



Center for Empirical Macroeconomics

Working Paper No. 59

**Fiscal policy in an endogenous growth model with
public capital: How important are transition
dynamics?**

by

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Abstract

This paper studies growth and welfare effects of fiscal policy in an endogenous growth model with productive public capital where we consider both distortionary and non-distortionary taxation in the model simultaneously. Analyzing the model it is assumed that the economy originally is on the balanced growth path when the government changes its fiscal parameters. The paper then studies growth and welfare effects of varying fiscal parameters both for the balanced growth path as well as for the economy on the transition path. It is demonstrated that the long-run effects may be different from effects on the transition path. Further, economic conditions explaining the results are presented.

JEL: E62, O41

Keywords: Fiscal Policy, Public Capital, Endogenous Growth, Transition Dynamics

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1 Introduction

A stylized fact of economic growth states that economies grow over time without a tendency for declining growth rates. This fact is taken into account by endogenous growth theory. In this type of models economies are characterized by constant growth rates of consumption, of capital and of GDP in the long-run. In order to prevent that growth rates decline over time the marginal product of capital must be constant. This can be achieved by positive externalities of investment, by human capital investment, by R&D spending which expand existing knowledge or by productive public spending which build up a public capital stock. Especially in the latter class of models the government plays an important role. It has to invest in a productive public capital stock in order to generate sustained per-capita growth.

Productive public capital has a long tradition in the economics literature. Arrow and Kurz [1] were among the first to present a formal model with that type of capital. However, their approach did not allow for sustained per-capita growth in the long-run. Futagami et al. [5] then presented an endogenous growth model with productive public capital which generates sustained per-capita growth in the long-run. Their model basically is a more general version of the simple approach presented by Barro [3]. The difference between these two models is that Futagami et al. assume that public investment does not affect aggregate production possibilities directly, as does Barro, but only indirectly by building up a stock of public capital which stimulates economic production.

One consequence of the model presented by Futagami et al. is that their model gives rise to transition dynamics, which does not hold for the Barro model. Both models, however, have in common that the budget of the government is balanced at any moment in time, as frequently assumed in this class of models.¹

As to the empirical relevance of public capital concerning the productivity of economies the results are not unambiguous. A frequently cited study is the paper by Aschauer [2], for

¹An exception is the model by Turnovsky [9], chap. 13, who allowed for public debt in his analysis.

example, who reports strong effects of public capital. Further, he states that public capital is dramatically more important than public investment as a flow variable. However, there are also studies which reach different conclusions. This is not too surprising because it is to be expected that the time period under consideration as well as the countries which are considered are important as to the results obtained. For a survey of the empirical studies dealing with public spending, public capital and the economic performance of countries see Sturm [8] and Pfähler et al. [7].

In this paper, we extend the model presented by Futagami et al. [5] and integrate a consumption tax and lump-sum transfers into the model. This is also novel because most contributions within this type of research only consider a distortionary income tax but not non-distortionary and distortionary taxes simultaneously in the model. Our goal, then, is to study the effects of fiscal policy as concerns economic growth and welfare where we will focus on effects of non-distortionary fiscal instruments because the influence of income taxation on growth and welfare have been extensively studied in the economics literature. See e.g. again Futagami et al. [5] who study effects of varying the income tax rate along the BGP and on the transition path or Greiner and Hanusch [6] who analyze the model on the BGP where they incorporate investment subsidies and transfer payments.

Further, we will place particular emphasis on the transition path from the previous balanced growth path to the new one and try to answer the question of whether the short-run outcome of fiscal policy differs from its long-run impacts. In this respect our approach is similar to the paper by Turnovsky [10] who, however, does not consider sustained per-capita growth. Turnovsky presents an exogenous growth model where fiscal policy has effects on the levels of economic variables but does not affect the growth rate in the long-run. Fiscal policy affects the growth rates of economic variables only on the transition path to the steady state.

The rest of the paper is organized as follows. In the next section we present the structure of our model and study its dynamics. In section 3 we derive growth and welfare

effects of fiscal policy for the model on the balanced growth path and section 4 analyzes effects of fiscal policy along the transition path. Section 5, finally, concludes.

2 The model and its dynamics

2.1 Structure of the model

Our model consists of a representative household with household production² and of the government. We start with the description of the household.

2.1.1 The household

The household maximizes the discounted stream of utility resulting from per-capita consumption over an infinite time horizon. The utility function is assumed to be logarithmic, $U(C(t)) = \ln C(t)$, and the household has one unit of labour which it supplies inelastically. The maximization problem, then, can be written as

$$\max_{C(t)} \int_0^{\infty} e^{-\rho t} \ln C(t) dt, \quad (1)$$

subject to³

$$\dot{K} = (1 - \tau)AK^\alpha H^{1-\alpha} + T_p - (1 + \tau_C)C. \quad (2)$$

ρ is the subjective discount rate and A is a technology index. C , K and H are consumption, private capital and public capital, respectively. All variables give per-capita quantities. $\tau \in (0, 1)$ and $\tau_C > 0$ are the income tax rate and the consumption tax rate and $T_p > 0$ gives lump-sum transfers.⁴ $(1 - \alpha)$ is the public capital share and α gives the private capital

²This is equivalent to a model with a representative firm which pays wages and interest equal to the marginal product of labour and of capital, respectively.

³In the following we delete the time argument t .

⁴In principle, $T_p < 0$ would be feasible implying that the household has to pay a lump-sum tax. However, we do not consider this case.

share. The dot gives the derivative with respect to time and we neglect depreciation of private capital.

To solve this problem we formulate the present-value Hamiltonian which is written as

$$\mathcal{H} = \ln C + \lambda((1 - \tau)AK^\alpha H^{1-\alpha} + T_p - (1 + \tau_C)C) \quad (3)$$

Necessary optimality conditions are given by

$$C^{-1} = \lambda(1 + \tau_C) \quad (4)$$

$$\dot{\lambda} = \rho\lambda - \lambda(1 - \tau)A\alpha K^{\alpha-1}H^{1-\alpha} \quad (5)$$

These conditions are also sufficient if the limiting transversality condition $\lim_{t \rightarrow \infty} e^{-\rho t} \lambda K = 0$ is fulfilled.

