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# A Keynesian Based Econometric Framework for Studying Monetary Policy Rules

by

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**Abstract**

In the framework of a Keynesian based monetary macro model we study the implications of targeting monetary aggregates or targeting the interest rate as two alternative monetary policy rules. Whereas the former targets the inflation rate indirectly, through the control of the money supply, the latter, also called the Taylor rule, implies direct inflation targeting. Our monetary macromodel exhibits: asset market clearing, disequilibrium in the product and labor markets, sluggish price and quantity adjustments, two Phillips curves for the wage and price dynamics and expectations formation which represents a combination of adaptive and forward looking behavior. The parameters of different model variants are estimated partly through single equation and partly through subsystem estimations for U.S. time series data 1960.1-1995.1. With the estimated parameters system simulations for the two monetary policy rules are performed and the stability as well as impulse-response properties of the two rules are explored.

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# 1 Introduction

Recently, in macroeconomics the quantitative study of monetary policy rules has been undertaken in a variety of frameworks. Such frameworks are, for example, the large-scale macroeconometric models (Fair, 1984 and the contributions collected in Taylor, 1999), the VAR (Bernanke and Blinder, 1992, Sims,1997, Fuhrer and Moore,1995) and the optimization based approach (Rotemberg and Woodford, 1997, 1999 and Christiano and Gust, 1999). Usually two alternative monetary policy rules have been considered, namely the monetary authority 1) targeting monetary aggregates or 2) targeting the interest rate. The former implies an indirect and the latter a direct inflation targeting. The latter rule originates in Taylor (1993) and has also been called the Taylor rule.[1] As has been shown historically, most central banks of OECD countries switched during the 1980s from the policy of controlling monetary aggregates to targeting inflation rates through controlling short -term interest rates.[2] The second type of monetary policy rule, the Taylor rule, has recently been given much attention and has extensively been evaluated in the context of macroeconometric frameworks, see Taylor (1999). This paper employs a small scale Keynesian integrated macromodel to evaluate the above monetary rules of central banks'.

The Keynesian monetary growth model presented and estimated here exhibits along the lines of Flaschel, Franke and Semmler (1997) asset market clearing, disequilibrium in product and labor market, sluggish price and quantity adjustments, two Phillips-curves for the wage and price dynamics and expectations formulation which represents a combination of adaptive and forward looking behavior. Moreover, as in Chiarella and Flaschel (2000, Ch.6), the current paper also includes real growth, inflationary dynamics and inventory adjustment. As to the historical tradition, on the demand side it is Keynesian, it makes use of Kaldor's distribution theory, uses the asset market structure as in Sargent's (1987, part I) Keynesian model, employs Malinvaud's (1980) investment theory, and a Metzler type inventory adjustment process[3] and uses an expectations mechanism which is forward and backward looking.[4]

More specifically, we consider a closed three sector economy (households, firms and government), where there exist five distinct markets; for labor, goods, money, bonds and equity (which are perfect substitutes of bonds).[5] In order to briefly summarize our model we use the following table. We employ the index $d$ to denote 'quantities demanded' and no index in the case of 'quantities supplied'. The symbols in the following table should be clear as to their economic meaning (a detailed list of the

[1] A rule of this type, however, can already be found in Fair (1984).

[2] See Svensson (1997). Although the German Bundesbank claims that it always had target the money supply there are several papers which show that in fact the Bundesbank has also used interest rate targeting, see Clarida and Gertler, (1995), Bernanke and Mihov (1998) and Clarida, Gali and Gertler (1997).

[3] See Franke and Lux (1993) and Franke (1996).

[4] See Groth (1988) and many recent contributions collected in Taylor (1999)

[5] We restrict ourselves to this standard, basic framework to stay, at least initially, very close to traditional foundations of Keynesian dynamics, see Sargent (1987, Ch.I-V) and Chiarella and Flaschel (2000, Ch.1).

notation employed is provided in the appendix).

| | Labor market | Goods market | Money market | Bonds market | Equities market |
|---|---|---|---|---|---|
| Households | $L$ | $C$ | $M^d$ | $B^d$ | $E^d$ |
| Firms | $L^d$ | $Y, I + \delta K$ | – | – | $E$ |
| Government | – | $G$ | $M$ | $B$ | – |

This is the basic structure of the closed economy model considered in this paper. Firms have desired capacity and desired inventories. Temporary deviation from those benchmarks are caused by unexpected changes in aggregate goods demand. A distinguishing feature of Keynesian models, in particular in contrast to equilibrium macromodels,[6] is that the consideration of under- or over-utilized capital besides an under- or over-utilized labor force is important. Yet, even in Keynesian tradition it has often been neglected, see Malinvaud (1980) for an exception in the context of a stationary economy.

Moreover, we include growth in a, although basic, but consistent way—a feature that is generally not often explored in the literature on Keynesian macrodynamics.[7] The fact that growth is lacking in most monetary disequilibrium macromodels is an important discontinuity in the development of the literature on such macroeconomics. Furthermore, our small scale model is complete in the sense that we consider all the major markets and define the financing conditions and budget restrictions of households, firms and the government. The model gives rise to nine interdependent laws of motion or – via a suitable assumption on wealth effects and tax collection – to a six-dimensional integrated dynamic system that neglects the dynamics of the government budget constraint in particular.

It should also be noted that all behavioral and technical relationships in the following model have been chosen to be linear as much as possible. It is not difficult to introduce into the model some well-known nonlinearities that have been used in the literature on real, monetary and inventory dynamics of Keynesian type. We use only unavoidable nonlinearities in the model. Such nonlinearities naturally arise from the growth rate formulation of certain laws of motion, certain unavoidable ratios and the multiplicative interaction of variables. Already on the basis of these most basic types of nonlinearities it can be shown that interesting dynamic properties will arise – without any 'bending of curves' often employed to tame the assumed explosive dynamical behavior of the partial submodels.

The model's dynamic features for the two policy regimes are explored for certain parameter constellations. We transform, and also extend, the continuous time model of Chiarella and Flaschel (2000, Ch.6) into a discrete time model. The general dynamic behavior of our system cannot be studied analytically with currently available techniques – apart from being able to make some basic statements about it. For the model

[6] See, for, example, Flaschel, Franke and Semmler (1997, Chs.1-4).

[7] Keynesian models of growth are rarely mentioned in surveys on the literature on monetary growth, as for example, in the recent survey article on 'Money, inflation and growth' by Orphanides and Solow (1990).

with money supply rule it is indicated, following Chiarella and Flaschel (1996, 2000, Ch.6), that for a certain range of parameter constellations interesting dynamics, for example, persistent cycles, may arise. On the other hand, the Taylor rule appears to add further stabilizing forces to this type of model, since it counteracts the destabilizing Mundell effect of inflationary expectations in a more direct way.

In order to match the model with the U.S. macroeconomic time series data we estimate key parameters through single equation or subsystem estimations using data from 1960.1-1995.1. In the estimation of the parameters for the wage-price dynamics and for the inventory dynamics as well as investment and consumption functions, expectations variables appear which are not observables. We can, however, transform the equations to be estimated and estimate the adjustment speeds involved in the expectations dynamics.

Given our parameter estimates we can explore the stability properties of our two policy rules and can study the question whether the impulse-response functions of our model variants match those of the data. Since both policy rules are defined here as feedback rules we find that they generate less instability than compared with studies that employ only auto regressive processes for the monetary policy.[8] Overall, our model is roughly able to replicate well known stylized facts obtained, for example, from VAR studies of macroeconomic variables. Our macroeconometric framework should be viewed only as first step toward describing the historical data and policy rules in a disequilibrium framework. Yet, it appears useful, since its results can compete with currently widely used equilibrium macromodels of RBC type.

The remainder of the paper is organized as follows. Section 2 introduces the small scale integrated monetary macromodel. Section 3 studies the steady state and the dynamics of the model, in intensive form. In section 4 we describe our econometric estimation strategy and report results from our estimations. Section 5 evaluates our results and section 6 concludes the paper. The appendices provide the notation used.

# 2 A Monetary Macrodynamic Model

In this section we reformulate and generalize the continuous time monetary macromodel as developed in Chiarella and Flaschel (2000, Ch.6) and extended in Chiarella, Flaschel, Groh and Semmler (2000), in discrete time which makes the time structure of the model much more transparent. We provide a structural form of the model that is theoretically coherent in its use of budget constraints, dating of activities and expectations and that – due to its specific extensions – can be investigated from the empirical point of view. The model is presented in terms of modules where each of them allows for a number of modifications that do not change the implied dynamics in significant ways, but make it more realistic from the empirical point of view.

We start with some notations. A complete list of notations is given in the appendix. Our Keynesian disequilibrium model uses in particular the following variables characterizing income distribution and asset allocation:

$$\text{1.} \qquad \underline{\text{Definitions (real remunerations, real wealth and rates of growth):}}$$

$$\omega_t \;=\; w_t/p_t, \;\; u_t = \omega_t/x_t, \;\; \rho_t^e = (Y_t^e - \delta K_{t-1} - \omega_t L_t^d)/K_{t-1}, \tag{1}$$

[8] See, for example, Christiano and Gust (1999).

$$
\begin{aligned}
W_t &= (M_{t-1} + B_{t-1} + p_{et}E_{t-1})/p_t, \quad p_b = 1. && (2)\\
\rho_t^n &= (Y_t^{dn} - \delta K_{t-1} - \omega_t L_t^n)/K_{t-1}, && (3)\\
Y_t^{dn} &= \bar{U}Y^p/(1 + n\beta_{n^d}), \quad Y_t^n = \bar{U}Y^p, \quad L_t^n = Y_t^n/x && (4)\\
\hat{z}_t &= \Delta z_t/z_{t-1} = (z_t - z_{t-1})/z_{t-1}, \quad \text{growth rate of a variable } \; z_t && (5)
\end{aligned}
$$

This set of equations represent real wages, $\omega_t$ and the wage share $u_t$, the expected real rate of return on capital, $\rho_t^e$, based on sales expectations $Y_t^e$ at $t-1$ for the present point in time $t$ and the definition of the current stock of real wealth $W_t$. Note that stocks that exist at time $t$ are indexed by $t-1$, while their actual reallocation and revaluation happens in $t$ and is thus indexed by $t$. Note also that secondary market components of our financial markets are integrated (later on) with the primary one, i.e., new issue and new demand for such assets, since there is no separation between primary and secondary markets (new issues and resale) in the present model type, see module 5. Current real wealth held by households in $t$ is here composed of money $M_{t-1}$, fixed price bonds $B_{t-1}$ ($p_b = 1$) and equities $E_{t-1}$ as in Sargent (1987)[9] and is determined on the basis of the current market prices for equities, $p_e$, and output $p$. Furthermore, current output is produced with the capital stock given at $t-1$ and with labor that is paid in $t$. Note furthermore that the definition of growth rates $\hat{z}$, and of first differences, is indexed forward in order to ease the presentation of the intensive form of the model later on. Note finally that we have added here the definition of the normal rate of return on capital, which is based on full capacity operation and which thus only varies with the real wage rate, in order to allow for an investment function (later on) that separates profitability effects from changes in actual activity levels.

Describing income distribution and savings along the line of Kaldor (1966)[10] we propose the behavior of households, represented by workers and asset holders, to be determined by the following set of equations. All behavioral equations are chosen as linear as possible. Only intrinsic 'natural' nonlinearities are allowed for at present. Later, extrinsic nonlinearities may be added in a systematic way.

