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**Keynesian Macroeconomics Without the LM Curve:  
Implications of Underlying Open Market Operations**

by

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# Keynesian Macroeconomics Without the LM Curve: Implications of Underlying Open Market Operations

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## **Abstract**

The paper relates to the burgeoning literature that combines an interest rate reaction function of the central bank with an IS equation and a Phillips curve relationship. It takes up the deterministic prototype model advocated by J.B. Taylor and D. Romer and, under the assumption of open market operations, makes the implied dynamics of bonds and high-powered money explicit. As a minor extension, consumption, *via* disposable income, is supposed to depend on the interest payments on bonds. The resulting dynamic system is possibly totally unstable, that is, no coefficients in the Taylor rule are able to achieve local stability. A numerical investigation demonstrates that stability as well as instability can be brought about by fairly reasonable parameter values. On the other hand, full convergence and divergence are both extremely slow. This implies that, practically, there is a whole continuum of stable equilibria, such that the bond dynamics can be said to exhibit near-hysteresis.

*JEL classification:* E 12, E 32, E 52.

*Keywords:* Taylor rule, government budget restraint, bond dynamics, local stability, hysteresis.

# 1 Introduction

The traditional IS–LM model and its various dynamic extensions have been designed to understand macroeconomic fluctuations. Regarding monetary policy, this framework is based on the supposition that the central bank targets the money supply. In recent times, however, a broad consensus has emerged that most central banks follow an interest rate rule. This observation calls for an alternative to the IS–LM approach which reverses the causality in the money market equilibrium condition. Rather than use money as an instrument and have the money market determine the rate of interest, monetary policy now sets the interest rate and the equilibrium condition serves to determine the corresponding quantity of the money supply. It is evident that the LM curve, which on the basis of a given money supply relates the market clearing interest rate to the output levels, becomes obsolete in this way. In a recent paper, David Romer (2000) emphasizes the advantages of this “Keynesian macroeconomics without the LM curve”. Besides addressing the weakness of IS-LM that it assumes money targeting, he, in particular, argues that the new approach also makes the treatment of monetary policy easier, in that it reduces the amount of simultaneity and gives rise to dynamics that are simple and reasonable.

Romer’s discussions and graphical illustrations amount to a condensed deterministic version of the (stochastic) models underlying a burgeoning branch of investigations surrounding the Taylor rule, which in addition to this rule employ a dynamic IS equation and a Phillips curve relationship. In fact, this outlines “a distinctive modern form of macroeconomics that is now being used widely in practice” (Taylor, 2000a, p.93). All these systems have in common that they do not include the movements of money and bonds that are implied by the open market operations enforcing the time path of the interest rate. These variables remain in the background since possible feedbacks from them on the rest of the economy are considered of secondary importance.

The present paper puts this assumption under closer scrutiny. Our starting point is a slight extension of the simple deterministic model put forward by Romer (2000) and Taylor (2000a, 2001), which exhibits a stable steady state position. In this context, the paper first makes the bond dynamics explicit, as it is derived from a government budget restraint and the motions of the monetary base. Asking for the conditions under which, in the appendix to the main system, bonds and money, too, converge to their steady state values, it is readily established that this should not be a great problem. Matters become more involved when subsequently aggregate demand in the model is differentiated by supposing that consumption depends on disposable income, part of which in turn is given by the interest payments. Though this provides a minimal feedback of bonds on the demand side, it turns out that local stability of the steady state is no longer easily warranted. There are meaningful sets of numerical parameters that cause the steady state

to be unstable, regardless of how strong the interest rate reactions in the central bank's policy function may be chosen.<sup>1</sup> Even more important, however, is the medium-run behaviour of the economy. Whether in the long-run stability prevails or not, here it is appropriate to characterize the dynamics of bonds as (near-) hysteresis.

The material is organized as follows. The next section recapitulates the Romer-Taylor setting in continuous time. Section 3 introduces bonds and high-powered money together with a money multiplier, a money demand function, and the government budget restraint. Section 4 derives the bond dynamics in intensive form, which is still a mere appendix to the inflation dynamics of the Romer-Taylor model, and establishes a condition for its stability. Section 5 modifies the IS equation; as has just been mentioned it makes consumption, *via* disposable income, dependent on the interest payments on bonds. In this way, the dynamics of bonds and the rate of inflation are interrelated. The analysis of the resulting two-dimensional system of differential equations leads to conditions for the local stability, or instability, of its steady state position. Since the corresponding inequalities are rather complicated, Section 6 takes some trouble to set up a numerical scenario. The subsequent investigation in Section 7 is first concerned with *ceteris paribus* parameter variations and how they affect local stability, and then with the system's dynamic properties over the medium-run. Section 7 concludes. An appendix finally collects the mathematical computations.

## 2 The IS–MP–IA model

In this section we formalize in a continuous-time framework the elementary adjustment mechanisms set forth in Romer (2000), or likewise in a recent textbook by Taylor (2001, Chs 24, 25). Romer christens this approach the IS–MP–IA model, where the acronyms MP and IA stand for monetary policy and inflation adjustments, respectively. As Taylor (2001, p.554) writes, this model combines Keynes's idea that aggregate demand causes departure of real GDP from potential GDP with newer ideas about central banking and how expectations and inflation adjust over time.

Let us begin with monetary policy MP, which means that the central bank sets the interest rate according to a Taylor rule (Taylor, 1993, p.202). Romer(2001, p.13) and Taylor (2001, p. 559) consider reactions of the interest rate to inflation only, but here we may just as well add an influence of economic activity. Thus, denote the nominal rate of interest by  $i$ , the rate of inflation by  $\pi$ , the output-capital ratio, which is invoked as a measure of capacity utilization, by  $y$ , and let

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<sup>1</sup>By construction, the model admits of no variable that might jump on the stable branch of a saddle point.

$r^o$  be the equilibrium real rate of interest,  $\pi^o$  the target level of inflation, and  $y^o$  the output-capital ratio in long-run equilibrium.  $\alpha_\pi$  being a positive,  $\alpha_y$  a nonnegative coefficient, we then have

$$i = r^o + \pi + \alpha_\pi(\pi - \pi^o) + \alpha_y(y - y^o) \quad (1)$$

In many (quarterly) discrete-time models or empirical estimations of interest rate reaction functions, expected inflation is used rather than the current inflation rate  $\pi$ . Usually these expectations refer to the next quarter and are supposed to be rational. As we are working in continuous time where the next period is infinitesimally near, this concept of inflationary expectations may presently be left aside. Another difference to the literature is that economic activity is commonly represented by the output gap. Since, however, the output gap and the (detrended) output-capital ratio show strong comovements over the business cycle, employing the output-capital ratio  $y$  in (1) fulfills the same role.<sup>2</sup>

Regarding the IS part of the model, we treat  $i - \pi$  as the real rate of interest and suppose that the temporary equilibrium condition for the goods market is already solved for the output-capital ratio.  $y$  may thus be represented as a decreasing function of  $i - \pi$ ,

$$y = f_y(i - \pi), \quad f'_y < 0 \quad (2)$$

To ensure existence of a long-run equilibrium, the central bank must set the real interest rate  $r^o$  in (1) at a suitable level that makes it compatible with this IS curve. In detail, with  $i^o := r^o + \pi^o$ , the function  $f_y$  is required to satisfy  $f_y(r^o) = f_y(i^o - \pi^o) = y^o$ .

Eqs (1) and (2) give rise to an inverse relationship between inflation and output. Accordingly,  $y$  can be directly conceived as a decreasing function of  $\pi$ ,

$$y = y(\pi), \quad \text{where } y(\pi^o) = y^o, \quad y_\pi := dy/d\pi < 0 \quad (3)$$

The negative slope is obvious for  $\alpha_y = 0$ , where (3) is immediately inferred from substituting (1) in (2). With  $\alpha_y > 0$ , (3) follows from a straightforward application of the Implicit Function Theorem (see the appendix).

The third building block of the model is IA, inflation adjustments. It assumes an accelerationist Phillips curve, which is to say that the rate of inflation is given at any point in time and shifts up (down) when real output is above (below) its natural level (Romer, 2000, p. 16; Taylor, 2001, pp. 566ff). In continuous time, the concept reads,

$$\dot{\pi} = f_\pi(y - y^o), \quad f_\pi(0) = 0, \quad f'_\pi > 0 \quad (4)$$

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<sup>2</sup>Adopting the Hodrick-Prescott filter, we have computed a correlation coefficient of 0.97 over the four major U.S. cycles between 1961 and 1991.

Incidentally, it is readily checked that this specification of inflation adjustments implies counter-cyclical motions of the price level around its trend, which by now appears to be a well established stylized fact of the business cycle in many industrialized countries.<sup>3</sup>

Eq. (4) completes the model. Clearly,  $y = y^o$ ,  $\pi = \pi^o$ ,  $i = i^o = r^o + \pi^o$  constitute a long-run equilibrium position. The return of the economy from disequilibrium back to steady state growth is also evident: a positive departure of the output-capital ratio from normal causes an increase in the rate of inflation, which causes the central bank to raise the real interest rate, which then moves the output-capital ratio back to normal. Substituting (3) in (4), the dynamics is formally described by one differential equation in the inflation rate  $\pi$ ,

$$\dot{\pi} = F(\pi) := f_{\pi}[y(\pi) - y^o] \quad (5)$$

By (3),  $\pi^o$  is a stationary point of (5). It is stable, even globally so, since  $F'(\pi) = f'_{\pi} y_{\pi} < 0$ . As Romer (2000, p. 18) concludes with his “Advantage 6” of the IS–MP–IA model: “The model’s dynamics are straightforward and reasonable.”

### 3 Bonds and high-powered money

With this section we begin to consider the implications of the interest rate variations for the monetary sector. Romer (2000, pp. 20–24) as well as Taylor (2001, pp. 559f) entertain the view that the central bank changes the interest rate through open market operations. In addition, Romer (p. 24) states explicitly that the corresponding adjustments of the stock of money, i.e., of the monetary base, have no further effect on aggregate demand, and the same holds true for the bonds that are bought or sold in the open market. In a first step, we accept this hypothesis and study the resulting bond dynamics, which is thus a mere appendix to the system of the previous section.

Let  $H$  be the stock of high-powered money and  $1/\mu$  the (constant) money multiplier, so that the money supply is given by  $M^s = H/\mu$ .  $M$  denotes money and  $B$  the outstanding fixed-price bonds with their variable interest rate  $i$  (the bond price normalized at unity).  $M$  and  $B$  may be the only financial assets in the economy. Money demand  $M^d$  is specified as a fraction  $f_m$  of  $M+B$  that decreases in the interest rate and (possibly) increases with economic activity as it is measured by the output-capital ratio:

$$M^d = f_m(i, y) \cdot (M + B), \quad f_{mi} < 0, \quad f_{my} \geq 0 \quad (6)$$

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<sup>3</sup>See, for example, Cooley and Ohanian (1991), Backus and Kehoe (1992), Fiorito and Kollintzas (1994).

where  $0 < f_m < 1$  and  $f_{mi} = \partial f_m / \partial i$ ,  $f_{my} = \partial f_m / \partial y$ . Hence, the money market equilibrium condition  $M^s = M^d$  reads  $H/\mu = f_m(i, y) (H/\mu + B)$ .