2.1.2 The government

The government in our economy receives tax income from income taxation and from taxing consumption it then uses for public investment, for transfer payments and for non-productive public spending, i.e.

$$\tau_C C + \tau AK^\alpha H^{1-\alpha} = T_p + C_p + \dot{H}, \quad (6)$$

with C_p non-productive public spending.

It should be noted that non-productive public spending does not affect utility of the household implying that it is a mere waste of resources. Defining $t_p (c_p) \in (0, 1)$ the share of total tax revenue used for transfers (for non-productive public spending) public capital evolves according to

$$\dot{H} = (1 - c_p - t_p) (\tau AK^\alpha H^{1-\alpha} + \tau_C C). \quad (7)$$

In the following we define $1 - t_p \equiv i_p$, with $i_p \in (0, 1)$. Raising i_p then implies that public investment rises and non-productive public spending declines. For $i_p = 1$ there is no non-productive public spending other than transfer payments. As for private capital we neglect depreciation of public capital.

2.2 The balanced growth path and the dynamics of the model

A balanced growth path (BGP) is defined as a path on which all variables grow at the same constant positive rate, i.e. $\dot{K}/K = \dot{C}/C = \dot{H}/H = \text{constant} > 0$. To analyze whether there exists a possibly unique BGP and to study its dynamics, we first derive the growth rates describing our economy. Using $T_p = t_p(\tau A K^\alpha H^{1-\alpha} + \tau_C C)$ we get the growth rate of private capital from (2) as

$$\frac{\dot{K}}{K} = (1 - \tau)Ah^{1-\alpha} - (1 + \tau_C)c + t_p(\tau_C c + \tau Ah^{1-\alpha}), \quad h(0) = h_0, \quad (8)$$

with $c \equiv C/K$ and $h \equiv H/K$. The growth rate of private consumption is obtained from (4) and (5) as

$$\frac{\dot{C}}{C} = (1 - \tau)A\alpha h^{1-\alpha} - \rho, \quad h(0) = h_0 \quad (9)$$

and the growth rate of public capital is given by (7),

$$\frac{\dot{H}}{H} = (i_p - t_p) \left(\tau A h^{-\alpha} + \tau_C \frac{c}{h} \right), \quad h(0) = h_0 \quad (10)$$

To analyze our economy around a BGP we differentiate c and h with respect to time giving

$$\dot{c} = c \left((1 - \tau)A\alpha h^{1-\alpha} - \rho + c(1 + \tau_C(1 - t_p)) - Ah^{1-\alpha}((1 - \tau) + \tau t_p) \right) \quad (11)$$

$$\begin{aligned} \dot{h} &= h \left((i_p - t_p) \left(\tau Ah^{-\alpha} + \tau_C \frac{c}{h} \right) + c(1 + \tau_C(1 - t_p)) - Ah^{1-\alpha}(1 - \tau(1 - t_p)) \right), \\ h(0) &= h_0. \end{aligned} \quad (12)$$

A rest point of (11)-(12) gives a BGP for our economy. Solving (11) with respect to c and inserting this term in (12) shows that there exist a unique h^* and c^* which solve (11)-(12).⁵ Further, the BGP is a saddle point.⁶ That is our economy is both globally and locally determinate,⁷ i.e. there exists a unique BGP and a unique $c(0)$ so that the economy converges to the BGP in the long-run.

In the next section we study effects of fiscal policy along the BGP.

⁵The * denotes BGP values.

⁶The derivation is in the appendix.

⁷For the definition of global and local determinacy see e.g. Benhabib and Farmer [4].

3 Fiscal policy on the balanced growth path

3.1 Growth effects of fiscal policy

The balanced growth rate is given by (9). This shows that the balanced growth rate negatively depends on the income tax rate τ and positively on the ratio of public to private capital $h \equiv H/K$. Since neither the consumption tax rate τ_C nor the transfer share t_p have distortionary effects they do not appear in (9) and these two fiscal parameters affect the balanced growth rate only through their effect on the ratio of public to private capital on the BGP, h^* . Consequently, a rise in the consumption tax rate will raise the balanced growth rate because more resources are available for productive public spending which raises the marginal product of private capital and, thus, increases the private investment share.⁸ The contrary holds for an increase in t_p . When more public resources are used for non-productive public spending this reduces the balanced growth rate. Table 1 below illustrates this for numerical values of the parameters.

As to variations of the income tax rate it can be seen from (9) that, on the one hand, a higher income tax rate reduces the balanced growth rate. On the other hand, a higher income tax rate raises the tax revenue and, thus, productive public spending. Consequently, the balanced growth rate is maximized when the positive growth effects just equals the negative one. For the model without a tax on consumption and without transfers this holds when the income tax rate just equals the elasticity of production with respect to public capital (cf. Futagami et al. [5]).

To gain additional insight into our model we resort to numerical examples. As a benchmark we use the following parameter values. The public capital share is set to 20 percent, i.e. $(1 - \alpha) = 0.2$, $A = 0.2$ and the discount rate is set to 5 percent, i.e. $\rho = 0.05$. A public capital share of about 20 percent is frequently used in endogenous growth models and also within the range of empirical estimates obtained (see e.g. [3] and

⁸See the phase diagrams in figures 1 and 3 in section 4.1.

[7]). The income tax rate and the consumption tax rate are set to $\tau = 0.15$ and $\tau_C = 0.15$. The transfer share is set to 1 percent, i.e. $t_p = 0.01$, and i_p is 5 percent, $i_p = 0.05$. Table 1 gives the balanced growth rate, denoted by g , for different values of the tax rates and of the transfer share with the other parameters set to their respective benchmark values.

Table 1. Balanced growth rate, g , for different values of fiscal parameters with the other parameters set to their benchmark values.

τ	g	τ_c	g	t_p	g
0.075	0.0227	0.15	0.0228	0.01	0.023
0.11	0.0229	0.2	0.0237	0.03	0.017
0.2	0.0221	0.25	0.0245	0.04	0.012

One realizes that higher consumption tax rates and smaller transfer payments imply higher balanced growth rates as mentioned above. Further, these results are robust, that is they hold for other values of i_p and t_p , too. For example, setting $i_p = 1$ ($i_p = 0.5$) and $t_p = 0.5$ ($t_p = 0.25$) gives the same results from a qualitative point of view.