2. Households (workers and asset-holders):

$$
\begin{aligned}
C_t &= (1 - s_w)(\omega_t L_t^d + r_t B_{t-1}^w/p_t - T_t^w)\\
&+ (1 - s_c)(\rho_t^e K_{t-1} + r_t B_{t-1}^c/p_t - T_t^c), && (6)\\
S_{pt} &= s_w(\omega_t L_t^d + r_t B_{t-1}^w/p_t - T_t^w) + s_c(\rho_t^e K_{t-1} + r_t B_{t-1}^c/p_t - T_t^c) && (7)\\
W_t + S_{pt} &= (M_t^d + B_t^d + p_{et}E_t^d)/p_t, && (8)\\
\hat{L}_{t+1} &= n_l = \text{const.} && (9)
\end{aligned}
$$

Aggregate consumption of households, $C_t$, see equ. (6), is based on differentiated saving ratios, $s_w, s_c$, of workers and pure asset holders. Workers save in the form of bonds and thus have real interest income of amount $r_t B_{t-1}^w/p_t$ in addition to their wage income $\omega_t L_t^d$. We assume for both types of households that their real taxes, $T_t^w, T_t^c$, are paid out of their income in a lump sum fashion (see module 4.). Equ. (7) provides the definition of real private savings, $S_{pt}$, of both workers and pure asset

[9] Assuming consols ($p_b = 1/r_t$) in place of the fixprice bonds assumed by Sargent (1987) does not significantly alter the dynamics of the private sector to be considered below, due to the neglect of interest income and wealth effects in the present formulation of the model.

[10] We do, however, not consider corporate saving as Kaldor did.

holders, which is, in equ. (8), allocated to the actual changes in the stock of money, of bonds and of equities. Equ. (8) thus states how real wealth and real savings act as budget restriction for aggregate stock demand for real money balances, real bond and real equity holdings of both workers and asset owners at time $t$ (Walras' law of stocks and flows). The supply of labor, $L_t$, is inelastic at each moment in time with a rate of growth, $\hat{L}_{t+1,}$ given by $n_l$, the natural rate of growth. Note here again that we use end of period indices to characterize growth rates of quantities and prices.

The production sector and the behavior of firms are described by the following set of equations:

$$
\begin{aligned}
&3. \quad \underline{\text{Firms (production, investment and inventory):}} \\
Y_t^p &= y^p K_{t-1},\ y^p = \text{ const. },\ U_t = Y_t/Y_t^p, && (10)\\
L_t^d &= Y_t/x,\ \hat{x} = n_x = \text{ const.},\ V_t = L_t^d/L_t = Y_t/(xL_t), && (11)\\
I_t/K_{t-1} &= i_1(\rho^m - \xi - (r_t^m - \pi_t^m)) + i_2(U_t - \bar{U}) + n, \quad n = n_l + n_x && (12)\\
S_{ft} &= Y_{ft} = Y_t - Y_t^e = \mathcal{I}_t, && (13)\\
Y_t^e &\neq Y_t^d = C_t + I_t + \delta K_{t-1} + G_t && (14)\\
\frac{p_{et}\Delta E_t}{p_t} &= I_t + Y_t^e - Y_t^d = I_t + \Delta N_t - \mathcal{I}_t, \\
\Delta E_t &= E_t - E_{t-1}, \quad \Delta N_t = N_t - N_{t-1} && (15)\\
\hat{K}_t &= \Delta K_t/K_{t-1} = I_t/K_{t-1}, \quad \Delta K_t = K_t - K_{t-1}. && (16)
\end{aligned}
$$

According to equ.s (10),(11), firms produce output, $Y_t$, in the technologically simplest way possible, via a fixed proportions technology characterized by the given potential output-capital ratio $y^p = Y_t^p/K_{t-1}$ and a ratio $x$ between actual output $Y_t$ and labor $L_t^d$ needed to produce the output, which grows in time with the given rate $n_x$. This simple concept of a fixed proportions technology exhibiting Harrod neutral technical progress allows for a straightforward definition of the rate of utilization of capital, $U_t$, and labor, $V_t$.[11] Note that current investment $I_t$ will not have a capacity effect in the current point in time $t$, i.e., capacity output is restricted by the capital stock $K_{t-1}$, and that labor is paid ex post, at $t$, from the proceeds obtained from current sales, $Y_t^d$.

In equ. (12) investment per unit of capital, $I_t/K_{t-1}$, is driven by two forces, the excess return of the normal rate of return on capital, $\rho_t^m$, over the real rate of interest, $r_t^m - \pi_t^m$, and the (then uncorrelated) deviation of actual capacity utilization $U_t$ from the normal or non-accelerating -inflation rate of capacity utilization $\bar{U}$. Note that all these rates are understood as medium run averages to be explained below. Note also that we have added a risk premium to the real rate of interest in comparison to the real rate of return on capital. There is also an unexplained trend term in the investment equation which is set equal to the natural rate of growth, plus the rate of technical progress, for reasons of simplicity, see also Sargent (1987, Ch.5) in this regard. An endogenous treatment of the trend term may however be desirable.[12]

Savings of firms, equ. (13), is equal to the excess of output over expected sales (caused by planned inventory changes). We assume in this model that expected sales

[11] Chiarella and Flaschel (2000, Ch.5) show how such an approach can be extended to the case of smooth factor substitution without substantial change of the model.

[12] See Chiarella, Flaschel, Groh and Semmler (2000) for a demonstration that the nature of the present approach to disequilibrium growth is not changed very much by the inclusion of, for example, endogenous technical change of Uzawa-Lucas-Romer type.

are the basis of firms' dividend payments (after deduction of capital depreciation, $\delta K_{t-1}$, and real wage payments, $\omega_t L_t^d$.)[13]. Equ. (14) shows the excess of expected demand over actual demand. In the present version of the model any such excess demand has to be financed by firms by issuing new equity (or gives rise to windfall profits if this excess is negative). It follows, as expressed in equ. (15), that the total amount of new equity issued by firms must equal the intended fixed capital investment and unexpected inventory changes, $Y_t^e - Y_t^d = N_t - N_{t-1} - \mathcal{I}_t$; compare our formulation of the inventory adjustment mechanism in module 6. Finally, equ. (16) states that (fixed business) investment plans of firms are always realized in this Keynesian (demand oriented) context, by way of corresponding inventory changes.

We now turn to a brief description of fiscal and monetary policy rules where the former are here still chosen in a way as simple as possible[14] in the context of a growing economy, since we want to concentrate on the behavior of the private sector of the economy and on monetary policy rules in the following (all fiscal variables are thus given magnitudes in the intensive form of the model). Government savings is defined in equ. (19) and the government budget restriction is given by equ. (22), which however is of no importance for the dynamics of the model due to our neglect of interest income and wealth effects.

4. Government (fiscal and monetary authorities):

$$t^w = \frac{T_t^w - r_t B_{t-1}^w/p_t}{K_{t-1}} = const., \quad t^c = \frac{T_t^c - r_t B_{t-1}^c/p_t}{K_{t-1}} = const. \tag{17}$$

$$G_t = gK_{t-1}, \quad g = const. \quad (T_t = T_t^w + T_t^c) \tag{18}$$

$$S_{gt} = T_t - r_t B_{t-1}/p_t - G_t = (t^w + t^c - g)K_{t-1}, \tag{19}$$

$$\hat{M}_t = \Delta M_t/M_{t-1} = \mu_t, \quad \Delta M_t = M_t - M_{t-1} \tag{20}$$

$$\mu_t = \mu_{t-1} + \beta_{m_1}(\bar{\mu} - \mu_{t-1}) + \beta_{m_2}(\bar{\pi} - \hat{p}_{t+1}) + \beta_{m_3}(\bar{U} - U_t), \quad \beta_{m_i} > 0 \tag{21}$$

$$\Delta B_t = p_t G_t + r_t B_{t-1} - p_t T_t - \Delta M_t, \quad \Delta B_t = B_t - B_{t-1}. \tag{22}$$

As money supply rule has been extended in comparison to earlier presentations of the macro model in order to be directly comparable to the interest rate policy rule to be described below.[15]

As stated we want to focus on monetary policy in this paper and we thus have stylized fiscal policy in as simple a way as possible. As regards to the monetary policy

[13]Further instruments for the financing decision of firms are considered in Franke and Semmler's (1999) model of the financial market. For simplicity we here, however, stick to the framework used in Sargent (1987, part I) and assume that firms do not borrow and retain no expected earnings

[14]We here assume that government expenditure per unit of capital is a given magnitude. The same holds for wage taxation and property income taxation, both net of interest, see Sargent (1987, Ch.5) for the same type of assumption, but also .

[15]Note that interest rate steering according to this money supply rule is fairly roundabout, since it involves all of the following static and dynamic equations (written for simplicity in continuous time):

$$\begin{aligned} r &= r_o + (h_1 y - m)/h_2, \quad m = M/(pK) \\ \hat{m} &= \mu - (\hat{p} + \hat{K}) \\ \dot{\mu} &= \beta_{m_1}(\bar{\mu} - \mu) + \beta_{m_2}(\bar{\pi} - \hat{p}) + \beta_{m_3}(\bar{U} - U) \end{aligned}$$

The Taylor interest rate policy rule, to be described below, is much more directly operating on the interest rate in order to steer economic activity and the rate of inflation.

we will explore alternative rules. Module 4 above assumes that the monetary authority, for controlling inflation, targets the supply of money, as represented in equ. (21).We formulate the money supply rule as feedback rule. The growth rate of the money supply, $\hat{M}$, is assumed to be steered towards a constant target term $\bar{\mu}$, but subject to temporary deviations when current inflation differs from the target level which in turn are subject to further deviations by a term that characterizes the current state of the business cycle. Too high inflation as compared to the target level thus, for example, induce the central bank to moderate its adjustment towards the growth target $\bar{\mu}$ and this the more the higher the activity in the business cycle whereby it is assumed that $\bar{\pi} = \bar{\mu} - n$.

As modern alternative to this money supply oriented policy we also investigate the Taylor rule according to which the monetary authority aims at setting the nominal rate of interest in response to deviations of the interest rate from its steady state value, the deviations of the actual rate of inflation, $\hat{p}_t$, from a target rate of inflation, $\bar{\pi}$,[16] and the deviations of the actual rate of capacity utilization from the target rate of capacity utilization, see equation (23) below. We also assume, as in Clarida, Gali and Gertler (1998), some interest rate smoothing in the application of the Taylor rule. This alternative rule, often called the central bank's reaction function, thus reads:

$$r_{t+1} = r_t - \beta_{r_1}(r_t - r_o) + \beta_{r_2}(\hat{p}_{t+1} - \bar{\pi}) + \beta_{r_3}(U_t - \bar{U}), \quad \beta_{r_i} > 0. \tag{23}$$

Note that the here employed rate of inflation is a forward rate of inflation[17] where we, in contrast to our use of expected medium run averages, disregard errors in expectations formation, see our presentation of the wage-price sector in module 7 of the model where myopic perfect foresight interacts with medium run expectations of inflation in the mutual interdependence of the wage and price setting process. Note finally that the above Taylor rule assumes that money demand is always realized at the nominal rate of interest set by the monetary authority. In view of the fiscal rules for government and either of the monetary rules for the central bank the issue of new bonds by the government (net of open market operations by the central bank) is then determined residually via equ. (22). This states that the resulting money and bond financing must exactly cover the deficit in government expenditure financing. This holds also for the Taylor rule.[18]

We now describe the asset market equilibrium conditions of the model:

5. Equilibrium conditions (asset-markets):

$$W_t + S_{pt} = (M_t^d + B_t^d + p_{et}E_t^d)/p_t, \tag{24}$$

$$M_t = M_t^d = h_1 p_t Y_t + h_2 p_t K_{t-1}(r_o - r_{t+1}) \tag{25}$$

[16] One has to assume, that $\bar{\pi} = \bar{\mu} - n$ holds in order to get a steady state solution for the model.

[17] Svensson (1997) suggests such a formulation of the inflation gap.

