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Search Unemployment in a Dynamic New Keynesian Model of the Business Cycle

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Abstract

The monetary transmission mechanism plays an important role in studying the persistent effects of monetary policy. At least since Chari et al. (2000) it is generally accepted that New Keynesian models of the business cycle display a 'persistence' problem.

In this paper, we follow the approach of Walsh (2002) and include search unemployment in a dynamic New Keynesian model of the business cycle in order to study the effects of a monetary shock. After deriving the equilibrium solution of the model, we study the behavior of the impulse response functions due to a monetary shock. To complete our analysis we confront the results of our simulation to time series evidence for the U.S., U.K. and Germany.

Our main result is that the implementation of search unemployment does improve the capability of the model to reproduce some stylized facts of the monetary transmission mechanism, however, to a lesser degree than expected.

JEL classification: E24, E32, J40, J63

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1   Introduction

In recent years, there has been considerable interest in the study of dynamic stochastic general equilibrium models of monetary economies. One main interest is to analyze how the implementation of nominal rigidities in otherwise standard dynamic stochastic general equilibrium models can generate adjustment patterns, that means impulse response functions, which are compatible to what have been found in the data, for instance, by Christiano et al. (1999), Favero (2001) or Leeper et al. (1996). These empirical studies suggest, that monetary disturbances generate inertial behavior of inflation and persistence in aggregate quantities: the pattern of hump shaped response of output and the gradual response of inflation are assessed as stylized facts to be explained by a model (see, e.g. Fuhrer (2000)).[1] While these rigidities seem to be important to understand why monetary impulses do impact real variables in the way described above, it is now well known, however, (see for instance Chari et al. (2000)) that dynamic stochastic general equilibrium models with nominal rigidities alone display a 'persistence problem', i.e. there is little persistence in the response of real economic activity due to nominal shocks (Dotsey and King (2001)).

This is true as long as one abstains from assuming rigidities that are generally considered as implausible. As spelled out by Bergin and Feenstra (2000), real effects in the data, tend to have longer life than can reasonably be assumed for the types of rigidities on the real side of the economy.

As the addition of nominal rigidities to standard dynamic general equilibrium models is not enough to reproduce the stylized facts, a modification of the real side of the model economy is indispensable.[2] For instance, Christiano et al. (2001) introduce habit persistence, adjustment costs in investment and variable capacity utilization. Beside capital utilization, Dotsey and King (2001) concentrate on 'a substantial role' for produced inputs and variations in labor supply. Both studies

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[1]It is well known that dynamic models without nominal rigidities fail to reproduce these adjustment patterns (see Walsh (1998), chapters 2 and 3).

[2]That nominal rigidities might probably not be enough to explain the non-neutrality of money has already been emphasized in a static environment by Ball and Romer (1990). They argue that a plausible degree of nominal rigidity has to be supplemented by the existence of real rigidities. Absent these real rigidities there is always a big incentive for the firm to adjust its nominal price in response to an monetary impulse, because marginal costs are sensitive to movements in factor demand and factor supply. See, for example, Gerke (2001) for an illustration. The elastic response of output to demand without increased marginal cost is termed 'real flexibilities' by Dotsey and King (2001)

succeed in improving the performance of the model to replicate salient features of the monetary transmission process. Christiano et al. (2001) and previous work of Jeanne (1998) and Huang and Liu (2002) demonstrate that the conditions of the labor market, especially the existence of wage rigidity, is likely to be crucial for the question at hand.

The present paper also focusses on the labor market. In particular, we follow Pissarides (1988) and Mortensen and Pissarides (1994) by assuming that labor cannot be gratuitously and instantaneously reallocated across firms. Therefore, we replace the frictionless labor market of the Walrasian model by introducing search and matching frictions.[3] In particular the work of Mortensen and Pissarides (1994, 1999b) and Pissarides (2000) has emphasized that the manner by which workers seek for jobs and firms look for new employees and the way how these agents are matched together is likely to have an important role in the propagation of economic disturbances. Therefore, it has to be expected that the adjustment processes of aggregate variables, following a real or nominal impulse, is, by some degree, determined by how efficiently the labor market generates new job matches.

Following Walsh (2002), this paper attempts to bridge the gap between two different strands of the literature, the pioneering work of Langot (1995), Merz (1995) and Andolfatto (1996) who study the implications of the search and matching frictions in the context of a standard Real Business Cycle Model, and Hairault and Portier (1993), Yun (1996) or Chari et al. (2000) and others who focus on price stickiness and monopolistic competition. The following analysis combines elements of both strands. We evaluate the dynamic effects of money growth shocks in a sticky price model with two sided search in the labor market. The combination of these two strands is potentially attractive because on the one hand the matching models based on the work of Mortensen and Pissarides (1994, 1999b) replicate important features of the business cycle, for instance the volatilities of job flows.[4] On the other hand, the sticky price models only succeed to replicate the dynamic patterns of output and inflation when prices are fixed for a period of time which is not in line with the empirical evidence. By implementing labor market search in a sticky price model, we investigate whether the interplay of price stickiness and labor

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[3]See, for instance, Mortensen and Pissarides (1999a) or Pissarides (2000) for an actual discussion of search and matching models of the labor market.

[4]See, for example, Cole and Rogerson (1999) for a detailed evaluation of this kind of model.

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market search improves the ability to account for the sluggish response of output and inflation without relying on implausible calibration. The particular role of such labor market frictions in the analysis of monetary policy shocks is already analyzed by Cooley and Quadrini (1999) or Walsh (2002). Complementary to these studies, we analyze the transmission mechanism of a monetary shock, when an aggregate capital stock is included and a positive money demand function is motivated by a 'money in the utility function' approach.[5] In this paper, we implement the basic approach of Merz (1995) into a framework which is based on Ireland (1997).

The remainder of this paper is organized as follows. Section two outlines the market structure of the model. Section three presents the symmetric equilibrium of our model. In section four we show the obtained impulse response functions of our numerical simulations. Section five concludes.

