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Cheap Talk, Gullibility, and Welfare in an Environmental Taxation Game*

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Abstract

We consider a simple dynamic model of environmental taxation that exhibits time inconsistency. There are two categories of firms, Believers, who take the tax announcements made by the Regulator to face value, and Non-Believers, who perfectly anticipate the Regulator's decisions, albeit at a cost. The proportion of Believers and Non-Believers changes over time depending on the relative profits of both groups. We show that the Regulator can use misleading tax announcements to steer the economy to an equilibrium that is Pareto superior to the solutions usually suggested in the literature. Depending upon the initial proportion of Believers, the Regulator may prefer a fast or a low speed of reaction of the firms to differences in Believers/Non-Believers profits.

Keywords: Environmental policy, emissions taxes, time inconsistency, heterogeneous agents, bounded rationality, learning, multiple equilibria, Stackelberg games.

Nous considérons un modèle dynamique de taxation environnementale qui souffre d'incohérence temporelle. Il y a deux catégories de firmes, Croyantes et Non-Croyantes. Les Croyantes considèrent que les taux de taxes annoncés par le Régulateur seront effectivement appliqués. Les Non-Croyantes former des anticipations parfaites mais coûteuses des choix du régulateur. La proportion de Croyantes et de Non-Croyantes change dans le temps en fonction des profits relatifs des deux groupes. Nous montrons que le Régulateur peut utiliser des annonces fausses pour guider l'économie vers un équilibre qui domine au sens de Pareto les équilibres habituellement suggérés dans la littérature. En fonction de la fraction initiale de Croyantes, le Régulateur peut préférer que les firmes réagissent vite ou lentement à la différence entre les profits des deux groupes de firmes.

Mots-Clefs: Politique environnementale, taxes sur les émissions, incohérence temporelle, agents hétérogènes, rationalité bornée, équilibres multiples, jeux de Stackelberg.

J.E.L. Classification: H23, H3, Q5, C69, C79, D62

1 Introduction

The use of taxes as a Regulatory instrument in environmental economics is a classic topic. In a nutshell, the need for regulation usually arises because producing causes detrimental emissions. Due to the lack of a proper market, the firms do not internalize the impact of these emissions on the utility of other agents. Thus, they take their decisions on the basis of prices that do not reflect the true social costs of their production. Taxes can be used to modify the prices confronting the firms so that the socially desirable decisions are taken.

The problem has been exhaustively investigated in static settings where there is no room for strategic interaction between the Regulator and the firms. Consider, however, the following situation: (a) The emission taxes have a dual effect, they incite the firms to reduce production *and* to undertake investments in abatement technology. This is typically the case when the emissions are increasing in the output and decreasing in the abatement technology; (b) Emission reduction is socially desirable, the reduction of production is not; and (c) The investments are irreversible. In that case, the Regulator must find an optimal compromise between implementing high taxes to motivate high investments, and keeping the taxes low to encourage production. The fact that the investments are irreversible introduces a strategic element in the problem. If the firms are naive and believe his announcements, the Regulator can insure high production *and* important investments by first declaring high taxes and reducing them once the corresponding investments have been realized. More sophisticated firms, however, recognize that the initially high taxes will not be implemented, and are reluctant to invest in the first place. In other words, one is confronted with a typical time inconsistency problem, which has been extensively treated in the monetary policy literature following Kydland and Prescott (1977) and Barro and Gordon (1983). In environmental economics, the time inconsistency problem has received yet only limited attention, although it frequently occurs. See among others Gersbach and Glazer (1999) for a number of examples and for an interesting model, Abrego and Perroni (1999), Batabyal (1996a), Batabyal (1996b), Dijkstra (2002), Marsiliani and Renstrőm (2000), Petrakis and Xepapadeas (2003).

The time inconsistency is directly related to the fact that the situation described above defines a Stackelberg game between the Regulator (the leader) and the firms (the followers). As noted in the seminal work of Simaan and Cruz (1973a), Simaan and Cruz (1973b), inconsistency arises because the Stackelberg equilibrium is not defined by mutual best responses. It implies that the follower uses a best response in reaction the leader's action, but not that the leader's action is itself a best response to the follower's. This opens the door to a re-optimization by the leader once the follower has played. Thus, a Regulator who announces that he will implement the Stackelberg solution is not credible. An usual conclusion is that, in the absence of additional mechanisms, the economy is doomed to converge towards the less desirable Nash solution.

A number of options to insure credible solutions have been considered in the literature – credible binding commitments by the Stackelberg leader, reputation building, use of trigger strategies by the followers, etc. See McCallum (1997) for a review in a monetary policy context. Schematically, these solutions aim at assuring the time consistency of Stackelberg solution with either the Regulator or the firms as a leader.Usually, these solutions are not efficient and can be Pareto-improved.