[18] The rate of change of the money supply $\mu$ implied by the Taylor rule reads (in terms of continuous time for simplicity):

$$\mu = \hat{M} = \hat{p} + \frac{h_1\dot{Y} + h_2\dot{K}(r_o - r) - h_2 K\dot{r}}{h_1 Y + h_2 K(r_o - r)}.$$

This expression differs considerably from the money supply rule of module 4. of the model. It must be inserted into the Government budget constraints (22) in order to determine the evolution of government debt in the case of the Taylor interest rate policy rule.

$$
\begin{aligned}
r_{t+1} &= \frac{p_{t+1}Y^e_{t+1} - \delta p_{t+1}K_t - w_{t+1}L^d_{t+1}}{p_{et}E_t} + \frac{(p_{e,t+1} - p_{et})E_t}{p_{et}E_t} \\
&= \frac{\rho^e_{t+1}p_{t+1}K_t}{p_{et}E_t} + \hat{p}_{e,t+1} \qquad (26) \\
B_t &= B_{t-1} + \Delta B_t = B^d_t, \quad E_t = E_{t-1} + \Delta E_t = E^d_t. \qquad (27)
\end{aligned}
$$

The source of the stock demands for financial assets is again shown in (24) as the aggregate real value of the existing stock at current market prices plus real savings of workers and the assets owning households. Money demand is specified as a simple linear function of nominal output, $p_tY_t$, and interest $r_{t+1}$ to be paid on the currently traded bonds in the next period ($r_o$ the steady state rate of interest), but with $K_t$ in place of $W_{t+1}$ as measure of real wealth. This equation determines the rate of interest for the period $[t, t+1]$ on the basis of predetermined values for the other variables of the money demand equation.[19]Note also that money market equilibrium (25) does not feed back into the rest of the model in the case of the Taylor monetary policy rule, in which case money supply is always adjusted in order to meet money demand at the nominal rate of interest $r_{t+1}$ set by the central bank. The form (25) of the money demand function is chosen in the above way in order to allow for a simple formula for the nominal rate of interest in the intensive form of the model. Nonlinear money demand functions with real wealth in the place of the capital stock would be more appropriate and thus should replace this simple function in further extensions of this model.[20]

Asset markets are assumed to clear at all times. Equ. (25) describes this assumption for the money market providing the equation for the current market rate of interest to be used for the payments of interest in the next point in time in the case of the money supply rule (21). Bonds and equities are assumed to be perfect substitutes (on the basis of expected or actual rates of return in $t+1$ which are formed in $t$ based on information known at $t$) when the trade on the stock market is performed, see equation (26). This equation assumes myopic perfect foresight and equates on this basis the interest rate with the expected rate of return on equities, i.e., the sum of the dividend rate of return and of the actual capital gains per share in the period $[t, t+1]$. Due to the neglect of wealth effects this equation does not feed back into the rest of the dynamics and is thus neglected in the following. Assuming that bonds and equities are perfect substitutes amounts to assuming, in the light of the assumed Walras's law of stocks and flows that the clearing of the money market implies that the bond and equity market are then cleared as well, with wealth holders accepting any reallocation of their wealth with respect to bonds and equities.

The disequilibrium in the goods market is an important driving force for the dynamics of our economy. This is described by the following set of equations:

## 6. Disequilibrium in the goods market (adjustment mechanism):

[19]This convention conforms with the definition of $\rho^e_{t+1}$ that we use in the determination of share prices below.

[20]The above simple money demand function can be obtained as a Taylor approximation of a general money demand function if it is assumed that money demand is homogeneous of degree 1 in income and wealth and if the variable $K_t$ is used as a proxy for the evolution of real wealth.

$$
\begin{aligned}
S_t &= S_{pt} + S_{gt} + S_{ft} = p_{et}\Delta E_t/p_t + \mathcal{I}_t = I_t + \Delta N_t, && (28)\\
Y_t^d &= C_t + G_t + I_t + \delta K_{t-1}, && (29)\\
&= (1-s_c)Y_t^e + (s_c - s_w)\omega_t L_t^d + \gamma K_{t-1}\\
&+ (i_1(\rho_t^m - \xi - (r_t^m - \pi_t^m)) + i_2(U_t - \bar{U}) + n + \delta)K_{t-1}\\
N_t^d &= \beta_{n^d} Y_t^e && (30)\\
\mathcal{I}_t &= nN_t^d + \beta_n(N_t^d - N_{t-1}), && (31)\\
Y_t &= Y_t^e + \mathcal{I}_t, && (32)\\
Y_{t+1}^e &= Y_t^e + nY_t^e + \beta_{y^e}(Y_t^d - Y_t^e) && (33)\\
N_t &= N_{t-1} + Y_t - Y_t^d. && (34)
\end{aligned}
$$

It is easy to check, by means of the presented budget equations and savings relationships, that the consistency of new money and new bonds flow supply and demand implies the consistency of the flow supply and demand for equity. Equ. (28) of this disequilibrium block of the model describes on this basis simple identities that can be related with the ex post identity of total savings $S_t$ and total investment $I_t^a$ for a closed economy. It is here added for accounting purposes solely. Equ. (29) defines aggregate demand, $Y_t^d$, which is assumed to be never constrained in the present model.

In equ. (31) desired inventories $N_t^d$ are assumed to be a constant fraction of expected sales, $Y_t^e$, and intended inventory investment, $\mathcal{I}_t$, is determined on this basis via the adjustment speed $\beta_n$ multiplied by the current gap between intended and actual inventories, $(N_t^d - N_t)$. The latter is augmented by a growth term that integrates in the simplest way the fact that this inventory adjustment rule is operating in a growing economy. Output of firms, $Y_t$, in equ. (32) is the sum of expected sales and planned inventory adjustments. Sales expectations are formed in a purely adaptive way, see equation (33). Finally, in eq. (34), actual inventory changes are given by the discrepancy between actual output, $Y_t$, and actual sales, $Y_t^d$.

We now turn to the last module of our model which is the wage-price module. It decomposes the standard across markets Phillips curve mechanism into two dynamic equations augmented by a law of motion for inflationary expectations formation concerning the medium run.

7. Wage-Price-Module (adjustment equations):

$$
\begin{aligned}
\hat{w}_{t+1} &= \beta_w(V_t - \bar{V}) + \kappa_w(\hat{p}_{t+1} + n_x) + (1-\kappa_w)(\pi_t + n_x), && (35)\\
\hat{p}_{t+1} &= \beta_p(U_t - \bar{U}) + \kappa_p(\hat{w}_{t+1} - n_x) + (1-\kappa_p)\pi_t, && (36)\\
\pi_{t+1} &= \pi_t + \beta_\pi(\hat{p}_{t+1} - \pi_t). && (37)
\end{aligned}
$$

Our above representation of the wage-price module of the model is based on fairly symmetric assumptions on the causes of wage- and price-inflation. Wage inflation for $[t, t+1]$, according to eq. (35), is driven, on the one hand, by a demand pressure component, given by the deviation of the actual rate of employment, $V_t$, from the NAIRU-rate, $\bar{V}$. On the other hand, it is driven by a cost push term, measured by a weighted average of the short-run future rate of price inflation, $\hat{p}_{t+1}$ (representing myopic perfect foresight) and an expected rate of inflation, $\pi_t$, which we interpret as concerning the medium run, both augmented by the growth rate of labor productivity. Similarly, in equ. (36), price inflation is driven by the demand pressure term, $(U_t -$

$\bar{U}$), $\bar{U}$ the NAIRU rate of capacity utilization, and a cost pressure term, represented by the weighted average of the short-run future rate of wage inflation $\hat{w}_{t+1}$, again allowing for perfect foresight in the short-run, to be diminished by the growth rate of labor productivity, and again the rate of inflation $\pi_t$ expected to hold over the medium-run.[21]The rate of inflation $\pi_t$, expected to hold over the medium run, is in turn determined by assuming that it follows a weighted average of past inflation rates, leading to an inflationary expectations mechanism as in (37).

We stress that we have assumed myopic perfect foresight as far as asset markets and short-run expectations in the wage-price mechanism are concerned. This is unproblematic for the Keynesian structure of the model as long as wage and price adjustment does not solely depend on these short run measures of cost-pressure, but is also paying attention to adaptively formed medium run or average developments of inflation thereby introducing inertia into the accelerator terms of the wage-price dynamics regarding upward or downward adjustments of wages and prices. The short-run accelerators coefficients in the wage and price Phillips curves are thus both smaller than one, which reduces the power of the myopic perfect foresight to a rather secondary issue, (though rational expectations are in fact assumed in order to put not too much weight on possible short-run errors in inflationary expectations). Yet, as far as sales expectations are concerned we still rely in this model on a simple adaptive expectations mechanism.

Short-run expectations of price and wage inflation (as said for reasons for simplicity without any error term) thus do not translate themselves one to one and immediately into wage claims or price level changes, but they are here further increased (diminished) if past inflation rates have been higher (lower) and / or if future inflation over the medium run is expected to be higher (lower) compared to what is currently the case. These aspects of our wage-price sector introduce inertia in a new way without violation of the condition that the labor market and the goods market must be balanced at the steady state. Assuming errors in the judgments on currently occurring wage and price inflation would make the model more realistic, but would not alter its dynamics significantly, since the important thing in this module is represented by the fact that the coefficients in front of current price and wage cost pressures are in general less than unity[22] (as was found in empirically oriented studies of the short-run accelerator term in the conventional price Phillips curve). We thus neglect errors in wage-price changes that are currently occurring and thus include into our model a perfectness that is generally considered a problem for conducting Keynesian type aggregate demand analysis, but which indeed is only an assumption of very secondary importance in the demand driven model of this paper.

It is obvious from this description of the model that it is, on the one hand, already a very general description of macroeconomic disequilibrium dynamics. On the other hand, it is still dependent on some simplifications with respect to financial markets[23] and the fiscal policy rules. This can be justified at the present stage by observing that many of its simplifying assumption are indeed typical for macrodynamic models

[21]A related determination of the wage-price dynamics by cost-push and demand pressure components can be found in Fair (1997); although in Fair wages are not really driven by a demand pressure component.

[22]One coefficient less than unity is in fact already sufficient.

[23]See for example, Franke and Semmler (1999) for the formulation of an advanced financial sector in a macromodel.

which attempt to provide a complete description of a closed economy, see in particular the model of Keynesian dynamics of Sargent (1987, Part I). Also, those incomplete specifications may not prevent us from successfully calibrating the model.

Lastly we want to remark that we have assumed in the investment function (12) as expression for the expected rate of inflation the medium-run rate determined in the wage price module of the model. Therefore we have to use a medium run time horizon in this investment behavior with respect to nominal interest and real profitability as well, which we here for reasons of simplicity determine as follows:

$$\begin{aligned} \rho_t^m &= \sum_{i=0}^{11} \delta_i^\rho \rho_{t-i}^n, \quad \sum_{i=0}^{11} \delta_i^\rho = 1, \\ r_t^m &= \sum_{i=0}^{11} \delta_i^r r_{t-i}, \quad \sum_{i=0}^{11} \delta_i^r = 1, \\ \pi_t^m &= \pi_t \quad \text{(as before)}. \end{aligned}$$

Here it is also appropriate to relabel the former variable $\pi_t$, by $\pi_t^m$, to clearly show where we use here concepts that refer to a medium run horizon. Note that such an extension of the model introduces further lags into the model that reflect the adjustment of expectations with respect to a medium-run horizon, of which however we expect that they do not alter the dynamics of the model significantly.

# 3 The Dynamics of the Private Sector under Alternative Monetary Policy Rules

In this section we first study in the context of our Keynesian dynamics a special case of the money supply rule (21) of the monetary authority where the nominal rate of interest is only indirectly influenced through the decision on the change in money supply by the central bank. We also make some observations on the case of the general money supply rule. After that we explore the dynamics of the macromodel in the case where the monetary authority follows the Taylor rule.