2 The Model

Market structure of the model

The structure of our model builds on the seminal papers of Blanchard and Kiyotaki (1987), Hairault and Portier (1993) and Ireland (1997). The modelling of the labor market is based on Merz (1995). We assume that the economy consists of a representative household, a representative firm which produces a final good, a continuum of firms producing intermediate goods and a monetary authority. The final good and capital services are exchanged in perfectly competitive markets. On the other hand, the intermediate goods and labor are traded under monopolistic competition and in a process exhibiting search externalities for both households and firms, respectively. These externalities arise because of trade frictions in the process in which households and firms exchange labor. In particular, the rate of contacts between firms and workers depends on the number of traders on both sides of the labor market. Furthermore, for each trader a positive externality exists if the number of traders on the opposite side of the market increases. For example, if the number of vacant jobs rises (decreases), an unemployed worker gets a new job with a higher (lower) probability. According to Merz (1995), we distinguish between two kinds of search processes. The first one describes the search of an unemployed worker for a new job

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[5]Cooley and Quadrini (1999) and Walsh (2002) assume endogenous job creation and destruction and motivate the money demand by cash in advance constraints.

at variable search intensities whereas the other one shows the firm offering a new job vacancy in order to create a new job. Both the search intensity and the creation of new vacancies take time and consume real resources.

The remainder of this section is organized as follows. In a first step, we outline the structure of the labor market. Then we describe the behavior of the representative household, the final good firm, the intermediate goods firms and of the monetary authority. In a last step we outline the interaction of intermediate goods firms and the households, i.e. wage bargaining procedure.

The Labor Market

The economy's labor force is assumed to be constant and is normalized to one. Let n_t denote the ratio of employed labor at time t, the unemployment rate follows straightforward: $u_t = 1 - n_t$.

At the aggregate level, employment evolves as

$$n_t = (1 - \tilde{\psi})n_{t-1} + \tilde{m}_t, \tag{1}$$

where $\tilde{m}_t$ denotes the number of job matches and $\tilde{\psi} \in (0,1)$ specifies the rate of exogenous job destruction. The number of job matches are generated by the so-called matching function:[6]

$$\tilde{m}_t = \tilde{m}(s_t u_{t-1}, v_t), \tag{2}$$

where s_t and v_t denote the search intensity of an unemployed worker and the number of vacancies posted by the intermediate goods firms, respectively. According to the literature (see for example Blanchard and Diamond (1989) or Pissarides (2000)) we assume that $\tilde{m}_t$ is linear homogeneous. This kind of matching function implies that the following transition probabilities from unemployment to employment depend only on the labor market tightness $\tilde{\Theta}_t = v_t/u_{t-1}$. These probabilities are defined as:[7]

$$\tilde{p}_t = \frac{\tilde{m}_t(u_{t-1}, v_t)}{s_t u_{t-1}} \quad \text{and} \quad \tilde{q}_t = \frac{\tilde{m}_t(u_{t-1}, v_t)}{v_t} \tag{3}$$

It can be seen easily that the probability $\tilde{p}_t$ for an unemployed worker decreases either by an increase in unemployment or search intensity. Furthermore, this probability increases by an increase in the market tightness which is driven by a higher

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[6]An extensive discussion of the matching function can be found in Pissarides (2000) or Petrongolo and Pissarides (2001).

[7]See, for example, Merz (1995), p. 274.

number of job vacancies. From the firm's point of view, the opposite holds for $\tilde{q}_t$. According to Merz (1995) the total search effort is defined as the result of the search intensity of the unemployed workers and the recruiting investments of the intermediate goods firms. Both activities help to increase the employment and are able to counteract the transition from employment to unemployment driven by the exogenous destruction rate $\tilde{\psi}$. Once firms and workers met, they are engaged in a Nash-bargaining process in order to set the wage rate (see below).

The household sector

We assume that the representative household consists of a large number of agents who pool their income and provide each agent with a complete insurance against variations in income due employment or unemployment. The household's preferences are defined over consumption, labor and real cash balances, where the optimal tupel is chosen with respect to a budget constraint. Preferences are described by the following utility function:

$$U = E_t \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - n_{t-1}, m_t), \tag{4}$$

where $\beta \in [0,1]$ denotes a constant discount factor, c_t and $m_t = \frac{M_t}{P_t}$ denote consumption and real cash balances, respectively. We assume further, that the household owns and accumulates the capital stock. Capital is rented to the intermediate goods sector for a payment $P_t r_t k_{t-1}$ of nominal interest. The evolution of physical capital, k_t, is specified as

$$k_t = (1 - \delta)k_{t-1} + I_t, \tag{5}$$

where $\delta \in (0,1)$ and I_t denote the depreciation rate and the household's investments, respectively.

The number of agents employed in period t follows as:

$$n_t = (1 - \tilde{\psi})n_{t-1} + \tilde{p}_t s_t u_{t-1}. \tag{6}$$

The budget constraint of the representative household is given by:

$$P_t(1 + r_t - \delta)k_{t-1} + P_t w_t n_{t-1} + M_{t-1} + \tau_t + \Pi_t = P_t c_t + P_t k_t + P_t c(s_t)(1 - n_{t-1}) + M_t. \tag{7}$$

Furthermore, the household receives a lump-sum transfer τ_t paid by the government and dividend payments from the continuum of intermediate goods producers, $\Pi_t =$

