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Modelling asymmetries and moving equilibria in unemployment rates

by

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Abstract:

The paper discusses a simple univariate nonlinear parametric time-series model for unemployment rates, focusing on the asymmetry observed in many OECD unemployment series. The model is based on a standard logistic smooth transition autoregressive (LSTAR) model for the first difference of unemployment, but it also includes a lagged level term. This model allows for asymmetric behaviour by permitting 'local' nonstationarity in a globally stable model. Linearity tests are performed for a number of quarterly, seasonally unadjusted, unemployment series from OECD countries, and linearity is rejected for a number of them. For a number of series, nonlinearity found by testing can be modelled satisfactorily by use of our smooth transition autoregressive model. The properties of the estimated models, including persistence of the shocks according to them, are illustrated in various ways and discussed. Possible existence of moving equilibria in series not showing asymmetry is investigated and modelled with another STAR model.

***Key words**:*

Persistence, nonlinearity, smooth transition regression, time series model, linearity test

1. Introduction

In most OECD countries the unemployment rates have increased markedly since the 1960s. The tendency of an unemployment rate to remain on a level it has reached is often called hysteresis. Blanchard and Summers (1987) demonstrated that a simple 'insider model' (see Lindbeck and Snower (1988a) for the insider-outsider theory of employment) leads to employment following a random walk with an error. They also showed that, under certain conditions, this result is obtained if the wage pressure from the outsiders (unemployed) does not depend on total unemployment but only on expected short-term unemployment. A popular way of investigating hysteresis or full persistence in unemployment rates has consequently been to test the null hypothesis that the unemployment rate has a unit root. An unemployment rate may also be thought of as remaining on a given level until it is dislodged and pushed to a new level by a shock or a series of shocks. The search and match model of Diamond (1982) is an example of a theory model that would cause employment to switch between multiple equilibria. According to that model, shocks to the economy may cause employment to get stuck in an equilibrium that is not maximizing employment and consumption. A time series characterization of such behaviour would require a nonlinear model.

If asymmetry is taken to mean that the unemployment rate increases faster than it decreases then hysteresis or multiple equilibria as such do not imply any asymmetry in unemployment rates. On the other hand, visual inspection of OECD unemployment rate series gives the impression that at least some of them may be asymmetric in this sense. This stylized fact has prompted economists to find explanations to it. Asymmetric adjustment costs of labour provide one such explanation. Several micro studies have shown that the costs of hiring and firing are not symmetric; see Hamermesh and Pfann (1996) for a survey. However, as Hamermesh and Pfann pointed out, at the moment it is not clear that asymmetry at the firm level implies asymmetry in the aggregates. On the other hand, Bentolila and Bertola (1990) argued that a microeconomic model with an asymmetric labour cost adjustment function explains much of the developments in the European unemployment rates after the first oil price shock in 1973. Another theory that might explain asymmetries is that of recessions as cleansing periods; see, for example, Caballero and Hammour (1994) and references therein. Empirical studies have shown that job destruction is highly asymmetric over the business cycle: jobs disappear at a higher rate during recessions than expansions. As this is not compensated by asymmetry in job creation, the result is asymmetry in employment. This seems to accord with the insider-outsider theory. Lindbeck and Snower (1988b) pointed out that under certain conditions, incumbent workers, i.e., insiders, would be able to prevent employment from rising during expansions. Furthermore, capital destruction has been discussed as a constraint creating asymmetries; see, for example, Bean (1989).

Modelling asymmetry would require nonlinear time series models. Various nonlinear models have been fitted to a number of unemployment rates or their differences or functions of them for varying reasons. Neftçi (1984) used unemployment series as a business cycle indicator when he investigated asymmetry of business cycles using a two-state Markov chain; see also Sichel (1989) and Rothman (1991), and Pfann (1993) for a survey. Parker and Rothman (1997), Rothman (1998), and Montgomery, Zarnowitz, Tsay and Tiao (1998) considered forecasting US unemployment with nonlinear models. A number of authors have illustrated new statistical theory by applying it to unemployment series. Hansen (1997) developed statistical inference for the threshold parameter in a threshold autoregressive (TAR) model and fitted a TAR model to a US unemployment rate series. Koop and Potter (1998) modelled the logistically transformed US unemployment rate by applying Bayesian techniques to TAR models. Brännäs and Ohlsson (1998) derived analytical temporal aggregation formulas for autoregressive asymmetric moving average (ARasMA) models and fitted an ARasMA model to monthly and quarterly Swedish unemployment rates. Tschernig (1996) checked his nonparametric model selection and estimation techniques by applying them to a set of German unemployment rates (original ratios and differences; seasonally unadjusted and adjusted).

Some work has also been done to test economic theories. Stock (1989) used US and UK data to test the time deformation idea: the dynamic properties of unemployment rate series are a function of the level of the series. With a suitable set of parameters, one could demonstrate how a series may get stuck at a high level as the work of Diamond (1982) would suggest. This leads to models that resemble TAR and switching regression models. Franses (1998) considered the hypothesis that seasonality varies with the business cycle using unemployment rates. Bianchi and Zoega (1998) took up the issue of multiple equilibria and studied their existence in 15 OECD unemployment series using a variant of the switching regression model with Markov switching introduced by Lindgren (1978). In their model, only the intercept is switching, the parameterization being identical to that in Hamilton (1989). Multivariate work includes the time-varying adjustment cost model of Burgess (1992a,b) and the work of Acemoglu and Scott (1994).

While many economic theories suggest that asymmetry may be a characteristic feature of unemployment rates there have not been many attempts at modelling this phenomenon in an explicit fashion. Typical macroeconomic models are vector autoregressions in which the unemployment rate is assumed to be nonstationary and cointegrated with a set of other macro variables. In this article we show that it is possible to account for asymmetries in unemployment rates by starting from the realistic assumption that the unemployment rate is a stationary, but possibly nonlinear, variable. We show that a simple modification of a well-known nonlinear model, the so-called smooth transition autoregressive (STAR) model, constitutes a useful model for capturing the asymmetry, when present, in quarterly unemployment series. In a number of cases, such a model is easy to interpret and fits the data much better than a linear autoregressive model. Our results should therefore have relevance also in the multivariate modelling of macroeconomic relationships and could be seen as a first step towards multivariate specifications.

We do not expect all our unemployment series to show asymmetry. For the series that do not exhibit asymmetry we inquire as Bianchi and Zoega (1998) did whether or not unemployment rates have been moving between different equilibria. Some economic theories indicate that possibility. Our approach differs from that of Bianchi and Zoega in the sense that we allow the transition from one equilibrium to the next to be smooth. Such cases of moving equilibria may again be characterized by STAR models of a particular kind. As an extra bonus, smooth changes in the seasonal pattern of unemployment rates can also be captured and parameterized in this framework, which gives new insight in the dynamics of these series.

The plan of the paper is as follows: In Section 2 we introduce the model and illustrate some of its important properties, and in Section 3 we briefly describe the testing and modelling procedure. After presenting the data set in Section 4 we turn to the empirical results in Sections 5 and 6. Section 7 contains conclusions.

2. The model

2.1 General

As discussed in the Introduction testing the null hypothesis of a unit root has been a common way of analysing unemployment rate series. It is tantamount to having hysteresis or full persistence as the null hypothesis. In a recent survey, Røed (1997) reported that the unit root hypothesis has rarely been rejected for any country or unemployment series. Some of the seasonally adjusted US monthly or quarterly unemployment rates seem to constitute the only exception to this pattern. The implication of this result is that the unemployment rate is fully persistent. In other words, in a univariate setting if we exclude the possibility of a drift, any unemployment rate is an equilibrium rate. Besides, as the tests assume linearity, at any level, be it 2% or 22%, say, the rate is as likely to go up as it is to move down from that level.

If an empirical analysis is started by testing hysteresis the empirical evidence Røed (1997) assembled indicates that one does not often get further to considering asymmetry. Our analysis has a different starting-point. We assume that the unemployment rate is a stationary process. This is a reasonable assumption as the rate is a bounded variable. Furthermore, our starting-point is that the process is even linear. The first thing we shall do is to test linearity against the alternative that the process is nonlinear. The nonlinearity is parameterized in a way which allows asymmetry in the behaviour of the unemployment rate. If linearity is rejected against such an alternative, we specify, estimate, and evaluate a nonlinear model for the unemployment rate and discuss the implications of the estimated model. Persistence will be among the issues to be considered. The possibility of multiple equilibria will also receive attention.

2.2 An artificial example

The idea of modelling asymmetries in unemployment rates suggested in this paper can be illustrated as follows. Figure 2.1 depicts a 'stylized unemployment series' without seasonality. The series grows rapidly and decreases rather slowly. A relatively long decrease is followed by another rapid increase. The time and magnitude scales are arbitrary. In fact, the series has been generated by the following simple nonlinear autoregressive model

$$\begin{aligned}\Delta y_t = {} & \mu_1 + \alpha_1 y_{t-1} + \left(\mu_2 + \alpha_2 y_{t-1}\right) \\ & \times \left(1 + \exp\left\{-\gamma\left(\Delta y_{t-1} - c\right)\right\}\right)^{-1} + u_t ,\end{aligned} \tag{2.1}$$

where $u_t \sim \text{nid}\left(0, \sigma_u^2\right)$, and μ_1=0, α_1=–0.01, μ_2=0.32, α_2=–0.20, γ=1000, c=0.05, and σ_u^2=0.02^2. As can be seen, the realization displays many characteristic features of an asymmetric unemployment series.

It is seen that (2.1) is an equation in levels although it has been parameterized using first differences. The process behaves as follows. When the value of the process (y_t) is near zero then the next observation (y_{t+1}) has a tendency to be near zero as well, unless a sufficiently large positive shock arrives. In the latter case, in the next period the high value increases the value of the logistic transition function which is a function of the first difference of y_t. Note that the value of γ, the steepness parameter, is so high that the value of the logistic function changes rapidly from zero to unity around the location parameter c as a function of Δy_{t-1}. Suppose the next value is unity. The next observation is thus generated by

$$\Delta y_{t+1} = 0.32 - 0.21 y_t + u_{t+1} \tag{2.2}$$

which means that it is likely to be clearly higher than the preceding one because the process is still way below the mean of (2.2) which is about 1.5. The (positive) mean is determined by $\alpha_1+\alpha_2<0$ and $\mu_1+\mu_2>0$. It is essential that $\alpha_2<0$ and $\mu_2>0$. Thus the growth continues until the realization approaches the mean of the above regime. Then any sufficiently large negative shock will slow down the process, and this slowdown shows in the transition function the next period. The process then may switch back to

$$\Delta y_{t+1} = -0.01 y_t + u_{t+1} \qquad (2.3)$$

corresponding to the value zero of the transition function. The drift towards the original zero level is weak, however, as the near-zero negative coefficient of y_t in (2.3) indicates. Thus the coefficient α_1 controls the speed of adjustment to the lower level. The parameter c determines the magnitude of the shock that will trigger growth in the realization when the process is decreasing or fluctuating at the bottom level. Note that (2.1) with the parameter values used in the example does not imply the existence of multiple equilibria.

We may add more lagged differences to (2.1) but the basic idea remains the same. The key is that the variable in the transition function is a difference while the process itself is expressed in levels and remains bounded in probability. In the following sections we shall generalize the model and discuss it more formally.

2.3 Asymmetry and unit roots tests

As mentioned before, it has been common to test the hypothesis of a unit root in unemployment series, and the hypothesis has generally been accepted. For this reason, before proceeding further it is important to know what happens if unit root tests are performed on series generated by a model of type (2.1). If the unit root hypothesis is solidly rejected the model cannot form a useful starting-point for modelling unemployment series because it would then contradict well-established empirical facts. As a first ”test” of (2.1) we therefore carried out a small simulation experiment applying a standard augmented Dickey-Fuller test. The data were generated both according to the LSTAR model (2.1) and according to a linear autoregressive model using the same random numbers in both cases. The linear generating process was simply

$$\Delta y_t = \alpha_1 y_{t-1} + u_t \qquad (2.4)$$

where $u_t \sim \text{nid}\left(0, 0.02^2\right)$. The lag-level parameters α_1 and α_2 were varied, using the values -0.01, -0.02, -0.05, -0.1 for α_1 and 0, -0.01, -0.1, and -0.2 for α_2. In order to generate realizations with peaks of approximately the same magnitude over the different combinations of α_1 and α_2, the value of μ_2 was adjusted from one experiment to the other. Figure 2.2 gives an example of the realizations produced by the different parameter combinations, using the sequence of random numbers that generated the realization in Figure 2.1. Series of 200, 500 and 1000 observations are generated. The lag length used in the augmented Dickey-Fuller test is determined by adding lags until the Ljung-Box test statistic no longer rejects the null hypothesis at the signficance level 0.05. In practice, the resulting lag length is almost always equal to one or two. The augmented Dickey-Fuller tests are computed with and without a constant and a trend.

The results of the simulations based on 10000 replicates each can be found in Tables 2.1-2.3. The figures in the tables are the relative rejection frequencies at the 5% level. The rightmost column of each table refers to the linear case, in which the data have been generated linearly by assuming $\alpha_2 = \mu_2 = 0$ in (2.1).