In this paper, we suggest a new solution to the time inconsistency problem in environmental policy. We show that tax announcements that are not respected can increase the payoff not only of the Regulator, but also of all firms, if these include any number of naive *Believers* who take the announcements at face value. Moreover, if firms tend to adopt the behavior of the most successful ones, a stable equilibrium may exist where a positive fraction of firms are Believers. This equilibrium Pareto-dominates the one where all firms anticipate perfectly the Regulator's action. To attain the superior equilibrium, the Regulator builds reputation and leadership by making announcements and implementing taxes in a way that generates good results for the Believers, rather than by pre-committing to his announcements.

This Pareto-superior equilibrium is not the only possible one. Depending upon the model parameters (most crucially: upon the speed with which the firms that follow different strategies react to differences in their respective profits, i.e., upon the *flexibility* of the firms) it may be rational for the Regulator to steer the Pareto-inferior fully rational equilibrium. This paper, thus, stresses the importance of the flexibility in explaining the policies followed by a Regulator, the welfare level realized, and the persistence or decay of private confidence in the Regulator's announcements.

The potential usefulness of employing misleading announcements to Pareto-improve upon standard game-theoretic equilibrium solutions was suggested for the case of general linear-quadratic dynamic games in Vallée et al. (1999) and developed by the same authors in subsequent papers. An early application to environmental economics is Vallée (1998). The Believers/Non-Believers dichotomy was introduced in Deissenberg and Alvarez Gonzalez (2002), who study the credibility problem in monetary economics in a discrete-time framework with reinforcement learning. A similar monetary policy problem has been investigated in Dawid and Deissenberg (2004) in a continuous-time setting akin to the one used in the present work.

The paper is organized as follows. In Section 2, we present the model of environmental taxation, introduce the imitation-type dynamics that determinate the evolution of the number of Believers in the economy, and derive the optimal reaction functions of the firms. In Section 3, we discuss the solution of the static problem one obtains by assuming a constant proportion of Believers. Section 4 is devoted to the analysis of the dynamic problem and to the presentation of the main results. Section 5 concludes.

2 The Model

We consider an economy consisting of a Regulator R and of a continuum of atomistic, profit-maximizing firms i with identical production technology. Time τ is continuous. To keep notation simple, we do not index the variables with either i or τ, unless useful for a better understanding.

In a nutshell, the situation we consider is the following. The Regulator can tax the firms in order to incite them to reduce their emissions. Taxes, however, have a negative impact on the employment. Thus, R has to choose them in order to achieve an optimal compromise between emissions reduction and employment. The following sequence of events occurs in every τ:

- R makes a non-binding announcement $t^a \geq 0$ about the tax level he will implement. The tax level is defined as the amount each firm has to pay per unit of its emissions.
- Given t^a, the firms form expectations t^e about the actual level of the environmental tax. As will be described in more detail later, there are two different ways for an individual firm to form its expectations. Each firm i decides about its level of investment v_i based on its expectation t_i^e.
- R chooses the actual level of tax $t \geq 0$.
- Each firm i produces a quantity x_i.
- The individual firms revise the way they form their expectations (that is, revise their beliefs) depending on the profits they have realized.

The Firms

Each firm produces the same homogenous good using a linear production technology: The production of x units of output requires x units of labor and generates x units of environmentally damaging emissions. The production costs are given by:

$$c(x) = wx + c_x x^2, \tag{1}$$

where x is the output, $w > 0$ the fixed wage rate, and $c_x > 0$ a parameter. For simplicity's sake, the demand is assumed infinitely elastic at the given price $\tilde{p} > w$. Let $p := \tilde{p} - w > 0$.

At each point of time, each firm can spend an additional amount of money γ in order to reduce its current emissions. The investment

$$\gamma(v) = c_v v^2, \tag{2}$$

with $c_v > 0$ a given parameter, is needed in order to reduce the firm's current emissions by $v \in [0, x]$. The investment in one period has no impact on the emissions in future periods. Rather than expenditures in emission-reducing capital, γ can therefore be interpreted as the additional costs resulting of a temporary switch to a cleaner resource – say, of a switch from coal to natural gas.

Depending on the way they form their expectations t^e, we consider two types of firms, Believers B and Non-Believers NB. The fraction of Believers in the population is denoted by $\pi \in [0, 1]$. Believers consider the Regulator's announcement to be truthful and set $t^e = t^a$. Non-Believers disregard the announcement and anticipate perfectly the actual tax level, $t^e = t$. Making perfect anticipations at any point of time, however, is costly. Thus, Non-Believers occur costs of $\delta > 0$ per unit of time.

The firms are profit-maximizers. As will become apparent in the following, one can assume without loss of substance that they are myopic, that is, maximize in every τ their current profit.

The Regulator R

The Regulator's goal is to maximize over an infinite horizon the cumulated discounted value of an objective function with the employment, the emissions, and the tax revenue as arguments. In order to realize this objective, it has two instruments at his disposal, the announced instantaneous tax level t^a, and the actually realized level t.

