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Stylized Facts of German Business Cycles: References for Macrodynamic Calibration

by

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Abstract

The paper investigates the cyclical components of several German macroeconomic key variables such as labour productivity, employment, the price level, real wages, the wage share, excess demand, and consumption. Computing the standard deviations of the detrended series and their cross correlations with the output gap, and comparing these statistics to US data, it is found that German and US cycles display similar comovements, while the variability may somewhat differ in a number of cases. Finally, a list of desirable cyclical features is put forward which may serve as a guideline to calibrating small or medium-sized models of the business cycle.

JEL classification: C 82, E 32.

Keywords: Business cycles, stylized facts, comovements, calibration, German data.

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1 Introduction

Even small macrodynamic models may easily become so complicated that one has to resort to computer simulations. This is all the more true for models of the business cycle and the study of their global dynamics. To obtain numerical values for the various parameters of such a model, two different approaches can be taken. The first and perhaps most obvious one is econometrics, that is, the model is decomposed into tractable subsystems and linear or nonlinear regressions of these building blocks are run. Especially for small models where there are only few variables on the right-hand side of a regression equation and, to maintain the model's simple structure, lags are not desirable, estimations are often fraught with difficulties because of the collinearity in the explanatory variables. Apart from that, the criterion of a good fit in the single equations does not necessarily mean that the parameters on the whole, when combined in the full model, are still able to generate a satisfactory dynamical behaviour.

The problems just indicated are addressed by the second method, which is called calibration and concentrates directly on the implications of the chosen parameters for the entire system. Here the numerical parameters themselves are derived from some *a priori* information, economic intuition, or are just set in a trial-and-error procedure. Then the model is simulated with these parameters and the cyclical features of the resulting dynamics are evaluated with a view to certain stylized facts of the business cycle. Thus, a set of parameter values is deemed successful if they give rise to a rough reproduction of some elementary time series statistics characterizing the variability and comovements of the economic variables. Given the insight that after all there is no such thing as a true model (which might be verified), and that therefore a model is chosen on the basis of the questions it allows to ask, and not because it is able to best mimic the data, a relative informal assessment of the model's performance is not only sufficient but also appropriate.[1]

Regarding the stylized facts to which a calibration may refer, corresponding statistical numbers and furthermore the graphical time series diagrams are by now readily available for the USA (for example, King and Rebelo, 1999; Stock and Watson, 1999). A similar collection of the properties of German business cycles is, however, still lacking. The present paper is a contribution to close this gap. Besides the output gap as the measure of the business cycle, it is concerned with the cyclical components of seven other macroeconomic key variables: labour productivity, employment, the price level, real wages, the wage share, excess demand, and consumption. The standard deviations of these series are computed to characterize their variability, and the cross correlations with the output gap to capture the comovements over the cycle. Subsequently, these

[1] It goes without saying that econometrics and calibration are to be thought of as two complementary, rather than exclusive, approaches.

statistics are compared to their counterparts from US data. As a succint summary, a list of desirable cyclical features is finally put forward which, qualitatively as well as quantitatively, may serve as a guideline to calibrating small or medium-sized models of the business cycle.

The paper is structured as follows. Section 2 introduces the variables we are concerned with and discusses their cyclical components on the basis of the visual impression from time series diagrams. Section 3 undertakes a quantitative analysis by computing the standard deviations and cross correlations and subsequently comparing them to US data. It also contains the just mentioned brief catalogue of desirable features that a cyclical macro model may exhibit. In addition, Section 4 gives some hints to the order of magnitude of a steady state output-capital ratio and inventory-sales ratio. Section 5 concludes.

2 General cyclical features

To start with, we need a measure of the business cycle. In this respect the output gap is most often referred to, that is, the percentage deviation of current aggregate output from potential output. In an *ex post* analysis, the latter may well be proxied by the trend component of the output series. The concept of the trend, however, is not unproblematic. Beginning with Nelson and Plosser (1982), it was argued that the trends in macroeconomic time series were stochastic, so that much of the variation that had been considered business cycles would actually be permanent shifts in trend. While this stochastic view of the world soon became predominant, the pendulum has in the meantime swung back from that consensus. Thus, from recent research on this issue it can be concluded "that at the very least there is considerable uncertainty regarding the nature of the trend in many macroeconomic time series, and that, in particular, assuming a fairly stable trend growth path for real output—perhaps even a linear deterministic trend—may not be a bad approximation" (Diebold and Rudebusch, 2001, p. 8).[2]

Against this background, we feel legitimated to work with the notion of a deterministic trend. Because of its flexibility and wide-spread application, the Hodrick-Prescott (HP) filter is adopted for detrending, with $\lambda = 1600$ as the standard smoothing parameter for quarterly data.[3]

[2] This short paper is a slightly revised version of the introductory chapter of their book on business cycles (Diebold and Rudebusch, 1999).

