

Wage Stickiness and Nonclearing Labor Market in Business Cycles *

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Abstract

This paper presents a dynamic optimization model of RBC type augmented by wage stickiness, and nonclearing labor market. Agents are required to adaptively optimize when facing constraints on the markets. Calibration for the U. S. economy shows that the model will produce higher volatility in employment. Moreover, it provides more reasonable cross-correlation of employment and wages with other macroeconomic variables and improves on the correlation of the technology shock with employment. Overall, the model fits the data better than the benchmark RBC model.

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1 Introduction

The real business cycle (RBC) model has become one of the major approaches in macroeconomics to explain observed economic fluctuations. Despite its rather simple structure, it can explain, at least partially, the volatility of some major macroeconomic variables such as output, consumption and capital stock. However, to explain the actual variation in employment the model generally predicts an excessive smoothness of labor effort in contrast to empirical data. The problem of excessive smoothness in labor effort and its failure of explaining the actual variation of employment is a well-known in the RBC literature since the RBC paradigm was put forward.¹ A recent empirical evaluation of this failure of the RBC model is given in Schmidt-Grohe (2001). There the RBC model is compared to indeterminacy models, as developed by Benhabib and his co-authors. Whereas in RBC models the standard deviation of the labor effort is too low, in indeterminacy models it turns out to be excessively high.

Another problem in RBC literature related to this is the cross correlation of output, labor effort, consumption and wages. As has been stated by Rotemberg and Woodford (1996) and Schmidt-Grohe (2001) the RBC model predicts that (a) forecastable movements in output, hours and consumption move in different directions when impacted by a permanent technology shock² whereas the data show that forecastable changes in those variables are positively correlated and (b) the overall movement of those three variables, responding to a technology shock, are highly correlated. A similar observation also holds, as we will show, for the variation of wages. We here can observe that the model also generally implies, for forecastable movements of the variables in the model, a high positive correlation of wages with output, consumption and a negative correlation with employment whereas in the data the latter correlation is positive. We want to argue in this paper that these problems appear to be considerably related to the specification of the labor market.

A further major puzzle in the RBC model is that the model often predicts a significantly high positive correlation between the technology shock and employment whereas empirical research demonstrates, at least at business cycle frequency, a negative or almost zero correlation. This puzzle is often named the technology puzzle (see King and Rebelo (1999) and Francis and

¹Critical evaluations of this issue include Summers (1986), Mankiw et al. (1985), Rotemberg and Woodford (1996).

²In fact, in the model hours should fall and consumption rise whereas in the forecastable change in these three variable one observes that these three series should be positively correlated, see Rotemberg and Woodford (1996).

Ramey (2003)).³

We would like to express the view here that the excessive smoothness of the variation in employment, the incorrect correlation of the macro variables and the positive correlation of the technology shock and employment essentially arise from an unrestricted consumption - leisure (employment) choice model where economic agents can, in an intertemporal setting freely and smoothly trade off consumption, leisure and employment.⁴ Indeed, in the context of the smooth and unconstrained intertemporal choice of RBC models there are three marginal conditions that ensure three equilibria to be established. These are

- (i) the Euler equation that ensures an equality in the intertemporal trade off of consumption in consecutive periods,
- (ii) the marginal rate of substitution equal to the real wage (the cost of trading off leisure against consumption is equal to the real wage),
- (iii) the optimizing of the firm ensures the equality of the marginal product of labor equal to the real wage.

Whereas the establishment of those equalities presumes frictionless labor markets⁵, actual labor markets are sluggishly adjusting. Thus, as recently discussed in many contributions, in order to approach the labor market puzzle in a real business cycle model, one thus has to make some improvement upon labor market specifications.⁶ As we argue in this paper one possible approach for such improvement is to allow for wage stickiness and a nonclearing labor market.^{7, 8}

An important research along the line of micro-founded Keynesian economics has been historically developed by New Keynesian analysis based on the sticky price and monopolistic competition. Attempts have now been

³The authors have extensively elaborated on this issue in Gong and Semmler (2006, ch. 5 and ch. 9).

⁴An earlier test of this assumption has been undertaken by Mankiw et al. (1986) who state that their empirical results "casts serious doubts on the premise of most classical macroeconomic models that observe a labor supply represents unconstrained choices given opportunities". (p. 241)

⁵As well as product and capital markets.

⁶See for example Hall (2005), Blanchard and Gali (2005) and Uhlig (2004).

⁷Recently there are many studies on EU-countries that allow for some sluggish in the labor market. Many of those studies are discussed in Ernst et al. (2006).

⁸Recently Gali, Gertler and Lopez-Salido (2003) have considered the welfare cost for the case when conditions (ii) does not hold, i.e. when the marginal rate of substitution differs from the real wage and thus from the marginal product of labor, given by (iii).

made recently that introduce the Keynesian features into a dynamic optimization model. Rotemberg and Woodford (1995, 1999), King and Wollman (1999), Gali (1999), Erceg, Henderson and Levin (2000), Woodford (2003), Smets and Wouters (2004) and Christiano, Eichenbaum and Evans (2005) present a variety of models with monopolistic competition and sticky prices.

On the other hand, there are models of efficiency wages where nonclearing labor market could occur.⁹ We shall remark that in those studies with nonclearing labor market, an explicit labor demand function is introduced from the perspective of the decision problem of the firm side. However, the decision rule with regard to labor supply in these models is often dropped because the labor supply no longer appears in the welfare function of the household. Consequently, the moments of labor effort become purely demand-determined. Implicitly, the labor supply in these models is assumed to be given exogenously, and normalized to 1. Hence nonclearing occurs in labor market if the demand is not equal to 1.¹⁰

In this paper, we will present a stochastic dynamic optimization model including Keynesian features along the line of the above consideration. In particular, we shall allow for wage stickiness and nonclearing labor market. However, unlike the other recent models of nonclearing labor market, we shall view the decision rule of the labor effort derived from a dynamic optimization problem as being a natural way to reflect the desired labor supply. Although we propose intertemporal decision of economic agents we presume that agents re-optimize once they face constraints on the market. In particular, we presume that households adaptively optimize once they have learned about market constraints.

The basic mechanism works as follows. First, the intertemporal decision of the household produces a notional labor supply but this labor supply cannot necessarily be made effective. Since we presume a Calvo type updating scheme for the partial adjustment of actual wages to the optimal wage, this creates sticky wages. Given the wage sequence the firms, following the above marginal rule (iii), adjust its notional demand for labor. Then, given the imbalance of the supply and demand for labor a decision rule will have to be

⁹See Danthine and Donaldson (1990, 1995), Benassy (1995) and Uhlig and Xu (1996) among others. A recently developed model of nonclearing markets of the French disequilibrium tradition, which resembles ours, can be found in Portier and Puch (2004). Uhlig (2004) also presumes that models with exogenous wage sequence at nonclearing market level will be better suited to match actual labor market movements.

¹⁰Another line of recent research on modeling unemployment in a dynamic optimization framework can be found in the work by Merz (1999) and Walsh (2002) among others, who employs search and matching theory to model the labor market. Yet, as shown recently, the search and matching models have difficulties to capture the volatility of the actual ratio of vacancies and unemployment, see Shimer (2005).

implemented to determine the actual employment. Subsequently, when the households face a constraint on the labor market, they have to adaptively optimize, i.e. re-optimize, to adjust their optimal consumption sequence to the labor market constraint.

The here given short sketch over our proposed mechanism will permit to improve on the above mentioned three puzzles. One of the advantages of our formulation, as will become clear, is that employment rules can be explored to specify the realization of actual employment when a nonclearing market emerges. Overall our presumed adaptive behavior ensures that indeed intertemporal decision¹¹ are taken, but also nonclearing of markets can be taken account of.

There is a similarity of our approach chosen here and the New Keynesian analysis. New Keynesian literature presents models with imperfect competition and sluggish price and wage adjustments. However, the market in this model variant is still assumed to be cleared since the producer supplies the output whatever the market demands is at the existing price. A similar consideration is also assumed to hold for the labor market. Here the wage rate is set optimally by a representative of the household according to the expected market demand curve for labor. Once the wage has been set, it is assumed to be sticky for some periods and only a fraction of wages are allowed to be changed with optimization in each period. Though in those models there exists a gap between the optimal wage and existing wage, the labor market is cleared since the household is assumed to supply the labor whatever the market demand is at the given wage rate. Such a quantity decision, as we will discuss in sect. 2 in the following analysis, may imply that the supplier no longer behaves optimally. Also as to our knowledge this model variant has not been rigorously put to a test of whether it can replicate actual labor market movements.

In sum, the model we present here allows for wage stickiness and non-clearing market. The nonclearing labor market requires quantity decisions. We wish to argue that the New Keynesian and our approach are complementary rather than exclusive, and therefore they can somewhat be consolidated as a more complete system for price and quantity determination within the Keynesian tradition.

The remainder of this paper is organized as follows. Section 2 provides a static analysis of price and quantity in our model of nonclearing market. Section 3 presents the structure of our dynamic model and the adaptive op-

¹¹One could, of course, also allow only for a fraction of the consumers adaptively optimizing and another fraction following some rule of thumb, see Gali, Lopez-Salido and Valles (2003).

timization mechanism. Section 4 calibrates the model for the U.S. economy. Section 5 concludes. The appendix contains some technical derivations of the model.

2 Nominal Stickiness and Noncleared Markets: A Static Analysis

Due to the quite intricate structure of our model, we shall first provide a static analysis on how price and quantity may be seen to be determined. We rely on Figure 1 to discuss, in a preliminary way, of how our approach relates to the New Keynesian view. We shall remark that the discussion concerning Figure 1 is rather general and can be viewed to be relevant for both a product market or a labor market.