As the discussion takes place in a growth context, the equilibrium condition has to be given in intensive form. Designating  $h = H/pK$ ,  $b = B/pK$  ( $p$  the price level,  $K$  the capital stock) and multiplying through by  $\mu$ , the equation becomes

$$(h + \mu b) f_m(i, y) - h = 0 \quad (7)$$

Since monetary policy uses the interest rate as its instrument, equality in (7) is not brought about by  $i$ , but by suitable combinations of the bond ratio  $b$  and the ratio of high-powered money  $h$ . For the further analysis it is useful to treat  $h$  in (7) as a function of  $i$ ,  $y$ , and  $b$ . Denoting it by  $f_h$ , we have

$$h = f_h(i, y, b), \quad \text{with } f_{hi} < 0, f_{hy} \geq 0, f_{hb} > 0 \quad (8)$$

The signs of the reactions to the *ceteris paribus* changes in these variables are obvious (similar as above,  $f_{hi} = \partial f_h / \partial i$ , etc.). The precise formulae are provided in the appendix.

Bonds and the monetary base are linked together in a second relationship, which is of a dynamic form. This is the government budget restraint,  $\dot{H} + \dot{B} = pG + iB - T$ . To reflect the idea of automatic stabilizers, though in a simple way, we allow for countercyclical movements of government spending  $G$ ; with two nonnegative parameters  $\gamma_k$ ,  $\gamma_y$  and  $Y^o = y^o K$  the level of normal output, it is given by  $G = \gamma_k K - \gamma_y (Y - Y^o)$ . As for nominal taxes  $T$ , a proportional tax rate  $\tau$  levied on total income  $pY + iB$  is assumed.<sup>4</sup> The intensive form of the budget identity is then calculated as

$$\dot{h} + \dot{b} = \gamma_k + \gamma_y y^o - (\gamma_y + \tau)y + (1 - \tau)ib - (h + b)(\pi + g) \quad (9)$$

where  $g$  designates the growth rate of the capital stock  $K$ .

To derive an ordinary differential equation from (9), a reduced-form representation of  $h$  has to be established, such that  $h$  is a function of the dynamic state variables of the system to be considered. For example, if the state variables are  $\pi$  and  $b$  and  $\dot{\pi} = F_\pi(\pi)$  as in (5), we have  $h = h(\pi, b)$  with partial derivatives  $h_\pi$  and  $h_b$ , which gives rise to  $\dot{h} = h_\pi \dot{\pi} + h_b \dot{b} = h_\pi F_\pi(\pi) + h_b \dot{b}$ . Substituting this expression in (9) finally allows one to resolve it into a differential equation of the form  $\dot{b} = F_b(\pi, b)$ . This is spelled out and subsequently examined for stability in the next section.

As this procedure appears rather technical, we may before briefly discuss the determination of bonds and high-powered money in a discrete-time setting; for simplicity with respect to original

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<sup>4</sup>Output  $Y$  may be thought of as net of capital depreciation.

levels. Let  $H_t, B_t$  be the beginning-of-period stocks and  $D = D(p_t Y_t, i_t B_t)$  the period  $t$  nominal deficit. Then the government budget restraint for period  $t-1$  is

$$(H_t - H_{t-1}) + (B_t - B_{t-1}) = D(p_{t-1} Y_{t-1}, i_{t-1} B_{t-1})$$

On the analogy to (8),  $H_t$  satisfies a functional relationship

$$H_t = f_h(i_t, Y_t, B_t)$$

Thus, there are two interrelated equations to compute the stocks  $H_t$  and  $B_t$ : one is the condition for money market equilibrium, the other the financing of the government deficit. A problem may be that  $H_t$  cannot be represented by an explicit function;  $H_t$  and  $B_t$  would then only be implicitly determined.

## 4 Bond dynamics in the IS–MP–IA model

When the IS–MP–IA model is augmented by high-powered money and bonds, in an appendix to eq. (5), we get another state variable besides the inflation rate  $\pi$ , namely, the bond ratio  $b$ . To establish the differential equation for  $b$ , we first take the output function  $y = y(\pi)$  in (3) and substitute it in the Taylor rule (1). In this way the interest rate is expressed as a function of  $\pi$ ,

$$i = i(\pi) := r^o + \pi + \alpha_\pi(\pi - \pi^o) + \alpha_y[y(\pi) - y^o] \quad (10)$$

Although  $dy/d\pi$  is negative, it is easily shown that the real interest rate always increases in response to an increase in inflation:  $di/d\pi > 1$  for all values of  $\alpha_y \geq 0$ .<sup>5</sup>

The two functions  $y = y(\pi)$  and  $i = i(\pi)$  can be plugged into (8), to the effect that the ratio of high-powered money  $h$  can be expressed as a function of  $\pi$  and  $b$ ,

$$h = h(\pi, b) := f_h[i(\pi), y(\pi), b] \quad (11)$$

After differentiating (11) with respect to time,  $\dot{h} = h_\pi \dot{\pi} + h_b \dot{b}$ , it remains to substitute the derivative in the government budget restraint (9) and solve the resulting equation for  $\dot{b}$ . In eq. (9) we assume that the capital growth rate  $g$ , like the output-capital ratio in (2), depends solely on the real interest rate, so that  $g$ , like  $y$  in (3), can also be expressed as a function of the rate of inflation,  $g = g(\pi)$ . Taking finally account of eq. (4) for  $\dot{\pi}$ , we obtain

$$\dot{b} = F_b(\pi, b) := \frac{1}{1 + h_b} \left\{ -h_\pi f_\pi [y(\pi) - y^o] + \gamma_k + \gamma_y y^o - (\gamma_y + \tau) y(\pi) + (1 - \tau) i(\pi) b - (h + b) [\pi + g(\pi)] \right\} \quad (12)$$

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<sup>5</sup>With  $y_\pi$  as computed at the beginning of the appendix, one has  $di/d\pi = 1 + \alpha_\pi + \alpha_y y_\pi = 1 + \alpha_\pi [1 - \alpha_y |f'_y| / (1 + \alpha_y |f'_y|)] = 1 + \alpha_\pi / (1 + \alpha_y |f'_y|)$ .



The long-run equilibrium level of  $b$ , which renders  $\dot{b} = 0$ , is computed as follows. Denote, to this end, all steady state values by a superscript ‘o’, in particular,  $g^o = g(\pi^o)$  and  $f_m^o = f_m(i^o, y^o)$  (recall  $i^o = r^o + \pi^o$ ). Eq. (7) gives  $h^o = \mu f_m^o b^o / (1 - f_m^o)$ , while with  $y(\pi^o) = y^o$ , the time derivative  $\dot{b}$  vanishes if  $\gamma_k - \tau y^o + (1 - \tau) i^o b^o - (h^o + b^o)(\pi^o + g^o) = 0$ . Solving the latter equality for  $b^o$  yields

$$b^o = \frac{\gamma_k - \tau y^o}{[1 + \mu f_m^o / (1 - f_m^o)](\pi^o + g^o) - (1 - \tau) i^o} \quad (13)$$

As it is made more precise in Section 6 below,  $\gamma_k - \tau y^o$  is reasonably positive. A positive value of  $b^o$  then requires the denominator in (13) to be positive. Given that  $\mu f_m^o / (1 - f_m^o)$  is fairly low, we may directly posit that the equilibrium nominal growth rate of the economy,  $\pi^o + g^o$ , exceeds the after-tax equilibrium rate of interest  $(1 - \tau) i^o$ . Since this supposition will also be invoked later on in the analysis, we set it up as

**Assumption 1.** *The tax rate  $\tau$  is less than  $\gamma_k / y^o$ . For the steady state values of the rates of interest, inflation, and real growth, the following inequality is satisfied,*

$$(1 - \tau) i^o < \pi^o + g^o$$

The bond dynamics converges locally to  $b^o$  if  $\partial F_b / \partial b < 0$  in (12). This partial derivative results like  $\partial F_b / \partial b = [(1 - \tau) i^o - (1 + h_b)(\pi^o + g^o)] / (1 + h_b)$ , where  $h_b = \partial h / \partial b$  is calculated as  $h_b = f_{hb} = \mu f_m^o / (1 - f_m^o)$  (cf. the appendix). Hence  $1 + h_b > 0$ , and  $\partial F_b / \partial b < 0$  if Assumption 1 applies. Considering eqs (5) and (12) together as a two-dimensional differential equations system in  $\pi$  and  $b$ , we have for the entries of its Jacobian matrix:  $j_{11} = \partial \dot{\pi} / \partial \pi < 0$ ,  $j_{22} = \partial \dot{b} / \partial b < 0$ , and  $j_{12} = \partial \dot{\pi} / \partial b = 0$ . Irrespective of the sign of the fourth entry  $j_{21} = \partial \dot{b} / \partial \pi$ ,  $\partial F_b / \partial b < 0$  ensures that the Jacobian has a negative trace and a positive determinant. These findings are summarized in Proposition 1, which allows us to conclude that the dynamics of bonds and the monetary base that takes place in the background of the IS–MP–IA model, is no great problem.

**Proposition 1.** *Suppose Assumption 1 holds true. Then the equilibrium point  $(\pi^o, b^o)$  of the IS–MP–IA model, eq. (5), with its associated bond dynamics, eq. (12), is locally asymptotically stable.*

## 5 Interest payments and their impact on stability

So far, bonds had no feedback on the real sector. On the other hand, bonds are bought by agents because of the interest receipts, which are an income item and so have an effect on demand, most

prominently on consumption. If furthermore total income is taxed proportionately as laid out in the government budget restraint above, then it is not seen how these interest payments should cancel out in the determination of aggregate demand. From this point of view it is almost a consistency requirement that interest payments, and thus bonds, enter explicitly into the model's IS equation.<sup>6</sup>

Besides better tractability of the model, an economic reason for disregarding the income effects of interest payments on aggregate demand is that they are a small part of total income and so may be expected to have a minor bearing on the dynamics only. With this section we embark on checking this intuition.

To set the stage, decompose aggregate demand into consumption, investment and government spending. Consumption  $C$  is supposed to be made up of a basic component that grows with the capital stock and a component that varies in line with disposable income, where the latter includes the interest payments. With a constant  $c_k > 0$  and  $c_y$  the marginal propensity to consume out of disposable income,  $0 < c_y < 1$ , we have

$$C = c_k K + c_y (1 - \tau) (Y + iB/p) \quad (14)$$

This type of consumption function that allows bonds to enter aggregate demand will be the only essential amendment to the IS–MP–IA model considered above.

For notational simplicity, the real interest rate is supposed to act merely on investment  $I$ . Other factors related to current output levels that may influence investment are likewise neglected.<sup>7</sup> The investment function is thus

$$I = f_g(i - \pi) \cdot K, \quad f_g' < 0 \quad (15)$$

As government spending has already been specified before, clearing of the goods market,  $Y = C + I + G$ , is fully determined. To obtain the IS equilibrium value of the output-capital ratio  $y$ , we first abbreviate the interest rate reaction function (1) as

$$j = j(y, \pi) := r^o + \pi + \alpha_\pi(\pi - \pi^o) + \alpha_y(y - y^o) \quad (16)$$

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<sup>6</sup>Another tax rule in macroeconomic modelling is to define (nominal) tax collections  $T$  net of transfers, and to include interest payments as part of those transfer payments. If additionally  $T/pK = \text{const.}$  is postulated and that it is after-tax income on which aggregate demand depends, bonds again disappear from the IS equation. This simplifying device is, for example, adopted by Sargent (1987) in his textbook on macroeconomic theory (cf. pp. 16f, 113f) and by many Keynesian-oriented macro models in its wake.

