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Center for Empirical Macroeconomics

Working Paper No. 49

Monetary Policy, Currency Unions and Open Economy Macrodynamics

by

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In this paper we extend an integrated closed-economy macrodynamic model to account for a large open economy in a currency area with fixed nominal exchange rates between the currencies. The major issues are the effectiveness and macrodynamic effects of monetary policy for countries in a pegged or fixed exchange rate system. The model assumes a leader-follower relationship as in Kenen (2002) where the dominant country pursues a monetary policy and the other countries adjust. We explore the effect of two policy rules – the monetary authority targeting the money supply or interest rate. The model augments Keynesian AS-AD growth dynamics by goods market disequilibrium, price and quantity adjustment processes and income distribution dynamics and is completely specified with respect budget and behavioral equations. The steady state of the resulting 7D dynamical system is asymptotically stable for sluggish responses to labor and goods market imbalances, if the Keynes-effect is strong and sales expectations sufficiently fast. Increasing adjustment speeds for wages, prices, inflationary expectations or inventories however implies loss of local stability by way of Hopf-bifurcations. Monetary policy rules augment the dimension of the considered dynamics by one (interest rate rule) or two (money supply rule) and may stabilize the economy to some extent if the adjustment speeds in these rules are chosen with care. Yet, behavioral nonlinearities are needed in addition to ensure economic viability over a larger range of the parameter values of the model.
Keywords: Keynes-Metzler-Goodwin growth, Currency unions and fixed exchange rates, Large open economy macrodynamics, Instability, Monetary policy rules, Employment cycles.
JEL Subject classification:E31, E32, E37, E52

1. Introduction

After the end of the Bretton Woods system and the transition to flexible exchange rates in the 1970s many countries have since then experienced a stronger volatility of their macroeconomic variables such as consumption, investment, real wages but also nominal wages and prices caused by the volatility of exchange rates. In recent times this has given rise to controversies concerning the proper choice of exchange rate systems for specific countries or regions. The choice is between flexible exchange rates, dirty floating, where monetary authorities occasionally intervene, pegged exchange rates or completely fixed exchange rates such as currency boards. Fixed exchange rates have also been established between countries in a currency union. In Europe, for example, for a long time period, a union of different currencies was established between 1979 and 1999 where exchange rates were supposed to be fixed within a certain band. Such a currency system with pegged exchange rates, the European Monetary System (EMS), was the dominant exchange rate arrangement for almost twenty years until in January 1999 the currency union with different currencies was replaced by a currency union with a single currency, the Euro. Currency unions with separated currencies but fixed exchange rates have been discussed to be established in North-America between Canada, U.S. and Mexico, in Asia between Asian countries, possibly including Japan.[1]

One of the major reasons for such regional system of fixed exchange rates is that countries with strongly integrated trade save considerable transaction costs when moving from highly volatile flexible to pegged exchange rates. Yet, the monetarists' objection against pegged exchange rates is that countries will lose monetary policy as stabilization instrument because monetary authorities are obliged to use instruments of monetary policy to keep the exchange rate constant. In the extreme, as monetarists tend to argue (see McCallum, 1996, Ch. 7) monetary policy becomes endogenous because it has to be devoted to keep the exchange rates fixed.

Yet, the experience of the EMS from 1979-1999, with the exception of the serious disturbance 1992, seems to have shown that pegged exchange rates can work and demonstrate that monetary policy can be conducted even by being devoted to two goals: exchange rate stabilization and stabilization of inflation

[1] Currency systems like the one above have recently been proposed by Robert Mundell of Colombia University, New York, see also Kenen (2002).

and output. Yet, note that such a currency arrangement seems to work where a large economy dominates the other economies included in the pegged exchange rate system with respect to monetary policy. We thus presume that the other economies will always adjust the nominal interest rate achieved by the domestic economy so as to keep the nominal exchange rates constant. Such a behavior of the other countries to follow a dominant country to adjust there interest rate within a pegged exchange rate region has been called the leader-follower model, see Kenen (2002). This may not always be convenient for the other economies but this is likely to happen and has happened in fact under the EMS where Germany dominated the currency system. For such a currency arrangement in North-America, the U.S. would be the dominant country and in Asia presumably Japan.

Surely, there are also disadvantages with this type of currency arrangements, yet one might want to demonstrate (1) how the macroeconomic dynamics work and (2) how policy – we here focus on monetary policy – can successfully be inacted even under a system of pegged exchange rates. To study these questions is important since, as above mentioned, large regions that are nowadays highly integrated through trade naturally tend to adopt pegged exchange rate systems between the integrated economies.

We will study those questions in a Keynesian framework for open economies with pegged exchange rates. We allow for disequilibria in the product and labor market, sluggish wage, price and output adjustments and the trade account responding – given that the nominal exchange rates are fixed with in a band – to real exchange rates. More specifically we consider a dominant large economy in the context of pegged exchange rates, and presume that 1) intermediate goods as well as private and public consumption demand respond to real exchange rates and 2) a wage and price Phillips-curve is impacted by real exchange rates. In this context then macroeconomic dynamics as well as effectiveness of monetary policy are studied. Concerning monetary policy we consider two rules – the monetary authority targeting the money growth rate or directly targeting the inflation rate (and output) through the Taylor rule.

Of course, we want to note that our analysis appears to be valid only if there are no major currency attacks which can lead either to major realignments of the currencies or to the abolition of the pegged system.[2] We therefore here only

[2] Such a major currency attack has occurred for the EMS in September 1992 and produced

discuss where the balance of payments gets into balance through some consistency assumptions on domestic flows so that there is no need for the Central Bank to intervene in the market for foreign exchange due to the assumed flow consistency between private households and the government. In this way disturbances in the balance of payments are still excluded from consideration.

Finally, we want to note that there are numerous studies, see for example Ball (1999), that explore monetary policy rules for open economies under flexible exchange rates. In those studies the domestic economy is only characterized by very stylized core equations for the macroeconomic dynamics namely by a simple IS-equation, a Phillips curve and an uncovered interest parity equation for the open economy. As to our knowledge the problem of monetary policy rules for a pegged exchange rate system in the context of a larger macroeconomic model has not been sufficiently addressed.

The remainder of this paper is organized as follows. Section 2 introduces in detail the open economy model of the dominant country under pegged exchange rates with product and labor market disequilibrium, wage and price Phillips-curve for an open economy with balance of payment equilibrium. Section 3 transforms the model into an intensive form representation so that the existence of the steady state equilibria as well as the macroeconomic dynamics around such balanced growth paths can be studied. In Section 4 a feedback motivated stability analysis of the interacting feedback channels of the dynamics is provided, similar to – but much more detailed than – the one in the main part of the preceding chapter. Section 5 explicitly studies the dynamic effects of the two active monetary policy rules, one based on the growth rate of the money supply and one – like the Taylor rule – based on the steering of the short-term nominal rate of interest. Section 6 concludes the paper.

2. The model in extensive form

In this section we present our model of large open economy macrodynamics with disequilibrium in the real markets, on its extensive form level, including all budget equations that underlie the stock reallocation decisions and flow demand and supply decisions of households, production and investment decisions of firms and the deficit-financed expenditure decisions of the government. We here intro-

a considerable currency crisis for the EU member states with subsequent realignment and a larger band.

duce a still simple model of international trade for a large open economy in a fixed exchange rate regime or area where the characteristics of the surrounding countries are basically given – with the exception of the foreign exchange rate that has to adjust to the domestic one in the considered situation of an economy that dominates the other ones in this respect.

Module 1 of the model provides some definitions of basic variables: the real wage ω, the expected rate of return on capital ρ^e (δ the depreciation rate), real financial wealth W (consisting of money, domestic and foreign bonds and equities) and the real exchange rate η.[3] The expected rate of return on physical capital is based on expected sales from which depreciation, real wages and real imports of firms have to be deducted. Firms therefore here make use of a three factor technology where besides capital K and labor L^d the imports J^d are used to produce real output Y. In addition to the measure of expected returns on capital we use normal returns ρ^n in the investment function of the model, which are based on normal usage of capacity $y_o = \bar{U}y^p$ and the normal sales to capital ratio y_o^d.[4] We note that the exchange rate e is a given magnitude in the considered model which implies that domestic and foreign fix price bonds (prices set equal to one in each currency for simplicity) can be considered as perfect substitutes if they earn the same nominal rate of interest.

1. Definitions (income distribution, real wealth, real exchange rate):

$$\omega = w/p, \ \rho^e = (Y^e - \delta K - \omega L^d - J^d/\eta)/K, \ \ \eta = p/(ep^*) \tag{1}$$

$$W = (M + B_1 + eB_2 + p_e E)/p, \ \ p_b = p_b^* = 1, \ \ e = const. \tag{2}$$

$$\rho^n = y_o^d - \delta - \omega y_o/x - jy_o/\eta, \quad y_o^d = \bar{U}^c y^p/(1 + n\alpha_{n^d}), \ \ y_o = \bar{U}^c y^p \tag{3}$$

Module 2 provides the equations for the household sector, consisting of workers and asset holders, with lump-sum taxes, T_w, concerning wage and interest income of workers and T_c, concerning the dividend and interest income received by asset holders, held constant net of interest per unit of capital, see the government module below, since fiscal policy is not a topic in the present paper. Asset demand is shown in general terms in equations (4) and (5), where only money demand is explicitly specified.[5] The wealth constraint for asset reallocations is

[3] Measured as amount of foreign goods currently traded for one unit of the domestic good.

[4] See the steady state calculations in section 3 for the derivation of the expressions for y_o^d, y_o.

[5] The formulation of money demand can be derived from a money demand function of type $M^d/p = m^d(Y, W, r)$ homogeneous of degree one in (Y, W). A Taylor expansion of $M^d/(pW) = m^d(Y/W, r)$ would yield (5). For simplicity we replace W by K in (5) in the developments

(4). Its implications are explicitly considered only in the case of money demand (5) which allows the usual LM-determination of the domestic nominal rate of interest. Domestic bonds and foreign bonds exhibit the same rate of interest, either through dominance of the domestic money supply steering of the nominal rate of interest and thus an assumed adjustment to the obtained interest rate by the other countries of this currency union or – later on – through an assumed Taylor rule steering of the nominal rate of interest in the dominant economy and again an adjustment to this rate in all other countries in the currency union. Domestic equities are also considered as perfect substitutes, see eq. (30), where equity prices are assumed to adjust such that returns are equalized with those on short-term bonds. The reallocation of interest bearing assets may thus be ignored, since asset holders accept any composition of such assets if money demand has adjusted to money supply by movements of the short-term rate of interest r (or supply to demand in the case of the Taylor rule).

Eqs. (6), (7) define the real disposable income of pure asset holders and of workers, respectively. The consumption by these two groups of the domestic goods C_1 and the foreign goods C_2 depends both in the case of asset owners and of workers on the real exchange rate η in the usual way, which is here formalized by means of the consumption ratio $\gamma(\eta)$ in front of the sum of the total consumption expenditures of workers and asset holders, based on given saving ratios s_w, s_c of these two groups of agents. Note that consumption of the foreign good is based on real income in domestic terms and must thus be transformed into units of the foreign commodity by means of the real exchange rate η. Aggregate domestic consumption C is defined in eq. (10).

Note finally that workers save in the form of money and domestic bonds, while asset holders also save in the form of foreign bonds and domestic equities. We thus in particular assume that only bonds are traded internationally. This is not a severe restriction in the present formulation of the model, since financial asset accumulation does not yet feed back here into the real part of the economy, due to the neglectance of wealth and interest rate effects in the consumption function of both workers and asset holders, see the tax collection rules in module 4 of the model. The model that we are investigating here thus still exhibits only a very traditional type of real-financial interaction basically based on the assumed simple LM-theory of the money market (or a Taylor interest rate policy rule,

below.

both to be introduced in a later section of the paper). Note however that the model allows for saving of workers and the accumulation of money and short-term domestic bonds by them. Note furthermore that we assume with respect to asset holders that all expected profits are paid out as dividend to which interest income here and abroad must be added to obtain their before tax total income.

