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Anthropogenic climate change, abatement and economic growth in a multi-region world

by

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Abstract

This paper studies the effects of global warming in a descriptive model of endogenous growth with multiple regions. It is assumed that deviations from the pre-industrial greenhouse gas concentration, which implies a change in the global surface temperature, negatively affect aggregate output and the marginal product of capital. The paper derives optimal abatement ratios in the non-cooperative world and for the cooperative case assuming that the growth rate is an endogenous variable. Further, the cooperative situation is compared to the outcome resulting when abatement shares are set such that marginal damages in each regions are equal.

Keywords: climate change, greenhouse gases, economic growth, abatement, multi-region world
JEL: E60, O41, Q28

1 Introduction

According to the Intergovernmental Panel on Climate Change (IPCC) it is very likely[1] that the 1990s was the warmest decade and 1998 the warmest year since 1861 (IPCC, 2001, p. 26) and the warming of the earth still continues. The reason for this phenomenon is emission of greenhouse gases (GHGs), like carbon dioxide (CO_2) or methane (CH_4), which has drastically increased in the 20th century and still continues to rise leading to higher concentrations of GHGs in the atmosphere. Higher GHG concentrations generate a rise in the average global surface temperature and make extreme weather events more likely. Further, it is likely that statistically significant increases in heavy and extreme weather events have occurred in many mid- and high latitude areas, primarily in the Northern Hemisphere.

In the economics literature numerous studies study the impact of environmental degradation on economic growth using endogenous growth models (for a survey see e.g. Smulders, 1995, or Hettich, 2000). Generally, these studies are rather abstract because they intend to derive general results. It is assumed that economic activities lead to environmental degradation and, as a consequence, reduce utility and/or production possibilities. The goal of these studies often is to analyze how public policy affects environmental conditions as well as the growth rate and welfare of economies.

However, as far as I know there exist only few economic studies which incorporate climate models in a growth model and study the effects of GHG emissions on the growth rate of economies. Instead, economic studies dealing with global warming are mostly cost benefit analysis which take the growth rate of economies as an exogenous variable. These studies then compute the discounted cost of reducing GHG emissions and confront them with the discounted benefit of a lower increase in GHG concentration and, as a consequence, of a smaller increase in average global surface temperature (see e.g. Nordhaus, 2000, or Tol, 2001, and for a survey IPCC, 1996).[2]

A great problem in studying the economic consequences of global warming is the uncertainty

[1]Very likely (likely) means that the level of confidence is between $90 - 99$ ($66 - 90$) percent.

[2]We do not go into the details of these studies. The interested reader is referred to the IPCC report (see IPCC, 1996).

as concerns the damages caused by a change of the earth climate. Nevertheless, there are analysis doing this. For example, the IPCC estimates that a doubling of CO_2, which goes along with an increase of global average surface temperature between 1.5 and 4.5 degree Celsius, reduces world GDP by 1.5 to 2 percent (see IPCC, 1996, p. 218). This damage is obtained for the economy in steady state and comprises both market and nonmarket impacts. Nonmarket impacts are direct reductions of people's welfare resulting from a climate change.

In this paper we intend to integrate a simple climate model in a descriptive model of endogenous growth in order to analyze the effects of GHG emissions and of abatement policies on economic growth for different regions of the world. As to the economic model we assume that the propensity to consume is given and makes a certain fraction of GDP. We then derive the optimal abatement ratio both for the non-cooperative and for the cooperative world. Further, we compare our results to what is obtained when the marginal damages equal marginal costs of abatement in a static framework.

The rest of the paper is organized as follows. In section 2 we present our general descriptive growth model. Further, we model GHG emissions and changes in average surface temperature using a simple energy balance model (EBM). Section 3 studies optimal abatement spending and the resulting growth rates in a non-cooperative world. Section 4 analyzes the cooperative world and section 5 compares our results with a world where instantaneous marginal damages equal marginal cost of abatement. Section 6, finally, concludes the paper.

2 A descriptive model of endogenous growth

We assume that aggregate production in region i, $i = 1, ..., n$, takes place according to the following per capita production function

$$Y_i(t) = A_i K_i(t) D_i(M(t) - M_o), \tag{1}$$

with $Y_i(t)$ per capita production in region i, A_i a positive constant, $K_i(t)$ a composite of human and physical capital. $D_i(M(t) - M_o)$ is the damage function giving the damage resulting from deviations of actual GHG concentration from the pre-industrial concentration, M_o. It should be mentioned that the assumption of a continuous damage function is only justified provided the increase in GHGs does not exceed a certain threshold. This holds because for higher increases of the GHGs catastrophic events may occur going along with extremely high economic costs which are difficult to estimate. Just one example is the break down of the Gulf Stream which would dramatically change the climate in Europe. Therefore, the analysis assuming a damage function only makes sense for increases of GHGs within certain bounds.