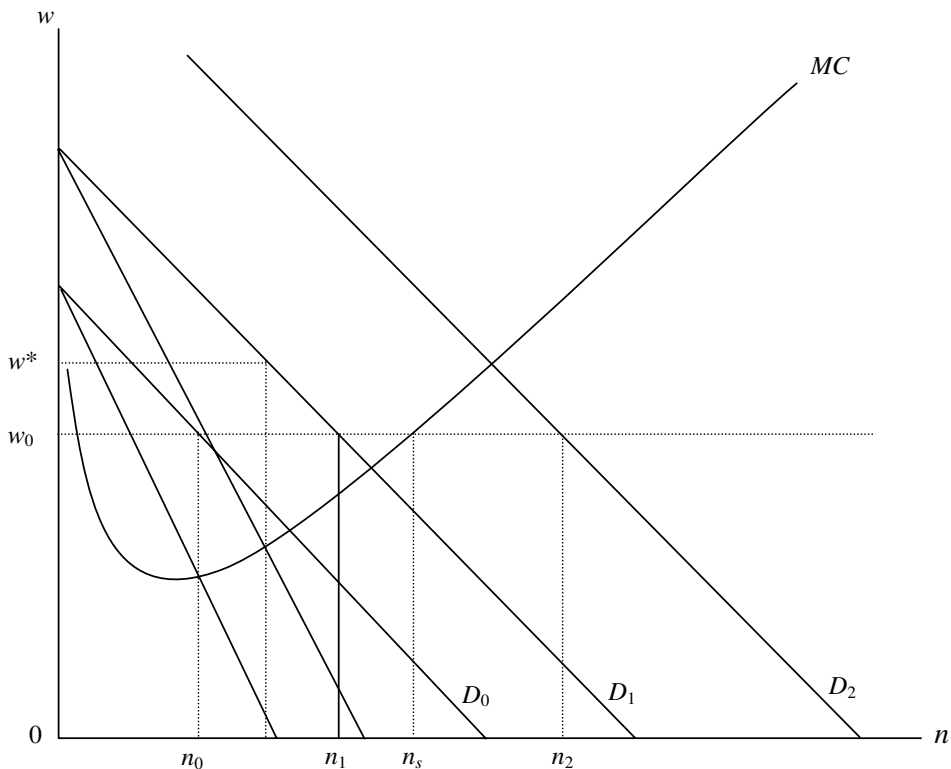


Figure 1: A Static Model with Sticky Price Wage and Disequilibrium

Suppose that the producer (or the household in the case of the labor market) has set up its price optimally according to the expected demand

curve D_0 . Let us denote this price as w_0 . Consider now the situation that the supplier's expectation on demand is not fulfilled. Instead of n_0 , the market demand at w_0 is n_1 . In this case, the household may reasonably believe that the demand curve should be D_1 and therefore the optimum price should be w^* . This raises the question whether the supplier should change the price from w_0 to the optimal w^* . Yet there may be nominal price and wage stickiness.

A common explanation for nominal price stickiness is that there are adjustment costs for the changing price¹² (or wage). This may provide the reason for the supplier (or household) to stick to the price (or wage) even if it is known that current price (or wage) may not be optimal. In the case of the labor market, one may also derive this stickiness from wage contracts as in Taylor (1980) with the contract period to be longer than one period.

In the spirit of Calvo (1983) we presume that the existence of adjustment costs entails that there exists, for the economy as a whole, a probability ξ , that a fraction of wages will be sticky and the other fraction $(1 - \xi)$ will be adjusted. In our dynamic model, as will be presented in the next section, this implies a partial adjustment process, such as

$$w_t = \xi w_{t-1} + (1 - \xi)w_t^*, \quad (1)$$

where w_t is the actual wage rate at period t while w_t^* is the optimal wage rate in t . Wage stickiness due to such a partial adjustment process for the wage has been presumed in many recent papers, see Gali and Blanchard (2005), Hall (2005) and Shimer (2005).¹³ In appendix I we will provide a detailed derivation of this partial adjustment process.

Given such wage stickiness, we shall now turn to the problem of how quantities are determined. The recent New Keynesian literature presumes that at the existing wage rate, the producer (or household) shall supply output (or labor effort) whatever the market demand is. With such a way of quantity determination, the market is regarded to be "cleared" and therefore the concept of disequilibrium as often appearing in the traditional Keynesian

¹²There could be the so-called menu cost for changing prices (though this seems more appropriate for the output price). There is also a reputation cost for changing prices, see Rotemberg (1982). In addition, changing the price (or wage) needs information, computation and communication, which may be also costly. See the discussion in Christiano, Eichenbaum and Evans (2005) and Zbaracki, Ritson, Levy, Dutta and Bergen (2000). All these efforts cause costs, which may be summarized as adjustment cost of changing the price or wage.

¹³We here make no further attempt to elaborate on some micro foundations of such partial adjustment process. For such an effort, see, for example Christophel and Linzart (2005).

literature, no longer appears here.

However, at the given price the producer's willingness to supply is n_s . In the case of labor market, the marginal cost curve, MC in figure 1, can be interpreted as marginal disutility of labor which has also an upward slope if we use the standard log utility function as in the RBC literature. Therefore, n_s can be understood as the household's willingness to supply labor. If we define the market demand and market supply in this standard way, nonclearing labor market can be a permissible phenomena.

On the recognition that a disequilibrium may still exist in a monopolistic competitive market where prices and wages adjust sluggishly, the quantity determination in the recent New Keynesian literature could be enriched by referring the traditional disequilibrium theory. Suppose in Figure 1, the actual demand at w_0 is n_2 and therefore the supplier may expect that demand curve will be D_2 . If we allow the supplier to provide output (or labor effort) whatever the market demand is, we find that the actual supply should be equal to n_2 . Yet, this supply is not optimal since at n_2 , the marginal cost is larger than the marginal revenue when the price is given. Or, in terms of our marginal condition (ii) in the introduction, the marginal rate of substitution could be greater than the real wage that the household receives, indicating a gap between those two which arises from a noncleared market. However, if we apply the short-side rule as in disequilibrium analysis (see Benassy 1975, 1984, 2003), we would find that the producer will choose n_s which is optimal in this case. On the other hand, if n_s is chosen this also implies some nonclearing markets.¹⁴

In the model we present in the next section, we shall consider a particular disequilibrium rule, we will call it a compromise rule, for the quantity determination given sticky price and wage as also assumed in the recent New Keynesian literature. We will find that the introduction of noncleared markets and disequilibrium rule will require a second step optimization, so an adaptive optimization due to market constraints, that arises after the first step. Next let us present our model structure.

3 An Economy with Sticky Wage and Non-clearing Labor Market

In order to be methodologically consistent, we also follow the usual assumptions of identical households and identical firms. Therefore we are considering

¹⁴Even if the New Keynesian do not allow for some disequilibrium rule, they probably would admit the second phenomena, see Gali, Gertler and Lopez-Salido (2003).

an economy that has two representative agents: the representative household and the representative firm. There are three markets in which the agents exchange their products, labor and capital. The household owns all the factors of production and therefore sells factor services to the firm. The revenue from selling factor services can only be used to buy the goods produced by the firm either for consuming or for accumulating capital. The representative firm owns nothing. It simply hires capital and labor to produce output, sells the output and transfers the profit back to the household.

Unlike the RBC model, in which one could assume an once-for-all market, we, however, in this model shall assume that the market to be re-opened at the beginning of each period t . This is necessary for a model with nonclearing markets in which adjustments should take place.

Let K_t denote for capital stock, N_t for per capita working hours, Y_t for output and C_t for consumption. Assume that the capital stock in the economy follow the transition law:

$$K_{t+1} = (1 - \delta)K_t + A_t K_t^{1-\alpha} (N_t X_t)^\alpha - C_t, \quad (2)$$

where δ is the depreciation rate; α is the share of labor in the production function $F(\cdot) = A_t K_t^{1-\alpha} (N_t X_t)^\alpha$; A_t is the temporary shock in technology and X_t the permanent shock that follows a growth rate γ . We follow the usual process to divide both sides of equation (2) by X_t so that

$$k_{t+1} = \frac{1}{1 + \gamma} \left[(1 - \delta)k_t + A_t k_t^{1-\alpha} (n_t \bar{N}/0.3)^\alpha - c_t \right], \quad (3)$$

where $k_t \equiv K_t/X_t$, $c_t \equiv C_t/X_t$ and $n_t \equiv 0.3N_t/\bar{N}$ with \bar{N} to be the sample mean of N_t . This indicates that all the variables are now stationary. Note that n_t is often regarded to be the normalized hours. The sample mean of n_t is equal to 30 %, which, as pointed out by Hansen (1985), is the average percentage of hours attributed to work.

3.1 The Wage Setting

Note that there are three commodities in our model and therefore there are three types of prices, the output price p_t , the wage rate w_t and the rental rate of capital stock r_t . One of them should serve as a numeraire, which we assume to be the output. This indicates that the output price p_t always equals 1 and thus the wage w_t and the rental rate of capital stock r_t are all measured in terms of the physical units of output.¹⁵ As to the rental rate of

¹⁵For our simple representative agent model without money, this simplification does not effect our major result derived from our model. Meanwhile, it will allow us to save some

capital r_t , it is assumed to be adjustable so as to clear the capital market. We can then ignore its setting. Indeed, as will become clear, one can imagine any initial value of the rental rate of capital when the firm and the household make the quantity decisions and express their desired demand and supply. This leaves us to focus the discussion only on the wage setting.