$\int_0^1 \Pi_t(i)di$. P_t denotes the price level and M_t and k_t the amount of money and capital, respectively, held by the household. $c(s_t)$ describes the search costs of an unemployed worker to find a new job. Finally, the real wage w_t results from a bargaining process between firms and workers (see below). Given eqn. (4), the households maximization problem follows as

$$\max_{c_t,I_t,m_t,s_t,k_t,n_t} E_t\Big\{\sum_{t=0}^{\infty}\beta^t u(c_t, 1-n_{t-1}, m_t)\Big\}$$

subject to the budget constraint and the evolution of employment, given by equations (7) and (6). The necessary first order conditions are[8]

$$\lambda_t = u_c(c_t, 1-n_{t-1}, m_t) \tag{8}$$

$$0 = \xi_t + \beta\big[\lambda_{t+1}r_{t+1} - \xi_{t+1}(1-\delta)\big] \tag{9}$$

$$\lambda_t = -\xi_t \tag{10}$$

$$0 = -\lambda_t c_s(s_t) - \zeta \tilde{p}_t \tag{11}$$

$$0 = \zeta_t + \beta\big[-u_n(c_{t+1}, m_{t+1}, 1-n_t)$$
$$+\lambda_{t+1}(w_{t+1} + c(s_{t+1})) - \zeta_{t+1}((1-\tilde{\psi}) - \tilde{p}_{t+1}s_{t+1})\big], \tag{12}$$

$$\lambda_t = u_m(c_t, 1-n_{t-1}, m_t) + \beta\frac{\lambda_{t+1}}{1+\pi_{t+1}}, \tag{13}$$

where λ_t, ξ_t and ζ_t denote Lagrange multipliers. Taking equations (9) and (10) we derive

$$-u_c(c_t, 1-n_{t-1}, m_t) + \beta\big[u_c(c_t, 1-n_{t-1}, m_t)(1+r_{t+1}-\delta)\big] = 0. \tag{14}$$

With (10) and (11) we rewrite (8) into[9]

$$0 = u_c(c_t, 1-n_{t-1}, m_t)c_s(s_t)$$
$$-\tilde{\beta}\big[-u_n(c_{t+1}, m_{t+1}, 1-n_t) + u_c(c_{t+1}, m_{t+1}, 1-n_t)(w_{t+1}+c(s_{t+1})) \tag{15}$$
$$+\frac{u_c(c_{t+1}, m_{t+1}, 1-n_t)c_s(s_{t+1})}{P_{t+1}}(1-\tilde{\psi}-\tilde{p}_{t+1}s_{t+1})\big].$$

The final good sector

The final good firm produces the final good, y_t, by taking $y(i)$ units of each intermediate good i as input at each period t. The production process is described by

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[8]Please note that all subscripts except t and $t+1$ denote partial derivatives.

[9]Equations (14) and (15) are the analogue part of the household's problem as in Merz (1995) equations (H1) and (H2).

the following constant returns to scale technology

$$y_t = \left[\int_0^1 y_t(i)^{\frac{(\theta-1)}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \tag{16}$$

with $\theta > 1$.

Equation (17) represents the respective maximization problem of the final good firm:

$$\max_{y_t(i)} P_t y_t - \int_0^1 P_t(i) y_t(i) di, \tag{17}$$

where $P_t(i)$ denotes the price of the intermediate good (i). From (17) the demand for the intermediate good results as

$$y_t^d(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\theta} y_t. \tag{18}$$

Because of the zero profit condition, the price level is determined as:

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}. \tag{19}$$

The intermediate goods sector

Each intermediate goods firm produces a distinct good $i \in [0,1]$ with labor and capital as inputs. These intermediate goods are imperfect substitutes and are sold in a market under monopolistic competition. In each period t, the intermediate goods producer chooses the number of job openings v_t, labor of the next period n_t, the number of capital services supplied by the representative household and the price $P_t(i)$. Because of the matching process of the previous period $t-1$ the number of workers employed at time t is given. Furthermore, the firms and workers are engaged in a wage bargaining process every period the job is productive. Analogue to Cooley and Quadrini (1999) this wage setting procedure specifies the hours worked in period t. It is conceivable that the individuals of the economy work overtime if they observe a positive shock.

According to Rotemberg (1982) each intermediate goods producer is faced with a quadratic cost function which describes the adjustment of its nominal price. This cost function is expressed as

$$\frac{\phi_P}{2} \left[\frac{P_t(i)}{P_{t-1}(i)} - 1 \right]^2 y_t. \tag{20}$$

Equation (20) highlights the notion that price changes might have negative effects on customer - firm relationships. These negative effects increase with the magnitude of the price change and the level of economic activity. Besides the costs of adjusting its nominal price, the intermediate goods producer also faces a linear cost function when offering new job vacancies av_t, with $a \geq 0$.

The production technology of the intermediate goods producer is assumed to be of the following form:

$$y_t(i) = z_t \big[k_{t-1}(i)\big]^{\alpha} \big[n_{t-1}(i)\big]^{1-\alpha}. \tag{21}$$

Note that the technology shock z_t is given by a stationary stochastic process

$$\log z_t = (1 - \psi) \log \bar{z} + \psi_z \log z_{t-1} + \epsilon_t^z, \tag{22}$$

with $\epsilon_t^z \sim i.i.d.\ \mathcal{N}(0, \sigma_z^2)$ and $\psi_z \in [0, 1]$.

The optimization problem of the intermediate goods producers is to maximize the present value of profits[10]

$$\max \quad E_t \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t \Pi_t(i)}{P_t}, \tag{23}$$

where $\beta^t \lambda_t / P_t$ denotes the marginal utility value of the representative household of an additional unit of profits during period t. The nominal profits of firm i, Π_t, are defined as:

$$\Pi_t(i) = P_t(i)y_t(i) - P_t w_t n_{t-1} - aP_t v_t - P_t r_t k_{t-1}(i) - P_t \frac{\phi_P}{2}\Big[\frac{P_t(i)}{P_{t-1}(i)} - 1\Big]^2 y_t. \tag{24}$$

Equation (23) is maximized subject to the following constraints:

$$y_t^s(i) \;=\; z_t\big[k_{t-1}(i)\big]^{\alpha}\big[n_{t-1}(i)\big]^{1-\alpha} = \Big[\frac{P_t(i)}{P_t}\Big]^{-\theta} y_t = y_t^d(i) \tag{25}$$

$$n_t \;=\; (1 - \tilde{\psi})n_{t-1} + \tilde{q}_t v_t, \tag{26}$$

where (25) follows from eqns. (18) and (21).Equation (26) denotes the evolution of employment from the firm's perspective. We obtain the following first order

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[10]Note that $\beta^t \lambda_t$ is a stochastic discount factor (pricing kernel). See Rotemberg and Woodford (1992), p. 1160 and 1168.