As to the damage function $D_i(M - M_o)$ we assume that it is C^2 and satisfies[3]

$$D_i(M - M_o) \begin{cases} = 1, & \text{for } M = M_o \\ < 1, & \text{for } M \neq M_o, \end{cases} \tag{2}$$

with derivative

$$\frac{\partial D_i(\cdot)}{\partial M} \equiv D_i'(\cdot) \begin{cases} > 0, & \text{for } M < M_o \\ < 0, & \text{for } M > M_o. \end{cases} \tag{3}$$

The per capita capital accumulation function is given by[4]

$$\dot{K}_i = A_i K_i D_i(\cdot)(1 - c_i - \tau_{B,i}) - (\delta_i + n_i)K_i, \tag{4}$$

with c_i the consumption share in region i and $\tau_{B,i}$ the abatement share. $n_i \in (0,1)$ is the population growth rate in region i and $\delta_i \in (0,1)$ is the depreciation rate of capital.

As concerns emissions of GHGs we assume that these are a by-product of production and expressed in CO_2 equivalents. So, emissions are a function of per capita output relative to per capita abatement activities. This implies that a higher production goes along with higher emissions for a given level of abatement spending. This assumption is frequently encountered in environmental economics (see e.g. Smulders, 1995). It should also be mentioned that the emission of GHGs does not affect production directly but only indirectly by raising the concentration of GHGs in the atmosphere which affects the climate of the Earth and which leads to a higher surface temperature and to more extreme weather situations. Formally, emissions in region i are described by

$$E_i = \left(\frac{a_i Y_i}{B_i}\right)^{\gamma_i} = \left(\frac{a_i}{\tau_{B,i}}\right)^{\gamma_i}, \tag{5}$$

[3]In the following we delete the time argument t.

[4]The dot over a variable gives the derivative with respect to time.

with B_i per capita abatement with $B_i = \tau_{B,i} Y_i$. $\gamma_i > 0$ and $a_i > 0$ are positive constants. The parameter a_i can be interpreted as a technology index describing how polluting a given technology is. For large values of a_i a given production (and abatement) goes along with high emissions implying a relatively polluting technology and vice versa.

Next, we describe the interrelation between economic activities and the change in the average global surface temperature. The simplest method of considering the climate system of the earth is in terms of its global energy balance which is done by so-called energy balance models (EBM). According to an EBM the change in the average surface temperature on earth is described by[5]

$$\frac{dT(t)}{dt}\, c_h \equiv \dot{T}(t)\, c_h = S_E - H_E(t) - F_N(t) + \beta_1\,(1-\xi)\, 6.3 \ln\frac{M}{M_o},\ T(0) = T_0, \qquad (6)$$

with $T(t)$ the average global surface temperature measured in Kelvin[6] (K), c_h the heat capacity[7] of the earth with dimension $J\, m^{-2}\, K^{-1}$ (Joule per square meter per Kelvin)[8] which is considered a constant parameter, S_E is the solar input, $H_E(t)$ is the nonradiative energy flow, and $F_N(t) = F\uparrow(t) - F\downarrow(t)$ is the difference between the outgoing radiative flux and the incoming radiative flux. S_E, $H_E(t)$ and $F_N(t)$ have the dimension Watt per square meter (Wm^{-2}). $F\uparrow$ follows the Stefan-Boltzmann-Gesetz which is

$$F\uparrow = \epsilon\, \sigma_T\, T^4, \qquad (7)$$

with ϵ the emissivity which gives the ratio of actual emission to blackbody emission. Blackbodies are objects which emit the maximum amount of radiation and which have $\epsilon = 1$. For the earth ϵ can be set to $\epsilon = 0.95$. σ_T is the Stefan-Boltzmann constant which is given by $\sigma_T = 5.67\, 10^{-8}\, Wm^{-2}K^{-4}$. Further, the ratio $F\uparrow / F\downarrow$ is given by $F\uparrow / F\downarrow = 109/88$. The difference $S_E - H_E$ can be written as $S_E - H_E = Q(1-\alpha_1)\alpha_2/4$, with $Q = 1367.5 Wm^{-2}$ the solar constant, $\alpha_1 = 0.3$ the planetary albedo, determining how much of the incoming energy is reflected by

[5]This part follows Roedel, 2001, chap. 10.2.1 and chap. 1. See also Henderson, 1987, and Gassmann, 1992. A more complex presentation can be found in Harvey, 2000.

[6]273 Kelvin are 0 degree Celsius.