Following the discussion in the previous section with regard to Figure 1, we shall now first discuss how the household chooses the optimal wage rate at period t , that is, w_t^* . We can express this determination by relying on the following model of dynamic optimization:

$$\max_{w_t^*, \{c_{t+i}\}_{i=0}^{\infty}} E_t \left[\sum_{i=0}^{\infty} (\xi\beta)^i U(c_{t+i}, n_{t+i}) \right] \quad (4)$$

subject to

$$k_{t+i+1} = \frac{1}{1+\gamma} [(1-\delta)k_{t+i} + f(k_{t+i}, n_{t+i}, A_{t+i}) - c_{t+i}]; \quad (5)$$

$$w_t^* = f_n(k_{t+i}, n_{t+i}, A_{t+i}). \quad (6)$$

Above, $U(\cdot)$ is the utility function which depends on consumption c_{t+i} and employment n_{t+i} ; $f(\cdot) \equiv A_{t+i}k_{t+i}^{1-\alpha}(n_{t+i}\bar{N}/0.3)^\alpha$ is the production function in a stationary form, which is implied by (3); $f_n(\cdot)$ is the marginal product of labor derived from $f(\cdot)$; β is the discount factor; $(1-\xi)$ is the probability that the wage rate w_t^* will be set in period $t+1$;¹⁶ and finally, E_t is the expectation operator. Note that here we have assumed that the household knows the production function $f(\cdot)$ and therefore knows the firm's demand curve for the labor. This indicates that the employment n_{t+i} should satisfy the first-order condition as expressed in (6) for all the possible future periods.

In this paper, we shall assume that the utility function takes the following standard form:

$$U(c, n) = \ln c + \theta \ln(1 - n). \quad (7)$$

Given such a utility function, the solution regarding w_t^* can be expressed by the following proposition:

Proposition 1 *Assume that $E_t A_{t+i} = A_t$, for $i = 0, 1, 2, \dots$ while $U(c_{t+i}, n_{t+i})$ is implied by (7). Then, the optimum wage rate w_t^* can be expressed as*

effort to explain the nominal price determination, a focus in the recent New Keynesian literature.

¹⁶Therefore, $(1-\xi)^i$ is the probability that w_t^* is set in period $t+i$.

$$w_t^* = \left[\frac{(A_t)^{1/(1-\alpha)} (\theta \alpha^{1/(1-\alpha)} - \alpha^{\alpha/(1-\alpha)})}{1 - \delta} \right]^{(1-\alpha)/\alpha} \quad (8)$$

provided $\theta > 1/\alpha$.

The proof of this proposition is provided in Appendix I. we shall remark that the restriction $\theta > 1/\alpha$ ensure that the solution with respect to w_t^* is real. Also note that for the empirical test, w_t^* should be regarded to be the wage rate that has been detrended by the permanent growth in labor productivity. This will be made clear in Appendix I. The same is true for w_t as expressed in equ. (1).

Given the optimal wage rate w_t^* as expressed in (8), the actual wage rate w_t is partially adjusted toward to optimal wage rate, w^* , and thus is given by equ. (1).

3.2 The First Step Decision of the Household

The next step in our multiple stage decision process is to model the first step decision of the households, given the price and wage that have been set up. We here define the household's notational demand and supply as those demand and supply that can allow the household to obtain the maximum utility under the condition that these demand and supply can be realized at the given set of prices. Although the household may realize that their national demand and supply may not be effective, such modelling is still necessary because it provides the basis for the household to bargaining with the employer, the firm, when disequilibrium occurs.

We can express the household's national decision as a sequence of output demand and factor supply $\{c_{t+i}^d, i_{t+i}^d, n_{t+i}^s, k_{t+i+1}^s\}_{i=0}^{\infty}$, where i_{t+i} is referred to investment. Note that here we have used the superscripts d and s to refer to the agent's desired demand and supply. The decision problem for the household to derive its demand and supply can be formulated as

$$\max_{\{c_{t+i}, n_{t+i}\}_{i=0}^{\infty}} E_t \left[\sum_{i=0}^{\infty} \beta^i U(c_{t+i}^d, n_{t+i}^s) \right] \quad (9)$$

subject to

$$k_{t+i+1}^s = (1 - \delta)k_{t+i}^s + f(k_{t+i}^s, n_{t+i}^s, A_{t+i}) - c_{t+i}^d. \quad (10)$$

For the given technology sequence $\{A_{t+i}\}_{i=0}^{\infty}$, equs. (9) and (10) form a standard intertemporal decision problem. The solution to this problem can

be written as:

$$c_{t+i}^d = G_c(k_{t+i}^s, A_{t+i}); \quad (11)$$

$$n_{t+i}^s = G_n(k_{t+i}^s, A_{t+i}). \quad (12)$$

We shall remark that although the solution appears to be a sequence $\{c_{t+i}^d, n_{t+i}^s\}_{i=0}^{\infty}$ only (c_t^d, n_t^s) along with (i_t^d, k_t^s) , where $i_t^d = f(k_t^s, n_t^s, A_t) - c_t^d$ and $k_t^s = k_t$, are actually carried into the market by the household for exchange due to our assumption of re-opening of the market.

3.3 The Quantity Decision of the Firm

Since the firm simply rents capital and hires labor on a period-by-period basis, the problem faced by the representative firm at period t is to choose the current input demands and output supplies (n_t^d, k_t^d, y_t^s) that maximizes the current profit. We presume that the firm has a perceived demand curve for its product. Thus given the output price, which is set at 1 as a numeraire, the firm has an expected constraint on the market demand for its product. We shall denote this expected demand as \hat{y}_t . This is somehow corresponding to n_1 or n_2 (when price equals w_0), as in Figure 1, if we refer it to the product market.

On the other hand, given the price of output, labor and capital stock $(1, w_t, r_t)$, the firm should also have its own desired supply y_t^* , which in Figure 1 corresponds to n_s . This desired supply is the amount that allows the firm to own the maximum profit on the assumption that all its output can be sold. Obviously, if the expected demand \hat{y}_t is less than the firm's willingness to supply, y_t^* , the firm will choose \hat{y}_t . Otherwise, it will choose y_t^* as is common in disequilibrium analysis.

This consideration indicates that the expectation on \hat{y}_t will become an important factor in determining the demand for factors. For the given production function, we thus find that the demand for labor and capital are both functions of price, technology and expectation:

$$k_t^d = k(r_t, w_t, 1, A_t, \hat{y}_t); \quad (13)$$

$$n_t^d = h(r_t, w_t, 1, A_t, \hat{y}_t). \quad (14)$$

We are now considering the transactions in our three markets. Let us first consider the two factor markets.

3.4 Transaction in the Factor Market

Given the quantity decision regarding the desired demand and supply from the household and the firm, we shall now discuss the transaction between

them. We have assumed the rental rate of capital r_t to be adjustable in each period and thus the capital market is cleared. This indicates that

$$k_t = k_t^s = k_t^d.$$

As concerning the labor market, there is no reason to believe that firm's demand for labor, as expressed in (14) should be equal to the willingness of the household to supply labor as determined in (12) given the way of wage determination as explained in sections 2 and 3.1. Therefore, we cannot regard the labor market to be cleared.

When the labor market is not cleared, we shall have to specify what rule should apply regarding the realization of actual employment.

Disequilibrium Rule: When disequilibrium occurs in the labor market either of the following two rules might be considered to be applied:

$$n_t = \min(n_t^d, n_t^s) \tag{15}$$

$$n_t = \omega n_t^d + (1 - \omega)n_t^s \tag{16}$$

where $\omega \in (0, 1)$.

Above, the first is the famous short-side rule when disequilibrium occurs, as discussed in sect. 2. As mentioned before, it has been widely used in the literature on disequilibrium analysis (see, for instance, Benassy 1975, 1984, among others). The second might be called the compromise rule. This rule indicates that when nonclearing of the labor market occurs both firms and workers have to compromise. If there is excess supply, firms will employ more labor than what they wish to employ.¹⁷ On the other hand, when there is excess demand, workers will have to offer more effort than they wish to offer.¹⁸ Such mutual compromises may be due to the institutional

¹⁷This case could also be brought about by firms by demanding the same (or less) hours per worker but employing more workers than being optimal. This case also corresponds to what is discussed in the literature as labor hoarding where firms hesitate to fire workers during a recession because it may be hard to find new workers in the next upswing, see Burnside et al. (1993). Altogether here a gap between the MRS and real wage would arise. Moreover, firms may be off their marginal product curve and thus this might require wage subsidies for firms as has been suggested by Phelps (1997).

¹⁸This could be achieved by employing the same number of workers but each worker supplying more hours (varying shift length and overtime work); for a more formal treatment of this point, see Burnside et al. (1993).

structures and moral standards of the society.¹⁹ Given the rather corporate relationship of labor and firms in some European countries, for example, this compromise rule might be considered a reasonable approximation. Such a rule that seems to hold for many countries was already discussed early in the economic literature, see Meyers (1968) and also Solow (1979).

We want to note that the unemployment we discuss here is different from unemployment as often discussed in those search and matching models. In our model, the unemployment is mainly due to some labor market stickiness and the insufficiency in the expected demand \hat{y}_t , which allows us to derive the demand for labor, see equ. (14), relative to the household's willingness to supply labor, given the institutional arrangements of the wage setting. On the other hand, frictional unemployment can arise from informational and institutional search and matching frictions where welfare state and labor market institutions may play a role. Yet the frictions in the institutions of the matching process are likely to explain only a certain fraction of observed unemployment.²⁰

3.5 The Adaptive Optimization and the Transaction in the Product Market

After the transactions in these two factor markets have been carried out, the firm will engage in its production activity. The result is the output supply, which can be expressed as

$$y_t^s = f(k_t, n_t, A_t). \quad (17)$$

Then the transaction needs to be carried out with respect to y_t^s . It is important to note that when the labor market is not cleared, households face

¹⁹Note that if firms are off their supply schedule and workers off their demand schedule, a proper study would have to compute the firms' cost increase and profit loss and the workers' welfare loss. If, however, the marginal cost for firms is rather flat (as empirical literature has argued, see Blanchard and Fischer, 1989) and the change of MRS is also low the overall loss may not be so high. The departure of the value function – as measuring the welfare of the representative household from the standard case – is studied in Gong and Semmler (2006, ch. 8). Results of this study show rather small effects.

