kalecki-Goodwin-Rose (KGR) accumulation and employment dynamics. The critical condition that separates these two cases from each other is given by: $i_\rho^c = \beta_y/(\beta_y - y_o) > 1$ or $\beta_y^c = y_o i_\rho/(i_\rho - 1)$ as far as steady state stability analysis is concerned. Note that the steady state of the reduced 3D dynamics is the same as the one calculated for the originally 4D dynamics (if their steady state value l_o is inserted into the 3D case).

On the basis of the assumed labor and goods market dominance in the third law of motion we indeed get for the sign structure in the Jacobian of the dynamics at the steady state (under the not very restrictive assumption $\beta_y - y_o > 0$ sufficiently large):

$$J = \begin{pmatrix} + & - & - \\ \pm & \pm & \pm \\ - & + & - \end{pmatrix} \quad or \quad \begin{pmatrix} - & - & + \\ \pm & \pm & \pm \\ - & + & - \end{pmatrix}.$$

We assume for the time being that the entry J_{23} is positive, i.e., that $\beta_{e\omega}$ is chosen sufficiently small.

In the first case we then get instability in particular if the dynamic multiplier process works with sufficient speed (β_y sufficiently large in the entry J_{11}) and thus obtain a Kaldor (1940) situation with respect to the search for bounding mechanisms (in a profit-led environment). In the other case, we have a wage-led situation in the goods market ($\dot{y}_\omega > 0$) and thus get instability in the assumed case of a labor-market led real wage dynamics if this feedback structure becomes sufficiently dominant (in the destabilizing interaction resulting from the entries J_{13}, J_{31} in the Jacobian J). In between these two situations are of course also parameter domains with local stability characteristics. The overall conclusion however is that certain bounding mechanisms (like downward money wage rigidity) have to be added in general in order to keep the dynamics an economically viable one.

Summing up we have on this level of generality a model type where trend investment and target employment of firms basically determine the long-run outcome and where the dynamics around the implied steady state positions is driven in an unstable or stable fashion by a Keynesian dynamic multiplier process that depends on income distribution and by an advanced type of wage-price spiral and the resulting real wage dynamics, with insider and outsider effects and an employment recruitment policy of firms in addition (that links inside employment rates with outside employment rates).

5 Feedback structures of KMGS and KGR dynamics. A brief comparison

In this section we briefly compare the model of this paper with the related K(eynes) M(etzler) G(oodwin) S(teindl) model introduced and investigated in Asada, Flaschel and Skott (2006). The purpose of this section is to show how these two related models compare and differ in their feedback channels and what feedback channels of a Kaleckian approach to accumulation dynamics are still missing in the KGR model of the present paper.

5.1 Feedback channels in KMGS growth

The original KMG model, see Chiarella and Flaschel (2000) and Chiarella, Flaschel and Franke (2005) for its derivation and detailed investigation, contained four important feedback chains, the interest rate channel (Keynes- vs. Mundell-effects), the real wage channel (normal vs. adverse Rose effects) and the Metzlerian inventory dynamics (which is of a multiplier-accelerator type). The KMGS model of Asada, Flaschel and Skott (2006) – like the present model – excludes two of these feedback chains (Keynes- and Mundell-effects), but introduces two new feedback chains: a dynamic Harrodian accelerator mechanism in fixed capital formation and a Kalecki-Steindl reserve army mechanism. These feedback chains interact with each other in the full 5D dynamics of the KMGS model and different feedback mechanisms can become dominant, depending on the parameters of the model.

1. The Keynes and Mundell effects: Neither the stabilizing Keynes effect nor the destabilizing Mundell effect is present in the KMGS model. The reason is simple: we have excluded there any influence of the real rate of interest on investment and consumption (and also wealth effects on consumption are ignored). Thus, although price inflation appears in the real wage dynamics, it does not affect aggregate demand.

2. A Metzler type inventory accelerator mechanism: The Metzlerian inventory adjustment process defines two laws of motion, one for sales expectations and one for inventory changes. The crucial parameters in these adjustment equations are the adjustment speeds of sales expectations and of intended inventory changes, respectively, where the first one tends to be stabilizing and second one destabilizing.

3. A Harrod type investment accelerator mechanism: This mechanism works in Asada, Flaschel and Skott (2006) through parameters in the investment equations. Increased capacity utilization leads to higher investment (both directly and via the gradual changes in the trend of capital accumulation), thereby leading to an increase in aggregate demand. As a result, sales expectations increase and produce a further rise in output and capacity utilization. Hence, a dynamic Harrodian multiplier-accelerator process here interacts with distribution effects and the Metzlerian inventory adjustment process. Trend investment can be seen as representing an investment climate – like the inflation climate – or as

slowly evolving 'animal spirits', and it may be reasonable to assume that the direct effect on current investment is stronger than the indirect effect on trend investment.

4. A Goodwin/Rose type reserve army mechanism: There are additional feedback channels of Goodwin-Rose type. The specification of aggregate demand in KMGS implied that the short-term effect of real wages on goods demand is positive (via workers' consumption). Hence, real wages will be further stabilizing if price flexibility with respect to demand pressure on the market for goods is sufficiently high and wage flexibility with respect to demand pressure on the market for labor is sufficiently low (the delayed negative effect of real wages on investment behavior will of course just establish the opposite conclusions).

5. A Kalecki/Steindl type reserve army mechanism (the conflict about full employment and its consequences:
This conflict is represented in the present paper by the parameter i_e. We here assume, as in Flaschel and Skott (2006), Asada, Flaschel and Skott (2006), that 'bosses dislike full employment'. Increases in the rate employment e thus exert a downward pressure in the investment demand function, leading to reduced economic activity and providing a check to further increases in the rate of employment.

The feedback channels 2-5 are summarized in Table 2.

1. *Metzlerian Accelerator Mechanism:* $y^e \overset{+}{\longmapsto} y \overset{+}{\longmapsto} y^d \overset{+}{\longmapsto} y^e$
2. *Harrodian Accelerator Mechanism:* $u \overset{+}{\longmapsto} \dot{a} \overset{+}{\longmapsto} \dot{u}$
3. *Goodwin/Rose Reserve Army Mechanism:* $\omega \overset{+:C(-:I)}{\longmapsto} u, e \overset{+/-}{\longmapsto} \dot{\omega}$
4. *Kalecki/Steindl Reserve Army Mechanism:* $e \overset{-}{\longmapsto} \dot{a} \overset{+}{\longmapsto} \dot{u} \overset{+}{\longmapsto} \dot{e}$

Table 2: The Feedback Structure of the KMGS model (u capacity utilization)

The full interaction of these feedback chains determines the stability of the interior steady state position of the considered model. Based on our partial analysis of the feedback channels, we expected and have confirmed in Asada, Flaschel and Skott (2006) that wage flexibility, fast inventory adjustment and fast investment trend adjustment are destabilizing, while price flexibility is stabilizing (if the corresponding Rose effect is tamed by assuming appropriate investment behavior). Manipulating the stabilizing parameters appropriately may thus help to create local stability or allow at least to ensure the boundedness and economic viability of the trajectories in the case of local instability.

5.2 The feedback structure of the KGR model of capital accumulation and employment dynamics

Comparing the feedback chains of the KGR model of this paper to the feedback channels just discussed first of all shows that Keynes and Mundell effects are ignored here too, since there is not yet a real rate of interest effect in aggregate goods demand (and since monetary policy, be it money supply oriented or interest rate oriented, is not yet an issue here).

