Journal of Economic Behavior & Organization
Vol. 57 (2005) 430–447

JOURNAL OF Economic Behavior & Organization

www.elsevier.com/locate/econbase

Economic growth and global warming: A model of multiple equilibria and thresholds

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Available online 23 May 2005

Abstract

We presume a simple endogenous growth model where global warming affects economic growth and analyze the dynamics of the competitive economy and of the social optimum. Our regulatory instrument is an emission tax rate. We demonstrate that for certain values of the emission tax ratio the competitive economy exhibits multiple equilibria and a threshold may exist that separates the domains of attraction for the growth paths. There exist paths to high growth rates and low temperature and low growth rates and high temperature. For the planner's problem the long-run equilibrium is unique unless the damage of global warming is very small.

JEL classification: C61; O13; Q30

Keywords: Thresholds; Multiple equilibria; Global warming; Endogenous growth

1. Introduction

According to the Intergovernmental Panel on Climate Change (IPCC) it is certain that the global average surface temperature of the earth has increased since 1861. Over the 20th century the temperature has increased by about 0.6 °C, and it is very likely[1] that the 1990s

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[1] Very likely (likely) means that the level of confidence is between 90 and 99 (66 and 90) percent.

0167-2681/$ – see front matter
doi:10.1016/j.jebo.2005.04.007

was the warmest decade since 1861 (IPCC, 2001, 26). Further, the rise in the average global surface temperature has been accompanied by an increase in heavy and extreme weather events, primarily in the Northern Hemisphere.[2] In general, changes in the climate may occur as a result of both internal variability within the climate system and as a result of external factors where the latter can be natural or anthropogenic. However, there is strong evidence that most of the climate change observed over the last 50 years is the result of human activities. Especially, the emission of greenhouse gases (GHGs), like carbon dioxide (CO_2) or methane (CH_4) just to mention two, are considered as the cause for global warming and these emissions continue to alter the atmosphere in ways that are expected to affect the climate.

In the economics literature, the effect of global warming is modelled mostly using integrated assessment models. These are computable general equilibrium models in which stylized climatic interrelations are taken into account by a climate subsystem incorporated in the model. Examples for this type of models are CETA (see Peck and Teisberg, 1992), FUND (see Tol, 1999), RICE and DICE (see Nordhaus and Boyer, 2000), WIAGEM (see Kemfert, 2001) or DART (see Deke et al., 2001). The goal of these studies, then, is to evaluate different abatement scenarios as to economic welfare and as to their effects on GHG emissions.[3] In analyzing implications of climate policies these models often neglect transition dynamics; instead it is assumed that the economy is in some steady state. Further, the growth rate of the economy is taken as exogenously given, and feedback effects of lower GHG concentrations in the atmosphere on economic growth are frequently neglected.

There is another important research direction, undertaken by scientists, that studies the impact of greenhouse gas emissions on climate change through the change of ocean circulations and that can also be related to our study. The papers by Deutsch et al. (2002) and Keller et al. (2000), for example, describe how the gulf stream and the North Atlantic current, part of the North Atlantic thermohaline circulation (THC), transport a large amount of heat from warm regions to Europe. As those papers show, due to the heating up of surface water, the currents could suddenly change and trigger a change in temperature. The THC collapse and the sudden cooling of regions would most likely have a strong economic impact on Europe and Africa. An event like this would have an impact on the climate in these regions and would also likely affect economic growth. Further results on THC mechanisms are given in Broecker (1997). In our modeling of the interaction of economic growth and climate change, we will leave aside this possible event, although it might exacerbate some of the results obtained in our paper.

The overall goal of our paper is different from the above studies. Our primary goal is not to evaluate different abatement policies as to their welfare effects, as the first type of studies do, nor modeling exacerbating events for global warming. We want to study, in the context of a simple endogenous growth model, the long-run effects of the interaction of global warming and economic growth and, in particular, the transitions dynamics that might occur with global warming. More specifically, we want to study the question of whether

[2] More climate changes are documented in IPCC (2001, 34).

[3] However, the results partly are very sensitive with respect to the assumptions made. See for example Popp (2003) who shows that the outcome in Nordhaus and Boyer (2000) changes when technical change is taken into account.

there possibly exist multiple equilibria and thresholds that separate basins of attraction for optimal paths to some long-run steady state. In order to study such a problem, we take a basic endogenous growth model as the starting point and integrate a simple climate model.

We should also like to point out some limitations of our model. We do not purport to use an elaborate up-to-date model describing the process of global warming. Instead, we confine our analysis to a basic energy balance model (EBM) that allows for feedback effects. As to those feedback effects we posit that the albedo of the earth is affected by increases in GHGs. Other possible feedback effects, such as a change in the flux ratio (see Section 1) for example, are neglected. We are aware that this limits in a way the relevance of our model. However, the qualitative outcome and the message of our paper remain the same with our simplified specification.