[3] In recent times, the band-pass (BP) filter developed by Baxter and King (1996) has gained in popularity. On the basis of spectral analysis, this procedure is mathematically more precise about what constitutes a cyclical component. The BP(6,32) filter preserves fluctuations with periodicities between six quarters and eight years, while eliminating all other fluctuations, both the low frequency fluctuations that are associated with trend growth and the high frequencies associated with, for instance, measurement error. More exactly, with finite data sets the BP(6,32) filter approximates such an ideal filter. As it turns out, for the time series with relatively low noise (little high

Though one may occasionally get the impression that this value is too low and makes the trend line nestle too closely against the actual time path, we maintain it in order to have a uniform detrending procedure and since it is almost exclusively used in the literature. Besides, possibly too narrow trend deviations need not be of great concern to us as their standard deviations will always be related to the standard deviation of the output gap.

Our output series is real gross value added. Quarterly data (from the DIW) for West Germany are available until 1993:2. Owing to the strong and certainly atypical effects from German unification, we should go no further than the year 1989. It seems, however, that the (trough-to-trough) cycle then underway, which began in 1982:4, was not fully completed at that time. It might also be suspected that the German business cycle, with growth at a relatively low level, has nearly died out in the second half of the 1980s, since there are no more cyclical features sufficiently standing out over these years. Unfortunately, simple detrending is an unsuitable instrument to decide on this issue because at the end of the sample period the computed trend may tend to unduly approach the actual series.[4] So, to be on the safe side, we choose to limit the sample period of our investigation already by the trough occurring in 1982:4. On the other hand, we let it begin with the first trough in the 1960s, which was in 1963:1. The time interval underlying the HP detrending procedure was, of course, longer: it extends from 1960:1 until 1989:4, so that there is no fear of end-of-period effects.

The time series of the resulting output gap is depicted in the upper panel of Figure 1.[5] The sample period 1963:1 – 1982:4 covers three trough-to-trough cycles with two further troughs in 1967:3 and 1975:1. The German business cycle over the span of time here considered has, therefore, an average length of six years and eight months.

We can thus turn to the macroeconomic variables whose comovements with the output gap we want to study. Let us begin with labour productivity z, specified as the ratio of output Y (i.e., real gross value added) over total hours worked L. Productivity has since long been counted a procyclical variable, for which it may it suffice to mention that Okun (1980, pp. 821f) lists it among his stylized facts of the business cycle. In the second panel of Figure 1, which shows the

frequency variation) the outcome of the HP 1600 and the BP(6,32) filter is almost the same. For real national US output, this is exemplified in King and Rebelo (1999, p. 933, fig. 1).

[4]We checked this problem with US output data for nonfinancial corporate business. Computing the HP 1600 trend over the period 1952 – 98 yielded trend deviations at the end of 1989 in the range of 2 per cent, whereas terminating the sample period at 1989:4 led to deviations between only 0 and 0.5 per cent.

[5]Needless to say that, as usual in a growth context, detrending applies to the logarithm of the output Y, so that what is exactly shown in Figure 1 are the differences $100 \cdot (\ln Y - \ln Y^o)$ (Y^o denoting the trend value of Y). Of course, the latter expression approximates the percentage deviations $100 \cdot (Y - Y^o)/Y^o$. The same holds true for the other time series that are growing over time. As for the time axis in Figure 1, the number 63.0 indicates quarter 1963:1, 63.25 indicates 1963:2, etc.