²⁰Yet, the background for most of the search and matching models is still smooth and frictionless intertemporal choice and "first best solutions". For a recent position representing this view, see Ljungqvist and Sargent (1998, 2003). For comments on this view, see Blanchard (2003), see also Walsh (2002) who employs search and matching theory to derive the persistence of real effects resulting from monetary policy shocks. For a further evaluation of 'first best solution' under sticky labor markets see Blanchard and Gali (2005).

constraints on the labor market and the previous consumption plan as expressed by (11) becomes invalid due to the improper budget constraint that leads to the improper transition law of capital (10), for deriving the plan. Therefore, the household will be required to behave adaptively and to construct a new consumption plan, which should be derived from the following optimization program:

$$\max_{(c_t^d)} U(c_t^d, n_t) + E_t \left[\sum_{i=1}^{\infty} \beta^i U(c_{t+i}^d, n_{t+i}^s) \right] \quad (18)$$

subject to

$$k_{t+1}^s = (1 - \delta)k_t + f(k_t, n_t, A_t) - c_t^d, \quad (19)$$

$$k_{t+i+1}^s = (1 - \delta)k_{t+i}^s + f(k_{t+i}^s, n_{t+i}^s, A_{t+i}) - c_{t+i}^d; \quad (20)$$

$$i = 1, 2, \dots$$

Note that in this optimization program the only decision variable is about c_t^d and the data includes not only A_t and k_t but also n_t , which is given by either (15) or (16). The actual employment, n_t is here a constraint. We can write the solution in terms of the following equation (see Appendix II for details):

$$c_t^d = G_{c2}(k_t, A_t, n_t). \quad (21)$$

Given this adjusted consumption plan, the product market should be cleared if the household demands the amount $f(k_t, n_t, A_t) - c_t^d$ for investment. Therefore, c_t^d in (21) should also be the realized consumption.²¹

4 Calibration for the U. S. Economy

This section provides an empirical study, for the U. S. economy, using our model as presented in the last section. For our empirical test, we consider two model variants: the benchmark RBC model, as the standard for comparison, and our model with nonclearing labor market. Specifically, we shall call the benchmark model as Model I and the model with nonclearing market as Model II.

²¹We have obtained some comments from participants of conferences, to explore an alternative closure of our model by allowing the condition (ii) in the introduction to hold, namely to let, for a given $\{n_t\}$, the MRS equal the real wage, determining the consumption, c_t . Yet, we think our closure is preferable since we can allow for intertemporal household decisions.

4.1 The Data Generating Process

For the benchmark model, the Model I, we shall first assume that the temporary shock A_t may follow an AR(1) process:

$$A_{t+1} = a_0 + a_1 A_t + \epsilon_{t+1}, \quad (22)$$

where ϵ_t is an independently and identically distributed (*i.i.d.*) innovation: $\epsilon_t \sim N(0, \sigma_\epsilon^2)$. The data generating process thus include (3), (22) as well as

$$c_t = G_{11}A_t + G_{12}k_t + g_1; \quad (23)$$

$$n_t = G_{21}A_t + G_{22}k_t + g_2; \quad (24)$$

$$w_t = \alpha(N/0.3)^{\alpha-1} A_t k_t^{1-\alpha} n_t^{\alpha-1}. \quad (25)$$

Note that here (25) is the wage variation that makes the demand for labor equal to the labor supply n in the standard model; (23) and (24) are the linear approximations to (11) and (12). The coefficients G_{ij} and g_i ($i = 1, 2$ and $j = 1, 2$) are all complicated functions of the model's structural parameters, α , β , among others. They are computed by a numerical algorithm using the linear-quadratic approximation method.²² Given these coefficients and the parameters in equation (22), including σ_ϵ , we can simulate the model to generate stochastically simulated data. These data can then be compared to the sample moments of the observed economy.

To define the data generating process for our model with sticky wages and nonclearing labor market, the Model II, we shall first modify (24) as

$$n_t^s = G_{21}A_t + G_{22}k_t + g_2. \quad (26)$$

On the other hand, the equilibrium in the product market after the adaptive optimization indicates that c_t^d in (21) should be equal to c_t . Therefore, this equation can also be approximated as

$$c_t = G_{31}A_t + G_{32}k_t + G_{33}n_t + g_3. \quad (27)$$

In the Appendix II, we provide the details how to compute the coefficients G_{3j} , $j = 1, 2, 3$, and g_3 .

Next we consider the demand for labor n_t^d as implied in (13) - (14). The following proposition concerns the derivation of n_t^d .

Proposition 2 *When the capital market is cleared, the demand for labor can be expressed as*

$$n_t^d = \begin{cases} (0.3/\bar{N}) (\hat{y}_t/A_t)^{1/\alpha} k_t^{(\alpha-1)/\alpha} & \text{if } \hat{y}_t < (\alpha A_t/w_t)^{\alpha/(1-\alpha)} k_t A_t \\ (\alpha A_t/w_t)^{1/(1-\alpha)} k_t (0.3/\bar{N}) & \text{if } \hat{y}_t \geq (\alpha A_t/w_t)^{\alpha/(1-\alpha)} k_t A_t \end{cases} \quad (28)$$

²²The algorithm that we used here is from Gong and Semmler (2006).

The proof of this proposition is provided in Appendix III. Finally, we assume that

$$\hat{y}_t = y_{t-1} \quad (29)$$

so that the expectation is fully adaptive to the actual output in the last period.²³

Thus, for Model II, the data generating process includes (1), (3), (8), (16), (22) and (26) - (29). Note that here we in this paper only consider the compromising rule as the realization when disequilibrium occurs in the labor market.²⁴

4.2 The Data and the Parameters

The data set used in this section as a sample economy for U. S. is taken from OECD.²⁵ It covers the period from 1960.1 to 2005.2 and will be available upon request. There are altogether 11 parameters in our models: α , γ , a_0 , a_1 , σ_ε , β , δ , θ , μ , ξ and ω . We first specify α at 0.66, which is standard as in Christiano and Eichenbaum (1992). γ is set to 0.0085, which is the average growth rate of GDP in the sample. These two parameters allows us to compute the data series of the temporary shock A_t . With this data series A_t , we estimate the parameters a_0, a_1 and σ_ε . The next three parameters β, δ and θ are also set to the standard value, again from Christiano and Eichenbaum (1992). For the new parameters μ, ξ and ω , we first specify μ at 0.0043, which is the average growth rate of the labor force in the United States. The parameter ξ is set to 0.9131, which is obtained by matching the wage sequence according to equation (1) with the sequence of w_t^* computed by (8) given the other related parameters as specified previously. Finally, ω is set to 0.3293. This is estimated by

$$\omega = \underset{t}{\operatorname{argmin}} \sum [n_t - (\omega n_t^d + (1 - \omega)n_t^s)]$$

which minimal the residual sum of square between actual employment and the model generated employment.²⁶ The estimation is executed by a

²³Of course, one can also consider other forms of expectation. One possibility is to assume expectation to be rational so that it is equal to the steady state of y_t . Indeed, we have also undertaken such empirical study, yet the result is less satisfying.

²⁴Note that here we only use the compromise rule for the determination of employment though implicitly we use the short side rule for the output supply. Empirically, the short side rule seems to be less satisfying than the compromise rule when we study the labor market disequilibrium. See the comparison of these two rules in Gong and Semmler (2003).

²⁵See OECD(2005).

²⁶both of which are detrended by HP-filter before matching.

conventional algorithm using grid search. Table 1 illustrates these parameters:

Table 1: The Parameters used in Calibration

α	0.66	σ_ϵ	0.3984	μ	0.0043
γ	0.0085	β	0.993	ξ	0.9131
a_0	0.7383	δ	0.0209	ω	0.3293
a_1	0.9894	θ	2		

Given the parameters in table 1 we can compute the parameters G_{ij} as stated in the linear decision rules for c_t and n_t of eqs. (23)-(24): We obtain

$$\begin{aligned} c_t &= 0.0420k_t + 22672.5730A_t + g_1 \\ n_t &= -2.3342 \cdot 10^{-9}k_t + 0.0022A_t + g_2. \end{aligned}$$

Given that k is very large the coefficients for k_t are very small.

One can plot the optimal policy reaction, c_t, y_t, ω_t and n_t , responding to the state variable k_t . Presuming that we have $k_t < k^*$, with k^* the steady state value, one would expect optimal consumption, output and wage to be lower than at the steady state, but moving up, and labor effort, n_t , above its steady state but moving down as k_t rises toward the steady k^* .

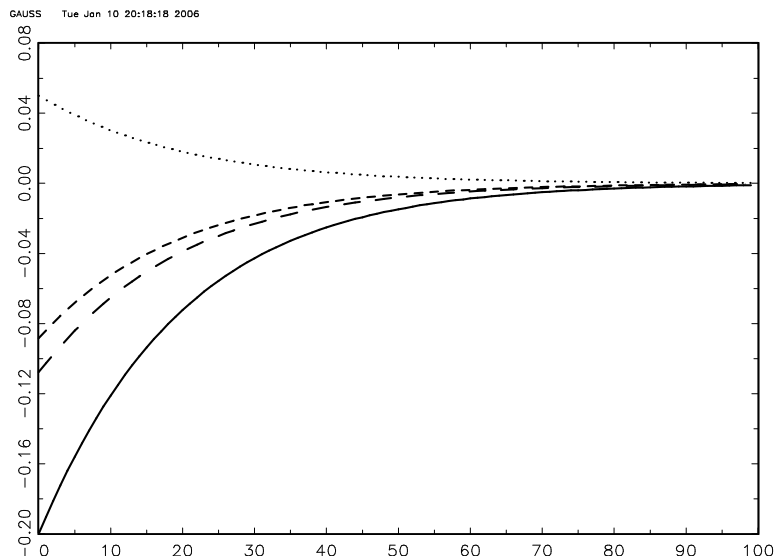


Figure 2: Optimal response of consumption (solid line), output (big dashed line) wage (small dashed line) and labor effort (dotted line) to k_t .