As concerns the income tax rate table 1 shows that the balanced growth rate is maximized for τ about 11 percent, i.e. $\tau = 0.11$ which is smaller than the elasticity of production with respect to public capital. In [5] and [3] the balanced growth rate is maximized if it equals the elasticity of aggregate production with respect to public capital. The different outcome is to be seen in the fact that the government in our model also levies a consumption tax. Therefore, the government has to set the income tax rate, which has a distortionary effect, lower compared to the situation where the income tax rate is the only source of public revenue if it wants to maximize the balanced growth rate. Consequently, when we set $\tau_C = 0$ ($\tau_C = 0.05$) the balanced growth rate reaches its maximum for $\tau = 0.2$ ($\tau = 0.17$.) It should be noted that the reverse also holds. That is the growth maximizing income tax rate is higher than the elasticity of aggregate production with

respect to public capital when the government has additional productive public spending like an investment subsidy for example as shown in Greiner and Hanusch [6].

As to the dependence of the growth maximizing income tax rate on the parameters i_p and t_p , increases in i_p tend to raise the growth maximizing income tax rate while increases in t_p tend to lower the growth maximizing income tax rate. Intuitively, this is obvious because with a higher share of productive public spending more government revenues lead to a higher balanced growth rate and vice versa. However, the quantitative effects are negligible, that is i_p and t_p do hardly influence the growth maximizing income tax rate. So, the balanced growth rate reaches its maximum for $\tau = 0.11$ ($\tau = 0.13$, $\tau = 0.14$, $\tau = 0.14$) for $i_p = 0.05$ ($i_p = 0.25$, $i_p = 0.5$, $i_p = 1$), with $t_p = 0.01$ and the other parameters set to their benchmark values. Setting $i_p = 1$ the balanced growth rate reaches its maximum for $\tau = 0.14$ ($\tau = 0.14$, $\tau = 0.13$) for $t_p = 0.01$ ($t_p = 0.5$, $t_p = 0.8$). For $i_p = 0.05$ variations in t_p do not affect the growth maximizing income tax rate at all. Next, we study welfare effects along the BGP.

3.2 Welfare effects on the BGP

To study welfare effects of fiscal policy along the balanced growth path we note that (1) on the BGP can be written as

$$F \equiv \int_0^\infty e^{-\rho t} \ln(c^* K^* e^{gt}) dt = \rho^{-1} \ln(c^* K^*) + g\rho^{-2}, \quad (13)$$

where g denotes the balanced growth rate and we normalize K^* to one, i.e. $K^* = 1$. (13) shows that welfare positively depends on the long-run growth rate and positively on the ratio of consumption to capital on the BGP, c^* . Thus, any fiscal policy which raises the balanced growth rate tends to increase welfare in the long-run, on the one hand. On the other hand, variations in the consumption tax rate and in the transfer ratio also affect the ratio c^* and, thus, welfare. The ratio on the BGP is obtained from (11) as

$$c^* = \frac{\rho}{1 + \tau_C(1 - t_p)} + A(h^*)^{1-\alpha} \left(\frac{(1 - \alpha)(1 - \tau) + \tau t_p}{1 + \tau_C(1 - t_p)} \right) \quad (14)$$

This shows that a higher $\tau_C (t_p)$ has a negative (positive) direct effect on c^* . However, an increase in $\tau_C (t_p)$ raises (reduces) h^* and h^* has a positive effect on c^* . So, the overall effect is ambiguous, i.e. an increase in $\tau_C (t_p)$ may lead to a higher or lower c^* .⁹ If a rise in $\tau_C (t_p)$ has a positive (negative) effect on c^* , it is to be expected that there exists an inverted U-shaped relation between long-run welfare and the consumption tax rate, where boundary solutions cannot be excluded.

To gain further insight we again resort to numerical examples. Tables 2 and 3 show the consumption-capital ratio on the BGP, c^* , and welfare on the BGP, F , for different parameter values.

Table 2. c^* and F for different values of τ_C and for different values of i_p , with $t_p = 0.01$ and with the other parameters set to their benchmark values.

$i_p = 0.05$			$i_p = 0.5$			$i_p = 1$		
τ_C	c^*	F	τ_C	c^*	F	τ_C	c^*	F
0	0.0673	-46.54	0	0.0752	-31.7	0.05	0.0754	-25.49
0.05	0.0645	-46.69	0.05	0.0723	-31.49	0.1	0.0725	-25.4
0.15	0.0595	-47.32	0.075	0.0709	-31.47	0.15	0.0698	-25.43
0.2	0.0573	-47.72	0.1	0.0695	-31.48	0.2	0.0672	-25.56

Table 2 shows that an increase of the consumption tax rate reduces the ratio c^* independent of i_p . The welfare maximizing value of τ_C , however, depends on the share used for public investment. For $i_p = 0.5$ ($i_p = 1$) welfare on the BGP is maximized for $\tau_C = 0.075$ ($\tau_C = 0.1$) and for $i_p = 0.05$ the boundary solution $\tau_C = 0$ turns out to maximize welfare in the long-run. One can see that the higher the public investment share the higher is the welfare maximizing consumption tax rate. The economic mechanism seems to be obvious. With a high public investment share a lot of the additional tax revenue, gained through a higher consumption tax rate, is used for productive public spending and leads to a strong

⁹The ambiguous effect can also be seen by applying the implicit function theorem to (11) and (12).

increase of the balanced growth rate. As a consequence, the increase in economic growth compensates for the decrease in the consumption-capital ratio c^* and welfare rises. Table 3 studies how the welfare maximizing consumption tax rate depends on the transfer share for a given i_p .

Table 3. c^* and F for different values of τ_C for different values of t_p with $i_p = 1$ and with the other parameters set to their benchmark values.

$t_p = 0.01$			$t_p = 0.25$			$t_p = 0.5$		
τ_C	c^*	F	τ_C	c^*	F	τ_C	c^*	F
0.05	0.0754	-25.49	0.15	0.0764	-25.57	0.35	0.0774	-25.67
0.1	0.0725	-25.4	0.2	0.0744	-25.44	0.5	0.0737	-25.42
0.15	0.0698	-25.43	0.3	0.0705	-25.41	0.6	0.0713	-25.4
0.2	0.0672	-25.56	0.4	0.0669	-25.58	0.65	0.0702	-25.42

Table 3 shows that, as in table 2, the consumption-capital ratio on the BGP declines when the consumption tax rate rises. Further, one realizes that the higher the transfer share t_p the higher is the consumption tax rate which maximizes welfare along the BGP. The reason is that with a higher transfer share a larger fraction of the tax revenue is spent for non-productive spending and the ratio of private capital to public capital, h^* , is small and, thus, the balanced growth rate. In this situation h^* and the growth rate are very sensitive with respect to an increase in the consumption tax rate. Therefore, a rise in τ_C has a strong effect on the balanced growth rate when transfer payments are high even if only a relatively small fraction of the tax revenue is used for productive public spending. The increase in the balanced growth rate, then, dominates the negative growth effect of a lower c^* .