For the smallest sample size, T=200 (Table 2.1), it is seen that the test version without a constant and a trend has the highest frequency of null hypothesis rejections, and for this test statistic the rejection frequencies for the nonlinear models are of roughly the same magnitude as for the linear ones. The differences between the test results for nonlinear and linear series using this test statistic become somewhat more pronounced when the sample size increases. For the top rows representing the asymmetric series the rejection frequencies for the nonlinear realizations are below those for the linear ones for most values of α_2 when T=1000 (Table 2.2). For the two other panels, i.e., the ADF tests with a constant and possibly a trend, the evidence for T=1000 is more mixed. By comparing the results for T=200 and T=1000 for one and the same model it seems that the ADF tests might be inconsistent against some LSTAR models we have simulated. Taken together, the results seem to suggest that at sample sizes often encountered in applied work, asymmetry as defined by (2.1) does not provide additional evidence against the unit root hypothesis compared to the corresponding linear model. Finding a unit root on the one hand and nonlinearity (asymmetry) on the other using the same data thus need not be a contradiction.

2.4 The LSTAR model

The simulation results of the previous section thus do not invalidate the idea of using the structure defined in (2.1) for modelling unemployment series. However, although (2.1) will form the core of the specification the model has to be extended before fitting it to data. The main reason for this is that it ignores the seasonal variation present in almost any unemployment series. Using deseasonalized series is not advisable because it is not known what may happen to a nonlinear asymmetric series when it is filtered by applying some standard seasonal adjustment procedure. At a first glance, extending (2.1) to explain seasonal variation does not seem difficult. We could just add seasonal dummy variables linearly to model (2.1), which we also do. But then, as mentioned in the Introduction, the seasonal pattern in unemployment rates may change over time due, for example, to gradual technological change such as making construction work less dependent on seasons. Taking this possibility into account is possible in the STAR framework and requires a nonlinear extension of the seasonal part of the model. Finally, there exists literature suggesting that seasonality in production series depends on the business cycle, for a review see Franses (1996, pp. 84-88). It is thus logical to expect similar effects in unemployment rates as well. For example, a rapid decrease in production leading to a strong nonseasonal increase in the unemployment rate may also have a temporary impact on the seasonal pattern of unemployment. Accounting for this effect requires another extension of the basic model (2.1).

The STAR type model accommodating all the above features can be defined as follows

$$\Delta y_t = \mu_1^0 + \mu_1^1 H_k\left(t^*;\gamma_1,\mathbf{c}_1\right) + \alpha_1 y_{t-1} + \sum_{j=1}^{p}\beta_{1j}\Delta y_{t-j} + \sum_{i=1}^{s-1}\left(\delta_{1i}^0 + \delta_{1i}^1 H_k\left(t^*;\gamma_1,\mathbf{c}_1\right)\right)d_{it} + \\ + \left\{\mu_2^0 + \mu_2^1 H_k\left(t^*;\gamma_1,\mathbf{c}_1\right) + \alpha_2 y_{t-1} + \sum_{j=1}^{p}\beta_{2j}\Delta y_{t-j} + \sum_{i=1}^{s-1}\left(\delta_{2i}^0 + \delta_{2i}^1 H_k\left(t^*;\gamma_1,\mathbf{c}_1\right)\right)d_{it}\right\} \\ \times G\left(\Delta_s y_{t-d};\gamma,c\right) + u_t \tag{2.5}$$

where y_t is the unemployment rate in percent, d_{it} denote seasonal dummies, i=1,2,..,s–1, s=4 for quarterly data, and $u_t \sim \mathrm{nid}\left(0,\sigma_u^2\right)$. Function $G\left(\Delta_s y_{t-d};\gamma,c\right)$ is related to modelling asymmetry and is defined as a logistic transition function of $\Delta_s y_{t-d}$ given by

$$G\left(\Delta_s y_{t-d};\gamma,c\right) = \left[1+\exp\left\{-\gamma\left(\Delta_s y_{t-d}-c\right)/\sigma\left(\Delta_s y_t\right)\right\}\right]^{-1},\ \gamma > 0 \tag{2.6}$$

where $\sigma\left(\Delta_s y_t\right)$ is the standard deviation of $\Delta_s y_t$ and $\gamma > 0$ is an identifying restriction. The value of the delay parameter d in (2.6) is generally unknown and has to be determined from the data.

Note that the transition variable in G is a difference as in (2.1). Enders and Granger (1998) had a similar construction for a threshold autoregressive (TAR) model; they called their model the Momentum TAR model. (The two-regime TAR model is a special case of the LSTAR model obtained as $\gamma\to\infty$ in G). Using the seasonal rather than the first difference as the transition variable is important. The asymmetries of primary interest in this article are those related to the business cycle, not those of the annual seasonal cycle in unemployment. Even without seasonality, a relatively long difference would normally be preferable to the first difference. If the transition variable is rather noisy then a first difference does not represent variations in the unemployment cycle, and smoothing in the form of a longer difference becomes necessary.

Modelling smooth changes in the seasonal pattern over time are the reason for combining the intercept and the seasonal dummies in (2.5) with another transition function

$$H_k\left(t^*;\gamma_1,\mathbf{c}_1\right)=\left(1+\exp\left\{-\gamma_1\prod_{j=1}^{k}\left(t^*-c_{1j}\right)\right\}\right)^{-1},\quad k=1,2,3 \tag{2.7}$$

where $t^* = t/T$ (time in the transition function is scaled between 0 and 1), $\gamma_1 > 0$, and with $\mathbf{c}_1 = \left(c_{11},\ldots,c_{1k}\right)'$, for k = 1,2,3 and, furthermore, $c_{1i} \leq c_{1j}$ for $i<j$. Function (2.7) with k=1 describes monotonic parameter change in time that becomes a single break when $\gamma_1 \to \infty$, whereas k=2 allows symmetric change around $(c_{11} + c_{12})/2$ with a structural break and a counterbreak as the limiting case as $\gamma_1 \to \infty$. Finally, k=3 allows nonmonotonic and nonsymmetric change, but H_3 may also be a monotonically increasing function of t. Function (2.7) thus offers plenty of flexibility in modelling changing seasonal patterns.

Finally, note that the seasonal dummies also appear in connection with function G, which enables us to model the effects of other than seasonal changes in the unemployment rate on the seasonal pattern. Franses (1998) also makes use of this idea. As mentioned above, such effects may be present in the series. It should be pointed out, however, that not all of the features listed above are present in every unemployment series considered. Model (2.5) represents the most general specification we are willing to consider, but most of our estimated models will be submodels of (2.5).

3. Testing linearity and modelling nonlinear series

A practical advantage of considering models of the smooth transition regression (STR) or autoregression (STAR) type here is that there exists a ready-made modelling procedure for STR and STAR models that can be applied to our unemployment series. The modelling cycle, fully described in Teräsvirta (1998), comprises three stages: testing linearity against STAR, if linearity is rejected specifying and estimating the STAR model, and evaluating the estimated model. In this case we do not, however, proceed directly with (2.5) but rather in stages. We treat the possible asymmetry expressed with the transition function G as being our primary interest. This means assuming first that seasonality is constant over time and that there exists just a single equilibrium, that is, $H_k \equiv 0$. The specification and estimation of the LSTAR model is carried out under this assumption. The assumption is tested when the estimated model is evaluated. If it is rejected, the model is augmented as (2.5) indicates and the parameters of the augmented model are estimated.

We begin with the following LSTAR model:

$$\begin{aligned}\Delta y_t = {} & \mu_1 + \alpha_1 y_{t-1} + \sum_{j=1}^{p} \beta_{1j} \Delta y_{t-j} + \sum_{j=1}^{s-1} \delta_{1j} d_{j,t} + \\ & + \left(\mu_2 + \alpha_2 y_{t-1} + \sum_{j=1}^{p} \beta_{2j} \Delta y_{t-j} + \sum_{j=1}^{s-1} \delta_{2j} d_{j,t} \right) \\ & \times \left(1 + \exp\left\{ -\gamma \left(\Delta_s y_{t-d} - c \right) / \sigma\left(\Delta_s y_{t-d} \right) \right\} \right)^{-1} + u_t, \gamma > 0\end{aligned} \tag{3.1}$$

where $u_t \sim \text{nid}\left(0, \sigma_u^2\right)$. The linearity test used here is the LM-type test described in Granger and Teräsvirta (1993, ch. 6) and Teräsvirta (1994, 1998). They are based on expanding the transition function in (3.1) into a Taylor series, merging terms and reparameterizing. The resulting auxiliary regression forms the base for the tests. As the Taylor expansion is of order zero under the null hypothesis of linearity (there is nothing to expand) the remainder does not affect the asymptotic distribution theory. To begin with, a linear model with a lag structure such that the errors can be assumed to be free from autocorrelation is selected. To this end, Δy_t is regressed on $y_{t-1}, \Delta y_{t-1}, \ldots, \Delta y_{t-p}$, seasonal dummies, and a constant, for various values of p, and the value minimizing an information criterion (AIC) is selected to be the null model. Linearity is tested against (3.1) with and without seasonal dummies in the nonlinear part of the model. This is done separately for different lags (d) of the seasonal difference of unemployment as the transition variable. If linearity is rejected for more than one value of d, the lag yielding the smallest p-value is selected. (See Teräsvirta (1994) for the motivation behind this selection rule.) If the p-values for a number of values of d are of similar magnitude, more than one model may be tentatively estimated and the final choice left to the evaluation stage. After rejecting linearity and choosing d the LSTAR model is estimated by nonlinear least squares. At this stage, the size of the model may be reduced by imposing exclusion restrictions whenever appropriate.

It should be pointed out that by performing a whole set of linearity tests the overall significance level is not under the control of the modeller. This may lead to erroneous rejections of linearity. Nevertheless, the tests in this context are used as model building devices rather than tests of an economic theory. An erroneous decision at the model specification stage may be revealed already when the model is estimated (convergence problems and meaningless parameter estimates) or at the evaluation stage at the latest. This is another reason for emphasizing the importance of model evaluation. The adequacy of an estimated model is checked by a set of misspecification tests in which model (3.1) is tested against more general alternatives. An LM test of serially uncorrelated errors is performed as well as the test of no autoregressive conditional heteroskedasticity. The hypothesis of no remaining nonlinearity is tested against an alternative of additional nonlinearity of the STR type. Furthermore, since constancy of parameters is a crucial assumption underlying the estimation of the model, this hypothesis is tested against an alternative of smoothly changing parameters. This is done for the case where the alternative covers all parameters except γ and c, but testing (3.1) against (2.5) is of course of particular interest. If model (3.1) is rejected then the parameters of (2.5) are estimated. The results of the different statistics from the three parameter constancy tests corresponding to k=1,2,3 in (2.7) help select k. These tests have the same structure as the linearity tests in that the new additive component defined by the alternative is approximated by Taylor-expanding the transition function to circumvent the lack of identification under the null hypothesis. For details, see Teräsvirta (1998), Eitrheim and Teräsvirta (1996), and Jansen and Teräsvirta (1996). Rejecting the general hypothesis may be taken to indicate misspecification and should in that case lead to respecifying the model.

4. Data

The data consists of quarterly, seasonally unadjusted unemployment rate series from a number of OECD countries. The availability of series of seasonally unadjusted unemployment figures starting in the early 1970s at the latest has determined the selection of the series. The countries included are USA, Canada, Japan, Australia, (West) Germany, Austria, Italy, Denmark, Finland, Norway, and Sweden. The German series ends in 1991 to avoid the effects of the German unification and the resulting strutural break in the series. The main data source is the *OECD Main Economic Indicators*, but series of higher quality have for some countries been obtained from other sources. (The complete data set is available from the authors.) The series are depicted on a comparable scale in Figure 4.1. It should be mentioned that none of the three ADF tests (no constant; constant, no trend; constant and trend) rejects the unit root hypothesis for any of the series at the 10% significance level).

5. Results: asymmetry

5.1 Linearity tests

As already explained, modelling the series begins with linearity tests. The main test results are summarized in Table 5.1. Linearity is not rejected against (3.1) for either the US, Japanese, Norwegian, or Canadian series. For all of the European unemployment series except the Norwegian one, and for the Australian one, linearity is rejected, and nonlinear model building is attempted for these series. For the Italian and Austrian series, however, the LSTAR model does not seem to be an appropriate alternative to linearity. In the Italian case we encounter convergence problems, and grid estimation indicates the presence of several local minima in the objective function. None of the estimated models seems to characterize any asymmetry, which, given the shape of the series in Figure 4.1, may not be surprising. The model for the Austrian series resulting from the specification procedure can be estimated in the sense that convergence is achieved. Nevertheless, parameter estimates, tests against misspecification, and other statistics indicate that the estimated model is not meaningful. In fact, Figure 4.1 already conveys the impression that asymmetry of the type we are interested in is not present in the Austrian series. We shall return to these two series in Section 6 as well as to those for which linearity was not rejected.