As figure 2 shows one indeed obtains, if $k_t < k^*$ an negative correlation of employment and consumption and a positive correlation of consumption, wage²⁷ and output as capital stock is $k_t < k^*$ but rising. The economic explanation is that with $k_t < k^*$ the marginal product and thus the real interest rate is high, consumption, output and wage are low but saving is high. On the other hand the short fall of capital and its high marginal product makes people not only to postpone consumption, but also leisure, and thus labor effort is high.

What we have explained with respect to a short fall of capital, $k_t < k^*$ of course also holds, if the technology shock is permanent so that the actual k_t has to be down-scaled by an increase in technology trend. Then the same paths for output, consumption and labor effort would arise as in figure 2.²⁸

²⁷Note that we obtain the movement of the wage directly from the equ. (25).

²⁸See also Rotemberg and Woodford (1996) for such an interpretation of a permanent technology shock. They also show in their work that the forecastable movement of the variables in the RBC model is incorrect as compared to the actual data., when for the

Overall, figure 2 shows us the proper cross-correlation that the standard Model I with smooth and frictionless optimizing behavior of the agents and market clearing would predict as a result of a permanent technology shock.

4.3 Calibration

Next we want to calibrate our two model variants I and II. We calibrate what Rotemberg and Woodford (1996) call the overall movements of the variables. We shall first remark that to generate the stationary series as required for the empirical test, we also have to divide the related data series (such as output, capital stock among others) by the permanent shock X_t . We set the initial condition for X_t to be 1,000,000. The same time, we also have to re-scale the wage series after it is divided by Z_t , the permanent shock in productivity. This re-scaling is necessary because we do not exactly know the initial condition of Z_t , which we also set to 1,000,000. We re-scale the wage series in such a way that the average wage series is equal to the average optimum wage series w_t^* as computed by (8).

Table 2 provides our calibration results from 5000 stochastic simulations. All time series are detrended by the HP-filter.

latter the forecastable movements are obtained by a VAR regression.

Table 2: Calibration of the Model Variants
(numbers in parentheses are the corresponding standard error)

	c_t	k_t	n_t	y_t	w_t
Standard Deviations					
Sample Economy	0.0098	0.0050	0.0106	0.0138	0.0109
Model I Economy	0.0046 (0.0005)	0.0024 (0.0004)	0.0036 (0.0003)	0.0096 (0.0010)	0.0061 (0.0006)
Model II Economy	0.0042 (0.0005)	0.0025 (0.0004)	0.0091 (0.002)	0.0102 (0.0009)	0.0032 (0.0005)
Correlation Coefficients					
Sample Economy					
Consumption (c_t)	1.0000				
Capital Stock (k_t)	0.0018	1.0000			
Employment (n_t)	0.7021	0.0130	1.0000		
Output (y_t)	0.9479	0.0001	0.8375	1.0000	
Wage (w_t)	0.2014	0.3182	0.0675	0.2287	1.0000
Model I Economy					
Consumption (c_t)	1.0000 (0.0000)				
Capital Stock (k_t)	0.31593 (0.0643)	1.0000 (0.0000)			
Employment (n_t)	0.8995 (0.0205)	-0.1261 (0.0331)	1.0000 (0.0000)		
Output (y_t)	0.9741 (0.0054)	0.0840 (0.0495)	0.9774 (0.0051)	1.0000 (0.0000)	
Wage (w_t)	0.9932 (0.0011)	0.2050 (0.0584)	0.9440 (0.0122)	0.9923 (0.0016)	1.0000 (0.0000)
Model II Economy					
Consumption (c_t)	1.0000 (0.0000)				
Capital Stock (k_t)	0.2523 (0.0818)	1.0000 (0.0000)			
Employment (n_t)	0.1848 (0.1097)	-0.2198 (0.1103)	1.0000 (0.0000)		
Output (y_t)	0.7983 (0.0411)	-0.1074 (0.0927)	0.7138 (0.0457)	1.0000 (0.0000)	
Wage (w_t)	0.7035 (0.0330)	0.7928 (0.0385)	0.2158 (0.1021)	0.4996 (0.0549)	1.0000 (0.0000)

Note that in this calibration we are here moving on to a comparison of the actual or overall movement of the variables in contrast to the forecastable

movement of the variables as discussed in sect. 4.3.

First we want to remark that the moment statistics from our sample economy are not much different from those in the standard data sets, such as the data set used in Christiano and Eichenbaum (1992), although we have also added the statistics of the wage sequence.²⁹ Secondly, our calibration for the Model I economy replicates the standard RBC model as discussed in the literature. Here we find the excessive smoothness of labor effort. For our time period, 1960.1 to 2005.2, we find 0.37 in the Model I Economy as the ratio of the standard deviation of labor effort to the standard deviation of output. This ratio is roughly 0.77 in the Sample Economy. The problem is somewhat better resolved in our Model II Economy with wage stickiness and nonclearing labor market. There the ratio is approximately 0.89.

Although we have improved on the volatility of labor effort in the Model II economy, we have to point out that the wage sequence in our model still turns out to be excessively smooth. In the sample economy, the ratio is about 0.79 as the standard deviation of wage to the standard deviation of output. This number is however 0.31 in the Model II economy. On the other hand, in the standard model, the Model I economy, the volatility of wage sequence is higher. Here the ratio is about 0.63. In our Model II economy the excessive smoothness of the wage sequence is largely due to the specification that the wage is determined somehow exogenously. Here we have posited that the wage is determined by its own lagged value along with the exogenous factor, the technology A_t .³⁰ It does not depend on those endogenous variables such as output among others. Though this specification does produce a very good match with the sample sequence (see Panel A and Panel B in Figure 3), its volatility will be unavoidably reduced if we do not include an additional shock, as in our calibration, with regard to the wage specification (see Panel C and Panel D in Figure 3).³¹ The standard deviation of this additional shock could in principle be computed by the residual generated from matching the sample sequence of the wage when we estimate ξ .

Given this consideration, we provide an additional calibration for our model II economy, but this time adding another innovation generated by a

²⁹Indeed, it is the inclusion of the wage sequence in our model that makes necessary the reconstruction of our data set.

³⁰Note that the optimum wage w_t^* as a partial determination of wage sequence (see equation 1) only varies with the technology, see equ. (8).

³¹Given the estimated ξ as reported in Table 1, the predicted wage in Figure 3 is expressed by the observed lagged wage and the observed optimum wage, the latter of which is determined by the observed technology via (8). On the other hand, the calibrated wage is explained by the lagged calibrated (or simulated) wage and the observed optimum wage.

disturbance of the wage equation. The equ. (1) can then be written as

$$w_t = \xi w_{t-1} + (1 - \xi)w_t + \nu_t$$

Here ν_t can be regarded as the second shock. When we estimate the ξ in that equation we get some residual which can be regarded as a sample of ν_t and therefore we can compute the standard deviation of ν_t . This is exactly similar to the procedure when we compute the standard deviation of technology shock. Note that we have reported the ξ in table 1.

Table 3 provides the result where we can find the volatility of the wage sequence has greatly improved.

Table 3: Calibration of Model II Economy with Additional Shock
(numbers in parentheses are the corresponding standard error)

	c_t	k_t	n_t	y_t	w_t
Standard Deviations	0.0042 (0.0005)	0.0025 (0.0004)	0.0091 (0.0003)	0.0102 (0.0010)	0.0159 (0.0017)
Correlation Coefficients					
Consumption (c_t)	1.0000 (0.0000)				
Capital Stock (k_t)	0.2571 (0.0814)	1.0000 (0.0000)			
Employment (n_t)	0.1775 (0.1058)	-0.2239 (0.1061)	1.0000 (0.0000)		
Output (y_t)	0.7963 (0.0406)	-0.1062 (0.0891)	0.7111 (0.0453)	1.0000 (0.0000)	
Wage (w_t)	0.1391 (0.1398)	0.1586 (0.1488)	0.0415 (0.1016)	0.0981 (0.1326)	1.0000 (0.0000)

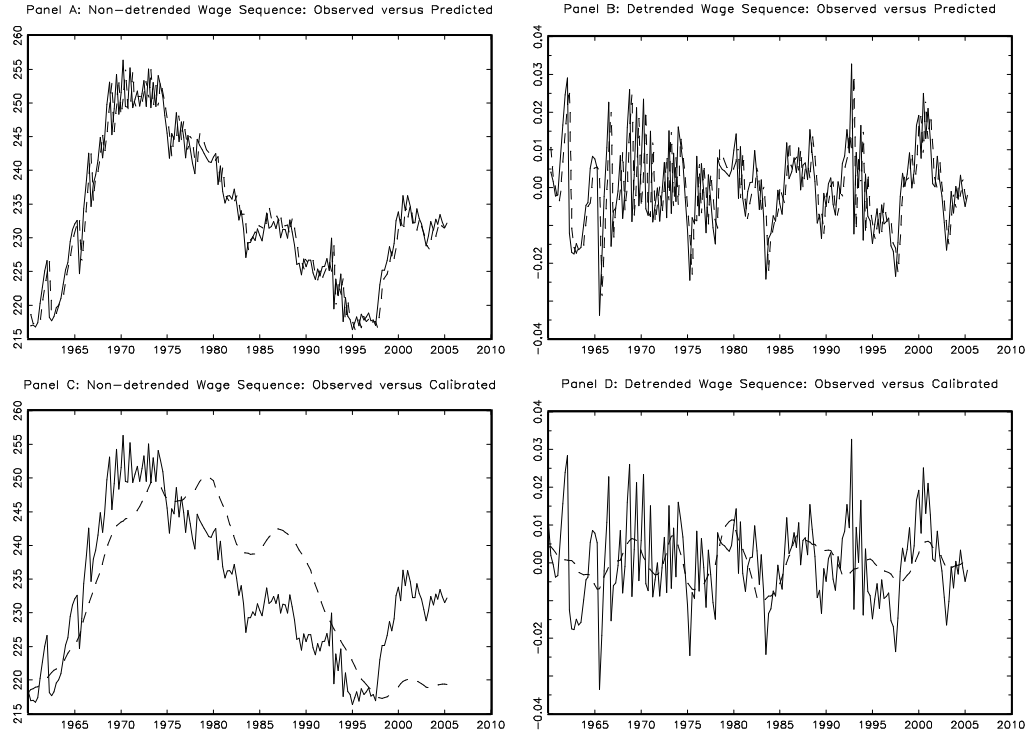


Figure 3: Matching of the wage sequence: Solid line the observed wage, dotted line the predicted or calibrated wage

Next, we can look at the cross-correlations of the macroeconomic variables, for both the one and two shock calibrations. In the Sample Economy, there are no significant correlations among macroeconomic variables except perhaps between output and consumption and between output and labor. Yet, in the Model I economy, we find that almost all economic variables are strongly correlated with each other, except the capital stock. We shall remark here that such an excessive correlation can be expected from other calibration exercises³² of the standard RBC model, but has, to our knowl-

³²See for example Schmidt-Grohe, where all the considered macro variables reveal very strong cross correlations.

edge, not explicitly been discussed in the RBC literature, including the recent study by Schmidt-Grohe (2001). Discussions have often been focused on the correlation with output. It can be seen as a success of our model that all cross-correlations have been significantly weakened in our Model II economy. This holds true for the one shock calibration (Table 2) but especially for the model with a second shock, see Table 3, resembling more the cross-correlations in the actual economy.