Next, we analyze welfare effects of varying the transfer share given the i_p . Table 4 shows welfare associated with different transfer payments where in the left part of the

figure we set $i_p = 0.05$ and in the right part we assume that the government spends all its revenues for public investment and for transfers, i.e. we set $i_p = 1$.

Table 4. Consumption share, c^* , and welfare, F , on the BGP for different values of fiscal parameters with the other parameters set to their benchmark values.

$i_p = 0.05$			$i_p = 1$		
t_p	c^*	F	t_p	c^*	F
0	0.0598	-46.34	0	0.07	-25.445
0.01	0.0595	-47.32	0.05	0.071	-25.406
0.03	0.0586	-50.09	0.1	0.072	-25.397
0.04	0.0577	-52.41	0.15	0.074	-25.418

Table 4 shows that two different cases are possible. First, an increase in transfer payments may lead to a lower consumption ratio on the BGP (and a smaller long-run growth rate as noted in the last section) implying that welfare is maximized for $t_p = 0$. This is the case on the left hand side of table 4. Second, a rise in the transfer share may increase the long-run consumption share. This implies that there exists a welfare maximizing transfer share as in the right part of table 4. The latter result is more likely to occur the larger i_p is. If i_p is high the public capital to private capital ratio on the BGP, h^* , is large and a rise in t_p has less drastic effects since a lot of public revenue is spent for productive investment so that the positive direct effect of t_p on c^* dominates the negative indirect one associated with a lower growth rate.

It should be noted that the value of t_p which maximizes welfare positively depends on the consumption tax rate. The reason is that a higher consumption tax rate implies a smaller c^* which reacts stronger to variations in t_p . For example, setting $\tau_C = 0.2$ welfare on the BGP is maximized for $t_p = 0.17$ for $i_p = 1$ and the other parameters as in table 4.

4 Growth and welfare effects along the transition path

In analyzing effects of fiscal policy on the transition path we confine our investigations to changes in the consumption tax rate and in the transfer share. We do this because variations of the income tax rate have been frequently analyzed, as mentioned in the Introduction, and the results in our framework are basically identical to those obtained. As in the last section we will first study growth effects.

4.1 Growth effects on the transition path

To study growth effects we resort to phase diagram analysis. We assume that the economy is on the BGP when the government changes the fiscal parameters. Figure 1¹⁰ shows the phase diagram for (11) and (12) and the effects of a once-and-for-all rise in τ_C at $t = 0$. From (11) and (12) we see that a rise in the consumption tax rate τ_C shifts both the $\dot{c} = 0$ isocline and the $\dot{h} = 0$ isocline to the right leading to a higher h^* and to a lower c^* . This implies that the balanced growth rate rises. Further, at $t = 0$ the ratio c jumps down to the stable manifold of the saddle point (the thick gray line in figure 1) leading to the new BGP (denoted as E_2 in figure 1) while h does not change at $t = 0$ because both private and public capital are fixed and react only gradually, in contrast to consumption. Over time, both the ratios c and h rise until the new BGP is reached. Note that in figure 1 we draw the stable manifold of the linearized system which is given by $c(t) = c^* + (h(t) - h^*)(v_{11}/v_{12})$, with v_{1i} , $i = 1, 2$, the elements of the eigenvector belonging to the negative eigenvalue $\mu < 0$.¹¹

¹⁰Figure 1 and figure 2 are drawn for the benchmark parameter values where τ_C rises from $\tau_C = 0.15$ to $\tau_C = 0.2$.

¹¹However, the $\dot{c} = 0$ and the $\dot{h} = 0$ isoclines are the ones of the original nonlinear system although they look linear.

Figure 1 about here

The growth rates on the transition path are shown in figure 2 where g_i gives the growth rate of variable $i = C, K, H$. First, we can state that along the transition path $\dot{C}/C > \dot{K}/K$ and $\dot{H}/H > \dot{K}/K$ hold because c and h rise over time. At time $t = 0$ the growth rate of consumption \dot{C}/C does not react to a change in τ_C since τ_C does not appear in (9) and since h is fixed. For $t > 0$, h rises implying an increase in \dot{C}/C until the new balanced growth path is reached. This implies that the growth rate of consumption gradually rises until the new balanced growth path is reached.

Since c initially jumps down to the stable manifold (see figure 1) and then rises we have $\dot{c} > 0$ implying $\dot{C}/C > \dot{K}/K$. Further, we know that \dot{C}/C does not change at $t = 0$ implying that \dot{K}/K declines. Over time \dot{K}/K increases until the new balanced growth path is reached which is larger than the old one. This implies that private saving temporarily declines because the higher consumption tax rate reduces disposable income. Thus, the consumption tax affects saving in the short-run by reducing disposable income and, thus, affects the accumulation of capital. Although the consumption tax rate does not have a substitution effect it has an influence on the growth rate of capital on the transition path through its income effect. This leads to an overshooting of the growth rate of private capital over its long-run BGP value.

As to the growth rate of public capital, (10) shows that the increase in τ_C tends to raise the growth rate of public capital while the induced decline in c tends to reduce the growth rate. However, we know that the growth rate of the private capital stock declines at $t = 0$. Since c declines this implies that the tax revenue rises at $t = 0$. Otherwise, the growth rate of private capital could not rise. Consequently, the growth rate of public capital \dot{H}/H increases at $t = 0$ since a certain part of the additional tax revenue is used for productive public investment. Over time, as h rises the growth rate of public capital declines and approaches the balanced growth rate as shown in figure 2. That is we observe an overshooting of the growth rate of public capital over the long-run value as a result of

the rise in the consumption tax rate.