The presentation of the results is organized as follows. We first consider the modelling results for one country, West Germany, in detail. This is done to allow the reader a glimpse of the estimated models and how an estimated model is evaluated. Furthermore, the different ways of highlighting the dynamic properties of the estimated models are introduced by using the West German LSTAR model as an example. Characteristic features of other successful models will be summarized after this discussion.

5.2 Germany

As linearity was rejected for the German series we estimated an LSTAR model of order 8, with d=1, and with seasonal dummies in both the linear and the nonlinear part of the model. The residuals from the estimated model exhibited no serial correlation. However, the tests of no additional nonlinearity indicated misspecification, and the null hypothesis of parameter constancy was strongly rejected in favour of an alternative with a time-varying intercept in the nonlinear part of the model, i.e., under the alternative, μ_2 is replaced by $\mu_2^0 + \mu_2^1 H_k\left(t^*;\gamma_1,\mathbf{c}_1\right)$. Since the test result in this case gives little information as to which one of the three alternative transition function specifications to choose, all three models were estimated. The most parsimonious specification, with k = 1, turned out adequately to capture the parameter variability detected by the test. The final estimated model has the form

$$
\begin{aligned}
\Delta y_t = & -0.013 y_{t-1} + 0.51\Delta y_{t-4} - 0.31\Delta y_{t-5} + 0.20\Delta y_{t-8} \\
& \quad (0.0081) \qquad (0.14) \qquad (0.080) \qquad (0.080) \\
& + 0.20 d_1 + 0.17 d_2 - 0.29 d_3 \\
& \quad (0.12) \quad (0.17) \quad (0.11) \\
& + [\ 0.58 + 1.27\ (\ 1 + \exp\{-2.74\,(t^* - 0.50\ )/\hat{\sigma}(t^*)\}\ )^{-1} \\
& \quad (0.21) \quad (0.61) \qquad\qquad (1.54) \qquad (0.072) \\
& - 0.15 y_{t-1} + 0.58\Delta y_{t-1} - 0.36\Delta y_{t-4} + 0.024 d_1 - 1.66 d_2 + 0.098 d_3\ ] \\
& \quad (0.051) \qquad (0.14) \qquad (0.22) \qquad (0.22) \quad (0.39) \quad (0.24) \\
& \times [\ 1 + \exp\{-4.29\,(\Delta_4 y_{t-1} - 0.31)/\hat{\sigma}(\Delta_4 y_t)\}\ ]^{-1} + \hat{u}_t \\
& \qquad\qquad (2.18) \qquad\qquad (0.12)
\end{aligned}
\tag{5.1}
$$

$s = 0.23$	skewness = 0.86	excess kurtosis = 2.29
LJB = 40.6 (1.5×10^{-9})	AIC = -2.78	SBIC = -2.34
LM = 0.15 (0.86)	SDR = 0.81	$R^2 = 0.92$

In this equation and the ones in Appendix 2, figures in parentheses below parameter estimates denote estimated standard errors, otherwise they are p-values of tests. Here, s is the estimated standard deviation of the residuals, LM denotes the LM statistic of no ARCH (Engle, 1982) computed with two lags, SDR denotes the ratio s / s_{LIN}, where $s_{LIN} = \sqrt{\frac{1}{T-k_{LIN}}\sum \hat{\varepsilon}_t^2}$, $\hat{\varepsilon}_t$ are the residuals from a regression of Δy_t on a constant, y_{t-1}, Δy_{t-j}, $j = 1, ..., p$, and seasonal dummies, and k_{LIN} is the number of parameters in this linear model. The Lomnicki-Jarque-Bera (LJB) test rejects normality. This result is due to a small number of large residuals. The standardization of the transition function exponent by dividing it by $\hat{\sigma}(\Delta_4 y_t)$ makes γ scale-free.

Equation (5.1) has 19 parameters and thus does not appear parsimonious. However, it should be noted that eight of them, almost one half, are directly related to modelling seasonality: the coefficients of the seasonal dummy variables and those of Δy_{t-4}. Modelling the smooth level shift in the peak of the unemployment rate takes another four parameters. The linear AR(8) model with seasonal dummy variables already had 13 parameters. Note that $\hat{\alpha}_2 < 0$ and, in this case, $\hat{\mu}_2^0 + \hat{\mu}_2^1 \hat{H}_1 > 0$, as we may expect from the example in Section 2.2. As for misspecification tests, the test statistics of no error autocorrelation do not indicate misspecification. None of the tests of no additive nonlinearity with seasonal dummies included now rejects at the 5% level. The tests of parameter constancy now all have clearly higher *p*-values than the previous tests, but there are still indications of parameter nonconstancy in the tests. Yet the remaining parameter variation is not easily captured by including more parameters into the time-varying subset of parameters in (5.1). (All these results are available from the authors upon request.) Despite the fact that the model is not entirely satisfactory, it is therefore maintained for the moment.

Figure 5.1 shows the observed unemployment level (upper panel) and the two estimated transition functions plotted over time (lower panel): The transition function for the model, taking values at or close to one during the upsurges, and the transition function of time for the varying intercept of the nonlinear part of the model. The latter is increasing smoothly over time allowing the peaks of the unemployment rate to increase over the observation period. Figure 5.2 shows the estimated transition function plotted against the transition variable with one dot for every observation in the sample (note that a single dot may represent more than one observation). The transition between the two extreme regimes is seen to be smooth. Figure 5.3 shows the residuals from the nonlinear model together with the residuals from the linear model used as a basis for linearity testing. The time series plot indicates that the major contribution of the nonlinear model is where the unemployment rate is growing. Elsewhere in the sample, gains from fitting a nonlinear model seem to be minor.

To illustrate the dynamic properties of (5.1), the persistence of shocks in particular, we estimated generalized impulse response (GIR) functions for this model. We refer the reader to Koop, Pesaran and Potter (1996) for a definition and discussion. The computational details can be found in Appendix 1. The estimated GIR functions are presented graphically in Figure 5.4. As in Skalin and Teräsvirta (1999), we make use of a suggestion by Hyndman (1995, 1996) and use highest density regions to show the distribution of functions up until 20 quarters ahead. The figure depicts three GIR functions. The first one is a GIR function based on all shocks, the second one is based on negative shocks greater than one residual standard deviation and the third one on corresponding positive shocks. The first GIR function shows that a shock to the unemployment rate can be quite persistent. After 20 quarters the density function still has not shrunk even close to a point. The other two figures reveal why. The positive shocks play a role here. A sufficiently large positive shock at a crucial moment may trigger a strong increase in the unemployment rate, which shows as pronounced multimodality (the highest density regions consist of a number of separate subintervals in Figure 5.4) in the GIR function. These positive shocks are very persistent. The corresponding graph for the negative shocks is not a mirror image of the one for positive shocks, which is another indication of the asymmetry built into the model. While the general conclusion about persistence is similar to that of the unit root testers the other implications of this analysis are not.

The persistence of the level of unemployment (hysteresis) may be illustrated by considering the deterministic extrapolation of the model. Figure 5.5 shows the trajectory obtained by extrapolating from the most recent values without adding noise. It turns out that the estimated model contains a limit cycle. The trajectory is a self-repeating asymmetric cycle fluctuating (ignoring seasonal variation) between 6 and 10 per cent. Extrapolations starting from earlier time-points in the 1960s and 1970s also show limit-cycle behaviour, but the amplitude of the cycle is less than from the 1980s onwards. Peel and Speight (1998) found a limit cycle in a somewhat different STAR model for the seasonally adjusted UK unemployment series. While a limit cycle in (5.1) may suggest the presence of endogenous cycles in the (West) German economy, a more prudent interpretation is that the unemployment rate of the country has been a strongly persistent variable.

5.3 Other countries

The other countries with an informative STAR model for the unemployment rate are Australia, Denmark, Finland, and Sweden. The estimated equations can be found in Appendix 2 while their properties are summarized in Table 5.2. This table also includes Germany for comparison. It is seen that four of the five models have the same general pattern: $\hat{\alpha}_2 < 0$ and $\hat{\mu}_2 > 0$. Finland constitutes the only exception. In the Finnish case the dynamics of the process are mainly characterized by the lagged first differences. In addition to Germany, Australia also has a model indicating that the expected rate of unemployment has been moving smoothly from a lower to a higher equilibrium over time. The models for Denmark and Finland have constant parameters which is equivalent to a single equilibrium whereas seasonality in the unemployment rate seems to have changed over time in Sweden. (A detailed study of the estimated equation would show that the change started in the mid-1980s.) In general, the estimated equations are different special cases of the general model (3.1). Another common feature in them is that the transition from the one extreme regime to the other is smooth. Denmark is closest to being an exception. This is seen from Figure 5.6 which shows the estimated transition functions for the four countries. (The one for Germany appears in Figure 5.2.)

As in the case of Germany we may consider the degree of persistence of shocks to the unemployment rate through estimated GIR functions. The graphs generated in the same way as for Germany can be found in Figure 5.7-5.10. (To conserve space, the graphs showing responses to symmetric shocks have been omitted.) It is seen that the unemployment rates in the three Scandinavian countries are quite persistent. Note that the Swedish GIR densities are remarkably peaked. The 50% highest density region is a very short interval, close to being a single point, whereas the 75% one is quite wide. Shocks to the Australian unemployment rate seem to be somewhat less persistent than those for the other four countries. On the other hand, the Australian LSTAR model contains a limit cycle, like the German model. But then, this is not necessarily contradictory. Extrapolating the deterministic counterpart ('skeleton'; Tong, 1990) of the stochastic model and shocking the stochastic one and following the effects of the shock(s) over time are two procedures that emphasize different dynamic aspects of the model.

6. Results: moving equilibria

In this section we shall reconsider those series for which either the tests did not reject symmetry (linearity) or the modelling effort failed to produce sensible results. The reason is that we are interested not only in asymmetry but also in the possibility that the unemployment rate has been moving between different equilibria under the observation period. This alternative can be investigated within the same framework as before using a simplified model. We specify the STAR model

$$\Delta y_t = \mu_1(t^*) + \alpha_1 y_{t-1} + \sum_{j=1}^{p} \beta_{1j} \Delta y_{t-j} + \sum_{i=1}^{s-1} \delta_{1i}(t^*) d_{it} + u_t \tag{6.1}$$

where

$$\mu_1(t^*) = \mu_1^0 + \mu_1^1 H_k(t^*;\gamma_1, c_1) \tag{6.2}$$

and

$$\delta_{1i}(t^*) = \delta_{1i}^0 + \delta_{1i}^1 H_k(t^*;\gamma_1, c_1),\ i = 1,2,3 \tag{6.3}$$

with $H_k(t^*;\gamma_1, c_1)$ defined as in (2.7) as our maintained model. It can be regarded as a special case of (2.7) (no asymmetry effects). It may also be seen as a linear AR model with a time-varying intercept and seasonal parameters. If $\mu_1^1 \neq 0$ and $H_k(t^*;\gamma_1, \mathbf{c}_1)$ is not constant in (6.2) then the process moves smoothly between equilibria as described by the transition function. If $\delta_{1i}^1 \neq 0$ in (6.3) for at least one i, while $H_k(t^*;\gamma_1, \mathbf{c}_1)$ is not constant then the seasonal pattern changes over time. Both types of transitions (level and seasonality) may be simultaneously present in the series.

When $\gamma_1 = 0$ in H_k, $\mu_1(t^*)$ and $\delta_{1i}(t^*)$ are constant over time. We model the remaining series by first testing this null hypothesis against $\gamma_1 > 0$. The null model is thus a pure AR(p) model augmented linearly by the seasonal dummies. The problem of (6.1) not being identified under the null hypothesis is again circumvented by expanding the transition function $H_k(t^*;\gamma_1, \mathbf{c}_1)$ into a first-order Taylor series, merging terms and reparameterizing. This yields an auxiliary regression equation, and the null hypothesis is that the terms containing *t* or its higher powers in this regression have zero coefficients. Under the null hypothesis the Taylor expansion is of order zero so that the remainder does not affect the distribution theory. The standard asymptotic theory applies; for details see Lin and Teräsvirta (1994) or Teräsvirta (1998). The tests are those called LM*k*, *k*=1,2,3, in these two references. If linearity (parameter constancy) is rejected then a STAR model of type (6.1-6.3) is fitted to the series. Parameter *k* is chosen by looking at test results and comparing estimated models in cases it was considered necessary to estimate them for different values of $k(\leq 3)$.

Results of the tests can be found in Table 6.1. Those shown in the table are for testing the constancy of the intercept and that of the coefficients of the seasonal dummy variables separately. It is seen that the only country for which neither null hypothesis is rejected at the 5% level of significance is the United States. For Austria and Canada there is evidence of the intercept not being constant over time. As to Italy, Japan, and Norway, the constancy of the seasonal pattern is rejected very strongly, but there is also evidence for change in the intercept.