In the derivation of the intensive form of the wage-price dynamics, module 7, we follow Chiarella and Flaschel (2000) and solve the two wage price equations (37), (38) for the two unknowns $\hat{w}_{t+1} - \pi_t - n_x, \hat{p}_{t+1} - \pi_t$ which gives rise to the following explicit expressions for these two variables:[24]

$$\hat{w}_{t+1} - \pi_t - n_x = [\beta_w(V_t - \bar{V}) + \kappa_w \beta_p (U_t - \bar{U})]/[1 - \kappa_w \kappa_p], \tag{38}$$

$$\hat{p}_{t+1} - \pi_t = [\kappa_p \beta_w (V_t - \bar{V}) + \beta_p (U_t - \bar{U})]/[1 - \kappa_w \kappa_p]., \tag{39}$$

These equations in turn imply for the dynamics of the share of wages $u_t = \omega_t / x$ the law of motion:

$$\hat{u}_{t+1} = \hat{w}_{t+1} - \hat{p}_{t+1} - n_x = [(1 - \kappa_p)\beta_w (V_t - \bar{V}) - (1 - \kappa_w)\beta_p (U_t - \bar{U})]/[1 - \kappa_w \kappa_p]. \tag{40}$$

[24] Note that the reduced form equ. (39) defines a Phillips-curve of the traditional across markets type, but one where also the rate of capacity utilization of firms is present besides the rate of employment, both in form of deviations from their NAIRU levels.

This statement, however, is only true when one neglects second order terms for example in the formula that relates the nominal rates of wage and price inflation with the growth rate of the real wage. Such second order terms are repeatedly neglected in all following calculations of the intensive form of the model. The above law (40) provides the first dynamical equation of this intensive form. Note also, that the formula for $\hat{p}_{t+1} - \pi_t$ is inserted into the following laws of motion of the intensive form of the model in various places.

Neglecting second order terms we get from the model of the preceding section the following autonomous six-dimensional dynamic system in the variables share of wages $u_t = \omega_t/x$, labor intensity in efficiency units $l_t = xL_t/K_{t-1}$,[25] real balances per unit of capital $m_t = M_t/(p_t K_{t-1})$, inflationary expectations $\pi_t^m$, sales expectations per unit of capital $y_t^e = Y_t^e/K_{t-1}$ and inventories per unit of capital $\nu_t = N_{t-1}/K_{t-1}$, which describe the laws of motion of the private sector of our economy.[26]

$$\begin{aligned}
\hat{u}_{t+1} &= \kappa[(1-\kappa_p)\beta_w(V_t - \bar{V}) + (\kappa_w - 1)\beta_p(U_t - \bar{U})], && (41)\\
i(\cdot) &= i_1(\rho_t^m - \xi - (r_t^m - \pi_t^m)) + i_2(U_t - \bar{U}) \\
\hat{l}_{t+1} &= -i(\cdot), && (42)\\
\hat{m}_{t+1} &= \mu - \pi_t - n - \kappa[\beta_p(U_t - \bar{U}) + \kappa_p\beta_w(V_t - \bar{V})] - i(\cdot), && (43)\\
\pi_{t+1}^m &= \pi_t^m + \beta_\pi[\kappa[\beta_p(U_t - \bar{U}) + \kappa_p\beta_w(V_t - \bar{V})]], && (44)\\
y_{t+1}^e &= y_t^e + \beta_{y^e}(y_t^d - y_t^e) - i(\cdot)y_t^e && (45)\\
\nu_{t+1} &= \nu_t + y_t - y_t^d - (i(\cdot) + n)\nu_t. && (46)
\end{aligned}$$

For output per capital $y_t = Y_t/K_{t-1}$ and aggregate demand per capital $y_t^d = Y_t^d/K_{t-1}$ we have the following expressions:[27]

$$\begin{aligned}
y_t &= (1 + n\beta_{n^d})y_t^e + \beta_n(\beta_{n^d}y_t^e - \nu_t) && (47)\\
y_t^d &= (1-s_w)(u_t y_t - t^w) + (1-s_c)(\rho_t^e - t^c) + i(\cdot) + n + \delta + g && (48)\\
&= (1-s_c)y_t^e + (s_c - s_w)u_t y_t + i(\cdot) + n + \delta + \gamma
\end{aligned}$$

We make use in addition of the expressions and abbreviations (in the case of a money supply rule):

$$\begin{aligned}
V_t &= l_t^d/l_t, \;\; U_t = y_t/y^p, \;\; l_t^d = xL_t^d/K_{t-1} = y_t \\
\rho_t^e &= y_t^e - \delta - u_t y_t, \;\; r_{t+1} = r_0 + (h_1 y_t - m_t)/h_2, \\
\rho_t^n &= y_t^{dn} - \delta - u_t y_t^n, \quad y_t^{dn} = \bar{U}y^p/(1 + n\beta_{n^d}), y_t^n = \bar{U}y^p \\
\mu_t &= \hat{M}_t = \mu_{t-1} + \beta_{m_1}(\bar{\mu} - \mu_{t-1}) + \beta_{m_2}(\bar{\pi} - \hat{p}_{t+1}) + \beta_{m_3}(\bar{U} - U_t) \\
\rho_t^m &= \sum_{i=0}^{11}\delta_i^\rho \rho_{t-i}^n, \quad \sum_{i=0}^{11}\delta_i^\rho = 1, \quad r_t^m = \sum_{i=0}^{11}\delta_i^r r_{t-i}, \quad \sum_{i=0}^{11}\delta_i^r = 1
\end{aligned}$$

We next show that the above dynamics has a uniquely determined steady state which is locally asymptotically stable under reasonable assumptions on the parameters

[25] $l_t^d = xL_t^d/K_{t-1}$.
[26] $\kappa = 1/(1 - \kappa_w\kappa_p)$.
[27] $\gamma = -(1-s_w)t^w - (1-s_c)(\delta + t^c) + g$.

of the dynamics and which loses its stability by way of a Hopf bifurcation if certain adjustment speeds become large enough.

**Proposition 1**

There is a unique steady-state solution or point of rest of the dynamics (41) – (46), fulfilling $u_0, l_0, m_0 \neq 0$, which is given by the following expressions:[28]

$$y_0 = \bar{U}y^p, \quad l_0^d = y_0, \quad l_0 = l_0^d/\bar{V}, \quad y_0^e = y_0^d = \frac{y_0}{1 + n\beta_{n^d}}, \tag{49}$$

$$u_o = \frac{s_c y_o^e - (\gamma + \delta + n)}{(s_c - s_w)y_o}, \quad \rho_0^e = y_o^e - \delta - u_o y_o, \tag{50}$$

$$m_0 = h_1 y_0, \quad \pi_0 = \bar{\pi} = \bar{\mu} - n, \quad r_0 = \rho_0^e + \pi_o, \quad \nu_0 = \beta_{n^d} y_0^e. \tag{51}$$

We assume that the parameters of the model are such that the steady state values for $u, \rho^e, r$ are all positive.[29] Note that this demands in particular that

$$s_w < (\gamma + n)/(y_o^e - \delta)$$

must hold true as upper limit for the savings rate of workers and that the case $s = s_c = s_w$ of a uniform savings rate as in traditional Keynesian textbook stories lies outside this range (since the wage share would be equal to $\infty$ then).

**Proof:** The proof basically rests on the fact that equ.s (41), (43), set equal to zero imply, combined with equations (42), (44), two independent linear equations in the unknowns $V_t - \bar{V}, U_t - \bar{U}$ which therefore are both zero in the steady state. The remaining steady state conditions are then easily obtained from these two equilibrium situations by setting the remaining right hand sides of (41) – (46) equal to zero. △

**Proposition 2**

*Assume that the parameters $\beta_w, \beta_p, \beta_{\pi^m}, \beta_n, h_2, i_1$ are all sufficiently small and the parameter $\beta_{y^e}$ sufficiently large. Then: The steady state of the dynamics (41) – (46), is locally asymptotically stable.*

Sluggish wage price adjustments (including expectations), low interest rate sensitivity of money demand and profitability sensitivity of investment demand, and a small inventory accelerator coupled with a fast multiplier process thus make the system convergent and thus provide a proper starting point for the investigation of its dynamics. A very long and detailed statement and proof of this proposition is provided in Köper

[28] Equations (50) show that the so-called Pasinetti paradox does not hold true here, since workers save in a particular type of financial asset here (savings deposits) and thereby influence the long run rate of profit in a specific way.

[29] The steady state values for the financial assets of our model are:

$$b_o = \frac{g - (t^w + t^c) - \mu m_o}{\mu}, b_o^w = \frac{s_w u_o y_o - t^w}{\mu}, b_o^c = b_o - b_o^w, q_o = (\frac{p_e E}{pK})_o = 1,$$

but are of no importance here since this part of the model does not yet influence the dynamics of the private sector, see Chiarella, Flaschel, Groh and Semmler (2000) for models that surpass the limitations of the present approach.

(2000) for the case of the continuous time limit of the model of this section, where it is also shown that loss of such stability always comes about by way of Hopf bifurcations and thus in particular in a cyclical fashion. Around the parameter value where the Hopf bifurcation occurs the system loses its local stability in general either by the birth of an attracting limit cycle after the bifurcation point has been passed (the super critical case) or the death of a repelling limit cycle when the bifurcation point is approached from below (the subcritical case). The occurrence of supercritical Hopf bifurcation, and thus of persistent and attracting limit cycles, is demonstrated numerically for a simple version of the dynamics in Chiarella and Flaschel (2000, Ch.6). These results also hold in the case of the Taylor interest rate policy rule, but are more difficult to obtain in the case of an active money supply rule, since this adds another differential equation to the model and makes the dynamics a seven dimensional one.

We now come to a discussion of the feedback mechanisms that are at work in the dynamics (41) – (46). They are composed of the interaction of the Keynes effect, the Mundell effect, the Metzlerian accelerator effect and the so-called Rose effect, but not yet Fisher debt effects, Pigou wealth effects, various types of accelerator mechanisms in the real and the financial part of the economy, see Chiarella, Flaschel, Groh and Semmler (2000) for a discussion of such further feedback chains. The figure 1 shows the mechanisms involved in the dynamics of this paper.

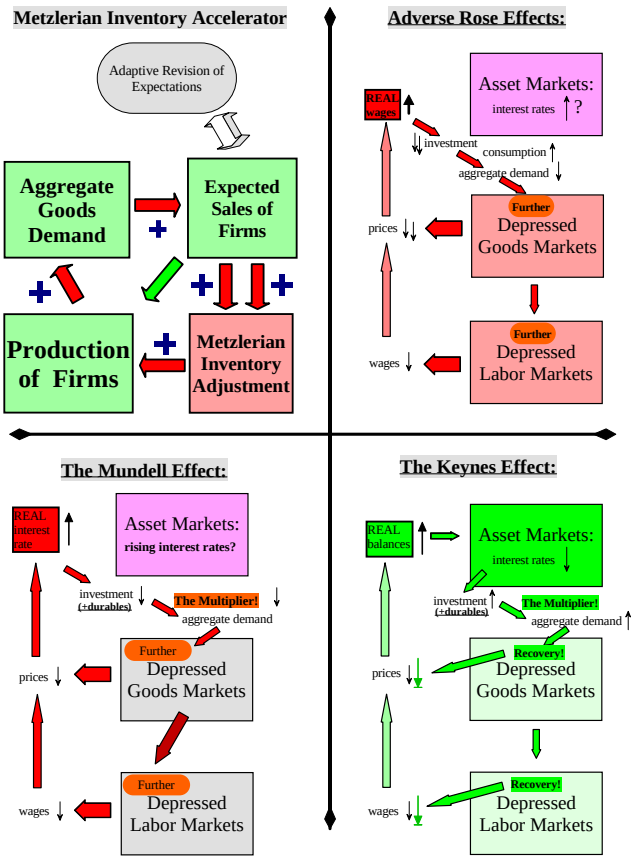


Figure 1: The feedback chains of the model

The Keynes effect, figure 1 bottom right, is of course well known and it basically

means that falling wages and prices increase real liquidity which ceteris paribus decrease the nominal rate of interest and thus aggregate demand and output of firms and employment. This counteracts a further fall in wages and prices and thus helps to stabilize the economy. The same conclusions of course holds for rising wages and prices. This, however, is the sole stabilizing mechanism in the model, since the Metzlerian inventory accelerator mechanism shown in figure 1 top left is only stabilizing when inventory adjustments are sluggish and – on this bases – the sales expectations mechanism sufficiently fast (coupled with a propensity to spend that is smaller than one), which in fact then provides but a rigorous form of the well known dynamic multiplier story.