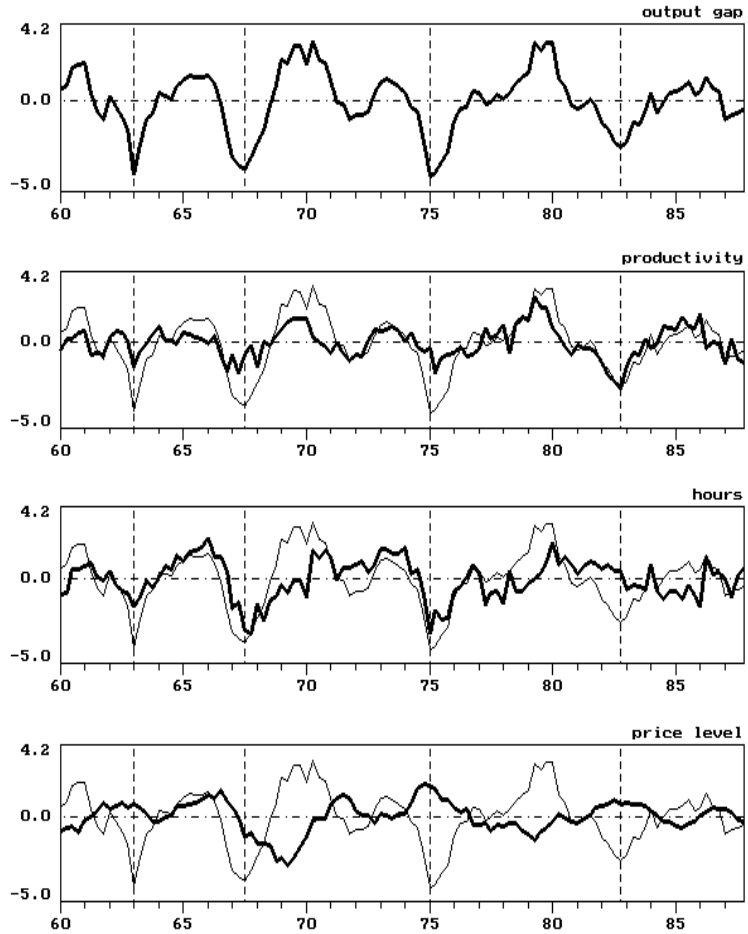

Figure 1: Cyclical components of empirical series 1.

Note: Variables measured in per cent of their trend values (HP 1600). The thin line reproduces the output gap series.

percentage deviations of z from its HP 1600 trend line z^o, there is indeed no question that this series displays a markedly procyclical pattern.

This result is noteworthy for macrodynamic modelling in two respects. First, many such models assume for the production technology that labour productivity is constant or grows at a constant rate. It follows from Figure 1 that these models, if they are meant to deal with business cycle behaviour, will overstate the procyclical variations of the employment rate. For example, denoting labour supply (in hours) by L^s and potential output by Y^p (which, as has been said above, may be captured by trend output Y^o), and identifying trend productivity z^o with Y^p/L^s, the employment rate, e, can be decomposed as $e = L/L^s = (Y/Y^p)(L/Y)(Y^p/L^s) = (Y/Y^p)\,/\,(z/z^o)$. From this identity it is seen that the employment rate would move one-to-one with capacity

utilization Y/Y^p in models with the abovementioned simplifying assumption, while in fact these fluctuations of e are weakened when, properly, procyclical productivity is taken into account.

The second aspect of the systematic variations of productivity concerns income distribution. It has here to be observed that it not only depends upon the real wage w/p (w being the nominal wage rate and p the price level), but also on z. The relationship is most straightforward if income distribution is represented by the wage share $v = wL/pY$, from which we get $v = (w/p)/z = (w/z^o p)/(z/z^o)$.[6] This decomposition clarifies that the impact of real wages on the wage share may be considerably modified by the countercyclical variations of $1/(z/z^o)$.

Regarding the first implication of the procyclicality of labour productivity, its attenuating influence on the motions of the employment rate is also borne out in the third empirical series in Figure 1, which displays the percentage deviations of hours L from the trend L^o. If we proxy labour supply L^s in hours by the trend series of hours worked L^o, then the variable in the third panel in Figure 1 can be interpreted as just the employment rate e (or as $100 \cdot (e-1)$, to be more accurate). The panel confirms that this employment rate is procyclical and (mostly) has a lower amplitude than the output gap, or capacity utilization Y/Y^p, for that matter.

Our next theme is price inflation. Since time series of quarterly rates of inflation are relatively noisy and so cannot be easily related to output variations with their high persistence, it is more illuminating to study the price level directly. Prices were formerly treated as procyclical, but by now there seems to be general consensus that their cyclical component rather tends to move inversely to economic activity. This is considered to hold for the USA as well as for the major industrialized countries in general; see, for example, Cooley and Ohanian (1991), Backus and Kehoe (1992), Fiorito and Kollintzas (1994). In fact, the bottom panel of Figure 1, which exhibits the trend deviations of the implicit deflator of gross value added, indicates a more or less countercyclical price level also for Germany. At least, the series can be interpreted this way if we concentrate on the comparatively large positive output gap in 1969/70 and 1979/80, and on the troughs in 1975 and 1982.