Finally by addressing the technology puzzle we shall investigate the temporary effect of the technology shock on labor effort. Table 4 reports the cross correlation of temporary shock A_t from our 5000 thousand stochastic simulation. As one can find there, the two models predict rather different correlations. In the Model I (RBC) Economy, technology A_t exerts temporary effects not only on consumption, wage and output, but also on employment, which are all significantly positive. Yet, in our Model II Economy with sticky wages and nonclearing labor market, we find that the correlation with employment is no longer significant. This is consistent with the widely discussed recent finding that technology has near-zero (or even negative) effect on employment.³³ In the context of our model this result is obtained because the product market is constrained, as posited in proposition 2, and therefore the technology shock is likely not to increase employment.

Table 4: Cross Correlations of Variables with the Technology Shock

	c_t	k_t	n_t	y_t	w_t
Model I Economy	0.9600 (0.005)	0.0401 (0.0453)	0.9858 (0.0033)	0.9990 (0.0002)	0.9860 (0.0028)
Model II Economy (without second shock)	0.9404 (0.0109)	-0.0724 (0.0609)	0.1763 (0.1016)	0.8107 (0.0363)	0.4317 (0.0238)
Model II Economy (with second shock)	0.9403 (0.0109)	-0.0730 (0.0612)	0.1773 (0.1023)	0.8107 (0.0366)	0.0836 (0.1352)

5 Some Conclusions

The benchmark RBC model has difficulties to explain the performance of the labor market. These difficulties are likely to be caused by the structure of the competitive general equilibrium model smoothly adjusting to the three marginal conditions as stated in sect. 1 of this paper. This modelling structure may restrict its usefulness to the real world, which perhaps is better represented by a model with sticky wage and nonclearing labor market. In

³³See Francis and Ramey (2003).

this paper, we present a dynamic general disequilibrium model of RBC type where agents are adaptively optimizing when facing constraints. Calibration for the U. S. economy shows that such a model variant will produce a higher volatility in employment, and thus fits the data better than the benchmark model. Moreover, it improves on the more reasonable cross correlation of wages, of the macroeconomic variables. Finally, it improves on the technology puzzle. Our result is consistent with a class of models along the line of New Keynesian tradition of wage stickiness. Yet, in contrast to the latter, we, however, allow for nonclearing labor market in addition to wage stickiness. This approach may help to treat labor market and business cycle puzzles coherently within a single model of dynamic optimization. It may also, as shown in Ernst et al. (2006) permit for a more consistent evaluation of labor market reforms as currently undertaken in EU-countries.³⁴

³⁴For a more detailed study of the effects of labor market reforms in the context of a general disequilibrium model of the above type, see Ernst et al. (2006).

6 Appendix

6.1 Appendix I: The Optimum Wage Rate (Proposition 1)

Let $X_t = Z_t L_t$, with Z_t to be the permanent shock resulting purely from productivity growth, and L_t from population growth. We shall assume that L_t has a constant growth rate μ and hence Z_t follows the growth rate $(\gamma - \mu)$. The production function can be written as $Y_t = A_t Z_t^\alpha K_t^{1-\alpha} H_t^\alpha$, where H_t equals $N_t L_t$ and can be regarded as total labor hours. We thus obtain the following first-order condition regarding the demand for hours:

$$W_t^* = \alpha A_{t+i} (Z_{t+i})^\alpha (K_{t+i})^{1-\alpha} (H_{t+i})^{\alpha-1},$$

where W_t^* is the optimum wage rate without detrending. Diving both sides by Z_{t+i} , we find that (6) can be written as

$$w_t^* = \alpha (\bar{N}/0.3)^{\alpha-1} A_{t+i} k_{t+i}^{1-\alpha} n_{t+i}^{\alpha-1}.$$

This equation is equivalent to (6) and allows us to derive

$$n_{t+i} = k_{t+i} (\eta A_{t+i} / w_t^*)^{1/(1-\alpha)} \quad (30)$$

where $\eta \equiv \alpha (\bar{N}/0.3)^{\alpha-1}$. Substitute (30) into (4) and (5), our dynamic optimization model can thus be expressed as

$$\max_{w_t^*, \{c_{t+i}\}_{i=0}^{\infty}} E_t \left[\sum_{i=0}^{\infty} \tilde{\beta}^i U(c_{t+i}, n(k_{t+i}, A_{t+i}, w_t^*)) \right] \quad (31)$$

subject to

$$k_{t+i+1} = \frac{1}{1+\gamma} [(1-\delta)k_{t+i} + f(k_{t+i}, n(k_{t+i}, A_{t+i}, w_t^*), A_{t+i}) - c_{t+i}], \quad (32)$$

where $\tilde{\beta} = \xi\beta$ and $n(\cdot)$ is implied by (30). Note that here

$$\begin{aligned} f(k_{t+i}, n(k_{t+i}, A_{t+i}, w_t^*), A_{t+i}) &= (\bar{N}/0.3)^\alpha A_{t+i} k_{t+i}^{1-\alpha} \left[k_{t+i} (\eta A_{t+i} / w_t^*)^{\frac{1}{1-\alpha}} \right]^\alpha \\ &= (\alpha / w_t^*)^{\alpha/(1-\alpha)} (A_{t+i})^{1/(1-\alpha)} k_{t+i}, \end{aligned}$$

while

$$U(c_{t+i}, n(k_{t+i}, A_{t+i}, w_t^*)) = \ln c_{t+i} + \theta \ln \left[1 - k_{t+i} (\eta A_{t+i} / w_t^*)^{1/(1-\alpha)} \right]$$

To derive the first-order condition for the problem (31) - (32), we set the Lagrange as

$$L = E_t \sum_{i=0}^{\infty} \tilde{\beta}^i \left\{ \ln c_{t+i} + \theta \ln \left[1 - k_{t+i} (\eta A_{t+i}/w_t^*)^{1/(1-\alpha)} \right] \right\} - E_t \sum_{i=0}^{\infty} \tilde{\beta}^{i+1} \lambda_{t+i+1} \left\{ k_{t+i+1} - \frac{1}{1+\gamma} \left[(1-\delta)k_{t+i} + \left(\frac{\alpha}{w_t^*} \right)^{\frac{\alpha}{1-\alpha}} (A_{t+i})^{\frac{1}{1-\alpha}} k_{t+i} - c_{t+i} \right] \right\}.$$

Taking the partial derivatives with respect to w_t^* , we obtain

$$E_t \sum_{i=0}^{\infty} \tilde{\beta}^i \left[\frac{\theta k_{t+i} (\eta A_{t+i})^{1/(1-\alpha)} (w_t^*)^{\frac{-2+\alpha}{1-\alpha}}}{(1-\alpha)(1-n_{t+i})} \right] + E_t \sum_{i=0}^{\infty} \left[\frac{-\alpha \tilde{\beta}^{i+1} \lambda_{t+i+1} (\alpha)^{\alpha/(1-\alpha)}}{(1+\gamma)(1-\alpha)} (A_{t+i})^{1/(1-\alpha)} w_t^{*-1/(1-\alpha)} k_{t+i} \right] = 0$$

By re-organizing while using the assumption $E_t A_{t+i} = A_t$, the above equation can further be written as

$$E_t \sum_{i=0}^{\infty} \tilde{\beta}^i \left[\frac{\theta k_{t+i} (\eta)^{1/(1-\alpha)}}{(1-n_{t+i})} \right] = E_t \sum_{i=0}^{\infty} \tilde{\beta}^i \frac{\alpha^{1/(1-\alpha)} \tilde{\beta} \lambda_{t+i+1}}{(1+\gamma)} w_t^* k_{t+i}. \quad (33)$$

From (30) and the assumption $E_t A_{t+i} = A_t$, we find that $E_t k_{t+i} (\eta)^{1/(1-\alpha)}$ can also be expressed as $(w_t^*/A_t)^{1/(1-\alpha)} E_t n_{t+i}$. This implies that (33) can be written as

$$w_t^* = \frac{(w_t^*/A_t)^{1/(1-\alpha)} E_t \sum_{i=0}^{\infty} \tilde{\beta}^i \frac{n_{t+i}}{1-n_{t+i}}}{E_t \sum_{i=0}^{\infty} \tilde{\beta}^i \left[\alpha^{1/(1-\alpha)} \frac{\tilde{\beta} \lambda_{t+i+1}}{(1+\gamma)} k_{t+i} \right]}. \quad (34)$$

Next, take the partial derivatives with respect to k_{t+i} :

$$\frac{-\theta (\eta A_{t+i}/w_t^*)^{1/(1-\alpha)}}{1-n_{t+i}} + \frac{\tilde{\beta} E_t \lambda_{t+i+1}}{1+\gamma} \left[(1-\delta) + (\alpha/w_t^*)^{\alpha/(1-\alpha)} (A_{t+i})^{1/(1-\alpha)} \right] = 0.$$