[7]The heat capacity is the amount of heat that needs to be added per square meter of horizontal area to raise the surface temperature of the reservoir by 1K.

[8]1 Watt is 1 Joule per second.

the atmosphere and $\alpha_2 = 0.3$ captures the fact that a part of the energy is absorbed by the surface of the Earth.

The effect of emitting GHGs is to raise the concentration of GHGs in the atmosphere which increases the greenhouse effect of the Earth. This is done by calculating the so-called radiative forcing which is a measure of the influence a GHG, like CO_2 or CH_4, has on changing the balance of incoming and outgoing energy in the Earth-atmosphere system. The dimension of the radiative forcing is Wm^{-2}. For example, for CO_2 the radiative forcing, which we denote as F, is given by

$$F \equiv 6.3 \ln(M/M_o), \tag{8}$$

with M the actual CO_2 concentration, M_o the pre-industrial CO_2 concentration and ln the natural logarithm (see IPCC, 2001, p. 52-53).[9] For other GHGs other formulas can be given describing their respective radiative forcing and these values can be converted in CO_2 equivalents. β_1 is a feedback factor which captures the fact that a higher CO_2 concentration affects for example atmospheric water vapour which has effects for the surface temperature on Earth. β_1 is assumed to take values between 1.1 and 3.4. The parameter ξ, finally, captures the fact that $\xi = 0.3$ of the warmth generated by the greenhouse effect is absorbed by the oceans which transport the heat from upper layers to the deep sea. In equilibrium, i.e. for $\dot{T} = 0$, (6) gives a surface temperature of about 288.4 Kelvin which is about 15 degree Celsius for the pre-industrial GHG concentration, i.e. for $M = M_o$.

The heat capacity of the Earth, c_h, is largely determined by the oceans since most of the Earth's surface is covered by seawater. Therefore, the heat capacity of the oceans is used as a proxy for that of the earth. c_h is then given by $c_h = \rho_w \, c_w \, d \, 0.7$, with ρ_w the density of seawater ($1027 \, m^{-3} \, kg$), c_w the specific heat of water ($4186 \, J \, kg^{-1} \, K^{-1}$) and d the depth of the mixed layer which is set to 70 meters. The constant 0.7 results from the fact that 70 percent of the Earth are covered with seawater. Inserting the numerical values, assuming a depth of 70 meters and dividing by the surface of the earth gives $c_h = 0.1497$.

Setting $\beta_1 = 1.1$ and assuming a doubling of CO_2 implies that in equilibrium the average surface temperature rises from 288.4 to 291.7 Kelvin, implying a rise of about 3.3 degree Celsius.

[9] The CO_2 concentration is given in parts per million (ppm).

This is in the range of IPCC estimates[10] which yield increases between 1.5 and 4.5 degree Celsius as a consequence of a doubling CO_2 concentration (IPCC, 2001, p. 67).

Summarizing this discussion the EBM can be rewritten as

$$\dot{T}(t)\, c_h = \frac{1367.5}{4}\, 0.21 - 0.95\, \left(5.67\, 10^{-8}\right) (21/109)\, T^4 + 4.851 \ln \frac{M}{M_o},\; T(0) = T_0. \tag{9}$$

The concentration of GHGs, M, evolves according to the following differential equation

$$\dot{M} = \beta_2 \sum_{i=1}^{n} E_i - \mu M, M(0) = M_0. \tag{10}$$

where μ is the inverse of the atmospheric lifetime of CO_2. As to the parameter μ we assume a value of $\mu = 0.1$.[11] β_2 captures the fact that a certain part of GHG emissions are taken up by oceans and do not enter the atmosphere. According to IPCC $\beta_2 = 0.49$ for the time period 1990 to 1999 for CO_2 emissions (IPCC, 2001, p. 39).

The economy is completely described by equations (4), (9) and (10), with emissions given by (5).

3 The non-cooperative world

In this section we analyze the non-cooperative world or the Nash equilibrium. Each region maximizes utility resulting from per capita consumption where we assume a logarithmic utility function. Thus, the optimization problem in each region $i = 1, ..., n$ is given by

$$\max_{\tau_{B,i}} \int_0^\infty e^{-\rho_i t} \ln(c_i A_i K_i D_i(\cdot)) dt \tag{11}$$

subject to (10) and (4) with $c_i A_i K_i D_i(\cdot) = C_i$ per capita consumption. ln denotes the natural logarithm and ρ_i is the discount rate.