Furthermore we use in KGR growth only a dynamic multiplier story (to describe Keynesian goods market quantity adjustment process) in place of a (possibly accelerating) Metzlerian inventory and sales expectations adjustment process. Nevertheless we can have unstable adjustment processes in the multiplier process too, if we assume that the propensity to spend in aggregate demand is larger than one. Besides no Metzlerian accelerator mechanism we also do not yet allow for Harrodian fixed capital investment accelerator processes.

As in the KMGS approach we have two reserve army mechanisms present in our KGR variant of this approach that are related of the work of Goodwin (1967) and Kalecki (1943) and that interact with Rose (1967) type real wage feedbacks:

$$\omega \xrightarrow{\pm} y^d \xrightarrow{+} y \xrightarrow{+} u_w = \frac{1}{xl_o}\frac{y}{e} \xrightarrow{+} e \xrightarrow{\pm} \hat{\omega}.$$

In this feedback channel, we have in addition (via our formulation of Okun's Law) a direct effect of real wages on the rate of employment and also a negative effect of the rate of employment onto its rate of change (via the workforce utilization target of firms) and finally of course the influence of the assumed labor supply reaction to capital stock growth. Furthermore there are opposing influences of economic activity y on the growth rate of real wages, via the goods market and price inflation dynamics and via the utilization ratio of the workforce within firms and money wage inflation dynamics. Finally there is also the ambiguous effect of changes of the rate of employment on the real wage, directly and positive through the external labor market and indirectly and partially negative through changes in workforce utilization within firms.

The Goodwin reserve army mechanism is working in this Keynesian aggregate demand environment if increase in real wages and decreases in profitability decrease aggregate demand (if this demand is profit-led) and if this decreases the rate of employment via decreases in capital stock utilization, thereby providing a check to further real wage increases. Yet, the Goodwin reserve army mechanism was initially a purely supply side phenomenon and is not easily identified in a Keynesian goods demand environment as we will see later on. Furthermore, the alternative reserve army mechanism, the one of Kalecki (1943), works independently of the shown real wage feedback channel, since it postulates that increases in the rate of employment directly decrease the growth rate of the capital stock which decreases aggregate demand, output and thereby provides a check to further increases in the rate of employment. While the real wage channel may give rise to a variety of (un-)stable feedback situations, the Kaleckian mechanism should by and large contribute to the stability of the considered steady state (though it may undermine this stability to a certain degree if labor supply is capital stock growth driven).

In closing this discussion we briefly stress again the importance of the assumed Okun-type two stage links from the goods market to the labor market which translate capital utilization ratio into labor force utilization ratios and changes in the rate of employment on the external labor market. Taking all these feedback mechanisms into account we intend to make use of them in the numerical section such that an assumed locally explosive adjustment process around the balanced growth path is turned into economically viable (bounded) dynamics through appropriate nonlinearities in the adjustment functions of the model. First of all we however consider a situation where under strong assumptions indeed a locally asymptotically stable and thus an attracting steady state can be shown to exist.

In the preceding section we have in addition investigated minor further destabilizing feedback chains caused by our assumptions on labor force growth that are however not of central importance in the present paper, since they are easily modified by way of other assumptions on this rate of growth. Nevertheless these feedback chains suggest that recruitment policies of firms (acting on the participation rate of the domestic labor force or on foreign labor markets) may indeed contribute to economic instability when coupled with certain further reaction patterns of the model.

5.3 A feedback-suggested local stability scenario

We now investigate the interaction of the feedback structures considered in the preceding subsection on the basis of the following additional assumption (besides the ones already made for real wage dynamics: $\hat{\omega}_y < 0, \hat{\omega}_e > 0$ and for the dynamic multiplier process $\dot{y}_y < 0$ $(\beta_y > y_o)$ which implied $\dot{y}_\omega > 0$).

Assumptions:

1. We first assume that $\beta_{eu}, \beta_{e\omega}$ are large enough to dominate the signs of the partial derivatives in the law of motion for the employment rate e.

2. Furthermore, we assume the parameter β_y to be large enough (and $i_\rho < 1$) such that the sign of $b = a_1(\beta_y)a_2(\beta_y) - a_3(\beta_y)$ in the Routh Hurwitz conditions is determined by $a_1(\beta_y)a_2(\beta_y)$, which is a quadratic function of β_y, while the determinant is only linear in β_y. The economy is wage led in such a situation and the dynamic multiplier stable from the perspective of the involved partial derivatives.

3. Finally, we assume that the parameters β_{we}, β_{wu} are small enough and thus not decisive for the sign of the determinant of the system at the steady state (the Rose feedback channel $J_{13}J_{21}J_{32}$ for this wage led economy is assumed as being sufficiently weak).

This in particular means that the influence of the $-i(\cdot)$ term in the equation for the adjustment of the employment rate, describing the effect of the recruitment policy of firms in their search for additional labor force supply, is not so strong that the direct effects of increased capacity utilization, increased external employment and an increasing real wage are overthrown.

For the considered Jacobian this in sum gives:

$$J = \begin{pmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{pmatrix} = \begin{pmatrix} - & - & + \\ + & - & - \\ - & \pm & - \end{pmatrix}.$$

In this Jacobian we have in the first row the negative multiplier effect $J_{11} < 0$, the Kaleckian reserve army effect $J_{12} < 0$, and the positive effect of a wage-led economy $J_{13} > 0$. In the second row we have the assumed dominance of the $\beta_e' s$ and a negative effect of real wages on (recruited) labor force growth, represented by J_{23}.

Indeed, under the assumed conditions, we can easily derive the validity of the Routh-Hurwitz conditions $(a_1, a_2, a_3, b > 0)$. We obviously have $a_1 > 0$ for the diagonal terms in J. With respect to a_2 only the term $J_{23}J_{32}$ can create instability problems. This term has however been assumed to be neglectable. This same assumption (on J_{23}) also allows to ignore a destabilizing effect in the determinant of J.

We thus in sum get that the parameters $\beta_y, \beta_e' s$ and $\beta_{w's}, i_\rho$ can be crucial for macroeconomic stability (with the former to be chosen sufficiently large and the latter sufficiently small. Yet, the opposite situation may only be one among many other where no local stability result may hold. It is therefore not unlikely that the steady state may be locally repelling and that the forces that can make such an economy a viable one must therefore be found in certain behavioral nonlinearities that limit the dynamics to economically meaningful domains when it departs too much from its steady state position.

Remark: The stability result just achieved will get lost in situations where insiders dominate the evolution of the real wage (both with respect to the rate of employment e and the rate of capacity utilization y), in particular if Blanchard and Katz error correction is weak, since an increasing parameter β_{wu} will imply a negative value for the a_2 Routh-Hurwitz polynomial parameter (if chosen sufficiently large). Insiders may therefore destabilize situation of economic prosperity if they last sufficiently long.

5.4 Consensus-driven economies: Attraction towards accepted steady state positions

The Kaleckian reserve army mechanism (like the Goodwinian one) may not be optimal for the stable evolution of modern market economies, both from the economic and the social point of view. Both mechanism correct results that are unwanted by firms and their owners through mass unemployment (and supporting economic policy) with all its consequences for the economic and social evolution of the considered society. We thus now introduce some further assumptions on the parameters of the model that characterize our model economy that imply its stable evolution around an steady state path that is satisfactory both from the workers' and the firms' point of view.