This allows us to obtain

$$\frac{\tilde{\beta} E_t \lambda_{t+i+1}}{1+\gamma} k_{t+i} = \frac{\theta (\eta A_{t+i}/w_t^*)^{1/(1-\alpha)} k_{t+i}}{(1-n_{t+i}) \left[(1-\delta) + (\alpha/w_t^*)^{\alpha/(1-\alpha)} (A_{t+i})^{1/(1-\alpha)} \right]} \quad (35)$$

Again using equation (30) and the assumption $E_t A_{t+i} = A_t$, we find from (35) that

$$E_t \left[\frac{\tilde{\beta} \lambda_{t+i+1}}{(1+\gamma)} k_{t+i} \right] = \frac{\theta}{(1-\delta) + (\alpha/w_t^*)^{\alpha/(1-\alpha)} (A_t)^{1/(1-\alpha)}} E_t \left[\frac{n_{t+i}}{1-n_{t+i}} \right]. \quad (36)$$

Substituting (36) into (34), we obtain

$$\begin{aligned} w_t^* &= \frac{(w_t^*/A_t)^{1/(1-\alpha)} E_t \sum_{i=0}^{\infty} \tilde{\beta}^i \frac{n_{t+i}}{1-n_{t+i}}}{\frac{\theta \alpha^{1/(1-\alpha)}}{(1-\delta) + (\alpha/w_t^*)^{\alpha/(1-\alpha)} (A_t)^{1/(1-\alpha)}} E_t \sum_{i=0}^{\infty} \tilde{\beta}^i \frac{n_{t+i}}{1-n_{t+i}}} \\ &= \frac{1}{\theta \alpha^{1/(1-\alpha)}} (w_t^*/A_t)^{1/(1-\alpha)} [(1-\delta) + (\alpha/w_t^*)^{\alpha/(1-\alpha)} (A_t)^{1/(1-\alpha)}]. \end{aligned}$$

Solving this nonlinear function for w_t^* , we obtain (8) as expressed in Proposition 1.

6.2 Appendix II: Adaptive Optimization and Consumption Decision

For the problem (18) - (20), we define the Lagrangian:

$$\begin{aligned} L &= E_t \left\{ [\log c_t^d + \theta \log(1 - n_t)] + \right. \\ &\quad \left. \lambda_{t+1} \left[k_{t+1}^s - \frac{1}{1+\gamma} [(1-\delta)k_t^s + f(k_t^s, n_t, A_t) - c_t^d] \right] \right\} + \\ &\quad E_t \left\{ \sum_{i=1}^{\infty} \beta^i [\log(c_{t+i}^d) + \theta \log(1 - n_{t+i}^s)] + \right. \\ &\quad \left. \beta^i \lambda_{t+i+1} \left[k_{t+i+1}^s - \frac{1}{1+\gamma} [(1-\delta)k_{t+i}^s + f(k_{t+i}^s, n_{t+i}^s, A_{t+i}) - c_{t+i}^d] \right] \right\}. \end{aligned}$$

Since the decision is only about c_t^d , we thus take the partial derivatives of L with respect to c_t^d , k_{t+1}^s and λ_{t+1} . This gives us the following first-order condition:

$$\frac{1}{c_t^d} - \frac{\lambda_t}{1+\gamma} = 0, \quad (37)$$

$$\frac{\beta}{1+\gamma} E_t \left\{ \lambda_{t+1} \left[(1-\delta) + (1-\alpha)A_{t+1} (k_{t+1}^s)^{-\alpha} (n_{t+1}^s \bar{N}/0.3)^\alpha \right] \right\} = \lambda_t, \quad (38)$$

$$k_{t+1}^s = \frac{1}{1+\gamma} [(1-\delta)k_t^s + A_t (k_t^s)^{1-\alpha} (n_t \bar{N}/0.3)^\alpha - c_t^d]. \quad (39)$$

Recall that in deriving the decision rules as expressed in (23) and (24) we have postulated

$$\lambda_{t+1} = Hk_{t+1}^s + QA_{t+1} + h, \quad (40)$$

$$n_{t+1}^s = G_{21}k_{t+1}^s + G_{22}A_{t+1} + g_2, \quad (41)$$

where H, Q, h, G_{21}, G_{22} and g_2 have all been resolved previously in the household optimization program. We therefore obtain from (40) and (41)

$$E_t\lambda_{t+1} = Hk_{t+1}^s + Q(a_0 + a_1A_t) + h, \quad (42)$$

$$E_t n_{t+1}^s = G_2k_{t+1}^s + D_2(a_0 + a_1A_t) + g_2. \quad (43)$$

Our next step is to linearize (37) - (39) around the steady states. Suppose they can be written as

$$F_{c1}c_t + F_{c2}\lambda_t + f_c = 0, \quad (44)$$

$$F_{k1}E_t\lambda_{t+1} + F_{k2}E_tA_{t+1} + F_{k3}k_{t+1}^s + F_{k4}E_t n_{t+1}^s + f_k = \lambda_t, \quad (45)$$

$$k_{t+1}^s = Ak_t + WA_t + C_1c_t^d + C_2n_t + b. \quad (46)$$

Expressing $E_t\lambda_{t+1}, E_t n_{t+1}^s$ and E_tA_{t+1} in terms of (42), (43) and $a_0 + a_1A_t$ respectively, we obtain from (45)

$$\kappa_1k_{t+1}^s + \kappa_2A_t + \kappa_0 = \lambda_t, \quad (47)$$

where, in particular,

$$\kappa_0 = F_{k1}(Qa_0 + h) + F_{k2}a_0 + F_{k4}(G_{22}a_0 + g_2) + f_k,$$

$$\kappa_1 = F_{k1}H + F_{k3} + F_{k4}G_{21},$$

$$\kappa_2 = F_{k1}Qa_1 + F_{k2}a_1 + F_{k4}G_{22}a_1.$$

Using (44) to express λ_t in (47), we further obtain

$$\kappa_1k_{t+1}^s + \kappa_2A_t + \kappa_0 = -\frac{F_{c1}}{F_{c2}}c_t^d - \frac{f_c}{F_{c2}}, \quad (48)$$

which is equivalent to

$$k_{t+1}^s = -\frac{\kappa_2}{\kappa_1}A_t - \frac{F_{c1}}{F_{c2}\kappa_1}c_t^d - \frac{\kappa_0}{\kappa_1} - \frac{f_c}{F_{c2}\kappa_1}. \quad (49)$$

Comparing the right side of (46) and (49) will allow us to solve c_t^d as

$$c_t^d = -\left(\frac{F_{c1}}{F_{c2}\kappa_1} + C_1\right)^{-1} \left[Ak_t + \left(\frac{\kappa_2}{\kappa_1} + W\right) A_t + C_2n_t + \left(b + \frac{\kappa_0}{\kappa_1} + \frac{f_c}{F_{c2}\kappa_1}\right) \right].$$

6.3 Appendix III: The Firm's Demand for Labor (Proposition 2)

Let us first consider the firm's willingness to supply y_t^* under the condition that the rental rate of capital r_t clears the capital market while the wage rate w_t is given. In this case, the firm's optimization problem can be expressed as

$$\max y_t^* - r_t k_t^d - w_t N_t^d$$

subject to

$$y_t^* = A_t (k_t^d)^{1-\alpha} (N_t^d)^\alpha.$$

The first-order condition tells us that

$$(1 - \alpha)A_t (k_t^d)^{-\alpha} (N_t^d)^\alpha = r_t, \quad (50)$$

$$\alpha A_t (k_t^d)^{1-\alpha} (N_t^d)^{\alpha-1} = w_t, \quad (51)$$

from which we can further obtain

$$\frac{r_t}{w_t} = \left(\frac{1 - \alpha}{\alpha} \right) \frac{N_t^d}{k_t^d}. \quad (52)$$

Since the rental rate of capital r_t is assumed to clear the capital market, we can thus replace k_t^d in the above equations by k_t . Since w_t is given, and therefore the demand for labor can be derived from (51):

$$n_t^d = \frac{0.3}{\bar{N}} \left(\frac{\alpha A_t}{w_t} \right)^{\frac{1}{1-\alpha}} k_t$$

Note that we have used the definition $N_t = n_t(\bar{N}/0.3)$ to express n_t^d in the above equation. We shall regard this labor demand as the desired demand on the basis that the firm's willingness supply y_t^* can be executed (or $\hat{y}_t > y_t^*$). This is indeed the second equation in (28). Given this n_t^d , the firm's willingness to supply y_t^* can be expressed as

$$\begin{aligned} y_t^* &= A_t k_t^{1-\alpha} (n_t^d \bar{N} / 0.3)^\alpha \\ &= A_t k_t \left(\frac{\alpha A_t}{w_t} \right)^{\frac{\alpha}{1-\alpha}}. \end{aligned} \quad (53)$$

Next, we consider the case that the firm's supply is constrained by the expected demand \hat{y}_t , or in other words, $\hat{y}_t < y_t^*$ where y_t^* is given by (53). In this case, the firm's profit maximization problem is equivalent to the following minimization problem:

$$\min r_t k_t^d + w_t N_t^d$$

subject to

$$\hat{y}_t = A_t (k_t^d)^{1-\alpha} (N_t^d)^\alpha. \quad (54)$$

The first-order condition will still allow us to obtain (52). Using equation (54) and (52), we obtain the demand for capital k_t^d and labor N_t^d as

$$k_t^d = \left(\frac{\hat{y}_t}{A_t} \right) \left[\left(\frac{w_t}{r_t} \right) \left(\frac{1-\alpha}{\alpha} \right) \right]^\alpha; \quad (55)$$

$$N_t^d = \left(\frac{\hat{y}_t}{A_t} \right) \left[\left(\frac{w_t}{r_t} \right) \left(\frac{\alpha}{1-\alpha} \right) \right]^{1-\alpha}. \quad (56)$$

Since the real rental of capital r_t will clear the capital market, we can replace k_t^d in (55) by k_t . Substituting it into (56) for explaining r_t , we obtain

$$n_t^d = \left(\frac{0.3}{\bar{N}} \right) \left(\frac{\hat{y}_t}{A_t} \right)^{1/\alpha} \left(\frac{1}{k_t} \right)^{(1-\alpha)/\alpha}.$$

This is the first equation in (28).