Figure 2 about here

Next we study growth effects of raising the transfer share t_p . In this case, an increase in t_p shifts the $\dot{c} = 0$ and the $\dot{h} = 0$ isoclines to the left leading to a smaller h^* , thus reducing the balanced growth rate, while c^* may rise or fall as demonstrated in table 4. The phase diagram is shown in figure 3 where we again employ the benchmark parameter values of the last section and where we raise t_p from 1 to 3 percent which implies a decline in c^* .

Figure 3 about here

Again, the economy adjusts in a way so that c jumps to the stable branch of the saddle point. Since both isoclines shift to the left c and h are to the right of the isoclines at $t = 0$ implying that both c and h decline over time. This means that $\dot{C}/C < \dot{K}/K$ and $\dot{H}/H < \dot{K}/K$ hold for all $t \in (0, \infty)$. The growth rates of consumption, of private capital and of public capital on the transition to the new BGP are shown in figure 4.

Figure 4 about here

At $t = 0$ the growth rate of consumption does not react to changes of transfer payments since t_p does not appear in (9) and since h is fixed. For $t \in (0, \infty)$ h declines and, consequently, \dot{C}/C decreases until the new balanced growth path is reached. The growth rate of private capital temporarily rises because higher transfer payments imply a positive income effect raising disposable income and, thus, saving and the accumulation of private capital. Over time, \dot{K}/K declines until the new long-run growth rate is reached. Note that $\dot{C}/C < \dot{K}/K$ holds for all $t \in (0, \infty)$. The growth rate of public capital, finally, declines at $t = 0$ since a rise in transfer payments implies that less resources are spent for productive public investment. Over time \dot{H}/H increases since h rises and, thus, the tax revenue. Again, we observe an overshooting of the growth rates of private and of public capital over the long-run balanced growth rate.

4.2 Welfare effects of fiscal policy

In this section we study welfare effects of fiscal policy taking into account transition dynamics. As above we assume that the economy is originally on the BGP and converges to the new BGP after the change in fiscal parameters. In particular, we are interested in the question of whether fiscal policy has the same welfare effects as for the model where we limited our analysis to the BGP.

To do so, we proceed as follows. First, we calculate welfare on the BGP for a given parameter constellation for different time horizons. Welfare on the BGP from $t = 0$ until $t = t_f$ is given by

$$F_1(t_f) \equiv \int_0^{t_f} e^{-\rho t} \ln(C(0)e^{gt}) dt = \ln(c(0)K_0) \left(\frac{1 - e^{-\rho t_f}}{\rho} \right) + g \left(\frac{1 - e^{-\rho t_f}(1 + \rho t_f)}{\rho^2} \right), \quad (15)$$

where g is again the balanced growth rate and $c(0)$ and K_0 are equal to their BGP values c^* and K^* where we normalize $K^* = 1$ as in the previous section. Second, we numerically compute welfare after a change in a fiscal parameter has occurred at time $t = 0$ according to

$$F_1(t_f) \equiv \int_0^{t_f} e^{-\rho t} \ln(C(t)) dt, \quad (16)$$

where $C(t)$ is computed by solving (9) numerically. Since capital is fixed at $t = 0$, K_0 is equal to the BGP value K^* before the change in the fiscal variable. Further, since $K_0 = K^* = 1$ we get $C(0) = c(0)$. To compute $C(t)$ from (9) we need $h(t)$ which we approximate by its linear approximation.¹² $C(0) = c(0)$ and $h(t)$ then are given by

$$c(0) = c^* + (h_0 - h^*) \begin{pmatrix} v_{11} \\ v_{12} \end{pmatrix} \quad (17)$$

$$h(t) = h^* + (h_0 - h^*)e^{\mu t}, \quad (18)$$

where h_0 is the ratio of public to private capital before the change of the fiscal parameter,

¹²In our numerical examples the deviations of the linear model from the nonlinear are small and the qualitative results do not change if the calculations are done for the nonlinear model. This is shown in an appendix which is available on request.

i.e. on the old BGP, and c^* , h^* are steady state values after the rise of the fiscal parameter, i.e. on the new BGP. v_{1i} , $i = 1, 2$, in (17)-(18) are the elements of the eigenvector belonging to the negative eigenvalue $\mu < 0$. This shows that $c(0)$ is uniquely determined.

We first calculate welfare on the transition path with our parameter values from the last section and compare the results to those obtained when we limited our considerations to the BGP. The welfare effects on the transition path resulting from a rise in τ_C are reported in table 5 for different time periods.

Table 5. Welfare effects of an increase in the consumption tax rate on the transition path for different time horizons.

$i_p = 0.05, t_p = 0.01$				$i_p = 1, t_p = 0.5$				
τ_C	$F_1(10)$	$F_1(25)$	$F_1(50)$	τ_C	$F_1(10)$	$F_1(25)$	$F_1(50)$	$F_1(100)$
0.15	-21.3818	-37.0253	-45.3054	0.15	-17.5346	-27.3618	-29.2931	-27.3424
0.2	-21.7002	-37.5794	-45.9672	0.2	-17.675	-27.5424	-29.3865	-27.3245

It should be recalled that a rise in the consumption tax rate reduces welfare on the BGP for a rise of τ_C from 15 percent to 20 percent for $i_p = 0.05$ and $t_p = 0.01$ (see table 2). For $i_p = 1$ and $t_p = 0.5$ welfare on the BGP rises with a higher τ_C until $\tau_C = 0.6$ before it declines (see table 3).

Table 5 gives welfare for different time horizons for the economy with $\tau_C = 0.15$ and $\tau_C = 0.2$. For $\tau_C = 0.15$ the economy is on the balanced growth path and $F_1(t_f)$ is computed according to (15). For $\tau_C = 0.2$ the economy jumps on the stable manifold with $c(0)$ given by (17) while h_0 and K_0 are fixed and then approach the new BGP as shown in figure 1.

In order to understand the results of table 5 one has to recall that, as a consequence of the rise in τ_C , the ratio c declines at $t = 0$ and the growth rate of consumption does not change at $t = 0$. Over time, both c and the growth rate of consumption rise until they converge to their new BGP values (see figures 1 and 2). This implies that the growth

rate of consumption on the transition path is smaller than the balanced growth rate and the decline in c is larger compared to the decline when only the BGP is considered. As a consequence, negative welfare effects of a rise in τ_C are more likely compared to BGP considerations and a decline of welfare if only the BGP is considered is sufficient for a decline on the transition path. In this case, welfare effects along the BGP path are the same as welfare effects on the transition path, from a qualitative point of view.