To find the optimum we construct the current-value Hamiltonian which is

$$\begin{aligned} H_i(\cdot) \;\; = \;\; & \ln(c_i A_i K_i D_i(\cdot)) + \lambda_{1,i} \left(\beta_2 \sum_{i=1}^{n} \left(\frac{a_i}{\tau_{B,i}} \right)^{\gamma_i} - \mu M \right) + \\ & \lambda_{2,i} (A_i\, K_i\, D_i(\cdot)(1 - c_i - \tau_{B,i}) - (\delta_i + n_i) K_i), \end{aligned} \tag{12}$$

[10] IPCC results are obtained with more sophisticated Atmosphere-Ocean General Circulation Models.

[11] The range of μ given by IPCC is $\mu \in (0.005, 0.2)$, see IPCC1, 2001, p. 38.

with $\lambda_{j,i}$, $j = 1, 2$, the shadow prices of M and K_i in region i respectively and $E = a_i^{\gamma_i} Y_i^{\gamma_i} B_i^{-\gamma}$ emissions. Note that $\lambda_{1,i}$ are negative while $\lambda_{2,i}$ are positive.

The necessary optimality conditions are obtained as

$$\frac{\partial H_i(\cdot)}{\partial \tau_{B,i}} = \lambda_{1,i}\beta_2(-\gamma_i)a_i^{\gamma_i}\tau_{B,i}^{-\gamma_i-1} - \lambda_{2,i}A_iK_iD_i(\cdot) = 0, \tag{13}$$

$$\dot{\lambda}_{1,i} = (\rho_i + \mu)\,\lambda_{1,i} - D_i'(\cdot)/D_i - \lambda_{2,i}\,A_i\,K_i\,D_i'(\cdot)(1 - c_i - \tau_{B,i}) \tag{14}$$

$$\dot{\lambda}_{2,i} = (\rho_i + \delta_i + n_i)\,\lambda_{2,i} - K_i^{-1} - \lambda_{2,i}\,A_i\,D_i(\cdot)(1 - c_i - \tau_{B,i}). \tag{15}$$

Further, the limiting transversality condition $\lim_{t\to\infty} e^{-\rho_i t}(\lambda_{1,i}M + \lambda_{2,i}K_i) = 0$ must hold.

From (13) we get the optimal abatement activities (as a ratio to GDP) in each region as

$$\tau_{B,i}^o = \left(\frac{\beta_2(-\lambda_{1,i})\gamma_i a_i^{\gamma_i}}{\lambda_{2,i}A_iK_iD_i(\cdot)}\right)^{1/(1+\gamma_i)} \tag{16}$$

(16) shows that $\tau_{B,i}^o$ is the higher the more polluting the technology in use is, which is modelled in our framework by the coefficient a_i. This means that economies with less clean production technologies have a higher optimal abatement share than economies with a cleaner technology. However, this does not mean that economies with a cleaner technology have higher emissions. This holds because, on the one hand, the higher abatement share may not be high enough to compensate for the more polluting technology. On the other hand, the second-best pollution tax rate also depends on $\lambda_{1,i}$, $\lambda_{2,i}$ and K_i. Further, from the expression for $\tau_{B,i}^o$ one realizes that the higher the absolute value of the shadow price of the GHG concentration, $|\lambda_{1_i}|$, the higher the abatement share has to be set.

In the following we will confine our investigations to the balanced growth path (BGP). A BGP is defined as follows[12]

Definition *A balanced growth path (BGP) is a path such that* $\dot{T} = 0$, $\dot{M} = 0$ *and* $\dot{K}/K = C_1$ *hold, with* $M \geq M_o$ *and* $C_1 > 0$ *a positive constant.*

This definition contains several aspects. First, we require that the GHG concentration and the temperature must be constant along a BGP. This is a sustainability aspect. Second, the growth rate of per capita capital is constant over time. It should be noted that this implies that

[12]In the following, steady state is used equivalently to balanced growth path.

the growth rates of per capita GDP and of per capita consumption are constant, too, and equal to that of capital. Third, we only consider balanced growth paths with a GHG concentration which is larger than or equal to the pre-industrial level. This requirement is made for reasons of realism. Since the GHG concentration has been rising monotonically over the last decades it is not necessary to consider a situation with declining GHG concentration.

To gain further insight into our model we use numerical calculations and we consider three regions. Two relatively highly developed regions where one region is producing with a relatively clean technology and the other uses a relatively polluting technology. One may think of the European OECD countries as the first region and of the USA as the second region. The third region is given by low income countries with a technology which is more polluting than the other two regions. We normalize $a_1 = 0.001$. a_2 is double as large as a_1, i.e. $a_1 = 0.002$, and a_3 is four times as large as a_1, i.e. $a_3 = 0.004$. These relations reflect about the situation in European OECD countries relative to the USA and relative to low income countries in 1995 (see Nordhaus and Boyer, 2000, table 3.1). γ_i, $i = 1, 2, 3$, is set to one in all three regions, i.e. $\gamma_i = 1$, $i = 1, 2, 3$.

