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Hold-Up and the Evolution of Investment and Bargaining Norms

by

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Abstract

The purpose of this paper is to explore the evolution of bargaining norms in a simple team production problem with two sided relationship specific investments and competition. The puzzle we wish to address is why efficient bargaining norms do not evolve even though there exist efficient sequential equilibria? Conditions under which stochastically stable bargaining conventions exist are characterized, and it is shown that the stochastically stable division rule is independent of the long run investment strategy. Hence, efficient sequential equilibria are not in general stochastically stable, a result that may help us understand why institutions, such as firms, may be needed to ensure efficient exchange in the context of relationship specific investments. We also find that increasing competition, while enhancing incentives, may also destabilize existing bargaining norms, and may explain why the introduction of market in transition economics may initially result in reduced output.

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1 Introduction

A starting point for modern contract theory is the Coase (1960) conjecture stating that in the absence of transactions costs, individuals should be able to bargain to efficient allocations, regardless of the original allocation of property rights. However, when individuals make investments that are both relationship specific and non-contractible before trade, then, as Grossman and Hart (1986) show, the allocation of property rights can affect the returns from these investments, and hence the efficiency of the relationship. This observation has led to the "property rights" view of the firm, in which ownership allocation and firm boundaries are viewed as a mechanisms that enhance productive efficiency (see Hart (1995)).

This view is not uncontroversial. Maskin and Tirole (1999) observe that incomplete contracts and hold-up do not preclude the *existence* of efficient contracts. They show that regardless of the property rights allocation, if agents have sufficient foresight, then there exist contracts that provide efficient investment incentives. By itself, this does not imply that agents necessarily choose efficient contracts from the set of incentive compatible contracts. Such a step typically relies upon some model of negotiation or equilibrium selection. Both Moore (1992) and Tirole (1999) worry that efficient contracts may be too complex to use in practice, and hence to explain observed contracting arrangements one should rely upon a more realistic model of behavior.[1]

One approach to this problem that has been particularly influential in the legal academy are evolutionary models of selection.[2] For example Ellickson (1991) has studied the system of property rights allocation in Shasta County, and concludes that over time individuals are likely to evolve efficient norms of behavior. Moreover, even when there is an existing *inefficient* legal rule, individuals are able to evolve rules that are more efficient and supersede the legally enforceable rule.

In this paper we use the evolutionary learning model of Young (1993a) and Kandori, Maillath, and Rob (1993) to evaluate the claim that through a process of experimentation and learning individuals select an efficient equilibrium in a simple bilateral trade model, in which both parties have an opportunity to make non-contractible, relationship specific investments. Specifically, we consider a situation in which each agent makes a non-contractible investment of cost c before entering a market. After entering the market, they are randomly matched, at which point they observe each other's productivity, and then play a Nash demand game to divide the gains from trade.[3] An important feature of the Nash demand game is that any division of the gains from trade is a potential Nash equilibrium, and hence the division rule selected can depend upon each party's contribution to the productivity of the match. This ensures that whenever it is efficient

[1]See discussion on page 773 of Tirole (1999).

[2]See Alchian (1950) for an account that is still well worth reading. Kim and Sobel (1995) make the point explicitly that even if one allows for communication one cannot be assured that an efficient allocation will be selected. They show that efficient allocations are selected in pure coordination games. When the common interest assumption fails (as in this paper), evolution with communication has no unique equilibrium.

[3]The rules of the Nash demand game are as follows. Given the gains from trade S, each person makes a demand d_i, and if $d_1 + d_2 \leq S$, then they recieve their demand. Otherwise they receive zero.

for both parties to invest, there exists a sequential equilibrium implementing the efficient allocation.[4]

Our model captures a phenomenon that is observed in many contractual settings. For example Macauley (1963) documents in the case of form contracts for the sale of goods that trading partners do not explicitly agree upon all the terms and conditions for trade. This is evidenced by that fact that the terms and conditions of the standard purchase and acceptance orders are often inconsistent with each other. In the event of a dispute contracting parties typically depend upon past practice. For example, if a supplier provides a good of substandard quality, then in the absence of specific contract term, the Uniform Commercial Code of the United States simply requires the two parties to reach an amicable adjustment to the price.

The problem is that if the resulting decrease in price is small, then the seller has reduced incentives to take precautions. Similarly, the buyer may not be able to accept the goods, in which case buyer and seller need to renegotiate the terms of the agreement to either delay shipment or terminate all together the agreement. In either case, there is a mutually observed event that results in a rent (or loss) that must be allocated between the two parties. In some cases the buyer and seller may have contract terms in place that explicitly allocate liability. Given that it is very common for there to be no contract term in place, the question then is whether or not over time an there will evolve an efficient bargaining norm governing the *ex post* division of the gains from trade. By bargaining norm, we mean an implicit agreement on how to divide the *ex post* gains from trade or renegotiation as a function of information available at the time of negotiation.

The first issue we address is the existence of a stable bargaining norm in a two sided version of models by Ellingsen and Robles (2002) and Troger (2002). They consider a model in which only one party makes an investment, followed by play of the Nash demand game. They show that there is an efficient stable equilibrium, with the feature that the investing party appropriates most of the gains from trade. This is a very interesting result, however we feel that the interpretation provided by the authors is somewhat misleading. These authors claim that their results demonstrate that holdup cannot be considered a stable feature of an environment with boundedly rational individuals. In particular, they claim that their results imply that stable division rules have the feature that individuals are rewarded according to their contribution. Troger (2002) states explicitly that their results support equity theory, described by Rabin (1998) as the theory which predicts "that people feel that those who have put more effort into creating resources have more claim on those resources."[5]

While equity might be correct, it does not follow from the results of Ellingsen and Robles (2002) and Troger (2002). In their model, like all evolutionary models based upon Young (1993a), experimentation ensures that all possible histories and strategies occur with positive probability. In particular, one eventually reaches an outcome corresponding to the investing player getting all the rents and making an efficient investment. To destabilize this outcome, it must be the case that the investing player can profitably deviate, but since the outcome is efficient, and she appropriates all the rents, this is not possible. [6]

[4]In a related model, Carmichael and MacLeod (2003) show that the efficient allocation rule is unique when there is sufficient diversity in preferences. In the subsequent discussion, when we use the term "equilibrium" by itself, we mean the sequential equilibrium of the game. Similarly, the term "stable equilibrium" refers to Peyton Young's notion of a stochastically stable equilibrium, which we define formally in section 4.

[5]As cited in Troger (2002), page 376. The orginal source is Rabin (1998), page 18.

[6]The fact that strategies are discrete implies that the actual results is a bit more complex. However, this arguement captures

Thus, what they have shown is not that each player is rewarded according to their contribution, but that the allocation of all the rents to one party is stable when that party is making all the investments. In particular, the additional value added by the investment may be relatively small compared to the gains from trade, but never the less, the stable outcome entails giving the investing party all the rents. Our preferred interpretation of their results is that they demonstrate that one does not need new organizational forms, such as firms, to enhance efficiency when relationship specific investments are one-sided.

This is a very interesting result, and can help us understand the conditions under which efficient property rights can evolve. It also provides a new way to interpret the evidence in Demsetz (1967) and Ellickson (1991) that documents the evolution of efficient property right norms. In both cases (fur trapping and the fencing of land), efficient allocations could be achieved when only one party makes an investment, and hence one observes the efficient evolution of property rules in this particular case.

The holdup model of Grossman and Hart (1986) is quite different. They are concerned with the case in which the allocation of bargaining power to one party creates a trade-off, namely it enhances incentives for one party at the expense of the other.[7] This issue goes back to the Alchian and Demsetz (1972) view of the firm as a problem of *team production,* namely one faces the problem of coordinating the efforts of several individuals, not just one person. We pay particular attention to complementary inputs, namely inputs for which the marginal product of investment by one person is much less than investment by two individuals. Our first result in the case of two sided relationship specific investments is that there exists no stable bargaining norm when investments are observable, but not contractible.[8]

The reason such a norm does not exist is that at an efficient equilibrium both parties make high investments, and hence there are no high-low matches in the long run equilibrium. This implies that beliefs regarding the outcome of high-low matches can drift due to the noise inherent in the evolutionary learning model, until eventually it is in one party's interest to enter the market with a low investment. A similar argument applies to the low-low equilibrium.

We solve this problem by observing that it is not reasonable to suppose that there is a deterministic relationship between investment and performance. For example, if a firm invests in worker training, there is always a chance that some of the workers do not acquire the skill. We formally model this effect by supposing there is a small probability that high investment results in low productivity, and conversely that a low investment may (with low probability) result in high output. This, seemingly small, modification of the investment game now ensures the existence of a stable bargaining norm, regardless of the investment strategies. This bargaining norm corresponds to the equal division rule, and hence we can conclude that equity theory is not consistent with a stable equilibrium when both parties contribute to the gains from trade, and there is some uncertainty regarding the link between investment and productivity. Moreover, the model predicts that individuals ignore sunk costs at the bargaining stage, as assumed in the standard

the essential intuition when the set of investment strategies is close to a continuous choice.

[7]Grossman and Hart (1986) state in their abstract that "When residual rights are purchased by one party, they are lost by a second party, and this inevitably creates distortions." Note that such distortions cannot arise in the case of one-sided investment when the investing party buys the asset. As they note, this observation goes back to the work of Alchian and Demsetz (1972), who emphasize the team nature of firm production.

[8]This result is suggested in Troger (2002) discussion his model, and we first proved it in Dawid and MacLeod (2001), a journal this is now unfortunately out of print.

hold-up model.

In our model investment is discrete, thus even if the bargaining norm is unique and given by the equal split rule, multiple investment equilibria are possible. In particular, when investments are complementary, there is a large set of parameter values for which it is an equilibrium for either both parties to make high investments or both to make low investments. The learning model can select between these equilibria, and we find that when the cost of investment is greater than half of the potential gains from going to the high investment equilibrium, then the low investment equilibrium is stable.

Therefore, we can conclude that in a simple holdup problem with two sided investment, when investments are complements and costs are sufficiently high, then the efficient equilibrium is not stable. Hence learning and experimentation over time does not necessarily lead to efficient bargaining norms. This result is consistent with the view articulated by Jean-Philippe Platteau (2000), who explicitly argues that the egalitarian norm, very common in Africa, can impede development. He cites the example of a fishery project at lake Kivu in southern Congo, where a poor village was given access to a new gill net technology.[9] Within the village, there was pressure to allow many individuals to use the new nets, resulting in inefficient maintenance and care of the equipment. In this case, even though the technology clearly improved total output, the existing norms made it impossible for the villagers to implement an efficient system to utilize this new technology.

It appears that the egalitarian norm is so stable, that it is very difficult to introduce organizational forms for which there is a strong respect for property (or "equity theory"). A standard economic prescription in these cases is the introduction of more competition. We consider the implications of this possibility by supposing that if trade does not occur, the individual may re-enter the market with their investments the next period, with a discount factor of δ. Varying δ from 0 to 1 parameterizes the model between the case of pure holdup and perfect competition.

We find that the introduction of an outside option does indeed enhance efficiency. However, there is a cost. When δ is close to 1, then individuals with high productivities currently in high-low matches, may prefer not to trade and re-enter the market next period in the hopes of meeting another high productivity individual. We find that merely the possibility of being better off is sufficient to destabilize the evolution of a stable bargaining norm.

Suppose it is an equilibrium for all individuals to make a low investment, and hence an individual with high productivity has a low probability of meeting a high productivity individuals the next period and hence should trade as soon as she enters the market, regardless of the productivity of her partner. Now, suppose she is better off not trading if she were sure to meet a high productivity person the next period, then we can show that no stable bargaining norm can evolve. The reason is similar to our previous non-existence results. When high-low matches are rare, then it is possible for beliefs to drift, with the consequence that there is always a chance that the high type believes she is better off waiting to trade with another high type.

This result provides an illustration of how the introduction of markets can have a potentially destabilizing effect upon bargaining norms. This observation appears to be consistent with the recent transition experience in Eastern Europe, as documented by Roland (2000). He notes that the period of transition from a planned to market economy in Eastern Europe entailed a great deal of disorganization and a decline in output as people

[9] See Mellard, Platteau, and Watongoka (1998). The case is discussed in detail in Platteau (2000), pages 200-201.

learn how to engage in trade in a new market environment. Our model provides a potential explanation for the decline in output. Namely, the mere possibility of better trades in the future, can lead to high types delaying trade, particularly if low types attempt to expropriate too much of the gains from trade created by the high types.

One of the messages of this paper is that a necessary condition for the evolution of norms is experience with matches at which these norms are exercised. Hence, our results suggest that even though there may exist efficient incentive contracts, if they are sensitive to the way payments are structured in events that are very infrequent, then they may not be stable, and may not work as expected in practice.

The agenda of the paper is as follows. The next section introduces the basic model, and it is shown that whenever high investment is efficient, there is a sequential equilibrium implementing the efficient allocation (which we simply call an *equilibrium*). Sections 4 introduces the formal stochastic learning model that is used to define the notion of a stochastically stable equilibrium, which simple call a *stable equilibrium.* This is followed by a discussion of how adding two sided investment to the model results in the non-existence of a stable equilibrium. A preliminary analysis of the stable equilibrium for our model is carried out in section 6. Section 7 considers the case of substitutes, where the marginal return from the first investment is greater than the second investment, while section 8 presents our results for complementary investments. The paper concludes with a discussion of the results.

2 The Model

We are interested in the kind of bargaining and investment norms which are developed endogenously in a population of adaptive agents. To examine this, we use an evolutionary bargaining model similar to Young (1993b) and Kandori, Maillath, and Rob (1993) as extended to incorporate investment by Troger (2002) and Ellingsen and Robles (2002). The basic idea underlying this approach is that individuals anonymously interacting in a population use a random sample of observed past behavior to build beliefs about current actions of their opponent. With a large probability they then choose the optimal strategy given their beliefs.

Consider a single population of identical agents who are repeatedly matched randomly in pairs to engage in joint production (or in a joint project). Every agent can make an investment, either high (H) or low (L), before entering the population that influences his type, and accordingly the joint surplus of the project.

This investment can be thought of as human capital, such as the acquisition of special skills needed for a project, though the framework is sufficiently general that any type of project specific investment might be considered. It can also be applied to situations where there are explicit contracts in place, but they are incomplete. In that interpretation an event occurs that is not covered by the contract, for example there is a defect in the product. Given that the product is defective, then the buyer is not longer under an obligation to accept delivery, but it may never the less be efficient to trade with the defective product, rather than order a new product. The question then is how is the new price determined. Notice, that this is a two sided investment problem because the sellers efforts affect the probability of a defect, while specific investments by the buyer, create a need to trade now, rather than wait for a replacement product.