With the upper two panels of Figure 2 we get information about income distribution, beginning with real wages. The controversy surrounding the comovements of the real wage rate is usually summarized by saying that, if anything, it moves (weakly) procyclical, rather than countercyclical. Results about the cyclical properties of the real wage appear to be quite sensitive to precisely how it is constructed, depending on the index in the denominator (p) and whether the numerator (w) includes various compensation items. As for w, we refer to gross wages,[7] while p

[6] $w/z^o p$ can be viewed as the real wage rate normalized, or 'deflated', by trend productivity. Thus, $w/z^o p$ as well as z/z^o can be stationary variables in the long-run.

[7] The behaviour of net wages will be briefly characterized further below.

is the same implicit output deflator as before, so that strictly speaking the present series w/p is the product wage. On the basis of this specification, the top panel of Figure 2 shows that the real wage rate follows fairly closely the cyclical pattern of the output gap, though perhaps with a certain and nonnegligible lag.

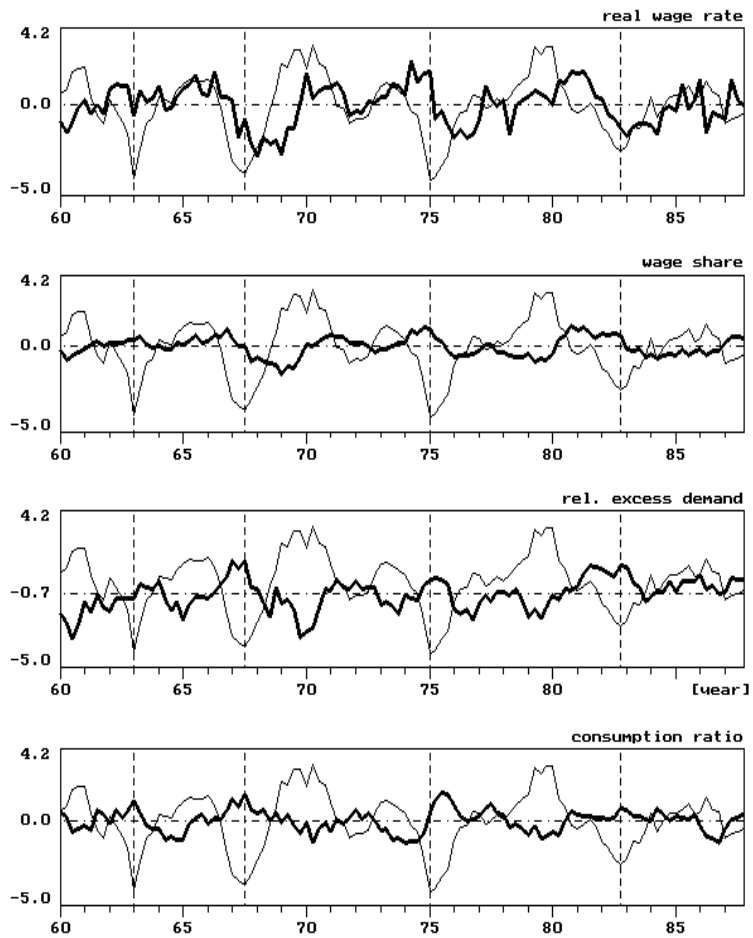

Figure 2: Cyclical components of empirical series 2.

Note: Top panel is deviation from trend (HP 1600) in per cent; panel 2 and 4 show the differences between the variables and their trend values (HP 1600) where both are measured in percentage points; relative excess demand (in %) is not detrended. The thin line again reproduces the output gap series.

The variable that more directly describes the distribution of income between workers and capital owners is the wage share v. It is only rarely mentioned in the discussion of typical features of a business cycle, which might in part be due to the special difficulties that one encounters for this variable in separating the cyclical from some intermediate quasi-trend behaviour. The trend deviations depicted in the second panel in Figure 2 may therefore be taken with some care. To

be exact, these percentage values are specified as $100 \cdot (v - v^o)$ (of course, v^o being the HP 1600 trend line of v).

From a visual inspection of this time series it might be tentatively said that the wage share has a more or less countercyclical tendency. Such a statement is tempting since, at a qualitative level, it is well compatible with the above decomposition of the wage share as $v = (w/z^o p)/(z/z^o)$: if both series $w/z^o p$ and z/z^o are procyclical and exhibit not too low a degree of persistence, then the wage share would be countercyclical if the trend deviations of labour productivity had a greater amplitude than the (productivity-deflated) real wage rate. The quantitative statistics computed below, however, will reveal that the argument is insufficient and the relationships between these three variables are more complex than this. On the other hand, the largely procyclical motions of real wages and productivity in the formula for v explain the relatively small fluctuations of the wage share.