As to the damage function we assume the following function

$$D_i = \left(1 + m_i(M - M_o)^2\right)^{-b_i}, \; m_i, b_i > 0, \tag{17}$$

which fulfills the requirements of (2). The damage caused by a higher GHG concentration is assumed to be the same for the first and second region and about three times as high in the third region for a doubling of GHGs. Therefore, we set $m_1 = m_2 = 0.013$, $b_1 = b_2 = 1$ and $m_3 = 0.087$, $b_3 = 0.5$. This implies that a doubling of GHGs goes along with a damage of about 1.3 percent in regions 1 and 2 and multiplying GHGs by 3.5 implies a damage of about 7 percent. For the third region the damage is 4 percent for a doubling of GHGs and about 19 percent when GHGs are multiplied by 3.5. These values roughly reflect the situation in European OECD countries, in the USA and in low income countries (see Hackl and Pruckner, 2003, table 1).

The discount rate is assumed to be the same in the three regions and we set $\rho_i = 0.03$, $i = 1, 2, 3$, and the population growth rates are assumed to be zero in the first two regions,

$n_1 = n_2 = 0$, and two percent in the third region, $n_3 = 0.02$.

The marginal propensity to consume is set to 80 percent in all three regions, $c_i = 0.8$, $i = 1, 2, 3$. The marginal product of capital in the second region is assumed to be larger than in the first region and the latter is larger than in the third region and we set $A_1 = 0.35$, $A_2 = 0.5$ and $A_3 = 0.25$. This implies a higher marginal product of capital in the second region compared to the first and third. Depreciation rates are set to $\delta_1 = \delta_2 = 0.04$ in regions 1 and 2 and $\delta_3 = 0.01$ in region 3. Thus, we acknowledge that depreciation of capital is higher in those regions with higher income.

Defining $\kappa_i \equiv K_i \cdot \lambda_{2,i}$, a BGP is given by the solution of the equations

$$0 = \kappa_i \left(\dot{K}_i / K_i + \dot{\lambda}_{2,i} / \lambda_{2,i} \right) \tag{18}$$

$$0 = \rho_i \, \lambda_{1,i} + \lambda_{1,i} \, \mu - D_i'(\cdot)/D_i - \lambda_{2,i} \, A_i \, K_i \, D_i'(\cdot)(1 - c_i - \tau_{B,i}^o) \tag{19}$$

$$0 = \beta_2 \sum_{i=1}^{3} \left(\frac{a_i}{\tau_{B,i}^o} \right) - \mu M, \tag{20}$$

with $\tau_{B,i}^o = \left((\beta_2 (-\lambda_{1,i}) a_i) / (\lambda_{2,i} A_i K_i D_i(\cdot)) \right)^{0.5}$, $i = 1, 2, 3$.

The balanced growth rate is given by $g_i \equiv A_i D_i(\cdot)(1 - c_i - \tau_{B,i}) - (\delta_i + n_i)$, with $\tau_{B,i}^o$ as above. In table 1 we give the result of our calculations for the three regions.[13]

Table 1. Optimal abatement shares, emissions and balanced growth rates for the three regions and GHG concentration as well as average global temperature (non-cooperative case)

$\tau_{B,1}^o$	E_1	g_1	$\tau_{B,2}^o$	E_2	g_2	$\tau_{B,3}^o$	E_3	g_3	$M^\star$	$T^\star$
0.0067	0.1499	0.025	0.0089	0.2237	0.052	0.0234	0.1711	0.01	2.67	293.1

This table shows that the region with the less clean production technology (region 2) has a higher abatement share than the region with the cleaner production technology (region 1) if damages caused by a rise in GHGs are the same, about 0.9 percent of GDP in region 2 compared to 0.7 percent in region 1. However, this does not mean that emissions in region 2 are smaller than in region 1. So, region 1 has fewer emissions than region 2. This means that the higher emission share cannot compensate for the less clean production technology.

[13]The $\star$ denotes values on the BGP.

Taking into account that both the production technology and the damages caused by a rise in GHGs are different (comparing regions 2 and 3) one can see that region 2 spends relatively less for abatement than region 3, 0.9 percent versus 2.3 percent. Further, region 2 has higher emissions than region 3 although it has a cleaner production technology. The reason for that outcome is to be seen in higher damages in region 3. This means that countries experiencing higher damages also tend to have higher abatement shares and fewer emissions.[14]

With no cooperation GHGs rise by about 2.7 of the pre-industrial level implying an increase in the average global surface temperature of 4.7 degrees Celsius for the parameter values we assume.