Specifically, we assume that demand pressure in the labor market (both inside and outside of the firm) does not influence the rate of wage inflation very much, i.e., the wage level is a fairly stable magnitude. Furthermore, the Kaleckian reserve army mechanism is absent from the model ($i_e = 0$). Moreover, the benchmark values for demand pressures and the employment policy of firms are all given magnitudes, consistent with each other and all sufficiently high to not imply labor market segmentation and significant disqualification of unemployed workers (this can be coupled with flexible hiring and firing policies then, i.e., the parameter β_{eu} may be chosen as large). Finally, but not really necessary, we may assume $\kappa_w, \kappa_p = 0$, i.e., cost-pressure on the market for labor as well as for goods is only relevant if it becomes permanent in nature.

These conditions imply as Jacobian matrix around such a 'satisfactory' steady state:

$$J = \begin{pmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{pmatrix} = \begin{pmatrix} - & 0 & + \\ + & - & \pm \\ - & 0 & - \end{pmatrix}.$$

It is easy to show for this case of a wage-led economy that all Routh-Hurwitz conditions are valid in this situation, i.e., the steady state of the economy is not only sufficiently good in nature, but also attracting. We thus have another case here where now a flexible labor market, a balanced workforce participation within firms and a balanced choice of working hours per week may be in harmony with each other and may work satisfactorily well in an environment that is close to balanced growth. Such consensus-driven economies can be usefully compared with economies that are equally flexible in their adjustment mechanisms, but subject to significant reserve army fluctuations as well as with economies where a variety of rigidities are working in addition.[6]

6 Local instability and global boundedness

We continue to investigate the 3D dynamics in the state variables y, e, ω :

$$\begin{aligned}
\dot{y} &= f(y,e,\omega) = -\beta_y(1-\frac{\omega}{x})y + (\beta_y - y)i(\cdot) + \beta_y \bar{a}, \\
\dot{e} &= g(y,e,\omega) = \beta_{eu}(\frac{y}{xl_o} - \tilde{u}_f(e)e) - \beta_{e\omega}(\omega-\omega_o)e - i(\cdot)e, \\
\hat{\omega} &= h(y,e,\omega), \quad h_y < 0,\ h_e > 0,\ h_\omega < 0, \quad \text{see eq. (13)}
\end{aligned}$$

where $i(\cdot) = i_\rho(\rho-\rho_o) - i_e(e-\bar{e}_f), \quad \rho = (1-\omega/x)y$ in order to turn now to the analysis of partial situations (in phases of prosperity or stagnation, respectively) where local steady state instability is sooner or alter tamed by certain turning points during the evolution of such boom or busts. We first consider reasons that lead to the instability of the balanced growth path of the investigated economy, before we study factors which may lead to turning points in these phases when the economy departs too much from the balanced growth path scenario.

6.1 Conflict-driven economies: Repelling steady state configurations

We here again consider the regime of a stable multiplier process, i.e., also of a wage-led economy, the empirically more relevant one and now indeed consider such a regime close to the steady state of the dynamics. We assume that the Kaleckian reserve army mechanism is not working close to the steady state ($i_e = 0$), that the parameter $\beta_{e\omega}$ in Okun's law is small close to balanced growth and that Blanchard and Katz error correction is weak around the steady state ($\beta_{w\omega} = 0, \beta_{p\omega} = 0$). Furthermore, insider effects are assumed to represent the dominant factor (dominating the $y-$effect) in the real wage dynamics and outsider effects dominate with respect to e in the money wage dynamics. The matrix of partial derivatives J of system (24)–(24) is therefore then characterized by

$$J = \begin{pmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{pmatrix} = \begin{pmatrix} - & 0 & + \\ + & - & + \\ + & + & 0 \end{pmatrix}.$$

[6] We add to this consideration that the investment function when estimated for the US-economy shows a significant negative influence of the rate of employment on the rate of capital accumulation I/K which indicates that this economy may not be of the type just considered (at least partially).

By and large the Rose real wage channel is thus the destabilizing force in the considered situation, augmented by a secondary destabilizing effect caused by the recruitment of labor supply according to the growth rate of the capital stock. These effects make the determinant of J unambiguously positive and thus imply local instability around the steady state by means of the Routh-Hurwitz conditions.

By contrast, a profit-led economy would have a negative determinant in the considered situation, but would in turn be unstable if the parameter β_y is chosen sufficiently large (giving rise to a positive trace thereby) or if the $\dot{\omega}_y$ effect is sufficiently weak or even negative (due to a larger parameter β_{py}), since the minors of order two of the Jacobian J then become all negative. This latter effect is again a destabilizing Rose or real wage effect, now situated in a profit-led environment.

Inspecting the assumptions made above from a broader perspective, the objective to establish divergent dynamics around the steady state is likely met for the considered KGR economy, i.e., that there will be repelling forces around its balance growth path in general (due to a variety of reasons like dominant insider behavior and more).

6.2 Kalecki-type upper turning points

In a situation of strong, but later on weakening economic growth (as it was given in the 1950's, 1960's and the early 1970's) we expect that the following sign structure in the considered Jacobian J is the relevant one (and assume $i_\rho \leq 1$ in particular).

$$J = \begin{pmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{pmatrix} = \begin{pmatrix} - & - & + \\ + & - & - \\ - & + & - \end{pmatrix}.$$

We thus consider here the stabilizing role of the Kaleckian reserve army mechanism (via $J_{12}J_{21}$ in particular), a stabilizing interaction between real wages and the rate of employment (via $J_{23}J_{32}$ in particular), a stabilizing feedback established via the dynamics of capacity utilization and wage-led goods demand (via $J_{13}J_{31}$ in particular) and only stabilizing eigen-feedbacks along the diagonal of the matrix J.

We again have a wage-led goods market dynamics, the dominance of the β_e terms in the employment dynamics and the dominance of the goods market with respect to utilization ratios and of the labor market with respect to employment rates as far as real wage dynamics are concerned. We then have a negative trace and only positive minors of order two for the matrix J. The only problematic term in the determinant of J is then given by the term $J_{13}J_{21}J_{32}$ representing again a destabilizing real wage channel within the considered dynamics. The other five products composing the determinant of J are all negative which suggests that the determinant is negative if this real wage channel does not work with extraordinary strength. A value of the parameter β_{ey} chosen sufficiently low (i.e., a recruitment policy of the firms that is sufficiently sluggish in the considered situation) may for example generate such a situation. For the Routh-Hurwitz condition

$$a_1a_2 - a_3 = (-trJ)(J_1 + J_2 + J_3) + \det J$$

we finally get that the only problematic term in this expression is given by $J_{12}J_{23}J_{31}$, since $J_{13}J_{21}J_{32}$ is positive and since all other terms in the determinant are contained in the expressions that form a_1a_2. Again it is very likely that this problematic term is dominated by the many expressions that form the remainder of a_1a_2. This is for example the case when the Kaleckian reserve army mechanism is working sluggishly (i_e small, i.e., it is only working when high employment rates are becoming persistent). One may assume here that this is coupled with a weak real wage effect on the recruitment policy of firms and also a weak entry J_{31} in the dynamics of real wages.

If these conditions become established the Routh-Hurwitz conditions would imply local stability of the steady state. Yet we assume these conditions to prevail in a boom phase far off the steady state in which case we can only speculate (and test this speculation numerically) that this contributes to global stability by implying an upper turning point for the considered phase of the long cycle in income distribution and factor utilization ratios.