condition for the intermediate goods firms:

$$\lambda_t r_t = \nu_t f_k(k_{t-1}(i), n_{t-1}(i), z_t) \tag{27}$$

$$\chi_t = -a\lambda_t \tilde{q}_t^{-1} \tag{28}$$

$$0 = \chi_t + \beta^t \left[-\lambda_{t+1} w_{t+1} + \nu_{t+1} f_n(k_t(i), n_t(i), z_{t+1}) - \chi_{t+1}(1 - \tilde{\psi}) \right] \tag{29}$$

$$0 = \lambda_t(1-\theta)\left[\frac{P_t(i)}{P_t}\right]^{-\theta}\frac{y_t}{P_t} - \lambda_t\phi_P\left[\frac{P_t(i)}{P_{t-1}(i)} - 1\right]\frac{y_t}{P_{t-1}(i)} \tag{30}$$

$$+\nu_t\theta\left[\frac{P_t(i)}{P_t}\right]^{-\theta-1}\frac{y_t}{P_t} + \beta E_t\left\{\lambda_{t+1}\phi_P\left[\frac{P_{t+1}(i)}{P_t(i)} - 1\right]y_{t+1}\frac{P_{t+1}(i)}{P_t(i)^2}\right\},$$

where ν_t and χ_t denote the respective Lagrange multipliers. In the case of a symmetric equilibrium, i.e. if $P_t = P_t(i)$, it follows from eqn. (30) that $\lambda_t/\nu_t \equiv \mu_t$, which represents the markup of the monopolistic firm. Furthermore, it can be shown for $\phi_P = 0$ that the markup is constant, i.e. $\mu_t = \theta/(\theta-1)$ (See Ireland (1997), p. 90).

The monetary authority

The monetary authority determines the money supply of the economy. In every period t, nominal money supply grows at a exogenous rate g_t, i.e. $M_t = (1+g_t)M_{t-1}$. The newly created money is paid to the household as a lump-sum transfer. The transfer satisfies:

$$\tau_t = M_t - M_{t-1} \tag{31}$$

By the definition of the growth rate of money, real balances ($m_t \equiv M_t/P_t$) can be expressed as

$$m_t = \frac{1+g_t}{1+\pi_t}m_{t-1}, \tag{32}$$

where π_t denotes the inflation rate at time t. With $\bar{g}$ as the steady state growth rate of money, we define $\bar{\omega}_t = g_t - \bar{g}$ as the deviation of the growth rate from its steady state. According to Walsh (1998) $\bar{\omega}$ is formulated as a stochastic process[11]

$$\bar{\omega}_t = \psi_{\bar{\omega}}\bar{\omega}_{t-1} + \phi_z z_{t-1} + \epsilon_t^{\bar{\omega}}, \tag{33}$$

with $\psi_{\bar{\omega}} \in (0,1]$ and $\epsilon_t^{\bar{\omega}} \sim i.i.d. \mathcal{N}(0, \sigma_{\bar{\omega}}^2)$. Furthermore, it is assumed that the individual knows about the realization of $\bar{\omega}_t$ and z_t when choosing its optimal values of consumption, leisure, real balances and capital in period t.

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[11]See Walsh (1998), p. 69. Note further that eqn. (33) is expressed in logs.

Wage setting

Once firms and workers meet, the wage is negotiated according to a Nash bargaining procedure. During this process firms and workers are considered as monopolists earning an economic rent if a job becomes productive. Therefore, this bargaining scheme allocates the rent surplus of a productive job between firms and workers.[12] For a worker j who matches to a firm i, the value of a job is given by the real wage $w_{j,i,t}$ net costs of search. On the other hand, the firm's value of a filled job follows from the difference between a worker's marginal product, the wages and the firm's advertising costs.

The net surplus of the household is given by

$$W^h = w_t + c(s_t) - u_{n_{t-1}}(c_t, m_t, 1 - n_{t-1}) + \frac{c_s(s_t)}{\tilde{p}_t}(1 - \tilde{\psi} - \tilde{p}_t s_t).$$

Note that the household's surplus consists of the wage rate, the search costs of the actual and the next period net the disutility of work. The net surplus of the intermediate goods producers follows as[13]

$$W^f = f_n(\cdot) - w_t + \frac{a}{\tilde{q}_t}(1 - \tilde{\psi}).$$

The Nash bargaining criterion is given by

$$w_t = \text{argmax} \ \left(W^h\right)^{\tilde{\xi}} \left(W^f\right)^{1-\tilde{\xi}}, \tag{34}$$

where $\tilde{\xi}$ denotes the bargaining strength of the worker. The wage results analogue to Cheron and Langot (2000):

$$w_t = \tilde{\xi}\Big[\frac{1}{\mu_t} f_n(k_t, n_{t-1}, z_t) + a\tilde{\Theta}_t\Big] + (1 - \tilde{\xi})\Big[\frac{u_{n_{t-1}}(\cdot)}{\lambda_t} - c(s_t)\Big]. \tag{35}$$

Comparing equation (35) to the wage setting in the Real Business Cycle models of Merz (1995) or Langot (1995), the only difference is the markup, due to monopolistic competition in the intermediate goods market.

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[12] *"Hence a realized job match yields some pure economic rent, which is equal to the sum of the expected search costs of the firm and the worker. Wages need to share this economic (local monopoly) rent, in addition to compensating each side for its costs from forming the job."* See Pissarides (2000), p. 15.

[13] The firm's and worker's marginal values of employment are obtained by applying the envelope theorem to the respective maximization problems. See e.g. Langot (1995) for an analogue approach.