Before partners start joint production or trade they bargain over the allocation of the joint surplus. If the bargaining does not lead to an agreement they split without carrying out the project and look for new partners. The effect of an investment stays intact as long as the agent has not carried out the project, it is however assumed that the investment is project specific and creates no additional revenue after the project has been carried out. Looking for a new partner for the project needs time and therefore payoffs from the next matching are discounted by a factor $\delta \in [0, \bar{\delta}]$. The more specific the project at hand is the longer is the search time and the smaller is δ. Hence, we interpret δ as a parameter measuring the project specificity, however it could be induced by any type of market frictions leading to search times. The value of trade t periods after the initial investment is $\delta^t U$, where U is the agents share of the gains from trade. When $\delta = 0$ the investment can only generate revenues in the current period and the model corresponds to one with purely relationship specific investment.

The sequence of decisions facing an individual are:

1. The agent, i, decides about her investment level $I_i \in \{h, l\}$, where the cost of investment is

$$c(I) = \begin{cases} c, & \text{if } I = h, \\ 0, & \text{if } I = l. \end{cases}$$

After the investment has been made the type $T_i \in \{H, L\}$ of the agent is determined. It is assumed that the probability of being a high type after having invested I is p_I, where $p_h > p_l$.

2. The agent is randomly matched with some partner and both observe each other's type. The types determine the size of the surplus, $S_{T_i T_j}$, where when convenient $S_H \equiv S_{HH}$, $S_A \equiv S_{HL} = S_{LH}$ and $S_L \equiv S_{LL}$, and satisfies $S_H \geq S_A \geq S_L > 0$.

3. Individual i makes a demand conditional upon her type and that of her partner j, denoted by $x_{T_i T_j} \in X_{T_i T_j}(k) = \left\{0, \alpha_{T_i T_j}, 2\alpha_{T_i T_j}, ..., k\alpha_{T_i T_j}\right\}$, $\alpha_{T_i T_j} = S_{T_i T_j}/k$, k is some large even number.

4. The payoff to individual i in this period is given by the rules of the Nash demand game:

$$U^i = \left\{ \begin{array}{ll} x^i_{T_i T_j}, & \text{if } x^i_{T_i T_j} + x^j_{T_j T_i} \leq S_{I_i I_j} \\ 0, & \text{if } x^i_{T_i T_j} + x^j_{T_j T_i} > S_{I_i I_j} \end{array} \right\} - c\left(I^i\right)$$

and similarly for player j. Agents are assumed to be risk neutral.

5. If agent i has traded in this period she leaves the population and is replaced by another individual. If there was no trade the individual stays in the population and goes again through steps 2 - 5 in the following period where future payoffs are discounted by a factor δ per period.

Throughout the analysis S_H and S_L are assumed fixed, while the degree of complementarity in investment, S_A, the cost of investment, c, and the discount rate δ are parameters that determine the nature of the investment problem.

Furthermore, we assume that the probability that the type differs from the investment level is symmetric and small, namely: $1 - p_h = p_l = \lambda$ for some small positive λ. This latter assumption plays an important role in the analysis because it ensures that even if all individuals carry out high investment, there is a strictly positive probability of having low types in the population. Hence each period there is the potential for trade between H and L types. As we shall see, the existence of such trades is a necessary condition for the evolution of a bargaining norm.

This is a one-population model where the only difference between individuals stems from their investment. Accordingly, in any uniform equilibrium where all individuals use identical strategies, the surplus has to be split equally in matches of partners with identical investments. We are concerned with the evolution of norms which are uniform equilibria, and hence in any norm the surplus has to be split equally between partners with identical investment[10]. Therefore, to simplify the analysis it is assumed here that when two high types meet or two low types meet they split the gains from trade equally if they trade, i.e. $x^i_{HH} = \frac{S_H}{2}, x^i_{LL} = \frac{S_L}{2}\ \forall i$. Although this has to hold true in any norm, our assumption is not completely innocent. In the absence of such an assumption we may also have a cyclical long-run phenomena where all individuals keep switching in a coordinated fashion between demanding more or less than half of the surplus in equal investment matchings. This would result in disagreement for half of the periods and a waste of parts of the surplus for the other half. Ruling out such phenomena makes the model much more tractable and allows us to focus on the question we are mainly interested in, namely the allocation of surplus in matches between partners with *different productivities* and the implication for investment incentives.

For most of the current analysis it shall be assumed that the discount factor δ is sufficiently small that it is always efficient to trade, regardless of the type of your partner, rather than wait. Hence the option to wait will act as a constraint on the current trade, an assumption that is discussed in more detail in the next section.

These assumptions greatly simplify the strategy space. When a player first enters the game she chooses $I \in \{h, l\}$, after which point she learns her type $T \in \{H, L\}$. Given her type, each period she needs to formulate only her demand when faced with a partner of a different type, since she adopts the equal split rule when faced with a partner of the same type. Formally, a strategy of the stage game is given by $(I, x_{HL}, x_{LH}) \in \{h, l\} \times X(k)^2$, where $X(k) = X_{LH}(k) = X_{HL}(k)$, but in every period other then the period she enters an agent only has to determine one action, namely x_{HL} if she is of type H or x_{LH} if she is of type L. For convenience let $x_H = x_{HL}$, denote the strategy of the high type when paired with a low type, while $x_L = x_{LH}$ is the strategy of a low type when paired with a high type. In what follows we will refer to the pair (x_H, x_L) as the bargaining strategy of an agent.

3 Equilibrium Analysis

Our goal is to understand the structure of the stochastically stable equilibria as a function of the cost of investment, c, the degree of investment complementarity, S_A, and the degree of investment specificity, modeled by δ. The purpose of this section is to characterize the uniform sequential equilibria in stationary

[10] Young (1993b) has shown in a two population model that the equal split is stochastically stable when both populations have identical characterisitcs. In his model contrary to ours there exist however conventions where the surplus is not split equally between the partners from the two populations although they have identical characteristics.

strategies of the population game[11] that result in high investment. It will turn out that if stochastically stable equilibria exist they are indeed in this class of equilibria.

Note that in the Nash demand game any strategy profile (x_H, x_L) such that $x_L + x_H = S_A$ is a Nash equilibrium. By a *bargaining norm* we mean a situation where all individuals have identical bargaining strategies of the form $(S_A - \hat{x}_L, \hat{x}_L)$ for some $\hat{x}_L \in [0, S_A]$.

Since the focus of this paper lies on the bargaining behavior in matches of different types we will make assumptions that guarantee that equal split trades always occur between equal types. Given our assumption that surplus is split equally between equal types if trade occurs, we only have to be concerned about the question whether equal types want to trade at all or rather wait for a different type. The maximal payoff a low type can get in the next period is S_A and therefore $\frac{S_L}{2} > \delta S_A$ is sufficient to guarantee trade between low types. For high types we must have $\frac{S_H}{2} > \delta S_A$ which clearly is a weaker condition. Hence we will assume throughout the paper that

$$\delta < \frac{S_L}{2S_A}. \tag{1}$$

Considering High-Low pairings and observe that for relatively high discount factors and strong complementarity between investments, even if a bargaining norm exists, one of the two partners would rather wait for a partner of identical type than to trade according to the bargaining norm. Given that in a High-Low pairing the high type expects a low bid of $\hat{x}_L$, the low type expects a high bid of $S_A - \hat{x}_L$ and given that both partners believe that they will meet an identical type in the following period, they will be willing to trade if

$$\begin{aligned} S_A - \hat{x}_L &> \delta S_H/2, \\ \hat{x}_L &> \delta S_L/2. \end{aligned}$$

The first condition ensures that the high type prefers trading with a low type, rather than waiting one period and trading with a high type. The second condition is the corresponding requirement for the low type. Adding these inequalities together implies the following necessary condition for trade to occur for HL matches:

$$\frac{2S_A}{S_L + S_H} > \delta. \tag{2}$$

Put differently, (2) implies that there exists a bargaining norm x_L such that individuals always trade in High-Low matchings regardless of their beliefs concerning the distribution of types in the population. Notice that condition (2) can not be binding, if investments are *substitutes.* Investments are *substitutes* if the marginal return from the first investment is greater than from the second investment:

$$\begin{aligned} S_A - S_L &> S_H - S_A, \\ \frac{2S_A}{S_L + S_H} &> 1. \end{aligned}$$

[11] This means that we consider scenarios where all individuals use identical strategies of the stage game every period and these strategies are constant over time.

Conversely, investments are complements if the marginal return from the second investment is larger:

$$\begin{aligned} S_A - S_L &< S_H - S_A, \\ \frac{2S_A}{S_L + S_H} &< 1. \end{aligned}$$

In this case, when δ is large it may be more efficient for HL pairs not to trade, and instead to delay trade until they meet a partner of the same type. For further reference, the requirement that there is a bargaining norm that implies trade in HL pairings regardless of the individual beliefs about the type distribution is summarized as the *trade condition:*

Definition 1 *The discount rate δ satisfies the* trade condition *if $\delta < \frac{2S_A}{S_L+S_H}$.*

It shall be shown below that this is a necessary condition for the existence of a stochastically stable bargaining norm when investments are complements. By a *norm* we mean a pair $\{I, \hat{x}_L\}$, with the interpretation that each agent selects the investment I upon entering the market, the low type demands $\hat{x}_L$, while the high type demands $\hat{x}_H = S_A - \hat{x}_L$. To economize on writing out the full set of strategies and payoffs, the notion of a self-enforcing norm is be defined as follows.

Definition 2 *A norm $\{H, \hat{x}_L\}$ is self-enforcing if:*

1. $(1-\lambda)(S_H/2 - \hat{x}_L) + \lambda\left((S_A - \hat{x}_L) - \frac{S_L}{2}\right) \geq c/(1-2\lambda)$,
2. $S_A - \hat{x}_L \geq \delta \frac{(1-\lambda)}{(1-\delta\lambda)} S_H/2$
3. $\hat{x}_L \geq \delta \frac{\lambda}{(1-\delta(1-\lambda))} S_L/2.$

The expected payoff of a person making a high investment assuming that trade is immediate and she meets a high type is $(1-\lambda) S_H/2 + \lambda \hat{x}_L$, while the result of no investment is $\lambda S_H/2 + (1-\lambda)\hat{x}_L$. If she meets a low type, the expected payoffs are $(1-\lambda)(S_A - \hat{x}_L) + \lambda S_L/2$ if she invests high and $\lambda(S_A - \hat{x}_L) + (1-\lambda)S_L/2$ if she invests low. Given the expected equilibrium fraction of high types in the market in any period is $(1-\lambda)$ a simple calculation yields condition 1. The second condition is the requirement that a person who is a high type prefers to trade with a low type, rather than wait until meeting a high type. The final condition requires the low type to prefer trading with a high type, rather then waiting until meeting a low type. This places a lower bound on $\hat{x}_L$. It is a straightforward exercise to show that for every self-enforcing norm there is a sequential equilibrium yielding this outcome for the trading game outlined above. A self-enforcing norm, $\{L, \hat{x}_L\}$, for low investment is defined in a similar fashion.

For much of the analysis the parameter λ is positive, but small. In the limit when $\lambda = 0$, a sufficient condition for the existence of a self-enforcing norm with high investment is that it is efficient.

Proposition 1 *Suppose it is strictly efficient for all agents to select high investment, $S_H - 2c > \max\{S_A - c, S_L\}$, then for all δ satisfying the trade condition a bargaining norm, $\hat{x}_L$, exists such that $\{H, \hat{x}_L\}$ is a self-enforcing norm for λ sufficiently small.*

This result demonstrates that when noise is small it is possible to support as an equilibrium high investment whenever it is efficient to do so. In contrast, the literature on the holdup problem assumes that the *ex post* division of the surplus is determined by the Nash bargaining solution, which in some cases induces inefficient investment. However the division implied by the Nash bargaining solution is only one among many sequential equilibria of the game. In general, one is able to conclude that for this game there are a large number of sequential equilibria, many of which induce efficient investment. The question then is whether or not the efficient equilibria are (stochastically) stable.

4 Learning Dynamics

Consider now the kind of bargaining and investment norms that are developed endogenously in a population of adaptive agents. Following Young (1993a) and Kandori, Maillath, and Rob (1993) it is assumed that agents sample the previous periods trades to build an empirical distribution regarding the investment and bargaining behavior of the other individuals in the population (see Young (1993b) for the application of this approach to the Nash bargaining game). Regarding the value of the outside option, agents believe that the distribution of low and high types in the economy is time stationary, a hypothesis that is consistent with the assumption that agents base current actions on past observations of the frequency of high types. It is also assumed that with a small probability they make mistakes in executing their optimal strategy given their beliefs regarding the play of the game described in section 2.

Our model consists of a single population of individuals who choose investment from $\{h, l\}$ upon entering the population and afterwards have to choose their action from the space $X(k)$ every period until they trade and leave the population. This choice is based on beliefs about distribution of types and bargaining behavior of the other individuals in the population. Each period every individual independently takes a random sample of m individuals from the previous period observing the type and the demand made at the bargaining stage. This sample is added to the memory of the individual thereby replacing some old observations[12].

Using the data in her memory each individual generates beliefs about the fraction of types H in the population and the distribution of demands made by other individuals in HL and LH pairings. Each of these beliefs is based on m observations, hence there is a finite set of possible beliefs we denote by B. For each $\beta \in B$ we denote by $\hat{p}(\beta)$ the estimated proportion of high types, by $\hat{F}_H(x_H, \beta)$ the estimated probability that x_H or less is demanded by a high type in a HL pairing and by $\hat{F}_L(x_L, \beta)$ the estimated probability that x_L or less is demanded by a high type in a LH pairing. Put differently, $\hat{F}_H$ and $\hat{F}_L$ are empirical distribution functions given the observations in the memory of the individual. It will turn out to be convenient to denote by $\mathcal{P}(z)$ the distribution function of point expectations z, i.e. $\mathcal{P}(z)(x) = 0$ for $x < z$ and $P(z)(x) = 1$ for $x \geq z$. When an agent leaves the market, her beliefs are passed on to the new agent entering the market to replace this agent. Beliefs in the first period are arbitrary.

The structure and time-line of the game with adaptive dynamics is summarized as follows (see also

[12]An exact mathematical description of the belief formation and learning dynamics considered as well as the associated belief and state spaces is given in Appendix A

Appendix A):

(i) At the beginning of the game beliefs are random, but when an individual leaves she is replaced by another agent with the same beliefs, say β.

(ii) Investment decisions are only made by agents entering the population in the current period. Given her beliefs, an agent chooses to invest if the expected gain from investment exceeds investment costs c under the assumption of optimal behavior on the bargaining stage. Then she draws her type, which is equal to her investment with probability $1-\lambda$.