Of course there may come supply side bottlenecks in existence in addition in the considered situation, given by the conditions $e \leq 1, y \leq y^p$ that may help to enforce upper turning points if these bounds to the employment rate and the ratios of capacity utilization are reached. Furthermore $\tilde{u}_f(e)$ may become steep at or close to the ceiling of absolute full employment $e = 1$.

6.3 Goodwin-type upper turning points?

In the original Goodwin (1967) growth cycle model one considers a Classical profit-led economy where aggregate Keynesian goods demand does not yet play a role and where therefore the term profit-led simply means that reduced profit slow down capital stock growth and the growth rate of the economy (and vice versa). This leads to increasing unemployment which sooner or later corrects income distribution again in favor of higher profitability. The turning points in the classical growth cycle model are therefore solely a consequence of changing income distribution, while production is always at full capacity.

This is different in the model type considered in this paper, since profit-led here means that the investment parameter i_ρ is so large (measured relative to the adjustment speed β_y) that at one and the same time we have that real wage increases act negatively on (Keynesian) aggregate goods demand and that the dynamic multiplier process – considered in isolation

– is unstable, due to the strong influence of income y on aggregate goods demand. Profit-led economies are therefore in this case plagued by partial multiplier instability which may be so strong that the trace of the Jacobian becomes positive in which case this partial instability dominates the overall outcome.

In the profit-led economy we have the following sign distribution in the Jacobian J in the case of a booming economy (for which we assume that insider-outsider effects are such that real wage growth depends both positively on the utilization ratio of the workforce within firms as well as on the employment rate characterizing the external labor market.[7]

$$J = \begin{pmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{pmatrix} = \begin{pmatrix} + & - & - \\ + & - & - \\ + & + & - \end{pmatrix}.$$

Manipulating (reducing appropriately) the speed of adjustment β_y characterizing the dynamic multiplier process (and the first row of the above matrix J) one can then first of all achieve that the trace of J becomes negative. In the same way one can also ensure that the principle minors of order two are all positive. Concerning the determinant of J we also get in this way (since J_{11}, J_{13} are thereby reduced simultaneously) that the expression $-J_{12}(J_{21}J_{33} - J_{23}J_{31})$ becomes the dominant term in $\det J$. Yet, this remaining term has contradicting signs in its two product expressions (the first one is negative and thus supportive, while the second one is positive and thus dangerous for economic stability). Manipulating the speed term β_y is thus not sufficient for stability and the creation of eigenvalues with only negative real parts.

There is finally again the Routh-Hurwitz condition $-trace\ J(J_1 + j_2 + j_3) + \det J$ to be considered. Here only the terms $J_{12}J_{23}J_{31} + J_{13}J_{21}J_{32})$ in $\det J$ can create problems, since all other ones cancel against some of the all positive terms in the $-trace\ J(J_1 + j_2 + j_3)$ expression. And with respect to these remaining terms only the second one is negative and thus problematic for stability. The two terms $J_{12}J_{23}J_{31}), J_{13}J_{21}J_{32}$ must therefore be made small relative to their respective counterparts in order to ensure the stability of the presently considered dynamics beyond which what can be achieved by lowering the adjustment speed of the dynamic multiplier appropriately in the range of a profit-led regime.

Closer inspection however again reveals that the size of J_{13} can again be manipulated via an appropriate reduction of the speed parameter β_y such that the remaining terms in $-trace\ \ J(J_1 + j_2 + j_3)$ will dominate the second of the above problematic terms. Moreover, choosing the parameter $\beta_{e\omega}$ in an appropriate range, may eliminate the final

[7] We also assume again that the β_e-terms are the dominant ones in our formulation of the dynamics of the employment rate.

problem for stability and thus turn all real parts of eigenvalues of the Jacobian J into negative magnitudes.

We stress however once again that the Jacobian has only been evaluated properly at the steady state and that this therefore only proves the local asymptotic stability of this steady state position. Yet, numerical experience with growth models of this type suggests that such results apply also far off the steady state position and thus at least give rise to the hope that one can enforce turning points in economic activity (in a period of accelerating growth) by assuming the above parameter restrictions to hold sufficiently far above the steady state position.

Overall however our finding is that Keynesian profit-led regimes may be plagued by multiplier instability to an extent that does not bring about trajectories that are bounded from above. Policy action – of the type described in Kalecki (1943), but also monetary policy – may therefore be needed to enforce upper turning points in such periods of strong economic growth and the significant changes in the capital-labor relationship that are implied thereby.

6.4 Rose-type lower turning points

In situations of a depressed economy we may by contrast find that the following sign structure in the Jacobian J may apply.

$$J = \begin{pmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{pmatrix} = \begin{pmatrix} - & 0 & + \\ + & - & + \\ - & 0 & 0 \end{pmatrix}.$$

The Kaleckian reserve army mechanism is surely absent in such a situation ($i_e = 0$) as may be the Blanchard and Katz error correction terms. Furthermore, the dependence of real wages on the rate of employment e may then be weak, since the $\beta_w' s$ are then sufficiently small. In such a situation we immediately get that the coefficients $a_1, a_2, a_3, a_1a_2 - a_3$ in the Routh-Hurwitz conditions are all positive. The basic stabilizing mechanism is then a normal real wage or Rose effect which stimulates the economy if real wages begin to increase due to falling price levels.

The considered situation would again imply local asymptotic stability around the steady state, but is here taken once again as an indication that there are tendencies for a recovery in the considered depressed state of the economy which give rise to a lower turning point in economic activity. This tentative result must of course be checked by numerical simulations of the model that based on behavioral nonlinearities that establish the considered situations for lower (and upper) turning points far off the steady state.

6.5 Goodwin-type lower turning points?

In the profit-led economy in its depressed phase we assume the following sign distribution in the Jacobian J.[8]

$$J = \begin{pmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{pmatrix} = \begin{pmatrix} + & 0 & - \\ + & - & + \\ + & 0 & 0 \end{pmatrix}.$$

We have assumed in this matrix that the Kaleckian reserve army mechanism is not working in the downward direction (since it only characterizes the economic – and political – aspects of full employment) and that the influence of the rate of employment on the dynamics of the real wage can be neglected in this state of the economy. Furthermore, Blanchard and Katz error correction is assumed to be sufficiently weak in the depression.

Dynamic multiplier instability can then be tamed by assuming that β_y is chosen such that the trace of J is negative and such that the term $J_{13}J_{31}$ dominates the principal minors of order two. In the considered situation we moreover always have that the determinant of J is negative and is part of the a_1a_2 expressions, implying that the Routh Hurwitz conditions can be fulfilled in this case by an appropriately low choice of the size of the parameter β_y within the profit-led goods demand regime. The case of lower turning points is thus easier to handle in this regime than the case of upper turning points, though, here too, much depends on the speed with which firms adjust their output decision towards their observation of aggregate goods demand.