<sup>7</sup>The only effect of adding an accelerator argument by making investment dependent on  $Y$  would be an increase in the multiplier. More precisely, the partial derivatives  $y_\pi$  and  $y_b$  in eq. (20) below would be higher since the term  $A_c$  defined in (19) would be lower.

and then define excess demand for goods, normalized by the capital stock, as a function  $E_y = E_y(y, \pi, b)$ , so that the IS equilibrium condition reads,

$$E_y(y, \pi, b) := c_k + c_y(1-\tau)[y + j(y, \pi)b] + f_g[j(y, \pi) - \pi] + \gamma_k - \gamma_y(y - y^o) - y = 0 \quad (17)$$

Equilibrium in (17) is brought about by variations of  $y$ . Accordingly, let  $y = y(\pi, b)$  be the value of the output-capital ratio at which, given  $\pi$  and  $b$ , excess demand  $E_y(y, \pi, b)$  vanishes. In this way, the accelerationist Phillips curve from eq. (5) becomes

$$\dot{\pi} = F_\pi(\pi, b) := f_\pi[y(\pi, b) - y^o] \quad (18)$$

that is, the motions of the rate of inflation are no longer independent of the evolution of government debt.

For negative reactions of output to inflation as they previously prevailed in eq. (3), it proves necessary that investment is rather sensitive. In detail, the negative effect on investment from an increase in the interest rate  $i$  must dominate the positive effect, through higher interest payments, on consumption:

**Assumption 2.**  $|f'_g| > c_y(1-\tau)b^o.$

Defining the terms

$$A_c := 1 - c_y(1-\tau) + \gamma_y, \quad A_g := |f'_g| - c_y(1-\tau)b^o \quad (19)$$

both of which are positive under Assumption 2, the partial derivatives of the IS output-capital ratio (evaluated at the steady state values) are computed as

$$y_\pi = \frac{c_y(1-\tau)b^o - \alpha_\pi A_g}{A_c + \alpha_y A_g}, \quad y_b = \frac{c_y(1-\tau)i^o}{A_c + \alpha_y A_g} \quad (20)$$

(these and the formulae to follow are again derived in the appendix). A *ceteris paribus* increase of the bond ratio  $b$  unambiguously raises output, because of the corresponding rise in interest receipts and thus consumption demand. In contrast, the effect of an increasing rate of inflation could go either direction. The function  $\pi \mapsto y(\pi, b)$ , which is the counterpart of the above function  $y = y(\pi)$  in eq. (3), is downward-sloping if and only if the inflation targeting coefficient  $\alpha_\pi$  in the Taylor rule is sufficiently high. Nevertheless, the numerical inspection in the next section demonstrates that the critical value of  $\alpha_\pi$  from when on, with  $\alpha_\pi$  increasing,  $y_\pi < 0$  obtains, tends to be relatively low unless the investment responsiveness  $|f'_g|$  is not too small. Hence the observation that weak reactions of monetary policy to inflation might be destabilizing, in that they do not avert a spiral of rising inflation and output, is remarkable, but it is not worrying since the interest

rate reactions can be reasonably believed to be strong enough. For better reference, we summarize this discussion in an extra proposition.

**Proposition 2.** *Given that Assumption 2 is fulfilled, so that  $A_g > 0$  in (19), the monetary policy rule diminishes the IS equilibrium output-capital ratio if, and only if, the inflation targeting coefficient  $\alpha_\pi$  exceeds a certain critical value  $\alpha_\pi^c$ . That is,*

$$y_\pi = \partial y / \partial \pi < 0 \quad \text{if, and only if,} \quad \alpha_\pi > \alpha_\pi^c := c_y(1-\tau)b^o / A_g .$$

When the interest rate function (1) or (16) is restricted to apply to the temporary equilibrium situations, one gets the reduced-form representation

$$i = i(\pi, b) := r^o + \pi + \alpha_\pi(\pi - \pi^o) + \alpha_y [y(\pi, b) - y^o] \quad (21)$$

whose partial derivatives are given by

$$i_\pi = 1 + \alpha_\pi + \alpha_y y_\pi > 1 , \quad i_b = \alpha_y y_b \geq 0 \quad (22)$$

It is thus seen that, irrespective of the concrete coefficients  $\alpha_\pi$ ,  $\alpha_y$ , the policy rule always leads to an increase in the real rate of interest if the inflation rate increases, even if  $y_\pi < 0$  and  $\alpha_y$  is large (this is verified in the appendix). As has just been shown, however, this property as such is not sufficient to cut down economic activity as a whole.

The next step in the analysis is to determine the ratio of high-powered money in its dependency on  $\pi$  and  $b$ . Referring to the function  $f_h = f_h(i, y, b)$  in (8), the reduced-form representation of  $h$  is

$$h = h(\pi, b) := f_h[i(\pi, b), y(\pi, b), b] \quad (23)$$

The procedure is very much the same as in Section 4, eq. (23) being a generalization of (11). Likewise, the motions of bonds are derived from substituting the time derivative  $\dot{h} = h_\pi \dot{\pi} + h_b \dot{b}$  in the government budget identity (9), solving it for  $\dot{b}$ , and using (18) for  $\dot{\pi}$ :

$$\begin{aligned} \dot{b} = F_b(\pi, b) := & \frac{1}{1 + h_b} \left\{ -h_\pi f_\pi [y(\pi, b) - y^o] + \gamma_k + \gamma_y y^o - (\gamma_y + \tau) y(\pi, b) \right. \\ & \left. + (1 - \tau) i(\pi, b) b - (h + b) [\pi + f_g(i(\pi, b) - \pi)] \right\} \end{aligned} \quad (24)$$

which may be compared with eq. (12) for  $\dot{b}$  in the previous section. To sum up, eqs (18) and (24) constitute a two-dimensional differential equations system in the inflation rate  $\pi$  and the bond ratio  $b$ , whose dynamics are now interrelated. The steady state position  $(\pi^o, b^o)$ , of course, remains the same.

To inquire into the local stability of the steady state, the Jacobian matrix of (18), (24) has to be set up. Define to this end,

$$\begin{aligned}
A_y &:= -h_\pi f'_\pi - (\gamma_y + \tau) + \alpha_y [(1-\tau)b^o + (h^o + b^o)|f'_g|] \\
A_b &:= (1-\tau)i^o - (1+h_b)(\pi_o + g^o) \\
A_\pi &:= (1+\alpha_\pi)(1-\tau)b^o + (h^o + b^o)(\alpha_\pi|f'_g| - 1) - h_\pi(\pi_o + g^o)
\end{aligned} \tag{25}$$

The Jacobian  $J$  can then be written as

$$J = \begin{bmatrix} f'_\pi y_\pi & f'_\pi y_b \\ \frac{A_y y_\pi + A_\pi}{1+h_b} & \frac{A_y y_b + A_b}{1+h_b} \end{bmatrix} \tag{26}$$

The reaction of the time rate of change of  $\pi$  to an increase in the level of inflation continues to be negative if, as pointed out in Proposition 2, the central bank in its policy rule pays adequate attention to inflation;  $j_{11} = \partial F_\pi / \partial \pi = f'_\pi y_\pi < 0$  then, which is the same result as in eq. (5) for the IS–MP–IA model. The response of inflation to changes in  $b$  indicates a possible source of destabilization, since higher government debt accelerates inflation;  $j_{12} = \partial F_\pi / \partial b = f'_\pi y_b > 0$ .

The partial derivatives of the function  $F_b$  in (24) are apparently more complicated than those of  $F_\pi$ . For the stability analysis it is also necessary to assess the sign of the denominator of the entries  $j_{21}$  and  $j_{22}$ . Actually, the derivative  $h_b$  turns out to be comparatively small. It is established in the appendix that  $1+h_b$  is positive if the the money demand is not excessively responsive to the interest rate. In explicit terms,  $1+h_b > 0$  if Assumption 3a is satisfied, while, considering more strictly  $h_b$  itself,  $h_b > 0$  if Assumption 3b is fulfilled. In particular, these statements hold regardless of the values chosen for the policy parameters  $\alpha_\pi$ ,  $\alpha_y$ . Note also that the denominator in Assumptions 3a and 3b contains the product of two terms,  $i^o$  and  $h^o + \mu b^o$ , which should be quite small. The expressions on the right-hand side will therefore be easily rather high.

$$\textbf{Assumption 3a.} \quad |f_{mi}| < \frac{[1 - (1-\mu)f_m^o] A_g}{c_y (1-\tau) (h^o + \mu b^o) i^o} .$$

$$\textbf{Assumption 3b.} \quad |f_{mi}| < \frac{\mu f_m^o A_g}{c_y (1-\tau) (h^o + \mu b^o) i^o} .$$

As for the second diagonal entry of the Jacobian,  $j_{22}$ , it can be concluded that Assumptions 1 and 3b imply  $A_b < 0$ . This type of inequality amounted to  $\partial \dot{b} / \partial b < 0$  in the IS–MP–IA model. Now, however, the additional term  $A_y y_b$  has to be taken into account, where  $A_y$  may take on either sign. In our numerical investigations there was nevertheless a strong tendency for  $j_{22}$  to be negative. The reason is that high values of  $\alpha_y$  increase  $A_y$ , but simultaneously decrease  $y_b$  at

the same order of magnitude; cf. eq. (20). On this basis we may say that normally both auto-feedbacks,  $\pi$  on  $\dot{\pi}$  and  $b$  on  $\dot{b}$ , are negative and thus stabilizing, just as they were in the IS–MP–IA model.

If we take  $j_{22} < 0$  for granted and concentrate on  $\alpha_\pi > \alpha_\pi^c$ , which implies  $y_\pi < 0$  and thus also  $j_{11} < 0$  and trace  $J < 0$ , the sign of the determinant of  $J$  becomes the decisive criterion for stability. Here the term  $A_y$  plays no more role since it cancels out:

$$\det J = f'_\pi (A_b y_\pi - A_\pi y_b) / (1+h_b) \quad (27)$$

The sign of  $h_\pi = f_{hi} i_\pi + f_{hy} y_\pi$  in the term  $A_\pi$  is given by  $f_{mi} i_\pi + f_{my} y_\pi$ . Hence  $h_\pi < 0$  under the condition on  $\alpha_\pi$  just stated.  $A_\pi$  itself, which is increasing in  $\alpha_\pi$ , is consequently positive if  $\alpha_\pi$  exceeds the value  $\tilde{\alpha}_\pi$  that renders the sum of the first two terms in  $A_\pi$  zero. In short,  $A_\pi > 0$  if  $\alpha_\pi > \max\{\alpha_\pi^c, \tilde{\alpha}_\pi\}$ . Computing  $\tilde{\alpha}_\pi = (h^o + \tau b^o) / [(1-\tau)b^o + (h^o + b^o)|f'_g|]$ , it is furthermore easily verified that  $\alpha_\pi^c > \tilde{\alpha}_\pi$  at least if  $c_y \geq (h^o + \tau b^o)/(1-\tau)b^o$ , an inequality that will be reasonably satisfied. It then follows that  $j_{11} < 0$  as an almost necessary condition for stability leads to  $A_\pi > 0$  in  $\det J$ . Thus, on the whole, both products  $A_b y_\pi$  and  $A_\pi y_b$  in (27) are positive.