In the next section we will compare this result to outcome in the cooperative world.

4 The cooperative world

In the cooperative world the optimization problem of the planner is given by[15]

$$\max_{\tau_{B,i}} \int_0^\infty e^{-\rho t} \sum_{i=1}^n w_i \ln(c_i A_i K_i D_i(\cdot)) dt \tag{21}$$

subject to (10) and (4) with $c_i A_i K_i D_i(\cdot) = C_i$ per capita consumption. ln again denotes the natural logarithm and ρ is the discount rate. w_i gives the weight given to different countries.

To find the optimum we construct the current-value Hamiltonian which is now written as

$$\begin{aligned} H(\cdot) \;=\; & \sum_{i=1}^n w_i \ln(c_i A_i K_i D_i(\cdot)) + \lambda_3 \left(\beta_2 \sum_{i=1}^n \left(\frac{a_i}{\tau_{B,i}} \right)^{\gamma_i} - \mu M \right) + \\ & \sum_{i=1}^n \lambda_{4,i} (A_i \, K_i \, D_i(\cdot)(1 - c_i - \tau_{B,i}) - (\delta_i + n_i) K_i), \end{aligned} \tag{22}$$

with λ_3 the shadow price of M and $\lambda_{4,i}$ the shadow prices of K_i. Again, λ_3 is negative while $\lambda_{4,i}$ are positive.

[14] This is seen more clearly when the regions have the same production technology as concerns pollution. Detailed calculations are available on request.

[15] We do not call this situation Pareto optimum because in the Pareto optimum the social planner would also determine the savings rate, which is exogenous in our context. Therefore, this solution is in a way second-best.

The necessary optimality conditions are obtained as

$$
\begin{aligned}
\frac{\partial H(\cdot)}{\partial \tau_{B,i}} &= \lambda_3 \beta_2 (-\gamma_i) a_i^{\gamma_i} \tau_{B,i}^{-\gamma_i - 1} - \lambda_{4,i} A_i K_i D_i(\cdot) = 0, && (23)\\
\dot{\lambda}_3 &= (\rho + \mu)\, \lambda_3 - w_i\, D_i'(\cdot)/D_i - \lambda_{4,i}\, A_i\, K_i\, D_i'(\cdot)(1 - c_i - \tau_{B,i}) && (24)\\
\dot{\lambda}_{4,i} &= (\rho + \delta_i + n_i)\, \lambda_{4,i} - w_i\, K_i^{-1} - \lambda_{4,i}\, A_i\, D_i(\cdot)(1 - c_i - \tau_{B,i}). && (25)
\end{aligned}
$$

Further, the limiting transversality condition $\lim_{t\to\infty} e^{-\rho t}(\lambda_3 M + \lambda_{4,i} K_i) = 0$ must hold.

From (23) we get the optimal abatement ratios as

$$
\tau_{B,i}^o = \left(\frac{\beta_2(-\lambda_3)\gamma_i a_i^{\gamma_i}}{\lambda_{4,i} A_i K_i D_i(\cdot)} \right)^{1/(1+\gamma_i)} \tag{26}
$$

(26) basically is equivalent to (16) with the exception that the shadow prices are different. This holds because in the cooperative world regions do not optimize separately.

To get further insight we proceed as in the last section. That is we consider three regions, insert numerical values for the parameters and then calculate the corresponding abatement shares, emissions, balanced growth rates as well as the rise in GHGs and in the average global surface temperature. The parameter values are as in the last section, with $\rho = 0.03$.

Defining $\kappa_i \equiv K_i \cdot \lambda_{4,i}$ a BGP is given by the solution of the following system of equations,

$$
\begin{aligned}
0 &= \kappa_i \left(\dot{K}_i/K_i + \dot{\lambda}_{4,i}/\lambda_{4,i} \right) && (27)\\
0 &= \rho\, \lambda_3 + \lambda_3\, \mu - D_i'(\cdot)/D_i - \lambda_{4,i}\, A_i\, K_i\, D_i'(\cdot)(1 - c_i - \tau_{B,i}^o) && (28)\\
0 &= \beta_2 \sum_{i=1}^{3} \left(\frac{a_i}{\tau_{B,i}^o} \right) - \mu M, && (29)
\end{aligned}
$$

with $\tau_{B,i}^o$ given by (26). Table 2 gives the result assuming equal weight to each region ($w_1 = w_2 = w_3 = 1$).