Private savings S_p of asset holders and workers together just absorb the actual changes in the money supply conducted by the government, the new equity issue of firms, part of the domestic new bond issue and also foreign bonds to some extent. We have to check later on that there is consistency in the absorption of flows and thus no obstacle for the supply of new money, new domestic bonds and the issue of new equities. Note that the flows shown in eq. (11) need not all be positive since we allow for flows out of the stocks of domestic and foreign bonds held domestically. Finally, labor supply L grows at a constant rate n_l, which – augmented by Harrod neutral technical change – is assumed to determine the trend growth rate in investment, sales expectations and inventories later on.[6]

2. Households (workers and asset-holders):

$$W = (M^d + B_1^d + eB_2^d + p_e E^d)/p \tag{4}$$

$$M^d = h_1 pY + h_2 pW(r_o - r), \quad \text{W reduced to K later on} \tag{5}$$

$$Y_w^D = \omega L^d + rB_{1w}/p - T_w, \quad s_w Y_w^D = \dot{M}_w + \dot{B}_{1w} \tag{6}$$

$$Y_c^D = \rho^e K + rB_{1c}/p + er^* B_2/p - T_c, \quad s_c Y_c^D = \dot{M}_c + \dot{B}_{1c} + e\dot{B}_2 + p_e \dot{E} \tag{7}$$

$$C_1 = \gamma(\eta)((1 - s_w)Y_w^D + (1 - s_c)Y_c^D), \quad \gamma(\eta) = \gamma_o + \gamma_1(\eta_o - \eta) \in (0, 1) \tag{8}$$

$$C_2 = \eta(1 - \gamma(\eta))((1 - s_w)Y_w^D + (1 - s_c)Y_c^D) \tag{9}$$

$$C = C_1 + (ep^*/p)C_2 = C_1 + C_2/\eta \tag{10}$$

$$S_p = Y_w^D + Y_c^D - C = s_w Y_w^D + s_c Y_c^D = (\dot{M} + \dot{B}_1 + e\dot{B}_2 + p_e \dot{E})/p \tag{11}$$

$$\hat{L} = n_l = \text{const.} \tag{12}$$

The third module concerns firms, modeled here with respect to their output and employment decision Y, L^d and their needs of imports for production. We thus have a three factor production technology and assume fixed proportions in production and thus strictly proportional relationships between capital K and

[6] Note her already, that due to our exclusion of wealth effects in consumption demand – and due to a particular choice of tax collection rules – in particular we do not have to consider the actual accumulation of financial assets in the intensive form dynamics of the private sector later on.

potential output Y^p, output Y and employment L^d as well as imported production factors J^d. On this basis we can also define unambiguously the capacity utilization rate of firms U^c and the rate of employment of the labor force V. Next, in eq.n (16) we describe the net investment decision of firms which is based on medium run values for the return differential between normal nominal profitability $\rho^n + \hat{p}$ and the nominal rate of interest r, $\hat{p}$ the rate of inflation. Excess returns $\epsilon = \rho^n + \hat{p} - (r + \xi) = \rho^n - (r + \xi - \hat{p})$ of firms, with ξ a given risk premium, transformed to such medium run values ϵ^m, interpreted as the currently prevailing investment climate, are one driving force for the investment decision, while the deviation of capacity from its normal value provides the short-run influence of the state of the business cycle on the investment decisions of firms. We assume next that the medium run values ϵ^m follow their short-run analogs in an adaptive fashion, representing the way how the medium run climate expression is updated in the light of the current experience of their short-run analogs. In the analysis of the model we will basically make use of the short-run excess variable in the investment function solely and leave the delayed influence of excess real profitability over the real rate of interest for numerical investigations of the model by and large.

The excess of expected sales Y^e over aggregate demand Y^d for the domestic commodity is shown next. Here, the index 1 is used in the usual way to denote the domestically produced commodity (also demanded by foreigners in amount Y_1^{d*}). Furthermore, we use $*$ to denote foreign demands and supplies. In equation (18) we state that the savings of firms are equal to their voluntary production of inventories which in turn is equal by definition to the excess of their production over their expected sales. Finally we have the financing condition of firms, eq.n (20), which states that all investment and all unintended inventory changes (windfall losses) are financed by the issue of new equities, which means that we do not yet allow for credit financing and the like. If $\dot{N} - \mathcal{I}$ is negative, firms do have windfall gains in the place of windfall losses and are using them for their investment financing and thus do not have to issue as many equities as their investment decision would in fact demand. Note again that expected profits are paid out as dividends and are thus not available for the financing of investment plans. The last equation of module 3., finally, states that we considering only Keynesian regimes as temporary positions of the economy, where in particular all investment orders are always fulfilled, i.e., firms never run out of inventories and indeed always serve aggregate demand, see module 6. of the model, see Chiarella, Flaschel, Groh and Semmler (2000, Ch.5) for a detailed modeling of this. Note that the present formulation of the sector of firms considers imported means of production only as inputs into production, not as part of the investment efforts of firms which are solely based on domestic commodities.

3. Firms (production-units and investors):

$$Y^p = y^p K,\ y^p = \text{ const. },\ \ U^c = Y/Y^p = y/y^p \ \ (y = Y/K) \tag{13}$$

$$L^d = Y/x,\ n_x = \hat{x} = \dot{x}/x = \text{ const.},\ \ V = L^d/L = Y/(xL) \tag{14}$$

$$J^d = jY,\ j = \text{ const.} \tag{15}$$

$$\frac{I}{K} = i_1\epsilon^m + i_2(U^c - \bar{U}^c) + n,\ n = n_l + n_x \tag{16}$$

$$\dot{\epsilon}^m = \beta_{\epsilon^m}(\epsilon - \epsilon^m),\quad \epsilon = \rho^n + \hat{p} - (r + \xi) = \rho^n - (r + \xi - \hat{p}) \tag{17}$$

$$\Delta Y^e = Y^e - C_1 - Y_1^{d*} - I - \delta K - G_1 = Y^e - Y^d \tag{18}$$

$$Y_f = S_f = Y - Y^e = \mathcal{I} \tag{19}$$

$$p_e\dot{E}/p = I + \Delta Y^e = I + (\dot{N} - \mathcal{I}) \tag{20}$$

$$\hat{K} = I/K \tag{21}$$

Module 4 describes the government sector of the economy in a way that allows for government debt in the steady state and for a simple monetary policy rule (to be modified later on). Government taxation of workers and asset holders income is such that taxes net of their interest rate receipts are held constant per unit of capital, see Rødseth (2000) for similar procedures. This simplification allows to treat tax policies as parameters in the intensive form of the model (due to our stress on the role of monetary policy rules) and removes in addition the impact of interest payments on wealth held in the form of bonds on the consumption decisions of both types of households. Government consumption per unit of capital is also assumed a parameter of the model, but is divided into domestic demand and demand for the foreign commodity in the same ratio as for the sector of households (which is thus varying uniformly across consuming sectors).[7] The definition of government savings is an obvious one, as is the growth rate for the money supply, assumed to equal the domestic steady state rate of real growth $n = n_l + n_x$ augmented by the steady state rate of inflation of the foreign country. Finally $\dot{B}$ describes the law of motion for government debt, which results

[7] We, however, neglect any influence of the real exchange rate on investment plans and in fact do not even consider the import of investment goods for the time being.

from the decision on taxation T, government consumption G and new money supply $\dot{M}$. Note that the central bank is not involved in foreign exchange market operations, since we can show later on that the balance of payments is balanced without any intervention from the monetary authority.

4. Government (fiscal and monetary authority):

$$T = T_w + T_c \tag{22}$$

$$t_w = \frac{T_w - rB_{1w}/p}{K} = const., \quad t_c = \frac{T_c - (rB_{1c} + er^* B_2)/p}{K} = const. \tag{23}$$

$$G = gK, \quad g = \text{const.} \tag{24}$$

$$G_1 = \gamma(\eta)G, \quad G_2 = \eta(1 - \gamma(\eta))G \tag{25}$$

$$S_g = T - rB/p - G \tag{26}$$

$$\hat{M} = n + \bar{\pi} = n + \hat{p}_o^* = \text{ const.} \quad \text{at first} \tag{27}$$

$$\dot{B} = pG + rB - pT - \dot{M} \tag{28}$$

The fifth module lists the equilibrium conditions for the four financial assets of the model: money, domestic and foreign bonds and equities. Due to the perfect substitutability assumptions (30), (31) it suffices to specify money demand explicitly, as wealth owners are indifferent to the allocation of the remaining terms, their domestic and foreign bond (which only have interest returns) and equity holdings (whose return consists of dividend returns and actual capital gains). Note that we have assumed in eq.n (31) that the dominant (the domestic) economy leads the surrounding ones with respect to interest rate formation (here based on its still simple money supply rule), i.e., the other economies will always adapt to the nominal interest rate achieved by the domestic economy, i.e., $r^* = r$ holds throughout. This type of monetary arrangement within a currency union has been called by Kenen (2002) the leader-follower model.

We stress that the model cannot be considered as being completely specified, since there may be more than one path for the accumulation of the financial assets as the model is formulated (which however does not matter for the real dynamics in its present formulation). Macroeconometric studies as in Powell and Murphy (1997) at this point assume for example that there is a fixed proportion according to which domestic and foreign bonds are accumulated in order to allow for a unique path in the accumulation of assets. Here we simply avoid this problem by stating again that the accumulation of financial assets does not yet matter for consumption demand.

5. Equilibrium conditions (asset-markets):

$$M = M^d = h_1 pY + h_2 pK(r_o - r) \tag{29}$$

$$r = \frac{\rho^e pK}{p_e E} + \hat{p}_e \tag{30}$$

$$r^* = r \tag{31}$$

Certainly our description of the asset markets[8] of a large open economy within in the EMU system with four types of financial assets is still fairly restrictive and must be improved in the future, see Chiarella, Flaschel, Franke and Skott (2002) for some attempts into this direction. Furthermore, stock adjustment mechanisms should be modeled in detail in future extensions of this model, when wealth accumulation is allowed to feed back into the overall dynamics.

Module 6 describes the adjustment process of output and inventories toward aggregate demand and desired inventories and is formulated as for the closed economy situation in Chiarella and Flaschel (2000). The only difference here is that actual savings are no longer identical to actual investments in capital goods and inventories, but are now obtained by adding the surplus in the current account to them (including the balance of the interest payment account), equal to the negative value of the capital account as we shall show below). Such accounting identities are added as consistency checks – in eq.n (37) – to the disequilibrium adjustment process that is considered in module 6. of our macrodynamic model. Note again that investment goods are only purchased from domestic production, while all other components of private domestic demand depend on the real exchange rate as described in the above modules. We thus only have index 1 commodities in this quantity adjustment process and the demand of foreigners for the domestic product Y_1^{d*} in addition.

The module 6. considers desired inventories N^d as proportion of adaptively adjusted expected sales Y^e and determines on this basis intended inventory changes as an adjustment of actual inventories N towards desired inventories (augmented by a term that accounts for trend growth). Production is then determined by the sum of expected sales and intended inventory changes (sales expectations Y^e being revised in a straightforward adaptive fashion, also augmented by a term that accounts for trend growth). Finally actual inventory

[8] In fact, only a traditional LM–equation for the domestic rate of interest is considered.

changes $\dot{N}$ are simply given by the excess of actual output over actual demand. which closes our description of the output and inventory adjustment mechanism of firms.[9]

6. Disequilibrium situation (goods-market):

$$Y^d = C_1 + I + \delta K + G_1 + Y_1^{d*} \qquad (Y^e \neq Y^d \text{ in general }) \tag{32}$$

$$\dot{Y}^e = \beta_{y^e}(Y^d - Y^e) + nY^e \tag{33}$$

$$\mathcal{I} = \beta_n(N^d - N) + nN^d, \quad N^d = \alpha_{n^d} Y^e \tag{34}$$

$$Y = Y^e + \mathcal{I} \tag{35}$$

$$\dot{N} = Y - Y^d = Y - Y^e + (Y^e - Y^d) = \mathcal{I} + \Delta Y^e \tag{36}$$

$$\begin{aligned} S &= S_p + S_f + S_g \\ &= I^a + (e\dot{B}_2 - \dot{B}_1^*)/p = I^a + NCX/p \\ &= I^a + \{Y_1^{d*} - (ep^*/p)(C_2 + G_2 + J^d)\} + \{er^* B_2/p - rB_1^*/p\} \\ &= I^a + NX + NFX/p, \quad I^a = I + \dot{N} \end{aligned} \tag{37}$$

Module 7 models the dynamics of the wage-price sector with two Phillips-curves for nominal wage and price inflation, $\hat{w}$ and $\hat{p}$, in the place of only one (for price inflation). This sector represents a considerable generalization of many other formulations of wage-price inflation, e.g. of models which basically only employ cost-pressure forces on the market for goods or a single across markets price Phillips curve.

Since workers consume both the domestic product and the foreign one, we have to use a weighted average $\hat{p}_c$ of domestic and foreign price inflation as cost-pressure term in the money wage Phillips curve. This weighted average is shown in eq.n (39). Here and everywhere, the weight is assumed to be given by the steady state value of $\gamma(\eta)$, which is γ_o, and thus not allowed to vary with the real exchange rate η. Note here also that the foreign inflation rate is assumed to be steady. Forming a concept of medium run cost of living inflation as shown in eq.n (40) therefore demands for no change as far as foreign price inflation is concerned. With respect to medium run inflation at home we use – as in the case of the investment climate – a measure π^m that is updated in an adaptive fashion, measuring the inflationary climate in which current price inflation is

[9] We note that demand for foreign goods $Y_2^d = C_2 + J^d + G_2$ is well defined, but does not feed back into the domestic dynamics and can thus be neglected in their investigations. This demand is assumed to be always satisfied by the surrounding economies.

operating. The average in the money wage eq.n (38), with weight κ_w, finally assumes that cost of living pressure in this PC is given by a weighted average of current (perfectly anticipated) cost of living inflation and the inflationary climate into which this index is embedded. Due to the openness of the considered economy we therefore now cost of living indices in the money wage PC and this in a way that does not pay attention only to their current rate of inflation. Besides cost pressure we have furthermore based the PC (38) also on demand pressure $V - \bar{V}$ in the usual way, $\bar{V}$ the NAIRU rate of employment.

In the price Phillips curve we use as measure of demand pressure the rate of capacity utilization U^c in its deviation from the normal rate of capacity utilization $\bar{U}^c$. Cost pressure is here given by wage inflation and import price inflation where we again form a weighted average. For analytical simplicity, see the footnote below, we use as weight the same parameter as for the consumer price index in the wage PC (and thus a uniform way in which import price inflation enters cost-pressure expressions). Furthermore, the inflationary climate in which the price PC is operating is given by a corresponding weighted average of domestic inflationary climate and the foreign one, again with the general weight γ_o for reasons of simplicity. The weight γ_o is therefore uniformly applied and might – because of this – be reinterpreted as the general accepted measure by which domestic rates of inflation and foreign ones are translated into averages driving domestic wage and price inflation. More general concepts for such averaging procedures can easily be adopted from the numerical as well as the empirical perspective, for example by paying attention to the fact that the variable-input cost-structure is in fact given by $\frac{ep^*J^d+wL^d}{pY} = j/\eta + \omega/x$.[10] Note also that labor productivity growth $n_x = \hat{x}$ has to be and has been added to the wage and price PC in appropriate ways.