The determination of the sign of  $\det J$ , therefore, requires a more detailed investigation of the single terms  $A_b$ ,  $A_\pi$  and  $y_b$ ,  $y_\pi$ . The task is impeded by the fact that  $y_\pi$  and  $A_\pi$  on the one hand, and  $y_b$  and  $A_b$  on the other hand, are of a similar order of magnitude and involve rather lengthy expressions. To handle them it is useful to introduce the terms  $\phi_o$  and  $\phi_\pi$ , into which in turn enter the abbreviations  $A_1$ ,  $A_2$ :

$$\begin{aligned} A_1 &:= \pi^o + g^o - (1-\tau) i^o + \mu f_m^o (\pi^o + g^o) / (1-f_m^o) > 0 \\ A_2 &:= (h^o + \mu b^o) (\pi^o + g^o) |f_{mi}| / (1-f_m^o) > 0 \\ \phi_o &:= -c_y (1-\tau) [A_1 b^o - i^o (h^o + \tau b^o - A_2)] \\ \phi_\pi &:= A_g A_1 - c_y (1-\tau) i^o [(1-\tau) b^o + (h^o + b^o) |f'_g| + A_2] \end{aligned} \quad (28)$$

where for the positive sign of  $A_1$ , Assumption 1 has been presupposed. After some tedious calculations the determinant of  $J$  can be decomposed as

$$\det J = f'_\pi (\phi_o + \phi_\pi \alpha_\pi) / \{ (1+h_b) |E_{yy}| \} \quad (29)$$

It may be observed that the slope  $f'_\pi$  of the Phillips curve, a possible responsiveness of money demand to output (the derivative  $f_{my}$ ), and the degree  $\gamma_y$  of countercyclicality in government expenditure have disappeared from the core expression  $\phi_o + \phi_\pi \alpha_\pi$  in (29). Even more important is the phenomenon that the sign of the determinant is independent of the policy coefficient  $\alpha_y$ , so only the inflation coefficient  $\alpha_\pi$  has a bearing on it.<sup>8</sup> On this basis we arrive at Proposition 3.

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<sup>8</sup> $\alpha_y$  has not been made explicit in eq. (27) for the determinant, but it is present in the derivatives  $h_\pi$  and  $h_b$

**Proposition 3.** *Suppose Assumptions 1, 2, and 3a are satisfied. Then the following holds for the dynamic process (18), (24).*

(a) *If  $\phi_\pi > 0$ , there exists a benchmark value  $\alpha_\pi^s > 0$  such that  $\alpha_\pi < \alpha_\pi^s$  implies instability of the steady state for all values of  $\alpha_y \geq 0$ , whereas  $\alpha_\pi > \alpha_\pi^s$  implies local asymptotic stability for all values of  $\alpha_y \geq 0$ .*

(b) *If  $\phi_o < 0$ ,  $\phi_\pi < 0$ , the steady state is unstable for all nonnegative values of  $\alpha_\pi$  and  $\alpha_y$ .*

The main result is that the monetary policy rule (1) can turn out to be completely unable to stabilize the local dynamics, although the bond dynamics has a negative feedback on itself ( $\partial \dot{b} / \partial b < 0$ ) and the central bank, by setting  $\alpha_\pi$  sufficiently high, achieves a negative feedback of inflation on itself ( $\partial \dot{\pi} / \partial \pi < 0$ ). Hence, it is the interaction between the inflation dynamics, that is, the interest rate–inflation–output nexus, and the consequences of monetary and fiscal policy in the financial sector, that is responsible for the instability of the long-run equilibrium. The feedbacks here involved are quite complex, which is also reflected in the irritating terms defined in (28). While this is an interesting theoretical finding, one might nevertheless question the practical relevance of the instability result, in the sense that only special and not too plausible sets of numerical parameters would actually produce it. This issue is taken up in the next section.

Another property of the model is that only the inflation targeting coefficient  $\alpha_\pi$  can stabilize the economy, whereas output targeting *via* the coefficient  $\alpha_y$  has no effect in this respect. This is somewhat surprising, just when he have pointed out the complex interactions in the model. In addition, there is a general tendency that  $\phi_o$  is negative and that the value of  $\alpha_\pi$  at which, with  $\phi_\pi > 0$ , the determinant of  $J$  becomes positive exceeds the value  $\alpha_\pi^c$  of Proposition 2. This means it is not sufficient for the central bank to accomplish negative reactions of output to an increase in inflation,  $y_\pi < 0$ , targeting of inflation must be stronger to also take account of the effects from the induced bond dynamics. However, rather than dwell into the troublesome algebraic details for this result to come about, we leave it to the next section’s numerical investigations.

It is furthermore worth mentioning that the sign of  $\det J$  in (29) remains also unaffected by fiscal policy in the form of the countercyclical expenditure coefficient  $\gamma_y$ . On the other hand, the coefficient does have some influence on the trace of  $J$  *via* the partial derivative  $y_\pi$  in  $j_{11}$ ; see eqs (19) and (20). Since, however, this effect is not very strong, it can be said that the strength of the government’s expenditure policy has practically no bearing on the stability question; certainly not if both  $\phi_o$  and  $\phi_\pi$  are negative.

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entering  $A_\pi$  and  $A_b$ , respectively. It is thus not obvious that  $\alpha_y$  eventually cancels out in  $\phi_o + \phi_\pi \alpha_\pi$ .

Note finally that if  $y_\pi < 0$  and also trace  $J < 0$ , but  $\alpha_\pi$  not high enough to stabilize the steady state, instability will be of the saddle point type. Nevertheless, as the model is constructed there is no scope for the rate of inflation to jump on the stable branch of the saddle point dynamics.

## 6 Setting up a baseline scenario

In this section we propose a scenario of numerical parameter values, which can then be used to get a feel for the algebraic expressions that entered the stability proposition in the previous section, and to inquire into the system's medium-run dynamics. Beginning with the equilibrium rates of interest, inflation, and real growth, they may be set at

$$i^o = 4.5 \% \quad \pi^o = 2 \% \quad g^o = 3 \% \quad (30)$$

2 per cent is the usual level for the target rate of inflation, while the bond rate is above the 4 per cent equilibrium value of the federal funds rate in Taylor's original formula (Taylor, 1993, p. 202). With a view to the later results it is, however, only a little bit higher. (30) clearly respects Assumption 1.

Next, consider the steady state ratios to output of money, bonds, and government expenditure as they result from US data. We choose  $M/pY = 0.15$  (see, for example, the diagram in Blanchard, 2000, p. 63),  $B/pY = 0.40$  (cf. Blanchard, 2000, p. 524, quoting from OECD Economic Outlook),  $G/Y = 0.20$  (cf. Table 1 in Alesina, 2000, p. 7). Furthermore, adopting the numbers for the reserve requirement ratio (0.10) and the proportion in which people hold money in currency (0.40) that are mentioned in Blanchard (2000, pp. 69, 71), and computing the corresponding money multiplier (which amounts to 2.17; *ibid.*, p. 73), we work with  $\mu = 0.46$ .

To relate  $M$ ,  $B$ ,  $G$  to the capital stock rather than output, we also need to have an idea about the output-capital ratio (though  $y$  nowhere shows up in the stability analysis itself). Here we make use of the capital stock series that can be extracted from the database provided by Ray Fair on his homepage<sup>9</sup> and set  $y^o = 0.90$ . Decomposing the ratio of high-powered money as  $h = H/pK = \mu(M/pY)(pY/pK)$ , and similarly so for the bond ratio  $b = B/pK$  and the ratio  $\gamma_k = G/Y$  when output is on its trend path ( $Y = Y^o = y^o K$ ), we so far have:

$$y^o = 0.90 \quad \gamma_k = 0.18 \quad \mu = 0.46 \quad h^o = 0.0621 \quad b^o = 0.360 \quad f_m^o = 0.2727 \quad (31)$$

The equilibrium proportion of the money holdings in (6) derives, of course, from  $f_m = M/(M + B) = (M/pY) / [(M/pY) + (B/pY)]$ . The reason for invoking  $\gamma_k$ , although it plays no role either

<sup>9</sup>The ULR is <http://fairmodel.econ.yale.edu>.



in the stability analysis, is that it serves us to compute the tax rate  $\tau$  that, given the steady state values already determined, is consistent with  $\dot{b} = 0$ . Equating the right-hand side of (12) to zero and solving it for  $\tau$  yields (omitting the steady state indication)

$$\tau = [\gamma_k + ib - (h+b)(\pi+g)] / (y + ib) = 19.11\% \quad (32)$$

This figure is close to the actual share of the government's total revenues to GDP (cf. Alesina, 2000, Table 2 on p. 8, which is quoted from the Congressional Budget Office), just as it should be.

We can thus turn to the reaction coefficients. The first one is the expenditure coefficient  $\gamma_y$  of the 'automatic stabilizer' in the government budget restraint. An orientation mark is provided by the rule of thumb that a 1% decrease in output leads to an increase in the deficit of 0.5% of GDP.<sup>10</sup> As the direct effect of output on the deficit is given by  $-(\gamma_y + \tau)Y$  and the tax rate  $\tau$  is approximately 20%, a reasonable value for  $\gamma_y$  is  $\gamma_y = 0.30$ .

The slope  $f'_\pi$  of the accelerationist Phillips curve (4) is determined as follows. Referring to the countercyclical variations of the price level around a Hodrick-Prescott trend, to capacity utilization  $u$  (which fluctuates around unity), and to the empirical standard deviations  $\sigma_u$  and  $\sigma_p$  of these time series, Franke (2001) argues for a ratio  $\sigma_p/\sigma_u = 0.50$  as a stylized fact of the business cycle. Countercyclical movements of the price level are generated by a differential equation  $\dot{\pi} = \tilde{\beta}_\pi(u - 1)$ . Simulating it under exogenous regular sine wave oscillations of  $u$ , the desired ratio of 0.50 is brought about by  $\tilde{\beta}_\pi = 0.45$ . Since  $u = y/y^o$ , the linear specification  $\dot{\pi} = \beta_\pi(y - y^o)$  of (4) is just a rescaling of the former equation, with  $\beta_\pi = \tilde{\beta}_\pi/y^o$ . These relationships motivate us to set  $f'_\pi = \beta_\pi = \tilde{\beta}_\pi/y^o = 0.45/0.90 = 0.50$ .

To infer the derivative  $f_{mi}$  of money demand with respect to changes in the rate of interest, we make reference to the interest elasticity  $\eta_{mi} = (\partial M^d/\partial i) / (M/i)$ . In this way,  $f_{mi} = \eta_{mi} f_m^o / i^o$ . Going back to the material discussed in Goldfeld (1976) and the short compilation in Boorman (1976, pp. 328–335), we decide on  $\eta_{mi} = -0.20$ , so that  $f_{mi} = -1.21$ . As has been noted above with respect to eq. (29), the reactions of  $M^d$  to changes in economic activity,  $f_{my}$ , are completely missing in the determinant of  $J$ . Because they only have a minor, almost negligible, bearing on the trace *via* the partial derivative  $h_b$  in entry  $j_{22}$ , we may just as well put  $f_{my} = 0$ .

A familiar order of magnitude for the marginal propensity to consume is  $c_y \approx 0.70$ . Regarding eq. (14), this intuition squares quite well with regressions of trend deviations of  $C/K$  on trend deviations of  $Y/K$ . Employing the Hodrick-Prescott filter, one obtains a slope coefficient  $\beta_c = 0.542$  over the sample period 1961–91.<sup>11</sup> Since the interest payments are only a small part of

<sup>10</sup>cf. Blanchard (2000, p. 526). In his regressions, Taylor (2000b, pp. 34ff) computes somewhat lower numbers.

<sup>11</sup>Slightly higher values of  $\beta_c$ , however, would allow the time series  $\beta_c Y/K$  to better trace out the turning points of  $C/K$ .

total income, the estimate would not be much affected if  $iB/pK$  were included in the regression. We may thus assume  $c_y(1-\tau) = \beta_c$  and, taking the tax rate  $\tau = 19.11\%$  from (32) into account, settle on  $c_y = 0.67$ .

A clue to setting the responsiveness of investment  $f'_g$  to variations of the real interest rate can be obtained from the literature. For example, Ball (1997, pp. 3 and 5) in his discrete-time annual model works with the dynamic IS equation  $x_{t+1} = 0.80x_t - 1.00(i_t - \pi_t) + \text{error term}$ , where  $x$  denotes the output gap (and the coefficients are justified by reference to further literature). The corresponding static relationship would be  $(1 - 0.80)x = -1.00(i - \pi)$ , so that a one-point rise in the interest rate diminishes the output gap by 1.25 per cent. The outcome of the estimates employed by Rudebusch and Svensson (1999, pp. 207f) in their quarterly model is similar. They get  $x_{t+1} = 1.16x_t - 0.25x_{t-1} - 0.10(i_t - \pi_t) + \text{error term}$  (the rates of interest and inflation are measured at annual rates). The multiplier of the real interest rate is here computed as  $-0.10/(1 - 1.16 + 0.25) = -1.11$ .