If welfare on the BGP rises, the result may change if the transition path is considered. So, welfare declines on the transition path for $i_p = 1$ and $t_p = 0.5$ and for a sufficiently small time horizon while welfare rises if the analysis is limited to the BGP. The reason for this outcome is that c first declines which reduces welfare. Only when the time horizon is sufficiently large ($t \geq 100$ in our numerical example) welfare rises and we get the same result from a qualitative point of view as for the model on the BGP. This holds because the positive welfare effect of this fiscal policy (a higher growth rate) only becomes effective at later periods.

Next we analyze welfare effects of increasing transfer payments on the transition path. Table 6 shows welfare effects of raising the transfer share from 0.1 to 0.15 for different time periods.

Table 6. Welfare effects of an increase in the transfer share on the transition path for different time horizons.

$i_p = 0.05$				$i_p = 1$				
t_p	$F_1(3)$	$F_1(10)$	$F_1(25)$	t_p	$F_1(10)$	$F_1(50)$	$F_1(75)$	$F_1(100)$
0.01	-7.7675	-21.3818	-37.0253	0.5	-17.5346	-29.2931	-28.1538	-27.3424
0.015	-7.7666	-21.3832	-37.0567	0.55	-17.4109	-29.2236	-28.1577	-27.3818

Again, we first recall that for the model on the BGP raising the transfer share from 1 to 1.5 percent and from 50 to 55 percent reduces welfare (see table 4 and the discussion following that table). Looking at welfare on the transition path one realizes that, in

contrast to the model on the BGP, welfare rises if the time horizon is sufficiently short (shorter than 3 and 50 years, respectively, in our example). The reason for this result is that the consumption-capital ratio increases at time $t = 0$, which implies a positive welfare effect, when the transfer share rises and declines only gradually over time. Further, the growth rate of consumption remains unchanged at $t = 0$ and also declines only gradually. On the BGP, however, the consumption-capital ratio immediately jumps to its new smaller value and the growth rate also takes its lower long-run value immediately. Therefore, an increase in transfer payments is more likely to generate a positive welfare effect on the transition path than on the BGP and the shorter the time horizon is over which welfare is considered, the more likely it is to get a positive welfare effect.

We should also say something about the convergence speed which is determined by the negative eigenvalue μ . In our example $\mu = -0.0353$ for $i_p = 0.05$ and $t_p = 0.01$. This implies that the convergence speed is about 3.5 percent. Setting $i_p = 0.5$ ($i_p = 1$) and $t_p = 0.1$ ($t_p = 0.5$) gives $\mu = 0.0683$ ($\mu = 0.0744$), i.e. a convergence speed of about 6.8 (7.4) percent. In general, it turns out that the higher (lower) productive (non-productive) public spending, the higher (lower) the speed of convergence.

5 Conclusions

This paper has presented an endogenous growth model with both distortionary and non-distortionary taxation and studied growth and welfare effects of fiscal policy. We could derive the following results.

1. Increases in non-distortionary taxes lead to a higher balanced growth rate in the long-run because a certain part of the additional tax revenue is used for productive public investment which spurs economic growth. The income tax rate maximizing balanced growth equals the elasticity of aggregate production with respect to public capital unless the government has additional sources of revenue. So, taxing consumption raises the tax revenue and, consequently, reduces the income tax rate which maximizes long-run growth.

2. Growth and welfare maximization may be different even if one confines the analysis to the balanced growth rate. This holds because a higher consumption tax rate reduce the consumption ratio on the BGP and has a negative effect on welfare. Therefore, there exists a welfare maximizing consumption tax rate. However, it must be underlined that boundary results cannot be excluded and the question of whether there are interior values of the consumption tax rate which maximize welfare depends on the parameter values of the model. A similar outcome was derived for the transfer share. There, however, a situation is feasible where both the balanced growth rate and the long-run consumption share decline when transfers are increased. Then, lower transfer payments always raise both long-run growth and welfare.

3. Analyzing fiscal policy on the transition path shows that there is overshooting of the growth rates of private and public capital when the consumption tax rate and the transfer share are increased. This overshooting occurs because fiscal policy immediately affects the income of the household and the tax revenue of the government while the private and the public capital stock adjust only gradually. Thus, the income effect going along with fiscal policy influences growth rates on the transition path even if fiscal policy does not lead to substitution effects. So, a rise in the consumption tax rate (transfer payment) implies a negative (positive) income effect leading to an initial decrease (increase) in the growth rate of private capital which rises (declines) over time as the new higher (lower) balanced growth rate is approached. For the growth rate of public capital the opposite holds because a higher consumption tax rate (transfer payments) implies a larger (smaller) tax revenue.

4. Comparing welfare effects of fiscal policy on the BGP and on the transition path we could show that welfare effects in the short-run may be contrary to those derived for the model on the BGP. So, an increase in transfer payments which reduces welfare in the long-run may raise welfare in the short-run because it goes along with higher consumption. Only when the time horizon is sufficiently long taking into account transition dynamics

yields the same results as the model on the BGP. An increase in the consumption tax rate yields the same welfare effects on the transition path as on the BGP if welfare on the BGP declines. The reason is that the negative welfare effect (decline in consumption) occurs immediately while the positive effect (higher growth rate as a result of more productive public spending) only becomes effective at later time periods. If long-run welfare rises as a result of a higher consumption tax rate it may nevertheless decline on the transition path if the time horizon is sufficiently short. In this case, only when the time horizon is long enough welfare effects on the transition path are the same as for the model on the BGP.

This paper has demonstrated that transition dynamics do matter. As concerns the growth rate we could demonstrate that the growth rates of economic variables may overshoot the long-run balanced growth path as a result of fiscal policy measures. Further, welfare on the transition path may be different from welfare effects in the long-run. Only when the time horizon is sufficiently large long-run effects coincide with effects on the transition path.