7. Wage-Price-Sector (adjustment equations and definitions):

$$\hat{w} = \beta_w(V - \bar{V}) + \kappa_w(\hat{p}_c + n_x) + (1 - \kappa_w)(\pi_c^m + n_x) \tag{38}$$

$$\hat{p} = \beta_p(U^c - \bar{U}^c) + \kappa_p\hat{c} + (1 - \kappa_p)\hat{c}^m \tag{39}$$

$$\hat{c} = \gamma_o(\hat{w} - n_x) + (1 - \gamma_o)\hat{p}_o^*, \quad \hat{c}^m = \gamma_o\pi^m + (1 - \gamma_o)\hat{p}_o^*$$

$$\hat{p}_c = \gamma_o\hat{p} + (1 - \gamma_o)\hat{p}_o^*, \quad \pi_c^m = \gamma_o\pi^m + (1 - \gamma_o)\hat{p}_o^* \tag{40}$$

[10] This variable input cost structure gives in fact rise to the same rate of growth expressions for c, c^m as they are employed in module 7., with the sole difference that the weight to be employed then is given by $(w/x)/(w/x + ep^*j)$ and is therefore in particular varying in time. As stated we however make use of the uniform weight γ_o in this paper throughout.

$$\dot{\pi}^m = \beta_{\pi^m}(\hat{p} - \pi^m) = \beta_{\pi^m}(\hat{p}_c - \pi_c^m) \quad (41)$$

$$\hat{p}_o^* = \bar{\pi} = const. \quad (42)$$

The remaining modules concern the openness of the economy. Since the exchange rate for Germany in the EU is a given magnitude we do not have to consider any Dornbusch type exchange rate dynamics in module 8 (which is thus void of content).

8. Exchange rate dynamics: Not existing

$$e = constant \quad (43)$$

Module 9, finally, describes the balance of payments Z. We first present real net exports NX (measured in terms of the domestic commodity) and then net capital exports (the export of liquidity) in nominal terms. Note here again that - though we specify all flows in and out of financial assets – they are not yet of relevance in the present model type, since interest and wealth effects are still suppressed in consumption behavior. Concerning nominal net interest payments,[11] we assume that they cross borders and thus appear as an item in the current account and in the balance of payments. We stress that the balance of payments must be balanced in our model, due to the assumptions to be made below concerning the flow restrictions of households, firms and the government. They essentially state that the new issue of money and equities are indeed (by assumption) absorbed by domestic households which means that the remainder of asset holders' savings goes into the purchase of domestic and foreign bonds (supplied by the government and foreigners in the amount necessary for flow consistency). Should domestic households demand more domestic bonds by their savings decision, these bonds are assumed to be supplied out of the stock, foreign asset holders hold, so that domestic households can always realize their concrete saving plans. Since new asset flows are regulated in this way we can show below that the balance of payments is always balanced (the current account is always the negative of the capital account) without any interference from the monetary authority due to the consistency assumptions made on money and equity issue. By contrast, the trade account need not be balanced even in the steady state, due to the fact that only domestic prices can adjust in the real exchange rate (which is too little to achieve a balanced trade account). There is therefore no need to intervene in foreign exchange markets of the part of the world that is here

[11] which are normally interpreted as net 'factor' exports NFX.

under consideration, if the foreign economy always supplies the flow amount of foreign bonds that is demanded by asset holders (and if it also supplies the flow of domestic bonds in the case the new issue of the government falls below what is demand by the domestic households[12]).

9. Balance of Payments:

$$NX = Ex - Im = Y_1^{d*} - (C_2 + G_2 + J^d)/\eta \tag{44}$$

$$NCX = e\dot{B}_2 - \dot{B}_1^*, \quad NFX = er^*B_2 - rB_1^* \tag{45}$$

$$\begin{aligned} Z &= pNX + NFX - NCX \\ &= \{pY_1^{d*} - ep^*(C_2 + G_2 + J^d)\} + \{er^*B_2 - rB_1^*\} - \{e\dot{B}_2 - \dot{B}_1^*\} \\ &= 0 \end{aligned} \tag{46}$$

Lastly, we collect the data needed from the 'foreign' economy. We already have assumed that inflation rates abroad are steady and fully anticipated, i.e. $\hat{p}_o^* = \pi_o^* = \bar{\pi} = const.$ We have also assumed that domestic monetary policy is not in conflict with this assumption, see module 4.[13] We assume finally for Y_1^{d*}, the demand of foreigners for the domestic good and thus for the export of the home country, that it is only a function of η if expressed per unit of capital, i.e.,

$$y_1^{d*} = Y_1^{d*}/K = y_1^{d*}(\eta) = \gamma_o^* + \gamma_1^*(\eta_o - \eta).$$

This closes the description of the equations of our macroeconomic growth dynamics of the large dominating open economy with under- or over-employment of labor and capital, with labor *and* goods-market in disequilibrium, but money market equilibrium and with a delayed adjustment of quantities as well as wages and prices. the reader is referred to the appendices for the consideration of the type of flow consistency that characterizes the considered economy in particular with respect to its balance of payments and also with to the pass-through effects that may hold with respect to foreign (and the exchange rate if it were allowed to vary).

[12] which moreover are indifferent between these two financial assets and therefore need not demand more than what is domestically supplied.

[13] We note that inflation is anchored by $\bar{\pi}$ and inflation in the rest of the countries, for example in the EU, while the nominal rate of interest is determined by domestic monetary policy of the dominant country.

3. Intensive form and steady state analysis

The extensive form model of section 2 can be reduced to an autonomous seven-dimensional dynamical system in the state variables $u = \omega/x$, the wage share, $l = xL/K$, the full employment output-capital ratio, $m = M/(pK)$, real balances per unit of capital, π^m, the inflationary climate, $y^e = Y^e/K$, sales expectations per unit of capital, $\nu = N/K$, inventories per unit of capital and finally ϵ^m the investment climate variable. The resulting system is set out in equations (47)–(53).

$$
\begin{aligned}
\hat{u} &= \kappa[(1-\gamma_o\kappa_p)\beta_w(V-\bar{V}) + (\gamma_o\kappa_w - 1)\beta_p(U^c - \bar{U}^c)] \\
&\quad + \kappa(\kappa_w - \kappa_p)\gamma_o(1-\gamma_o)(\hat{p}_o^* - \pi^m) && (47)\\
\hat{l} &= -i_1\epsilon^m - i_2(U^c - \bar{U}^c) && (48)\\
\hat{m} &= \hat{M} - \hat{K} - \hat{p} = \bar{\pi} + \hat{l} - \hat{p}, \quad m = \frac{M}{pK} \quad \text{or} && (49)\\
\hat{p} &= \kappa[\beta_p(U^c - \bar{U}^c) + \gamma_o\kappa_p\beta_w(V-\bar{V})] + \kappa(1+\gamma_o\kappa_p)(1-\gamma_o)(\hat{p}_o^* - \pi^m)] + \pi^m \\
\dot{\pi}^m &= \beta_{\pi^m}(\hat{p} - \pi^m) && (50)\\
\dot{y}^e &= \beta_{y^e}(y^d - y^e) + \hat{l}y^e && (51)\\
\dot{\nu} &= y - y^d - (n - \hat{l})\nu && (52)\\
\dot{\epsilon}^m &= \beta_{\epsilon^m}(\epsilon - \epsilon^m), \quad \epsilon = \rho^n + \hat{p} - (r + \xi) && (53)
\end{aligned}
$$

where output per unit of capital $y = Y/K$ and aggregate demand per unit of capital $y^d = Y^d/K$ are given by

$$
\begin{aligned}
y &= (1 + n\alpha_{n^d})y^e + \beta_n(\alpha_{n^d}y^e - \nu) && (54)\\
y^d &= c + g + i(\cdot) + y_1^{d*}(\eta) && (55)\\
&= \gamma(\eta)[(1-s_w)(uy - t_w) + (1-s_c)(\rho^e - t_c) + g] + y_1^{d*}(\eta) + i(\cdot) \text{ with} \\
i(\cdot) &= i_1\epsilon^m + i_2(U^c - \bar{U}^c) + n + \delta && (56)
\end{aligned}
$$

In the above we have employed the following abbreviations $V = y/l$, $U^c = y/y^p$, the employment rate and the rate of capacity utilization, $\rho^e = y^e - \delta - uy - jy/\eta$, the currently expected rate of profit, $\rho^n = y_o^d - \delta - uy_o - jy_o/\eta$, the normal rate of profit, $\epsilon = \rho^n + \hat{p} - (r + \xi)$ normal excess profitability, $r = r_0 + \frac{h_1 y - m}{h_2}$, the nominal rate of interest, $\eta = \frac{p}{ep_o^*} = \frac{m^* l}{m}, m^* = \frac{M}{ep^* xL} = const.$, the real exchange rate, determined by historical conditions, and $\kappa = (1 - \gamma_o^2 \kappa_w \kappa_p)^{-1}$.

With respect to the aggregate demand function y^d we have the partial derivatives (at the steady state):

$$\begin{aligned} y^d_{y^e} =& \gamma_o[(s_c - s_w)u_o(1 + n\alpha_{n^d}) + (1 - s_c)(1 - j(1 + n\alpha_{n^d})/\eta_o] + i_2(1 + n\alpha_{n^d})/y^p \\ &+ i_1\beta_p(\cdot)_{y^e} - i_1 r_{y^e} \\ y^d_\eta =& -\gamma_1[c_o + g] - \gamma_1^* + [\gamma_o(1 - s_c)]jy_o/\eta_o^2 + i_1 jy_o/\eta_o^2 \end{aligned}$$

in the case where $\epsilon = \epsilon^m$ holds true. In the case $\beta_\epsilon < \infty$, however, the i_1-terms have to be removed from these partial derivatives, since the influence of y^e, η on $i_1(\cdot)$ is then a delayed one. We assume throughout this paper that this latter case is characterized by $y^d_{y^e} < 1$, for $i_2 = 0$, and $y^d_\eta < 0$, which are natural assumptions from a Keynesian point of view. Note here that the parameters h_2 [due to $r_{y^e} = h_1(1 + n\alpha_{n^d})/h_2$], and β_p can be used in the case $\beta_\epsilon = \infty(\epsilon^m = \epsilon)$ to enforce either $y^d_{y^e} < 1$ for i_2 or $y^d_{y^e} > 1$, if this is desirable in certain more general situations. We also assume throughout the paper that the expected rate of profit ρ^e depends positively on the expected sales volume y^e close to the steady state.

This dynamical system represents in its first block (eqs. 47, 48) the real growth dynamics, describes with its second block (eqs. 49, 50) the nominal inflationary dynamics, provides thirdly (eqs. 51, 52) the inventory dynamics and lastly (eq.n 53) the adjustment of the investment climate.

Since prices concern the denominator in the real wage and wage share dynamics, the dependence of $\hat{u}$ on the rate of capacity utilization must obviously be negative, while the rate of utilization of the labor force acts positively on the real wage and the wage share dynamics. This law of motion can easily be derived from the wage and price PC of module 7 as was shown in an appendix to the preceding section.

Eq. (48) describes the evolution of the full employment output capital ratio $l = xL/K$ as determined by natural growth with rate n and investment per unit of capital $\hat{K} = I/K$, the latter also depending on trend growth in investment n,[14] on the real rate of return differential ϵ^m expected over the medium-run and on the state of excess demand in the market for goods as reflected by the term

[14] for reasons of simplicity, in order to avoid the addition of one further law of motion.

$U^c - \bar{U}^c$. Taken together, eqs. (47), (48), describe growth and income distribution dynamics in a way that closely related to the medium-run dynamics considered in Solow and Stiglitz (1968) and Malinvaud (1980). Its real origins are however in Rose's analysis of the employment cycle Rose (1967) and the extensions provided in Rose (1990).

The subdynamics (49), (50) are the monetary dynamics of our model and represents a general representation of Cagan (1956) type dynamics.

Eq. (51) describes the change in sales expectations as being governed by trend growth and by the observed expectational error (between aggregate demand y^d and expected sales y^e both per unit of capital). Similarly, eq. (52) states that actual inventories N change according to the discrepancy between actual output y and actual demand y^d (which in our Keynesian context is never rationed). These subdynamics represents an extension of Metzlerian ideas, see Franke and Lux (1993) and Metzler (1941) to a growing economy as in Franke (1996).

We stress that we want to keep and have kept the model as linear as possible, since we intend to concentrate on its intrinsic nonlinearities[15] at first (later again augmented by one basic extrinsic nonlinearities, but not yet by nonlinearities in the behavioral relationships).

This ends the description of the intensive form of our Keynesian monetary growth model, which exhibits sluggish adjustments of prices, wages and quantities (and corresponding to this the occurrence of over- or under-utilized labor and capital in the course of the business cycles that it generates).