To relate these findings to the present framework, it does not suffice to note the strong correlation between the output gap and the output-capital ratio, but the fluctuations have also to be compared in size. If both  $y = Y/K$  and  $\ln Y$  are detrended by the usual Hodrick-Prescott filter, then over the period 1961–91 the output gap exhibits a standard deviation of 1.81 percentage points, while the standard deviation of the output-capital ratio is 1.85. The numerical reactions of  $x$  and  $y$  to changes in the interest rate can therefore be directly compared with each other.<sup>12</sup>

The multipliers  $y_i := \partial y / \partial i$  mentioned above do not yet invoke the Taylor rule. The same kind of multiplier as it results from the IS equation (17) is given by  $y_i = -A_g/A_c = -[|f'_g| - c_y(1-\tau)b] / [1 - c_y(1-\tau) + \gamma_y]$ . Adopting for  $c_y, \tau, b, \gamma_y$  the values already determined, we look for a value  $f'_g$  such that  $y_i$  lies between  $-1.11$  and  $-1.25$ . This leads us to set  $f'_g = -1.10$ , which brings about  $y_i = -1.19$ . For a better overview, this and the other coefficients are collected in an extra equation:

$$\begin{aligned} \gamma_y &= 0.30 & f_{my} &= 0.00 & f_{mi} &= -1.21 & (\eta_{mi} &= -0.20) \\ f'_\pi &= 0.50 & c_y &= 0.67 & f'_g &= -1.10 \end{aligned} \tag{33}$$

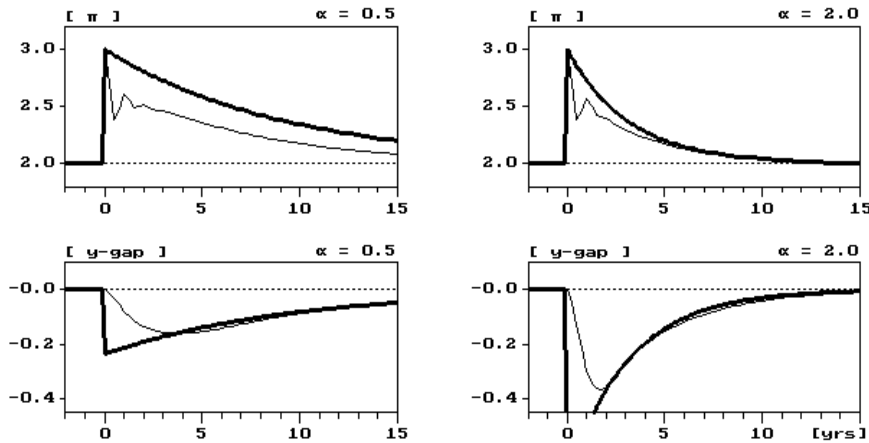
It is immediately seen that the coefficients  $f'_g$  and  $c_y$  satisfy Assumption 2. Furthermore, with the resulting  $A_g = 0.905$  in (19), the right-hand sides of Assumptions 3a and 3b are as high as 139 and 20, respectively. That is, the money demand function with its interest coefficient  $f_{mi}$  has no trouble at all to pass the required inequalities. It is also clear that Assumptions 2 and 3

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<sup>12</sup>Going into the details of footnote 6 in Rudebusch and Svensson (1999, p. 207), it might be inferred that the measure of the output gap underlying their regressions displays somewhat wider oscillations. This would mean that the coefficient  $f'_g$  established in a moment may also be taken a bit higher.

tolerate a great variety of deviations from eqs (31) and (33).

Given that the interest rate variations of the central bank take effect *via* the investment function, the investment reaction coefficient  $f'_g$  is of particular importance. Our choice of  $f'_g = -1.10$  has relied on translating the core of a dynamic IS relationship into a static IS equation. The consequences when the output solutions to the latter are fed into the accelerationist Phillips curve with  $f'_\pi = 0.50$  may thus not be exactly clear. At the end of this section we therefore evaluate this part of our numerical scenario by fixing the bond ratio, simulating the convergence of the (linearized) partial system  $\dot{\pi} = f_\pi[y(\pi, b^o) - y^o]$ , and comparing it to the quantitative adjustments as they result from another system studied in the literature.



*Figure 1:* Convergence paths after shock to inflation  $\pi$ .

*Note:* ‘y-gap’ denotes the output gap (in percentage points). Bold lines are the time paths of system  $\dot{\pi} = f_\pi[y(\pi, b^o) - y^o]$ , thin lines are those of the Rudebusch-Svensson model.  $\alpha = 0.5$  (and 2.0) indicates  $\alpha_y = \alpha_\pi = 0.5$  (and 2.0, respectively).

To begin with the linear differential equation  $\dot{\pi} = f'_\pi \cdot [y(\pi, b^o) - y^o]$ , let us first adopt Taylor’s (1993, p.202) common reference values  $\alpha_y = \alpha_\pi = 0.50$  in the monetary policy function. With the above setting of, especially,  $f'_g$  and  $c_y$  one computes  $y_\pi = -0.2126$  in eq. (20). This gives rise to an eigen-value  $\lambda = f'_\pi y_\pi = -0.50 \cdot 0.2126 = -0.1063$ , which indicates a fairly slow convergence speed. The bold line in the upper-left panel of Figure 1 illustrates the long adjustment time for  $\pi$  after a one-point shock to the rate of inflation at time  $t = 0$  (time on the horizontal axis is measured in years). The lower-left panel shows the time path of the corresponding output gap  $[y(\pi, b^o) - y^o]/y^o$ . In contrast, the thin lines in the two panels are the inflation rate and the output gap that are obtained, after the same shock, from the quarterly model estimated by Rudebusch

and Svensson (1999) mentioned above, which exhibits four lags of the inflation rate in the Phillips curve and two lags of the output gap in the IS equation (the numerical equations are reproduced in the appendix). Here convergence of the inflation rate is somewhat faster, though it still takes a veritable lapse of time. On the other hand, after a transitional phase of about three years the time path of the output gap is remarkably close to that of our model.

One result of the Rudebusch-Svensson paper is that a Taylor-type interest rate reaction function like eq. (1) performs nearly as well in minimizing a central bank's loss function as more ambitious rules (cf. *ibid.*, pp. 227ff). This, however, requires considerably higher coefficients  $\alpha_y$  and  $\alpha_\pi$ .<sup>13</sup> We thus did a second simulation with both  $\alpha_y$  and  $\alpha_\pi$  raised to a value of 2.00. As is seen in the two right-hand panels of Figure 1, these coefficients speed up convergence. Moreover, apart from the first few years, our differential equation and the Rudebusch-Svensson model generate very similar convergence paths. These sketches may serve to gain more confidence in the numerical parameters underlying our IS equation and the Phillips curve.

## 7 Numerical analysis

On the basis of the numerical scenario (30)–(33), we can now assess the scope for local stability or instability of system (18), (24) in greater detail. First, the condition for the derivative  $y_\pi$  to become negative is not very restrictive;  $\alpha_\pi^c$  from Proposition 2 is given by 0.216. So, in words, the policy rule induces a decline in output in response to an increase in the rate of inflation as soon as the inflation targeting coefficient  $\alpha_\pi$  exceeds  $\alpha_\pi^c = 0.216$ . By the same token, entry  $j_{11}$  in the Jacobian will be negative if  $\alpha_\pi > 0.216$ .

Turning to the trace of  $J$ , consider next the second diagonal entry  $j_{22}$ . At the critical value  $\alpha_\pi = \alpha_\pi^c$ ,  $j_{22}$  is already negative — for all nonnegative values of the other policy parameter  $\alpha_y$ . For example,  $j_{22}$  increases from  $-0.0261$  to  $-0.0031$  as  $\alpha_y$  rises from 0 to 10 ( $j_{22}$  still remains below zero at  $\alpha_y = 100$ ). The reason for  $j_{22} \cong A_y y_b + A_b < 0$  is that though  $A_y$  (defined in (25)) changes from negative to positive as  $\alpha_y$  rises, the composite term  $A_y y_b$ , which thus turns positive, too (since  $y_b > 0$  from (20)), is always dominated by the expression  $A_b$ , which is negative (cf. (25) and the remark on Assumption 3b).<sup>14</sup>

Since, as we may just claim for brevity, things for  $j_{22}$  are not very different for other values

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<sup>13</sup>Their exact size depends on the particular weights in the loss function for the variability of inflation, output, and changes in the interest rate.

<sup>14</sup>In finer detail,  $A_b$  increases marginally from  $-0.0222$  for  $\alpha_y = 0$  to  $-0.0218$  when  $\alpha_y = 10$ . At the same time, the increase of  $A_y$  from  $-0.260$  to  $7.295$  is mainly offset by the corresponding decrease of  $y_b$  from  $0.0322$  to  $0.0025$ , such that  $A_y y_b$  increases relatively weakly, from  $-0.0084$  over  $0.0024$  (at  $\alpha_y = 0.50$ ) to  $0.0181$ .

of  $\alpha_\pi$ , a negative trace of  $J$  may be taken for granted for at least all  $\alpha_\pi \geq \alpha_\pi^c = 0.216$ . So we have to study the determinant of  $J$ , that is, the expression  $\phi_o + \phi_\pi \alpha_\pi$  from eq. (29). Numerically, one here computes  $\phi_o = -0.0016$ ,  $\phi_\pi = 0.0012$  (rounded).  $\det J$  is therefore an increasing function of  $\alpha_\pi$ , which is negative at  $\alpha_\pi = 0$  and becomes positive when  $\alpha_\pi = -\phi_o/\phi_\pi = 1.314$ .

In conclusion, the central bank succeeds in stabilizing the steady state equilibrium, if it sets  $\alpha_\pi > 1.314$ . Notice that  $\alpha_\pi^s = 1.314 > 0.216 = \alpha_\pi^c$ . This means it is not sufficient for the policy rule to take care of  $y_\pi < 0$ , inflation targeting must rather be stronger. In fact, in the present example the central bank must raise  $\alpha_\pi$  above Taylor's reference value of  $\alpha_\pi = 0.50$ .