3 Equilibrium Solution

In the symmetric equilibrium where $\int_0^1 P_t(i)di = P_t$ the following conditions hold, too:

$$\int_0^1 n_{t-1}(i)di = n_{t-1} \tag{36}$$

$$\int_0^1 k_{t-1}(i)di = k_{t-1} \tag{37}$$

$$\int_0^1 y_t(i)di = y_t. \tag{38}$$

Because of the equilibrium conditions given by eqns (36) to (38) the aggregate resource constraint is given by

$$y_t = c_t + I_t + c(s_t)(1 - n_{t-1}) + av_t + \frac{\phi_P}{2}\Big(\frac{P_t}{P_{t-1}} - 1\Big)y_t \tag{39}$$

An equilibrium of this economy is a set of variables

$$\Omega_t = \big\{k_t, n_{t-1}, s_t, \tilde{p}_t, \tilde{q}_t, \tilde{m}_t, v_t, u_{t-1}, m_t, \mu_t, c_t, y_t, I_t, r_t, w_t, \tilde{\Theta}_t, \pi_t, z_t, \bar{\omega}_t\big\}$$

with the following properties: equations (8) to (13) determine the solution of the household's maximization problem. Furthermore, equations (27) to (30) solve the problem of the intermediate goods firms. The remaining equations (1), (2), (3), (5), (21), (22), (32), (33), (35) and (39) and the definition of the labor market tightness $\Theta_t = v_t/(1 - n_{t-1})$ close the model.

In order to calibrate the model we assume the following specifications

$$y_t(i) = z_t\big[k_{t-1}(i)\big]^\alpha\big[n_{t-1}(i)\big]^{1-\alpha} \tag{40}$$

$$\tilde{m}_t = \frac{u_{t-1}v_t}{\big[(s_tu_{t-1})^{\tilde{\lambda}} + v_t^{\tilde{\lambda}}\big]^{\frac{1}{\tilde{\lambda}}}} \tag{41}$$

$$u(\cdot) = \frac{\big(c_tm_t^b\big)^{1-\Phi}}{1 - \Phi} - \frac{n_{t-1}^{1-\nu}}{1 - \nu} \tag{42}$$

$$c(s_t) = c_0 \cdot s_t^{\tilde{\eta}}. \tag{43}$$

In determining the matching function (41) we follow den Haan et al. (2000) in order to ensure that the matching probabilities, $\tilde{p}_t$ and $\tilde{q}_t$ are bounded in an interval between 0 and 1. Furthermore, the specification of the utility function is taken from Fischer (1979) and Walsh (1998), respectively. The household's search effort is modelled according to Merz (1995).

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In order to simulate the model we follow Uhlig (1999) and log-linearize the system around its steady state and solve the system by the method of undetermined coefficients.

4   The Monetary Transmission Mechanism

We begin with the calibration of the model. Table 1 below reports the parameter specification we applied in our numerical simulations:

### Table 1: Parameter Specification

$\bar{N}$	$\bar{U}$	$\bar{Z}$	α	δ	β	$\bar{R}$	$\tilde{\eta}$
0.95	$1 - \bar{N}$	1	0.30	0.025	0.99	$1/\beta$	1.0
c_0	$\tilde{\psi}$	$\tilde{\lambda}$	$\tilde{\xi}$	a	$\tilde{\Theta}$	b	ν
0.005	$\bar{\tilde{M}}/\bar{N}$	0.5	0.5	0.05	0.20	0.005	-1.25
$\bar{g}$	Φ	ψ_z	ψ_u	ϕ	σ_z	σ_u	θ
1.0125	0.5	0.95	0.5	-0.15	0.007	0.00216	6
$\bar{\mu}$	$\bar{\pi}$	ϕ_P	$\bar{\Omega}$	$\bar{S}$			
$\theta/(\theta-1)$	$\Theta - 1$	3.95	0.15	1			

We assume a steady state employment ratio of $\bar{N} = 0.95$. The parameters that describe firms' recruiting costs and workers' search costs are set analogous to Merz (1995), $c_0 = 0.005$ and $a = 0.05$. We assume linear search costs for the household and set $\tilde{\eta} = 1$, accordingly. In specifying the labor market properties we have to set the parameter of the matching function $\tilde{\lambda}$, the market tightness $\tilde{\Theta}$ and the bargaining power of workers $\tilde{\xi}$. Setting $\tilde{\lambda} = 0.5$ we obtain steady state ratios for $\tilde{p}$ and $\tilde{q}$ equal to 0.48 and 0.10. Although these values are lower than reported by den Haan et al. (2000) ($p = 0.70/q = 0.12$), they also seem reasonable.[14] $\tilde{\Theta} = 0.20$ is set higher as reported in the literature (e.g. Langot (1995): $\tilde{\Theta} = 0.1044$) but consistent with empirical results for Germany. Setting $\tilde{\xi} = 0.5$ ensures symmetric bargaining power of firms and workers. The remaining parameter $\tilde{\psi}$ follows immediately from the steady state condition. This approach ensures, from our point of view, a consistent

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[14]Following van Ours and Ridder (1992), $\tilde{p} < 0.5$ indicates labor markets with a duration of unemployment for more than 6 months. As reported by the OECD (2000) a duration of more than 6 months accounts for more than 55% of unemployed workers in Germany and France.

calibration of this model.[15] The parameters ρ, δ and β are set according to the business cycle literature (see e.g. Ireland (1997)). The remaining parameters are specified according to Ireland (1997) and Walsh (1998). In particular, setting $\bar{g} = 1.0125$ ensures an annual growth rate of the money stock of 5%. Furthermore, $b = 0.005$ restricts the ratio of the money stock per GDP to 20 %. The value of $\phi_P = 3.95$ ensures an amount of adjustment costs of 0.03% of GDP. We choose $\nu = -1.25$ and $\Phi = 0.5$, which implies an intertemporal elasticities of labor and consumption of 0.8 and 2.0, respectively. By assuming $\theta = 6$ the markup is 20%. The parameters ψ_z and ψ_u determine the autocorrelation of the technology and the nominal shock. Furthermore, σ_z and σ_u denote the respective standard deviations of these shocks.