Proposition 1

The dynamical system (47) – (53) has a unique interior steady state given by:

$$V_o = \bar{V}, \quad U_o^c = \bar{U}^c \tag{57}$$

$$y_o = \bar{U}^c y^p \quad l_o = y_o/\bar{V} \tag{58}$$

$$\pi_o^m = \bar{\pi} \quad = \hat{w}_o - n_x = \hat{p}_o = \hat{p}_o^* \tag{59}$$

[15] In view of the linear structure of the assumed technological and behavioral equations, the above presentation of our model shows that its nonlinearities are, on the one hand, due to the necessity of using growth laws in various equations and, on the other hand, to multiplicative expressions for some of the state variables of the form $uy, y/l$ and $\hat{l}y$. Though therefore intrinsically nonlinear of the kind of the Rössler and the Lorenz dynamical system, our 7D dynamics may, however, still be of a simple type, since these nonlinearities do not interact with all of its 7 equations and are thus not as densely related with each other as in the Rössler and the Lorenz dynamics.

$$y_o^e = y_o^d = y_o/(1+n\alpha_{n^d}), \quad \nu_o = \alpha_{n^d} y_o^e \tag{60}$$

$$r_o = \rho_o^n + \bar{\pi}, \quad m_o = h_1 y_o \tag{61}$$

$$\eta_o = m^* l_o / m_o \tag{62}$$

$$u_o = \frac{\gamma_o[-(1-s_w)t_w + (1-s_c)(y_o^e - \delta - \frac{jy_o}{\eta_o} - t_c) + g] + y_1^{d*}(\eta_o) + n + \delta}{(s_c - s_w)y_o} \tag{63}$$

$$\rho_o^n = \rho_o^e = y_o^e - \delta - u_o y_o - jy_o/\eta_o, \quad \epsilon_o = \epsilon_o^m = 0 \tag{64}$$

We assume throughout this paper that parameters are chosen such that all steady state values shown are economically meaningful. A plausible first condition into this direction is that $s_w < s_c$ holds true (which we assume to be always the case). We stress that $\eta_o = m^* l/m_o$, $m^* = \frac{M}{ep^* xL}$ is basically supply side determined and is in particular not related to goods market equilibrium conditions.

Proof:

Setting the laws of motion (47) – (53) equal to zero, and neglecting the zero solution of the employed growth laws, we get first of all:

$$\epsilon_o = \epsilon_o^m, y_o^d = y_o^e, \hat{p}_o = \pi_o^m = \bar{\pi} = \hat{p}_o^*.$$

The equations (47), (50) imply on this basis that the rates of utilization on the labor and the goods market must equal their normal rates, which in turn gives $y_o = \bar{U}^c y^p, l_o = y_o/\bar{V}$. Equation (52) implies $y_o = y_o^d + n\nu_o$ which taken together with

$$y_o = y_o^e(1+n\alpha_{n^d}) + \beta_n(\alpha_{n^d} y_o^e - \nu_o)$$

implies the steady state values for y^e, ν. Equation (48) then implies – on the basis of $U^c = \bar{U}^c$ – the equality $r_o = \rho_o^n + \bar{\pi}$ which in turn implies $\epsilon_o = \epsilon_o^m = 0$. The LM-equation $r = r_o + (h_1 y - m)/h_2$ then implies $m_0 = h_1 y_o$, which also provides us with the steady state value $\eta_o = m^* l_o/m_o$ of the real exchange rate. By the definition of ρ^n we furthermore have $\rho_o^n = \rho_o^e$. The steady value of the wage share u finally follows from the goods market equilibrium equation $y_o^e = y_o^d$ by solving this linear equation for u_o. ∎

The proposition 1 states that the steady state of the dynamics (47) – (53) is basically of supply-side nature. Income distribution is adjusted, however, such that the goods market clears which also provides the steady state value of the real rate of interest. Demand-side aspects thus only concern income distribution, i.e., the determination of the rate of profit, the wage share and the rate of interest and are therefore of secondary importance as far as the steady state behavior of the quantities of the considered dynamical model is concerned.

4. Feedback motivated stability analysis

As the model is formulated we can distinguish four important feedback chains which we here describe in isolation from each other. Of course, these feedback channels interact with each other in the full 7D dynamics and various of them can become dominant when model parameters are chosen appropriately.

1. The Keynes effect: We here assume $IS - LM$ equilibrium in order to explain this well-known effect in simple terms. According to LM equilibrium the nominal rate of interest r depends positively on the price level p. Aggregate demand and thus output and the rates of capacity utilization therefore depend negatively on the price level implying a negative dependence of the inflation rate on the level of prices through this channel. A high sensitivity of the nominal rate of interest with respect to the price level (a low parameter h_2, the opposite of the liquidity trap) thus should exercise a strong stabilizing influence on the dynamics of the price level (49) and on the economy as a whole, which is further strengthened if price and wage flexibility increase. [16]

2. The Mundell effect: We again assume $IS - LM$ equilibrium again in order to explain this less well-known (indeed often neglected) effect. Since net investment depends (as is usually assumed) positively on the expected rate of inflation π^m, aggregate demand and thus output and the rates of capacity utilization depend positively on this expected inflation rate (in the medium term). This implies a positive dependence of $\hat{p} - \pi^m$ on π^m and thus gives rise to a positive feedback from the expected rate of inflation on its time rate of change if β_p, β_w are chosen sufficiently large, see eq. (50). Faster adjustment speeds of inflationary expectations will therefore destabilize the economy in this situation.

[16] The same argument applies to Pigou or wealth effects which are however not present here.

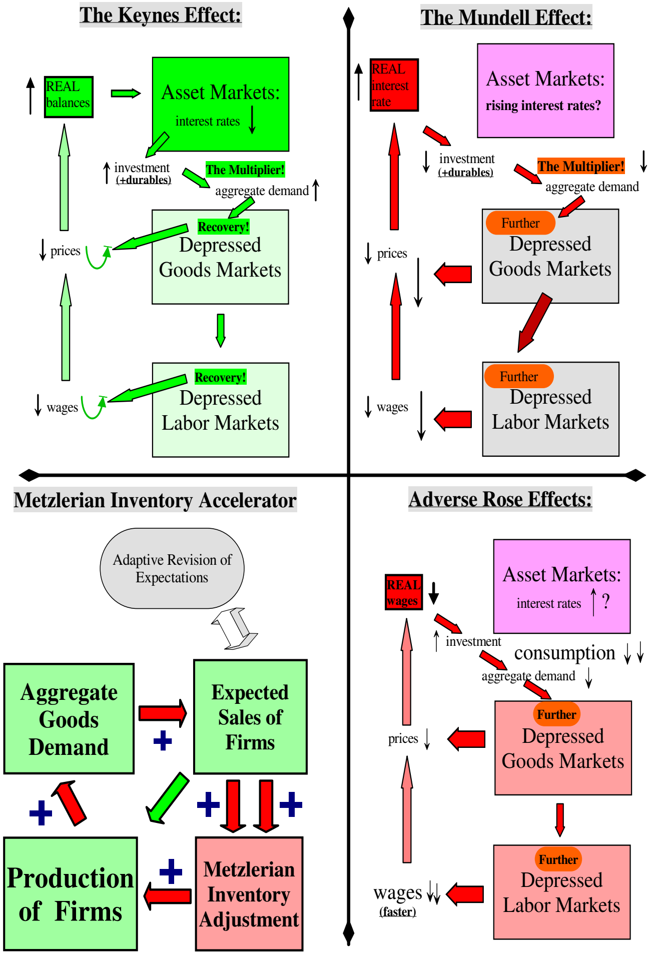

Figure 1: The feedback channels of the model.

3. The Metzler effect: As eq. (54) shows, output y depends positively on expected sales y^e and this the stronger the higher the speed of adjustment β_n of planned inventories. The time rate of change of expected sales in eq. (51) therefore depends positively on the level of expected sales when the parameter β_n is chosen sufficiently large. Flexible adjustment of inventories coupled with a high speed of adjustment of sales expectations thus lead to a loss of economic stability. There will, of course, exist other situations where an increase in the latter speed of adjustment may increase the stability of the dynamics.

4. The Rose effect: In order to explain this effect we assume again IS –

LM equilibrium and now also $\epsilon^m = \epsilon(\beta_{\epsilon^m} = \infty)$. We know from our above presentation of aggregate goods demand that output and in the same way the rate of employment and the rate of capacity utilization will depend positively or negatively on real wages, due to their opposite effects on the consumption of workers and on investment (and consumption out of profits). According to the law of motion for real wages (47) we thus get a positive or negative feedback effect of real wages on their rate of change, depending on the relative adjustment speed of nominal wages and prices. Either price or wage flexibility will therefore always be destabilizing, depending on investment and saving propensities, $i_1, s_c > s_w$ with respect to the expected rate of profit and workers' income. The destabilizing Rose effect (of whatever type) will be weak if both wage and price adjustment speeds β_w, β_p are low. The effects just discussed are summarized in figure 1.

Remark: Note with respect to these partial feedback channels that they are here – in comparison to the closed economy as considered in Chiarella and Flaschel (2000) and Chiarella, Flaschel, Groh and Semmler (2000) – enhanced by further price level effects that primarily concern exports and imports and thus affect aggregate demand through changes in the real exchange rate. This additional route should – as the Keynes effect – be a stabilizing one, since a rise in the domestic price level here reduces aggregate demand directly (in addition to the nominal interest rate channel) and thus provides a check to further increases in the level of prices. Furthermore, price level dynamics now in addition influence wage share dynamics (if $\kappa_w \neq \kappa_p$ holds true) via their delayed influence on the domestic inflationary climate which should exercise a stabilizing influence on the economy if wage earners are more short-sighted than firms (if $\kappa_w > \kappa_p$ holds true). This climate term in addition now plays a smaller role in domestic price inflation in comparison to the closed economy, since it is weakened here by an additional negative term in the reduced-form price Phillips curve. Finally, demand pressure forces are reduced in the present framework, since they are now being multiplied with the portion γ_o that represents the fraction of domestic demand for the domestic good in total domestic consumption demand.

This brief discussion of the basic 2D feedback mechanisms[17] in our full 6D

[17] Note that the present formulation of the model does not yet allow for the occurrence of Pigou effects or Fisher debt effects, where the first effect would strengthen the stabilizing potential of the Keynes effect and where the second one would be an addition to the destabilizing potential of the Mundell effect. The inclusion of these effects will give 'private wealth' an important role in the integrated dynamics and will also make the consideration of the government budget

dynamics (with $\epsilon^m = \epsilon, \quad \beta_{\epsilon^m} = \infty$) on balance suggests that increases in the speeds of adjustment of the dynamics will generally be bad for economic stability or viability. Exceptions to this rule are given by either wage or price flexibility and by the sales expectations mechanism, if inventories are adjusted sufficiently slowly. Of course, we do not have IS equilibrium in the full 6D dynamics as it was assumed above. This however simply means that the discussed effects work more indirectly, with some lag, due to the delayed interaction of aggregate demand, expected sales and output decisions. Mathematically speaking the above destabilizing effects will thus not always appear in the trace of the Jacobian of the system, but will be hidden somewhat in the principal minors that underlie the calculation of the Routh-Hurwitz conditions for local asymptotic stability.

Proposition 2

Assume that the parameters $\beta_w, \beta_p, \beta_n, \beta_{\pi^m}, h_2$ are chosen sufficiently small and the parameter β_{y^e} sufficiently large. Assume furthermore $\beta_{\epsilon^m} = \infty$, i.e., $\epsilon^m = \epsilon$. With respect to proposition 1 there then holds: The interior steady state of the reduced dynamics (47) – (52) is locally asymptotically stable.

Proof:

We start with the 4D subdynamics for u, m, y^e, l, isolated from the 2 remaining laws of motion by choosing $\beta_n, \beta_{\pi^m} = 0$ and by fixing π^m at $\pi_0^m = \hat{p}_o^*$ (where this variables then cannot depart from). The variable ν, by contrast, will move, but does not feed back into this 4D system if $\beta_n = 0$. This allows us to ignore the influences of π^m- and $\nu-$ related rows and columns in the 6D Jacobian at the steady state of the considered dynamics.

Let us first investigate the case where $\beta_w, \beta_p = 0$ holds in addition. For the coefficients $a_1, \ldots, a_4$ of the characteristic polynomial of the Jacobian J of the dynamics at the steady state we then get:

$$a_1 > 0, a_2 > 0, a_3 = 0, a_4 = 0,$$

if β_{y^e} is chosen sufficiently large and h_2 sufficiently small. By choosing h_2 sufficiently small we can indeed obtain the condition $y^d_{y^e} < 1$ for this partial derivative of the aggregate demand function y^d, since the term $-h_1(1+\alpha_{n^d})/h_2$ in this partial derivative will then dominate all other expressions in it. The coefficient $J_{55} = \beta_{y^e}(y^d_{y^e} - 1) + \hat{l}_{y^e} y^e_0$ in the trace of J will then be negative and in addi-

restraint a necessity.

tion dominate all other entries in the trace of J, if β_{y^e} is sufficiently large, i.e., $a_1 = -$ trace J is positive under these circumstances.

In the case $\beta_p, \beta_w = 0$, the 4D subsystem is in fact represented by the differential equations:[18]

$$\begin{aligned}
\hat{u} &= 0 \\
\hat{l} &= -i_1\epsilon - i_2(U^c - \bar{U}^c) \\
\hat{m} &= \hat{l} \\
\dot{y}^e &= \beta_{y^e}(y^d - y^e) + \hat{l}y^e
\end{aligned}$$

Only the identical combinations $\hat{l}, \dot{y}^e$ and $\hat{m}, \dot{y}^e$ will therefore give rise to positive principal minors of J of order 2, while all others must be zero. These two minors are in the presently considered situation identical and characterized by

$$\begin{vmatrix} \hat{l}_l & \hat{l}_{y^e} \\ \dot{y}^e_l & \dot{y}^e_{y^e} \end{vmatrix} = \begin{vmatrix} - & + \\ - & - \end{vmatrix} > 0,$$

since the investment terms in aggregate demand can be removed from the second row without changing this determinant and since $\hat{l}_{y^e}$ can be made positive by choosing h_2 in the investment function sufficiently small. Note here also that $\hat{l}$ and $\dot{y}^e$ both depend negatively on η and thus on l via $\eta = m^* l/m$. Due to this, we therefore get $a_2 > 0$, as asserted. The above 4D system finally immediately implies that a_3 and a_4 must both be zero.