The last two variables in Figure 2 are concerned with goods demand, where we concentrate on the series of excess demand. This emphasis derives from our interest in theoretical models that not only allow for disequilibrium on the goods market, but also consider its repercussion effects back on aggregate demand. In low-dimensional models along Metzlerian lines, where disequilibrium is buffered by inventories and inventory investment responds to a gap between actual and desired inventories, it is well-known that the inventory accelerator thus established easily gives rise to cyclical behaviour and may furthermore be destabilizing.[8] The destabilizing effects are perhaps less dramatic in more elaborate models, but even then inventories will remain a central variable. It is therefore important to have more reliable information about the cyclical variations of excess demand, whose flows govern the evolution of the stock of inventories.

The time series shown in the third panel of Figure 2 is relative excess demand, denoted as ξ. Writing Y^d for aggregate demand, it represents the percentage ratio $\xi = 100 \cdot (Y^d - Y)/Y$. The empirical series is constructed from the changes in real inventories, ΔN, simply by making use of the identity $\Delta N = Y - Y^d$, while for Y again gross value added (GVA) is taken. ξ is thus directly computed as $-100 \cdot \Delta N/\text{GVA}$.

It should be pointed out that even in a long-run equilibrium position, ξ will be different from zero. This is due to the fact that a constant proportion of inventories to the other quantity variables in a growing economy implies that a certain fraction of production goes to inventories. Hence, total output must exceed final aggregate demand Y^d, so that ξ is negative. This law also holds out of equilibrium as some average over time. In fact, the time average of ξ over our sample period amounts to -0.69%. This value is drawn out in the panel as the dot-dashed line in the

[8] Contemporaneous versions of such Metzlerian models are Franke and Lux (1993) and Franke (1996).

third panel.[9]

The variations of the actual values of ξ are rather regular, which is perhaps somewhat surprising since ξ is a ratio of two fluctuating variables. Specifically, the pattern we observe is of a fairly consistent countercyclical nature.

The countercyclical motions of relative excess demand are easily recognized to be brought about by the largest component of aggregate demand, i.e., by consumption C. The bottom panel of Figure 2 displays the trend deviations of the consumption ratio C/Y, whose countercyclicality is similarly pronounced as that of ξ. In detail, with $c := C/Y$ and c^o its trend, the series in Figure 2 is $100 \cdot (c - c^o)$.

3 A quantitative assessment

Figures 1 and 2 have shown qualitative features that a macro model for German business cycles should seek to take into account. In this section we take a step further and add quantitative information to the visual impression. This concerns the size of the fluctuations on the one hand, and the timing of the turning points of the variables on the other. The first facet is measured by the standard deviations, which are expressed in terms of the standard deviation of the output gap. The comovements of the variables are quantitatively characterized by their cross correlations with output. These statistics are all collected in Table 1. In the course of the discussion, we will also compare them with their counterparts of the American business cycle.[10]

Let us first consider the output gap itself, whose standard deviation over the sample period amounts to 1.75%. This is the same order of magnitude as in the US, where over the four major trough-to-trough cycles from 1961:1 to 1991:4 one computes a standard deviation of the GDP gap of 1.77%.[11] Also regarding persistence as described by the autocorrelation coefficients up to the fifth order, the two output gap series are very much alike. The average cycle period is somewhat longer in the US: seven years and nine months over the full sample period, and seven years and four months over the subperiod until 1981.

The procyclicality of labour productivity finds clear expression in a contemporaneous cross correlation coefficient with output of 0.66. Reckoning in a lead of z of one or two quarters, the

[9] A closer look at the series suggests that ξ has a slight upward tendency, but this phenomenon does not need to concern us here.

[10] The US time series we investigate are constructed from the database that is made available by Ray Fair on his homepage (`http://fairmodel.econ.yale.edu`), with a description being given in Appendix A of his US Model Workbook.