The objective function is given by:

$$\begin{aligned}
&\Phi(t^a, t) = \int_0^\infty e^{-\rho\tau} \phi(t^a, t) d\tau \\
&:= \int_0^\infty e^{-\rho\tau} \left[k(\pi x^b + (1-\pi)x^{nb}) - \kappa(\pi(x^b - v^b) \right. \\
&\left. +(1-\pi)(x^{nb} - v^{nb})) + t(\pi(x^b - v^b) + (1-\pi)(x^{nb} - v^{nb})) \right] d\tau,
\end{aligned} \tag{3}$$

where x^b, x^{nb}, v^b, v^{nb} denote the optimal production respectively investment chosen by the Believers B and the Non-Believers NB, and where k and κ are strictly positive weights placed by R on the average employment respectively on the average emissions (Remember that output and employment are in a one-to-one relationship in this economy). The strictly positive parameter ρ is a social discount factor.

Belief Dynamics

The firms' beliefs (B or NB) change according to a imitation-type dynamics, see Dawid (1999), Hofbauer and Sigmund (1998). The firms meet randomly two-by-two, each pairing being equiprobable. At each encounter the firm with the lower current profit adopts the belief of the other firm with a probability proportional to the current difference between the individual profits. This gives rise to the dynamics:

$$\dot{\pi} = \beta\pi(1-\pi)(g^b - g^{nb}), \tag{4}$$

where g^b and g^{nb} denote the profits of a Believer and of a Non-Believer:

$$\begin{aligned} g^b &= px^b - c_x(x^b)^2 - t(x^b - v^b) - c_v(v^b)^2, \\ g^{nb} &= px^{nb} - c_x(x^{nb})^2 - t(x^{nb} - v^{nb}) - c_v(v^{nb})^2 - \delta. \end{aligned}$$

Notice that $\dot{\pi}$ reaches its maximum for $\pi = \frac{1}{2}$ (the value of π for which the probability of encounter between firms with different profits is maximized), and tends towards 0 for $\pi \to 0$ and $\pi \to 1$ (for extreme values of π, almost all firms have the same profits). The parameter $\beta \geq 0$, that depends on the adoption probability of the other's strategy, measures the speed of the information flow between Bs and NBs. It may be interpreted as a measure of willingness to change strategies, that is, of the flexibility of the firms.

Equation (4) implies that by choosing the value of (t^a, t) at time τ, the Regulator not only influences the instantaneous social welfare but also the future proportion of Bs in the economy. This, in turn, has an impact on the future social welfare. Hence, although there are no explicit dynamics for the economic variables v and x, the R faces a non-trivial inter-temporal optimization problem.

Optimal Decisions of the Firms

Since the firms are atomistic, each single producer is too small to influence the dynamics of π. Thus, the single firm does not take into account any inter-temporal effect and, independently of its true planing horizon, *de facto* maximizes its current profit in every τ. Each firm chooses its investment v after it has learned t^a but *before* t has been made public. However, it fixes its production level x *after* v and t are known. The firms being price takers, the optimal production decision is:

$$x = \frac{p-t}{2c_x}. \tag{5}$$

The thus defined optimal x does not depend upon t^a, neither directly nor indirectly. As a consequence, both Bs and NBs choose the same production level (5) as a function of the realized tax t alone, $x^b = x^{nb} = x$.

The profit of a firm given that an investment v has been realized is:

$$\tilde{g}(v;t) = \frac{(p-t)^2}{4c_x} + tv.$$

When the firms determine their investment v, the actual tax rate is not known. Therefore, they solve:

$$\max_v [\tilde{g}(v;t^e) - c_v v^2],$$

with $t^e = t$ if the firm is a NB and $t^e = t^a$ if the firm is a B. The interior solution to this problem is:

$$v^b = \frac{t^a}{2c_v}, \quad v^{nb} = \frac{t}{2c_v}. \tag{6}$$

The net emissions $x - v$ of any firm will remain non-negative after the investment, i.e., $v \in [0, x]$ will hold, if:

$$p \geq \frac{c_v + c_x}{c_v} \max[t, t^a]. \tag{7}$$

Given above expressions (5) for x and (6) for v, it is straightforward to see that the belief dynamics can be written as:

$$\dot{\pi} = \beta\pi(1-\pi)\left(\frac{-(t^a-t)^2}{4c_v} + \delta\right). \tag{8}$$

The two effects that govern the evolution of π become now apparent. Large deviations of the realized tax level t from t^a induce a decrease in the stock of believers, whereas the stock of believers tends to grow if the cost δ necessary to form rational expectations is high.

Using (5) and (6), the instantaneous objective function ϕ of the Regulator becomes:

$$\phi(t^a, t) = (k - \kappa + t)\frac{p-t}{2c_x} + \frac{\kappa - t}{2c_v}(\pi t^a + (1-\pi)t). \tag{9}$$

3 The static problem

In the model, there is only one source of dynamics, the beliefs updating (4). Before investigating the dynamic problem, it is instructive to cursorily consider the solution $^{\$}$ of the static case in which R maximizes the integrand in (3) for a given value of π.