(iii) Each period the following steps are repeated until exit occurs:

1. At the beginning of every period t the individual randomly samples the types of m individuals from the previous period. This is used to update beliefs $b_t^i \in B$.
2. With probability $\varepsilon > 0$ the individual selects an action randomly from $X(k)$, under the uniform distribution. This noise process is *i.i.d.* between individuals and periods. With probability $1-\varepsilon$ the individual determines which demand maximizes the expected payoff under her beliefs if she is matched with a different type.
3. Agents are randomly paired, and their payoffs are determined. If the partners are of identical type, there is an equal split, otherwise they chose the actions determined at stage 2.
4. If trade occurs, both agents leave and are replaced with identical agents who begin at step (ii). If not, step (iii) is restarted.

Given that an agent's action is completely determined by her beliefs $b_t^i \in B$, and type $T^i \in \{H, L\}$[13], the state at time t is characterized by a distribution over beliefs and types, and accordingly there is a finite state space we call $\mathcal{S}$. The learning process described above defines a time homogeneous Markov process $\{\sigma_t\}_{t=0}^{\infty}$ on the state space $\mathcal{S}$. Although, even for $\epsilon > 0$, the transition matrix is not positive, the following lemma shows that the process is irreducible and aperiodic.

Lemma 1 *For $\epsilon > 0$ the Markov process $\{\sigma_t\}_{t=0}^{\infty}$ as defined above is irreducible and aperiodic.*

Hence, for $\epsilon > 0$ there exists a unique limit distribution $\pi^*(\epsilon)$ over $\mathcal{S}$, where $\pi_s^*(\epsilon)$ denotes the probability of state s. Following a standard approach in evolutionary game theory we consider the limit distribution for small values of ϵ and in particular characterize the states whose weight in the limit distribution stays positive as the mutation probability ϵ goes to 0. Such states are called stochastically stable:

Definition 3 *A state $s \in \mathcal{S}$ is called stochastically stable if $\lim_{\epsilon \to 0} \pi_s^*(\epsilon) > 0$. We say that a set is stochastically stable if all his elements are stochastically stable.*

[13]We look at the process after all incoming agents have made their investment decisions, but before they are paired and therefore the type of all agents is determined.

The reason why this concept is of interest is that for small ϵ the process spends almost all the time in stochastically stable sets. Hence, characterizing the stochastically stable outcome means characterizing the long run properties of the evolutionary process. To identify stochastically stable states it is necessary to first identify the minimal absorbing sets of the process for $\epsilon = 0$. It is well known that the set of stochastically stable states is a subset of the union of these so called limit sets. Formally, a limit set is defined as follows:

Definition 4 *A set $\Omega \subseteq \mathcal{S}$ is called a limit set of the process if for $\epsilon = 0$ the following statements hold:*

$$\forall s \in \Omega \;\; I\!P(\sigma_{t+1} \in \Omega | \sigma_t = s) = 1$$

$$\forall s, \tilde{s} \in \Omega \;\; \exists z > 0 \;\; s.t. \;\; I\!P(\sigma_{t+z} = \tilde{s} | \sigma_t = s) > 0.$$

In the following sections we will characterize the stochastically stable sets and discuss the implied investment and bargaining norms.

The question we address is the emergence of a unique, efficient and stable bargaining norm in which all individuals follow the same investment strategy, and have the same expectations regarding how to divide the gains from trade. This is formally defined by:

Definition 5 *A state s induces the bargaining norm x_L if all individuals have beliefs $\beta \in B$ that place probability one on the demand by their partner being x_L or $S_A - x_L$, depending upon their type in HL matches.*[14] *If all stochastically stable states induce the same bargaining norm we say that this bargaining norm is stable.*

Therefore, we shall say that a bargaining norm does not exist at a state s if there is heterogeneity in the beliefs of the agents regarding the terms of trade between high and low types. Let us now consider the constraints that the outside options place upon feasible bargaining norms.

5 Deterministic Investment Effects

Before we explore the stochastically stable norms of the model described above we discuss briefly the importance of our assumption that investment effects are stochastic ($\lambda > 0$) for the evolution of bargaining norms. This is particularly important since the results of Ellingsen and Robles (2002) and Troger (2002) show that in cases of one-sided investment the stable bargaining norm always induces efficient investment when $\lambda = 0$[15].

Dawid and MacLeod (2001) study the two-sided investment model presented for the case $\lambda = \delta = 0$. Their findings concerning the evolution of bargaining norms can be summarized as follows (compare Proposition 4 and Proposition 7 in Dawid and MacLeod (2001)):

Proposition 2 *For deterministic investment effects ($\lambda = 0$) and relationship specific investments ($\delta = 0$) the stochastically stable set always includes states where individuals have heterogenous beliefs about bargaining behavior. Hence, there is no stable bargaining norm.*

[14] Formally $\hat{F}_H(\beta) = \mathcal{P}(S_A - x_L)$, and $\hat{F}_L(\beta) = \mathcal{P}(x_L)$).

[15] Also, they only consider the case of relationship specific investments ($\delta = 0$).

The assumption that $\delta = 0$ is not crucial for this finding and the result would still hold for $\delta > 0$. The intuition is that, once the investment strategies in the population are uniform, any bargaining norm which might exist at that point will be slowly destroyed. Under identical investment strategies and deterministic investment effects the only way a pairing between a high and a low type might occur is that at least one of the two has mutated. A mutant may not follow the bargaining norm and hence at least half of the demands in high low pairings are completely random and do not follow any bargaining norm. Since all individuals use these demands to update their beliefs, any uniform consistent point beliefs which might have existed in the population will be destroyed, and beliefs about bargaining behavior between high and low types keeps drifting around in the space of possible beliefs. Therefore, stable bargaining norms between high and low types cannot evolve for deterministic investment effects.

This drift of beliefs is also present in the scenario with one-sided investment and eventually leads to an outcome where investors who invest efficiently get a sufficiently large part of the surplus that they have no incentive to change investment regardless of their beliefs about the allocation of surplus for other investment levels. In the case of two-sided investments one of the two partners will always have incentives to change her investment level if she believes that such a change increases her fraction of the surplus by a sufficient amount. Therefore, for the case of two sided deterministic investments there are not only no stochastically stable bargaining norms but also investment levels are in general inefficient.

6 Stochastically Stable norms

The findings reported in the previous section suggest that the uncertainty of investment effects should have an important positive role for the evolution of stochastically stable bargaining norms. For $\lambda > 0$ high-low matches occur with positive probability even after an investment norm has been established and therefore the drift of beliefs which is responsible for the continuous destruction of norms in the deterministic case cannot occur. In this section we return to the case $\lambda > 0$ and explore whether we always get stochastically stable bargaining norms.

A necessary condition for the evolution of a norm is that the terms of trade between high and low types result in outcomes that are better than their respective outside options. By simply waiting for a partner with the same type an agent can guarantee a non-negative expected payoff, where the size of the expected payoff depends on the agents' beliefs about the distribution of types and the value of δ. We say that a bargaining norm is compatible with $\hat{p}$ and δ if both parties are better off than their respective expected outside option. Denote the set of all bargaining norms which are compatible with all $\hat{p} \in [0, 1]$ for a certain discount factor by $\mathcal{C}(\delta)$. We get the following lemma, where $\alpha = \alpha_{LH} = \alpha_{HL}$ is the minimum unit of account for dividing the surplus, as defined in the game form of section 2.

Lemma 2 *The set $C(\delta)$ is non-empty for sufficiently small α if and only if the trade condition holds.*

Let us now characterize the limit sets in this framework. Once a bargaining norm x_L, which is compatible with δ, is reached, in the absence of mutations all low types always demand x_L against high types and high types always demand $S_A - x_L$ against low types. Hence, beliefs can never change once such a state has been reached. If beliefs are heterogeneous there is always a positive probability that all agents observe identical samples and beliefs become homogeneous and compatible. However, after a bargaining norm has been reached the distribution of agent types may change between two periods, even if the investment behavior is constant. The randomness of the outcome from investment implies that all distributions of H and L types are possible. Hence, if there is a bargaining norm that is not compatible with all $p \in P$, eventually it will be disrupted. This suggests that the limit sets correspond to norms in $\mathcal{C}(\delta)$, when it is not empty. Given that the trade condition holds $\mathcal{C}(\delta)$ is never empty and we always have bargaining norms as the possible long run outcome of our process. This is formalized in the following lemma.

Lemma 3 *Suppose that k is sufficiently large and the trade condition holds. Then for each $x_L \in \mathcal{C}(\delta)$ there exists a limit set $\Omega(x_L)$ consisting of all $s \in \mathcal{S}$ such that $s_\zeta > 0$ only if $\zeta = (T, \beta)$ for some some $T \in \{H, L\}$ and some β such that $\hat{F}_H(\cdot, \beta) = \mathcal{P}(S_A - x_L)$ and $\hat{F}_L(\cdot, \beta) = \mathcal{P}(x_L)$.*

If the trade condition does not hold then $\mathcal{C}(\delta) = \emptyset$, and the outside option of waiting for an equal type always becomes binding for some $\hat{p}$. In the following proposition we show that in such a scenario bids never settle down at a compatible norm but there occur persistent fluctuations driven by the fluctuations in the $\hat{p}^i$. Long run bargaining behavior is then characterized by ergodic behavior on a set of different bids.

Proposition 3 *Suppose that the trade condition does not hold, then for sufficiently large m, n, and k there is a unique stochastically stable set $\mathcal{L}$ where beliefs about demands as well as induced actual demands do not coincide for all individuals in all states contained in $\mathcal{L}$. Accordingly, there exist no stochastically stable bargaining norms.*

When investments are complements, there is a $\bar{\bar{\delta}} < 1$, such that for all $\delta > \bar{\bar{\delta}}$ the trade condition is not satisfied. This demonstrates that if the market is sufficiently competitive and investments are complements, then it is not possible for a bargaining norm to evolve. This does not imply that increasing market competition results in inefficiency. When the trade condition does not hold, then LL and HH matches are the most likely trades, and hence if high investment is strictly efficient, $(S_H/2 - c > S_L/2)$ individuals often find it in their interests to invest.

The question now concerns the nature of the fair division rule when it is efficient for HL pairs to trade, and therefore the remainder of the paper assumes that the trade condition is satisfied. Under this assumption we always have a whole set of potential long run norms to be reached. Clearly the investment incentives depend on which of these norms are reached in the long run. The characterization of the limit sets in lemma 3 shows that productivity types will keep fluctuating even after bargaining behavior has settled down at a norm. Investment incentives however do not only depend on the bargaining norm but also on the distribution of productivity types in the population. So, in order to understand the long-run evolution of investment behavior of the individuals we first have to examine which of the possible bargaining norms is selected in the long run and then given this bargaining behavior have to study the dynamics of productivity type distributions under our investment rule. The long run properties of the productivity type distribution together with the bargaining norm then determine which investment decisions are made in the long run. In the following two sections this analysis is carried out separately for the case where investments are substitutes and complements.

7 The Case of Substitutes

7.1 Stochastically Stable Bargaining norms

We know already that the trade condition always holds if investments are substitutes which implies that in the case of investment substitutes there always exist bargaining norms as a potential long-run outcome of our dynamic process.

The fact that individuals cannot perfectly determine the productivity of their investments has two important implications. First, every period there is a strictly positive probability of both types existing in the market, and thus there are always with positive probability HL trades occurring in the market which can be used to update the believes of individuals. Second, regardless of the investment decisions of individuals, any distribution of types has strictly positive probability. On the other hand, transitions between bargaining norms have to be triggered by (in general multiple simultaneous) mutations. Hence, for small mutation probabilities bargaining norms adjust more slowly, and are more stable than the realized distribution of types. This implies that the stochastically stable bargaining norm is *independent* of the long run investment behavior and therefore also independent of investment costs c. The next proposition provides a rigorous proof of this fact and derives the properties of the bargaining norm that arises in the long run. It turns out that the qualitative properties of the process depend on the degree of substitutability between investments. We say that investments are *weak substitutes* if $\frac{1}{2}(S_H + S_L) \leq S_A \leq \bar{S} := S_H - \frac{\delta}{2}(S_H - S_L)$. For $\bar{S} < S_A \leq S_H$ investments are *strong substitutes.*

Proposition 4 *For sufficiently large m, n the limit of the stochastically stable sets of the process $\{\sigma_t\}$ for $k \to \infty$ can be characterized as follows:*

(a) If investments are weak substitutes every stochastically stable state induces the bargaining norm

$$\hat{x}_A^S = \frac{S_A}{2} - \frac{\delta}{2(2-\delta)}(S_A - S_L).$$

(b) If investments are strong substitutes every stochastically stable state induces the bargaining norm

$$\hat{x}_A^S = \frac{S_A}{2} - \frac{\delta}{4}(S_H - S_L).$$

Following definition 5 we will refer to the bargaining norm induced by all stochastically stable states as the stable bargaining norm. In the absence of outside options ($\delta = 0$) the equal split rule is the unique, stable bargaining norm, regardless of investment levels.

7.2 Investment Norms with Substitutes

From proposition 4 we infer that the level of long run investment does not affect the outcome of bargains. This greatly simplifies the analysis of investment behavior in the long run. We can determine investment behavior as a function of the bargaining norm and then simply insert the stable bargaining norm. For a given bargaining norm $\hat{x}_L$ investment is optimal iff

$$\begin{aligned}&(1-\lambda)\left(\hat{p}\frac{S_H}{2}+(1-\hat{p})(S_A-\hat{x}_L)\right)+\lambda\left(\hat{p}\hat{x}_L+(1-\hat{p})\frac{S_L}{2}\right)-c\\ \geq\ &(1-\lambda)\left(\hat{p}\hat{x}_L+(1-\hat{p})\frac{S_L}{2}\right)+\lambda\left(\hat{p}\frac{S_H}{2}+(1-\hat{p})(S_A-\hat{x}_L)\right).\end{aligned}$$

Taking into account that $(S_H+S_L)/2-S_A<0$ this gives the following condition for high investment to be optimal:

(3) $$\hat{p}\leq p^*(\hat{x}_L;\lambda):=\frac{S_A-\hat{x}_L-S_L/2-c/(1-2\lambda)}{S_A-S_H/2-S_L/2}.$$

As pointed out above, to analyze the dynamics of investment for a given bargaining norm we consider the evolution of type distributions over time. Given a current distribution of types the distribution of types in the following period in general depends on the outcome of the stochastic sampling procedure for all agents, which gives the beliefs $\hat{p}(b_t^i)$ and therefore influences the investment decisions, and the actual realization of types given the investment decision. This can be described by a Markov process $\{\tilde{\sigma}_t\}_{t=0}^{\infty}$ on the state space $\tilde{S}=\{0,1/n,2/n,\ldots,1\}$. For $\lambda>0$ the process is irreducible and aperiodic. The unique limit distribution is denoted by $\tilde{\pi}^*(\lambda)$. The following lemma characterizes the limit distribution for small values of λ.