[11] Adopting the output of nonfinancial corporate business as the relevant variable, the standard deviation slightly increases to 1.81%.

		cross correlations between Y at time t and x at time						
Series x	σ_x/σ_Y	$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$
Y	$--$	0.47	0.71	0.89	1.00	0.89	0.71	0.47
z	0.57	0.57	0.68	0.73	0.66	0.50	0.29	0.04
L	0.75	0.23	0.45	0.65	0.82	0.79	0.71	0.59
p	0.58	−0.43	−0.37	−0.27	−0.12	0.09	0.29	0.45
w/p	0.68	−0.08	0.10	0.31	0.46	0.63	0.74	0.75
v	0.36	−0.48	−0.40	−0.26	−0.10	0.18	0.44	0.64
ξ	0.54	−0.48	−0.57	−0.56	−0.49	−0.36	−0.14	0.08
C/Y	0.41	−0.28	−0.46	−0.61	−0.74	−0.71	−0.57	−0.41

Table 1: Descriptive statistics for cyclical components of quarterly series, 1963:1 – 82:4.

Note: All series except ξ detrended by Hodrick-Prescott (with smoothing factor $\lambda = 1600$). First five series are trend deviations in per cent, v and C/Y (in %) are the differences to their trend values. ξ, too, is a percentage ratio. σ designates the standard deviations.

correlation is even a bit stronger. The comovements of z with Y are thus more pronounced than in the US, where at a lead of two and three quarters the same coefficients are 0.49 and 0.50, respectively.[12] So productivity may be said to be strongly procyclical in Germany and fairly procyclical in the US. This and other findings are summarized in the synopsis of Table 2. With $\sigma_z/\sigma_Y = 0.57$, the oscillations of z are also noticeably wider in Germany; in the US the same ratio is only 0.51.

Working hours L vary much in line with output, the contemporaneous correlation being 0.82. Though this is already a high coefficient, the relationship is still more intense in the US, where the coefficient amounts to 0.89 (and to 0.90 at a one-quarter lag for L). The trend deviations of hours and output are, however, not one-to-one. The standard deviation of L is just three quarters that of Y. According to this number, the variations of hours are considerably smoother in Germany than in the US, since for the latter a ratio $\sigma_L/\sigma_Y = 0.95$ is obtained.

[12] As has just been indicated, the sample period for the US data is 1961:1 – 1991:4 and output is represented by GDP. If instead output of nonfinancial corporate business is taken, the cross correlations and relative standard deviation are only marginally modified. Since this similarly holds true in the following, it will suffice to report the results relating to the GDP gap.

Variable	cyclicality GER	cyclicality USA	relative std. dev. GER	relative std. dev. USA
z	$+++$	$++$	0.57	0.51
L	$+++$	$+++$	0.75	0.95
p	$-$	$---$	0.58	0.59
w/p	$+++$	$++$	0.68	0.59
v	...	...	0.36	0.44
ξ	$--$	$--$	0.54	0.37
C/Y	$---$	$---$	0.41	0.40

Table 2: Synopsis of cyclical features.

Note: One, two, or three plus (minus) signs indicate that the variable is weakly, fairly, or strongly procyclical (countercyclical); the dots indicate that such a relationship does not prevail. Standard deviations are specified as a ratio of the respective output gap standard deviation.

Prices move markedly countercyclically in the US. At a lead of one and two quarters, cross correlations coefficients -0.66 and -0.69 are computed. For Germany, by contrast, we only get a correlation of -0.43 at a lead of three quarters. Or one might argue the price level is rather procyclical, since at a lag of three (or five) quarters we get a positive correlation with output that is as high as 0.45 (or even 0.60). With a view to the US results and the evidence referenced in the previous section, we nevertheless prefer to characterize the German price level as being weakly countercyclical. This evaluation is additionally justified if we cancel the 1960s in the sample period and reduce it to the years 1969 – 1982. Then at a lead of 0, 1, 2, 3, quarters the coefficients result like -0.50, -0.61, -0.65, -0.60, respectively, which is close to the US values just mentioned.

Regarding the size of the price fluctuations the two countries show practically no difference, the ratios σ_p/σ_Y being 0.58 and 0.59, respectively.

Table 1 points out more clearly than the time series diagram in Figure 2 that the basically procyclical motions of the real wage rate exhibit a significant lag of one up to five quarters. Allowing for this delay, however, the correlation with output comes out really strong (the correlation for a four- and five-quarter lag is 0.73 and 0.63). We may add that the relationship is a bit weaker when net (rather than gross) wages are adopted; at lags of three and four quarters the cross correlation coefficients are then 0.64 and 0.68.