From (9), one recognizes easily that at the optimum t^a will either take its highest possible value or be zero, depending on wether $\kappa - t^{\$}$ is positive or negative. The case $\kappa - t^{\$} < 0$ corresponds to the uninteresting situation where the regulator values tax income more than emissions reduction and thus tries to increase the volume of emissions. We therefore restrict our analysis to the environmentally friendly case $t^{\$} < \kappa$. Note that (7) provides a natural upper bound $\bar{t}^a$ for t, namely:

$$\bar{t}^a = \frac{pc_v}{c_v + c_x}. \tag{10}$$

Assuming that the optimal takes the upper value $\bar{t}^a$ just defined (the choice of another bound is inconsequential for the qualitative results), the optimal tax $t^{\$}$ is:

$$t^{\$} = \frac{1}{2}(\kappa + \bar{t}^a - \frac{c_v k}{c_v + c_x - c_x \pi}). \tag{11}$$

Note that $t^{\$}$ is decreasing in π : As π increases, the announcement t^a becomes a more powerful instrument, making a recourse to t less necessary. The requirement $\kappa > t^{\$}$ is fulfilled for all π iff:

$$c_v(k + \kappa - p) + c_x \kappa > 0. \tag{12}$$

In the reminder of this paper, we assume that (12) holds.

Turning to the firms profits, one recognizes the difference $g^{nb} - g^b$ between the NB's and B's profits is increasing in $|t^{\$} - t^{a\$}|$:

$$g^{nb} - g^b = \frac{(t^{\$} - t^{a\$})^2}{4c_v} - \delta. \tag{13}$$

For $\delta = 0$, the profit of the NBs is always higher that the profit of the Bs whenever $t^a \neq t$, reflecting the fact the latter make a systematic error about the true value of t. The profit of the Bs, however, can exceed the one of the NBs if the learning cost δ is high enough. Since $t^{a\$}$ is constant and $t^{\$}$ decreasing in π, and since $t^{\$} < t^{a\$}$, the difference $t^{a\$} - t^{\$}$ increases in π. Therefore, the difference between the profits of the NBs and Bs, (13), is increasing in π.

Further analytical insights are exceedingly cumbersome to present due to the complexity of the functions involved. We therefore illustrate a remarkable, generic feature of the solution with the help of Figure 1.[1] Not only the Regulator's utility ϕ increases with π, so do also the profits of the Bs and NBs. For $\pi = 0$, the outcome is the Nash solution of a game between the NBs and the Regulator. This outcome is not efficient, leaving room for Pareto-improvement. As π increases, the Bs enjoy the benefits of a decreasing t, while their investment remains fixed at $v(\bar{t}^a)$. Likewise, the NBs benefit from the decrease in taxation. The lowering of the taxation is made rational by the existence of the Believers, who are led by the R's announcements to invest more than they would otherwise, and to subsequently produce more.

The only motive that could lead R to reduce the spread between t and t^a, and in particular to choose $t^a < \bar{t}^a$, lies in the impact of the tax announcements on the beliefs dynamics. Ceteris paribus, R prefers a high proportion of Bs to a low one, since it has one more instrument (that is, t^a) to influence the Bs than to control the NBs. A high spread $t^a - t$, however, implies that the profits of the Bs will be low compared to those of the NBs. This, by (4), reduces the value of $\dot{\pi}$ and leads over time to a lower proportion of Bs, diminishing the instantaneous utility of R. Therefore, in the dynamic problem, R will have to find an optimal compromise between choosing a high value of t^a,which allows a low value of t and insures the Regulator a high instantaneous utility, and choosing a lower one, leading to a more favorable proportion of Bs in the future.

4 Dynamic Analysis

4.1 Characterization of the optimal paths

As pointed out earlier, the Regulator faces a dynamic optimization problem because of the effect of his current action on the future stock of believers. This problem is given by:

$$\max_{0 \le t^a(\tau), t(\tau)} \Phi(t^a, t) \quad \text{subject to (8).}$$

[1]The parameter values underlying the figure are $c_v = 5,\ c_x = 3,\ \delta = 0,\ p = 6,\ k = 4,\ \kappa = 3$. The figure is qualitatively robust with respect to changes in these values.

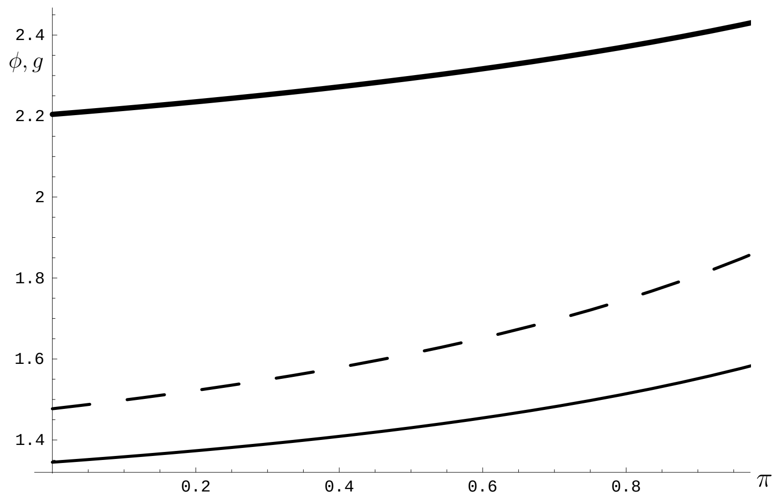

Figure 1: Profits of the believers (solid line), of the non-believers (dashed line), and Regulator's utility (bold-line) as a function of π.