The remainder of the paper is organized as follows. In Section 2 we start with a description of facts concerning GHG emissions and changes in average surface temperature of the earth using a simple energy balance model. Section 3 introduces the competitive version of our growth model. In this section we first present the structure of our model, analyze its dynamics and, then, study the question of how robust these results are and perform some comparative statics. Section 4 presents and analyzes the social planner's problem, and Section 5, finally, concludes the paper.

2. GHG emissions and the change in average global surface temperature

We begin with a description of current state of the knowledge concerning GHG emissions and the change in global average surface temperature. The simplest method of considering the climate system of the earth is in terms of its global energy balance, which is done by so-called energy balance models (EBM). According to an EBM the change in the average surface temperature on earth is described by[4]

$$\frac{\mathrm{d}T(t)}{\mathrm{d}t}c_{\mathrm{h}} \equiv \dot{T}(t)c_{\mathrm{h}} = S_{\mathrm{E}} - H(t) - F_{\mathrm{N}}(t), \quad T(0) = T_0, \tag{1}$$

with $T(t)$ the average global surface temperature measured in Kelvin[5] (K), c_{h} the heat capacity[6] of the earth with dimension $\mathrm{J\,m^{-2}\,K^{-1}}$ (Joule per square meter per Kelvin)[7] which is considered a constant parameter, S_{E} is the solar input, $H(t)$ is the non-radiative energy flow, and $F_{\mathrm{N}}(t) = F\uparrow(t) - F\downarrow(t)$ is the difference between the outgoing radiative flux and the incoming radiative flux. S_{E}, $H(t)$ and $F_{\mathrm{N}}(t)$ have the dimension Watt per square meter ($\mathrm{W\,m^{-2}}$). t is the time argument which will be omitted in the following as long as no

[4] This section follows Roedel (2001, chapters 10.2.1 and 1) and Henderson-Sellers and McGuffie (1987, chapters 1.4 and 2.4). See also Gassmann (1992). A more complex presentation can be found in Harvey (2000).

[5] 273 K are 0 °C.

[6] The heat capacity is the amount of heat that needs to be added per square meter of horizontal area to raise the surface temperature of the reservoir by 1 K.

[7] 1 W is 1 J/s.

ambiguity can arise. $F\uparrow$ follows the Stefan–Boltzmann–Gesetz, which is

$$F\uparrow = \epsilon \sigma_T T^4, \tag{2}$$

with ϵ the emissivity that gives the ratio of actual emission to blackbody emission. Blackbodies are objects that emit the maximum amount of radiation and that have $\epsilon = 1$. For the earth ϵ can be set to $\epsilon = 0.95$. σ_T is the Stefan–Boltzmann constant that is given by $\sigma_T = 5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$. Further, the flux ratio $F\uparrow/F\downarrow$ is given by $F\uparrow/F\downarrow = 109/88$. The difference $S_E - H$ can be written as $S_E - H = Q(1 - \alpha_1(T))/4$, with $Q = 1367.5\,\mathrm{W\,m^{-2}}$ the solar constant, $\alpha_1(T)$ the planetary albedo, determining how much of the incoming energy is reflected to space.

According to Henderson-Sellers and McGuffie (1987) and Schmitz (1991) the albedo $\alpha_1(T)$ is a function that negatively depends on the temperature on earth. This holds because deviations from the equilibrium average surface temperature have feedback effects that affect the reflection of incoming energy. Examples of such feedback effects are the ice-albedo feedback mechanism and the water vapour ‘greenhouse’ effect (see Henderson-Sellers and McGuffie, 1987, chapter 1.4). With higher temperatures a feedback mechanism occurs, with the areas covered by snow and ice likely to be reduced. This implies that a smaller amount of solar radiation is reflected when the temperature rises tending to increase the temperature on earth further. Therefore, Henderson and McGuffie (1987, chapter 2.4) and Schmitz (1991, 194) propose a function as shown in Fig. 1.

Fig. 1 shows $1 - \alpha_1(T)$, that part of energy that is not reflected by earth. For the average temperature smaller than T_l the albedo is a constant, then the albedo declines linearly, so that $1 - \alpha_1(T)$ rises until the temperature reaches T_u from which point on, the albedo is constant again. Here, we should like to point out that other feedback effects may occur, such as a change in the flux ratio of outgoing to incoming radiative flux for example. However, we do not take into account these effects since the qualitative result would remain the same.