Figure 1 shows the impulse response functions of employment, output and consumption due to a positive shock in money.
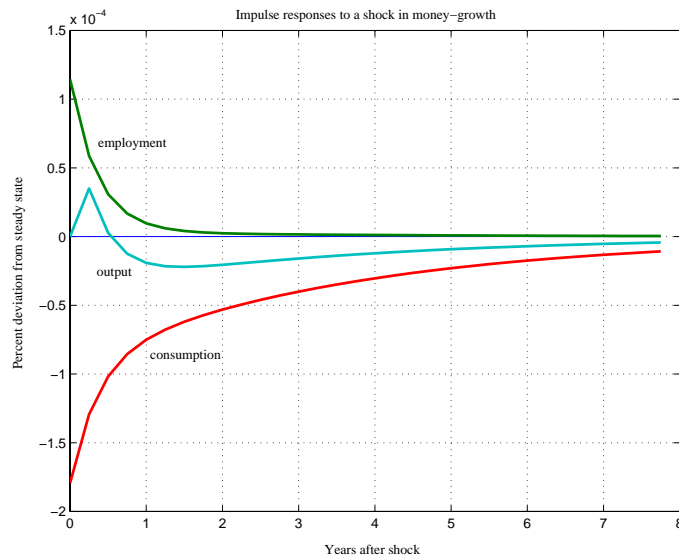
Figure 1: Consumption, Output, Employment

Although the impact of a 1% increase in the growth rate of nominal money supply is very low ($\times 10^{-4}$), the response of y shows the expected hump- shaped pattern, i.e. output does not react immediately. However, the output response is not very persistent. Compatible with the specification of the labor market, the impulse response of employment displays a persistent reaction, due to the low value of $\tilde{\psi}$.

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[15]For example, den Haan et al. (2000) set $q = 0.70$, $p = 0.12$ and $\tilde{\psi} = 0.10$. Applying these settings in our simulations we obtained no qualitative differences in our results.

Interestingly, consumption and investment (see figures 1 and 2) react negatively in response to the shock. This behavior can be explained by the existence of the costs associated with the search effort, i.e. the recruiting costs and the costs of price adjustments (see the aggregate resource constraint, eqn. (39)).
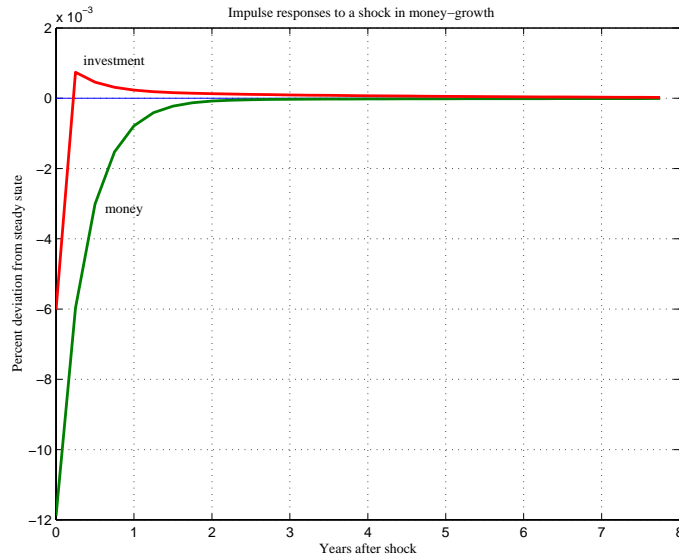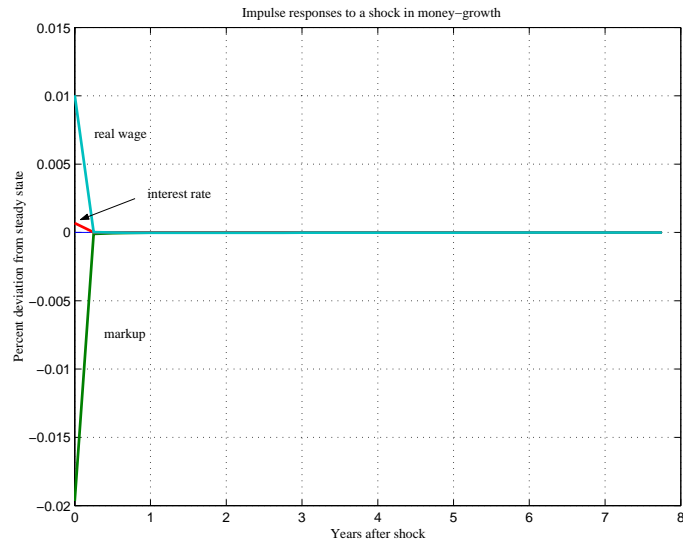

Figure 2: Real Cash Balances, Investment

A stark indication of the lack of significant price sluggishness is visualized by the strong response of inflation, which returns to the steady state value only one period after the shock (see figure 3).
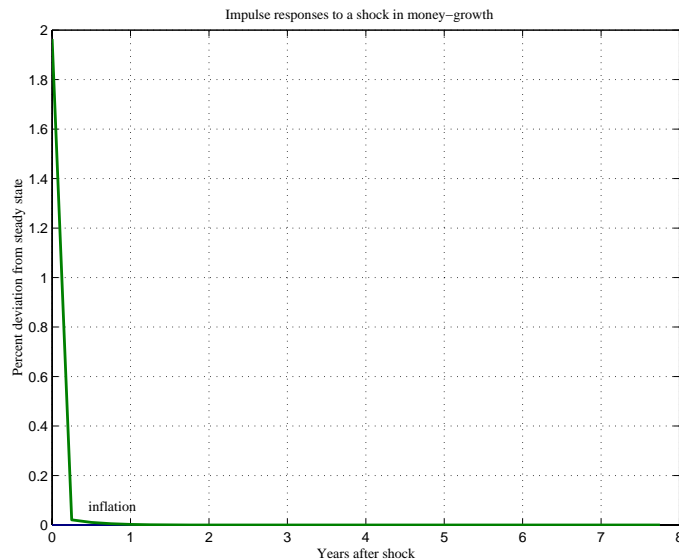

Figure 3: Inflation

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Figure 4: Markup, Interest Rate, Real Wage

The immediate response of the intermediate goods producers is shown by the adjustment pattern of the markup. Only one period after the impulse the markup is back at its steady state value. This shows that the intermediate goods producers have a strong incentive to adjust their prices. This also explains the immediate response of inflation.