We consequently estimated STAR models for all series except the US unemployment rate. The estimated models are presented in Appendix 3. The results of the misspecification tests did not in all cases indicate model adequacy, but the *p*-values were improved across the board for the series that we had tried to model previously. Detailed results are available from the authors. The estimated models are best interpreted by considering the graphs of the moving parameters multiplied by the seasonal dummies,

$$\hat{\mu}_1^0 + \hat{\mu}_1^1 H_k\left(t^*;\hat{\gamma}_1,\hat{\mathbf{c}}_1\right) + \sum_{i=1}^{s-1}\left(\hat{\delta}_{1i}^0 + \hat{\delta}_{1i}^1 H_k\left(t^*;\hat{\gamma}_1,\hat{\mathbf{c}}_1\right)\right)d_{it} \qquad (6.4)$$

over time. These can be found in Figures 6.1-6.5. The Austrian and Canadian models display a smooth transition in the level of unemployment over time. For Japan, there is a very slow climb in the level, but the most striking feature in the graph is the strong decrease in the amplitude of the seasonal variation from the early 1960s to the 1990s.

The Italian results indicate that the unemployment rate has slowly moved from an equilibrium level in the early 1970s to a higher equilibrium by the late 1980s; compare this with the discussion in Bentolila and Bertola (1990). At the same time, the seasonal pattern has undergone a considerable change. In the Norwegian series there is a very rapid rise in the level of the unemployment rate in 1988, and it is accompanied by a sharp increase in the amplitude of the seasonal variation.

It is interesting to compare these results with those in Bianchi and Zoega (1998), BZ for short; see their Figure 2. For Austria BZ discover five different equilibrium levels, each higher than the previous one with one exception, the period 1988(2)-1990(3), for which a lower equilibrium prevailed. This accords reasonably well with our results with a smooth transition. For Canada the results of BZ indicate a switch to a higher mean level in the early 1980s: compare this with Figure 6.2 according to which a smooth transition to a higher equilibrium is completed by that time. (BZ also find a temporary drop in the equilibrium rate in 1987-1990.) According to BZ, the Japanese unemployment series has had four equilibrium levels since 1970, each higher than the previous one. Again, this accords well with our monotonically increasing level. As BZ use seasonally adjusted series they miss the temporal development in the seasonal patterns that our STAR model picks up. This is of course true also for Norway. But then, while our results for Norway indicate a rapid level change in 1988, those in BZ also indicate a single switch in the equilibrium level at the same time. This also agrees with the findings of Akram (1998) who, however, claims another period of 'high' unemployment in 1982(4)-1984(3). Finally, BZ do not estimate any local mean level shift for the US which again is in line with our test results. For Italy, the results are different. Originally BZ find two equilibria of which the latter one is higher (their period is 1970(1)-1995(3)) with a break in 1983(2). Next, when they assume a break in this quarter and test the constancy of the intercept in a linear AR model with a Chow test against this break the null hypothesis of no break is not rejected. This may not be surprising, however, because the Chow test against a single break is not a powerful test against smooth structural change. Be that as it may, a break in 1983 may not be a convincing finding, which BZ acknowledge. Figure 5.1 clearly shows that the Italian unemployment rate began a steady climb already in the mid-1970s and that the growth continued to the late 1980s. Our results correspond to this visual impression.

Thus, with the exception of Italy, our results look rather similar to those in BZ. One may argue, however, that the similarities are superficial because the models are completely different and, besides, the series are not the same. The approach of BZ appears to accord well with the idea that large shocks may cause shifts in the equilibrium rate of unemployment. Our approach may seem more like presenting stylized facts because the transitions are described with deterministic functions of time. Nevertheless, it may be argued that what we see in Figures 6.1-6.5 does have a 'shock interpretation'. A smooth transition in the level may be a result of a series of shocks of the same sign. It may also be that while only large shocks cause switches in the equilibrium level, the transition from the old level to a new one initiated by such a shock may take time and be smooth. Viewed through a hidden Markov model, such a transition may appear as a sequence of discrete equilibria of either increasing (as is the case here) or decreasing order.

Our results also accord well with those of the standard unit root tests. It is well known, see, for example, Perron (1989) or Hendry and Neale (1991) for discussion and Monte Carlo evidence, that structural breaks or, for that matter, smooth structural change, bias test results towards accepting the null hypothesis. In Section 2.4 we have already seen that asymmetry of the type defined in (2.1) may make it difficult to reject the unit root hypothesis. As neither of the hypotheses, a unit root or (6.1), can be taken as the 'true model', the unit root test results and the idea of smooth structural change just characterize the same reality from two different angles.

7. Discussion

The results of Section 4 indicate that many unemployment series are asymmetric and can be adequately characterized by an LSTAR model of type (3.1). The fact that the estimates of the crucial parameters in the LSTAR model for a number of countries are similar and correspond to our expectations as Table 5.2 suggests is encouraging as it reduces the risk that our findings are spurious. The estimated equations also indicate that shocks to the system are rather persistent. This accords with the result of the unit root tests, but the other implications of accepting the unit root hypothesis, including symmetry, are quite different from what our approach leads to. For the remaining series discussed in Section 5, the results are also in harmony with those of the unit root tests. On the other hand, they yield interesting information about how the equilibrium level of unemployment has moved over the years. Even there, many countries display similar results and the STAR model is successfully fitted to all series for which linearity is rejected. As these models characterize the unemployment rate as stationary around a nonlinear trend they also underline the high persistence of these rates.

As is at least implicitly clear from previous sections, the information set used for the modelling is the history of the processes. This excludes any extraneous information and naturally shapes up the results. On the other hand, one may argue that, for example, the large increases in the unemployment rates in Sweden and Finland in the beginning of the 1990s have been caused by extraordinary, even unique, events that will not occur again. Because these increases constitute the single most dominating event in both series, this information may lead to treating them as outlying observations which do not fit into the general pattern. Doing so and modelling the corresponding observations as outliers could have an impact on the results and conclusions; for an illuminating discussion of this issue see Teräsvirta (1997) and van Dijk and Franses (1997). It should be noted, however, that neutralizing the dominating observations by dummy variables is a remarkably strong assumption. It implies that the presumed unique events only affect the level of the process and leave the dynamics intact.

The LSTAR models handling the asymmetry in unemployment rates have practical implications. When unemployment rates are being forecast using these models the estimated forecast densities are asymmetric. This is important information for the policymakers. To take an example, suppose that point forecasts indicate a decrease in unemployment. However, the attached forecast densities make it clear to the policymakers that the probability of erring to one side (less rapid decrease or even increase) is greater than erring to the other (even faster decrease). For comparison, the linear unit root models basically predict that the present unemployment rate will prevail and, besides, the forecast density is symmetric around each point forecast.

The results of the analysis are also relevant to multivariate modelling. If the univariate analysis leads to the result that the unemployment rate can be modelled as a stationary nonlinear process this has implications to multivariate model bulding. The straightforward linear vector autoregressions in which there exists a cointegrating relationship between the unemployment rate and a set of I(1) variables such as real wages and real output may be misspecified. This is because they exclude all nonlinearities from the start, and it is assumed that the unemployment rate is an I(1) variable. In such a situation, respecting the stylized facts and starting from the properties found in univariate series could be a preferable approach. Even if a linear model ultimately provides an adequate description of the relationships of interest it may be better to find that out by starting out with the asymmetry assumption when appropriate. In addition to parameterizing and highlighting stylized facts in unemployment series, our univariate analysis may serve as a starting-point for such an approach.

Appendix 1. Generalized impulse response functions and highest density regions

In this appendix we report how we estimated generalized impulse response (GIR) functions for our models. The reader is referred to Koop, Pesaran and Potter (1996) for a general definition of the function. All available observations (i.e., all possible vectors of the necessary lags) of the time series are used once (without sampling) as 'histories'. For each history, 100 initial shocks are drawn randomly with replacement from a subset of the residuals from the estimated STAR model, formed either of all residuals, of all residuals greater than one residual standard error, or of all residuals less than one negative residual standard error. For each combination of history and initial shock, 800 replicates of a 21-step prediction sequence (0,1,...,20 step ahead) are generated with and without the selected initial shock in the first step, and using randomly drawn residuals as noise everywhere else. For every horizon, the means over the 800 replicates of the two prediction sequences are computed, and the vector of 21 differences between the two means forms an observation of the GIR.

The observations of the GIR form the basis of the highest density regions used for a graphical representation of the generalized impulse response densities. For each one of the 21 horizons, the response density is estimated with a kernel algorithm. The points used in the kernel algorithm are a random sample from the GIR. For instance, in the case of Australia, with 111 'histories', this is a random sample from 11100 GIR values for each horizon. The 50% and 75% highest density regions are then estimated using the density quantile method as described in Hyndman (1996).

Appendix 2. Estimated asymmetry models

Finland

$$\begin{aligned}\Delta y_t = & -\underset{(0.012)}{0.0031} y_{t-1} + \underset{(0.074)}{0.32} \Delta y_{t-3} + \underset{(0.13)}{0.35} \Delta y_{t-5} - \underset{(0.084)}{0.26} \Delta y_{t-6} \\ & + \underset{(0.12)}{0.85} d_1 - \underset{(0.11)}{0.59} d_2 - \underset{(0.12)}{0.0049} d_3 \\ & + \left[-\underset{(0.023)}{0.017} y_{t-1} + \underset{(0.093)}{0.67} \Delta y_{t-1} + \underset{(0.11)}{0.50} \Delta y_{t-4} - \underset{(0.18)}{0.90} \Delta y_{t-5} + \underset{(0.084)}{0.26} \Delta y_{t-6} \right] \\ & \times \left[1 + \exp\{ -\underset{(1.57)}{2.87} (\Delta_4 y_{t-2} + \underset{(0.30)}{0.060}) / \hat{\sigma}(\Delta_4 y_t) \} \right]^{-1} + \hat{u}_t \end{aligned} \tag{A2.1}$$

s = 0.33	skewness = 0.013	excess kurtosis = -0.11
LJB = 0.078 (0.96)	AIC = -2.13	SBIC = -1.86
LM = 5.85 (0.0037)	SDR = 0.91	R^2 = 0.86

Denmark

$$\begin{aligned}\Delta y_t = & -\underset{(0.011)}{0.023} y_{t-1} + \underset{(0.11)}{0.47} \Delta y_{t-1} + \underset{(0.10)}{0.54} \Delta y_{t-4} - \underset{(0.12)}{0.23} \Delta y_{t-5} - \underset{(0.076)}{0.17} \Delta y_{t-6} + \underset{(0.088)}{0.20} \Delta y_{t-8} \\ & + \underset{(0.16)}{0.40} d_1 - \underset{(0.22)}{0.20} d_2 + \underset{(0.24)}{0.70} d_3 \\ & + \left[\underset{(0.44)}{1.18} - \underset{(0.055)}{0.11} y_{t-1} + \underset{(0.16)}{0.39} \Delta y_{t-1} - \underset{(0.17)}{0.95} \Delta y_{t-2} - \underset{(0.19)}{0.71} \Delta y_{t-5} + \underset{(0.076)}{0.17} \Delta y_{t-6} - \underset{(0.23)}{0.68} \Delta y_{t-8} \right] \\ & \times \left[1 + \exp\{ -\underset{(10.33)}{13.37} (\Delta_4 y_{t-1} - \underset{(0.099)}{1.22}) / \hat{\sigma}(\Delta_4 y_t) \} \right]^{-1} + \hat{u}_t \end{aligned} \tag{A2.2}$$

s = 0.36	skewness = -0.23	excess kurtosis = 0.52
LJB = 1.85 (0.40)	AIC = -1.89	SBIC = -1.44
LM = 6.35 (0.0026)	SDR = 0.82	R^2 = 0.91

Australia

$$\Delta y_t = \underset{(0.12)}{0.36} - \underset{(0.016)}{0.08} y_{t-1} + \underset{(0.20)}{0.16} d_{1t} - \underset{(.089)}{0.56} d_{2t} - \underset{(0.18)}{0.51} d_{3t} \tag{A2.3}$$
$$+ (\underset{(0.25)}{0.96} d_{1t} + \underset{(0.23)}{0.51} d_{3t}) \left(1 + \exp\{-\underset{(0.43)}{2.48}(t^* - \underset{(0.019)}{0.28}) / \hat{\sigma}(t^*)\}\right)^{-1}$$
$$+ \left[-\underset{(0.085)}{0.30} y_{t-1} + \underset{(0.86)}{3.67} \times \left(1 + \exp\{-\underset{(0.43)}{2.48}(t^* - \underset{(0.019)}{0.28}) / \hat{\sigma}(t^*)\}\right)^{-1}\right]$$
$$\times \left(1 + \exp\{-\underset{(1.29)}{3.36}(\Delta_4 y_{t-1} - \underset{(0.20)}{0.74}) / \hat{\sigma}(\Delta_4 y_t)\}\right)^{-1} + \hat{u}_t$$

s = 0.32	skewness = 0.48	excess kurtosis = 0.20
LJB = 4.39 (0.11)	AIC = -2.17	SBIC = -1.85
LM = 2.67 (0.07)	SDR = 0.85	R^2 = 0.82

Sweden

$$\Delta y_t = -\underset{(0.014)}{0.034} y_{t-1} - \underset{(0.11)}{0.35} \Delta y_{t-1} + \underset{(0.083)}{0.27} \Delta y_{t-4} - \underset{(0.069)}{0.22} \Delta y_{t-6} + \underset{(0.076)}{0.28} d_1 - \underset{(0.062)}{0.16} d_2 + \underset{(0.098)}{0.15} d_3 \tag{A2.4}$$
$$+ \underset{(0.56)}{1.75} d_3 \times \left[1 + \exp\{-\underset{(3.39)}{4.18}(t^* - 1) / \hat{\sigma}(t^*)\}\right]^{-1}$$
$$+ \left[\underset{(0.83)}{2.02} - \underset{(0.12)}{0.42} y_{t-1} + \underset{(0.28)}{1.31} \Delta y_{t-1} + \underset{(0.44)}{1.82} \Delta y_{t-6}\right]$$
$$\times \left[1 + \exp\{-\underset{(0.54)}{1.95}(\Delta_4 y_{t-1} - \underset{(0.26)}{1.75}) / \hat{\sigma}(\Delta_4 y_t)\}\right]^{-1} + \hat{u}_t$$

s = 0.25	skewness = 0.54	excess kurtosis = 0.58
LJB = 8.23 (0.016)	AIC = -2.67	SBIC = -2.34
LM = 2.90 (0.059)	SDR = 0.82	R^2 = 0.75

Note: The model is estimated with fixed $\hat{c}_1 \equiv 1$.