The Hamiltonian of the problem is given by:

$$\begin{aligned} & H(t^a, t, \pi, \lambda) \\ = \; & (k - \kappa + t)\frac{p - t}{2c_x} + \frac{\kappa - t}{2c_v}(\pi t^a + (1 - \pi)t) \\ & + \lambda\beta\pi(1 - \pi)\left(-\frac{1}{4c_v}(t^a - t)^2 + \delta\right), \end{aligned}$$

where λ denotes the co-state variable. The Hamiltonian is concave in (t, t^a) iff:

$$2\lambda\beta(1 - \pi)(c_v + c_x) - \pi c_x > 0. \tag{14}$$

The optimal controls * are then given by:

$$t^*(\pi, \lambda) = \frac{\lambda\beta(1 - \pi)(c_v(\hat{p} - k + \kappa) + c_x\kappa) - c_x\kappa\pi}{2\lambda\beta(1 - \pi)(c_v + c_x) - c_x\pi} \tag{15}$$

$$t^{a*}(\pi, \lambda) = \frac{\lambda\beta(1 - \pi)(c_v(\hat{p} - k + \kappa) + c_x\kappa) - c_v(\hat{p} - k - \kappa) + c_x\kappa(1 - \pi)}{2\lambda\beta(1 - \pi)(c_v + c_x) - c_x\pi} \tag{16}$$

Otherwise, there are no bounded optimal controls. In what follows we assume that (14) is satisfied along the optimal path. It can be easily checked that this is the case at the equilibrium discussed below.

The difference between the optimal announced and realized tax levels is given by:

$$t^{a*} - t^* = \frac{\kappa - t^*}{\lambda\beta(1-\pi)}. \tag{17}$$

Hence the optimal announced tax exceeds the realized one if and only if $t^* < \kappa$. As in the static case, we restrict the analysis to the environmentally friendly case $t^* < \kappa$ and assume that (12) holds.

According to Pontriagins' Maximum Principle an optimal solution $(\pi(\tau), \lambda(\tau))$ has to satisfy the state dynamics (8) plus:

$$\dot{\lambda} = \rho\lambda - \frac{\partial H(t^{a*}(\pi,\lambda), t^*(\pi,\lambda), \pi, \lambda)}{\partial \pi}, \tag{18}$$

$$\lim_{\tau\to\infty} e^{-\rho\tau}\lambda(\tau) = 0. \tag{19}$$

In our case the co-state dynamics are given by:

$$\dot{\lambda} = \rho\lambda - \frac{\kappa - t}{2c_v}(t^a - t) - \lambda\beta(1-2\pi)\left(-\frac{1}{4c_v}(t^a - t)^2 + \delta\right). \tag{20}$$

In order to analyze the long run behavior of the system we now provide a characterization of the steady states for different values of the public flexibility parameter β. Due to the structure of the state dynamics there are always trivial steady states for $\pi = 0$ and $\pi = 1$. For $\pi = 0$ the announcement is irrelevant. Its optimal value is therefore indeterminate. The optimal tax level is $t = \kappa$. For $\pi = 1$, the concavity condition (14) is violated and the optimal controls are indefinite. In the following, we restrict our attention to $\pi < 1$. In Proposition 1, we first show under which conditions there exists an interior steady state where Believers coexist with Non-Believers. Furthermore, we discuss the stability of the steady states.

Proposition 1 *Steady states and their stability:*

(i) For $0 < \beta \leq \frac{\rho}{2\delta}$ there exists no interior steady state. The steady state at $\pi = 0$ is stable.

(ii) For $\beta > \frac{\rho}{2\delta}$ there exists a unique stable interior steady state $^+$ with:

$$\pi^+ = 1 - \frac{\rho}{2\beta\delta}. \tag{21}$$

The steady state at $\pi = 0$ is unstable.

Proof: We prove the existence and the stability of the interior steady state ($\dot{\pi} = 0,\ \dot{\lambda} = 0$). The claims about the boundary steady state at $\pi = 0$ follow directly.