Table 2. Optimal abatement shares, emissions and balanced growth rates for the three regions and GHG concentration as well as average global temperature (cooperative case)

$\tau_{B,1}^o$	E_1	g_1	$\tau_{B,2}^o$	E_2	g_2	$\tau_{B,3}^o$	E_3	g_3	$M^\star$	$T^\star$
0.011	0.092	0.025	0.013	0.155	0.052	0.026	0.153	0.012	1.96	291.6

Comparing the outcome of the cooperative case with the non-cooperative one it is realized that the rise in GHGs is smaller and, consequently, the increase in the temperature smaller. GHGs rise by about the factor 2 implying an increase in temperature by 3.2 degrees Celsius. This is due to higher abatement shares in the cooperative world and, as a consequence, to smaller emissions in each region. As to the qualitative results we see that they do not differ from the last section. So, given the same damages of a rise in GHGs the region with the cleaner production technology has a smaller abatement share but also fewer emissions. Again, the higher abatement share in the region with the more polluting technology cannot compensate for the less clean technology.

It can also be seen that emissions are clearly smaller than in the non-cooperative case. In region 1 emissions are 38 percent smaller, in region 2 30 percent and in region 3 there are 10 percent fewer emissions compared to the non-cooperative world.

If damages caused by the increase in GHGs differ the region with the higher damage spends relatively more for abatement and has fewer emissions than the region with smaller damages (region 2 versus region 3). In this case, emissions in the region with the higher damages are smaller than in the region where damages are smaller. Further, growth rates are about the same in the cooperative world and in the non-cooperative world (growth rates are the same in regions 1 and 2 and slightly higher in region 3).

In table 3 we study our model assuming that utility in region 3 gets a weight which is double the weight given to utility in regions 1 and 2, i.e. $w_3 = 2w_1 = 2w_2 = 2$. The justification for this can be seen in the requirement that low income countries should receive higher weight.

Table 3. Abatement shares, emissions and balanced growth rates for the three regions and GHG concentration as well as average global temperature ($w_3 = 2w_1 = 2w_2 = 2$)

$\tau^o_{B,1}$	E_1	g_1	$\tau^o_{B,2}$	E_2	g_2	$\tau^o_{B,3}$	E_3	g_3	$M^\star$	$T^\star$
0.013	0.077	0.025	0.015	0.13	0.051	0.022	0.181	0.013	1.9	291.5

Table 3 shows that now region 3 has a smaller abatement share and higher emissions if utility of that region gets a higher weight compared to the case where all three regions get the same weight. The other two regions, however, have higher abatement shares and smaller emissions. Further, the growth rates in the regions remain basically unchanged. The overall increase in GHGs is also slightly smaller compared to the case of equal weight to all three regions.

5 Equal marginal damages in each region

In this section we consider our world economy where abatement ratios are set such that marginal damages in steady state are equal in each region given that the steady state GHG concentration attains an exogenously determined level (marginal damage rule). The economic mechanism justifying this assumption can be emission permits which are traded between regions.

Technically, we proceed as follows. From (10) and (5) we get in steady state

$$M^\star = \frac{\beta_2}{\mu} \sum_{i=1}^{n} \left(\frac{a_i}{\tau_{B,i}} \right)^{\gamma_i} \tag{30}$$

This gives damages in each region as

$$D_i = \left(1 + m_i \left(\frac{\beta_2}{\mu} \sum_{i=1}^{n} \left(\frac{a_i}{\tau_{B,i}} \right)^{\gamma_i} - M_o \right)^2 \right)^{-b_i} \tag{31}$$

Abatement shares, $\tau_{B,i}$, then are obtained as the solution of the equations

$$\frac{\partial D_i}{\partial \tau_{B,i}} = \frac{\partial D_j}{\partial \tau_{B,j}},\ i \neq j$$

subject to $M^\star = \bar{M}$, where $\bar{M}$ is an exogenously fixed level of GHGs. The balanced growth rate is again given by (4)/K.

To get insight in our model we compare the outcome of the cooperative world (with equal weights) with that where marginal damages are equalized. Table 4 shows the result where $M^\star$ is set to $M^\star = \bar{M} = 1.96$.

Table 4. Abatement shares, emissions and balanced growth rates for the three regions and GHG concentration as well as average global temperature with equal marginal damages

$\tau^o_{B,1}$	E_1	g_1	$\tau^o_{B,2}$	E_2	g_2	$\tau^o_{B,3}$	E_3	g_3	$M^\star$	$T^\star$
0.009	0.112	0.026	0.013	0.159	0.053	0.031	0.129	0.01	1.96	291.6

One realizes that the marginal damage rule generates almost the same outcome as obtained in the cooperative case as concerns economic growth. The only difference is that the growth rates in regions 1 and 2 are slightly larger while it is smaller in region 3 compared to the cooperative case. However, it must be underlined that the quantitative differences are only small. As to emissions one realizes that world-wide emissions do not change[16] but emissions in the regions are different compared to the cooperative world. So, emissions in regions 1 and 2 are 22 and 2.6 percent higher, respectively, and in region 3 they are 15.7 percent lower compared to the cooperative world.