The partial Mundell effect, bottom left in figure 1, is a real rate of interest effect on the economy (with the nominal rate of interest kept fixed then). Falling wage and prices and thus deflation increases the real rate of interest and thus reduces aggregate demand (investment and consumption). The resulting decline in the output and the employment of firms gives further momentum to the ongoing deflation and thus implies a deflationary spiral if no other mechanism – such as the Keynes effect – stops this deflationary tendency. Of course, this destabilizing effect also works in inflationary environments with increasing demand, output and employment and thus increasing inflation where expected inflation is changing into the direction of actual inflation.

Finally, though known since long, the Rose (1967) real wage effect is rarely discussed in the literature. It encompasses four possibilities one of which is shown in figure top right. Assume again that wages and prices are falling, but that prices are falling faster than wages. The real wage is therefore rising and is assumed in the figure to depress investment more than it increases consumption demand. The initial depressed situation on the markets for goods is therefore deepening and thus leads to further declines in prices and wages. This again gives rise to a deflationary spiral if the considered process repeats itself. In the case of consumption demand is increasing more than investment demand is decreasing we get the opposite conclusion and thus improvements on the market for goods and for labor that move the economy out of the depression. This is a normal Rose effect in contrast to the adverse one considered beforehand. Of course when wages are falling faster than prices we just get the opposite of what has just been said, and thus a normal Rose effect followed by an adverse one.

In order to evaluate those effects on aggregate demand we need to estimate the aggregate goods demand function from a reduced form representations of consumption and investment demand. The functions, to be estimated in the next section, are

$$\begin{aligned} c+g &= (1-s_c)y_t^e + (s_c - s_w)u_t y_t + \gamma = a_1 y_t^e - a_2 u_t y_t + a_3 \\ i+\delta &= -(i_1/y_o)u_t^m - i_1(r_t^m - \pi_t^m) + i_2 U_t + i_1(y_o^e - \delta) - i_2\bar{U} + n + \delta \\ &= -b_1 u_t^m - b_2(r_t^m - \pi_t^m) + b_3 U_t + b_4 \end{aligned}$$

Estimating the parameters $a_i, b_j$ of the above two equations will provide us with just enough equations from which the parameters of the aggregate demand function can be calculated and inference on the above stability problems can be made.

Through the subsequent estimation we will get the partial derivatives of $y^d$, our aggregate demand function, with respect to sales expectations, the current wage share, the nominal rate of interest and the expected rate of inflation (both medium run

values) and finally the level of inventories per unit of capital. We hereby make use of the relationship

$$y_t = (1 + n\beta_{n^d})y_t^e + \beta_n(\beta_{n^d}y_t^e - \nu_t)$$

between output and sales expectations and inventories. The coefficients of the aggregate demand function are related as follows:[30]

$$\begin{aligned} y^d_{y^e} &= (1 - s_c) + (s_c - s_w)u_o(1 + n\beta_{n^d} + \beta_n\beta_{n^d}) + (i_2/y^p)(1 + n\beta_{n^d} + \beta_n\beta_{n^d}) \approx 0.98 \\ y^d_u &= (s_c - s_w)y_o \approx 0.53, \quad y^d_{r^m} = -i_1 \approx -0.16, \quad y^d_{\pi^m} = i_1 \approx 0.16 \end{aligned}$$

We thus can guess that the Metzlerian type of quantity adjustment process will be stable, since aggregate demand increases (less than one), due to an increase in sales expectations and lead in turn to an increase in sales expectations that is less than the initial increase. Of course, such a statement is still only an intuitive and partial one and must be based on an investigation of the Jacobian of the dynamics at the steady state in order to be proved. Wage share adjustment by contrast is not stabilizing from such a partial perspective if wages respond stronger to changes in economic activity than prices, since aggregate demand, and thus sales expectations and output respond positively to an increase in real wages and the wage share, which due to the dominance of wage flexibility gives rise to further increases in real wages and the wage share. Next, increases in the nominal interest rate, with inflationary expectations being given, decrease aggregate demand, and thus sales expectations and economic activity, which reduces the pressure on the price level and thus on the nominal rate of interest, which thus is a stylizing feedback chain, the Keynes-effect in fact. By contrast, increases in inflationary expectations, with the nominal rate of interest now being given, increase aggregate demand, sales expectations and economic activity, and thus give rise to further increases in inflation and expected inflation, an unstable feedback mechanism we discussed under the name of the Mundell effect.

The question arises which one of these two effects, which both work through the real rate of interest channel, will be the dominant one in the presently considered situation. To give a tentative answer to this question we temporarily disregard the use of medium run moving average in the investment function and assume as real rate expression in the investment function the short-run version $r_t - \hat{p}_{t+1}$. This gives rise to the formula

$$r_t - \hat{p}_{t+1} = r_o - (1/h_2)m_t + \pi_t^m + (\frac{h_1 y^p}{h_2} - \kappa[\beta_p + \kappa_p\beta_w])U_t + const.$$

if we disregard the difference between $V_t$ and $U_t$ as measures of economic activity as is often done. The stabilizing Keynes effect is thus the dominant one if $h_2$ is chosen sufficiently small, since an increase in economic activity and the price level will then increase the real rate of interest unambiguously and thus lead to counteracting changes in economic activity. By contrast, high adjustment speeds for prices (and wages) will imply that increases in economic activity (as measured by $U_t$) will decrease the real rate of interest and thus lead to further increases in economic activity. In this case the destabilizing Mundell effect is the dominant one, in particular if there are expectations that respond quickly to changes in the inflation rate.

[30]The subsequent preliminary discussion of the stability properties of our model uses parameters that are based on estimates as undertaken in section 4.

Finally we have a negative effect of inventory accumulation on aggregate demand, sales expectations and output, given by:

$$y_\nu^d = -(i_2/y^p - (s_c - s_w)u_o)\beta_n$$

which can be viewed as potentially destabilizing since a decrease in aggregate demand piles up inventories which decreases goods demand even further. Note here in addition that this process becomes the stronger the higher the speed of adjustment of inventories becomes. Such an increase furthermore can destabilize the quantity adjustment process considered above in addition since the partial derivative $y_{y^e}^d$ becomes larger than one if the parameter $\beta_n$ is made sufficiently large.

Yet, for the parameter values of the next section the quantity adjustments are all stabilizing, while the real wage and real interest rate adjustments are destabilizing. In sum this gives rise to local instability for the steady state of our model of monetary growth, here still with a constant rate of growth of the money supply, since the price adjustment processes dominate the quantity adjustment processes. The question thus becomes what changes have to made to the model in order to make its steady state attracting or – if this is not possible – in order to bound the dynamics to economically meaningful domains when it departs too much from the steady state. Note that decreases in wage flexibility are unambiguously stabilizing since they make the adverse Rose effect less pronounced (or disappear) and since they reduce the destabilizing power of the Mundell effect. By contrast, decreased price flexibility reduces the destabilizing potential of the Mundell effect, but makes the adverse Rose effect a stronger one. While decreasing $\beta_w, h_2, \beta_n$ is thus always good for stability in the considered situation, the same does not hold true for decreases (or increases) in price flexibility $\beta_p$.

Quantity adjustment thus appears to be stable, distributional adjustments unstable, and the Mundell effect seems to dominate the Keynes effect at the interest rate sensitivity measured in the next section (where we obtain $h_2 = 2.14$). The longer the time horizon in the excess profitability measure in investment the stronger this short-run destabilizing mechanism becomes. The question, therefore, arises how active monetary policy – our generalized money supply rule or the Taylor interest rate rule – has to be to bring stability to an economy which appears to slightly explosive (slightly above the Hopf bifurcation point) in their cyclical dynamics. We only claim here that anti-inflationary policy rules, of both types, can indeed stabilize the dynamics of the private sector and make them convergent. This has been shown by numerical simulations of the theoretical model in Flaschel, Gong and Semmler (2000), but will be considered in this paper only from the empirical perspective on the basis of the empirical estimates in the now following section.

# 4 Estimation of the Model Parameters

This section discusses how we estimate the structural parameters of the model. These parameters are also used to simulate the model. We shall first remark that it is technically impossible, and also not necessary, to estimate all the parameters according to the reduced intensive form as expressed in (41) - (48). The system includes many expected variables which are not observable. Although the equations are all expressed in linear form, the parameters often appear in multiplicative form and hence are nonlinearly

related. What facilitates our estimation is the fact that we treat the entire system as being recursive or block recursive. This allows, whenever possible, to estimate the parameters by a single equation (either in reduced form or in structural form). Only for those parameters that appear in a simultaneous system, such as in the price-wage dynamics, we use the standard method, for example two stage least square (2SLS) to estimate the parameters. We shall remark that such an estimation strategy can also be found in Christiano and Eichenbaum (1992) who use such a strategy for a large system with many structural parameters.

We can divide all the estimated structural parameters into the 7 subsets. Table 1 provides the estimates and the standard errors.

Table 1: The Estimates of Structural Parameters*)

| | | |
|---|---|---|
| Set 1 | Sales Expectation | $\beta_{y^e} = 1.2610$ $\beta_n = 0.0414$<br>(0.1067) (0.0105)<br>$\beta_{n^d} = 0.4691$<br>(0.0203) |
| Set 2 | Price-Wage Dynamics | $\beta_w = 0.0958$ $\beta_p = 0$<br>(0.0285) (0.0000)<br>$\beta_x = 0.4702$ $\beta_\pi = 0.6537$<br>(0.0520) (0.1753)<br>$\kappa_p = 0.3430$ $\kappa_w = 0.9081$<br>(0.0834) (0.1387) |
| Set 3 | Consumption Function | $s_w = 0.0510$ $s_c = 0.6230$<br>(0.0258) (0.0654)<br>$\gamma = 0.0829$<br>(0.0145) |
| Set 4 | Investment Function | $i_1 = 0.1363$ $i_2 = 0.0340$<br>(0.0509) (0.0076)<br>$\xi = 0.1500$<br>(0.0069) |
| Set 5 | Money Demand Function<br>(money supply rule only) | $h_1 = 0.1769$ $h_2 = 2.1400$<br>(0.0028) (0.1771) |
| Set 6 | Reaction Functions of<br>Monetary Authority | $\beta_{r_1} = 0.0463$ $\beta_{r_2} = 0.0781$<br>(0.0315) (0.0327)<br>$\beta_{r_3} = 0.0148$ $\beta_{m_1} = 0.5524$<br>(0.0056) (0.0814)<br>$\beta_{m_2} = 0.0499$ $\beta_{m_3} = 0.0481$<br>(0.0938) (0.0168) |
| Set 7 | Other Parameters | $r_0 = 0.0221$ $y_p = 0.5091$<br>(0.0089) (0.0167)<br>$\overline{\pi} = 0.0074$ $\overline{\mu} = 0.0154$<br>(0.0119) (0.0095)<br>$\overline{U} = 0.8231$ $\overline{V} = 0.9403$<br>(0.0468) (0.0164)<br>$\delta = 0.0486$ $n_l = 0.0049$<br>(0.0034) (0.0029)<br>$n_x = 0.0032$ $n = 0.0081$<br>(0.0081) (0.0079) |

*)Standard errors are in the parenthesis

Before we elaborate on how we have estimated these parameters we shall first remark that in equ. (36) we have set $\beta_p$ to zero in our estimation of the price-wage dynamics. The estimated $\beta_p$ is close to zero and not significant according to the estimation procedure described below.[31] Given this result, we can expect that the standard demand-supply forces in determining prices and wages do not appear to be empirically significant, at least according to U. S. time series data. The estimation appear to support the markup theory of pricing.