7 Numerical examples

We have shown in the preceding section that the steady state of the considered KGR dynamics may likely be an unstable one and that there are a variety of possibilities that may nevertheless keep the resulting dynamics bounded during prosperity or depression and thus economically viable. The actual occurrence of such bounding mechanism may change in time and thus be confined to certain episodes in the evolution of capitalistic market economies. Moreover, monetary or fiscal policy may also be of importance in the factual explanations of the occurrence of such regime changes in the postwar evolution of industrialized economies (for example from the welfare state to Reagonomics and economic and social deregulation), see the next section in this regard. The relevance of the model

[8] We assume now that only the β_{eu}-term is dominant in our formulation of the dynamics of the employment rate (but that the role of $\beta_{e\omega}$ is neglectable).

of this paper is therefore not so much given by the implication of a unique cyclical pattern of capital accumulation and employment dynamics around its steady state position, but by its flexibility to explain a variety of partial scenarios in the evolution of capitalism after World War II. Clearly, though this is welcome from an economic point of view, the mathematical and numerical analysis of the dynamics is thereby made more complicated and unattractive as compared to other cycle models, like the Goodwin (1967) growth cycle dynamics, and thus less appealing from the mathematical point of view.

In this section we briefly consider in addition to our analytical results some numerical simulations of the model of this paper on its 3D reduced form level. We simplify the model somewhat by assuming fixed ratios $\bar{e}, \bar{u}_w, \bar{y}, \bar{e}_f, \bar{u}_f$ for the various benchmark comparisons in the wage and price PC, in the investment function and in Okun's law and assume in addition a situation where the steady state value of y is equal to $\bar{y}$. We thus make use in this section of the following parameter set:[9] $\beta_{we} = 0.5, \beta_{wu} = 0.6, \kappa_w = .5, \bar{e} = 0.9, \bar{u}_w = 0.9, \beta_{py} = 0.385, \kappa_p = 0.5, \bar{y} = 0.9, \beta_y = 1.2, \beta_{ey} = 0.3, \bar{u}_f = 0.9, \beta_{ew} = 1.5, x = 2, y^p = 1, i_\rho = 1.5, i_e = 0.1, \bar{a} = 0.3, \bar{e}_f = 0.9.$

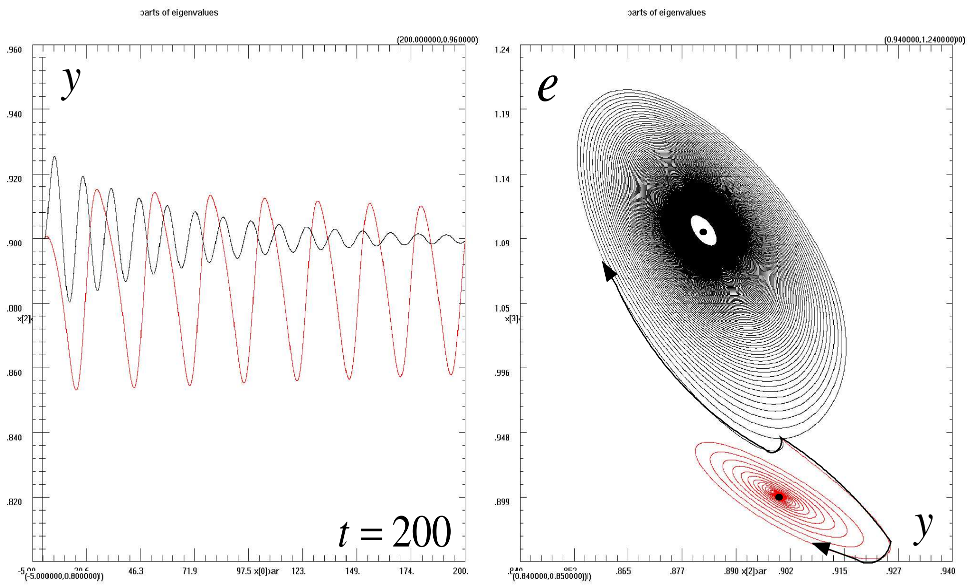

Figure 1: Damped fluctuations in Kaleckian investment and employment dynamics

In figure 1 we show for wage adjustment speeds $\beta_{we} = 0.5, \beta_{we} = 1$ the damped oscillations that result from these 2 adjustment speeds. The unexpected result of this simulation run is that there are indeed multiple stead state solution as the model is formulated, one

[9]We note with respect to this parameter set that the parameter $\bar{a} = \rho_o$ may be considered as much too high from the empirical point of view, though it also represents a profit share of 30 percent. Yet, since this rate also represents the growth rate of the economy it is of course of exceptional size. The model therefore needs reformulation, by adding deductions from profits that drive a sufficient wedge between its rate of profit and the resulting rate of growth.

economically meaningful solution for the lower adjustment speed and one that runs into supply bottlenecks in the labor market and thus cannot remain of the form shown in the fluctuation around the higher steady state value for e in figure 1. We thus get that increasing wage flexibility with respect to demand pressure on the external labor market may destabilize the steady state considered in this paper and lead to cyclical convergence to another interior steady state with a higher NAIRU level $\bar{e}$ on the labor market and a lower NAIRU level $\bar{y}$ on the market for goods. Such results hold for a large range of adjustment speeds β_y on the market for goods and can also occur when the parameter β_{we} is decreased instead of its increase considered above. Note finally that the movements in capacity utilization and the rate of employment are far from being positively correlated due to our extended formulation of Okun's law.

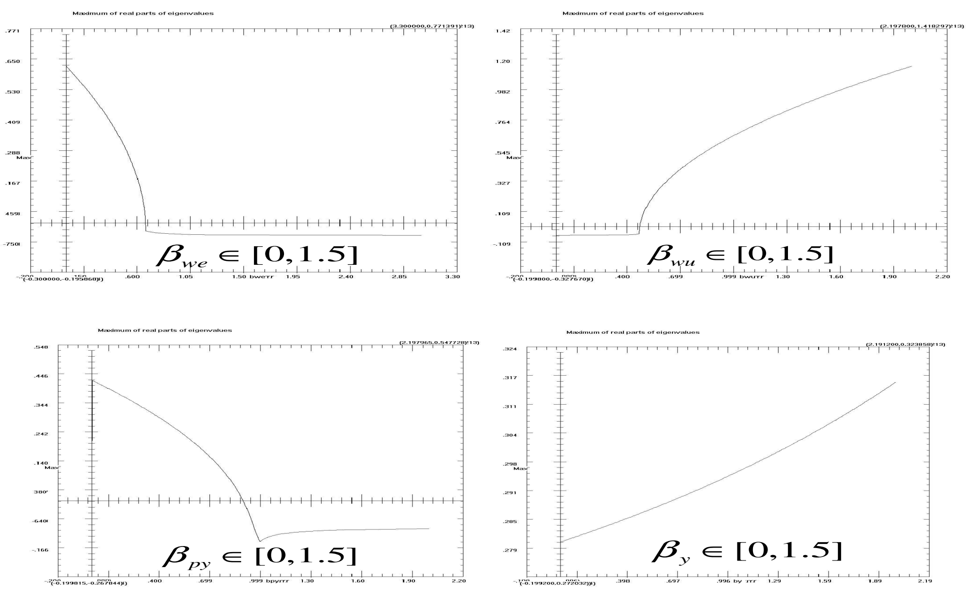

Figure 2: Eigenvalue diagrams for selected speeds of adjustment.