We next show that a_3 becomes positive, while a_4 still stays zero, when β_p is made slightly positive. In this case the 4D dynamics reads

$$\begin{aligned}
\hat{u} &= -\kappa(1 - \gamma_0\kappa_w)\beta_p(U^c - \bar{U}^c) \\
\hat{l} &= -i_1\epsilon - i_2(y/y^p - \bar{U}^c) \\
\hat{m} &= -\kappa\beta_p(y/y^p - \bar{U}^c) + \hat{l} \\
\dot{y}^e &= \beta_{y^e}(y^d - y^e) + \hat{l}y^e
\end{aligned}$$

The law of motion for $\hat{m}$ is obviously a linear combination of those for $\hat{u}$ and $\hat{l}$, i.e., $a_4 = |J| = 0$ remains to be true. The principal minors of order 3 of the 4D matrix J are obtained by considering the following combinations of the above four differential equations:

$$1. : \hat{u}, \hat{l}, \hat{m}; \quad 2. : \hat{u}, \hat{l}, \dot{y}^e; \quad 3. : \hat{u}, \hat{m}, \dot{y}^e; \quad 4. : \hat{l}, \hat{m}, \dot{y}^e.$$

[18] $\epsilon = \rho^n + \hat{p} - (r + \xi)$!

In case 1. we get for their Jacobian J at the steady state: $|J| = 0$, just as for the 4D Jacobian.

In case 2. we can modify – without change in sign of the determinant of the corresponding Jacobian – the right–hand sides of the laws of motion for u, l, y^e to

$$\begin{array}{lcl} -U^c & & -y^e \\ -i_1(\cdot) & \text{and thus to} & +u - l, \\ c(\cdot) + y_1^{d*}(\cdot) - y^e & & +u - l \end{array}$$

due to the influence of $\eta = m^* l/m$ in both the investment function $i_1(\cdot)$ and the consumption function $c(\cdot)$ of workers and asset holders (and government consumption g). The sign of the determinant of the Jacobian of subsystem u, l, y^e at the steady state is therefore given by the sign of

$$\begin{vmatrix} 0 & 0 & - \\ + & - & 0 \\ + & - & 0 \end{vmatrix} = - \begin{vmatrix} + & - \\ + & - \end{vmatrix} = ?$$

and thus indeterminate. We note however that the determinant belonging to the subsystem u, l, y^e does not depend on h_2, since the first row of its Jacobian is of the form $(0, 0, -)$ and can thus be used to remove the terms obtained from $r = r_0 + \frac{h_1 y(y^e) - m}{h_2}$ the other rows without changing this principal minor.

In case 3., the combination $\hat{u}, \hat{m}, \dot{y}^e$, gives in the same way rise to the sequence of reduced dynamics, all with the same sign for their 3D minor:

$$\begin{array}{lclcl} -U^c & & -y^e & & -y^e \\ -i_1\epsilon & \rightarrow & -i_1\epsilon & \rightarrow & +u - m\,. \\ y^d - y^e & & c(\cdot) + y_1^{d*}(\cdot) & & +u + m \end{array}$$

Note here that the term '$-m$' is generated by making h_2 sufficiently small, since the r–component in ϵ will then dominate the term jy/η in ϵ which would imply '$+m$'. The sign of the considered minor is therefore in this case given by

$$\begin{vmatrix} 0 & 0 & - \\ + & - & 0 \\ + & + & 0 \end{vmatrix} = - \begin{vmatrix} + & - \\ + & - \end{vmatrix} < 0$$

Finally, in case 4.: $\hat{l}, \hat{m}, \dot{y}^e$, we get, when calculating the sign of the corresponding minor:

$$\begin{array}{lclcl} -i_1(\cdot) - i_2(\cdot) & & -i_1(\cdot) & & -m \\ -\beta_p(\cdot) & \rightarrow & -y^e & \rightarrow & -y^e \\ y^d - y^e & & c(\cdot) + y_1^{d*}(\cdot) & & -l \end{array}$$

and thus again

$$\begin{vmatrix} 0 & - & 0 \\ 0 & 0 & - \\ - & 0 & 0 \end{vmatrix} = - \begin{vmatrix} 0 & - \\ - & 0 \end{vmatrix} < 0$$

We note that the last two minors are linearly increasing functions of $1/h_2$, since they contain the expression $i_1 r_m = -i_1/h_2$ due to the partial differentiation with respect to m. They therefore can be made as negative as needed and thus will make the sum of the considered 4 minors negative and thus a_3 positive when h_2 is chosen sufficiently small.

We have by now arrived at the situation where $a_1, a_2, a_3 > 0, a_4 = 0$ and $a_1, a_2 - a_3 > 0$ holds if β_p is chosen sufficiently small, whereby a_3 is made as small as desirable. There remains to be shown that a_4 becomes positive for $\beta_w > 0$, since this implies the remaining Routh–Hurwitz condition

$$(a_1 a_2 - a_3) a_3 - a_1^2 a_4 > 0$$

if β_w and thus a_4 are chosen sufficiently small. We thus have that the interior steady state of the full 4D subsystem u, l, m, y^e is locally asymptotically stable for β_{y^e} sufficiently large, h_2 sufficiently small and β_p, β_w sufficiently small in addition.

In order to prove that $a_4 > 0$ holds, we have to determine again the sign of the determinant of the Jacobian of these full 4D subdynamics at the steady state. This is done in the meanwhile standard way by reducing the equations representing the laws of motion for the state variables u, l, m and y^e as follows:

$$\begin{array}{lclcl} +V - U^c & & V & & -l \\ i_1\epsilon - i_2 U^c & & -i_1\epsilon & & -m \\ & \rightarrow & & \rightarrow & \\ -V - U^c & & -U^c & & -y^e \\ c(\cdot) + y_1^{d*}(\cdot) - y^e & & c(\cdot) + y_1^{d*}(\cdot) & & u \end{array}$$

This gives

$$\text{sign } |J| = \begin{vmatrix} 0 & - & 0 & 0 \\ 0 & 0 & - & 0 \\ 0 & 0 & 0 & - \\ + & 0 & 0 & 0 \end{vmatrix} = + \begin{vmatrix} 0 & - & 0 \\ 0 & 0 & - \\ + & 0 & 0 \end{vmatrix} = + \begin{vmatrix} 0 & - \\ + & 0 \end{vmatrix} > 0$$

and thus $a_4 > 0$ as asserted.

By now we have thus proved local stability for the 6D system where π^m is frozen at its steady state value and where v does not yet feedback into the 4D subdynamics. In order to get local asymptotic stability for the full 6D dynamics we proceed in the usual way by making first β_n slightly positive and show that the corresponding 5D minor is negative. Thereafter we make β_{π^m} slightly positive and show that the resulting 6D minor is positive in sign. The eigenvalues corresponding to these two extensions were zero initially. They become negative in this stepwise fashion, when β_n and β_{π^m} are made slightly positive, since eigenvalues depend continuously on the parameters of the considered dynamics and since the determinant (of the Jacobian at the steady state) is given by the product of corresponding eigenvalues. We have shown that 4 eigenvalues have negative real parts under the stated assumptions. Making β_n slightly positive and knowing for the corresponding 5D minor $|J| < 0$ implies the negativity for λ_5, the fifth eigenvalue (with λ_6 still being zero). In the same way, making β_{π^m} slightly positive with $|J| < 0$ for the resulting 6D minor implies that λ_6 must become negative due to $|J| = \lambda_1\lambda_2\lambda_3\lambda_4\lambda_5\lambda_6$. There thus remains to be shown that $|J| < 0$ in the 5D case and $|J| > 0$ in the 6D case holds true.

Case $\beta_n > 0, \beta_{\pi^m} = 0$:

In order to determine the sign of the determinant of the 5D Jacobian of this 5D system of differential equations we proceed as usual and get (h_2 sufficiently small):

$$
\begin{array}{llllll}
\hat{U} = \ldots & & V - U^c & & V & \\
\hat{l} = \ldots & & -i_1(\cdot) - i_2(\cdot) & & -i_1(\cdot) & \\
\hat{m} = \ldots & \to & -V - U^c & \to & -U^c & \\
\dot{y}^e = \ldots & & y^d - y^e & & c(\cdot) + y_1^{d*}(\cdot) - y^e & \\
\dot{\nu} = \ldots & & y - y^d - n\nu & & y - y^e - n\nu &
\end{array}
$$

$$
\begin{array}{llllll}
& -l & & -l & & -l \\
& -i_1(\rho^n - r) & & +u - m & & -m \\
\to & -y \hat{=} - y^e + \nu & \to & -y^e & \to & -y^e \\
& c(\cdot) + y_1^{d*}(\cdot) - y^e & & u + m & & +u \\
& -y^e - n\nu & & -n\nu & & -\nu
\end{array}
$$

and thus as determinant

$$
\begin{vmatrix} 0 & - & 0 & 0 & 0 \\ 0 & 0 & - & 0 & 0 \\ 0 & 0 & 0 & - & 0 \\ + & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & - \end{vmatrix} = - \begin{vmatrix} 0 & - & 0 & 0 \\ 0 & 0 & - & 0 \\ 0 & 0 & 0 & - \\ + & 0 & 0 & 0 \end{vmatrix} = - \begin{vmatrix} 0 & - & 0 \\ 0 & 0 & - \\ + & 0 & 0 \end{vmatrix} = \begin{vmatrix} 0 & - \\ + & 0 \end{vmatrix} < 0.
$$

The fifth eigenvalue thus must be negative, when the first four exhibit negative real parts and the change in β_n is sufficiently small.

Case $\beta_n > 0, \beta_{\pi^m} > 0$:

This case is relatively straightforward, since $\hat{l}$ can be added to $\hat{m}$ and the result to the $\dot{\pi}^m$ law such that this law is modified to

$$
\dot{\pi}^m \quad \hat{=} \quad - \beta_{\pi^m}\pi^m - const.\pi^m \quad \hat{=} \quad - \pi^m
$$

with $const. > 0$ without change in the sign of the determinant that is considered. With this modified law we can remove all π^m entries in the 6D Jacobian of the full 6D dynamics up to $J_{66} < 0$ ($J_{16}, \ldots, J_{56} = 0$) and thus obtain from what is already known: $|J| > 0$. As before the eigenvalue λ_6 must become negative then, if $\beta_{\pi^m} = 0$ ($\lambda_6 = 0$) is made slightly positive. This closes the proof of proposition 2. ∎

Proposition 3

Assume that the import–output ratio of firms is such that $j < m^* l_0/(h_2 y_0) = m^*/(h_2 \bar{V})$ holds true. Then:

1. *The determinant of the Jacobian J at the steady state of the dynamics (47) – (52) is always positive (for all sizes of adjustment speed parameters).*
2. *The system can only lose stability by way of cycles, i.e., more specifically, by way of non-degenerate Hopf–bifurcations in general.*

Proof:

1. As usual, the laws of motion that constitute the dynamical system can be simplified in the following way without change in the sign of the considered determinant ($\eta = m^* l/m$):

$$\begin{array}{lll}
\hat{u}: & V - U^c & \to V & \to -l \\
\hat{l}: & -(i_1(\cdot) + i_2(\cdot)) & \to -i_1(\cdot) & \to +uy + jy/\eta - r(y,m) \\
\hat{m}: & -\hat{p} = -V - U^c & \to -U^c = -y/y^p & \to y^e \\
\dot{\pi}^m: & -\pi^m & \to -\pi^m & \to -\pi^m \\
\dot{y}^e: & y^d - y^e & \to c(\cdot) + y_1^{d*}(\cdot) & \to +u + m \\
\dot{\nu}: & y - y^e - n\nu & \to -y^e - n\nu & \to -\nu
\end{array}$$

Evaluated at the steady state the expression $jy/\eta - r(y,m), \eta = m^* l/m$ gives as partial derivative with respect to m

$$jy_0/(m^* l_0) - 1/h_2 = j\bar{V}/m^* - 1/h_2$$

which is negative under the assumption made in the proposition. In the $\hat{l}, \dot{y}^e$ rows we therefore get

$$\begin{array}{ll}
\hat{l}: & +u - m \quad \to -m \\
\dot{y}^e: & l + u + m \to +u
\end{array}$$

and thus again a single permutation of indices that determines $|J|$:

$$|J| = (-1)^2 \begin{vmatrix} 0 & - & 0 & 0 \\ 0 & 0 & - & 0 \\ 0 & 0 & 0 & - \\ + & 0 & 0 & 0 \end{vmatrix} = (-1)^6 \begin{vmatrix} 0 & - \\ + & 0 \end{vmatrix} > 0.$$

2. Obvious from what is known about Hopf–bifurcation. ■

Compared to the closed economy (where $j = 0$ holds) we thus have to put a restriction on this coefficient if we want to preserve the feature of the closed economy case. We note however that the determinant of the matrix of partial derivatives of $u_1(\cdot)$ and $c(\cdot) + y_1^{d*}(\cdot)$ with respect to u, m need only to be positive in order to ensure the stated results, which is a much weaker condition than the one used in the proposition.[19] Be that as if may, valuation effects on imported intermediate goods can become a problem for the assertion of proposition 3 if they become to pronounced. This problem can of course only arise if the assumptions of proposition 2 are invalidated.