Equation (8) implies that, in order for $\dot{\pi} = 0$, to hold one must have:

$$(t^{aS} - t^*)^2 = 4c_v\delta. \tag{22}$$

That is, taking into account (17):

$$t^* = \kappa - 2\lambda^*\beta(1-\pi^*)\sqrt{c_v\delta.} \tag{23}$$

Equation (22) implies that $\dot{\lambda} = 0$ is satisfied iff:

$$\rho\lambda - \frac{\kappa - t^*}{2c_v}(t^{a*} - t^*) = 0. \tag{24}$$

Using (17) for $t^{a*} - t^*$ in (24) we obtain the condition:

$$t^* = \kappa - \sqrt{2\rho\beta(1-\pi^*)c_v}. \tag{25}$$

Combining (23) and (25) shows that

$$2\lambda^*\beta(1-\pi^*)\sqrt{c_v\delta} = \sqrt{2\rho\beta(1-\pi^*)c_v}$$

must hold at an interior steady state. This condition is equivalent to:

$$\pi^+ = 1 - \frac{\rho}{2\beta\delta}. \tag{26}$$

For $\beta \leq \frac{\rho}{2\delta}$, (26) becomes smaller or equal zero. Therefore, an interior steady state is only possible for $\beta > \frac{\rho}{2\delta}$.

Using (21, 15, 23), one obtains for the value of the co-state at the steady state:

$$\lambda^* = \frac{\delta\beta(c_x(2\sqrt{c_v\delta} + \kappa) + c_v(k + \kappa - p)) - \rho c_x\sqrt{c_v\delta}}{2\rho\beta\sqrt{c_v\delta}(c_v + c_x)}. \tag{27}$$

To determine the stability of the interior steady state we investigate the Jacobian matrix J of the canonical system (8, 20) at the steady state. The eigenvalues of J are given by:

$$e_{1,2} = \frac{tr(J) \pm \sqrt{tr(J)^2 - 4det(J)}}{2}.$$

Therefore, the steady state is saddle point stable if and only if the determinant of J is negative. Inserting (15, 16) into the canonical system (8, 20), taking the derivatives with respect to (π, λ) and then inserting (21, 27) gives the matrix J. Tedious calculations show that its determinant is given by:

$$det(j) = C(-\sqrt{\delta\beta}(c_v(k+\kappa-p)+c_x\kappa)-c_x\sqrt{c_v}(2\beta\delta-\rho)),$$

where C is a positive constant. The first of the two terms in the bracket is negative due to the assumption (12), the second is negative whenever $\beta > \frac{\rho}{2\delta}$. Hence, $det(J) < 0$ whenever an interior steady state exists, implying that the interior steady state is always stable. For $\beta = \frac{\rho}{2\delta}$ this stable steady state collides with the unstable one at $\pi = 0$. The steady state at $\pi = 0$ becomes stable for $\beta < \frac{\rho}{2\delta}$. □

Since there is always only one stable steady state and since cycles are impossible in a control problem with a one-dimensional state, we can conclude from Proposition 1 that the stable steady state is always a global attractor. Convergence to the unique steady state is always monotone. The long run fraction of believers in the population is independent from the original level of trust. From (26) one recognizes that it decreases with ρ. An impatient Regulator will not attempt to build up a large proportion of Bs since the time and efforts needed now for an additional increase of π weighs heavily compared to the future benefits. By contrast, π^+ is increasing in β and δ. A high flexibility β of the firms means that the cumulated loss of potential utility occurred by the Regulator *en route* to π^+ will be small and easily compensated by the gains in the vicinity of and at π^+. Reinforcing this, the Regulator does not have to make Bs much better off than NBs in order to insure a fast reaction. As a result, for β large, the equilibrium π^+ is characterized by a large proportion of Bs and provide high profits respectively utility to all players. A high learning cost δ means that the Regulator can make the Bs better off than the NBs at low or no cost, implying again a high value of π at the steady state.

Note that it is never optimal for the Regulator to follow a policy that would ultimately insure that all firms are Believers, $\pi^+ = 1$. There are two concurrent reasons for that. On the one hand, as π increases, the Regulator has to deviate more and more from the statically optimal solution $t^{a\$}(\pi)$, $t^{\$}(\pi)$ to make believing more profitable than non-believing. On the other, the beliefs dynamics slow down. Thus, the discounted benefits from increasing π decrease.

For $\rho = 0.8,\ c_v = 5,\ c_x = 3,\ \delta = 0.15,\ p = 6,\ k = 4,\ \kappa = 3$ e.g., the profits and utility at the steady state $\pi^+ = 0.733333$ are $g^b = g^{nb} =$

1.43785, $\phi = 2.33043$. This steady state Pareto-dominates the fully rational equilibrium with $\pi = 0$, where $g^{nb} = 1.32708$, $\phi = 2.30844$. It also dominates the equilibrium attained when the belief dynamics (8) holds but the Regulator maximizes in each period his instantaneous utility ϕ instead of Φ. At this last equilibrium, $\pi = 0.21036$, $g^b = g^{nb} = 1.375$, $\phi = 2.23689$. Note that the last two equilibria cannot be compared, since the latter provides a higher profit to the firms but a lower utility to the Regulator.