Real wages in the US can be said to be procyclical, too, though the comovements are less

pronounced than in Germany. On the other hand, there is no lag in the US but perhaps a slight lead. The highest correlation coefficients are 0.56, 0.57, 0.52 at a lead of 0, 1, 2 quarters, respectively.[13] Real wages in Germany are also more procyclical in the sense that they generate a higher (relative) standard deviation than in the US. In both countries, nevertheless, the ratios $\sigma_{w/p}/\sigma_Y$ are quite large: 0.68 for Germany and still 0.59 for the US.

One possibility of describing the cyclical features of the wage share in Figure 2 was to characterize it as a countercyclical variable. Table 1 confirms this insofar as the cross correlation at a lead of three quarters is -0.48. The same value is computed at leads of four and five quarters. In contrast, a distinctly positive correlation is seen to obtain at a lag of three quarters. Moreover, the coefficient even increases above 0.64 at longer lags: at a four- and five-quarter lag it amounts to 0.75 and 0.77, respectively. These correlations, the positive as well the negative ones, suggest very strongly that the wage share moves systematically with the output gap, but undergoes a systematic phase shift of almost a quarter of a cycle. It is interesting to note that such a phase shift is typical for the income distribution dynamics in growth cycle models of Goodwinian type.

Incidentally, while a phase shift is a reasonable interpretation of the distribution of the cross correlation coefficients, there is much less evidence of such a regularity in the US data; at least it does not seem to prevail over the full thirty years of the sample period that so far we have always referred to.

Recalling the previous section's identity $v = (w/z^o p)/(z/z^o)$ for the wage share, it is not surprising that the standard deviation of v falls significantly short of that of the real wage itself. Though $\sigma_{w/p}/\sigma_Y$ is larger in Germany than in the US (0.68 vs. 0.59), Germany has a smaller ratio σ_v/σ_Y (0.36 vs. 0.44). The reason for this are the abovementioned lower trend deviations of labour productivity in the US.

Countercyclicality of relative excess demand ξ was already plainly visible in Figure 2, and the coefficients in Table 2 confirm a fairly well-developed negative correlation with output. They additionally reveal a minor lead of one or two quarters. With -0.59 the US have a very similar value of the largest (in modulus) coefficient, but there excess demand exhibits neither a lead nor a lag. A more important difference is that excess demand in the US is less variable relative to output than in Germany; the ratio of the standard deviations is $\sigma_\xi/\sigma_Y = 0.37$ as opposed to the German ratio 0.54.[14]

[13] Because of less clearer evidence of procyclicality in other literature, it should be mentioned that the nominal (gross) wage rate extracted from Fair's US database (cf. footnote 10) was augmented by a uniform 50% wage premium for overtime work. This is a rough-and-ready procedure we have taken over from Fair himself.

[14] The German σ_ξ may be somewhat reduced if the slight upward trend of ξ to be recognized in Figure 2 were removed.

Though it is not obvious from the time series in Figure 2, the countercyclicality of the consumption ratio C/Y is even more pronounced, and it is strictly synchronous with output. Remarkably, consumption in the US shows practically the same quantitative features. The contemporaneous cross correlation coefficient is -0.78, vs. -0.74 for Germany, the ratio of the standard deviations $\sigma_{C/Y}/\sigma_Y$ is 0.40 vs. 0.41 for Germany.

At the end of this section we may recall the original motivation for setting up stylized facts of the German business cycle, namely, that they can represent a yardstick against which a business cycle model is to be calibrated. The above discussion was oriented toward just this goal. We succintly summarize it in Table 3 which, for each of the seven macroeconomic variables investigated, gives a range of the cyclical statistics that a model containing these variables should attempt to meet. The lagging behaviour would most easily be captured in a deterministic model with relatively smooth oscillations, where it may be directly the interval between the turning points of the output gap and the variable in question. In a stochastic setting, one would again refer to the maximal cross correlations, if the noise in the time series proves to be too strong.

variable x	σ_x/σ_y	Lag x
z/z^o	0.55 – 0.60	-0.50 – 0.00
$e - e^o$	0.70 – 0.80	0.00 – 0.75
$-(p/p^o)$	0.55 – 0.60	-0.75 – 0.00
ω/ω^o	0.65 – 0.70	0.25 – 1.25
$v - v^o$	0.30 – 0.40	$\approx 1/4$ cycle
$-(\xi - \xi^o)$	0.50 – 0.60	-0.75 – 0.00
$-[C/Y - (C/Y)^o]$	0.35 – 0.40	-0.25 – 0.50

Table 3: Desirable features of macrodynamic oscillations on the basis of German data.

Note: Reference series y is the output gap. e is employment rate, ω the real wage. Superscript 'o' denotes steady state or (for z, p, ω) trend values. The lags are measured in years.