Next we explain how we have obtained those estimates as expressed in Table 1. We start from below. The parameters in Set (7) are those parameters that can be either

[31]In Fair (1997) also the coefficient $\beta_w$ (see our equ. (35)) is insignificant so that he has neglected the impact of unemployment on wages in his regression

expressed in terms of an average, or are defined in a single structural equation with a single parameter. This allows us to apply moments estimation by matching the first moments of the model and the related data. The parameters in Set (6) are estimated by applying OLS directly to (21) and (23).

To estimate the parameters in Set (5), we use equ. (25) divided by $p_t k_{t-1}$.Then we obtain from this

$$r_{t+1} - r_0 = a_1 y_t + a_2 m_t \tag{52}$$

where $r_0$ is given in Set 6. The OLS regression on (52), gives us the estimated parameters $a_1$ and $a_2$. By setting $a_1 = \frac{h_1}{h_2}$and $a_2 = \frac{-1}{h_2}$, we then obtain the estimated $h_1$ and $h_2$. Since the structural parameters $h_1$ and $h_2$ appear multiplicatively in $a_1$ and $a_2$, we are not able to obtain the standard deviations directly from the OLS regression. We therefore treat these estimates of $h_1$ and $h_2$ as being nonlinear least square (NLS) estimates and use the method as discussed in Judge et al. (1988:508-510) to derive their standard deviations. We use the Gauss procedure GRADP to calculate the derivative matrix that is necessary to derive the variance-covariance matrix of the estimated parameters. We shall remark that the same principle is also applied to other similar cases whenever parameters appear in multiplicative form or NLS is applied.

The remaining parameters are more complicated to estimate. For their estimations we need, either directly or indirectly, the expectation variables that are not observables. Let's first discuss how we estimate the parameters related to sales expectation, i.e., Set (1). We estimate this parameter set based on the consideration that actual and predicted $y_t$ can be matched as close as possible via equation (47). This gives

$$y_t = b_1 y_t^e + b_2 \upsilon_t \tag{53}$$

Here we should regard the time series $y_t^e$ as being a function of $\beta_{y^e}$ via the adaptive rule (45)[32], given the initial condition $y_0^e$, which we set here to be $y_0$. We therefore can construct an objective function $f(\beta_{y^e})$ :

$$f(\beta_{y^e}) = e_y(\beta_{y^e})' e_y(\beta_{y^e}) \tag{54}$$

where $e_y(\beta_{y^e})$ is the error vector of OLS regression on (53) at the given $\beta_{y^e}$ and hence the series $y_t^e$. Minimizing $f(\beta_{y^e})$ by applying an optimization algorithm, we obtain the estimate of $\beta_{y^e}$. Given the estimate of $\beta_{y^e}$ and hence the series $y_t^e$ the OLS is applied to (53). This gives us the estimates of $b_1$ and $b_2$. By setting $b_1 = 1 + (n + \beta_n)\beta_{n^d}$ and $b_2 = -\beta_n \beta_{n^d}$ with $n$ given in Set (7), one then obtains the estimates of $\beta_n$ and $\beta_{n^d}$. Apparently, all these estimates can be regarded as a NLS, and therefore the standard deviation can be derived in a similiar way as discussed in Judge et al. (1988: 508-510).

Next, we discuss how we estimate parameter set (2). Given the time series $\pi_t$, the structural parameters $\beta_p, \beta_w, \beta_x, \kappa_p$and $\kappa_w$ can be estimated by the method of two stage least square (2SLS). The first stage is the OLS regression of the following reduced form (derived from 38 and 39):

$$\widehat{w}_{t+1} - \pi_t = w_1(V_t - \overline{V}) + w_2(U_t - \overline{U}) + w_3 n_{x,t+1} \tag{55}$$

$$\widehat{p}_{t+1} - \pi_t = p_1(V_t - \overline{V}) + p_2(U_t - \overline{U}) \tag{56}$$

[32]with $i(\cdot)$ set to 0

This will yield instrument variables for $\widehat{w}_{t+1}$ and $\widehat{p}_{t+1}$ in the right side of the following structural equations to which our second stage of OLS regression will be applied:

$$\widehat{w}_{t+1} - \pi_t = \beta_w(V_t - \overline{V}) + \kappa_w(\widehat{p}_{t+1} - \pi_t) + \beta_x n_{x,t+1} \tag{57}$$

$$\widehat{p}_{t+1} - \pi_t = \beta_p(U_t - \overline{U}) + \kappa_p(\widehat{w}_{t+1} - \pi_t - \beta_x n_{x,t+1}) \tag{58}$$

However, these estimations are based on the assumption of given time series $\pi_t$, whose dynamics is governed by the adaptive rule (37). We therefore shall first, as in the case of $y_t^e$, estimate $\beta_\pi$ to obtain $\pi_t$. The difference is now that we have to match both $\widehat{w}_{t+1}$ and $\widehat{p}_{t+1}$ and thus a set of weighting coefficients is needed. Since both $\widehat{w}_{t+1}$and $\widehat{p}_{t+1}$ are measured in terms of growth rates, it is reasonable to assume an equal weight in matching $\widehat{w}_{t+1}$ and $\widehat{p}_{t+1}$. This consideration allows us to construct the objective function:

$$f(\beta_\pi) = \left[ \begin{array}{cc} e_w(\beta_\pi)' & e_p(\beta_\pi)' \end{array} \right] \left[ \begin{array}{c} e_w(\beta_\pi) \\ e_p(\beta_\pi) \end{array} \right] \tag{59}$$

where $e_w$ and $e_p$ are the error vectors of $\widehat{w}_{t+1}$ and $\widehat{p}_{t+1}$ with respect to the 2SLS estimation for (55) - (56) and (57)-(58) respectively. An optimization algorithm is then applied to minimize $f(\beta_\pi)$ to obtain the NLS estimate of $\beta_\pi$

Once $\beta_{y^e}$ and $\beta_\pi$ are estimated we can construct the time series $y_t^e$ and $\pi_t$. This not only allows us to estimate the parameters in the equations for sales expectations and the price-wage dynamics but this is also necessary to estimate the parameters in the consumption and investment functions. To estimate the consumption function we use an OLS regression for

$$c_t + g_t = c_0 + c_1 y_t^e + c_2 u_t y_t \tag{60}$$

The structural parameters are obtained by setting $c_0 = \gamma$, $c_1 = 1 - s_c$, $c_2 = s_c - s_w$. The OLS regression equation for the investment function takes the form:

$$i_t - (n + \delta) = i_i(\rho_t^m - \xi - (r_t^m - \pi_t^m)) + i_2(U_t - \overline{U}) \tag{61}$$

For the above, $n, \delta$ and $\overline{U}$ are given in Set 7. $\xi$ is estimated by the method of moments, i.e., setting the mean of $\rho_t^m - \xi - (r_t^m - \pi_t^m)$ to 0.

Given the parameter estimates of our model, reported in Table 1 we can evaluate the performance of our macroeconometric models for the above stated monetary policy rules.

# 5 Evaluating the Macroeconometric Model and the Monetary Policy Rules

Employing our estimated parameters, we report in figures 2-3 the actual and predicted macroeconomic time series generated from some key behavioral functions. [33] One can observe that most macroeconomic variables are well predicted.

[33]Note that in this exercise the fitted line is obtained by simulating not the entire system of equations, but the corresponding behavioral functions using the estimated parameters.

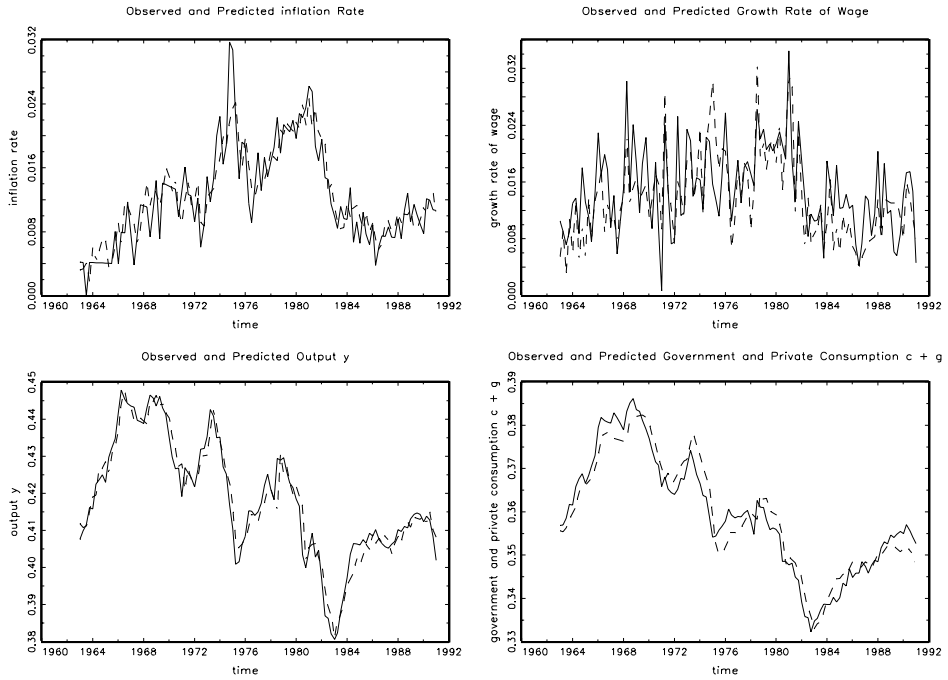


Figure 2: Observed and predicted variables

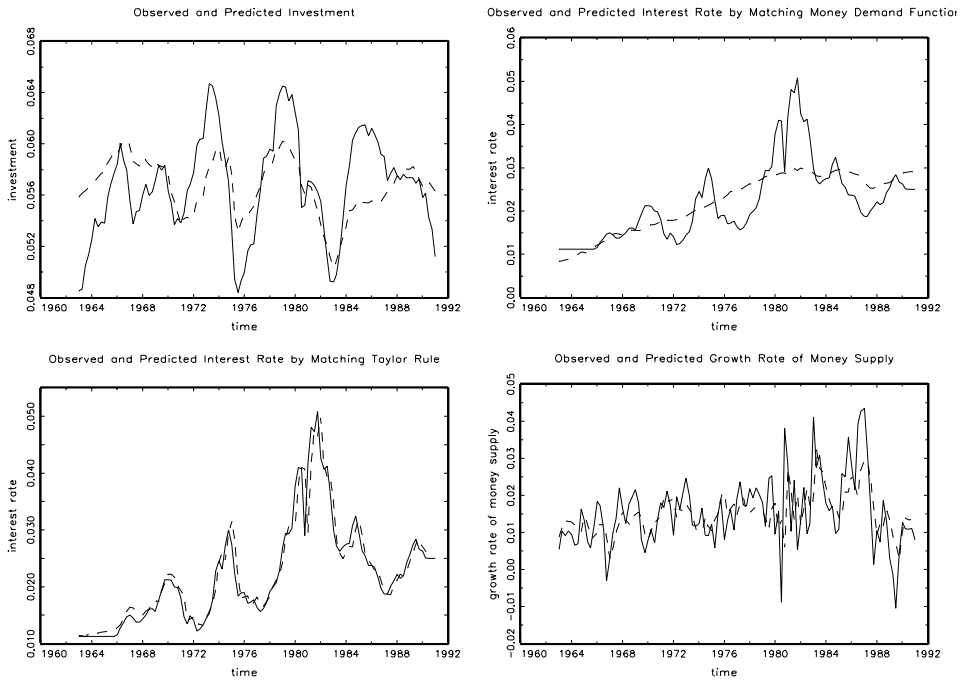


Figure 3: Observed and predicted variables

The fit, however, is less successful for investment. It is even less successful for the interest rate derived from the money demand function. This will create a difficulty for the exercise to simulate the impact of the money supply rule, which shall be discussed below. However, we shall remark that the parameters that we estimate here for the money demand function are statistically significant. This indicates that the explanatory variables, $y_t$ and $m_t$, do have some power to explain the interest rate $r_{t+1}$. Yet, admittedly there may be a better explanation for it (which may take, for example, a nonlinear form). The same argument may also be applied to the investment function.