Appendix 3. Estimated moving equilibria models

Austria

$$\Delta y_t = -\underset{(0.22)}{0.29} - \underset{(0.041)}{0.17} y_{t-1} - \underset{(0.058)}{0.14} \Delta y_{t-2} + \underset{(0.047)}{0.67} \Delta y_{t-4} + \underset{(0.054)}{0.12} \Delta y_{t-10} + \underset{(0.11)}{0.13} d_1 - \underset{(0.21)}{0.60} d_2 + \underset{(0.17)}{0.51} d_3 \quad \text{(A3.1)}$$
$$+ \underset{(0.46)}{1.70} \times \left[1 + \exp\{ -\underset{(0.68)}{2.23} (t^* - \underset{(0.026)}{0.29})^2 / \hat{\sigma}(t^*) \} \right]^{-1} + \hat{u}_t$$

s = 0.25	skewness = 0.38	excess kurtosis = 0.69
LJB = 5.67 (0.059)	AIC = -2.70	SBIC = -2.45
LM = 6.77 (0.0016)	SDR = 0.95	R^2 = 0.97

Italy

$$\Delta y_t = -\underset{(0.27)}{0.011} - \underset{(0.064)}{0.41} y_{t-1} + \underset{(0.077)}{0.083} \Delta y_{t-2} + \underset{(0.076)}{0.29} \Delta y_{t-4} + \underset{(0.072)}{0.19} \Delta y_{t-5} + \underset{(0.075)}{0.27} \Delta y_{t-8} \quad \text{(A3.2)}$$
$$+ \underset{(0.38)}{0.33} d_1 - \underset{(0.36)}{0.28} d_2 + \underset{(0.39)}{0.67} d_3 + \left(\underset{(0.76)}{4.74} - \underset{(0.48)}{0.40} d_1 + \underset{(0.46)}{0.11} d_2 - \underset{(0.47)}{0.86} d_3 \right)$$
$$\times \left[1 + \exp\{ -\underset{(1.64)}{11.56} (t^* - \underset{(0.017)}{0.27})^3 / \hat{\sigma}(t^*) \} \right]^{-1} + \hat{u}_t$$

s = 0.42	skewness = –0.60	excess kurtosis = 2.22
LJB = 35.49 (1.96×10^{-8})	AIC = -1.64	SBIC = -1.32
LM = 10.041 (8.85×10^{-5})	SDR = 0.91	R^2 = 0.66

Canada

$$\Delta y_t = \underset{(0.30)}{0.71} - \underset{(0.029)}{0.11} y_{t-1} + \underset{(0.081)}{0.49} \Delta y_{t-1} + \underset{(0.081)}{0.21} \Delta y_{t-4} - \underset{(0.078)}{0.17} \Delta y_{t-15} \tag{A3.3}$$

$$+ \underset{(0.074)}{0.24} \Delta y_{t-16} + \underset{(0.063)}{0.16} \Delta y_{t-19} - \underset{(0.064)}{0.24} \Delta y_{t-20} + \underset{(0.23)}{0.71} d_1 - \underset{(0.25)}{1.70} d_2 - \underset{(0.16)}{0.31} d_3$$

$$+ \underset{(0.34)}{0.70} \times \left[1 + \exp\{ - \underset{(1.80)}{2.56} (t^* - \underset{(0.13)}{0.30}) / \hat{\sigma}(t^*) \} \right]^{-1} + \hat{u}_t$$

s = 0.35	skewness = 0.82	excess kurtosis = 1.53
LJB = 26.26 (1.99×10^{-6})	AIC = -1.99	SBIC = -1.67
LM = 0.31 (0.73)	SDR = 0.94	R^2 = 0.89

Japan

$$\Delta y_t = - \underset{(0.19)}{0.13} - \underset{(0.037)}{0.076} y_{t-1} + \underset{(0.089)}{0.16} \Delta y_{t-3} + \underset{(0.089)}{0.17} \Delta y_{t-4} \tag{A3.4}$$

$$+ \underset{(0.34)}{0.78} d_1 - \underset{(0.21)}{0.24} d_2 + \underset{(0.11)}{0.15} d_3 + \left(\underset{(0.24)}{0.29} - \underset{(0.48)}{0.51} d_1 + \underset{(0.27)}{0.24} d_2 - \underset{(0.14)}{0.12} d_3 \right)$$

$$\times \left[1 + \exp\{ - \underset{(1.55)}{1.24} (t^* - \underset{(0.21)}{0.37}) / \hat{\sigma}(t^*) \} \right]^{-1} + \hat{u}_t$$

s = 0.11	skewness = 0.035	excess kurtosis = –0.38
LJB = 0.88 (0.64)	AIC = -4.35	SBIC = -4.074
LM = 2.79 (0.065)	SDR = 0.90	R^2 = 0.86

Norway

$$\Delta y_t = \underset{(0.16)}{0.0012} - \underset{(0.053)}{0.22} y_{t-1} + \underset{(0.098)}{0.26} \Delta y_{t-2} + \underset{(0.14)}{0.52} d_1 + \underset{(0.14)}{0.62} d_2 + \underset{(0.13)}{0.76} d_3 \qquad \text{(A3.5)}$$

$$+ \left(\underset{(0.22)}{0.64} + \underset{(0.21)}{0.79} d_1 - \underset{(0.21)}{0.18} d_2 - \underset{(0.23)}{0.55} d_3 \right)$$

$$\times \left[1 + \exp\{ - \underset{(440.41)}{93.98} \; (t^* - \underset{(0.011)}{0.59}) / \hat{\sigma}(t^*) \} \right]^{-1} + \hat{u}_t$$

s = 0.34	skewness = –0.19	excess kurtosis = 0.10
LJB = 0.59 (0.74)	AIC = -2.01	SBIC = -1.69
LM = 0.11 (0.89)	SDR = 0.87	R^2 = 0.68

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Tables

Table 2.1. ADF tests on data generated with nonlinear model: Relative rejection frequencies at the 5% level. *T*=200, 10000 replicates

a. No constant, no trend					
	α_2				
α_1	0	-0.01	-0.10	-0.20	Linear
-0.01	0.15	0.16	0.17	0.17	0.14
-0.02	0.19	0.19	0.18	0.17	0.26
-0.05	0.56	0.58	0.66	0.82	0.76
-0.10	0.91	0.92	0.95	0.99	0.99
b. Constant, no trend					
	α_2				
α_1	0	-0.01	-0.10	-0.20	Linear
-0.01	0.07	0.07	0.07	0.08	0.06
-0.02	0.12	0.13	0.13	0.13	0.09
-0.05	0.24	0.24	0.28	0.35	0.31
-0.10	0.64	0.65	0.75	0.89	0.81
c. Constant and trend					
	α_2				
α_1	0	-0.01	-0.10	-0.20	Linear
-0.01	0.09	0.09	0.08	0.07	0.05
-0.02	0.08	0.08	0.07	0.07	0.07
-0.05	0.14	0.14	0.15	0.16	0.18
-0.10	0.36	0.37	0.52	0.63	0.58

Table 2.2. ADF tests on data generated with nonlinear model: Relative rejection frequencies at the 5% level. *T*=1000, 10000 replicates

a. No constant, no trend					
	α_2				
α_1	0	-0.01	-0.10	-0.20	Linear
-0.01	0.47	0.53	0.63	0.72	0.77
-0.02	0.91	0.87	0.92	0.99	0.99
-0.05	0.71	0.67	0.80	0.99	0.99
-0.10	0.48	0.54	0.74	0.99	0.99
b. Constant, no trend					
	α_2				
α_1	0	-0.01	-0.10	-0.20	Linear
-0.01	0.43	0.55	0.75	0.83	0.30
-0.02	0.90	0.90	0.94	0.99	0.84
-0.05	0.83	0.79	0.83	0.99	0.99
-0.10	0.58	0.62	0.74	0.99	0.99
c. Constant and trend					
	α_2				
α_1	0	-0.01	-0.10	-0.20	Linear
-0.01	0.19	0.29	0.47	0.55	0.18
-0.02	0.77	0.81	0.89	0.96	0.59
-0.05	0.84	0.79	0.83	0.99	0.99
-0.10	0.58	0.62	0.74	0.99	0.99

Table 5.1. Smallest *p*-values of linearity tests for various values of delay *d* against logistic smooth transition autoregression, 11 unemployment rate series

Series	Order of AR model	Linearity test *without* seasonal dummies in the nonlinear part		Linearity test *with* seasonal dummies in the nonlinear part	
		d	*p*-value	*d*	*p*-value
Sweden 1962:1-1996:3	6	1	1.4×10^{-7}	1	3.8×10^{-7}
Germany 1960:1-1991:4	8	1	0.0074	1	0.00037
Finland 1960:1-1996:3	6	1 2	0.0032 0.0027	1 2	0.0062 0.0027
Denmark 1970:1-1995:4	8	1	0.017	1	0.046
Austria 1960:1-1995:4	6	1	3.5×10^{-6}	1	1.9×10^{-6}
Italy 1960:1-1995:4	9	3	0.025	3	0.011
Australia 1966:3-1995:4	5	Linearity not rejected*		Linearity not rejected	
Norway 1972:1-1997:3	8	Linearity not rejected		Linearity not rejected	
USA 1960:1-1995:4	6	Linearity not rejected		Linearity not rejected	
Canada 1960:1-1995:4	20	Linearity not rejected		Linearity not rejected	
Japan 1960:1-1995:4	4	Linearity not rejected		Linearity not rejected	

Note: The results are based on the first order Taylor series approximation of the transition function. See Teräsvirta (1998) for details of the test. The significance level 0.05 is used as a criterion for rejecting linearity.

* Since the smallest *p*-value for the test statistic based on the third order Taylor series approximation is small (*p*-value=0.008) for Australia, linearity is rejected and a nonlinear model specified and estimated although none of the *p*-values for the first-order tests is less than 0.05.

Table 5.2. Estimated intercepts and coefficients of the lagged level variable in the nonlinear part of the model, and the estimated location parameter *c*, in the five LSTAR models

Series	$\hat{\alpha}_2$	$\hat{\mu}_2$	$\hat{c}$	Limit cycle	Transition
Australia	-0.30 (0.085)	$\underset{(0.86)}{3.67}\,\hat{H}_1$	0.74 (0.20)	yes	smooth
Denmark	-0.11 (0.055)	1.18 (0.44)	1.22 (0.099)	no	smooth
Finland	-0.017 (0.023)		-0.06 (0.30)	no	smooth
Germany	-0.15 (0.051)	$\underset{(0.21)}{0.58} + \underset{(0.61)}{1.27}\,\hat{H}_1$	0.31 (0.12)	yes	smooth
Sweden	-0.42 (0.12)	2.02 (0.83)	1.75 (0.26)	no	smooth

Table 6.1. *p*-values of the parameter constancy tests against smooth structural change (a) in the intercept, (b) in the coefficients of the seasonal dummy variables in linear autoregressive models for series for which linearity is not rejected

Series	AR order	Test results, *p*-values, alternative with time-varying					
		(a) intercept			(b) seasonal dummy coefficients		
	p	*LM1*	*LM2*	*LM3*	*LM1*	*LM2*	*LM3*
Austria	6	0.048	0.020	0.009	0.88	0.82	0.046
	13	0.034	0.014	0.017	0.86	0.96	0.71
Italy	9	0.011	0.038	0.022	0.29	0.031	0.00015
USA	6	0.25	0.072	0.099	0.63	0.23	0.41
Canada	20	0.073	0.010	0.019	0.20	0.37	0.39
Japan	4	0.012	0.042	0.092	1.3×10^{-5}	0.00012	0.00034
Norway	8	0.054	0.16	0.029	0.0033	0.0027	0.00017

Note: For a detailed description of the tests, see Lin and Teräsvirta (1994) or Teräsvirta (1998).