Thus, our calculations show that the marginal damage rule basically does not change the outcome as concerns the balanced growth rate of the regions compared to the cooperative case. However, there are changes as concerns the level of GHG emissions up to 20 percent which is non-negligible.

6 Conclusion

In this paper we have derived optimal abatement spending reducing GHG emissions for a world where regions do not cooperate and for the case of cooperation. The novelty of our approach consists in assuming that damages of rising GHGs negatively affect production and the growth rate in economies. Thus, we take into account feedback effects of climate changes on economic growth. Further, we compared the cooperative situation with the situation where abatement shares are set such that marginal damages are equal in all regions. We could derive the following results.

1. Optimal abatement spending implies about a doubling of the GHG concentration if regions cooperate. In case of non-cooperation GHGs multiply by about 2.7.

2. Cooperation leads to smaller GHG emissions and basically unchanged growth rates compared to non-cooperation. If welfare in the regions receive different weights the region with the higher weight has a marginally higher growth rate and a smaller (more) abatement share (emissions) compared to the case when all countries receive the same weight.

[16]Of course, this is due to the requirement that GHGs are the same as in the cooperative world.

3. Regions with cleaner production technologies have smaller abatement shares but also fewer emissions, ceteris paribus. Further, regions experiencing higher damages from a rise in GHGs have higher abatement shares and smaller emissions.

4. If abatement shares are set such that marginal damages are equalized in the regions the growth rates in the regions are basically the same as in the cooperative case. However, abatement shares and emissions in the regions may be different from those obtained in cooperation.

To finish a word of caution must be said as to the relevance of our quantitative results. So, the result that the concentration of GHGs double crucially depends on the assumption of continuous damage functions. It is true that these functions give damages which are considered as plausible. However, catastrophic events which may occur when the GHG concentration exceeds a certain threshold are not considered.

As to future research, it would be interesting to study costs and benefits of keeping the atmospheric GHG concentration within certain bounds. In any case, this should be done taking into account the interrelation between economic growth and GHG emissions. This is an important fact which is neglected by most studies dealing with anthropogenic climate change and its economic consequences.

References

Gassmann, F. (1992), 'Die wichtigsten Erkenntnisse zum Treibhaus-Problem', in Schweizerische Fachvereeinigung für Energiewirtschaft (ed.), *Wege in eine* CO_2 *- arme Zukunft*, Zürich: Verlag der Fachvereine, 11-26.

Hackl, F., Pruckner, G.J. (2003) 'How global is the solution to global warming? Economic Modelling, **20**: 93-117.

Harvey, D. L. D. (2000), *Global Warming - The Hard Science*, Harlow: Prentice Hall, Harlow.

Henderson-Sellers, A. and K. McGuffie (1987), *A Climate Modelling Primer*, Chichester: John Wiley.

Hettich, F. (2000), *Economic Growth and Environmental Policy*, Cheltenham: Edward Elgar.

IPCC (2001), *Climate Change 2001: The Scientific Basis, IPCC Third Assessment Report of Working Group I* (available on internet, http://www.ipcc.ch).

IPCC (1996), *Climate Change 1995: Economic and Social Dimensions of Climate Change, Contribution of Working Group III to the Second Assessment Report of the IPCC*, Cambridge: Cambridge University Press.

IPCC (1990), *Climate Change. The IPCC Scientific Assessment*, Cambridge: Cambridge University Press.

OECD (1995), *Global Warming - Economic Dimensions and Policy Responses*, Paris.

Nordhaus, W. D. and J. Boyer (2000), *Warming the World. Economic Models of Global Warming*, Cambridge, Ma.: The MIT-Press.

Roedel, W. (2001), *Physik unserer Umwelt - Die Atmosphäre*, Berlin: Springer-Verlag.

Sachverständigenrat zur Begutachtung der gesamtwirtschaftlichen Lage (2001), *Chancen auf einen höheren Wachstumspfad*, Stuttgart: Metzler-Poeschel.

Smulders, S. (1995), 'Entropy, environment, and endogenous growth', *International Tax and Public Finance* **2**: 319-340.

Stockey, N. L. (1998), 'Are there limits to growth?', *International Economic Review* **39**: 1-31.

Tol, R. S. J. (2001), 'Equitable cost-benefit analysis of climate change', *Ecological Economics* **36**: 71-85.