4 A brief note on two steady state ratios

The main effort in calibration is devoted to the implications of 'deep' technology or preference parameters in mainstream macroeconomic modelling, or of behavioural parameters such as adjustment speeds or reaction intensities in (modern '*ad-hoc*') Keynesian disequilibrium models. In contrast, the ratios of the macro variables that would prevail on a deterministic long-run equilibrium growth path can mostly be set from the data directly. In this section we give some hints as to the order of magnitude of two steady state ratios connecting flows and stocks, namely, the output-capital ratio and, what would be important for models with goods market disequilibrium, the inventory-sales ratio.

Regarding the output-capital ratio Y/K, we think of the capital stock (K) installed in the non-financial corporate business (NFCB) sector as the relevant notion of firms' capital, which, in particular, excludes the housing and government sector. Annual data from DIW are available for the real net capital stock in the industry sector and the trade and transport sector, and we take the relationships in these two sectors as a proxy for the whole NFCB sector. Also available are the shares of these two sectors in total real gross value added, which provides us with the corresponding levels of output (Y). On this basis one obtains the figures listed in Table 4 for selected years in our sample period, where the output flow is measured at an annual rate. Thus, an output-capital ratio of $Y/K = 0.70$ or somewhat higher would be a reasonable choice to work with.

year :	1960	1970	1980	1985	1989
Y/K :	1.067	0.840	0.716	0.687	0.703

Table 4: German output-capital ratio proxying the NFCB sector.

The time series of relative excess demand ξ was constructed from the changes in real inventories. On the other hand, information about the level of these inventories is not available. We can, however, get a clue about a steady state relationship from some elementary considerations. Consider, to this end, the inventory-sales ratio,

$$\beta_{ny} \quad := \quad N \,/\, Y^d$$

(N denotes inventories and Y^d sales or aggregate demand, if we neglect rationing). With the real variables growing at rate g, inventory changes being governed by the continuous-time identity $\dot{N} = Y - Y^d$, and β_{ny} being a constant coefficient under equilibrium growth, we have $\beta_{ny}\, gY^d =$

$\beta_{ny}\dot{Y}^d = \dot{N} = Y - Y^d$, or $Y = (1+\beta_{ny}g)Y^d$. The definition of excess demand reads $\xi = (Y^d - Y)/Y$, which gives $Y = Y^d/(1+\xi)$. Equating the two expressions for Y and solving the resulting equation for β_{ny}, we then arrive at

$$\beta_{ny} = \frac{-\xi}{(1+\xi)\,g}$$

When introducing the variable ξ, it was indicated that it exhibits a negative time average of $\xi = -0.69\%$. Assuming an annual growth rate $g = 3\%$, the inventory-sales ratio in a steady state position turns out to be $\beta_{ny} = 0.232$ (with $g = 2.5\%$, it would rise to $\beta_{ny} = 0.278$).

Confidence in this outcome of the calculation may be gained by referring to R. Fair's database, which was mentioned previously in fn 10 and contains directly an inventory series N as well as a sales series Y^d in the NFCB sector. Employing these data, the ratio N/Y^d attains values between 0.20 and 0.23.

Other ratios involving inventories can now be readily computed from β_{ny}. For example, the inventory-capital ratio may be one of the state variables in the intensive form of a Keynesian disequilibrium model. Invoking the above output-capital ratio $y := Y/K$, the steady state ratio N/K is obtained from the decomposition $N/K = (Y/K)(N/Y) = y \cdot N/[(1+\beta_{ny}g)Y^d]$, which yields $N/K = \beta_{ny}\,y/(1+\beta_{ny}g)$.

5 Conclusion

While the cyclical behaviour of the US economy is empirically by now fairly well analyzed, at a level such that the numerical characterizations can directly serve as a yardstick for the performance of small and medium-sized models of the business cycle, a comparable range of results for Germany is still lacking. The present paper has set out to close this gap. Comparing the cyclical components of several macroeconomic variables in Germany and USA, both of which were determined by the same methodology, it turned out that the cycles in the two countries are not so much different as far as the comovements are concerned. The extent of variability, on the other hand, may differ in some cases. This finding suggests that a numerical model cannot simultaneously apply to both countries. If, however, the calibration of a model has largely proved successful with respect to one country, it should not be too problematic to recalibrate it in order to match the cyclical features of the other country. The different values in the structural parameters at which one thus arrives might, then, provide an explanation for the differences in German and US business cycles.

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