Appendix

On the existence, uniqueness and stability of the BGP

To show that there exists a unique BGP we proceed as follows. From (11) we compute c on the BGP as

$$c^* = \frac{\rho}{1 + \tau_C(1 - t_p)} + A(h^*)^{1-\alpha} \left(\frac{(1 - \alpha)(1 - \tau) + \tau t_p}{1 + \tau_C(1 - t_p)} \right)$$

where we neglect the economically meaningless stationary state $c^* = 0$. Inserting this term in (12) gives after simplification

$$\frac{\dot{h}}{h} = \rho + h^{-1} \left(\frac{\tau_C \rho (i_p - t_p)}{1 + \tau_C(1 - t_p)} \right) + Ah^{-\alpha} (i_p - t_p) \left(\frac{\tau_C(1 - \alpha(1 - \tau)) + \tau}{1 + \tau_C(1 - t_p)} \right) - Ah^{1-\alpha} \alpha(1 - \tau),$$

with $\lim_{h \rightarrow 0} \dot{h}/h = \infty$, $\lim_{h \rightarrow \infty} \dot{h}/h = -\infty$ and $\partial(\dot{h}/h)/\partial h < 0$. Since $c^* > 0$ and $\dot{H}/H > 0$ hold for $h^* > 0$ existence and uniqueness of the BGP is proved.

To analyze stability of the BGP we calculate the Jacobian matrix and recall that a negative determinant of that matrix is sufficient and necessary for saddle point stability. The Jacobian matrix corresponding to (11)-(12) is given by

$$J = \begin{bmatrix} c^*(1 + \tau_C(1 - t_p)) & c^*(1 - \alpha)A(h^*)^{-\alpha}((\alpha - 1)(1 - \tau) - \tau t_p) \\ (i_p - t_p)\tau_C + h^*(1 + \tau_C(1 - t_p)) & a_{22} \end{bmatrix},$$

with $a_{22} = (i_p - t_p)\tau A(-\alpha)(h^*)^{-\alpha} - (1 - \alpha)A(h^*)^{1-\alpha}(1 - \tau(1 - t_p)) - (i_p - t_p)\tau_C c^*/h^*$

The determinant of this matrix can be calculated as $\det J = -c^*((1 + \tau_C(1 - t_p))\alpha(i_p - t_p)\tau A(h^*)^{-\alpha} + (1 + \tau_C(1 - t_p))(i_p - t_p)c^*\tau_C(h^*)^{-1} - ((1 - \tau) + \tau t_p)(i_p - t_p)\tau_C(1 - \alpha)A(h^*)^{-\alpha} + (1 - \alpha)A(h^*)^{-\alpha}\alpha(1 - \tau)(i_p - t_p)\tau_C + (1 - \alpha)A(h^*)^{1-\alpha}\alpha(1 - \tau)(1 + \tau_C(1 - t_p)))$.

From $\dot{h}/h = 0$ we can derive the following relation: $-((1 - \tau) + \tau t_p)(i_p - t_p)\tau_C A(h^*)^{-\alpha} = (i_p - t_p)\tau_C(h^*)^{-1}\rho - A(h^*)^{-\alpha}\alpha(1 - \tau)\tau_C(i_p - t_p) + \tau A(h^*)^{-\alpha}(i_p - t_p)(1 + \tau_C(1 - t_p)) + \rho(1 + \tau_C(1 - t_p)) - A(h^*)^{1-\alpha}\alpha(1 - \tau)(1 + \tau_C(1 - t_p))$.

Inserting this expression in $\det J$ gives

$$\det J = -c^*(1 + \tau_C(1 - t_p)) \left((i_p - t_p)(\tau_C c^*(h^*)^{-1} + \tau A(h^*)^{-\alpha}) + \rho - \alpha A(h^*)^{1-\alpha}\alpha(1 - \tau) + C_1 \right),$$

with C_1 a term containing only positive elements. We know that $g = (i_p - t_p)(\tau_C c^*(h^*)^{-1} + \tau A(h^*)^{-\alpha})$ and $g + \rho = A(h^*)^{1-\alpha}\alpha(1 - \tau)$. Using this the determinant can be rewritten as

$$\det J = -c^*(1 + \tau_C(1 - t_p)) (g(1 - \alpha) + \rho(1 - \alpha) + C_1) < 0.$$

Thus, the model is saddle point stable.