Remark: For the full 7D dynamics, where the law of motion for ϵ^m is included ($\beta_{\epsilon^m} < \infty$) one can easily show, when the assumption of proposition 3 holds:

$$\begin{array}{lll}
\hat{u} & \to V & \to -l \\
\hat{l} & \to -i_1\epsilon^m & \to -\epsilon^m \\
\hat{m} & \to -U^c & \to -y^e \\
\dot{\pi}^m & \to -\pi^m & \to -\pi^m \\
\dot{y}^e & \to c(\cdot) + y_1^{d*}(\cdot) & \to +u + m \to +u \\
\dot{\nu} & \to y - y^e - n\nu & \to -\nu \\
\dot{\epsilon}^m & \to \epsilon & \to -u + m \to +m
\end{array}$$

Thus:

$$sign|J| = sign \begin{vmatrix} 0 & - & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & - \\ 0 & 0 & 0 & 0 & - & 0 & 0 \\ 0 & 0 & 0 & - & 0 & 0 & 0 \\ + & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & - & 0 \\ 0 & 0 & + & 0 & 0 & 0 & 0 \end{vmatrix}$$

[19] $(c(\cdot) + y_1^{d*}(\cdot))_\eta = -\gamma_1[c_0 + g] - \gamma_1^* + \gamma_0(1 - s_c)jy_0\eta_0^2$

$$\S PP = sign \begin{vmatrix} 0 & - & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & - \\ 0 & 0 & 0 & - & 0 \\ + & 0 & 0 & 0 & 0 \\ 0 & 0 & + & 0 & 0 \end{vmatrix} < 0$$

We thus get that a sluggish adjustment of ϵ^m (as well as one that is infinitely fast) will allow for proposition 2 for the full 7D (or the 6D) dynamics. Intermediate adjustment speeds may, however, give rise to problems with respect to local stability assertions (which – if true – demands for a rationalization).

5. Monetary policy rules for inflation targeting in the dominant country

Let us first consider the case where the monetary authority attempts to control the rate of inflation (and economic activity) by manipulating the growth rate μ of the money supply, here in the following way [20]

$$\dot{\mu} = \beta_{\mu_1}(\bar{\mu} - \mu) + \beta_{\mu_2}(\bar{\pi} - \hat{p}) + \beta_{\mu_3}(\bar{U}^c - U^c), \quad \bar{\pi} = \bar{\mu} - n = \hat{p}_0^*.$$

With this rule, the central bank attempts to steer the actual inflation rate $\hat{p}$ towards the target rate $\bar{\pi}$ by lowering the growth rate of money supply if $\hat{p}$ exceeds $\bar{\pi}$ (and vice versa). This restrictive policy is the stronger, the higher economic activity is at present, measured by the (negative of the) capacity utilization gap $U^c - \bar{U}^c$. In order to avoid too strong fluctuations in the growth rate of the money supply, there is also some smoothing of the fluctuations of μ measured by the adjustment parameter β_{μ_1}.

The immediate consequence of a changing growth rate μ of money supply M is that $m^* = \frac{M}{ep^*xL}$ is no longer constant in time, but now changing according to the law

$$\hat{m}^* = \mu - \hat{p}_0^* - n.$$

The 6D dynamics of proposition 2 is thus now 8 dimensional through the above adoption of a money supply rule, by the addition of the new state variables m^* and μ which influence the 6D dynamics through the real exchange rate $\eta =$

[20] choosing $\beta_{\mu_1} = \beta_{\mu_2} = \beta_{\mu_3} = 0$ and $\mu = \bar{\mu}$ leads us back to the 6D dynamics of proposition 2.

m^*l/m. This situation suggests that it may now be reasonable to use the state variable η in the place of m, since η is representing inflation more directly than $m = \frac{M}{pK}$ (where also capital accumulation is involved). We therefore now use the definition $m = m^*l/\eta$ in the place of $\eta = m^*l/m$ in the 6D dynamics of proposition 2, which enters these dynamics by way of the LM curve $r = r_0 + (h_1y - m)/h_2$ and make use of the law of motion for the real exchange rate η with $\hat{\eta}$ given by $\hat{p} - \hat{p}_o^*$ in the place of the law of motion for real balance per unit of capital m. The augmented 8D Jacobian resulting from these extensions and modifications can then be characterized as follows.

$$J = \left(\begin{array}{cccccc|cc}
J_{11} & & \ldots & & & J_{16} & 0 & 0 \\
\cdot & & & & & \cdot & - & 0 \\
\cdot & & & & & \cdot & 0 & 0 \\
\cdot & & & & & \cdot & 0 & 0 \\
\cdot & & & & & \cdot & + & 0 \\
J_{61} & & \ldots & & & J_{66} & + & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & + \\
 & -\hat{p} & & & -U^c & & 0 & -
\end{array}\right)$$

Proposition 4

1. Assume that $\beta_{\mu_2} = \beta_{\mu_3} = 0$ holds. Then: The eigenvalues $\lambda_1, \ldots, \lambda_6$ of the 6D dynamics are augmented by $\lambda_7 < 0, \lambda_8 = 0$ in the full 8D dynamical system.

2. The same holds true if $\beta_{\mu_2}, \beta_{\mu_3}$ are made slightly positive, i.e., for a fairly passive monetary rule.

Proof: Straightforward. ■

The considered 8D extension of the 6D dynamics of proposition 2 suffers however from a variety of weaknesses. As just stated it is subject to zero–root hysteresis, implying path–dependent convergence and steady state indeterminacy (with respect to m^*). With respect to the proof strategy of proposition 2 we moreover have that the starting subdynamics is now already fairly involved (6D!), since $\dot{y}^e$ now depends on m^*, l, η, u and μ. Finally, the ‘output gap’ (here the capacity utilization gap) is in fact two–times involved in the employed money supply rule, one time explicitly and one time implicitly as one of the determinants of the inflation gap.

All this suggest that this money supply rule is perhaps not the final solution to the monetary control of the dynamics under investigation. A first improvement of the employed rule is in fact given when the term $-\beta_{\mu_3} i(\cdot)$ is used in the place of $\beta_{\mu_3}(\bar{U}^c - U^c)$. Deviations of net investment I from the trend nK are here viewed to allow for a more restrictive monetary policy if positive than in the opposite case. In a Keynesian model, investment plays a central role in its dynamics and should therefore receive particular attention by the monetary authorities, in particular when the output or capacity gap is already paid attention to (via β_{μ_2}).

When this revised money supply rule is employed, it is easy to show that zero–root hysteresis vanishes and that a stable 6D dynamical system will give rise to stable 8D dynamics (including the m^*, μ dynamics) if $\beta_{\mu_1}, \beta_{\mu_2}$ are made slightly positive again. Proposition 4.1 continues to hold, but since we now have the influence of $-\hat{p}, \hat{l}$ in the 8th row of J, we get for

$$\begin{pmatrix} J_{77} \; J_{78} \\ J_{87} \; J_{88} \end{pmatrix} = \begin{pmatrix} 0 \; + \\ - \; - \end{pmatrix}$$

when $-\hat{p}, \hat{l}$ are removed from the last row by means of the $\hat{\eta}, \hat{l}$ laws of motion. $|J|$ is therefore now positive, implying eight eigenvalues with negative real parts if this holds for the 6D subdynamics and if $\beta_{\mu_2}, \beta_{\mu_3}$ are only slightly increased.

Due to the need to start stability investigations (as the ones of proposition 2) immediately with the 6D situations u, m, y^e, m^*, μ, l there remains however the problem that such money growth rules are much too indirect, unclear in their implications and also maybe problematic should the h_2–parameter in the LM curve $r - r_0 + (h_1 y - m)/h_2$ no longer be small enough. The application of money growth rules may therefore be problematic from the theoretical point of view.

We therefore now consider a Taylor interest rate policy rule of – in the light of the above experience – the following type:

$$\dot{r} = \beta_{r_1}(r_0 - r) + \beta_{r_2}(\hat{p} - \bar{\pi}) + \beta_{r_3} i(\cdot)$$

This rule states that a positive inflation gap $\hat{p} - \bar{\pi}$ is counteracted by an increase in the nominal rate of interest r (and vice versa) and this the stronger, the better the investment climate, measured by $i(\cdot) = I/K - n$, in fact is. There is again a smoothing term, here interest rate smoothing, that attempts to prevent too large fluctuations in the nominal rate of interest r. We here already stress that the inclusion of the investment climate into policy considerations will in fact help to remove adverse Rose effects from the local dynamics.

In the case of the above interest rate policy rule, we consider the dynamics $\hat{u}, \hat{l}, \dot{\pi}^m, \dot{y}^e, \dot{\nu}$ as provided by equations (47), (48), (50), (51), (52) with $\epsilon = \epsilon^m$ and the usual algebraic equations in addition, but now with

$$\begin{aligned}
&\hat{\eta} = \hat{p} - \hat{p}_0^* \text{ in the place of } \hat{m} \\
&\dot{r} = \beta_{r_1}(r_0 - r) + \beta_{r_2}(\hat{p} - \bar{\pi}) + \beta_{r_2} i(\cdot) \text{ in the place of } \hat{m}^*, \dot{\mu}
\end{aligned}$$

and $m = m^* l/\eta$ as an appended equation (or simply $m = h_1 y + h_2(r_0 - r)$).

Since we want to establish local asymptotic stability for a broader range of policy parameters β_{r_i} we start stability analysis by means of the following subdynamics, where we assume $\beta_w = \beta_n = \beta_{\pi^m} = 0 \quad (\pi^m \equiv \pi_0^m)$ initially in order to isolate these dynamics from the rest, represented by the state variables l, ν, π^m. The subdynamics to be investigated thus reads

$$\begin{aligned}
\hat{u} &= \kappa(\gamma_0 \kappa_w - 1)\beta_p(U^c - \bar{U}^c) \\
\hat{\eta} &= \kappa \beta_p (U^c - \bar{U}^c) + \pi_0^m \\
\dot{y}^e &= \beta_{y^e}(y^d - y^e) + \hat{l} y^e, \quad \hat{l} = -i(\cdot) \\
\dot{r} &= \beta_{r_1}(r_o - r) + \beta_{r_2}(\hat{p} - \bar{\pi}) + \beta_{r_2} i(\cdot)
\end{aligned}$$

Proposition 5

Assume that β_{r_2}, β_{r_3} are chosen sufficiently large. For the characteristic polynomial $\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4$ of the Jacobian J of these dynamics at the steady state there then holds

$$a_1, a_2, a_3 > 0, a_4 = 0, a_1 a_2 - a_3 > 0, (a_1 a_2 - a_3)a_3 - a_1^2 a_4 > 0$$

if β_{y^e} is chosen sufficiently large, β_p given, i.e., we have three eigenvalues with negative real part and one eigenvalue that is zero, according to the Routh–Hurwitz theorem on local asymptotic stability.

Proof:

We have

$$y^d_{y^e} = \gamma_0[(s_c - s_w)u_0(1 + n\alpha_{n^d}) + (1 - s_c) - j(1 + n\alpha_{n^d})/\eta_0 + i_2/y^p] < 1$$

by assumption and thus $y^d_{y^e} - 1 < 0$. Trace J can therefore be made as negative as desirable by choosing β_{y^e} sufficiently large, since the only other coefficient in trace J is given by $-\beta_{r_1}$.

For the sign structure of the Jacobian J we altogether have (β_{y^e} large enough):

$$J = \begin{pmatrix} 0 & 0 & - & 0 \\ 0 & 0 & + & 0 \\ \pm & - & - & - \\ 0 & 0 & + & - \end{pmatrix}$$

There is therefore one principal minor of order 2 in the formation of the coefficient a_2 that is problematic in sign ($J_{13}J_{31}$). We however have $J_{43} = \beta_{r_2}\kappa\beta_p/y^p + \beta_{r_3}i(\cdot)_{y^e} > 0$ and thus can dominate this problematic minor by choosing β_{r_2}, β_{r_3} sufficiently large (or β_p – and thus the Rose–effect – sufficiently small). In fact in order to have a stability statement for all β_{r_2}, β_{r_3} it is in fact preferable to chose β_p small, just as in proposition 2.

Considering the principal minors of order 3 we get from the above that two of them are negative, while all others are zero. The two negative ones are represented by:

$$\begin{vmatrix} 0 & + & 0 \\ - & - & - \\ 0 & + & - \end{vmatrix} < 0 \quad ; \quad \begin{vmatrix} 0 & - & 0 \\ \pm & - & - \\ 0 & + & - \end{vmatrix} < 0$$

We note that the latter determinant is negative, since $\beta_{r_3}i(\cdot)$ allows to remove the -sign from the $\pm$ entry. We thus get $a_3 > 0$.

Obviously, $a_4 = 0$ must hold true. There thus remains to be proved that $a_1a_2 - a_3$ holds true for all β_{r_i}'s. To prove this, it suffices to note that a_1, a_2 and a_3 all depend positively and linearly on β_{y^e}. This implies that $a_1a_2 - a_3$ is a positive quadratic function of β_{y^e} which must become positive if β_{y^e} is chosen sufficiently large. This ends the proof of proposition 5. ■

We stress that $\beta_{r_3}i(\cdot)$ was of importance in removing the $\pm$ ambiguity from the a_3 calculations. A policy rule with the capacity gap in the place of this investment climate term would not have been able to do this and would thus run into the danger of not taming the Rose effect effectively. We note also that all coefficients a_1, a_2, a_3 of the Routh–Hurwitz polynomial depend positively on $\beta_{r_1}, \beta_{r_2}, \beta_{r_3}$.