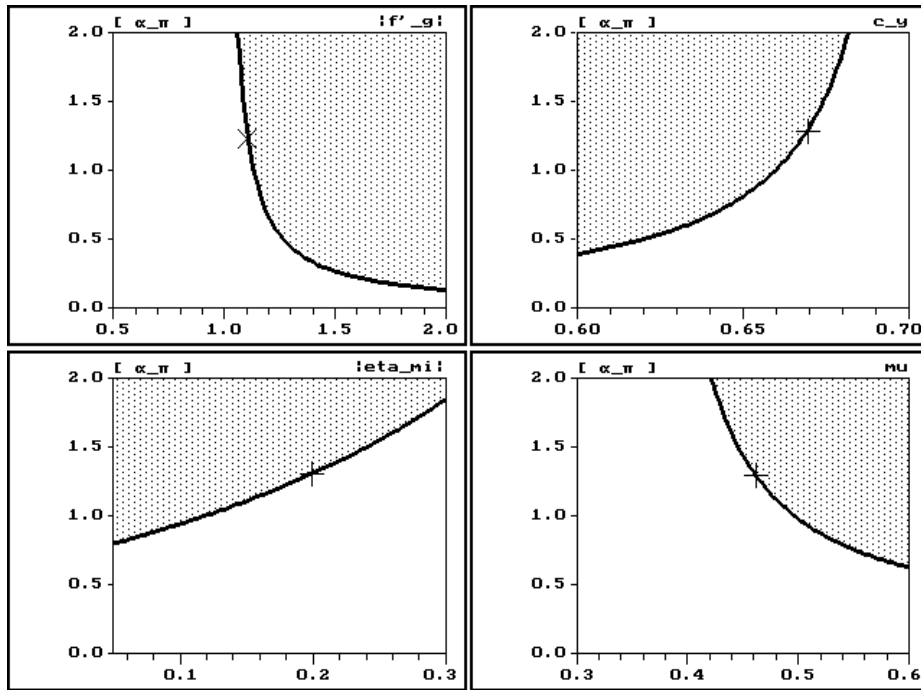
Before proceeding with a sensitivity analysis of this kind of result, we notice that the critical value  $\alpha_\pi^c = c_y(1-\tau)b^o/A_g$  does not change much, if at all, under a very wide range of parameter variations. It, moreover, turns out that  $\phi_o$  remains negative. Consequently,  $\phi_\pi > 0$  is required for the determinant in (29) to become positive, and for the steady state to become locally stable. If  $\phi_\pi > 0$ , then, except for extremely large deviations of some of the parameters from the scenario (30)–(33), the above phenomenon is maintained. We may thus point out:

**Observation 1.** *The borderline value  $\alpha_\pi^s$  (if it exists), from when on local stability prevails, tends to exceed the value  $\alpha_\pi^c$  from Proposition 2, from when on the output reactions  $y_\pi$  are negative. That is,  $\alpha_\pi > \alpha_\pi^c$  is a necessary, but not a sufficient, condition for the stability of the steady state position.*

Because of its central role for stability, let us have a closer look at the ‘slope’ coefficient  $\phi_\pi$  of the determinant, which perhaps appears to be somewhat low. By (28),  $\phi_\pi$  is the difference between two positive terms  $\phi_{\pi g} := A_g A_1 = 0.0201$  and  $\phi_{\pi c} := c_y(1-\tau)i^o [(1-\tau)b^o + (h^o + b^o)|f'_g| + A_2] = 0.0189$  (rounded). There should be several coefficients or steady state values a modest change of which yields  $\phi_{\pi g} - \phi_{\pi c} < 0$ . One example is a reduction of the investment reaction intensity. If it happens to be  $f'_g = -0.90$  (instead of  $-1.10$ ),  $A_g$  in (19) decreases and one computes  $\phi_{\pi g} = A_g A_1 = 0.0157$ .  $\phi_{\pi c}$  decreases, too, but less so;  $\phi_{\pi c} = 0.0168$ . The outcome is  $\phi_\pi = \phi_{\pi g} - \phi_{\pi c} = -0.0012$ . Since the ‘intercept’ term  $\phi_o$  is not affected by variations of  $f'_g$ ,  $\det J$  as determined in (29) is negative for all  $\alpha_\pi \geq 0$ . Under these circumstances, the central bank is no longer able to accomplish convergence toward the steady state.

A stronger responsiveness of investment, on the other hand, contributes to a stabilization of the economy. Thus, consider  $f'_g = -1.30$ . Here  $\phi_{\pi g}$  increases more than  $\phi_{\pi c}$ :  $\phi_{\pi g} = 0.0250$  and  $\phi_{\pi c} = 0.0209$ , so that the difference  $\phi_\pi$  rises from the previous 0.0012 to 0.0036.  $\phi_o$  remaining unchanged, the ratio  $\alpha_\pi^s = -\phi_o/\phi_\pi$  declines by a factor of almost 3, down to  $\alpha_\pi^s = 0.445$ . This threshold is now slightly below Taylor's reference value of 0.50.

Already with the information given, it can be critically asked for the speed of convergence or divergence. Before we turn to this issue, we investigate the stabilizing or destabilizing implications of other *ceteris paribus* parameter variations. A first set of reaction coefficients is presented in Figure 2. The upper-left panel extends the selective calculations for the investment responsiveness  $f'_g$ . To each value  $f'_g$ , the bold line indicates the corresponding value of the inflation targeting coefficient  $\alpha_\pi^s$ , if it exists, that ensures local stability for  $\alpha_\pi > \alpha_\pi^s$ . Hence the dotted area is the set of all pairs  $(|f'_g|, \alpha_\pi)$  that, given the other numerical parameters in (30)–(33), render the steady state locally stable. As  $\alpha_\pi^s$  decreases with  $|f'_g|$  rising, a higher responsiveness of investment to changes in the real interest rate is certainly stabilizing, while at a responsiveness not much less than the baseline value  $f'_g = -1.10$ , the economy is always unstable.



*Figure 2:* Local stability under variations of  $|f'_g|$ ,  $c_y$ ,  $|\eta_{mi}|$ ,  $\mu$ .

*Note:* Other parameters as set in scenario (30)–(33). Points in dotted area imply stability (instability otherwise), cross indicates  $\alpha_\pi^s$  when all parameters are taken from (30)–(33).

Once the possibility of total instability is recognized, the stabilizing potential of a higher investment responsiveness might seem intuitively clear, because the same change in the interest rate has a stronger impact on aggregate output (in the first round, so to speak) and on the adjustments of the rate of inflation (in the second round). However, given the complicated expressions in eq. (29) for  $\det J$ , which reflect the interaction of the inflation and bond dynamics, there are

additional mechanisms at work that are eventually responsible for the stabilizing effects of  $|f'_g|$ .

Another parameter that directly influences the reactions of aggregate output in response to changes in the real interest rate is the marginal propensity to consume,  $c_y$ . Since a higher propensity increases the Keynesian multiplier,  $c_y$  might be expected to have similar effects as  $|f'_g|$ . The upper-right panel of Figure 2 reveals that this presumption is false: higher values of  $c_y$  are destabilizing rather than stabilizing. Furthermore, already at a familiar propensity like  $c_y = 0.70$  there are no more realistic coefficients  $\alpha_\pi$  by which the central bank could bring about stability ( $\alpha_\pi^s = 9.16$  in this case).

The lower-left panel of Figure 2 demonstrates that a higher (modulus of the) interest elasticity of money demand,  $\eta_{mi}$ , is destabilizing. In comparison to the former two examples, however, the effect is quite moderate. The reason is that in the Jacobian matrix, the coefficient  $f_{mi}$  that corresponds to  $\eta_{mi}$  only shows up in the partial derivatives  $h_\pi$  and  $h_b$ , whose impact on the entries  $j_{21}$  and  $j_{22}$  (via  $A_y$  and  $A_\pi$ ) proves to be relatively minor.<sup>15</sup>

According to the fourth panel of Figure 2, the economy is destabilized by a higher money multiplier, i.e., a lower value of  $\mu$ . This observation could have some significance for a more explicit modelling of the financial sector. Recall that setting the money multiplier at  $1/\mu = 2.17$  was based on the standard formula that includes the reserve requirement ratio and the proportion in which people hold money in currency. If, for simplicity, the modelling of a banking sector disregards money holdings in currency, the money multiplier would directly be given by the reciprocal of the reserve requirement ratio, so that  $\mu \leq 0.10$ . At least in the present limited framework, this value would be much too low to possibly give rise to stability.

In a second set of experiments we study *ceteris paribus* variations of steady state magnitudes. To begin with the upper-left panel in Figure 3, which considers government debt, it is seen that higher indebtedness endangers stability. For better comparability, reference is made to the equilibrium ratio of bonds to nominal output rather than to the bonds-capital ratio  $b^o$ . It can thus be said that certainly total instability would prevail if government debt were at a European scale, with  $B/pY$  being at a 50 or 60 per cent level.

Regarding the other financial asset, the ratio of money holdings  $M/pY = 0.15$  can perhaps be deemed to be somewhat low. The second panel in the upper-right corner of Figure 3 shows that higher ratios would be no problem; they would, mildly, enhance the stability prospects of the economy.

The equilibrium rate of interest of the baseline scenario was set quite arbitrarily at  $i = 4.5\%$ . The lower-left panel makes us aware that this choice is not innocent: a few basis points

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<sup>15</sup>For example, reducing the interest elasticity to  $|\eta_{mi}| = 0.10$  only changes  $j_{21}$  from 0.2064 to 0.2187 and  $j_{22}$  from  $-0.0162$  to  $-0.0186$ .

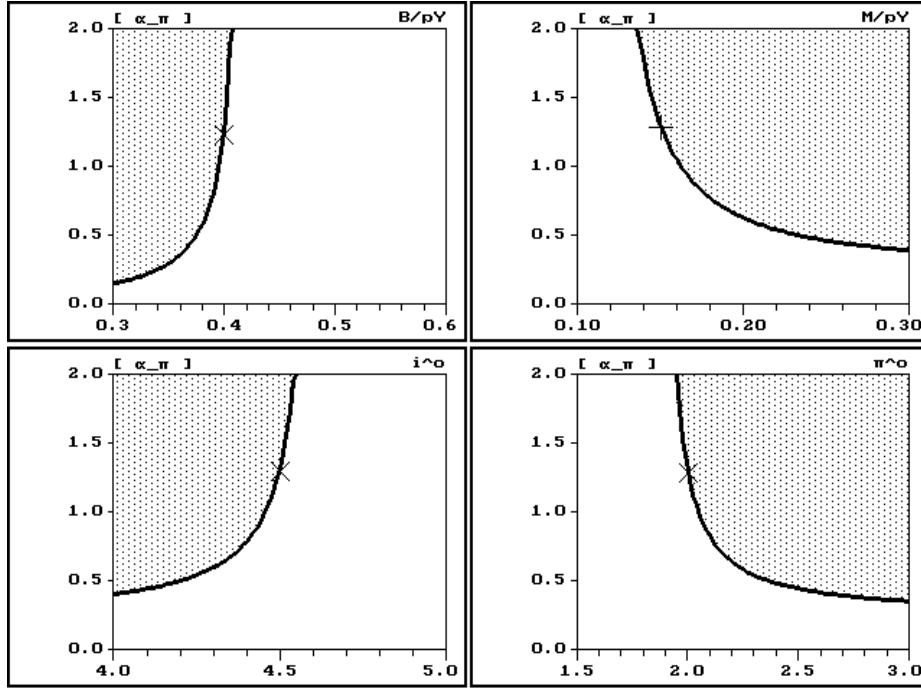


Figure 3: Local stability under variations of  $(B/pY)^o$ ,  $(M/pY)^o$ ,  $i^o$ ,  $\pi^o$ .

more destabilize the economy completely. Incidentally, the problem is not Assumption 1, which remains satisfied for  $i \leq 6.2\%$ . Referring to the terms  $\phi_{\pi g}$  and  $\phi_{\pi c}$  that were introduced above, a higher interest rate ( $i^o = 4.7\%$ , say) rather decreases  $\phi_{\pi g}$  (from 0.0201 to 0.0187) and increases  $\phi_{\pi c}$  (from 0,0189 to 0.0197), such that  $\phi_{\pi} = \phi_{\pi g} - \phi_{\pi c}$  quickly becomes negative (the ‘intercept’  $\phi_o$  raises slightly but stays below zero).

Lastly, the lower-right panel of Figure 3 could be viewed against recent debates on whether the central bank may have a target rate of inflation of less than 2%. The main apprehension is here the risk of a ‘liquidity trap’, when at low inflation or even deflation the central bank, owing to the nonnegativity constraint on  $i$ , can no longer sufficiently reduce the real rate of interest. These problems are, of course, completely absent in the local analysis of the present economy. The panel shows that targeting for lower inflation can also destabilize the economy in very different ways.

The findings of this numerical stability analysis may be briefly summarized in a second ‘observation’.