Figure 2.1. One realization of the process (2.1) (solid); the corresponding realization of the linear model (2.3) (dashed); and the transition function (solid)

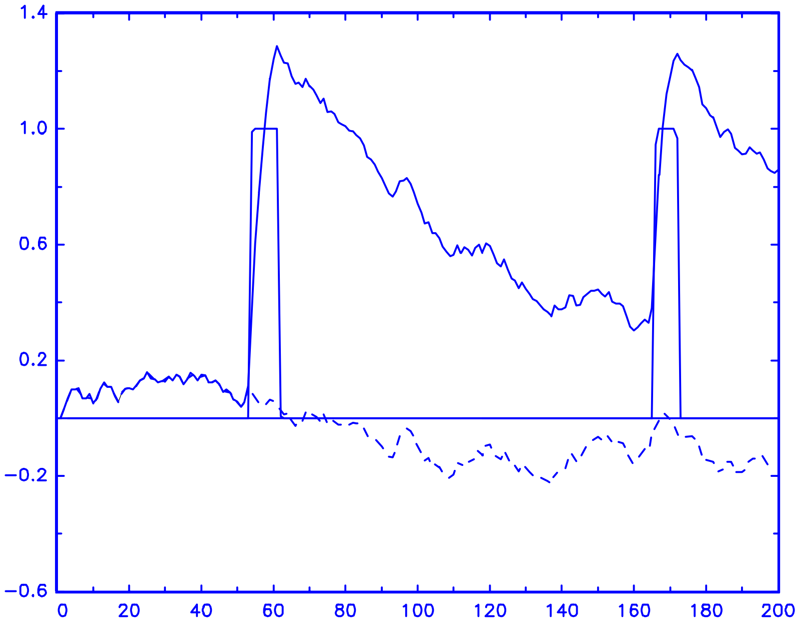


Figure 2.2. Examples of realizations generated with (2.1) (solid) for various values of α_1 and α_2, examples of realizations generated with the linear model (2.7) (dashed), and the transition function (solid)

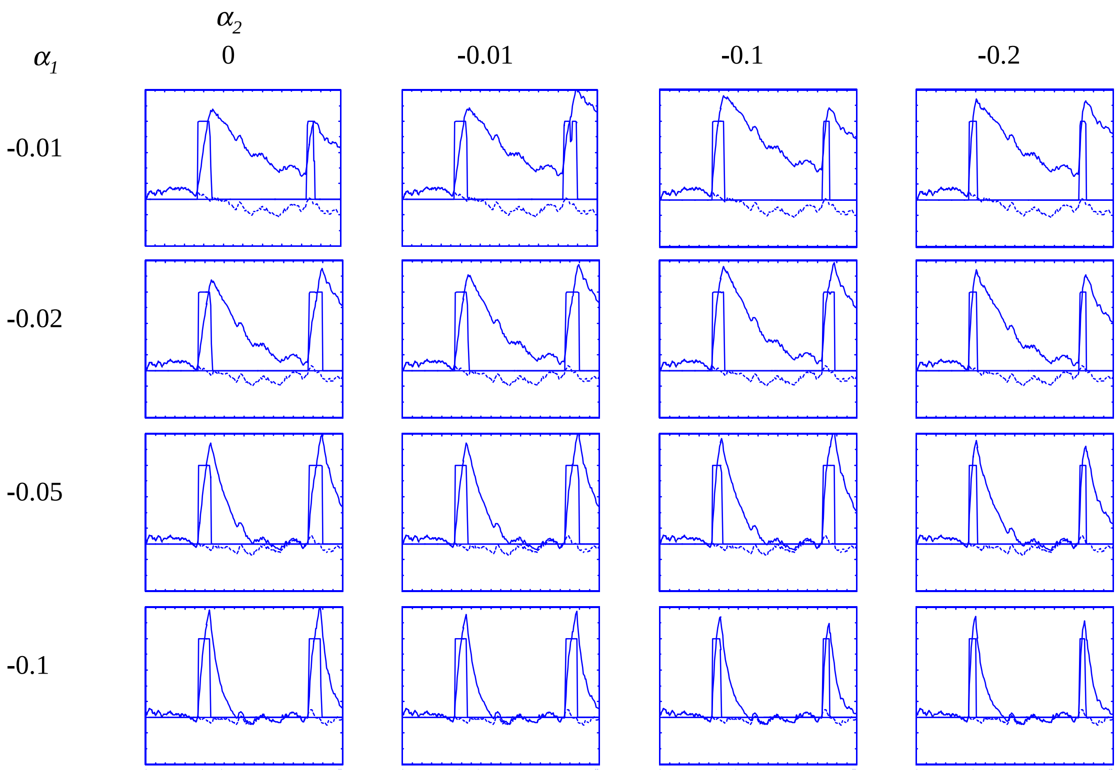
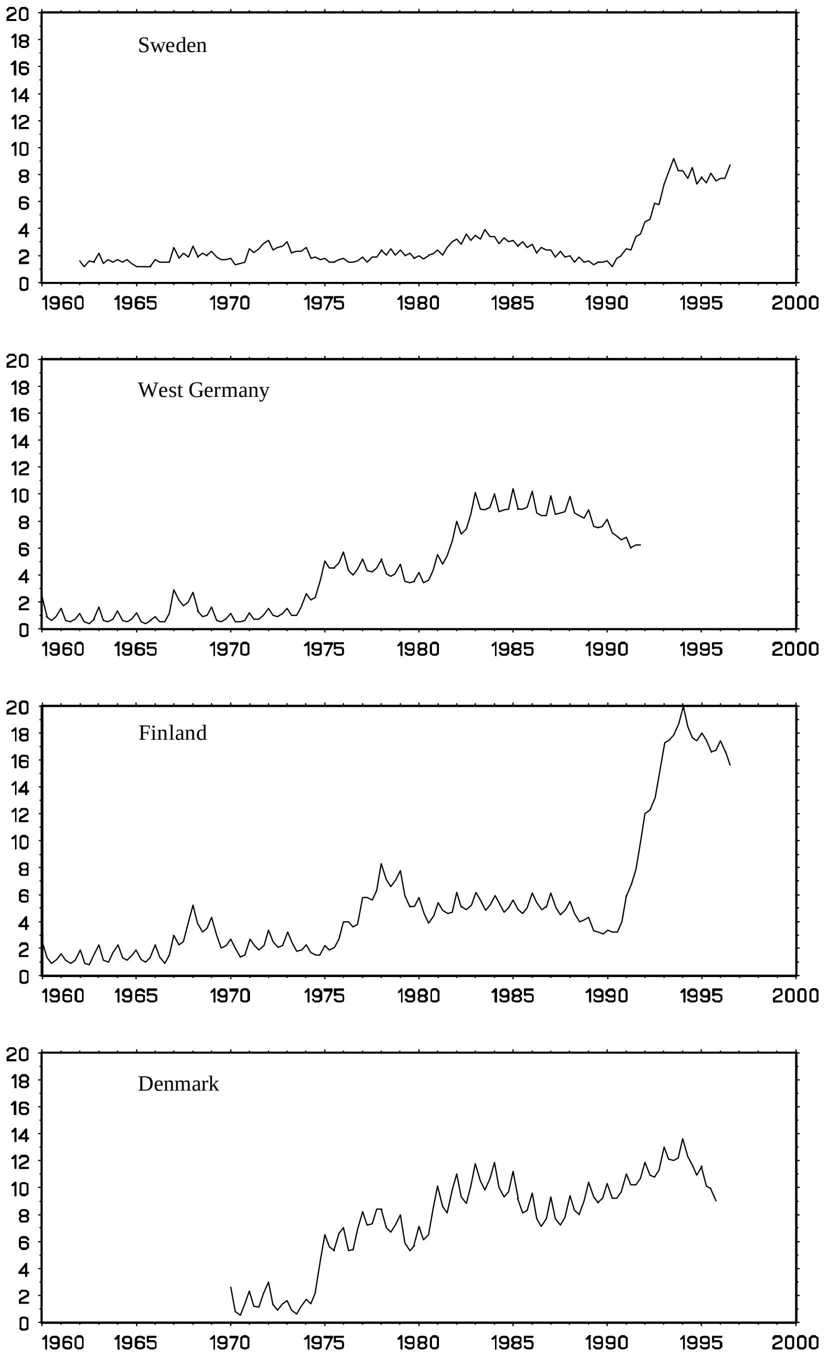
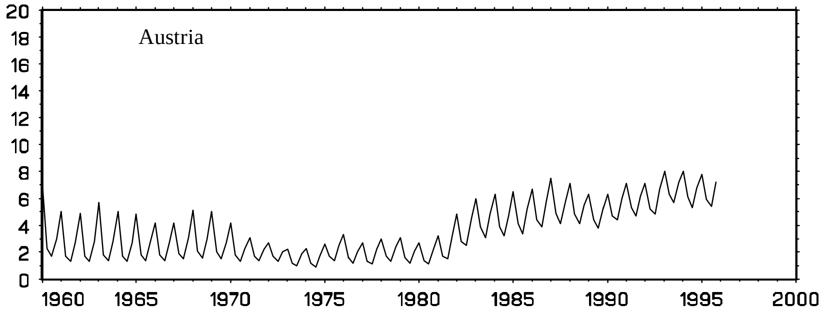


Figure 4.1. Unemployment rates by country




Austria
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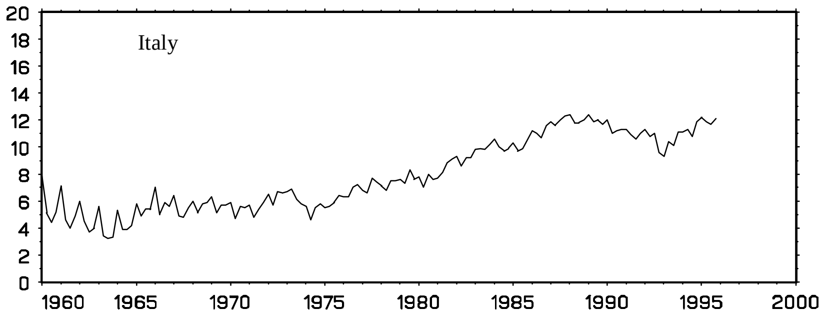
Italy
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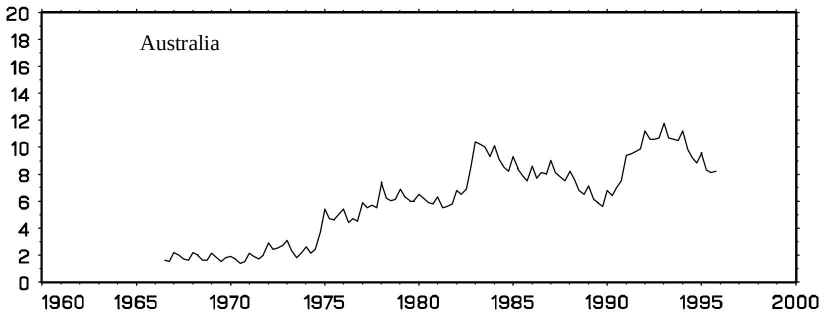
Australia
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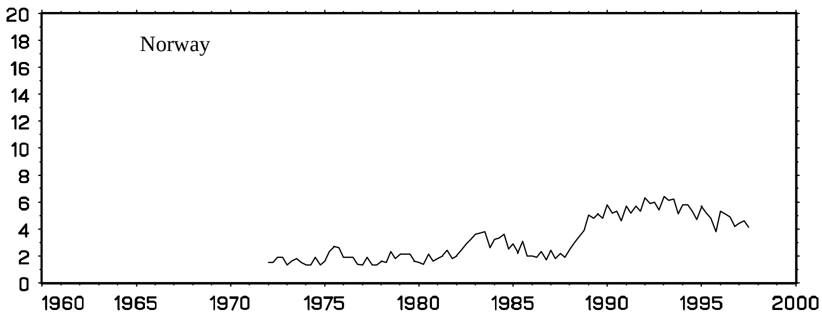
Norway
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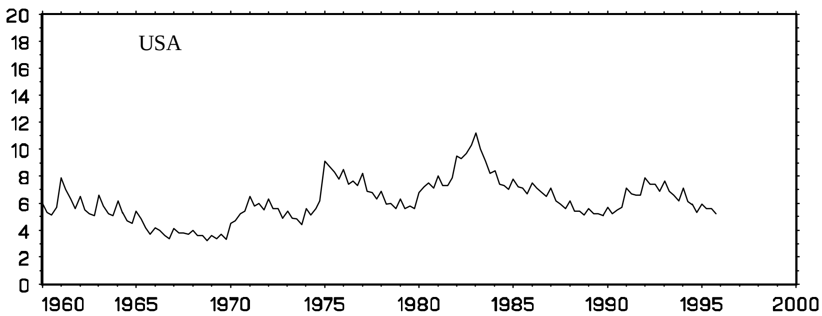
USA
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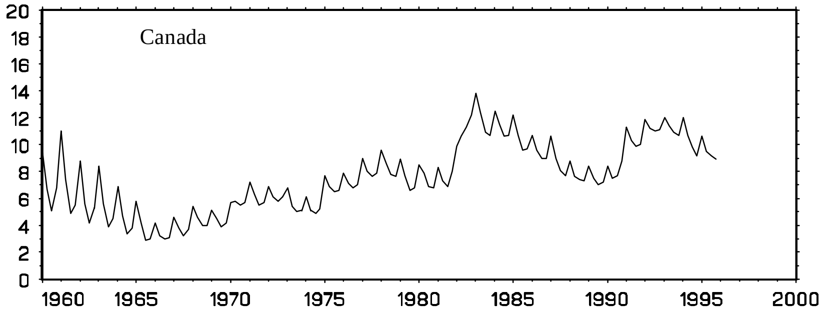
Canada
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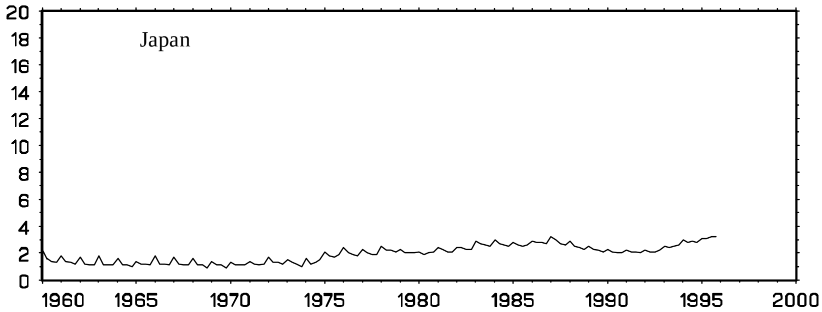
Japan
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Figure 5.1. Germany. Observed values (upper panel). Estimated transition functions over time (lower panel): (a) transition function $\hat{G}$ over time (irregular curve), (b) transition function $\hat{H}_1$ (logistic curve)