The adverse Rose effect (a negative dependence of aggregate demand y^d on u

via the investment term $i(\cdot)$, coupled with flexible prices and fixed wages: $\beta_w = 0$) endangers the validity of the Routh–Hurwitz conditions in two ways: via $J_{13}J_{31}$ in a_2 and – possibly – via the second of the 3D minors in a_3 as shown in the proof. Yet, due to $\beta_{r_3} i(\cdot)$ in the interest rate policy rule, the term $i(\cdot)$ can be removed from the 3D minor without changing its sign, i.e., this latter influence of an adverse Rose–effect is suppressed by the choice of the monetary policy rule. This however does not apply to the 2D minor containing the $J_{13}J_{31}$ expression, since the fourth row (the Taylor rule) is not involved in the formation of this minor. Yet, since $J_{33}J_{44} - J_{34}J_{43} > 0$ contains $\beta_{r_2} + \beta_{r_3}$ (in J_{43}), this latter 2D minor can be used to dominate the $J_{13}J_{31}$–term (in the case where J_{31} is negative) by choosing these interest rate policy parameters sufficiently large. The possibility of an unstable private sector can therefore be kept latent in the case of our choice of an interest rate policy rule.[21] This result holds for $\beta_w = \beta_n = \beta_{\pi^m} = 0$, i.e., for the neglectance of Rose effects based on wage flexibility, of Metzlerian accelerator instability and of accelerator effects in the price Phillips curve. The 4D subdynamics must therefore now be extended again in order to investigate whether stability can also be ensured for larger subdynamics or even the full 7D dynamics.

Proposition 6

We consider the situation of proposition 5, but now for $\beta_w > 0$ and with β_{r_1}, j chosen sufficiently small.

1. The state variable l then feeds back and enlarges the 4D dynamics of proposition 5 by one dimension to 5D.

2. The steady state of these 5D dynamics is locally asymptotically stable for β_w chosen sufficiently small, if it is stable in the limit $\beta_w = 0 \quad (\lambda_5 = 0)$.

Proof:

In the limit we have for the Routh–Hurwitz polynomial of the 5D Jacobian J the conditions $a_1, a_2, a_3 > 0, a_4, a_5 = 0$ and $a_1a_2 - a_3 > 0, (a_1a_2 - a_3)a_3 - a_1^2a_4 > 0$ and for the final combination of such Routh–Hurwitz coefficients again a positive value, due to $a_1a_2 - a_3, a_3 > 0$. If therefore suffices to show that a_4, a_5 become slightly positive if β_w is made slightly positive, since this will preserve the positivity of the combined Routh–Hurwitz terms.

For the determinant of the Jacobian J of the 5D dynamics we get at the steady state for $z = (u, \eta, y^e, r, l)'$ as reduced form the representation

[21] This does not hold true if the capacity gap is used in the place of $i(\cdot)$.

$$\begin{vmatrix} \partial\hat{u}/\partial z & \to \partial V/\partial z - \partial U^c/\partial z \\ \partial\hat{\eta}/\partial z & \to \partial V/\partial z + \partial U^c/\partial z \\ \partial(y^d - y^e)/\partial z & \to \partial(c(\cdot) + y_1^{d*}(\cdot))/\partial z \\ \partial\dot{r}/\partial z & \to (0,0,0,-1,0) \\ \partial\hat{l}/\partial z & \to -\partial(i_1(\cdot) + i_2(\cdot))/\partial z \end{vmatrix}$$

This representation of the sign of $|J|$ can again be further simplified without change in this sign to

$$-\begin{vmatrix} \partial V/\partial z \\ \partial U^c/\partial z \\ \partial(c(\cdot) + y_1^{d*}(\cdot)/\partial z \\ -\partial\rho^n/\partial z \end{vmatrix}$$

and from there to

$$-\begin{vmatrix} 0 & 0 & 0 & - \\ 0 & 0 & + & 0 \\ + & - & 0 & 0 \\ + & - & 0 & 0 \end{vmatrix} \overset{j=0}{=} \begin{vmatrix} 0 & 0 & 0 & - \\ 0 & 0 & + & 0 \\ + & - & 0 & 0 \\ + & 0 & 0 & 0 \end{vmatrix} > 0$$

We thus get $|J| < 0$ and therefore $a_5 > 0$ if the influence of intermediate imports is sufficiently weak.

The calculation of a_4 seems at first to be a formidable tasks, since it involves the determination of $|J|$ for the following set of subsystems of the 5D dynamics:

1.	2.	3.	4.	5.
u	u	u	u	η
η	η	η	y^e	y^e
y^e	y^e	l	l	l
l	r	r	r	r

Setting β_{r_1} equal to zero, however, makes minors 3. and 5. irrelevant, since the η, l–laws of motion can be used then to reduce the right–hand side of $\dot{r}$ to zero. Minor 2. also implies a zero determinant, since both $\hat{u}$ and $\hat{\eta}$ only depend on y^e in this case. Minor 1. can be obtained with respect to sign from

$$\begin{array}{lll}
\hat{u}:\ V & \to -l & \to -l \\
\hat{\eta}:\ U^c & \to +y^e & \to +y^e \\
\dot{y}^e: c(\cdot)+y_1^{d*}(\cdot) & \to +u-\eta & \to -\eta \\
\hat{l}:\ -i_1(\cdot) & \to +u & \to +u
\end{array}$$

i.e., by calculating

$$\begin{vmatrix} 0 & 0 & 0 & - \\ 0 & 0 & + & 0 \\ 0 & - & 0 & 0 \\ + & 0 & 0 & 0 \end{vmatrix} = (-1)^2 \quad \begin{vmatrix} 0 & 0 & + \\ 0 & - & 0 \\ + & 0 & 0 \end{vmatrix} = (-1)^2 > 0.$$

Minor 4., finally, can be obtained with respect to its sign from

$$\begin{array}{lll}
\hat{u}:\ V-U^c & \to V & \to -l \\
\dot{y}^e: c(\cdot)+y_1^{d*}(\cdot) & \to c(\cdot)+y_1^{d*}(\cdot) & \to +u \\
\hat{l}:\ -i(\cdot) & \to -i_1(\cdot) & \to +r \\
r:\ V+U^c & \to U^c & \to +y^e
\end{array}$$

which gives

$$\begin{vmatrix} 0 & 0 & - & 0 \\ + & 0 & 0 & 0 \\ 0 & 0 & 0 & + \\ 0 & + & 0 & 0 \end{vmatrix} = - \begin{vmatrix} + & 0 & 0 \\ 0 & 0 & + \\ 0 & + & 0 \end{vmatrix} = - \begin{vmatrix} 0 & + \\ + & 0 \end{vmatrix} > 0$$

Choosing j and β_{r_1} sufficiently small thus implies $a_4 > 0$ as asserted.

We thus have $a_4, a_5 > 0$ besides $a_1, a_2, a_3 > 0, a_1a_2 - a_3 > 0$ and get all further Routh–Hurwitz conditions validated if β_w is made positive (and not too large). We note that no restrictions were placed on the sizes of β_{r_2}, β_{r_3} which therefore can be chosen as desired by economic policy without making the 5D dynamics locally unstable. However, putting a brake on the evolution of r by making β_{r_1} large may endanger the stability of the steady state if interest rate smoothing becomes too strong. ∎

Proposition 7

Assume β_{r_1} positive. We enlarge the 5D dynamics by making β_{π^m} and then β_n slightly positive

1. *The Routh-Hurwitz polynomial coefficients a_6 and then a_7 become positive*
2. *All mixed Routh–Hurwitz conditions become positive due to small changes in their negative terms (formerly zero by assumption).*

We thus have that stability is preserved for small changes in β_{π^m} and β_n.

Proof:

1. Making use of $\hat{\eta}$, or $\hat{p}$, the law of motion for $\dot{\pi}^m$ can be reduced to $-\pi^m$ which implies $a_6 = |J| > 0$ if $|J|$ was negative for the 5D dynamics.
Similarly, reducing $\hat{\eta}$ to $+U^c$ or $+y = +y^e - \nu$ allows to reduce

$$\begin{aligned}\dot{\nu} &= y^d - y - (n - \hat{l})\nu \\ &\rightarrow y - y^e - n\nu \text{ (by means of } \dot{y}^e, \hat{l}) \\ &\rightarrow -y^e - n\nu \\ &\rightarrow -\nu\end{aligned}$$

which implies $|J| < 0$ ($a_7 > 0$) for the 7D case if the 6D case is characterized by $|J| > 0$.
2. Straightforward. ■

We thus in sum find local asymptotic stability if β_{y^e} is chosen sufficiently large, $\beta_p, \beta_w, \beta_{\pi^m}, \beta_n, \beta_{r_1}$ sufficiently small, β_{r_2}, β_{r_3} sufficiently large and j not too large.

Proposition 8

Assume $\beta_{r_1} > 0$. The determinant $|J|$ of the Jacobian of the full 7D system is always negative (for all sizes of the adjustment speed parameters in particular¿) if the coefficient j, characterizing intermediate imports, is sufficiently small.

Proof:

As usual, we can reduce the right–hand sides of the 7 differential equations to

$$\begin{array}{llll}\hat{u}: & V & \rightarrow -l & \rightarrow -l \\ \hat{l}: & -i_1(\cdot) & \rightarrow +u - \eta & \rightarrow +u \\ \hat{\eta}: & U^c & \rightarrow +y^e - r & \rightarrow +y^e \\ \dot{y}^e: & c(\cdot) + y_1^{d*}(\cdot) & \rightarrow +u - \eta & \rightarrow -\eta \\ \dot{\nu}: & -y^e - \nu & \rightarrow -\nu & \rightarrow -\nu \\ \dot{r}: & -r & \rightarrow -r & \rightarrow -r\end{array}$$

if $-\eta$ in the second row is made sufficiently small by choosing j sufficiently small. We thus get

$$\begin{vmatrix} 0 & - & 0 & 0 & 0 & 0 & 0 \\ + & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & + & 0 & 0 \\ 0 & 0 & 0 & - & 0 & 0 & 0 \\ 0 & 0 & - & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & - & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & - \end{vmatrix} = (-1)^3 \begin{vmatrix} 0 & - & 0 & 0 \\ + & 0 & 0 & 0 \\ 0 & 0 & 0 & + \\ 0 & 0 & - & 0 \end{vmatrix} > 0.$$

■

Corollary:

The steady state of the 7D dynamics can only lose or gain local asymptotic stability in a cyclical fashion, i.e. in general, by way of non-degenerate Hopf–bifurcations if j chosen sufficiently small.

We note here that the destabilizing Mundell–effect is more directly involved in the considered dynamics than, e.g., in Chiarella and Flaschel (2000, Ch.6). Investors have myopic perfect foresight with respect to price inflation $\hat{p}$. The investment function – as part of aggregate demand – in fact reads

$$i_1(\rho^n+\kappa[\beta_p(U^c-\bar{U}^c)+\gamma_0\kappa_p\beta_w(V-\bar{V})]+\kappa(1+\gamma_0\kappa_p)(1-\gamma_0)(\hat{p}_0^*-\pi^m)+\pi^m-r)+i_2(U^c-\bar{U}^c)+n$$

which implies $y^d_{y^e} > 1$ and thus an unstable dynamic multiplier process if β_p is sufficiently large. This process works independently of and in addition to the conventional Mundell–type indirect accelerator effect as described by the following feedback chain

$$\pi^m \uparrow: \rightarrow y^d \uparrow \rightarrow y^e \uparrow \rightarrow y \uparrow \rightarrow U^c, V \uparrow \dot{\pi}^m \uparrow$$

Note that the derivatives of the functions shown are all positive and thus give rise to positive feedback mechanisms throughout.

If β_p or β_{π^m} are sufficiently large it may be impossible for the interest rate policy rule to suppress their destabilizing potential completely, here abstracting still from further destabilizing processes such as the Rose real wage channel and the Metzler accelerator. The question arises what will keep the economy viable when the destabilizing feedback chains become dominant and make the interior steady state of the full 7D dynamics locally unstable.

6. Conclusions and Outlook

In this paper we have chosen a Keynesian disequilibrium framework for studying the role of monetary policy for a large country in a currency union – for example the German economy – under a pegged exchange rate system. Disequilibrium is allowed in the product and labor markets (with both sluggish price and quantity adjustments in the market for goods), whereas the financial markets are always cleared. Corresponding to the price and quantity adjustment processes, there are fluctuating rates of capacity utilization for both labor and capital. We have considered, in addition, inflationary expectations that are a combination of adaptive medium-run and perfect short-run forward looking ones and thus could allow for perfect foresight without being bothered by saddlepoint instabilities and the rational expectations solution adopted in such an environment. When, however, stability is lost in such a framework, by choosing the adjustment speeds of the model sufficiently large, the adoption of a kinked Phillips curve will restore economic viability in the large despite the local explosive nature of the dynamics around the steady state.

The main objective of the paper was to study, on the basis of such a model, the macroeconomic dynamics and the effects of recently discussed alternative monetary policy rules for economies not under flexible exchange rates – as for example, in Ball (1999) – but under a pegged exchange rate system. These policy rules are (1) the money supply rule or (2) direct inflation targeting by controlling the interest rate by the monetary authority. We considered the stability properties of the private sector of the economy and demonstrated the implication of those policy rules for overall macroeconomic stability. In view of what has been shown above we expect similar outcomes here, i.e., the adjustment speeds in the money growth and the interest rate policy rules must be chosen with sufficient care (in a limited domain) in order to give rise to damped fluctuations around the steady state of the economy. This in particular holds true in the case of an unstable private sector where such parameter may lie in a corridor (bounded away from zero) which is difficult to determine for the monetary authority. It may therefore be unknown whether increases or decreases in the parameters in from of the targets of the monetary authority are necessary in order to stabilize the persistent or explosive fluctuations generated by the private sector.