Lemma 4 *When investments are substitutes, then given a bargaining norm $\hat{x}_L$, the long run distribution of types for sufficiently small λ can be characterized as follows:*

(a) $p^*(\hat{x}_L;0)\leq 0$: no individual ever invests and $\lim_{\lambda\to 0}\tilde{\pi}_0^*(\lambda)=1$.

(b) $p^*(\hat{x}_L;0)>1$: all individuals always invest and $\lim_{\lambda\to 0}\tilde{\pi}_1^*(\lambda)=1$.

(c) $p^*(\hat{x}_L;0)\in(0,1)$: $\lim_{\lambda\to 0}\tilde{\pi}_1^*(\lambda)=\lim_{\lambda\to 0}\tilde{\pi}_0^*(\lambda)=0.5$.

In case (a) we say that $\hat{x}_L$ induces a no-investment norm, in (b) $\hat{x}_L$ induces a full investment norm, and in case (c) we say that $\hat{x}_L$ induces cyclical investment. By cyclical investment we mean that in one period everybody invests, and in the next period nobody invests. What is happening is when all individuals invest, it is optimal not to invest, and vice verso.[16] It should also be pointed out here that even if we consider the limit of long run stochastically stable sets for $\lambda\to 0$ this should be interpreted as the long run properties of the process if both ϵ and λ are small but λ is of an order of magnitude larger than the mutation probability ϵ. In other words, the transition between bargaining norms is always assumed to be much slower than the transition between investment patterns. Using the previous lemma it is straight forward to describe the investment behavior which is induced by the stochastically stable bargaining norms. We just have to insert the stochastically stable bargaining norm $\hat{x}_L^s$ into p^* and apply lemma 4.

[16]Note that for substitutes in cases where $p^*(\hat{x}_L;0)>1$ the action H is dominant at the investment stage for small λ and all heterogeneity in types is created by deviations of the actual type from investment. Therefore, it is easy to see that a bargaining convention $\hat{x}_L$ induces an investment convention if and only if there is a $\lambda^*>0$ such that the convention $\{H,\hat{x}_L\}$ is stable for all $\lambda<\lambda^*$.

Proposition 5 *Assume that m, n and k are sufficiently large.*

(a) If investments are weak substitutes the stochastically stable bargaining norm induces full-investment for $c < c^1$, no-investment for $c > c^2$ and cyclical investment for $c \in [c^1, c^2]$, where

$$\begin{aligned} c^1 &= \tfrac{1}{2(2-\delta)}(\delta(S_A - S_L) + (2-\delta)(S_H - S_A)) \\ c^2 &= \tfrac{1}{2-\delta}(S_A - S_L). \end{aligned}$$

(b) If investments are strong substitutes the stochastically stable bargaining norm induces full-investment for $c < c^3$ and cyclical investment for $c \geq c^3$, where

$$c^3 = \frac{1}{4}(\delta(S_H - S_L) + 2(S_H - S_A)).$$

Notice that when $\delta = 0$, then $c_1 = (S_H - S_A)/2$, but in the case of substitutes it is efficient for both parties to invest whenever $c < (S_H - S_A)$. Therefore we obtain under-investment in some cases.

In the case of weak substitutes the gain from investing at the bargaining norm is:

$$\begin{aligned} S_H/2 - x_L &= \frac{(S_H - S_A)}{2} + \frac{\delta}{2(2-\delta)}(S_A - S_L), \\ &\geq \frac{(S_H - S_A)}{2}. \end{aligned}$$

Therefore, the outside option increases the gains from investing, regardless of whether it is binding at the equilibrium. However, for weak substitutes, it never increases incentives to the point that the gains from investing are equal to the full marginal gains, given by $(S_H - S_A)$. On the other hand, if investments are strong substitutes and the gains from the second investment are very small (case (b) above) the stochastically stable norm indeed induces full investment whenever this is efficient. This is formalized in the following corollary.

Corollary 1 *If investments are strong substitutes the stochastically stable bargaining norm induces full-investment for all values of c where full investment is efficient.*

Proof. It is straight-forward to check that $c^3 \leq S_H - S_A$ under these assumptions. ■

In the summary of results below we will also provide a graphical illustration of these findings.

7.3 Impact of the Outside Option

Considering proposition 4, the effect of the outside options on the bargaining norm might be quite surprising. Notice, that in this model the outside option is introduced only as a constraint on the set of possible bargaining agreements, and hence one might expect the outside option principle to apply (see Binmore, Rubinstein, and Wolinsky (1986)). In that case if $x_L > \delta S_L/2$ and $S_A - x_L > \delta S_H/2$, then x_L should not depend on either S_H or S_L, yet we find that that for all $\delta > 0$ the stochastically stable bargaining norm depends upon at least one of the outside options, and that the low types share is always strictly increasing in S_L, a result that is consistent with Binmore, Proulx, Samuelson, and Swierzbinski (1995) who report results from a bargaining game with drift. This might raise the question whether the efficiency result of corollary 1 is a simple implication of the difference in threat point payoffs of the two types.

To address this question let us denote by $\hat{x}_L^N$ the allocation consistent with the Nash bargaining solution between a high and a low type where both have beliefs $\hat{p} = 1$ and the expected payoffs in the following period are treated as a threat point. This allocation has to satisfy

$$\hat{x}_L^N = \delta \hat{x}_L^N + \frac{1}{2}\left(S_A - \delta \hat{x}_L^N - \delta \frac{S_H}{2}\right),$$

and therefore we get

$$\hat{x}_L^N = \frac{S_A}{2} - \frac{\delta(S_H - S_A)}{2(2-\delta)}. \tag{4}$$

Comparing this expression with the stochastically stable bargaining norms from proposition 4, simple calculations show that under our assumption of $S_A > (S_H + S_L)/2$ we always have $\hat{x}_L^N > x_A^S$. Accordingly, the investment incentives in a population of investors under the stochastically stable bargaining norm are not only larger than under the equal split rule but also larger than under Nash bargaining with the outside options as threat points.

To understand this result intuitively we have to realize that the long run stability of the bargaining norms are determined by their resistance to change in scenarios where deviations from the norm have the highest chance of altering the norm. Bargaining norms are more easily destabilized in scenarios with low investment in the population since the expected losses from disagreement when not giving in to demands of deviators from the norm are the largest under this investment pattern. If investments are substitutes a high type has a lot of bargaining power in an environment of low types and hence the stochastically stable bargaining norm gives a larger part of the surplus to the high types than they would get if the norm had been evolved in a population of mostly high types. Hence, the stochastically stable norm allocates more to the high types than the Nash bargaining solution in an environment of high types would. Although developed in low investment scenarios, the stochastically stable bargaining norm is adhered to even if in the long run everyone invests, and hence it facilitates the development of full investment norms.

This discussion implies that the evolutionary learning facilitates the development of full investment. In the following corollary we compare the stochastically stable outcome to the notion of a self-enforcing norm under the equal split rule and the Nash bargaining solution:

Corollary 2 *(a) If c satisfies*

$$c \leq \frac{1}{2}(S_H - S_A),$$

then for λ sufficiently small the stochastically stable norm induces full investment, and $\{H, S_A/2\}$ as well as $\left\{H, \hat{x}_L^N\right\}$ are self-enforcing norms.

(b) If c satisfies

$$\frac{1}{2}(S_H - S_A) < c \leq \frac{1}{2-\delta}(S_H - S_A)$$

then for λ sufficiently small the stochastically stable norm induces full investment, $\left\{H, \hat{x}_L^N\right\}$ is a self-enforcing norm but $\{H, S_A/2\}$ is not a self-enforcing norm.

(c) If c satisfies

$$\frac{1}{2-\delta}(S_H - S_A) < c < \frac{\delta}{4}(S_H - S_L) + \frac{1}{2}(S_H - S_A),$$

then for λ sufficiently small the stochastically stable norm induces full investment, but neither $\{H, S_A/2\}$ nor $\left\{H, \hat{x}_L^N\right\}$ are self-enforcing norms.

Note that it follows from this corollary that the Nash bargaining solution $\hat{x}_L^N$ never implies efficient investment in the sense that there is always a range of cost values c where high investment is efficient but $\left\{H, \hat{x}_L^N\right\}$ is not a self enforcing norm. As we know from corollary 1, the stochastically stable norm does induce efficient investment if investments are strong substitutes. These results demonstrate that the decrease of the size of the hold-up region under the evolutionary dynamics compared to the equal split rule is not a simple implication of the existence of outside options. Rather, the dynamic interplay between investment and bargaining decisions is responsible for the increased long run investment in our evolutionary setting[17].

8 The Case of Complements

Consider now the case of complementary investments, where $(S_H + S_L)/2 \geq S_A \geq S_L$. We know already from proposition 3 that no norms evolve if the trade condition is violated. Therefore, we assume throughout this section that the trade condition holds.

Under this condition there is again a unique stochastically stable bargaining norm which is independent of beliefs regarding the fraction of high types in the market. The properties of this bargaining norm depend on the degree of complementarity of investments. Analogous to the substitutes case we call investments weak complements if $(S_H + S_L)/2 \geq S_A \geq \underline{S} := \frac{\delta}{2}((2-\delta)S_H + S_L)$, and strong complements if $\underline{S} > S_A \geq S_L$.

Proposition 6 *Suppose the trade condition holds, then for sufficiently large m, n the limit of the stochastically stable sets of the process $\{\sigma_t\}$ for $k \to \infty$ can be characterized as follows:*

(a) If investments are strong complements the stochastically stable bargaining norm is

$$\hat{x}_A^S = S_A - \delta \frac{S_H}{2}.$$

(b) If investments are weak complements the stochastically stable bargaining norm is

$$\hat{x}_A^S = \frac{S_A}{2} - \frac{\delta}{2(2-\delta)}(S_A - S_L).$$

Case (a) occurs when the outside option for the high type is binding for $\hat{p} = 1$. This can only happen if $\underline{S} > S_L$. A necessary and sufficient condition for this to apply is:

$$\delta \geq \frac{S_L}{S_H}.$$

One of the implications of the stochastic stability criteria is that, as in the case of substitutes, the existence of an outside options *always* increases the payoff for the high type relative to the equal division solution. On

[17] If we restricted the model to only a single possible investment level the resulting stochastically stable bargaining convention would exactly match the Nash bargaining solution with the outside option as threat point.

the other hand, it can be easily verified that the Nash bargaining solution with the outside option as threat point, given by (4), gives a smaller allocation of the surplus to low types compared to the stochastically stable norm and therefore provides higher investment incentives.

One can explore the maximum incentives possible, while ensuring the existence of a bargaining norm by supposing that the trade condition is satisfied with equality, namely $\delta = \frac{2S_A}{S_L+S_H}$. In this case

$$\begin{aligned}
\underline{S} &= \frac{\delta}{2}((2-\delta)S_H + S_L) \\
&= \frac{\delta\,(1-\delta)}{2}S_H + \delta(S_H+S_L)/2 \\
&= \frac{\delta\,(1-\delta)}{2}S_H + S_A > S_A,
\end{aligned}$$

and hence we are in the case of strong complements, and the stable bargaining norm is given by:

$$\begin{aligned}
\hat{x}_A^S &= S_A - \delta\frac{S_H}{2}, \\
&= S_A\frac{S_L}{S_L+S_A}.
\end{aligned}$$

This result illustrates the effect that the low payoff plays in determining the bargaining norm. When S_L is close to zero (the payoff in the absence of trade), then with sufficient competition one obtains first best incentives, while ensuring the existence of a bargaining norm. When $\delta = \frac{2S_A}{S_L+S_H}$ and $c < S_A$, then low investment is not an equilibrium for $S_L = 0$, and we have high investment in this case. However, in other cases, both high and low investment choices may be equilibria given the bargaining norm. The next sections explores which these equilibria is stable.

8.1 Investment Norms with Complements

Taking the bargaining norm as fixed, investment decisions have the structure of a coordination game when investments are complements. Incentives are larger for $\hat{p} = 1$ than for $\hat{p} = 0$. This implies that if the bargaining norm induces no investment for $\hat{p} = 1$, no individual will invest any more, once the bargaining norm has been established regardless of their beliefs $\hat{p}$ – a no-investment norm is induced. On the other hand, if investment is optimal at $\hat{p} = 0$, everyone invests under the stochastically stable bargaining norm – a full investment norm is induced. However, if investment is optimal for $\hat{p} = 1$ and no investment is optimal for $\hat{p} = 0$, both the homogeneous state corresponding to full investment and the homogeneous state corresponding to no investment are locally stable states in the sense that the process never leaves each of these states as long as high investment always implies high types and low investment always implies low types. In such a scenario the threshold p^* defined in (3) is in $(0,1)$ and investment is optimal if and only if $\hat{p} \geq p^*(\hat{x}_L)$.

Investment effects are assumed to be stochastic, and therefore there is always a positive probability that the process wanders from a non-investment to a full investment state and vice versa. As in the case of substitutes, for a given bargaining norm the evolution of the distribution of high and low types is described by a Markov process $\{\tilde{\sigma}_t\}_{t=0}^{\infty}$ on the state space $\tilde{S}$. This process has a unique limit distribution $\tilde{\pi}^*(\lambda)$. We are again interested in the the properties of this limit distribution if the probability of deviations of productivity types from investment becomes small with the understanding that λ is still an order of magnitude larger than ϵ.

Lemma 5 *Assume that* $0 < \lambda < 0.5$ *and* m *and* n *are and sufficiently large and a bargaining norm* x_L *is given. Then, for* $p^*(x_L, \lambda) > (<) 0.5$ *we have* $\sum_{i<n/2} \tilde{\pi}^*_{i/n}(\lambda) > (<) \sum_{i>n/2} \tilde{\pi}^*_{i/n}(\lambda)$. *Furthermore, we have* $\lim_{\lambda\to 0} \tilde{\pi}^*_0(\lambda) = 1$ *if* $p^*(x_L, 0) > 0.5$ *and* $\lim_{\lambda\to 0} \tilde{\pi}^*_1(\lambda) = 1$ *if* $p^*(x_L, 0) < 0.5$.

According to this lemma $p^*(x_L, 0) < 0.5$ implies that in the long run the probability to have a majority of high types is larger than the probability to have a majority of low types and as λ goes to zero the probability to see only high types goes to one. Since investment decisions have the structure of a coordination game here this lemma basically rephrases well known results by Kandori, Maillath, and Rob (1993). We say that a no-investment norm is induced if the threshold $p^*(x_L, 0)$, is larger than 0.5 and that a full investment norm is induced if this inequality holds the other way round. Using this we get the following characterization of the investment norms induced by stochastically stable bargaining norms.