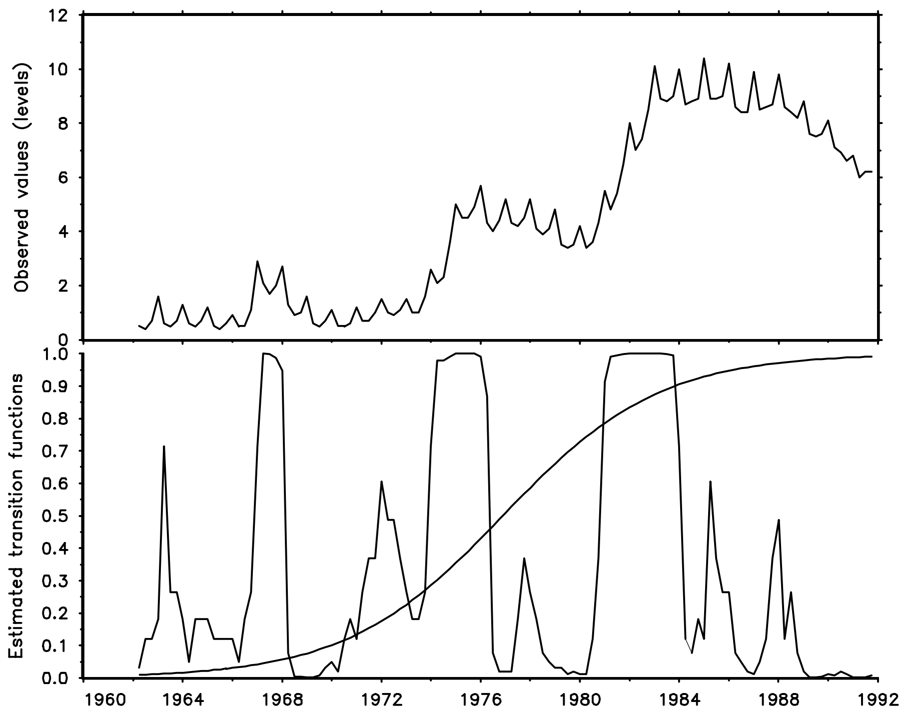


Figure 5.2. Germany. Estimated transition function, $\hat{G}$, as a function of the transition variable. Each dot represents at least one observation.

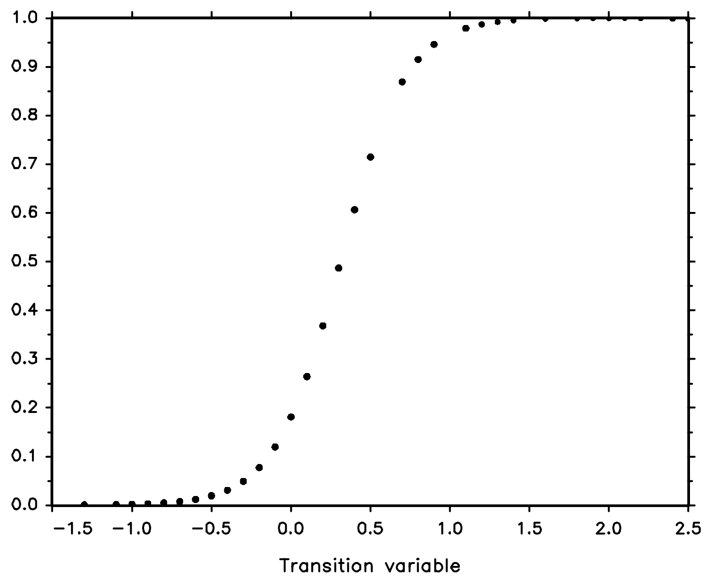


Figure 5.3. Germany. Residuals of the estimated LSTAR model (5.1) (dashed line) and the corresponding linear AR model (dotted line).

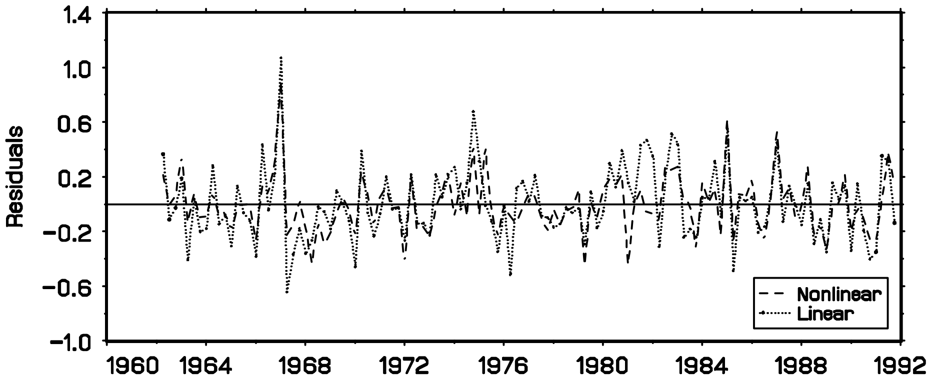
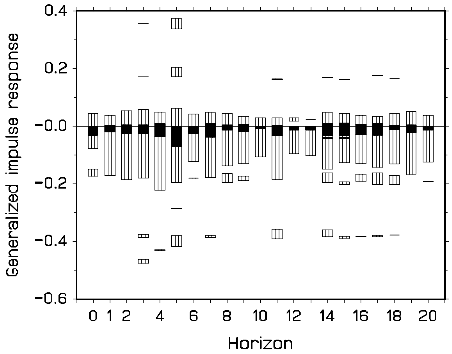
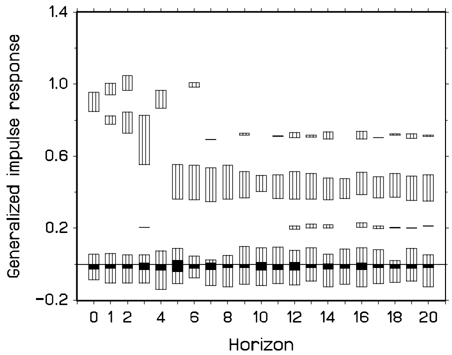


Figure 5.4. Germany: 50% (black) and 75% (hatched) highest density regions for the generalized impulse response to shocks

Response to symmetric shocks



Response to positive shocks, greater than 1 error s.d.



Response to negative shocks, less than –1 error s.d.

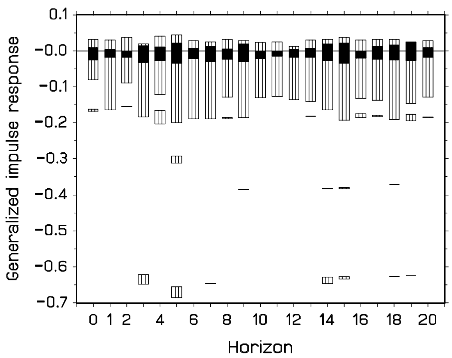


Figure 5.5. Germany. Deterministic extrapolation of the estimated LSTAR model (5.1) (horizon 50 years)

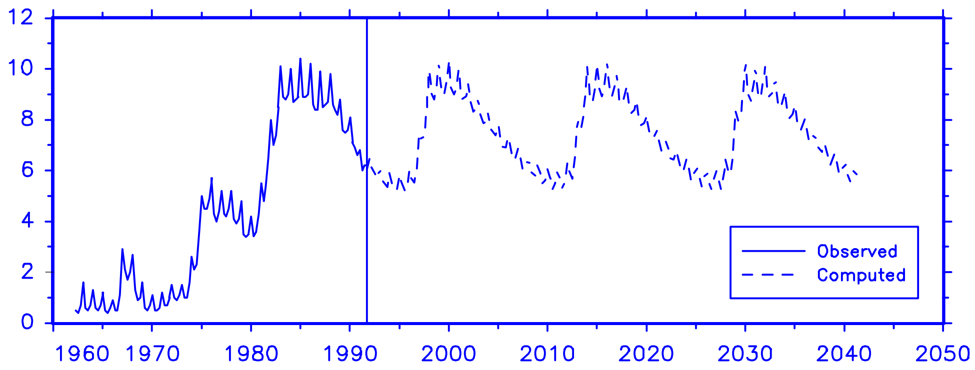
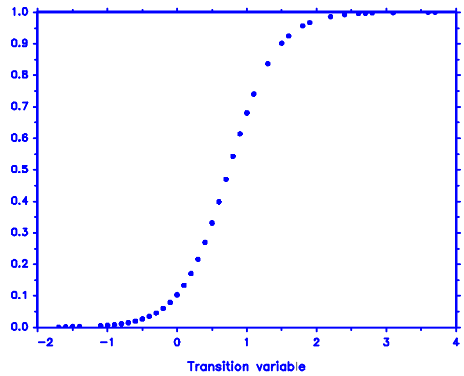
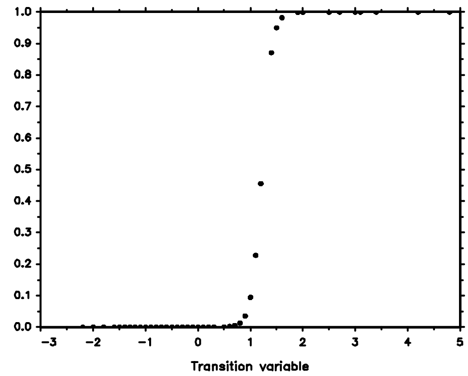


Figure 5.6. Estimated transition functions, $\hat{G}$, as functions of the transition variables. Each dot represents at least one observation

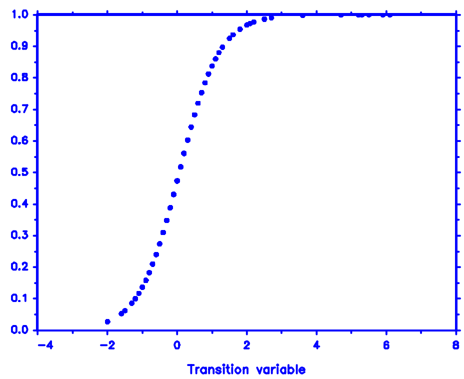
a. Australia



b. Denmark



c. Finland



d. Sweden

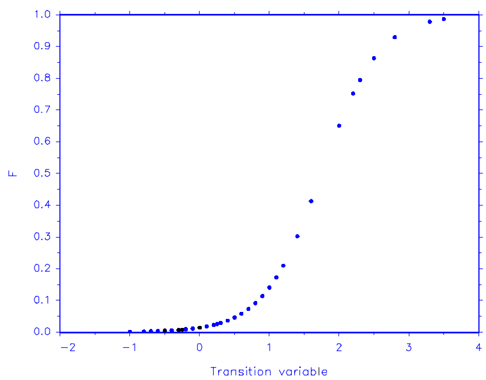


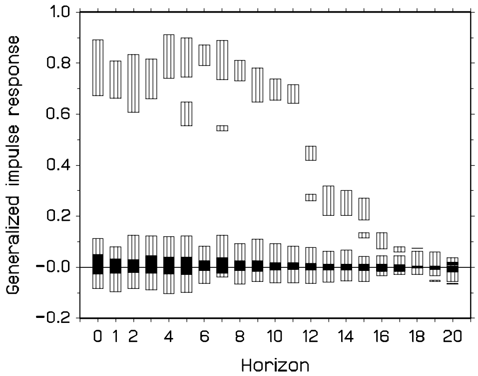
Figure 5.7.