The response of the labor market to a monetary shock is reported by figures 5 to 7.
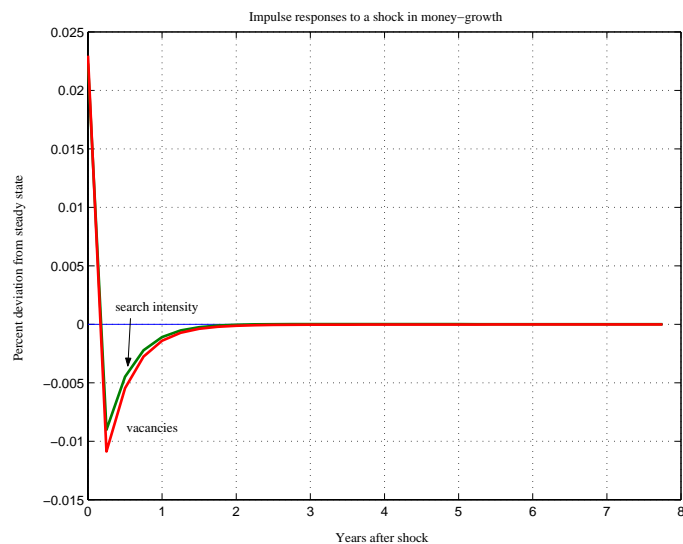
Figure 5: Search Intensity, Job Vacancies

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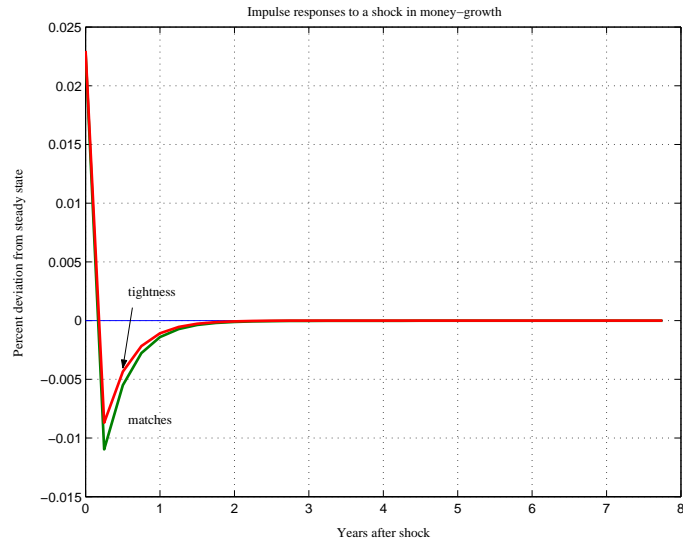
Figure 6: Job Matchings, Tightness $\tilde{\Theta}$

Regarding figures 5 and 6 we observe that households and firms increase their search intensity or their number of job vacancies (figure 5). Furthermore, a similar response is obtained for the number of matches and the labor market tightness (see figure 6). We interpret these results, in line with figures 3 and 4, as a lack of persistence.
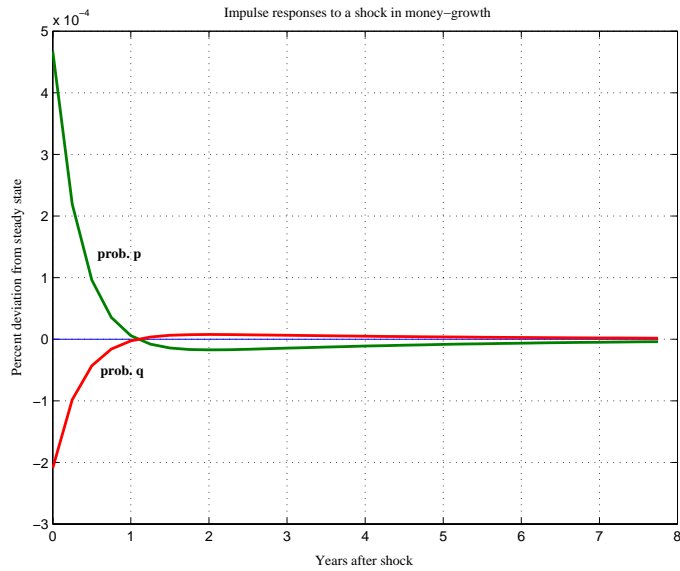
Figure 7: Matching probabilities, p,q

Considering figure 7 it can be seen that the probability for unemployed workers to find a job increases, whereas the probability of filling a vacant job decreases.

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However, this last result may not be robust as slight variation in the intertemporal elasticity of consumption, for example, flips the chart. The lack of persistence remains.

The displayed impulse response functions of output, employment, consumption and investment are robust for a wide range of alternative parameter settings. The same is true for the response of the markup, the interest rate, the real wage and inflation. The notable exception is search intensity and vacancies which display a persistent response when $\tilde{\psi}$ (the exogenous separation rate) converges to 1.

To complete our analysis, we compare our results with time series data for the U.S., U.K. and Germany.[16] In particular, the stylized facts for the U.S. and U.K. shown in table 2 reproduce the empirical correlations of inflation, unemployment and vacancies reported by Cheron and Langot (2000) or Millard et al. (1999) very well.[17]

Table 2: Cross Correlations I

	y	c	u	π	v		y	c	u	π	v
			U.S.						*U.K.*		
y	—	0.96	-0.26	0.25	0.44	—	0.62	-0.56	-0.06	0.58
c		—	-0.12	0.16	0.27		—	-0.23	-0.31	0.18
u			—	-0.20	-0.76			—	-0.16	-0.77
π				—	0.20				—	-0.08
v					—					—
			Germany						*Model*		
y	—	0.78	-0.48	-0.04	0.38	—	0.84	-0.95	-0.29	0.10
c		—	-0.23	-0.03	0.19		—	-0.72	-0.13	-0.00
u			—	-0.03	-0.60			—	0.56	-0.39
π				—	-0.08				—	-0.98
v					—					—

[16]All time series used in this examination are real quarterly time series data. For the U.S. we applied data from 1964.2 to 1996.4, for the U.K. we used a data set from 1964.2 to 1997.1 and for Germany data are taken from 1964.2 to 1998.2. All time series are taken from the OECD Main Economic Indicators and OECD Economic Outlook and Projections, published in the OECD Statistical Compendium, CD ROM Rel. 2002 / 2. Before applying the HP -filter, the data were detrended by the 16+ Population.