Proposition 7 *Assume that* m, n *and* k *are sufficiently large, the trade condition holds, and investments are complements, then the stochastically stable bargaining norm induces full investment if* $c < c^4(S_A, \delta)$ *and no-investment for* $c > c^4(S_A, \delta)$, *where*

$$c^4(S_A, \delta) = \begin{cases} \frac{1}{4}(S_H - S_L) + \frac{1}{2}(\delta S_H - S_A) & \text{if } S_A \leq \underline{S}, \\ \frac{1}{4}(S_H - S_L) + \frac{\delta}{2(2-\delta)}(S_A - S_L) & \text{if } S_A > \underline{S}. \end{cases} \tag{5}$$

In the case of investment complements high investment is efficient (for small λ) whenever $c < (S_H - S_L)/2$, but is only stochastically stable for $c < c_4 < (S_H - S_L)/2$. If investments are complements there always remains a hold-up region with inefficient investments in the long run. It follows from the coordination game structure of the investment stage that a bargaining norm $\hat{x}_L$ does not necessarily induce a high investment norm even if $\{H, \hat{x}_L\}$ is a self-enforcing norm. An implication of this, especially when compared to the case of substitutes, is that the set of parameters for which a full investment norm is self-enforcing under the equal split rule might be *larger* than the set of parameter values for which high investment is part of a stochastically stable equilibrium. To see this, notice that if $S_A > \delta S_H$ and $c < \frac{1}{2}(S_H - S_A)$ the norm $\{H, S_A/2\}$ is self-enforcing for sufficiently small λ. Comparing this bound on investment costs with c_4 we get

Corollary 3 *For*

$$S_A < \min\left[S_L + 2(1-\delta)S_H, \frac{(2-\delta)S_H + (2+\delta)S_L)}{4}\right]$$

the interval $[c^4(S_A, \delta), (S_H - S_A)/2]$ *is non-empty and if* c *belongs to this interval* $\left\{H, \frac{S_A}{2}\right\}$ *is a self-enforcing norm for sufficiently small* λ, *but there is no stochastically stable norm with full investment.*

The fact that there is a self-enforcing norm with high investment norm does not imply uniqueness of the equilibrium. Even if a high investment norm is self-enforcing there might be a coexisting low investment equilibrium with the corresponding equilibrium selection problem. To deal with the equilibrium selection problem we could use the concept of risk dominance as an equilibrium selection device and say that a bargaining norm induces high investment only if the full investment is the risk dominant equilibrium at the investment stage[18]. It is straightforward to check then that the maximal investment costs inducing high investment under the equal split rule is always below c^4. So, taking into account the coordination problems arising at the investment stage, the equal split rule again provides less investment incentives than the stochastically stable norm. The Nash bargaining solution, on the other hand, provides for the case of investment complements always larger incentives than the stochastically stable norm[19].

Overall these results illustrate that in the case of complements, stochastic stability implies that the holdup problem is even more severe than would be predicted with in a standard incomplete contract model with renegotiation.

9 Summary of the Results

Our results can be summarized with figures illustrating the relationship between the cost and productivity parameters, $\{S_A, c\}$ and the equilibrium investment level. Consider first the case in which investments are purely specific, namely $\delta = 0$. In this case the equilibrium bargaining norm is always the equal split rule. Hence, the only possible candidate equilibria are those investment levels that are also equilibria when agents use the Nash bargaining solution. The possible outcomes are illustrated in figure 1 below, where $S_L = 1$ and $S_H = 2$.

—— Figure 1 Goes Here ——

The illustrated trapezoid region represents all the $\{S_A, c\}$ combinations for which it is efficient for there to be full investment. Holdup occurs in the region above the diagonal line between $\{S_H, 0\}$ and $\{S_L, (S_H - S_L)/2\}$, because for these values high investment is not a stable outcome $(S_A/2 > S_H/2 - c)$. Notice that when investments are substitutes then in the region between the lines $C1$ and $C2$ investments cycles between high and low.

In the region below the diagonal line, even though all parameter combinations have a self-enforcing high investment norm, when investments are complements, then only the region below line $C4$ entails high investment at the stochastically stable equilibria. These results are in contrast to the results of Ellingsen and Robles (2002) and Troger (2002) who obtain the efficient outcome when there is investment by one party alone. Our results suggest that the allocations result in even less investment than suggested by Grossman and Hart (1986).[20] Suppose that one could allocated all bargaining power to one party, then the condition

[18] The use of risk dominance as the selection criterium is appropriate because it is well known that the risk dominant equilibrium coincides with the stochastically stable one in coordination games.

[19] This can be easily checked by realizing that the condition for HH to be the risk dominant equilibrium is $p^*(\hat{x}_L^N, \lambda) < 0.5$ and inserting the corresponding expressions in order to calculate the upper bound on c.

[20] See also Che and Hausch (1999) who show that with two sided investment and efficient renegotiation it is not possible to implement the first best. Their model is different than ours since it precludes the possibility of agents inefficiently refusing to trade when demands are not compatible. In our model such efficient renegotiation is an implication of the model, not an assumption.

for high investment by the owner is given by:

$$S_A - c \geq S_L.$$

The region for which allocating all bargaining power to one party is more efficient than the stable equilibria in the market with endogenous bargaining norms is illustrated in figure 2. Thus, we may conclude that the allocation of stable property rights to one party, even though both parties can make investments, can enhance efficiency.

—— Figure 2 Goes Here ——

The alternative solution to the allocation of bargaining power is to increase competition. The effect of the outside option on investment is illustrated in figure 3 for the case in which the discount factor is $\delta = 0.5$, and we continue to suppose $S_L = 1$ and $S_H = 2$. In this case line $C4$ now moves up, and hence the set of cases for which investment is high increases. Moreover, in the case of substitutes one now has high investment in a region where there would not be high investment in either the standard holdup model (the lower line) or under Nash bargaining with an outside option (the upper line), as discussed in corollary 2. Also notice that there is the possibility of overinvestment when investments are highly substitutable (S_A is close to S_H).

—— Figure 3 Goes Here ——

The model also provides some insights into the interplay between market competition and fair division norms. In our model, individuals who are similar use the equal split rule, that is HH and LL matches. The issue of what constitutes a fair division becomes an interesting question precisely when individuals vary in observable ways. When shielded from competition we find that the equal split rule remains the stochastically stable norm, however, when the trade condition is not satisfied, then high types prefer not to associate with low types, and there is no opportunity for a bargaining norm to evolve. In this case, the outside option increases the incentives to invest, but also results in a breakdown of agreements between low and high types, which may leave unmatched individuals in the market.[21]

One should probably not take the analogy too far, but this result does seem consistent with the impact of markets on agrarian economies. In the absence of markets all family members work on the farm, and hence the notion of unemployment is simply not well defined. This is no longer the case when there are markets because individuals spend time looking for better matches, resulting in observed unemployment. In those cases, market prices are substituted for the norms that have evolved to divide the gains from family labor.

[21]See MacLeod and Malcomson (1993), Felli and Roberts (2000) and Cole, Mailath, and Postlewaite (2000) who provide conditions under which competition, even if imperfect, may provide incentives for efficient two party investment.

10 Discussion

In the literature on contracts and organization there is tension between research on mechanism design demonstrating that efficient allocations can be implemented under a wide variety of situations, and the fact that in practice one rarely observes many of these mechanisms.[22] An example of this tension is the recent contribution of Maskin and Tirole (1999) demonstrating that one can achieve the first best in the hold-up model of Grossman and Hart (1986). Yet, the work of Grossman and Hart (1986) built upon a long literature based upon the idea that ownership and the allocation of property rights do matter for economic performance.[23]

The issue then is how can one formally capture the intuition some contractual arrangements - allocation of bargaining power to one agent, the equal division rule - seem stable, while others, such as the threat to punish an individual for a small deviation from an agreement, seem inherently unstable. The purpose of this paper is to begin to address this question by applying the evolutionary learning approach of Young (1993a) and Kandori, Maillath, and Rob (1993) to a simple model in which individuals make relationship specific investments, bargain over the terms of trade and then trade or move back into the market.

Our results shed light on how information and potential competition can affect the stability of equilibria. Our first observation is that the previous work by Troger (2002) and Ellingsen and Robles (2002) can be interpreted as showing that allocating all bargaining power to one agent is stochastically stable in the case of one sided investment. As we have shown in Dawid and MacLeod (2001), one can make this an if and only if statement. When one adds two sided investment to their model, there is no stable equilibrium. The reason is that the efficient equilibrium depends upon off equilibrium actions that occur with zero probability, and hence stable beliefs regarding the consequences of those actions do not evolve.

This result may shed light on the tension between the economic approach to contract and legal practice. Economists have long argued that courts should enforce contracts as written based upon the idea that voluntary contracts are an expression of individual intent. In practice, the courts often over-rule explicit contract term in favor of terms consistent with past practice, or as a function of past judicial decisions for similar cases. In addition, parties may consciously leave a contract incomplete and instead relying upon courts to fill in missing terms should there be litigation.[24]

The common principal in both cases is that past practice is often a more binding constraint upon a contract than express terms. Moreover, when deciding upon which terms to include in a contract, parties can be more certain of terms that have been previously tested in court. For this reason the American Institute of Architects publishes a guide to contract cases that arise in their use of the AIA form contracts, called the *Legal Citator*, and highlights cases in which the terms of the contract have been enforced as written.[25]

[22]See Moore (1992) a review of the mechanism design literature, and Tirole (1999) for an evaluation of its relevance for contract theory.

[23]See for example the work of Coase (1937), Alchian and Demsetz (1972), Klein, Crawford, and Alchian (1978), and Williamson, Wachter, and Harris (1975).

[24]See MacNeil (1974) industry practice, and its role in the law.

[25]Stein, Steven G. M. ed. *The American Institute of Architect Legal Citator*. New York: Matthew Bender, 2003 - updated annually. See www.aia.org for more information regarding the legal citator and the forms that the AIA sells for use in the construction industry.

The *Legal Citator* plays a role that is not too dissimilar to the role of beliefs in our model. The citator is merely a subsample of past cases, which attorneys can use to help their clients write enforceable contracts. Notice, that in this context a contract is enforceable *because* at some point in the past a court ruled that it is enforceable. This is quite different from the way an economist thinks about a contract, who includes a clause for a contingency because she anticipates the consequence of that contingency. In practice, a lawyer includes a contingency because the event has occurred in the past, and she wishes to protect her client should that event occur again in the future.

We show that norms can evolve for the division of the gains (or losses) in any event only if they occur with sufficient frequency. In our model we achieved this by supposing investment has a stochastic effect on productivity and find that this implies the existence of a stable bargaining norm that is independent of the value of the match.

This implies that when investments are complementary, the level of investment is lower than even the level predicted by the hold-up model. The fact that the low investment equilibrium is stable implies that experimentation and learning are not likely to lead one to a more efficient equilibrium. In the case of substitutes, we find that the level of investment is in general higher than is predicted by the hold-up model. In either case, we find that the egalitarian bargaining norm is stable, and hence results are consistent with Platteau (2000)'s claim that the egalitarian norms that are prevalent in rural Africa may be a barrier to the progress in that region. In particular, the results emphasize the idea that the egalitarian norm is most problematic in a production setting where one depends upon the complementary inputs of several individuals.

The economist's solution to this problem is to increase competition. If investments are interpreted as human capital investments, then increased competition means that individuals should have several potential buyers for any skills that they acquire. We explore the implications of increased competition by giving individuals the option to delay trade into the future in the expectation of finding a better match. Thus by increased competition we mean thicker markets *ex post*, as opposed to increased competition at the *ex ante* stage, when individuals make their investment decisions.[26]

We find that adding such competition does indeed enhance efficiency, however when the market is very competitive, there is a tendency towards associative matching (only high-high and low-low matches trade). As a consequence, we again have a breakdown in the existence of stable bargaining norms for the high-low trades, and it is possible for the market to be less efficient than it would be in the absence of competition. This result provides another way to think about the downturn in productivity observed during the transition in Eastern Europe, as documented in Roland (2000).

If increased competition is expected to endure, then individuals have an incentive to find ways to match more efficiently. We do not explore this possibility in this paper, and focus only upon the question of the evolution of efficient bargaining norms. We find that this questions by itself leads to a rich theory of implicit agreements and their potential breakdown that may help better understand how contracts and organizations are likely to work in practice with boundedly rational agents. It would be interesting in future work to explore the evolution of markets themselves so we may better understand the conditions under which specific

[26]MacLeod and Malcomson (1993), Cole, Mailath, and Postlewaite (2001) and Felli and Roberts (2000) have shown in a hold-up model imperfect competiton may in some case be sufficient to implement the first best.

institutions, such as privately owned firms, can allow the economy to evolve better functioning markets.

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Appendix A: Description of the Learning Dynamics

Sampling, memory and belief formation:

The memory of an individual consists of the following data:

1. m observations of types of individuals where all these observations stem from $t-1$. Let $\hat{p}_t^i \in P := \{0, 1/m, 2/m, \ldots, 1\}$ denote the fraction of individuals in this sample with $T_{i,t-1} = H$.
2. m observations of demands made by high and low types in HL matches. Since in general the sample taken in period t consists of fewer than m HL matches some older observations may remain in the sample. The oldest data is dropped as new observations are inserted. This sample is used to estimate the empirical distribution functions $\hat{F}_H(.)$ and $\hat{F}_L(.)$. Both of these empirical distribution functions are elements from the finite set

$$\mathcal{F} = \{F : X(k) \to \{0, 1/m, 2/m, ..., 1\} \,|F(x) \text{ is increasing, } F(S_A) = 1\}.$$

The set of all possible beliefs of an agent is then given by $B = P \times \mathcal{F}^2$.

Expected payoffs:

The expected payoff of an agent with type H or L choosing $a \in X(k)$ under beliefs $\beta \in B$, is given recursively by:

$$\begin{aligned} U_L(a, \beta) &= (1 - \hat{p}(\beta)) S_L/2 + \hat{p}(\beta) \left(\hat{F}_H(S_A - a, \beta) a + \delta \left(1 - \hat{F}_H(S_A - a, \beta) \right) U_L(a, \beta) \right), \\ U_H(a, \beta) &= \hat{p}(\beta) S_H/2 + (1 - \hat{p}(\beta)) \left(\hat{F}_L(S_A - a, \beta) a + \delta \left(1 - \hat{F}_L(S_A - a, \beta) \right) U_H(a, \beta) \right). \end{aligned}$$

Investment Decision

Given beliefs $b_t^i \in B$ agent i entering the population in period t chooses to invest if:

$$\max_{(x_L, x_H) \in X(k)^2} (1 - \lambda) U_H\left(x_H, b_t^i\right) + \lambda U_L\left(x_L, b_t^i\right) - c \geq \max_{(x_L, x_H) \in X(k)^2} (1 - \lambda) U_L\left(x_L, b_t^i\right) + \lambda U_H\left(x_H, b_t^i\right).$$

Determination of demands in step 2. of the timeline

Individual i chooses $a_t^i \in X(k)$ to maximize $U_{T^i}\left(a_t^i, b_t^i\right)$, given her type $T^i \in \{L, H\}$ and beliefs $b_t^i \in B$. When indifferent over demands she chooses the smallest demand. The agent's strategy is uniquely defined by her beliefs and type. Hence, we write $a_t^i = \alpha\left(T^i, b_t^i\right)$.