Yet, whereas the fit for the interest rate derived from the money demand function does not replicate the variation in the interest rate but solely the trend of the interest rate the estimated investment function at least partially captures the variation in investment. Given that empirical estimates notoriously fail to properly capture money demand and investment functions we may view our estimates for those two functions still a relative success given our limited aim to study the effects of monetary policy rules in a simple model.

If we simulate our macroeconometric model with the estimated parameters as reported in Table 1 for both, assuming that either the actual interest rate is determined by the money supply rule or the Taylor rule, we obtain figures 4 and 5.

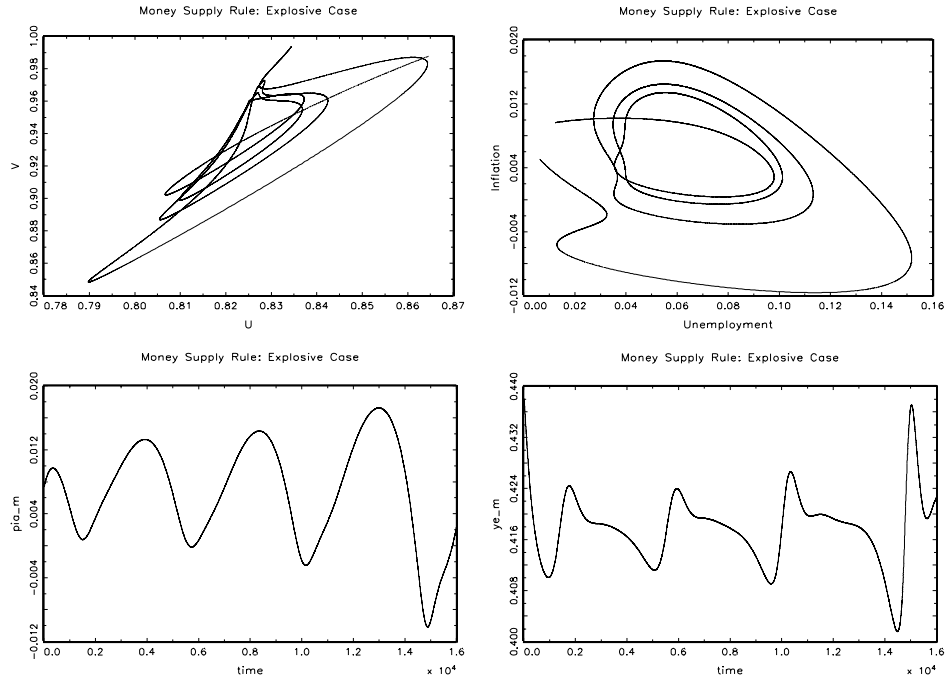


Figure 4: Simulation of the model with money supply rule (unstable case

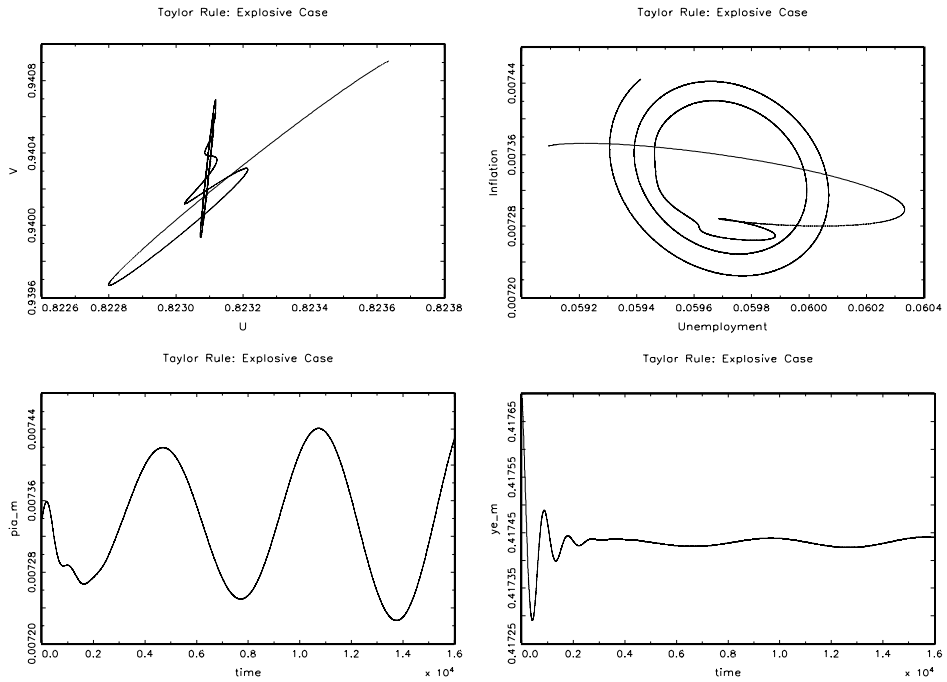


Figure 5: Simulation of the model with Taylor rule (unstable case)

For both policy feedback rules the macroeconomic variables exhibit a slight instability altough the instability occurs less for the Taylor rule, compare figures 3 and 4. When we, however, (strongly) increase the reactions of the money supply rule and the interest rate reaction to the output gap and inflation gap in the Taylor rule both rules lead to convergence results (although cyclically fluctuating).

The convergence results are depicted for the money supply rule in figure 6 and for the Taylor rule in figure 7.

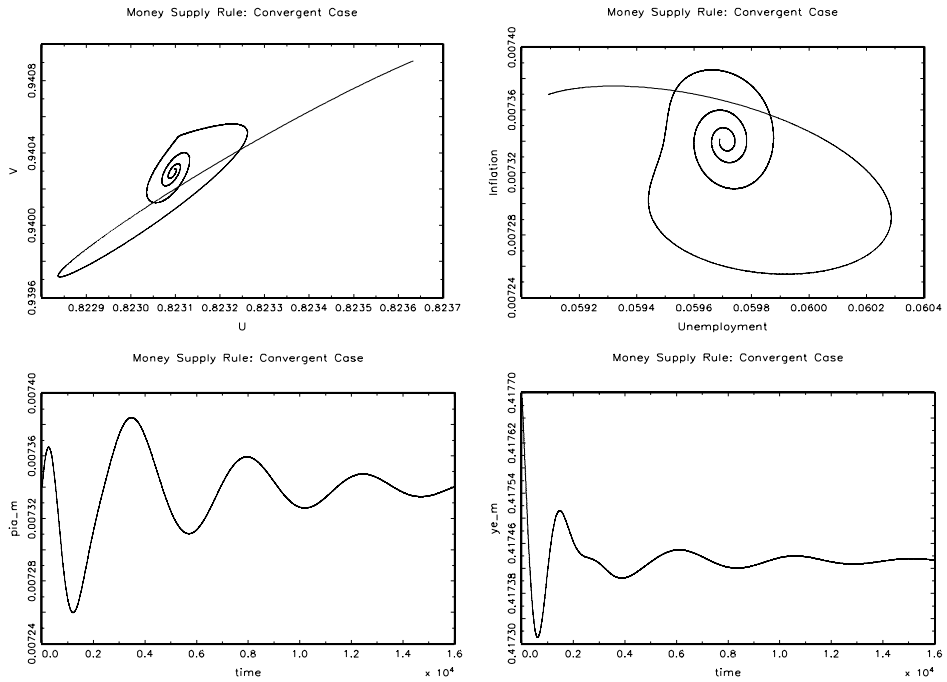


Figure 6: Simulation of the model with money supply rule (stable case)

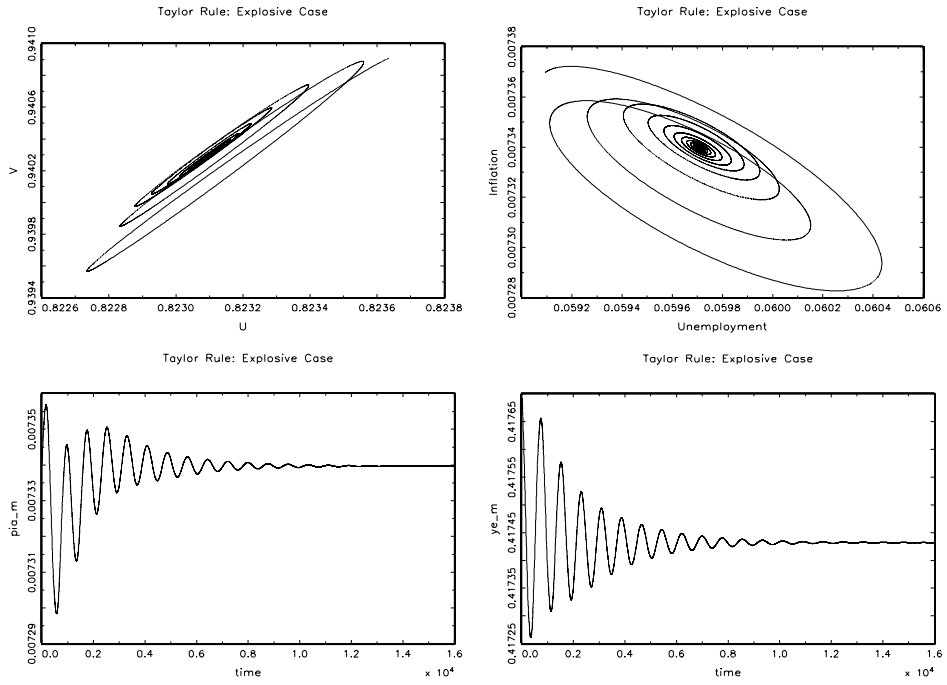


Figure 7: Simulation of the model with Taylor rule (stable case)

The possible instability generated by monetary policy rules have much been the topic of recent studies on monetary policy, see the various contributions in Taylor (1999). Christiano and Gust (1999), for example, show, although in an optimizing framework that if the Taylor rule puts to much emphasis on the output gap indeterminacy and instability of macroeconomic variables may be generated. Instability also occurs under their version of the money supply rule. Yet, in their formulation of the money supply rule they use an AR(2) process to stylize a money supply process - there is no feedback of the money supply to other economic variables such as, for example, in our case to the inflation and output gaps. We also have, for reason of comparison, employed such an AR(2) process for the money supply and indeed obtained too completely unstable paths of the macro variables. This complete instability can only be overcome by feedback rules as we have formulated for our money supply and Taylor rules.

Finally we want to study whether our model exhibits typical impulse-response functions well known from many recent macroeconomic studies, see for example Christiano, Eichenbaum and Evans (1994), and Christiano and Gust (1999). In those studies macro variables respond to liquidity shocks as follows. In the short run with liquidity increasing the interest rate falls, capacity utilization and output rises, employment rises and, due to sluggish price responses, prices slowly rise with a delay. Very similar responses can be seen in the context of our model variants for both money supply shocks, figure 8 and direct interest rate shocks (through the Taylor rule), figure 9.