In figure 2 we consider eigenvalue diagrams – showing the maximum real part of eigenvalues as function of selected adjustment speeds – in order to show where the steady state becomes locally unstable (when the considered real part becomes positive). We can see from this figure that increasing the parameters β_{we}, β_{py} is stabilizing the economy, while we have to decrease the parameter β_{wu} in order to achieve this. The parameter β_y, when increased, is not stabilizing the economy, in contrast to what one would have expected from the partial derivative of $\dot{y}$ with respect to y, which is negative. Our base parameter set is chosen such that our steady state solution is unstable – as is indicated by the eigenvalue diagram for β_y, but leads nevertheless to convergent fluctuations as figure 1 shows, yet convergence to a steady state for which the eigenvalues have not been calculated here.

In figure 3 finally, we show further eigenvalue diagrams, now for the benchmark levels that define NAIRU type utilization rates in the labor and in the goods market as well as the benchmark value for the employment rate in firms' investment behavior. We obtain from the se diagrams that higher NAIRU levels in the labor market are destabilizing the economy further, while a higher NAIRU level in the goods market reduces the explosiveness of the steady state considered in this paper, given through the choice of our base parameter set. Moreover, the result we here obtain for increases in the benchmark level $\bar{e}_f$ by which firms judge whether there is pressure on them due to changing work conditions is in fact stabilizing the economy (while increases in the reaction coefficient i_e – not shown – is not). We conclude that the implications of the dynamics considered in this paper can be numerous and need not confirm what is suggested through partial reasoning (concerning isolated entries of the Jacobian matrix of the dynamics at the steady state).

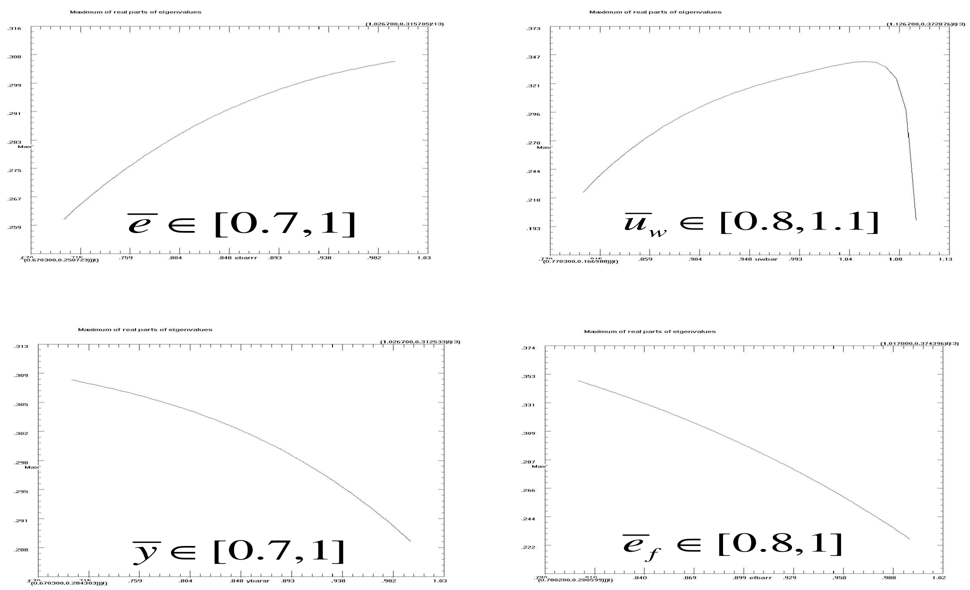

Figure 3: Eigenvalue diagrams for selected benchmark values of the model.

8 Political aspects of the Kaleckian investment and employment cycle

In this paper we have shown that the private sector of a capitalist economy may generate long cycles in investment in employment with specific upper and lower turning points, possibly depending on the specific historical period to be investigated by this model type. Missing so far in the model's laws of motion was however any influence from monetary or fiscal policy, the first due to our complete neglect of interest rates and financial markets' dynamics and the second, since neither government expenditure not taxes or other means of financing this expenditure were taken into account so far. Yet, besides the purely economic aspects of full employment and resulting investment and employment cycles, the political aspects of these issues – as they were briefly and brilliantly described in Kalecki (1943) – must also be taken into account in any thorough discussion of such cycle mechanisms.

The analysis of the dynamics of the private sector we have studied in this paper is moreover in one important respect still partial and thus not yet complete even on this level, since it does not investigate what is happening on the nominal side of the model. We have derived in section 3 the reduced-form real wage dynamics that results from our wage price spiral mechanism, but have not yet considered explicitly the underlying reduced form equations for money wage and price level inflation rates (from which the real wage dynamics has been derived). With respect to nominal price inflation in particular, the reduced-form expression explaining the forces behind price inflation reads:[10]

$$\begin{aligned}\hat{p} = \frac{\dot{p}}{p} &= \pi^c + \kappa[\beta_{py}(y-\bar{y}) + \beta_{p\omega}\ln(\omega/\omega_o) \\ &\quad +\kappa_p(\beta_{we}(e-\bar{e}) + \beta_{wu}(u_w - \tilde{u}_w(e)) - \beta_{w\omega}\ln(\omega/\omega_o))]\end{aligned}$$

In order to allow for steady inflation rates the term following π^c in the above formula must be zero in the steady state which gives for the output capital ratio in such a situation:

$$y_{oo} - \bar{y} = \frac{\kappa_p(\beta_{we}(e_o - \bar{e}) + \beta_{wu}(\tilde{u}_f(e_o) - \tilde{u}_w(e_o))}{\beta_{py}} = \frac{(1-\kappa_w)\kappa_p}{1-\kappa_p}(y_o - \bar{y})$$

A comparison with the steady state value of y_o however immediately reveals that this output - capital ratio is equal to the steady state ratio only in very special cases (concerning the weights in the cost pressure terms of both wage and price inflation), while we will have smaller or larger values of this y compared to y_o in general (for example $y_{oo} = \bar{y} + 0.5(y_o - \bar{y})$ in the balanced case where $\kappa_w = \kappa_p = 0.5$). If the real part in the economy is in the steady state, we may thus have persistently falling inflation rates ($\hat{p} < \pi^c$) or rising inflation rates ($\hat{p} < \pi^c$) in the nominal part of the economy, a process that surely cannot go on for ever. Political pressure may therefore come into being that may induce an adjustment of $\bar{e}, \bar{e}_f$ and of $\tilde{u}_{wo}, tildeu_{fo}$ towards each other, respectively (or may fail to do so). In this case we then get that

$$y_o = \bar{y}, \quad \bar{e} = \bar{e}_f, \quad u_{wo} = u_{fo}$$

will be established, i.e., there is only one NAIRU on the external and the internal labor market of firms and also with respect to the capital stock utilization ratio and these common benchmark levels are - as the model is presently formulated and applied – completely determined by the benchmark utilization and employment ratios as they are set by firms, if firms cannot be forced to alter their target levels in this respect.

[10] A similar expression applies to the reduced-form wage level inflation rate, but need not be discussed explicitly unless wage policy is taken into account in addition to monetary and fiscal policy.