State space of the process

The state space is given by all possible distributions of n individuals over the set $C = \{H, L\} \times B$:

$$\mathcal{S} = \{s \in [0, 1]^{|C|} | \sum_{c \in C} s_c = 1, \; n s_c \in \mathbb{N}_0 \; \forall c \in C\},$$

Appendix B: Proofs

Proof of Proposition 1:

Efficiency implies $S_H - c > S_A > 0$, therefore if one sets $\hat{x}_L = 0$, then conditions 1 and 3 for a self-enforcing norm are strictly satisfied for $\lambda = 0$. The trade condition implies that $S_A > \delta\left(S_L + S_H\right)/2 > \delta S_H/2$ and therefore condition 2 is strictly satisfied. Given that the expressions in the definition of a self-enforcing norm are continuous for small λ, these conditions are satisfied for small λ. $\square$

Proof of Lemma 1:

Let s and s' be two arbitrary states in $\mathcal{S}$. We show that there is a positive multi-step transition probability from s to s' and a positive one-step transition probability from s' to s'. This then implies that the process is irreducible and aperiodic.

Assume that $\sigma_t = s$. With positive probability the bargaining strategy of all agents at time t is such that all agents carry out the project (some mutations of bargaining strategies might be needed) and leave the population. Hence, with positive probability in period $t+1$ the types of all agents in the population are determined anew and with a positive probability the resulting distribution of types matches exactly the one in s'. Every period there is positive probability that the distribution of types stays like that. If there are both high and low types in s' it is straight-forward to see that any set of observations needed to create empirical distribution functions which have positive weight in s' can be created by multiple mutations of bargaining behavior of the agents given the type distribution. In case there are only high or only low types in s' consider the transition where first all but one agent get the type required in s', then all the observations needed to create all the beliefs in s' are created by mutations and finally the single agent with a different type leaves the population and changes her type. In any case there is a positive probability that s' is reached in multiple steps. Furthermore, since there is always a positive probability that all agents only observe matches between the same types during a period and therefore do not change their beliefs, there is a positive probability that the process stays in s' once it has reached s'. Hence, the process is irreducible and aperiodic. $\square$.

Proof of Lemma 2:

By waiting for an partner of same type, an agent with beliefs β expects a payoff of $\frac{\hat{p}(\beta)}{1-\delta(1-\hat{p}(\beta))}\frac{S_H}{2}$ if she is of type H and $\frac{1-\hat{p}(\beta)}{1-\delta\hat{p}(\beta)}\frac{S_L}{2}$ if she is of type L. Hence, a bargaining norm (x_L) is compatible with $\hat{p}$ and δ if $x_L \in [\underline{x}_L(\hat{p}), \bar{x}_L(\hat{p})]$, where $\underline{x}_L(\hat{p}) \in X(k)$ such that

$$\underline{x}_L(\hat{p}) - \alpha < \frac{\delta(1-\hat{p})}{1-\delta\hat{p}}\frac{S_L}{2} \leq \underline{x}_L(\hat{p})$$

and $\bar{x}_L(\hat{p}) \in X$ such that

$$\bar{x}_L(\hat{p}) \leq S_A - \frac{\delta\hat{p}}{1-\delta(1-\hat{p})}\frac{S_H}{2} < \bar{x}_L(\hat{p}) + \alpha.$$

Considering the monotonicity of these expressions with respect to $\hat{p}$ we get $\mathcal{C}(\delta) = [\underline{x}_L(0), \bar{x}_L(1)]$. Simple calculations now establish that $\mathcal{C}(\delta) \neq \emptyset$ for sufficiently small α if and only if $\delta < \frac{2S_A}{S_L+S_H}$. This is exactly the trade condition, hence the Lemma. $\square$

Proof of Lemma 3:

First we show that all the sets given in the Lemma are limit sets, i.e. we have to show that for $\epsilon = 0$ they are absorbing and for each pair of states in such a set there is a positive (multi-step) transition probability.

It follows from the definition of $\mathcal{C}(\delta)$ that if $x \in \mathcal{C}(\delta)$ and all individuals have point beliefs β such that $\hat{F}_L(\cdot, \beta) = \mathcal{P}(x)$, $\hat{F}_H(\cdot, \beta) = \mathcal{P}(S_A - x)$, all individuals have the optimal bargaining strategy $x_L = x, x_H = S_A - x$. Therefore, in the absence of mutations these point beliefs can never be altered and therefore $\Omega(x)$ is absorbing. Furthermore, since in every period every distribution of types has a positive probability regardless of the actual investment behavior, and so also for every $\hat{p} \in P$ there is a positive probability that a sample yielding such an estimator is observed, all possible distributions of types and $\hat{p}$ can be reached with positive probability. Hence, the set $\Omega(x)$ is connected, which implies that it is a limit set.

To prove that these are the only limit sets, we show that from every state which is not in one of the limit sets described above there is a positive probability to reach one of these sets. This comes down to showing that a homogeneous bargaining norm which is consistent with all $\hat{p} \in P$ can always be reached with positive probability. The transition can go as follows: assume $\sigma_t = s$ for some arbitrary state $s \in \mathcal{S}$. With positive probability there are at least m low types in σ_{t+1} and with positive probability at $t+2$ there is some pairing of a low type agent a_L and a high type agent a_H with bids $\tilde{x}_H, \tilde{x}_L$, where a_L has beliefs β such that $\hat{p}(\beta) = 0$ and accordingly $\tilde{x}_L \geq \frac{\delta S_L}{2}$. With positive probability this pairing is repeated m times from period $t+2$ till $t+m-1$ and one agent, we call him b_H, in the population samples all these pairings but no other high-low pairings. Accordingly, at $t+m$ she has beliefs such that $\hat{F}_L(\cdot, \beta_{t+m}) = \mathcal{P}(\tilde{x}_L)$. Furthermore, there is a positive probability that the beliefs of a_L (or the agent who replaces her) only observes high-high meetings during this period and her beliefs stay unchanged. Furthermore there is a positive probability that a_L and b_H are matched in periods $t+m$ till $t+2m-1$. In each such matching the two bids are $\tilde{x}_L$ of a_L and $S_A - \tilde{x}_L$ of b_H. Again, there is a positive probability that all individuals sample only these high/low pairings during periods $t+m$ to $t+2m-1$. Then in $t+2m$ all agents have beliefs such that $\hat{F}_L(dot, \beta_{t+2m}) = \mathcal{P}(\tilde{x}_L), \hat{F}_H(\cdot, \beta_{t+2m}) = \mathcal{P}(S_A - \tilde{x}_L)$. If $S_A - \tilde{x}_L \geq \frac{\delta S_H}{2}$ we have $\tilde{x}_L \in \mathcal{C}(\delta)$ and the proof of (a) is complete.

If $S_A - \tilde{x}_L < \frac{\delta S_H}{2}$, there is a positive probability that in period $t+2m+2$ there is a high type with $\hat{p} = 1$. This agent then makes a bid $\tilde{\tilde{x}}_H$ such that $\tilde{\tilde{x}}_H - \alpha < \frac{\delta S_H}{2} \leq \tilde{\tilde{x}}_H$ and the same arguments as above imply that there is a positive probability that a homogeneous state will evolve where all agents hold beliefs β such that $\hat{F}_L(\cdot, \beta) = \mathcal{P}(S_A - \tilde{\tilde{x}}_H), \hat{F}_H(\cdot, \beta) = \mathcal{P}(\tilde{\tilde{x}}_H)$. Since $\delta < \frac{2S_A}{S_H + S_L}$ implies $\frac{\delta S_H}{2} > S_A - \frac{\delta S_L}{2}$, we have $S_A - \tilde{\tilde{x}}_H \in \mathcal{C}(\delta)$ for sufficiently small α.

Proof of Proposition 3:

Define

$$
\begin{aligned}
\mathcal{B}_L(\delta) &= \{x_L \in X | \exists p \in P \text{ s.t. } x_L = \underline{x}_L(p) \text{ or } \exists p \in P \text{ s.t. } x_L = \bar{x}_L(p)\} \\
\mathcal{B}_H(\delta) &= \{x_H \in X | \exists p \in P \text{ s.t. } x_H = S_A - \bar{x}_L(p) \\
&\qquad \text{or } \exists p \in P \text{ s.t. } x_H = S_A - \underline{x}_L(p)\}
\end{aligned}
$$

as the set of all demands which lie just above the outside option for some $\hat{p} \in P$ and the best responses to that. The larger m is the larger these sets are and for sufficiently large m we simply have $\mathcal{B}_H(\delta) =$

$X \cap \left[S_A - \frac{\delta S_L}{2}, \frac{\delta S_H}{2}\right]$ and $\mathcal{B}_L = X \cap \left[S_A - \frac{\delta S_H}{2}, \frac{\delta S_L}{2}\right]$.

To prove our claim we show that for sufficiently large m, n and k the unique stochastically stable set of the process $\{\sigma_t\}$ is a set $\mathcal{L}$ where for all states $s \in \mathcal{L}$ we have $s_\zeta > 0$ only if $\zeta = (T, \beta)$ for some $T \in \{H, L\}$ and some β such that $\text{supp}(\hat{F}_H(\cdot, \beta)) \in \mathcal{B}_H(\delta)$, $\text{supp}(\hat{F}_L(\cdot, \beta)) \in \mathcal{B}_L(\delta)$.

Assume $\sigma_t = s$ for an arbitrary state $s \in \mathcal{S}$. Assume further that there are at least m low types and at least m high types in the population (if this is not true, there is a positive probability that at least m low and high types will be in the population within two periods). Then, there is a positive probability that in period $t+1$ all low types have beliefs $\hat{p}^i = 0$ and at least m are matched with high types. The resulting demands at $t+1$ of these low types are larger or equal to $\underline{x}_L(0)$. There is a positive probability that at least m high types observe these m demands in $t+2$ and that the same m high types in period $t+3$ have beliefs such that $\hat{p}(\beta^i) = 1$ and are matched with low types. Since for these individuals we have $\hat{F}_L(\cdot, \beta) = \mathcal{P}(\underline{x}_L(0))$ and $\underline{x}_L(0) > \bar{x}_L(1)$ the outside option is binding for all these high types and they demand $x_H = S_A - \bar{x}_L(1)$ in period $t+3$. With positive probability these m demands are sampled by all agents in $t+4$ and hence all agents have beliefs such that $\hat{F}_H(\cdot, \beta) = \mathcal{P}(S_A - \bar{x}_L(1))$. With positive probability these beliefs stay unchanged till $t+5$ whereas the belief about the type distribution changes to $\hat{p}(\beta) = 0$. With positive probability in $t+5$ now at least m low type agents are matched with high types and since $\underline{x}_L(0) > \bar{x}_L(1)$ their outside option is binding and their demands are $x_L = \underline{x}_L(0)$. With positive probability all agents sample the demands of these m low types in $t+6$ and hence all agents have beliefs β such that $(\hat{p}(\beta) = 0,\ \hat{F}_H(\cdot, \beta) = \mathcal{P}(S_A - \bar{x}_L(1)), \hat{F}_L(\cdot, \beta) = \mathcal{P}(\underline{x}_L(0)))$. We denote this state by $\tilde{s}$. The fact that there exists a positive multi-step transition probability from every state to $\tilde{s}$ implies that the Markov chain has a single limit set which includes $\tilde{s}$. Obviously, this single limit set consists of all states which can be reached with positive probability from $\tilde{s}$. Taking into account that every demand of a high type where the outside option is binding has to be in $\mathcal{B}_H$ and that the best response of a high type with some beliefs $\hat{F}_L$ with support in $\mathcal{B}_L$ and $\hat{p} \in P$ must lie in $\mathcal{B}_H$ as well, shows that all demands of high types have to be in $\mathcal{B}_H$ once $\tilde{s}$ has been reached. Similarly for a low type. Accordingly, given that $\epsilon = 0$, any observation outside $\mathcal{B}_H \times \mathcal{B}_L$ has probability zero once $\tilde{s}$ has been reached before. This shows that $\mathcal{L}$ is the only limit set in the state space which implies that this set has to be stochastically stable. □

Proof of Proposition 4:

We have to determine which of the limit sets characterized in Lemma 3 are stochastically stable. We use the radius modified coradius criterion introduced in Ellison (2000). For a union of limit sets Ω the radius $R(\Omega)$ is defined as the minimum number of mutations needed to get to a state outside the basin of attraction of Ω with positive probability. The modified coradius $CR^*(\Omega)$ is defined as follows: consider an arbitrary state $x \notin \Omega$ and a path $(z_1, z_2, \ldots, z_T)$ from x to Ω where $L_1, L_2, \ldots, L_r \subset \Omega$ is the sequence of limit sets the path goes through (this implies $L_r \subseteq \Omega$). We define the modified costs of this path by

$$c^*(z_1, \ldots, c_T) = c(z_1, \ldots, z_T) - \sum_{i=2}^{r-1} R(L_i),$$

where $c(z_1, \ldots, z_T)$ gives the number of mutations needed on the path $(x_1, \ldots, z_T)$. Denoting by $c^*(x, \Omega)$

the minimal modified costs for all paths from x to Ω we define the modified coradius as

$$CR^*(\Omega) = \max_{x \notin \Omega} c^*(x, \Omega).$$

Ellison (2000) proves that every union of limit sets Ω with $R(\Omega) < CR^*(\Omega)$ contains all stochastically stable states.

In what follows we calculate the radius and modified coradius of the bargaining norms described in Lemma 3. In the case of substitutes the limit sets are of the form $\Omega(x_L)$ for $x_L \in \mathcal{C}(\delta)$. Let $\tilde{x}_L$ be an arbitrary bargaining norm with $\tilde{x}_L \in \mathcal{C}(\delta)$. To destabilize the norm upwards either a sufficient number of high types have to mutate to a x_H smaller than $S_A - \tilde{x}_L$, in the extreme case $x_H = 0$, such that the best response of a high type who has sampled all these mutants becomes $x_L = S_A$, or a sufficient number of low types have to mutate to $x_L = \tilde{x}_L + \alpha$ such that the best response of a high type who has sampled all these mutants becomes $x_H = S_A - \tilde{x}_L - \alpha$, where $\alpha = \frac{S_A}{k}$. As has been demonstrated in Young (1993b), for sufficiently small α the second of these two possibilities yields transitions with a lower number of mutations (the number goes to zero as α goes to zero). Similar arguments hold for a downwards destabilization and therefore in order to leave a norm $\tilde{x}_L$ with the minimal necessary number of mutations either the path to $\tilde{x}_L + \alpha$ or the path to $\tilde{x}_L - \alpha$ has to be taken. We define by $c_+(x_L)$ the minimal number of mutations needed to get to $\tilde{x}_L + \alpha$ and by $c_-(x_L)$ the minimal number of mutations needed to get to $\tilde{x}_L - \alpha$. We first calculate $c_+(\tilde{x}_L)$.