This ranking of equilibria is robust with respect to parameter variations. A clear message emerges. As we contended at the beginning of this paper the suggested solution Pareto-improves on the static Nash equilibrium. This solution implies both a beliefs dynamics among the firms and a farsighted Regulator. A farsighted Regulator without beliefs dynamics is pointless. Beliefs dynamics with a myopic Regulator lead to a more modest Pareto-improvement. But it is the combination of beliefs dynamics and farsightedness that Pareto-dominates all other solutions.

4.2 The Influence of Public Flexibility

An interesting question is whether the Regulator would prefer a population that reacts quickly to profit differences, shifting from Believing to Non-Believing or vice versa in a short time, or if it would prefer a less reactive population. In other words, would the Regulator prefer a high or a low value of β?

From Proposition 1 we know that the long run fraction of Believers is given by:

$$\pi^* = \max \left[0, 1 - \frac{\rho}{2\beta\delta} \right].$$

A minimum level $\beta > \frac{\rho}{2\delta}$ of flexibility is necessary for the system to converge towards an interior steady state with a positive fraction of Bs. For β greater than $\frac{\rho}{2\delta}$, the fraction of Bs at the steady state increases with β, converging towards $\pi = 1$ as β goes to infinity. One might think that, since the Regulator always prefers a high proportion of Bs at equilibrium, he would also prefer a high value of β. Stated in a more formal manner, one might expect that the value function of the Regulator, $V^R(\pi_0)$, increases with β regardless of π_0. This, however, is not the case. The dependence of $V^R(\pi_0)$ on β is non-monotone and depends crucially on π_0. An analytical characterization of $V^R(\pi_0)$ being impossible, we use a numerical example to illustrate that point. The results are very robust with respect to parameter variations.

Figure 2 shows the steady state value $\pi^* = \pi^*(\beta)$ of π for $\beta \in [1, 30]$.

Figure 3 compares[2] $V^R(0.2)$ and $V^R(0.8)$ for the same values of β. The other parameter values are as before $\rho = 0.8,\ c_v = 5,\ c_x = 3,\ \delta = 0.15,\ p = 6,\ k = 4,\ \kappa = 3$ in both cases.

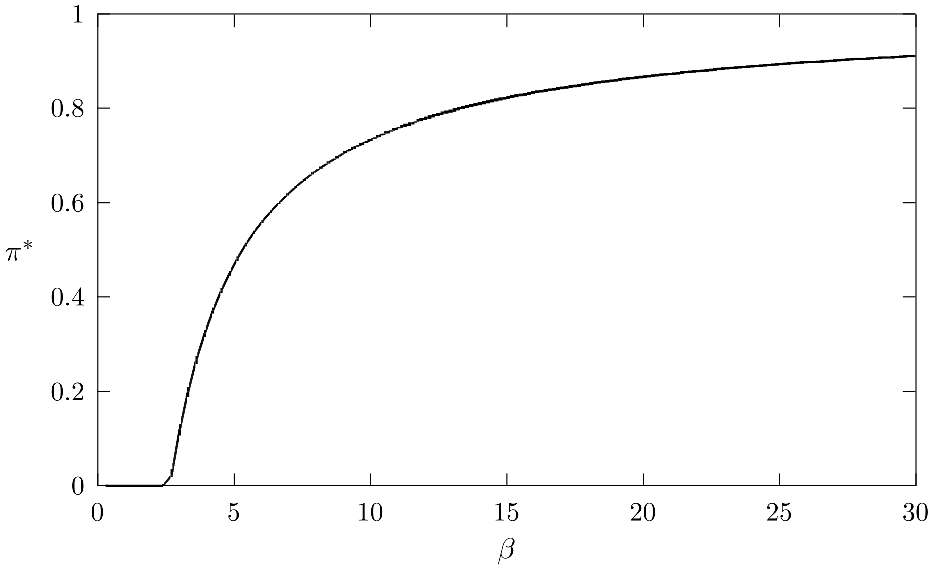

Figure 2: The proportion of believers at the steady state for $\beta \in [1, 30]$.

Figure 3 reveals that one always has $V^R(0.8) > V^R(0.2)$, reflecting the general result that $V^R(\pi_0)$ is increasing in π_0 for any β. Both value functions are not monotone in β but U-shaped. Combining Figure 2 and Figure 3 shows that the minimum of $V^R(\pi_0)$ is always attained for the value of β at which the steady state value $\pi^*(\beta)$ coincides with the initial fraction of Believers π_0. This result is quite intuitive. If $\pi^*(\beta) < \pi_0$, it is optimal for the Regulator to reduce π over time. The Regulator does it by announcing a tax t^a much greater than the tax t he will implement, making the Bs worse off than the NBs, but also increasing his own instantaneous benefits ϕ. Thus, R prefers that the convergence towards the steady state be as slow as possible. That is, V^R is decreasing in β. On the other hand, if $\pi^* > \pi_0$, it is optimal for R to increase π over time. To do so he must follow a policy

[2]The numerical calculations underlying this figure were carried out using a powerful proprietary computer program for the treatment of optimal control problems graciously provided by Lars Grüne, whose support is most gratefully acknowledged. See Grüne and Semmler (2002).