8.1 Monetary policy

We now however assume that the monetary authority can influence economic activity, via its interest rate policy, by making investment depending on the actual real rate of interest in the following way:

$$I/K = i(\cdot) + \bar{a} = i_\rho(\rho - \rho_o) - i_e(e - \bar{e}_f) - i_r((r - \hat{p} - \rho_o) + \bar{a} \tag{24}$$

As interest rate policy of the central bank we assume in addition the following classical type of Taylor rule:

$$\begin{aligned} r^* &= \rho_o + \hat{p} + \alpha_p(\hat{p} - \bar{\pi}) && (25) \\ \dot{r} &= \alpha_r(r^* - r) && (26) \end{aligned}$$

The target interest rate of the central bank r^* is here made dependent on the steady state real rate of interest (the real rate of profit) augmented by actual inflation back to a nominal rate, and is as usually dependent on the inflation gap. With respect to this target there is then assumed an interest rate smoothing of strength α_r. In the present paper we will only consider an extreme case of such a Taylor interest rate policy rule, namely the limit case $\alpha_r = \infty$ of no interest rate smoothing (the other limit case being an interest rate peg). In this case we have

$$\begin{aligned} r &= \rho_o + \hat{p} + \alpha_p(\hat{p} - \bar{\pi}) \\ I/K &= i_\rho(\rho - \rho_o) - i_e(e - \bar{e}_f) - i_r\alpha_p(\hat{p} - \bar{\pi}) + \bar{a} \\ \hat{p} &= \pi^c + \kappa[\beta_{py}(y - \bar{y}) + \beta_{p\omega}\ln(\omega/\omega_o) \\ &\quad +\kappa_p(\beta_{we}(e - \bar{e}) + \beta_{wu}(u_w - \tilde{u}_w(e)) - \beta_{w\omega}\ln(\omega/\omega_o))] \end{aligned}$$

The end result of such an extension of the model towards a treatment of monetary policy in its context is that the function $i(\cdot)$ is now extended by a third term whereby the negative influence of the employment rate on the rate of investment is enhanced and augmented by similar effects of u_w and y on it (with the same negative sign) and finally that the inflation climate π^c and its adaptive revision given by the law of motion

$$\dot{\pi}^c = \beta_{\pi^c p}(\hat{p} - \pi^c)$$

is now feeding back into the real part of the economy.

We see that the negative influence of economic activity (measured by the employment rate and now also the utilization ratios of both labor and capital) on the rate of investment is thereby made a stronger one, which possibly adds further stability to the dynamics according to our discussion of its turning points (as a substitute for the traditional Keynes-effect in models of exogenous money supply). Furthermore, the indirect inclusion of the inflation climate among the determinants of the rate of investment now adds a Mundell effect to the considered dynamics, but a stabilizing one, since – due to our formulation of monetary policy – increasing inflation and thus an increasing inflationary climate exercise a negative influence on investment and thus on economic activity and thus provide a check to further inflation. Monetary policy may thus add further stability to the considered dynamics, in particular on the nominal side of its working which is now fully integrated into them.

More important however is the impact of the added monetary policy on the steady state behavior of our economy.[11] In order to provide a simple illustration for this assertion we disregard here the Blanchard / Katz error correction terms in the wage and price Phillips curves. The above law of motion for the inflation climate and the law of motion for real wages (24) now provide to independent equations for the determination of the steady state values of e, y. Assuming for the influence of insider effects on the wage inflation rate

$$\tilde{u}_w(\bar{e}) = \bar{y}/(x l_o \bar{e}), \quad i.e.,$$

a consistency requirement for the benchmark value of the utilization ratio of the workforce in the steady state, gives for the steady state values of both e, y the values $\bar{e}, \bar{y}$ (since $\rho_o = \bar{a}$ continues to hold true). From the $\dot{e} = 0$-equation we then furthermore get the steady state value for labor intensity l_o. Finally, the determination of e_o through the level $\bar{e}_f$ now disappears from the set of steady state conditions, which now imply by means of $i(\cdot) = 0$:

$$i_e(e_o - \bar{e}_f) + i_r(\hat{p}_o - \bar{\pi}) = 0$$

This gives now an equation for the determination of the steady state rate of inflation in its deviation from the target rate of the central bank (and it also provides the steady state value of the nominal rate of interest i_o by $\rho_o + \hat{p}_o$).

The addition of interest rate effects on investment behavior and a specific interest rate policy rule thus imply that steady state levels of the employment rate and the utilization ratios of both workers and capital are now given by their NAIRU levels and no longer by the target rate of firms for their control of the social structure of accumulation. Furthermore, this latter target $\bar{e}_f$ now determines the steady state rate of inflation (together with the NAIRU $\bar{e}$ and the target rate $\bar{\pi}$ of the central bank and it makes this rate the higher the less strict the employment target of firms in their investment function becomes. Lowering the NAIRU $\bar{e}$ itself can be seen in addition to imply the steady state rate of inflation due to the relationship

$$\hat{p}_o = i_e(\bar{e}_f - \bar{e})/i_r + \bar{\pi}, \quad i.e.,$$

[11] We expect the steady state to be locally unique, since its defining 5 equations should imply a Jacobian with non-zero determinant, but cannot say anything on global uniqueness for the moment.

the model exhibits a long run Phillips curve that is negatively sloped (and which allows for zero steady state inflation if $\bar{e}_o = \bar{e}_f + i_r\bar{\pi}/i_e$ holds true).

The case of an interest rate policy becomes considerably more complex if a Tobinian portfolio sector is added to the model whose return characteristic influence investment behavior in addition. Monetary policy must then work its way through the portfolio substitution process and it moreover then depends on the way long-term interest, Tobin's q and other important variables determined by portfolio choice and asset accumulation feedback onto the investment decision of firms. Such complications for the conduct of monetary policy must however be left for future research here, see Chiarella, Flaschel, Hung and Semmler (2006) on these matters. The direct control of investment behavior in this model type as well as in models of the New Keynesian variety represents however only a first step into an analysis of the implications of monetary policy for economic activity and capital accumulation.

8.2 Fiscal policy

Concerning fiscal policy Kalecki (1943, ch.12) gives a variety of reasons why industrial leaders may be opposed to full employment achieved that is possibly by it. In view of our model and its formulation of this opposition even on the level of private investment formation it is however somewhat astonishing to note that a government expenditure function of fairly Keynesian type may – like monetary policy – provide further strength to the negative impact of the rate of employment on the rate of capital accumulation and may therefore be in line with what the industrial leaders may have instinctively or consciously (through their common insight) established as reaction an increase of the rate of employment towards the rate of (absolute) full employment. In order to substantiate this assertion we assume as fiscal policy rule the following expenditure function:

$$G/Y = (G/Y)_o - \alpha_g(e - \bar{e}_g), \quad i.e., \quad g = G/K = (G/Y)_o y - \alpha_g(e - \bar{e}_g)y.$$

where we assume that the component of $(G/Y)_o = (T/Y)_o$ in national income is purely tax-financed (and the remainder through changes in government debt and open market operations of the central bank which again impact on the portfolio choice of asset holders not yet considered in this paper). Part of the above expenditures may be infrastructure expenditures which expand the capital stock like private investment and can thus be added to the $i(\cdot)$ term again (as in our formulation of monetary policy), while the remainder may represent unproductive government expenditures to be added to the aggregate demand expression y^d directly. In both cases however we introduce a term into the goods market dynamics which enhances the working of the term $i_e(e - \bar{e})$, if $bare_g = \bar{e}_f$ is assumed. A proper Keynesian fiscal policy rule (whereby government spending is increased in during periods of stagnation and decreased in periods of prosperity) thus works in principle as industrial leaders would prefer it to work as far as situations of an increasing employment rate is concerned.