The number of mutations needed to destabilize a norm also depends on the beliefs $\hat{p}$. We first show that the minimal number of mutants either occurs at $\hat{p} = 0$ or at $\hat{p} = 1$. Consider a low type whose beliefs $\hat{F}_H$ attach probability q to $x_H = S_A - \tilde{x}_L + \alpha$ and $1 - q$ to $x_H = S_A - \tilde{x}_L$. Denote by v the expected discounted payoff of this individual given that he faces a high type and bids $x_L = \tilde{x}_L$ whenever facing a high type. Taking into account that he will always trade immediately when he meets another low type we get

$$v = (1-q)\tilde{x}_L + \delta q \left(\hat{p} v + (1-\hat{p}) \frac{S_L}{2} \right)$$

and

$$v(q; \hat{p}) := \frac{(1-q)\tilde{x}_L + \delta q (1-\hat{p}) S_L/2}{1 - \delta q \hat{p}}.$$

Note that this expression is monotonic in $\hat{p}$ for $\hat{p} \in [0,1]$ (increasing or decreasing). The minimal number of mutations needed to destabilize the norm is given by $\lceil m\tilde{q} \rceil$, where $\tilde{q}$ is the minimal q such that:

$$v(q; \hat{p}) < \tilde{x}_L - \alpha$$

holds for some $\hat{p} \in [0,1]$. Since the right hand side is constant in q and $\hat{p}$ and the left hand side is monotonous in $\hat{p}$ for all q the minimal q is either attained at $\hat{p} = 0$ or at $\hat{p} = 1$.

With $\hat{p} = 0$ we get

$$v(q; 0) = (1-q)\tilde{x}_L + \delta q \frac{S_L}{2},$$

which gives

$$q > q_{1-}(\tilde{x}_L) := \frac{\alpha}{\tilde{x}_L - \delta \frac{S_L}{2}}.$$

For $\hat{p} = 1$ we have

$$v(q, 1) = \frac{1-q}{1-\delta q}\tilde{x}_L.$$

Accordingly, the norm can be destabilized downwards if

$$q < q_{2-}(\tilde{x}_L) := \frac{\alpha}{\tilde{x}_L(1-\delta) + \delta\alpha}.$$

Comparing the two we see that $q_{1-}(\tilde{x}_L) < q_{2-}(\tilde{x}_L)$ if and only if $\tilde{x}_L > \frac{S_L}{2} + \alpha$. All-together we have

$$c_-(\tilde{x}_L) = \begin{cases} q_{1-}(\tilde{x}_L), & \text{if } \tilde{x}_L \geq \frac{S_L}{2} + \alpha, \\ q_{2-}(\tilde{x}_L), & \text{if } \tilde{x}_L < \frac{S_L}{2} + \alpha. \end{cases}$$

Similar reasoning for destabilizations upwards shows that for a high type, who is matched with a low type and who believes that a fraction q of low types demands $x_L = \tilde{x}_L + \alpha$ and a fraction $1-q$ of low types demands $x_L = \tilde{x}_L$, has the following expected payoff from demanding $x_H = S_A - \tilde{x}_L$:

$$\begin{aligned} w(q;0) &= \tfrac{1-q}{1-\delta q}(S_A - \tilde{x}_L) \\ w(q,1) &= (1-q)(S_A - \tilde{x}_L) + \delta q \tfrac{S_H}{2}. \end{aligned}$$

This implies

$$c_+(\tilde{x}_L) = \begin{cases} q_{1+}(\tilde{x}_L), & \text{if } \tilde{x}_L \geq S_A - \frac{S_H}{2} - \alpha, \\ q_{2+}(\tilde{x}_L) & \text{if } \tilde{x}_L < S_A - \frac{S_H}{2} - \alpha. \end{cases}$$

where

$$\begin{aligned} q_{1+} &= \frac{\alpha}{(S_A - \tilde{x}_L)(1-\delta) + \delta\alpha} \\ q_{2+} &= \frac{\alpha}{S_A - \tilde{x}_L - \delta\frac{S_H}{2}}. \end{aligned}$$

The function c_- is decreasing in $\tilde{x}_L$ whereas c_+ is increasing in this variable which implies that they have a unique intersection. We denote this intersection point by $\hat{x}_L$. Clearly at this point $\min[c_-, c_+]$ is maximized. For

$$\delta \leq \frac{2(S_H - S_A)}{S_H - S_L} \tag{6}$$

$\hat{x}_L$ lies on the intersection of q_{1-} and q_{1+} and is given by

$$\hat{x}_L = \frac{S_A}{2} - \frac{\delta}{2(2-\delta)}(S_A - S_L - 2\alpha). \tag{7}$$

To establish (a) we first observe that under the assumptions made in (a) the condition (6) holds and $\hat{x}_L \in \left[\frac{\delta S_L}{2}, S_A - \frac{\delta S_H}{2}\right]$ for small α. Hence, there exists a $\hat{\hat{x}}_L \in \mathcal{C}(\delta)$ that maximizes $\min[c_+, c_-]$ over $\mathcal{C}(\delta)$ and whose distance from $\hat{x}_L$ is smaller than α. Taking into account Lemma 3 this in particular implies that there is a limit set corresponding to the bargaining norm $\hat{\hat{x}}_L$.

From the arguments above it follows that for every $x_L \in \mathcal{C}(\delta)$ with $x_L < \hat{\hat{x}}_L$ we have for the radius of the limit set $\Omega(x_L)$: $R(\Omega(x_L)) = \lceil mc_+(x_L) \rceil$ and for every $x_L \in \mathcal{C}(\delta)$ with $x_L > \hat{\hat{x}}_L$ we have $R(\Omega(x_L)) =$

$\lceil mc_-(x_L) \rceil$. From every limit set $\Omega(x_L)$ there is a path to $\Omega(\hat{\hat{x}}_L)$ along a graph g which connects every limit set $\Omega(x_L)$ where $x_L < \hat{\hat{x}}_L$ with $\Omega(x_L + \alpha)$, and every limit set $\Omega(x_L)$ where $x_L > \hat{\hat{x}}_L$ with $\Omega(x_L - \alpha)$. This implies that

$$CR^*(\Omega(\hat{\hat{x}}_L)) \leq \max_{x_L \in \mathcal{C}(\delta) \setminus \{\hat{\hat{x}}_L\}} R(\Omega(x_L)).$$

For sufficiently large m we have $R(\Omega(\hat{\hat{x}}_L)) > R(\Omega(x_L))$ for all $x_L \in \mathcal{C}(\delta) \setminus \{\hat{\hat{x}}_L\}$ and therefore $R(\Omega(\hat{\hat{x}}_L)) > CR^*(\Omega(\hat{\hat{x}}_L))$. Using the radius-modified coradius criterion we can conclude that the limit set corresponding to $\hat{\hat{x}}_L$ is stochastically stable. For $k \to \infty$ we have $\hat{\hat{x}}_L \to \hat{x}_L$ and get (a). Exactly the same arguments establish (b), where it has to be taken into account that in this case $\hat{x}_L$ lies at the intersection of q_{1-} and q_{2+} which is given by

$$\hat{x}_L = \frac{S_A}{2} - \frac{\delta}{4}(S_H - S_L)$$

□

Proof of Lemma 4:

Parts (a) and (b) are trivial. To prove part (c) we denote by $Q(\lambda) = [q_{ij}(\lambda)] i, j \in \tilde{S}$ the one-step transition matrix of the Markov process $\{\tilde{\sigma}_t\}$. It can then easily be established that $\lim_{\lambda \to 0} q_{i0} + q_{i1} > 0$ for all $i \in \tilde{S}$. Furthermore, at state $i = 0$ no individual can sample any high types and hence we have $\hat{p}(\beta) = 0$ for all individuals and accordingly all choose high investment. Therefore $\lim_{\lambda \to 0} q_{01} = 1$ and by the same reasoning $\lim_{\lambda \to 0} q_{10} = 1$. Therefore, the only limit set for $\lambda \to 0$ is $\{0, 1\}$ which implies that $\lim_{\lambda \to 0} \tilde{\pi}_i^*(\lambda) = 0$ for all $i \in \tilde{S} \setminus \{0, 1\}$. Using this we get from the Chapman-Kolmogoroff equation at state 0

$$\tilde{\pi}_0^*(\lambda) \left(\sum_{i \in \tilde{S} \setminus \{0\}} q_{0,i} \right) = \sum_{i \in \tilde{S} \setminus \{0\}} q_{i,0} \tilde{\pi}_i^*(\lambda)$$

that $\lim_{\lambda \to 0} \tilde{\pi}_0^* = \lim_{\lambda \to 0} \tilde{\pi}_1^* = 0.5$. □

Proof of Proposition 6:

The proof of (b) is identical to the proof of part (a) of proposition 4. To proof (a) we again follow the proof of proposition 4 but observe that for $S_A < \frac{\delta}{2}((2 - \delta)S_H + \delta S_L)$ we have $\hat{x}_L > S_A - \delta \frac{S_H}{2}$. Therefore the point which maximizes $\min[c_+, c_-]$ over $\mathcal{C}(\delta)$ is given by $\hat{\hat{x}}_L$ where $\hat{\hat{x}}_L \leq S_A - \delta \frac{S_H}{2} < \hat{\hat{x}}_L + \alpha$. Stochastic stability of the limit set $\Omega(\hat{\hat{x}}_L)$ is established analogous to the proof of proposition 4 but here we have $\hat{\hat{x}}_L \to S_A - \delta \frac{S_H}{2}$ for $k \to \infty$. □

Proof of Lemma 5:

We show the proposition for $p^*(x_L) > 0.5$, the other case analogous.

We denote again the one-step transition matrix of the process$\{\tilde{\sigma}_t\}$ by $Q = [q_{ij}(\lambda)] i, j \in \tilde{S}$ We can write these transition probabilities as

$$q_{ij} = \binom{n}{j} \beta_i^j (1 - \beta_i)^{n-j},$$

where

$$\beta_i = (1 - \lambda) s(mp^*(x_L); i) + \lambda(1 - s(mp^*(x_L); i))$$

is the probability that a randomly chosen individual is of high type. Note that for a given bargaining norm the investment decision only depends on the number of high types sampled by an individual in the current period. We denote by

$$s(mp^*(x_L);i) = \sum_{k \geq mp^*} \binom{m}{k} \left(\frac{i}{n}\right)^k \left(1 - \frac{i}{n}\right)^{m-k}$$

the probability that an individual samples more than $mp^*(x_L)$ high types in a population with i high types. Since we are dealing with the case of investment complements here, this is the probability of high investment.

Note first that

$$\begin{aligned}
1 - s(mp^*;i) &= \sum_{k<mp^*} \binom{m}{k} \left(\frac{i}{n}\right)^k \left(1-\frac{i}{n}\right)^{m-k} \\
&= \sum_{k>m(1-p^*)} \binom{m}{k} \left(\frac{i}{n}\right)^{m-k} \left(1-\frac{i}{n}\right)^{k} \\
&> \sum_{k>mp^*} \binom{m}{k} \left(1-\frac{n-i}{n}\right)^{m-k} \left(\frac{n-i}{n}\right)^{k} \\
&= s(mp^*;n-i),
\end{aligned}$$

where the inequality follows from $p^* > 0.5$. Using this we get that for $\lambda < 0.5$

$$\begin{aligned}
1 - \beta_{n-i} &= 1 - (1-\lambda)s(mp^*;n-i) - \lambda(1 - s(mp^*;n-i)) \\
&= 1 - \lambda - s(mp^*;n-i)(1-2\lambda) \\
&> 1 - \lambda - (1 - s(mp^*;i))(1-2\lambda) \\
&= (1-\lambda)s(mp^*;i) + \lambda(1 - s(mp^*;i)) \\
&= \beta_i.
\end{aligned}$$

This means that in a population with i high types the probability that an individual becomes a high type is smaller than the probability that an individual becomes a low type in a population with i low types. In particular, this implies that the probability that at least z individuals become high types in state $\frac{i}{n}$ is smaller than the probability that at least z individuals become low types in state $\frac{n-i}{n}$ for all z. We denote by $\mathcal{L} = \left\{0, \frac{1}{n}, \ldots, \frac{n-2}{2n}\right\}$, $\mathcal{H} = \left\{\frac{n+2}{2n}, \ldots, \frac{n-1}{n}, 1\right\}$, $\tilde{\mathcal{L}} = \mathcal{L} \cup \left\{\frac{n}{2}\right\}$ and by $\tilde{\mathcal{H}} = \mathcal{H} \cup \left\{\frac{n}{2}\right\}$. Furthermore we denote by q_{iL} the transition probability from state i into the set $\mathcal{L}$ and analogous the transition probabilities into the other sets defined above. The arguments above imply that

$$q_{iL} > q_{n-iH} \quad \text{and} \quad q_{i\tilde{L}} > q_{n-i\tilde{H}} \;\forall i.$$

Note that both $(\mathcal{L}, \tilde{\mathcal{H}})$ and $(\tilde{\mathcal{L}}, \mathcal{H})$ are partitions of the state space, therefore under the stationary distribution the flows between $\mathcal{L}$ and $\tilde{\mathcal{H}}$ must be identical in both directions and so have to be the flows between $\tilde{\mathcal{L}}$ and $\mathcal{H}$. This gives

$$\sum_{i=0}^{n/2-1} \tilde{\pi}_i^* q_{i\tilde{H}} = \sum_{i=0}^{n/2} \tilde{\pi}_{n-i}^* q_{n-iL} \tag{8}$$

$$\sum_{i=0}^{n/2} \tilde{\pi}_i^* q_{iH} = \sum_{i=0}^{n/2-1} \tilde{\pi}_{n-i}^* q_{n-i\tilde{L}} \tag{9}$$

Our claim can now be easily shown by contradiction. If $\sum_{i=0}^{n/2-1} \tilde{\pi}_i^* \leq \sum_{i=0}^{n/2-1} \tilde{\pi}_{n-i}^*$ we can use $q_{iH} < q_{n-iL}$ for all i to derive that