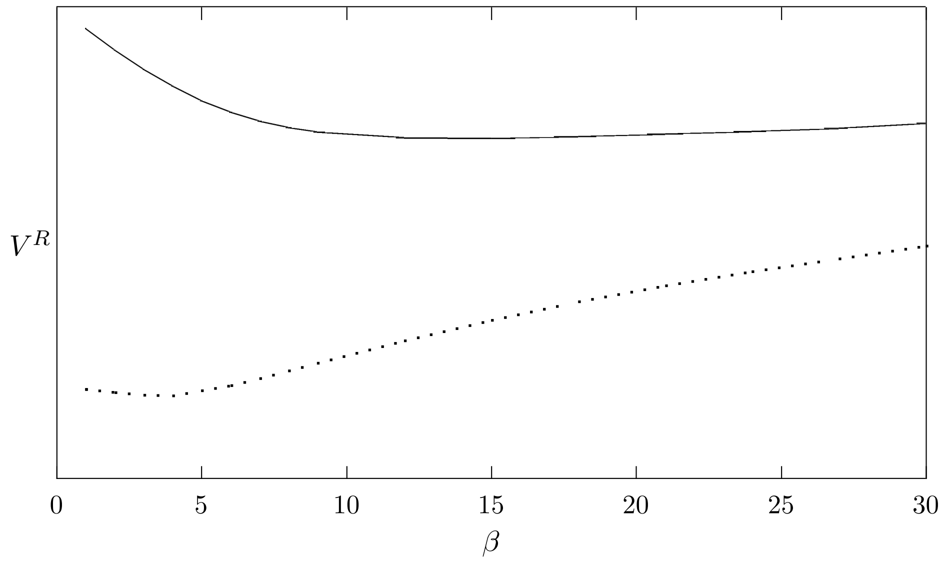

Figure 3: The value function of the Regulator for $\pi = 0.2$ (dotted line) and $\pi = 0.8$ (solid line) for $\beta \in [1, 30]$.

that makes the Bs better off than the NBs, but is costly in terms of his instantaneous objective function ϕ. It is therefore better for R if the firms react fast to the profit difference. The value function V^R increases with β. Summarizing, the Regulator prefers (depending on π_0) to be confronted with *either* very flexible *or* very inflexible firms. In-between values of β provide him smaller discounted benefit streams.

Whether a low or a high flexibility is preferable for R depends on the initial fraction π_0 of Believers. Our numerical analysis suggests that a Regulator facing a small π_0 prefer large values of β, whereas he prefers a low value of β when π_0 is large. This result may follow from the specific functional form used in the model rather than reflect any fundamental property of the solution.

If $\beta = 0$, the proportion of Bs remains fixed over time at π_0 – any initial value of $\pi_0 \in (0, 1)$ corresponds to a stable equilibrium. The Pareto-improving character of the inner equilibrium then disappears. Given $\beta = 0$, condition (14) is violated. The Regulator can announce any tax level without having to fear negative long term consequences. Thus, it is in his interest to exploit the gullibility of the Bs to the maximum. To obtain a meaningful, Pareto-improving solution, some flexibility is necessary that assures that the firms are not kept captive of beliefs that penalize them. Only then will the Regulator be led to take into account the Bs interest.

5 Conclusions

The starting point of this paper is a situation frequently encountered in environmental economics (and similarly in other economic contexts as well): If all firms are rational Non-Believers who make perfect predictions of the Regulator's actions and discard the Regulators announcements as cheap talk, standard optimizing behavior leads to a Pareto-inferior outcome, although there are no conflict of interest between the different firms and although the objectives of the firms and of the Regulator largely concur. We show that, in a static world, the existence of a positive fraction of Believers who take the Regulator's announcement at face value Pareto-improves the economic outcome. This property crucially hinges on the fact that the firms are atomistic and thus do not anticipate the collective impact of their individual decisions.

The static model is extended by assuming that the proportion of Believers and Non-Believers changes over time depending on the difference in the profits made by the two types of firms. The Regulator is assumed to recognize his ability to influence the evolution of the proportion of Believers by his choice of announced and realized taxes, and to be interested not only in his instantaneous but also in his future utility. It is shown that a rational Regulator will never steer the economy towards a situation where all firms are Believers. However, his optimal policy may lead to a stable steady state with a strictly positive proportion of Believers that is Pareto-superior to the equilibrium where all agents perfectly anticipate his actions. Prerequisites therefore are a sufficiently patient Regulator and firms that occur sufficiently high costs for building perfect anticipations of the government actions and/or are suitably flexible, i.e., switch adequately fast between Believing and Non-Believing. The conjunction of beliefs dynamics for the firms *and* of a farsighted Regulator allows for a larger Pareto-improvement than either only beliefs dynamics or only farsightedness. Depending upon the initial proportion of Believers, the Regulator is best off if the firms are very flexible or very inflexible. Intermediate values of the flexibility parameter are never optimal for the Regulator.

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