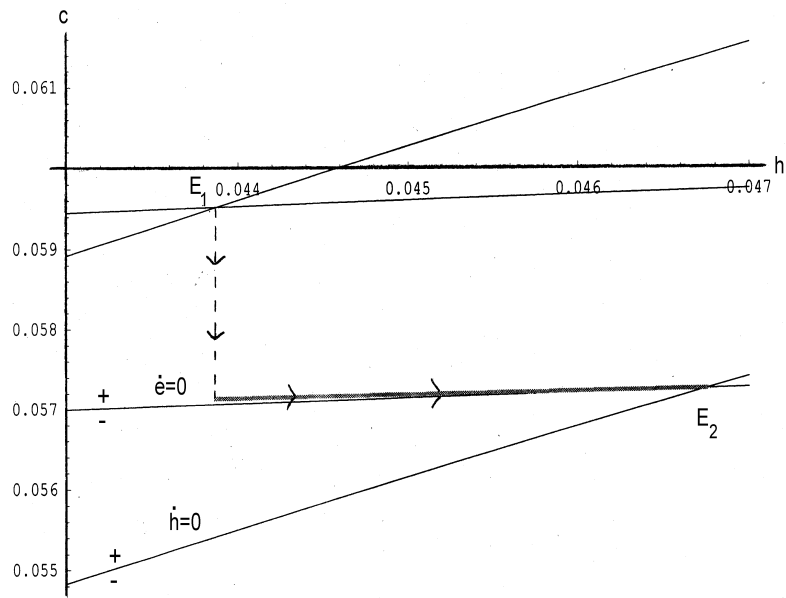


Figure 1

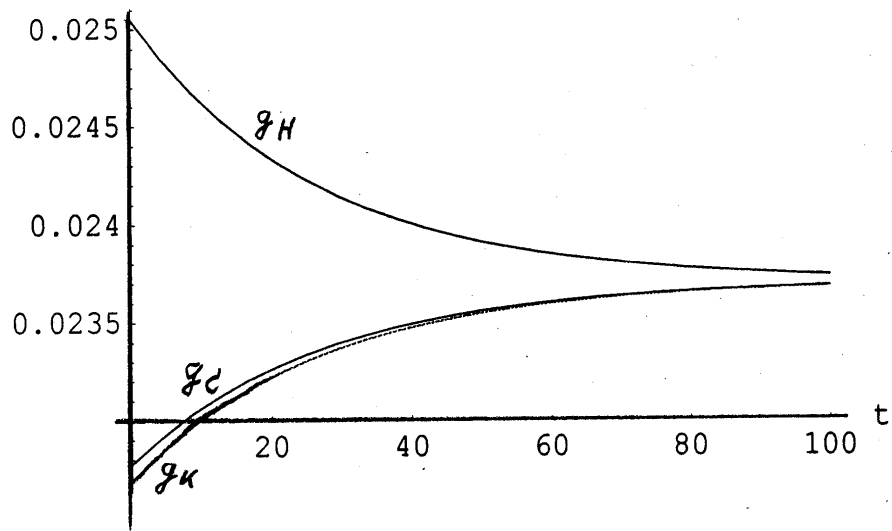


Figure 2

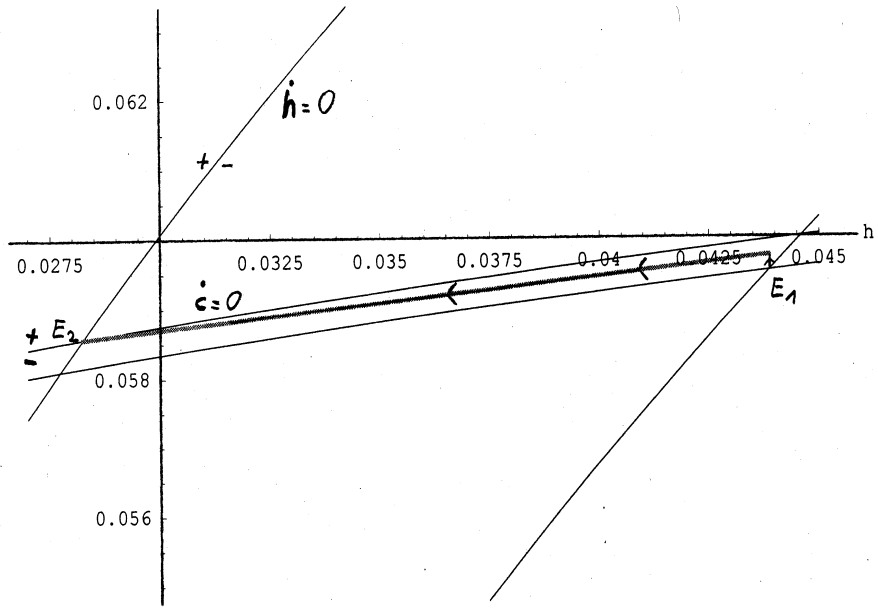


Figure 3

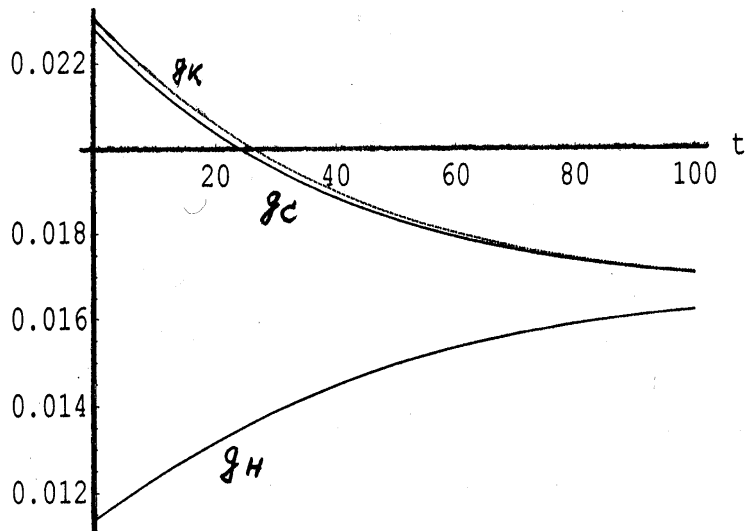


Figure 4

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Fiscal policy in an endogenous growth model with public capital:

How important are transition dynamics?

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Appendix available on request

To demonstrate that the linear system is a good approximation of the nonlinear one we compute welfare for the examples in the left part of tables 5 and 6 for the linear approximation (as in the paper) and for the nonlinear system. To find the stable manifold of the nonlinear system we proceed as follows:

1. First we start in an ϵ -environment around the rest point of the dynamic system and solve the differential equations (10) and (11) by backward integration.

2. We look for that t_0 which gives $h(t_0) = h^*$, with h^* the BGP value of h for the model with the original parameter values (e.g. for $\tau_C = 0.15$ in figure 1 and in the left part of table 5). For example, in figure 1 this gives the value for h^* in equilibrium E_1 . Further, we note the value of c for $t = t_0$, i.e. $c(t_0)$. This gives the starting values for h and c on the stable manifold after the change of the fiscal parameter (e.g. for $\tau_C = 0.2$ in figure 1 and on the left hand side of table 5).

3. We set $t_0 = 0$ and integrate (10) and (11) forward with starting values $h(t_0)$ and $c(t_0)$. Together with $K_0 = 1$ this gives $C(0)$ and the time path for $h(t)$ which are used to compute $C(t)$ and then the integral (16).

Table A1, which corresponds to the left part of table 5, gives the values for $F_1(t_f)$ for the linear and for the nonlinear model. One realizes that the differences are small. In particular, the qualitative results in the paper remain the same.

Table A1. Parameter values are set to their benchmark values as in the paper (except for τ_C), in particular $i_p = 0.05$, $t_p = 0.01$.

Linear Model				Nonlinear Model			
τ_C	$F_1(10)$	$F_1(25)$	$F_1(50)$	τ_C	$F_1(10)$	$F_1(25)$	$F_1(50)$
0.2	-21.7002	-37.5794	-45.9672	0.2	-21.7007	-37.5805	-45.969

Table A2, which corresponds to the left part of table 6, gives the values for $F_1(t_f)$ for the linear and for the nonlinear model. Again the differences are small and the qualitative results in the paper do not change when the nonlinear model is applied.

Table A2. Parameter values are set to their benchmark values as in the paper (except for t_p), in particular $i_p = 0.05$.

Linear Model				Nonlinear Model			
t_p	$F_1(3)$	$F_1(10)$	$F_1(25)$	t_p	$F_1(3)$	$F_1(10)$	$F_1(25)$
0.015	-7.7666	-21.3832	-37.0567	0.015	-7.7669	-21.3842	-37.059