**Observation 2.** *With respect to the local stability of the steady state position, the following parameter changes are stabilizing: a higher responsiveness of investment to interest rate variations,  $|f'_g|$ ; a lower propensity to consume,  $c_y$ ; a lower interest*



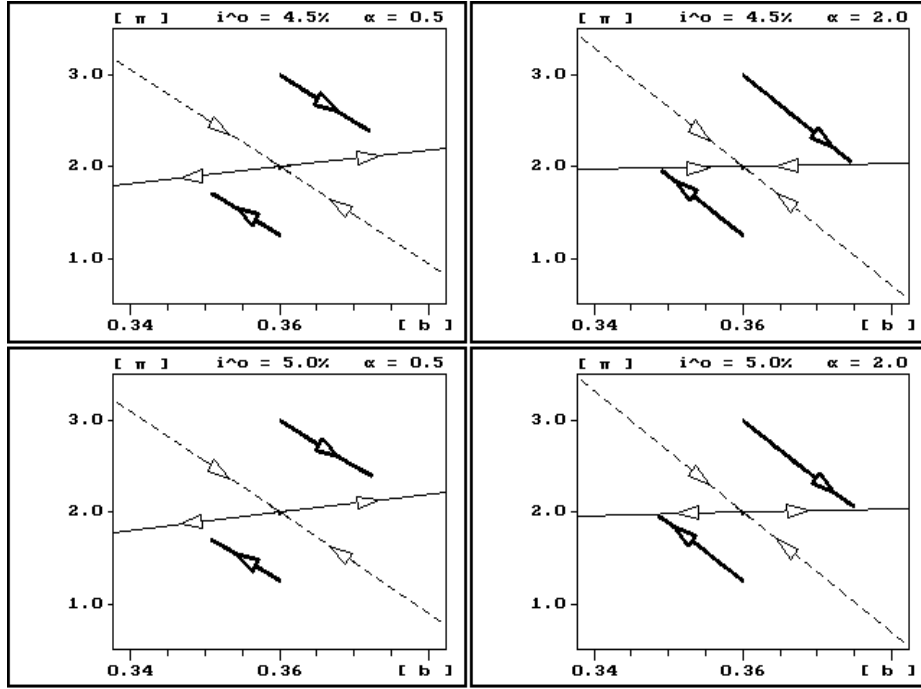
*elasticity (in absolute value) of money demand,  $|\eta_{mi}|$ ; a lower money multiplier, i.e., higher values of  $\mu$ . As for the steady state magnitudes, stabilizing are also: a lower ratio of government debt  $b^o$  or  $(B/pY)^o$ ; a higher money-output ratio  $(M/pY)^o$ ; a lower rate of interest  $i^o$ ; a higher target rate of inflation  $\pi^o$ .*

*Local stability as well as instability can be brought about by fairly reasonable sets of parameter values.*

The analysis so far was concerned with checking the stability conditions, whether local stability prevails or not. These results should, however, be complemented by an investigation of the speed of convergence or divergence. As a matter of fact, it might be suspected that the adjustments are rather slow. One easily infers from the above numerical examples that  $\det J \cong \phi_o + \phi_\pi \alpha_\pi$  is very close to zero, which means that likewise one of the eigen-values of the Jacobian, designate it  $\lambda_1$ , is nearly zero. In addition, the second eigen-value  $\lambda_2$  always falls short of this one, so that, after possibly a phase of transition, the speed of convergence or divergence is eventually determined by  $\lambda_1$ . To take up the example with the three investment reaction coefficients  $f'_g = -0.90, -1.10, -1.30$ , and setting  $\alpha_\pi$  as high as  $\alpha_\pi = 2$ , the leading eigen-value is computed as  $\lambda_1 = 0.0027, -0.0004, -0.0023$ , respectively (while  $\alpha_y$  discernibly changes the second-eigen-value, it has no effect on the first four significant digits of  $\lambda_1$ ). A similar order of magnitude obtains for alternative values of  $\alpha_\pi$  and also for quite different values of the other parameters. It must therefore be concluded that convergence of  $\pi$  and  $b$  toward the equilibrium, as well as divergence from it, takes place at a speed that is far below any time scale worth thinking of.

We have thus to ask for the economy's dynamic behaviour in the medium-run. Consider to this end the phase diagram in the  $(b, \pi)$ -plane of the linearized system (18), (24) in the upper-left corner of Figure 4, which has the baseline scenario with the Taylor coefficients  $\alpha_y = \alpha_\pi = 0.50$  underlying. As is already known, the steady state is still a saddle point at these values. The unstable manifold is drawn as the solid thin line with the outward-pointing arrows, the dashed thin line is the stable manifold given by the (translated) eigen-vector associated with the second eigen-value  $\lambda_2$ .

The bold lines depict two trajectories that are initiated by a positive and negative shock to the rate of inflation in the equilibrium position. The trajectories run over 10 years, where the arrow heads give the state of the economy after 5 years. It is thus seen that it takes more than ten years to reach the unstable manifold. In the meantime, the trajectories are determined by the second eigen-value, in the sense that the speed of change is basically given by  $\lambda_2 < 0$ , i.e. by a factor  $e^{\lambda_2 t}$ , and  $(b, \pi)$  moves parallel to the corresponding eigen-vector. Consequently, after the supply shock assumed, the rate of inflation adjusts back toward its target level, while the bond



*Figure 4:* Phase diagrams of the linearized system (18), (24).

*Note:* Bold lines are trajectories over 10 years (over 5 years at tip of arrow); solid (dashed) thin lines are the paths given by the leading eigen-value  $\lambda_1$  (by  $\lambda_2$ , respectively).  $\alpha = 0.5$  (2.0) stands for  $\alpha_y = \alpha_\pi = 0.5$  (2.0).

ratio begins to diverge from the steady state value. Over a reasonable span of time, this is what characterizes the dynamics.

When, eventually, the system approaches the unstable eigen-vector and the eigen-value  $\lambda_1$  takes over, the motions are so slow that hardly any significant change is visible over the next few decades. So, for all practical reasons, the solid thin line can be regarded as a continuum of equilibria. Observe that even for larger deviations of  $b$  from  $b^o$ , the corresponding ‘equilibrium’ rate of inflation remains close to  $\pi^o$ .

Things are essentially the same if  $\alpha_y$  and  $\alpha_\pi$  are raised to 2.0, which renders the steady state stable. This situation is represented in the upper-right panel of Figure 4. The eigen-vectors associated both with  $\lambda_1$  and  $\lambda_2$  change very little. Though in the long-run all trajectories (except those starting on the dashed thin line) are attracted by the solid thin line, which then carries the economy back to  $(b^o, \pi^o)$ , these adjustments are again completed only over an extremely long period of time. Practically this locus can again be viewed as an equilibrium set. Indeed the main difference to the first panel is the more rapid convergence to this set.

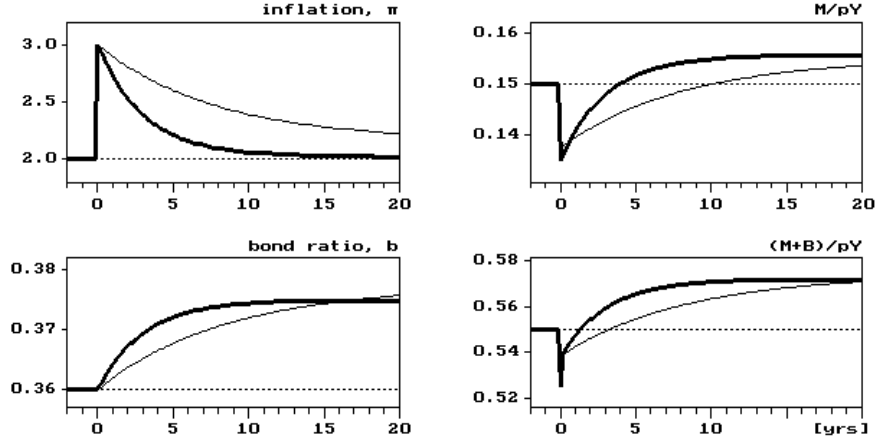
The lower two panels of Figure 4 are based on a higher equilibrium rate of interest,  $i^o = 5\%$ , which implies an unstable steady state for all  $\alpha_\pi$ . The dynamic features are nevertheless very similar. For  $\alpha_y = \alpha_\pi = 2.0$  the movements on the eigen-vector associated with  $\lambda_1$  now point outward, but this hardly matters because they are still so slow. Note also that the eigen-vectors as well as the time paths are almost indistinguishable from those in the upper row of Figure 4. We thus summarize:

**Observation 3.** *The trajectories of system (18), (24) in the  $(b, \pi)$ -plane are attracted by a set  $E$ , which can practically (i.e., over several decades) be regarded as a continuum of equilibria. The values of the inflation rate on this geometric locus are all close to the target level  $\pi^o$ .*

*The speed at which  $(b, \pi)$  approaches the set  $E$  is basically governed by the policy parameter  $\alpha_\pi$ , where higher values of  $\alpha_\pi$  speed up convergence.*

The dynamic features of the baseline scenario may finally be illustrated by the time series diagrams in Figure 5, where the bold lines depict the time paths obtained for  $\alpha_y = \alpha_\pi = 2.0$ . To put them into perspective, the thin lines are the time series resulting from  $\alpha_y = \alpha_\pi = 0.5$ . They show the same qualitative behaviour, though the adjustments are somewhat slower. So let us consider the bold lines, which correspond to the trajectory in the upper half-plane of the upper-right panel in Figure 4. They, in particular, demonstrate that all motions have nearly ceased after 10 years (when the trajectory in Figure 4 has almost reached the quasi-equilibrium set  $E$  given by the eigen-vector of  $\lambda_1$ ). A comparison of the inflation time series in Figure 5 with the upper-right panel in Figure 1 shows that the two adjustment paths of  $\pi$  with the bond dynamics frozen (in Figure 1) and integrated (in Figure 5) are virtually identical. The same holds true for the output gaps,  $[y(\pi, b) - y^o]/y^o$  (not shown) *vis-à-vis*  $[y(\pi, b^o) - y^o]/y^o$  in Figure 1, since the partial derivative  $y_b = \partial y / \partial b$  are really small.

Figure 5 makes it once again clear that after the supply shock to the inflation rate that disturbs the economy from the steady state position, inflation is led back to its target level, whereas bonds persistently diverge. That is, the bond ratio settles on a seemingly new equilibrium value, which is quite distinct from  $b^o$ . To which one, it is easily conceivable, depends on the size of the shock to  $\pi$ . Likewise, if another shock occurs to  $\pi$  in the course of its adjustment toward  $\pi^o$ , the bond ratio will converge to still another quasi-equilibrium level, depending also on the specific value  $b$  has attained when the shock occurred. What has thus been briefly described is the phenomenon of hysteresis (“history matters”). Technically, near-hysteresis could have already been inferred from recognizing the near-zero eigen-values of the Jacobian.



*Figure 5:* Time series of baseline scenario simulations.

*Note:* Bold lines result from adopting  $\alpha_y = \alpha_\pi = 2.0$ , thin lines from  $\alpha_y = \alpha_\pi = 0.5$ .

The two panels on the right of Figure 5 demonstrate that what has been said about the bond ratio similarly applies to the alternative financial asset, money, as well as to total wealth (within our limited setting)  $M + B$ , both here related to nominal output.<sup>16</sup> Interestingly, the shock to  $\pi$  first diminishes the money-output ratio. In the sequel,  $M/pY$  rises and eventually overshoots the steady state ratio, without returning to it. The same pattern is obtained for the sum of the two assets,  $(M + B)/pY$ .