The real difference between such policies and the intentions of industrial leaders therefore may be found in the fixing of the benchmark level $\bar{e}_g$ from which onwards government expenditure will be larger than the tax-financed portion. Considering fiscal policy in Germany under Chancellor Willy Brandt for example one can even find evidence for a target rate $\bar{e}_g$ equal to '1' and considering the behavior of other governments in this period surely a rate much above the level that industrial leaders will consider as consistent with their view on the evolution of the social structure of accumulation (that does not increase – but will in fact even reduce – workforce participation, does not decrease labor market flexibilities and the like. The opposition of firms and of their managers may be more directed against the level of $\bar{e}_g$, then against the fact that government pursues a Keynesian policy in the ups and downs of the business cycle which may help to stabilize their goods demand around a level compatible with their view of a convenient level of unemployment on the external labor market and his implications for the labor market within firms.

Note that the simplest way to introduce the above fiscal policy into the model is to assume that $(G/Y)_o = (T/Y)_o$ represents government consumption and is financed by taxing wages and the rest of government expenditure is assumed to be debt financed and used for investment purposes, since this leaves total goods consumption unchanged and adds the cyclical component of government expenditures again to the investment behavior without changing its determinants significantly (if $\bar{e}_g = \bar{e}_f$ is assumed). This latter assumption may be justified in steps – by way of a long phase of significant unemployment on the labor market which removes the tendencies concerning workforce participation, workforce firing and workday reductions established during the prosperity phase of the Kaleckian investment and employment cycle. If fiscal policy is moreover conducted symmetrically over the considered cycle the evolution of government debt may only become a problem if downturns are long and upswings are short. In such a case a more conservative (asymmetric) expenditure rule may also be established which does not give depressions less weight than upswings as far as countercyclical expenditure management is concerned. In this way the social structure of accumulation may be adjusted towards a growth path that allows again for a prosperity period sooner or later. Note here however again that we have so far bypassed all problems that may be implied by the working of the financial markets and the portfolio choice of asset holders – nationally or even more in periods of rapid globalization – for the real macroeconomic activity on the goods and the labor markets.

9 Conclusions

We have considered in this paper a model of the Keynes/Kalecki-Goodwin/Rose variety with an advanced type of wage-price spiral, a simple dynamic multiplier story – based on a theory of aggregate demand that includes Kaleckian (1943, ch.12) opposition of industrial leaders to the establishment of long phases of full employment on the labor market – and an advanced type of Okun's law linking goods and labor markets. We have investigated extensively the reduced form 3D real dynamics implied by the model, their local and global stability features and the partial feedback structures that underlie these stability considerations. Of particular interest was the case of local instability of the steady state solution and the derivation of various scenarios that may allow for upper as well as lower turning points from a global point of view, in order to make the considered dynamics an economically viable one. In section 7 we moreover provided some numerical illustrations of the model. In the preceding section we finally pointed to some problems on the nominal side of the considered economy that were not yet feeding back into the real dynamics and that demanded therefore for an extension of the model concerning interest effects on aggregate demand and a Taylor interest rate policy rule in order to tame the explosive dynamics shown to exist in the nominal part of the dynamics. Besides monetary policy we there also briefly considered the role of fiscal policy of a Keynesian type which augmented the private sector of our Keynes/Kalecki-Goodwin/Rose economy in a simple and straightforward way.

Kalecki's (1943) essay on the political aspects of full employment was thus one of the focal points of this paper, yet was still treated here in a very basic format, since the paper was primarily concentrated on the economic aspects of full employment. These concerned the role of longer phased cycles of prosperity and stagnation, as already considered in Asada, Flaschel and Skott (2006), based on a theory of aggregate demand that included Kalecki (1943, ch.12) opposition of industrial leaders to the establishment of long phases of full employment on the labor market, and cyclical evolutions in the kind and degree of the welfare state and its various aspects, quite in contrast to what is normally investigated under the heading 'business fluctuations'. We by and large view the employed workforce, the insider, to represent the dominant element that destabilizes long-lasting situations of full employment, leading in particular to reactions of industrial leaders and the government (including the central bank) that bring about upper turning points in economic activity and subsequent strong recessions or even depression in such booming economies. Yet, we have seen that a variety of further reasons may be involved in explaining actual pronounced turning points in economic activity, upper as well as lower ones, in particular as whether the real wage dynamics are goods or labor market lead and the goods and labor market dynamics are wage or profit led. There may therefore specific historical reason be involved that explain lasting changes in labor market and inflation regimes which makes the analysis conducted in this paper considerably less closed than comparable models of the ordinary business cycle.

An important further topic of the paper was, once it is assumed that the rate of employment on the labor market is adjusting only gradually to rate of capacity utilization of firms, that one has to introduce the utilization ratio of the workforce in order to get a meaningful chain of events that lead from the utilization of the capital stock of firms to this rate of employment. As in a capillary system there are two labor market pressures for firms, one outside and one inside measure, which may be treated and manipulated in different ways by firms during phases of prosperity or depression. Such an extension of Okun's law is a necessary step in any approach that wants to formulate the link from goods to labor market on a theoretically coherent basis and was thus nearly a compelling step in the present framework. During phases of prosperity, however, the dominance of insiders in the wage-price spiral may lead to the partially destabilizing situation that real wage changes may depend negatively on the rate of employment and positively on the rate of capacity utilization, in contrast to what is expected in a framework that does not allow for insider effects in the dynamics of goods and labor markets, where therefore employment is always immediately adjusted to the activity level implied by the working of the market for goods.

A final issue that was only treated briefly in this paper concerned the distinction between conflict-driven and consensus-driven economies. The first type of economy exhibits a repelling steady state path and thus demands for nonlinearities a least far off the steady state in order to keep its trajectories bounded. Such economies generate order by more or less large scaled cycles in employment and economic activity, that are endogenously generated and subject to supportive monetary or fiscal policy. The question here is whether this is a socially acceptable way of running the economy (disregarding efficiency issues) or whether ways have to be found to allow the economy to progress to a situation which we have called a consensus-driven economy in this paper.

Consensus-driven and conflict-driven economies can be further differentiated as shown in the table 3. There we distinguish, besides 'Scandinavian type' consensus-driven economies and strictly conflict-driven countries, between developed economies which are very flexible, but exhibit an unstable steady state position and those which are fairly inflexible around a stable depressed steady state situation.

	high steady state	low steady state
stable steady state	Nordic consensus-driven economy	Kaleckian market economy type I
unstable steady state	Kaleckian market economy type II	Southern conflict-driven economy

Table 3: Four types of market economies.

They may be even further differentiated on the basis of the details of their steady state position. Such and further questions must be left for future research here however. In these and other respects the paper thus provides an introduction into Kaleckian long phased cycles (not business cycles as he has characterized the situation) with more or less welfare state features in their prosperity phase and more or less deregulation processes in their stagnation phase. We hope that the classifications provided in this paper will stimulate further work on the Kaleckian (1943) type of reserve army mechanism as it has been distinguished here from a Goodwin (1967) type of reserve army mechanism (that is driven by the conflict about income distribution).

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11 Appendix: Routh Hurwitz stability conditions

Theorem (Routh-Hurwitz stability conditions for a three-dimensional system)

All of the roots of the characteristic equation of the Jacobian of a dynamical system

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$$

have negative real parts if and only if the set of inequalities

$$a_1 > 0, \quad a_3 > 0, \quad b = a_1 a_2 - a_3 > 0 \tag{27}$$

is satisfied.

Remark on theorem A.5:

The inequality $a_2 > 0$ is always satisfied if the set of inequalities (27) is satisfied.