[17]Some slight differences belong to the different time intervals considered in the above mentioned articles.

In a next step we examine if our model is able to reproduce the so-called 'Dunlop-Tharsis' - Observation, i.e. we analyze the correlation between output and real wage and real wage and employment, respectively.

Table 3: Cross Correlations II

	U.S.[a]	Model
$corr(y,w)$	0.21 / 0.04	0.99
$corr(w,n)$	0.08	0.95

[a]The empirical correlation coefficients of the U.S. are taken from Danthine and Kurmann (2002) and Cheron and Langot (2000), respectively

Comparing the empirical correlations with our model we conclude that most of the correlation are qualitatively compatible with the data. However, quantitatively there are some marked differences. For example, the negative correlation between output and inflation is higher in absolute value compared to Germany and the U.K. (note that this relation is positive for the U.S.). On the other side, the negative correlation between vacancies and unemployment (Beveridge curve) is smaller than the empirical ones. A prototypical Phillips curve relationship which is exhibited in the empirical correlations is not present in the model. The result of table 3 is, that our model produces a much higher correlation between output and real wage and output and employment, respectively, than it is reported by the data.[18]

In summary, we interpret the success of the model as mixed. Compared to standard New Keynesian models of the business cycle with Walrasian labor markets the model only slightly (if at all) improves. In some dimensions (see the responses of consumption and investment in figures 1 and 2) the model is not in accordance with empirical impulse response functions.

5 Conclusion

In this paper, we introduced search unemployment into the framework of a dynamic New Keynesian model of the business cycle. The main difference of this model to

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[18]The correlation coefficients decrease to 0.72 and 0.48, respectively, if the workers bargaining power, $\tilde{\xi}$, converges to 0.

the literature, for example Cheron and Langot (2000) or Walsh (2002), is that it motivates the price stickiness and the introduction of money differently. In particular, we apply a 'money in the utility function' approach rather than assuming cash in advance constraints. Furthermore, we assume exogenous job creation and destruction, whereas, for example, Cooley and Quadrini (1999) or Walsh (2002) model this process endogenously.

However, the analysis of section four has shown that the obtained results are compatible to the work of Walsh (2002) or Cheron and Langot (2000). The model reproduces some stylized facts of the business cycle, e.g. a hump - shaped response of output due to a shock in money growth or a Beveridge curve relation. On the other hand, a Phillips curve relation and the 'Dunlop-Tharsis'-observation which is reported by the empirical literature is not reproduced by the model. To analyze why other models reproduce such a relation is left for future research.

Because of the high complexity of the model, further studies of the robustness are necessary. For example, comparing the responses to a shock in technology and comparing the results with the one of Walsh (2002), we observe a positive impact of technology on employment, which is not compatible, for instance, to Walsh (2002) (see figure 9 in the appendix). However, we observe a negative response of employment due to a shock in technology if we set $\Phi = 2.0$, i.e. if we decrease the intertemporal elasticity of consumption.

Furthermore, a detailed empirical analysis is still missing. In particular, a comparison of empirical impulse response functions to the results of section four could give further insights into the monetary transmission mechanism.

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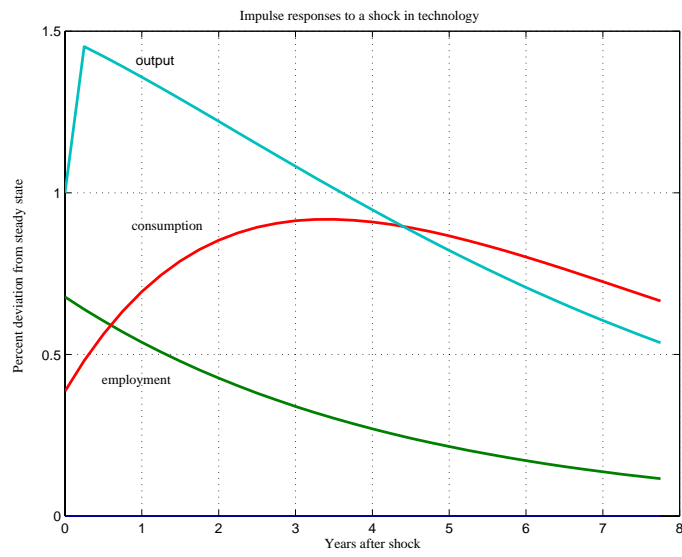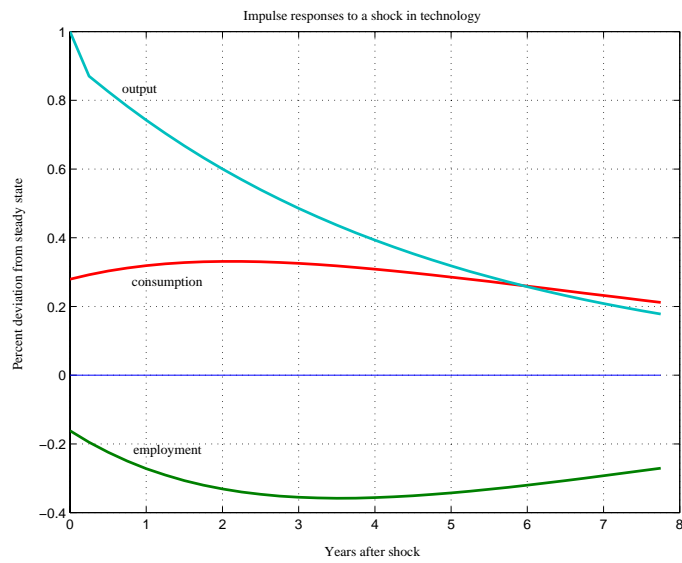# A   Impulse Responses to a Shock in Technology

Figure 8: Impulse Responses, $\Phi = 0.5$

Figure 9: Impulse Responses, $\Phi = 2.0$