Australia. 50% (black) and 75% (hatched) highest density regions for the generalized impulse response to shocks

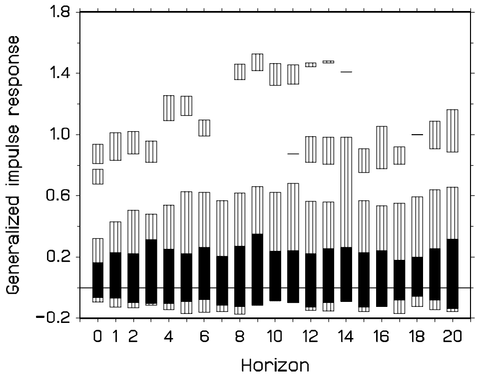
Figure 5.8.

Denmark 50% (black) and 75% (hatched) highest density regions for the generalized impulse response to shocks

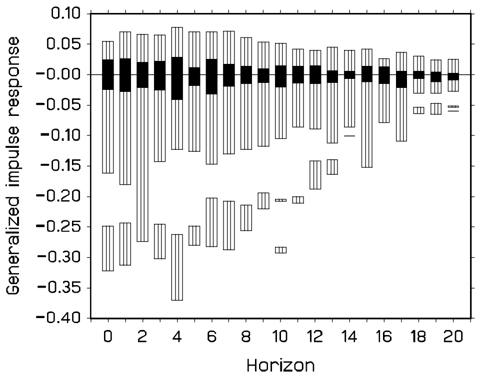
Response to positive shocks, greater than 1 error s.d.



Response to positive shocks, greater than 1 error s.d.



Response to negative shocks, less than –1 error s.d.



Response to negative shocks, less than –1 error s.d.

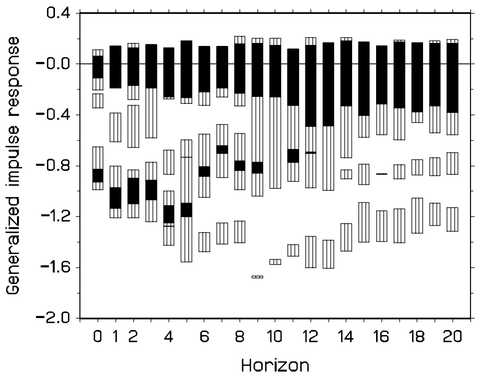
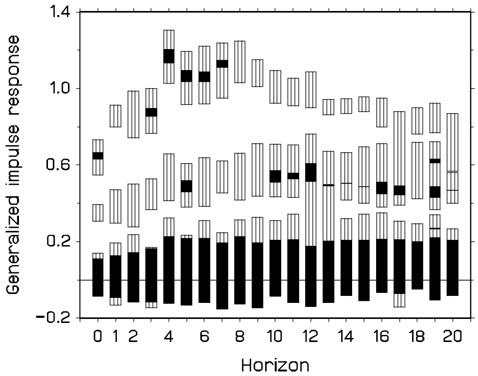


Figure 5.9.

Finland. 50% (black) and 75% (hatched) highest density regions for the generalized impulse response to shocks

Response to positive shocks, greater than 1 error s.d.



Response to negative shocks, less than –1 error s.d.

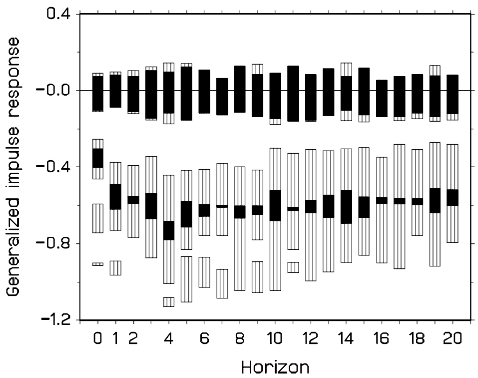


Figure 5.10.

Sweden. 75% (hatched) highest density regions for the generalized impulse response to shocks

Response to negative shocks, less than –1 error s.d.

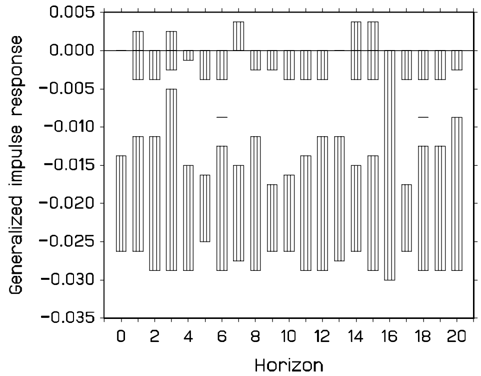


Figure 6.1
Austria. Observed values (upper panel) and moving intercept (lower panel)

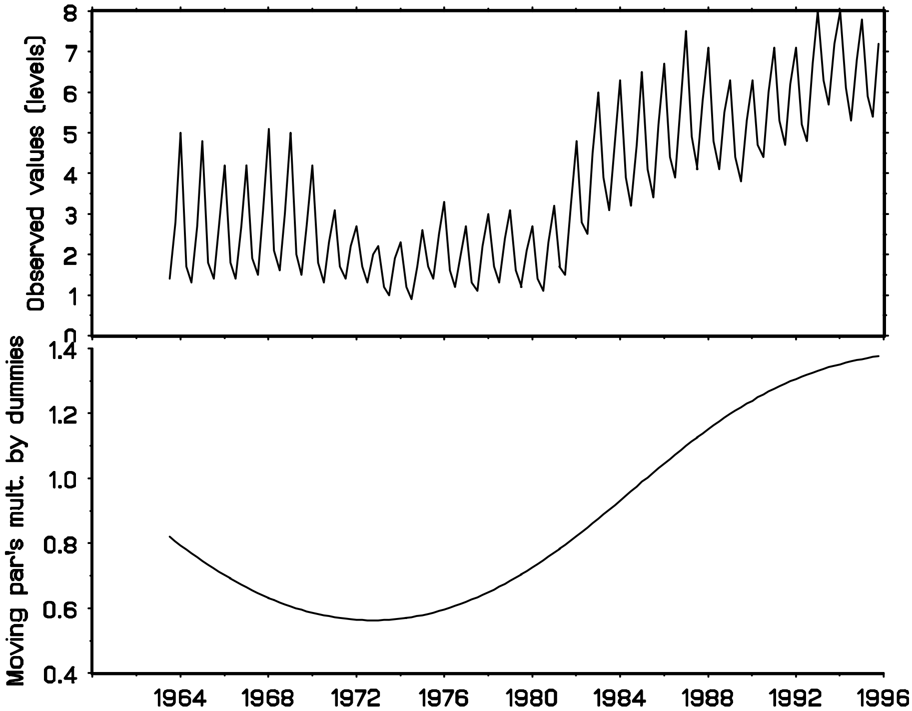


Figure 6.2
Canada. Observed values (upper panel) and moving intercept (lower panel)

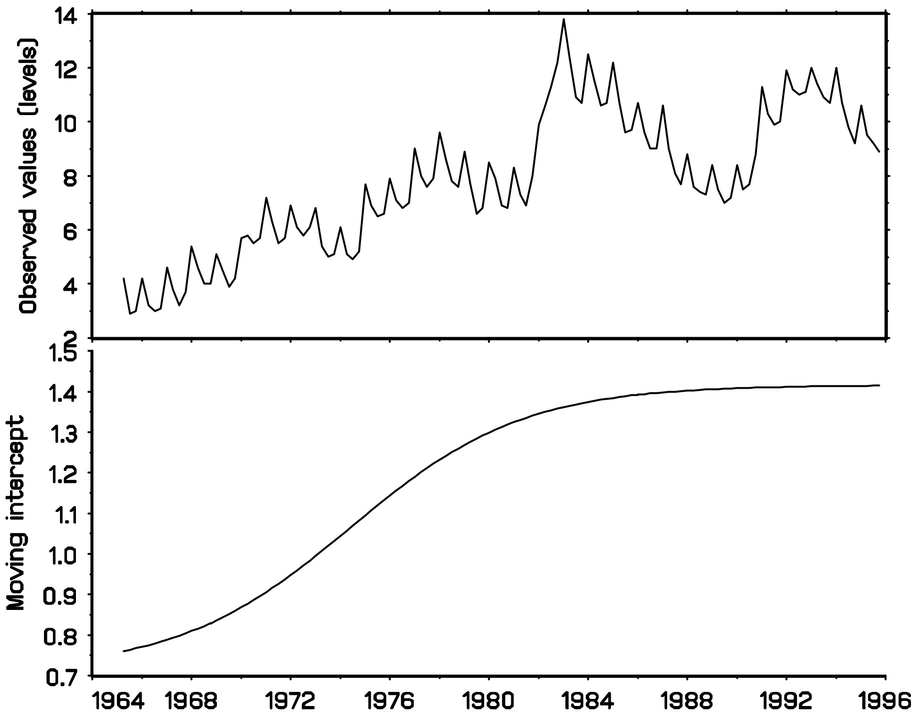


Figure 6.3
Japan. Observed values (upper panel) and moving parameters multiplied by seasonal dummies (lower panel)

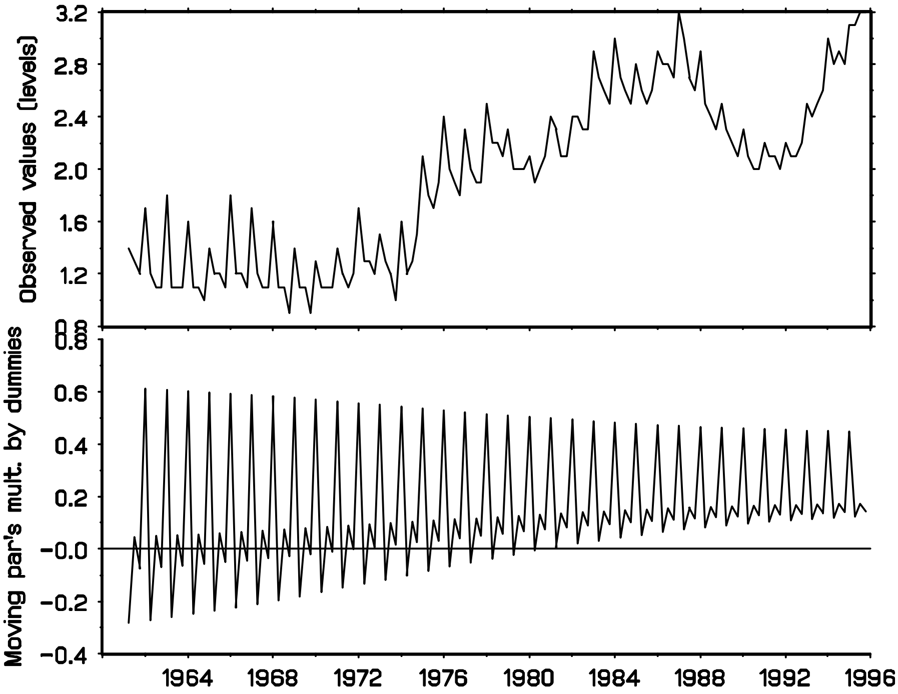


Figure 6.4
Norway. Observed values (upper panel) and moving parameters multiplied by seasonal dummies (lower panel)

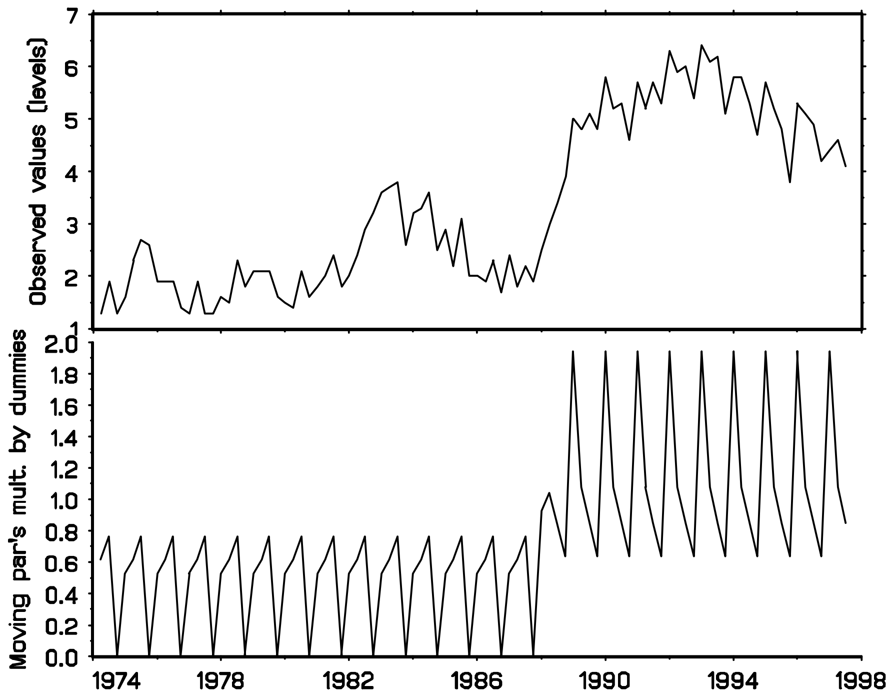


Figure 6.5
Italy. Observed values (upper panel) and moving parameters multiplied by seasonal dummies (lower panel)