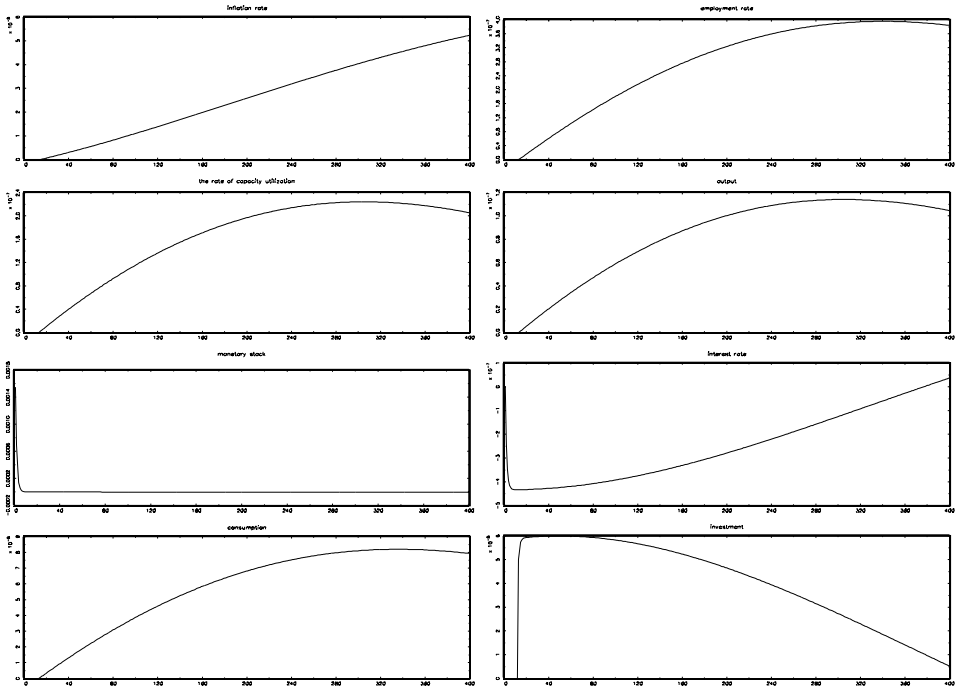


Figure 8: Impulse responses for money supply rule

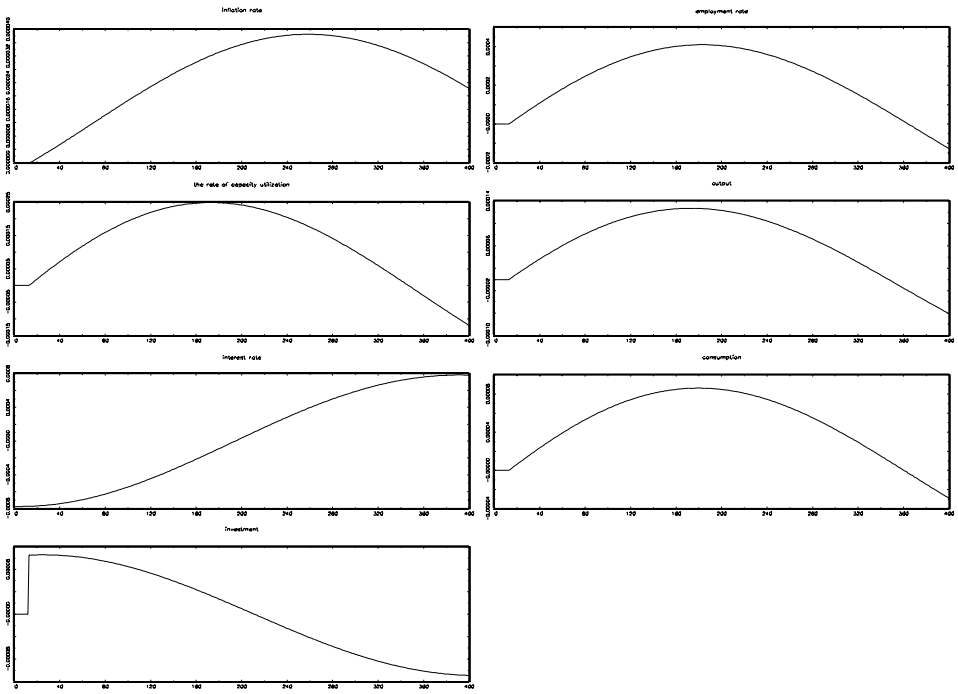


Figure 9: Impulse-responses for Taylor-rule

Note that we have shown the trajectories in deviation form from the steady state. For the money supply rule, figure 8, we have assumed that first there is an out of steady state increase in the growth of money supply. This gives rise to an interest rate fall, rise of employment, utilization of capacity, investment, consumption and, with a delay, a rise in the inflation rate. Finally in the long run all variables, although cyclically, move back to their steady state levels. Similar results can be observed in figure 9 for the Taylor rule except there we displace the interest rate through a shock from its steady state value. The interest rate is decreased but it moves back in the direction of its steady state value. The other variables also respond as one would expect from VAR studies of macroeconomic variables. With the fall of the interest rate there is a rise in capacity utilization, output, employment, investment and consumption and, again with a delay, a rise in the inflation rate. The latter can be observed from the fact that the inflation rate peaks later than the utilization of capacity, output and employment. Overall, our model is roughly able to replicate well known stylized facts obtained from VAR studies of macroeconomic variables.

# 6 Conclusions

In the paper we have chosen a Keynesian based macro econometric framework for studying macro dynamics and monetary policy. In our framework disequilibrium is

allowed in the product and labor markets whereas the financial markets are always cleared. There are sluggish price and quantity adjustments and expectations formation represents a combination of adaptive and forward looking behavior. We consider two monetary policy rules. These policy rules are (1) the money supply rule and (2) the interest rate targeting by the monetary authority. We demonstrate the implication of those policy rules for a monetary macro model of Keynesian type, and study how the private sector behaves under those alternative policy rules. We estimate the parameters of the model employing U.S. macroeconomic time series data from 1960.1-1995.1.

Based on the estimation of the parameters, obtained partly from subsystems partly from single equations, we study, using simulations, the dynamic properties for economies which employ either the money supply or the Taylor rule. As we could show with respect to volatility of the macroeconomic variables the model with the Taylor rule seems to perform better in the sense that it gives rise to a faster convergence of macroeconomic variables. The impulse-response functions for our two model variants show roughly the same features as empirical impulse-response functions based on VAR studies show. This shows that our disequilibrium model can compete with currently widely used equilibrium macro models.

Of course, more empirical work needs to be done in order to confirm or evaluate the findings of this paper. Yet, AS-AD disequilibrium models that include a treatment of income distribution, the role for aggregate demand and economic growth, have not yet been discussed in the theoretical and applied literature to a sufficient degree and thus deserve more attention than they have received so far.

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# 8 Appendices

## 8.1 Appendix 1: Notations

The model of this paper is based on the following basically standard macroeconomic notation:

A. Statically or dynamically endogenous variables:

| | |
|---|---|
| $Y_t$ | Output |
| $Y_t^d$ | Aggregate demand $C_t + I_t + \delta K_{t-1} + G_t$ |
| $Y_t^e$ | Expected aggregate demand (from $t-1$ to $t$) |
| $N_t$ | Stock of inventories |
| $N_t^d$ | Desired stock of inventories |
| $\mathcal{I}_t$ | Desired inventory investment |
| $L_t^d$ | Level of employment |
| $C_t$ | Consumption |
| $I_t$ | Fixed business investment |
| $I_t^p$ | Planned total investment $I_t + \mathcal{I}_t$ |
| $I_t^a$ | Actual total investment $= I_t + N_t - N_{t-1}$ total investment) |
| $r_t$ | Nominal rate of interest (from $t-1$ to $t$, price of bonds $p_b = 1$) |
| $p_{et}$ | Price of Equities |
| $S_{pt}$ | Private savings |
| $S_{ft}$ | Savings of firms ($= Y_{ft}$, the income of firms) |
| $S_{gt}$ | Government savings |
| $S_t = S_{pt} + S_{ft} + S_{gt}$ | Total savings |
| $T_t$ | Real taxes (w: of workers, c: of asset holders) |
| $G_t$ | Government expenditure |
| $\rho_t^e$ | Rate of profit (Expected rate of profit) |
| $V_t = L_t^d / L_t$ | Rate of employment |
| $Y_t^p$ | Potential output |
| $\Delta Y_t^e = Y_t^e - Y_t^d$ | Sales expectations error |
| $U_t = Y_t / Y_t^p$ | Rate of capacity utilization |
| $K_{t-1}$ | Capital stock used for production in $t$ |
| $w_t$ | Nominal wages |
| $p_t$ | Price level |
| $\pi_t (= \pi_t^m)$ | Expected rate of inflation for $[t-1, t]$ (average over the medium–run) |
| $\rho_t^m$ | Medium-run Average of Rate of profit |
| $r_t^m$ | Medium-run Average of Rate of Interest |
| $p_t^*$ | The $p^*$ concept of the FED |
| $L_t$ | Normal labor supply |
| $M_t$ | Money supply (index d: demand) |
| $B_t$ | Bonds (index d: demand, w: workers, c: asset owners) |
| $E_t$ | Equities (index d: demand) |
| $W_t$ | Real Wealth |

| | |
|---|---|
| $\omega_t$ | Real wage ($u_t = u_t/x_t$ the wage share) |
| $\nu_t = N_t/K_{t-1}$ | Inventory-capital ratio |

B. Parameters

| | |
|---|---|
| $\bar{V} \in (0,1)$ | 1. NAIRU-type normal utilization rate concept (of labor) |
| $\bar{U} \in (0,1)$ | 2. NAIRU-type normal utilization rate concept (of capital) |
| $\delta$ | Depreciation rate |
| $\mu$ | Growth rate of the money supply |
| $n = n_l + n_x$ | Natural growth rate (labor supply growth plus productivity growth) |
| $i_{1,2} > 0$ | Investment parameters |
| $h_{1,2} > 0$ | Money demand parameters |
| $\beta_w \geq 0$ | Wage adjustment parameter |
| $\beta_p \geq 0$ | Price adjustment parameter |
| $\beta_\pi \geq 0$ | Inflationary expectations adjustment parameter |
| $\beta_{r_i} \geq 0$ | Parameters of Interest Rate Policy (the Taylor Rule, $i = 1,2,3$) |
| $\bar{\pi}$ | Target Inflation Rate of the FED |
| $\beta_{n^d} > 0$ | Desired Inventory output ratio |
| $\beta_n \geq 0$ | Inventory adjustment parameter |
| $\beta_{y^e} \geq 0$ | Demand expectations adjustment parameter |
| $\kappa_{w,p} \in [0,1], \kappa_w\kappa_p \neq 1$ | Weights for short– and medium–run inflation |
| $\kappa$ | $= (1 - \kappa_w\kappa_p)^{-1}$ |
| $y^p > 0$ | Potential output–capital ratio ($\neq y$, the actual ratio) |
| $x = x_t > 0$ | Output–labor ratio ($n_x$ its rate of growth) |
| $t^w, t^c$ | Wage and property income taxes net of interest per unit of capital |
| $g$ | Government expenditures unit of capital |
| $s_c \in [0,1]$ | Savings–ratio (out of profits and interest) |
| $s_w \in [0,1]$ | Savings–ratio |
| $\delta_i' s$ | Weights in adaptively formed expectations |
| $\gamma$ | constant in aggregate demand per unit of capital |

C. Mathematical notation

| | |
|---|---|
| $\hat{z}_t = \frac{z_t - z_{t-1}}{z_{t-1}}$ | Growth rate of $z$ for the interval $[t-1, t]$ |
| $\Delta$ | Difference Operator ($\delta E_t = E_t - E_{t-1}$) |
| $r_o, etc.$ | Steady state values |
| $y_t = Y_t/K_t, etc.$ | Real variables in intensive form |
| $m_t = M_t/(p_t K_{t-1})$ | Nominal variables in intensive form |