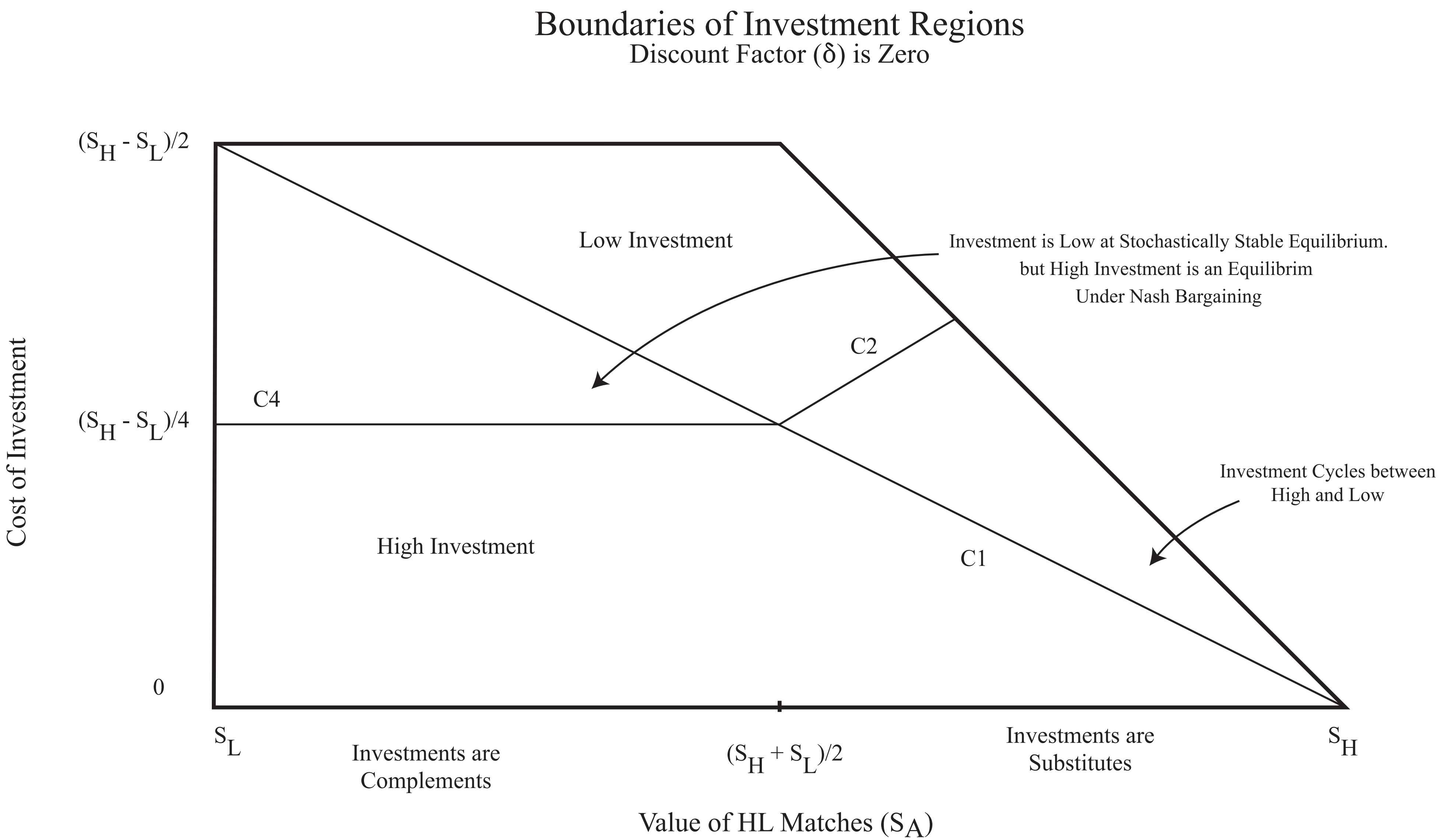
$$\sum_{i=0}^{n/2} \tilde{\pi}_i^* q_{iH} < \sum_{i=0}^{n/2} \tilde{\pi}_{n-i}^* q_{n-iL}$$

Since (8, 9) have to hold this implies

$$\sum_{i=0}^{n/2-1} \tilde{\pi}_{n-i}^* q_{n-i\tilde{L}} < \sum_{i=0}^{n/2-1} \tilde{\pi}_i^* q_{i\tilde{H}}$$

But we also have $q_{i\tilde{H}} < q_{n-i\tilde{L}}$ for all i and therefore this inequality contradicts our assumption that $\sum_{i=0}^{n/2-1} \tilde{\pi}_i^* \leq \sum_{i=0}^{n/2-1} \tilde{\pi}_{n-i}^*$. Accordingly, we must have $\sum_{i=0}^{n/2-1} \tilde{\pi}_i^* > \sum_{i=0}^{n/2-1} \tilde{\pi}_{n-i}^*$

To show that $\lim_{\lambda \to 0} \tilde{\pi}_0^* = 1$ we can again apply the radius-modified coradius criterion. For $\lambda = 0$ there are two limit sets, namely $\{0\}$ and $\{1\}$. In order to invest high an individual has to sample at least $\lceil mp^* \rceil$ high types. Therefore the radius of $\{0\}$ is given by $R(\{0\}) = \lceil mp^* \rceil$. On the other hand, the state where maximal the number of mutations is needed to have a positive transition probability into $\{0\}$ is the state 1 and therefore we have $CR^*(\{0\}) = \lceil m - mp^* \rceil$. For $p^* > 0.5$ this implies that $R(\{0\}) > CR^*(\{0\})$ for sufficiently large m and therefore $\lim_{\lambda \to 0} \tilde{\pi}_0^* = 1$. □



Boundaries of Investment Regions
Discount Factor (δ) is Zero
(S_H - S_L)/2
(S_H - S_L)/4
0
Cost of Investment
Low Investment
Investment is Low at Stochastically Stable Equilibrium.
but High Investment is an Equilibrim
Under Nash Bargaining
C2
C4
C1
Investment Cycles between
High and Low
High Investment
S_L
(S_H + S_L)/2
S_H
Investments are
Complements
Investments are
Substitutes
Value of HL Matches (S_A)

Figure 1

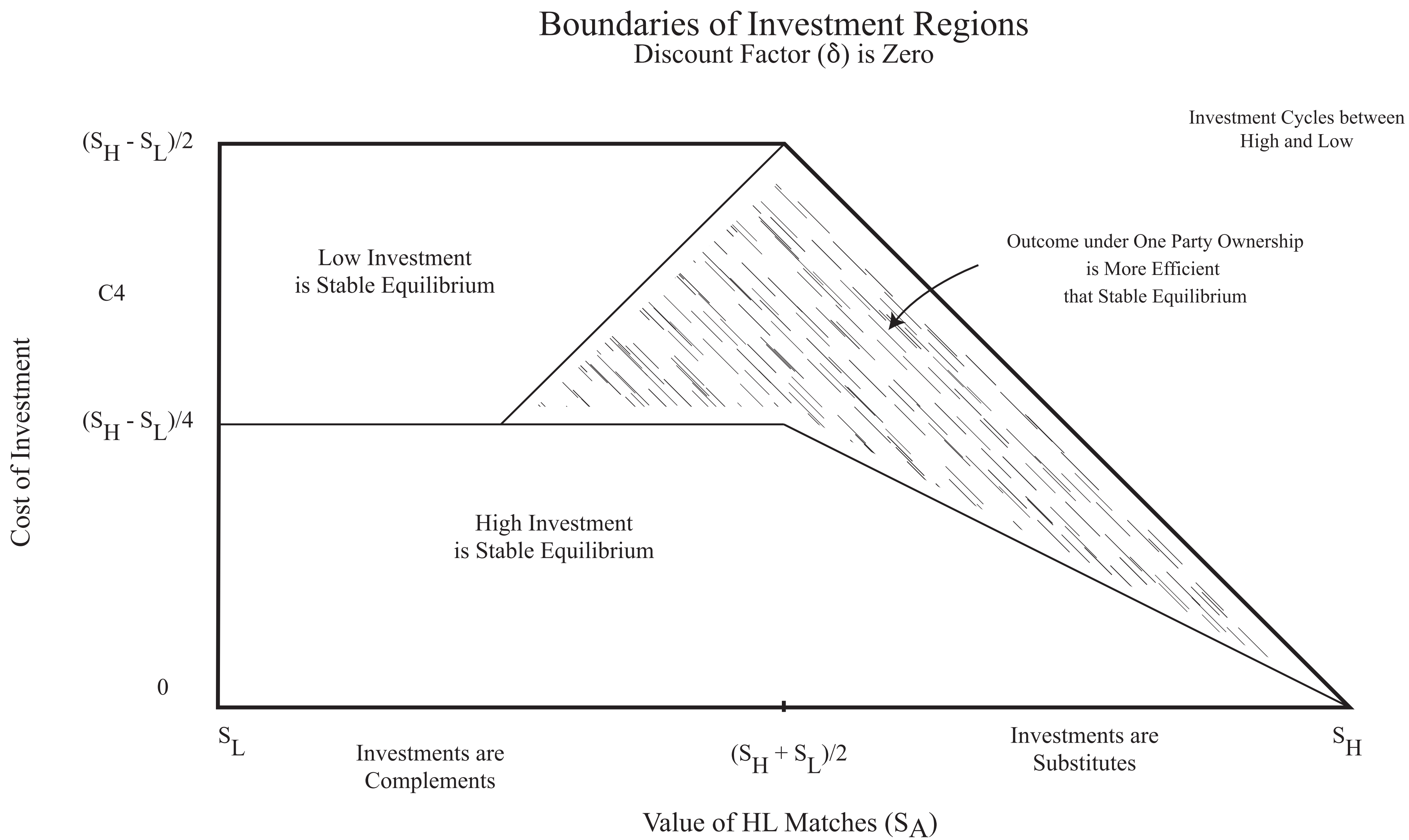

Boundaries of Investment Regions
Discount Factor (δ) is Zero
Investment Cycles between
High and Low
(SH - SL)/2
C4
(SH - SL)/4
0
Low Investment
is Stable Equilibrium
Outcome under One Party Ownership
is More Efficient
that Stable Equilibrium
High Investment
is Stable Equilibrium
Cost of Investment
SL
Investments are
Complements
(SH + SL)/2
Investments are
Substitutes
SH
Value of HL Matches (SA)

Figure 2

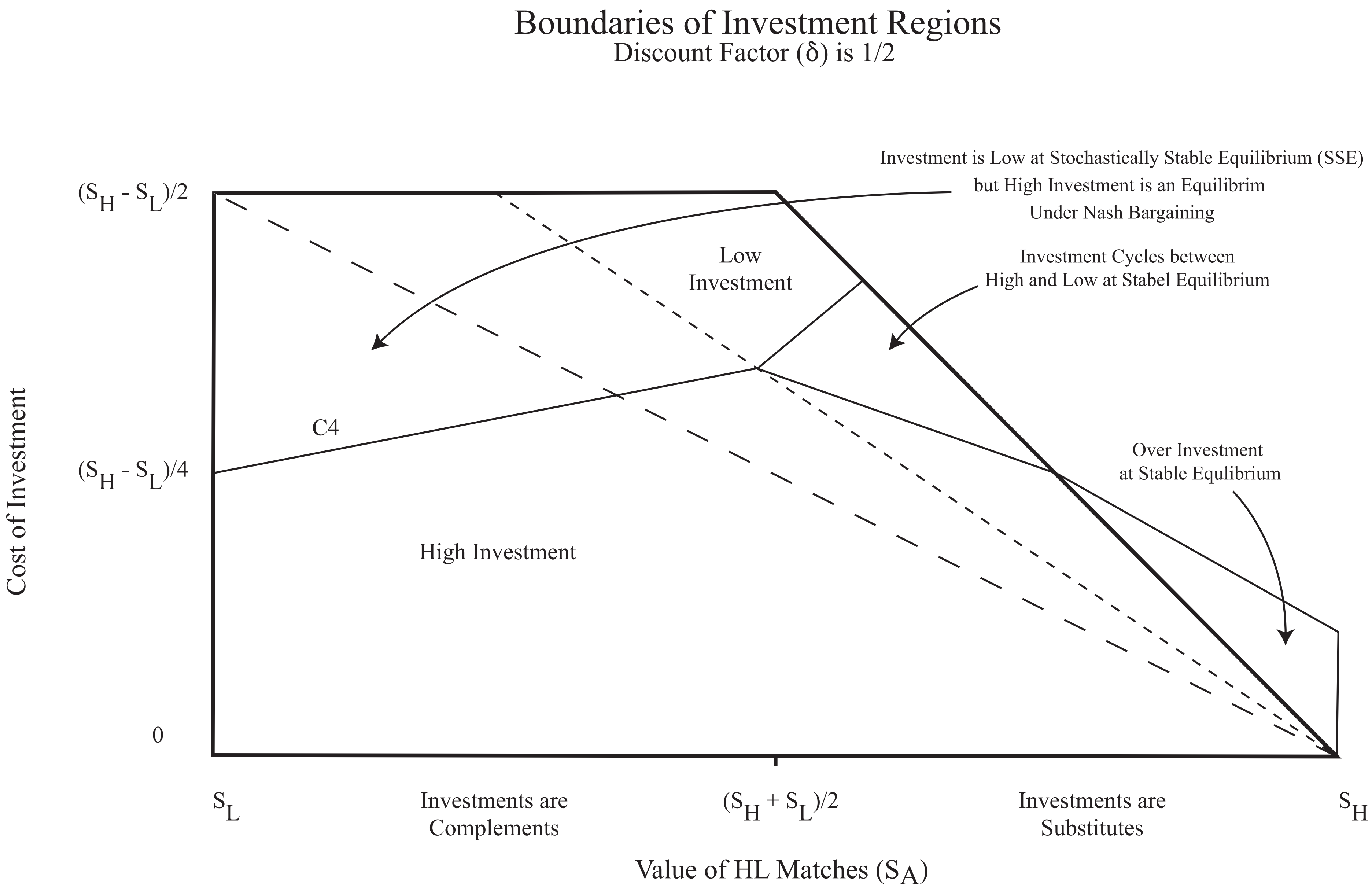

Boundaries of Investment Regions
Discount Factor (δ) is 1/2
Investment is Low at Stochastically Stable Equilibrium (SSE)
but High Investment is an Equilibrim
Under Nash Bargaining
$(S_H - S_L)/2$
Low
Investment
Investment Cycles between
High and Low at Stabel Equilibrium
C4
$(S_H - S_L)/4$
Over Investment
at Stable Equlibrium
Cost of Investment
High Investment
0
S_L
Investments are
Complements
$(S_H + S_L)/2$
Investments are
Substitutes
S_H
Value of HL Matches (S_A)

Figure 3