References

- [1] Benassy, J. P. (1975): "Neo-Keynesian Disequilibrium Theory in a Monetary Economy," *Review of Economic Studies*, Vol. 42, 502 - 523.
- [2] Benassy, J.-P. (1984): *The Economics of Market Disequilibrium*, New York: Academic Press.
- [3] Benassy, J.-P. (1995): "Money and Wage Contract in an Optimizing Model of the Business Cycle," *Journal of Monetary Economics*, Vol. 35: 303 - 315.
- [4] Benassy, J.-P. (2003): *The Macroeconomics of Imperfect Competition and Nonclearing Markets*, Cambridge: MIT Press.
- [5] Blanchard, O. and S. Fischer (1989): "Lectures on Macroeconomics", Cambridge, MIT-Press.
- [6] Blanchard, O. (2003): "Comments on Ljungqvist and Sargent" in *Knowledge, Information, And Expectations in Modern Macroeconomics*, edited by P. Aghion, R. Frydman, J. Stiglitz and M. Woodford, Princeton University Press, Princeton: 351 - 356.
- [7] Blanchard, O. and J. Gali (2005): "Real Wage Rigidities and the New Keynesian Model", mimeo, MIT.
- [8] Burnside, A. C., M. S. Eichenbaum and S. T. Rebelo (1993): "Labor Hoarding and the Business Cycle," *Journal of Political Economy*, Vol. 101, 245 - 273.
- [9] Calvo, G. A. (1983): "Staggered Contracts in a Utility Maximization Framework," *Journal of Monetary Economics*, Vol. 12: 383 - 398.
- [10] Christiano, L. J. and M. Eichenbaum (1992): "Current Real Business Cycle Theories and Aggregate Labor Market Fluctuation," *American Economic Review*, June: 431-472.
- [11] Christiano, L. J., M. Eichenbaum and C. Evans (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy", *Journal of Political Economy*, vol. 113, no. 1: 1-45.
- [12] Christophel, K. and T. Linzart (2005): "The Role of Real Wage Rigidity and Labor Market Frictions for Unemployment and Inflation Dynamics", ECB Working paper Series, no. 556.

- [13] Danthine, J. P. and J. B. Donaldson (1990): "Efficiency Wages and the Business Cycle Puzzle," *European Economic Review*, Vol. 34, 1275 - 1301.
- [14] Danthine, J. P. and J. B. Donaldson (1995): "Non-Walrian Economies," in T. F. Cooley edited, *Frontiers of Business Cycle Research*, Princeton: Princeton University Press.
- [15] Erceg, C. J., D. W. Henderson and A. T. Levin (2000): "Optimal Monetary Policy with Staggered Wage and Price Contracts," *Journal of Monetary Economics*, Vol. 46: 281 - 313.
- [16] Ernst, E., G. Gong, W. Semmler and L. Bukuviciute (2006), "Quantifying the Impact of Structural Reforms, European Central Bank, working paper, forthcoming.
- [17] Francis, N. and V.A. Ramey (2003): "The Source of Historical Economic Fluctuations: An Analysis using Long-Run Restrictions", University of California, San Diego.
- [18] Gali, J. (1999): "Technology, Employment, and the Business Cycles: Do Technology Shocks Explain Aggregate Fluctuations?" *American Economic Review*, Vol. 89: 249 - 271.
- [19] Gali, J., M. Gertler and J.D. Lopez-Salido (2003): "Markups, Gaps, and the Welfare Costs of Business Fluctuations", mimeo, Dept. of Economics, New York University.
- [20] Gali, J. J.D. Lopez-Salido and J. Valls (2003): "Rule of Thumb Consumers and the Design of Interest Rate Rules", mimeo, Universitat Pompeu Fabra.
- [21] Gali, J. and O. Blanchard (2005), "Real Wage Rigidities and the New Keynesian Model", mimeo, MIT.
- [22] Gong, G. and W. Semmler (2003): "Business Cycles, Wage Stickiness and Nonclearing Labor Market," Paper presented in the 9th International Conference of the Society for Computational Economics and Finance, Seattle, U.S.A.
- [23] Gong, G. and W. Semmler (2006): "Stochastic Dynamic Macroeconomics - Theory and Empirical Evidence", Oxford/New York, Oxford University Press.

- [24] Hall, R.E. (2005):: “Employment Fluctuations with Equilibrium Wage Stickiness”. *The American Economic Review*, vol. 95, no. 1: 50-66.
- [25] Hansen, G. H.(1985): “Indivisible Labor and BusinessCycle,” *Journal of Monetary Economics*, Vol.16, 309 - 27.
- [26] King, R. G. and S. T. Rebelo (1999): “Resuscitating Real Business Cycles,” in *Handbook of Macroeconomics, Volume I*, edited by J. B. Taylor and M. Woodford, Amsterdam/New York, Elsevier Science.
- [27] King, R. G. and A. L. Wolman (1999): “What should the Monetary Authority do when Prices are sticky?” in *Monetary Policy Rules*, edited by J. Taylor, Chicago: The University of Chicago Press.
- [28] Ljungqvist, L. and T. Sargent (1998): “The European Unemployment Dilemma,” *Journal of Political Economy*, Vol. 106, 514 - 550.
- [29] Ljungqvist, L. and T. Sargent (2003): “European Unemployment: From a Worker’s Perspective,” in *Knowledge, Information, and Expectations in Modern Macroeconomics*, edited by P. Aghion, R. Frydman, J. Stiglitz and M. Woodford, Princeton University Press, Princeton, 326 - 350.
- [30] Malinvaud, E. (1994): ”Diagnosing Unemployment”, Cambridge, Cambridge University Press.
- [31] Mankiw, N. G. (1985): “Small Menu Cost and Larger Business Cycles, a Macroeconomic Model of Monopoly,” *Journal of Quaterly Economics*, Vol. 100, 529-539.
- [32] Mankiw, N.G., J.J. Rotemberg and L. Summers (1985): “Intertemporal Substitution in Modern Macroeconomics”, *The Quarterly Jurnal of Economics*, vol. 100, no. 1: 225-251.
- [33] Merz, M. (1999): “Heterogenous Job-Matches and the Cyclical Behavior of Labor Turnover,” *Journal of Monetary Economics*, Vol. 43: 91-124
- [34] Meyers, R.J. (1968) ”What Can We Learn from European Experience, in Unemployment and the American Economy?”, ed. by A.M. Ross, New York: John Wiley & Sons, Inc.
- [35] OECD (2005): *OECD Economic Outlook*, OECD
- [36] Phelps, E. (1997): *Rewarding Work*, Cambridge: MIT-Press

- [37] Portier, F. and L. A. Puch (2004): “The Welfare Cost of Business Cycles in an Economy with Nonclearing Markets,” Working paper, University of Toulouse.
- [38] Rotemberg, J. (1982): “Sticky Prices in the United States,” *Journal of Political Economy*, Vol. 90: 1187-1211.
- [39] Rotemberg, J. and M. Woodford (1995): “Dynamic General Equilibrium Models with Imperfectly Competitive Product Markets,” in *Frontiers of Business Cycle Research*, edited by T. F. Cooley, Princeton: Princeton University Press.
- [40] Rotemberg, J., J. and M. Woodford (1996): “Real Business-Cycle Models and the Forecastable Movements in Output”, *The American Economic Review*, vol. 86, no. 1: 71-89.
- [41] Rotemberg, J. and M. Woodford (1999): “Interest Rate Rules in an Estimated Sticky Price Model,” in *Monetary Policy Rules*, edited by J. Taylor, Chicago: the University of Chicago Press.
- [42] Shimer, R. (2005): “The Cyclical Behavior of Equilibrium Unemployment and Vacancies”, *The American Economic Review*, vol. 95, no. 1: 25-49.
- [43] Smets, F. and R. Wouters (2004): “Comparing Shocks and Frictions in US and Euro Area Business Cycles. A Bayesian DSGE Approach,” European Central Bank, Working paper no. 391, September.
- [44] Schmidt-Grohe, S. (2001): “Endogenous Business Cycles and the Dynamics of Output, Hours and Consumption,” *American Economic Review*, Vol. 90: 1136-1159.
- [45] Solow, R. (1979): “Another Possible Source of Wage Stickiness,” *Journal of Macroeconomics*, Vol. 1: 79-82
- [46] Summers, L. H. (1986): ”Some Skeptical Observations on Real Business Cycles Theory”, *Federal Reserve Bank of Minneapolis Quarterly Review*, Vol. 10, p.23-27.
- [47] Taylor, J. B. (1980): “Aggregate Dynamics and Staggered Contracts,” *Journal of Political Economy*, Vol. 88: 1 - 24.
- [48] Uhlig, H. (2004): ”Macroeconomics and Asset Markets: Some Mutual Implications, Humboldt University, Working paper.

- [49] Uhlig, H. and Y. Xu (1996): “Effort and the Cycle: Cyclical Implications of Efficiency Wages,” mimeo, Tilburg University.
- [50] Walsh, L. E. (2002): “Labor Market Search and Monetary Shocks,” Working Paper, University of California, Santa Cruz.
- [51] Woodford, M. (2003): Interest Rate and Prices, Princeton University Press.
- [52] Zbaracki, M. J., M. Ritson, D. Levy, S. Dutta and M. Bergen (2000): “The Managerial and Customer Costs of Price Adjustment: Direct Evidence from Industrial Markets,” Manuscript, Wharton School, University of Pennsylvania