We have meanwhile estimated the model employing German macroeconomic time series data from 1970.1-1991.1 in Flaschel, Gong and Semmler (2003) and have studied the impulse-response functions for our macrodynamic model. Based on the estimation of the parameters, obtained partly from subsystems and partly from single equations, we obtained typical impulse response reactions from the Keynesian perspective from the high order dynamics under consideration. we intend furthermore to study, using VAR methodology, the proper comovements of the variables by employing either the money supply or the Taylor rule. The results so far obtained in Flaschel, Gong and Semmler (2003) largely confirm what one knows from other, low dimensional, VAR-studies. As we can also show there with respect to containing instabilities, the model variant with the Taylor feedback rule is superior in terms of stabilizing inflation rates and output. Yet, as shown in theoretical study in section 5 in this paper, choosing too strong policy reactions may again be destabilizing the economy.

7. Appendices

Appendix 1: Proof of flow consistency

We here consider and prove the following identities:

1. $S = I + \dot{N} + NCX/p$
2. $S = I + \dot{N} + NX + NFX/p,$ i.e.
3. $Z = NX + NFX/p - NCX/p = 0$

on the basis of the budget constraints provided in the modules on household, firm and government behavior. We first consider the relationships between real savings and its allocation to financial asset, and consider thereafter the sources of aggregate savings and its relationships to total investment and the current account. With respect to the definitions of NX, NFX, NCX the reader is referred to module 9. above. We stress that Y^d denotes the total demand for the domestically produced good and Y the domestic output of this commodity.
We have first

– –

$$\begin{aligned}
S_p &= Y_w^D + Y_c^D - C \\
&= s_w Y_w^D + s_c Y_c^D \\
&= (\dot{M} + \dot{B}_1 + e\dot{B}_2 + p_e \dot{E})/p \\
S_f &= Y_f = Y - Y^e = \mathcal{I} = I + \dot{N} - p_e \dot{E}/p \\
S_g &= T - rB/p - G = -(\dot{M} + \dot{B})/p
\end{aligned}$$

This implies:

$$\begin{aligned}
S &= S_p + S_f + S_g = I + \dot{N} + [e\dot{B}_2 - (\dot{B} - \dot{B}_1)]/p = I + \dot{N} + NCX/p \\
&= \text{Actual investment + Capital Account Balance}
\end{aligned}$$

– –

Secondly, there holds:

$$\begin{aligned}
S_p &= \omega L^d + \rho^e K - T + rB_1/p + er^* B_2/p - (C_1 + C_2/\eta) \\
&= Y^e - \delta K - T - J^d/\eta + rB_1/p + erB_2/p - C \\
S_f &= Y - Y^e
\end{aligned}$$

$$
\begin{aligned}
S_g &= T - rB/p - G \\
S &= S_p + S_f + S_g \\
&= Y - Y^d + Y^d - C - G - \delta K - J^d/\eta + er^* B_2/p - r(B - B_1)/p \\
&= \dot{N} + I + Y_1^{d*} - (C_2 + G_2 + J^d)/\eta + er^* B_2/p - r(B - B_1)/p \\
&= \text{Actual investment + Current Account Balance} \\
&= I + \dot{N} + NX + NFX/p
\end{aligned}
$$

– – – – –– – – – – – –– ––

The balance of payments is therefore always balanced without need of an intervention from the central bank, despite the assumption of a given nominal rate of exchange. This is due to the assumption that households always accept the new inflow of money and bonds resulting from the government budget constraint which establishes flow consistency as was just shown. The situation showing up in the balance of payments of our large open economy is thus still of a very tranquil type.

Appendix 2: Wage-price dynamics and foreign inflation (or exchange rate) pass-through:

The wage and price inflation curves (38), (39) can be reduced to two linear equations in the unknowns $\hat{w} - \pi^m - n_x, \hat{p} - \pi^m$:

$$
\begin{aligned}
\hat{w} - \pi^m - n_x &= \beta_w(V - \bar{V}) + (1 - \gamma_o)(\hat{p}_o^* - \pi^m) + \gamma_o \kappa_w (\hat{p} - \pi^m) \\
\hat{p} - \pi^m &= \beta_p(U^c - \bar{U}^c) + (1 - \gamma_o)(\hat{p}_o^* - \pi^m) + \gamma_o \kappa_p (\hat{w} - \pi^m - n_x)
\end{aligned}
$$

These equations can be easily solved and imply the following reduced form expressions for these two unknowns. Note that the reduced form expression now hide our assumption of myopic perfect foresight of wage earners and firms (on their corresponding cost-pressure item). It therefore appears as if in particular the reduced-form price PC purely backward looking which however is not true in the underlying structural equation. Note also that the reduced-form PC's all depend on the parameters of the wage-price modules in fairly mixed ways.

$$
\begin{aligned}
\hat{w} - \pi^m - n_x &= \kappa[\beta_w(V - \bar{V}) + (1 - \gamma_o)(\hat{p}_o^* - \pi^m) + \gamma_o \kappa_w[\beta_p(U^c - \bar{U}^c) + (1 - \gamma_o)(\hat{p}_o^* - \pi^m)]] \\
\hat{p} - \pi^m &= \kappa[\beta_p(U^c - \bar{U}^c) + (1 - \gamma_o)(\hat{p}_o^* - \pi^m) + \gamma_o \kappa_p[\beta_w(V - \bar{V}) + (1 - \gamma_o)(\hat{p}_o^* - \pi^m)]]
\end{aligned}
$$

These equations in turn immediately imply for the dynamics of the wage share u the following law of motion

$$\hat{u} = \kappa[(1-\gamma_o\kappa_p)\beta_w(V-\bar{V})+(\gamma_o\kappa_w-1)\beta_p(U^c-\bar{U}^c)]+\kappa(\kappa_w-\kappa_p)\gamma_o(1-\gamma_o)(\hat{p}_o^*-\pi^m)$$

where κ is given by$1/(1-\gamma_o^2\kappa_w\kappa_p)$ which is always well-defined and positive if $\gamma_o < 1$ holds true. The second equation also provides the reduced-form law of motion for the deviation of domestic price inflation from its medium-run level which has to be inserted into the law of motion for real balances per unit of capital and the inflationary climate expression in the following derivation of the interacting laws of motion of the intensive form of the model.

The above reduced-form expressions for both wage and price inflation show that there is more than complete pass-through of foreign price inflation or (if this happens) of a devaluation of the domestic currency (in which case one has to use $\hat{e}+\hat{p}_o^*$ in the place of only $\hat{p}_o^*$) to consumer price inflation. Here, if demand pressures on the market for goods and for labor are considered as given magnitudes. In the case of domestic price inflation (wage inflation) we have as term in front of the term $\hat{p}_o^*(+\hat{e})$ the fraction

$$0 < \alpha_p = \frac{1+\kappa_p\gamma_o}{1-\kappa_p\gamma_o\kappa_w\gamma_o} < 1, \quad (0 < \alpha_w = \frac{1+\kappa_w\gamma_o}{1-\kappa_w\gamma_o\kappa_p\gamma_o} < 1).$$

It can be shown by taking first derivatives, i.e., by demonstrating that import price inflation is passed through to domestic price inflation (wage inflation) the weaker, the smaller the corresponding weight κ_p (κ_w) becomes. The pass-through effect is therefore the strongest for $\kappa_p = 1 (\kappa_w = 1)$. where it obviously is smaller than one.

We thus have that the consumer price level rises by more than just $(1-\gamma_o)\hat{p}_c^*$, since domestic inflation is also increasing due to the import-cost pressure experienced by firms. Furthermore, also wage inflation is increasing and thus adds to this cost pressure via domestic factor-price increases. Finally, also the wage share here depends on foreign price inflation, positively if $\kappa_w > \kappa_p$ and negatively if the reverse inequality holds.

As the model is formulated we have straightforward quantity effects of the real exchange rate through their impact effects on the domestic demand for the foreign good and through their impact on exports. Furthermore, wage and price dynamics (as well as the dynamics of income distribution, the latter for $\kappa_w \neq \kappa_p$) is changed through cost pressure effects on wage earners and firms (and the inflationary expectations this gives rise to). Finally. we also have that profitability, interest and investment also depend import price dynamics and imported inflation, These however are all channels by which the dynamics of the domestic economy is modified through the foreign rate of inflation (or the exchange rate should it become variable). These feedback links from the foreign economies to the domestic one are most easily seen in the intensive form presentation of the model that is provided in the next section.[22]

Appendix 3: Notation

Note here that we use $*$ to denote foreign activities and an index 2 for foreign commodities. Furthermore an index w refers to worker households and index c to pure asset holders. The index d stands for demand, while the corresponding supply symbol does not have an index at all (in order to simplify notation). Finally, we use β_x expressions to denote the speed of adjustment of a variable x.

A. Statically or dynamically endogenous variables:

Y	Output
Y^e	Expected sales
Y_w^D, Y_c^D	Disposable income of workers and asset-holders
L^d	Employment
J^d	Imports
V	Rate of employment ($\bar{V}$ NAIRU rate)
U^c	Rate of capacity utilization ($\bar{U}^c$ NAIRU rate)
C_1	Consumption of the domestic good (index 1: good originates from country 1 = domestic economy)
$C_2 \geq 0$	Consumption of the foreign good (index 2: good originates from country 2 = foreign economy)
I	Intended (= realized) fixed business investment
$\mathcal{I} \geq 0$	Planned inventory investment (existing stock = N)
I^a	Total investment (including actual inventory changes)
$I^a = I + \dot{N}$	Actual total investment
r	Nominal rate of interest (price of bonds $p_b = 1$)
p_e	Price of equities
$S = S_p + S_f + S_g$	Total savings
S_p	Private savings
S_f	Savings of firms (= Y_f, the income of firms)
S_g	Government savings

[22] Note here finally that we always have $\hat{p}_c = \hat{p} + (1 - \gamma_o)\hat{\eta}$ by definition.

$T = T^w + T^c$	Real taxes
G	Government expenditure
ρ^e	Expected rate of profit (before taxes)
$V = L^d/L$	Rate of employment ($\bar{V}$ the employment–complement of the NAIRU)
K	Capital stock
w	Nominal wages
p	Price level
p_c	Consumers' price index
π^m	Inflationary climate for domestic prices
π^m_c	Inflationary climate for consumer prices
p_b	Price of domestic bonds
p_e	Price of domestic equities (not traded)
e	Exchange rate (units of domestic currency per unit of foreign currency)
ϵ	Expected excess profitability $(\rho^e + \hat{p} - (r + \xi))$
ϵ^m	Investment climate expression
M	Money supply (index d: demand, growth rate μ_0)
L	Labor supply
B	Domestic bonds, of which B_1 and B_1^* are held by domestic and foreign asset-holders, respectively (index d: demand)
B^*	Foreign bonds, of which B_2 and B_2^* are held by domestic and foreign asset-holders, respectively (index d: demand)
E	Equities (index d: demand)
W	Real domestic wealth
ω	Real wage ($u = \omega/x$ the wage share)
$\Delta Y^e = Y^e - Y^d$	Expectations error on the goods market
$Ex \geq 0$	Exports in terms of the domestic good
$Im \geq 0$	Imports in terms of the domestic good
$NX = Ex - Im$	Net exports in terms of the domestic good
NFX	Net factor export payments
NCX	Net capital exports
Z	Surplus in the balance of payments
$\eta = p/(ep^*)$	Real exchange rate (measured in Goods*/Goods)
$nx = NX/K$	Net exports per unit of capital

B. Basic parameters of the model:

δ	Depreciation rate
ξ	Risk premium rate

i_1, i_2	Investment parameters
h_1, h_2	Money demand parameters
$n = n_l + n_x$	Natural growth rate (including productivity growth)
μ	Steady growth rate of money supply
β_w	Wage adjustment parameter
β_p	Price adjustment parameter
β_{π^m}	Inflation climate adjustment speed
β_{ϵ^m}	Investment climate adjustment speed
β_{y^e}	Sales expectations adjustment speed
β_n	Inventory adjustment speed
α_{n^d}	Inventory-sales ratio
$\kappa_w, \kappa_p \in [0,1]$	Weights of short– and long–run inflation ($\kappa_w \kappa_p \neq 1$)
y^p	Output–capital ratio
x	Output–labor ratio ($\hat{x} = n_x$)
t^w, t^c	Tax policy parameters
$s_c \in [0,1]$	Savings–rate (pure asset-holders)
$s_w \in [0,1]$	Savings–rate (workers)
γ_o	Share of the domestic good in domestic consumption
$g = G/K$	Fiscal policy parameter
$j = J^d/Y$	fixed import-output ratio

C. Further notation

$\dot{x}$	Time derivative of a variable x
$\hat{x}$	Growth rate of x
y', y_x	Total and partial derivatives
$r_o, etc.$	Steady state values ($\bar{r}$ parameters which may differ from r_o)
$y = Y/K, etc.$	Real variables in intensive form
$m = M/(pK), etc.$	Nominal variables in intensive form
$\nu = N/K$	Inventory-capital ratio
$\bar{\pi}$	Inflation target of the central bank

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