## 8 Conclusion

The present paper relates to the burgeoning literature that combines an interest rate reaction function of the central bank with an IS equation and a Phillips curve relationship. For simplicity, this approach disregards the dynamics of bonds and high-powered money that are implied by this type of monetary policy, and possible feedbacks that may emanate from them. By contrast, taking up the deterministic prototype model of Taylor (2000a, 2001) and Romer (2000) and supposing in line with their background discussion that the interest rates are enforced by open market operations, we have made the evolution of these financial assets explicit. A channel was furthermore introduced through which bonds act on aggregate demand, the assumption being that

<sup>16</sup>The time path of  $M/pY$  is obtained from the decomposition  $M/pY = (H/\mu pK)(pK/pY) = h/\mu y$  and the linear approximation  $z = z^o + z_\pi(\pi - \pi^o) + z_b(b - b^o)$ ,  $z = y, h$ .

consumption depends on disposable income, and that the latter comprises the interest payments on bonds.

The integration of these concepts into the Taylor-Romer framework gave rise to a two-dimensional differential equations system. A first result of the subsequent local stability analysis was the possibility of total instability. That is, whatever values for the inflation and output coefficients in the Taylor rule the central bank may adopt, the steady state position is always unstable. The outcome is somewhat surprising since the (one-dimensional) Taylor-Romer model is unambiguously stable, our innovation of the feedback of bonds on the demand side is minimal, and also the auto-feedback of bonds themselves is a negative one. A careful numerical investigation established that both stability and instability can be brought about by meaningful parameter configurations.

A second main finding of the numerical analysis was a near-zero eigen-value of the Jacobian matrix, which in fact prevailed over all parameter variations considered. It is also the maximal eigen-value, the other one being distinctly negative. So, in the medium-run, the trajectories are attracted by a set which can be characterized as continuum of quasi-equilibria, since the motions have then virtually ceased on it. Consequently, the dynamics can be said to exhibit (near-) hysteresis.

Specifically, if the economy experiences a supply shock in the steady state position, the rate of inflation turns back toward its target level, at a speed roughly comparable with that in the models alluded to above. Over the same time horizon, however, bonds and money diverge from their steady state ratios. We take this behaviour as an indication that the neglect of possible feedbacks of the financial assets on the real side of the economy (other than the interest payment effects here considered) may not be fully consistent. Larger variations of the array of financial assets are likely to affect other financial rates of return, in addition to the bond rate of interest, and some of them, or wealth variables themselves, should finally impact on aggregate demand. Feedbacks from a financial sector modelled in greater detail may therefore not just be interesting, they may also be important in assessing the virtues of (alternative) monetary policy rules.<sup>17</sup>

In conclusion, we may refer to the “Keynesian macroeconomics without the LM curve” mentioned in the title of the paper and likewise to the more general (discrete-time and stochastic) approach based on IS mechanisms, some Phillips curve and an interest rate reaction function. When introducing money and bonds into this framework, the implicit question was for possible destabilization effects. This question can now be answered at three different levels. First, in the long-run, the economy is not safe from instability, as there are realistic parameter scenarios of even

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<sup>17</sup>Strictly speaking, the present assets are still a short-cut because the concept of the money multiplier usually presupposes the existence of loans granted to the private sector by commercial banks.

total instability. This long-run was, however, found to be unduly long. Second, with respect to the medium-run and the usual key variables, convergence of output back toward normal and inflation back toward its target value is not essentially endangered by the present feedbacks from the bond dynamics. Third, these adjustments imply larger deviations of bonds and money. From this feature it can be suspected that also models with a richer financial sector will generate significant variations of the financial assets, which eventually should affect the rest of the economy. This third level provides a field for future research concerning the efficiency of monetary policy rules.

## Appendix

To show  $y_\pi = dy/d\pi < 0$  in eq. (3), write  $j(y, \pi)$  for the right-hand side of (1) and define the function  $F_y = F_y(y, \pi) := y - f_y[j(y, \pi) - \pi]$ , which has partial derivatives  $\partial F_y/\partial y = 1 - \alpha_y f'_y > 0$ ,  $\partial F_y/\partial \pi = -\alpha_\pi f'_y > 0$ . Applying the Implicit Function Theorem to the equation  $F_y(y, \pi) = 0$  yields  $dy/d\pi = -(\partial F_y/\partial \pi) / (\partial F_y/\partial y) < 0$ .

The formulae for the partial derivatives in (8) are most easily derived by applying the Implicit Function Theorem to eq. (7). Omitting here and in the following the superscript ‘o’ that indicates the steady state values, this yields,

$$f_{hi} = \frac{(h + \mu b) f_{mi}}{1 - f_m} \quad f_{hy} = \frac{(h + \mu b) f_{my}}{1 - f_m} \quad f_{hb} = \frac{\mu f_m}{1 - f_m}$$

To obtain the partial derivatives of the IS output-capital ratio  $y = y(\pi, b)$  from eq. (17), compute the partial derivatives of the excess demand function  $E_y$ ,

$$E_{yy} = -A_c - \alpha_y A_g \quad E_{y\pi} = c_y(1 - \tau)b - \alpha_\pi A_g \quad E_{yb} = c_y(1 - \tau)i$$

and get  $y_\pi = -E_{y\pi}/E_{yy}$ ,  $y_b = -E_{yb}/E_{yy}$  from, again, the Implicit Function Theorem. To verify that  $i_\pi = 1 + \alpha_\pi + \alpha_y y_\pi > 1$  in (22), observe that  $\alpha_\pi + \alpha_y y_\pi > \alpha_\pi + \alpha_y \alpha_\pi A_g / (A_c + \alpha_y A_g) > \alpha_\pi - \alpha_\pi = 0$ .

The partial derivatives of the function  $h = h(\pi, b)$  in (23) are

$$\begin{aligned} h_\pi &= f_{hi} i_\pi + f_{hy} y_\pi \\ &= (h + \mu b) [(1 + \alpha_\pi) f_{mi} + (f_{my} + \alpha_y f_{mi}) y_\pi] / (1 - f_m) \\ h_b &= f_{hi} i_b + f_{hy} y_b + f_{hb} \\ &= [(h + \mu b) (f_{my} + \alpha_y f_{mi}) y_b + \mu f_m] / (1 - f_m) \end{aligned}$$

To determine the sign of  $h_b$  and  $1 + h_b$ , abbreviate  $A_i := c_y(1 - \tau)i$  and consider the expression  $a = a(\alpha_y) := \alpha_y y_b = \alpha_y A_i / (A_c + \alpha_y A_g)$ . The function  $a(\cdot)$  is increasing with an upper limit  $A_i/A_g$  as  $\alpha_y \rightarrow \infty$ . With  $f_{my} \geq 0$  we then have  $1 + h_b \geq [1 - f_m + (h + \mu b)a(\alpha_y)f_{mi} + \mu f_m] / (1 - f_m)$ , and

the term in square brackets is positive if  $|f_{mi}| < [1 - (1-\mu)f_m] / (h+\mu b)a(\alpha_y)$ . The right-hand side of this inequality is decreasing in  $\alpha_y$ , and for  $\alpha_y \geq 0$  is larger than  $[1 - (1-\mu)f_m]A_g / (h+\mu b)A_i$ . This proves  $1+h_b > 0$  if Assumption 3a is fulfilled.

Similarly,  $h_b > 0$  if  $(h+\mu b)a(\alpha_y)f_{mi} + \mu f_m > 0$ , or  $|f_{mi}| < \mu f_m / (h+\mu b)a(\alpha_y)$ , which is satisfied if  $|f_{mi}| < \mu f_m A_g / (h+\mu b)A_i$ , the expression in Assumption 3b.

To verify (29) for  $\det J$ , we take up eq. (27) and prove that  $|E_{yy}|(y_\pi A_b - y_b A_\pi) = \phi_o + \alpha_\pi \phi_\pi$ . Define to this end  $A_{i\pi} := (1-\tau)i - (\pi+g)$  and  $A_{cg} := c_y(1-\tau)b - \alpha_\pi A_g$ . Then, with eqs (20), (25) and  $E_{yy}$  as determined above,

$$\begin{aligned}
|E_{yy}| y_\pi A_b &= A_{cg} [A_{i\pi} - h_b(\pi+g)] \\
&= A_{cg} A_{i\pi} - A_{cg} \mu f_m (\pi+g) / (1-f_m) \\
&\quad - A_{cg} (h+\mu b) (f_{my} + \alpha_y f_{mi}) c_y (1-\tau) i (\pi+g) / (1-f_m) |E_{yy}| \\
&= A_{cg} [A_{i\pi} - \mu f_m (\pi+g) / (1-f_m)] \\
&\quad - \frac{A_{cg} (h+\mu b) (f_{my} + \alpha_y f_{mi}) c_y (1-\tau) i (\pi+g)}{(1-f_m) |E_{yy}|} \\
- |E_{yy}| y_\pi A_b &= -c_y(1-\tau)i [(1+\alpha_\pi)(1-\tau)b + (h+b)(\alpha_\pi |f'_g| - 1) \\
&\quad + (h+\mu b)(1+\alpha_\pi)|f_{mi}|(\pi+g) / (1-f_m) \\
&\quad - (h+\mu b) (f_{my} + \alpha_y f_{mi}) A_{cg} (\pi+g) / (1-f_m) |E_{yy}|] \\
&= -c_y(1-\tau)i \{ -\tau b - h + \alpha_\pi [(1-\tau)b + (h+b)|f'_g|] \\
&\quad + (h+\mu b)(\pi+g)|f_{mi}| / (1-f_m) + \alpha_\pi (h+\mu b)(\pi+g)|f_{mi}| / (1-f_m) \} \\
&\quad + \frac{c_y(1-\tau)i (h+\mu b) (f_{my} + \alpha_y f_{mi}) A_{cg} (\pi+g)}{(1-f_m) |E_{yy}|}
\end{aligned}$$

The two fractions cancel out. Then, referring to (26), note that  $A_{i\pi} - \mu f_m(\pi+g)/(1-f_m) = -A_1$ .

We thus remain with

$$\begin{aligned}
|E_{yy}| (y_\pi A_b - y_b A_\pi) &= -c_y(1-\tau)bA_1 + \alpha_\pi A_g A_1 + c_y(1-\tau)i (h + \tau b - A_2) \\
&\quad - \alpha_\pi c_y(1-\tau)i [(1-\tau)b + (h+b)|f'_g| + A_2] \\
&= -c_y(1-\tau) [A_1 b - i (h + \tau b - A_2)] \\
&\quad + \alpha_\pi \{ A_g A_1 - c_y(1-\tau)i [(1-\tau)b + (h+b)|f'_g| + A_2] \} \\
&= \phi_o + \alpha_\pi \phi_\pi
\end{aligned}$$

Finally, to turn to the Rudebusch-Svensson (1999) model mentioned at the end of Section 6, its quarterly equations for the inflation rate and the output gap read, in their deterministic part (*ibid*, p.208):

$$\begin{aligned}
\pi_{t+1} &= 0.70 \pi_t - 0.10 \pi_{t-1} + 0.28 \pi_{t-2} + 0.12 \pi_{t-3} + 0.14 y_t \\
y_{t+1} &= 1.16 y_t - 0.25 y_{t-1} - 0.10 (\bar{i}_t - \bar{\pi}_t)
\end{aligned}$$

where the bar over  $i$  and  $\pi$  indicates backward-looking four-quarter averages,  $\bar{i}_t = (1/4) \sum_{k=0}^3 i_{t-k}$  ( $i$  and  $\pi$  are measured at annual rates). The coefficients result from an estimation over the sample period 1961:1–96:2 (the standard errors are reported as 1.009 for  $\pi$  and 0.819 for  $y$ , the Durbin-Watson statistic is in both cases close to 2).

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