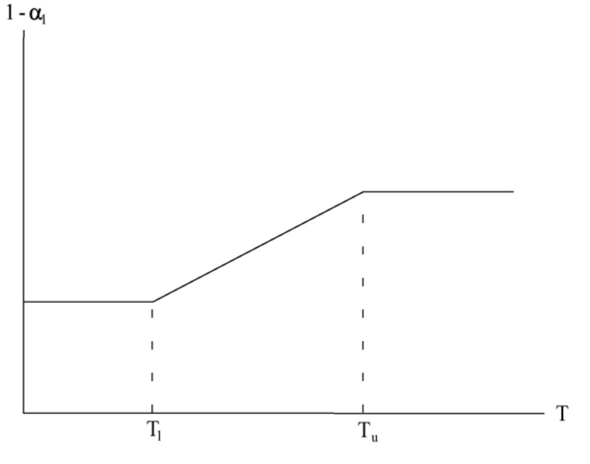

Fig. 1. Albedo as a function of the temperature.

Summarizing this discussion the EBM can be rewritten as

$$\dot{T}(t)c_{\mathrm{h}} = \frac{1367.5}{4}(1-\alpha_1(T)) - 0.95(5.67\times 10^{-8})\left(\frac{21}{109}\right)T^4, \quad T(0)=T_0. \tag{3}$$

According to Roedel (2001), $(1-\alpha_1(T)) = 0.21$ holds in equilibrium, for $\dot{T}=0$, giving a surface temperature of about 288 K which is about 15 °C. c_{h} is the heat capacity of the earth. Since most of the earth's surface is covered by seawater, c_{h} is largely determined by the oceans. Therefore, the heat capacity of the oceans is used as a proxy for that of the earth. The numerical value of this parameter[8] is $c_{\mathrm{h}} = 0.1497\,\mathrm{J\,m^{-2}\,K^{-1}}$.

The effect of emitting GHGs is to raise the concentration of GHGs in the atmosphere, increasing the greenhouse effect of the earth. This is done by calculating the so-called radiative forcing, which is a measure of the influence a GHG, such as CO_2 or CH_4, has on changing the balance of incoming and outgoing energy in the earth–atmosphere system. The dimension of the radiative forcing is $\mathrm{W\,m^{-2}}$. For example, for CO_2 the radiative forcing, which we denote as F, is approximately given by

$$F = 6.3\ln\frac{M}{M_0}, \tag{4}$$

with M the actual CO_2 concentration, M_{o} the pre-industrial CO_2 concentration and ln the natural logarithm (see IPCC, 1996, 52–53).[9] For other GHGs other formulas can be given describing their respective radiative forcing and these values can be converted in CO_2 equivalents. Incorporating (4) in (3) gives

$$\begin{aligned}\dot{T}(t)c_{\mathrm{h}} = {}& \frac{1367.5}{4}(1-\alpha_1(T)) - 0.95(5.67\times 10^{-8})\left(\frac{21}{109}\right)T^4 \\ & + (1-\xi)6.3\ln\frac{M}{M_0}, \quad T(0)=T_0.\end{aligned} \tag{5}$$

The parameter ξ captures the fact that a certain part of the warmth generated by the greenhouse effect is absorbed by the oceans which transport the heat from upper layers to the deep sea. We set $\xi = 0.23$.

The concentration of GHGs M, finally, evolves according to the following differential equation

$$\dot{M} = \beta_1 E - \mu M, \qquad M(0) = M_0. \tag{6}$$

E denotes emissions, and μ is the inverse of the atmospheric lifetime of CO_2. As to the parameter μ we assume a value of $\mu = 0.1$.[10] β_1 captures the fact that a certain part of GHG emissions are taken up by oceans and do not enter the atmosphere. According to IPCC $\beta_1 = 0.49$ for the time period 1990–1999 for CO_2 emissions (IPCC, 2001, 39). Table 1 gives a survey of the parameters used in our EBM.

[8] For more details concerning the calculation of this parameter see Harvey (2000).

[9] The CO_2 concentration is given in parts per million (ppm).

[10] The range of μ given by IPCC is $\mu \in (0.005, 0.2)$, see IPCC (2001, 38).

Table 1
Important parameters used in the EBM

Parameter	Meaning	Adopted value	Unit
c_h	Heat capacity	0.1497	$\mathrm{J\,m^{-2}\,K^{-1}}$
σ_T	Stefan–Boltzmann constant	5.67×10^{-8}	$\mathrm{W\,m^{-2}\,K^{-1}}$
Q	Solar constant	1367.5	$\mathrm{W\,m^{-2}}$
ξ	Part of temperature rise absorbed by oceans	0.23	Percentage
β_1	Emissions absorbed by oceans	0.49	Percentage
μ	Inverse of atmospheric lifetime of GHG	0.1	Percentage

3. The competitive economy

In this section we present our economic framework. We start with the description of the structure of our economy.

3.1. *The structure of the economy*

We consider an economy where one homogeneous good is produced. Further, the economy is represented by one individual with household production who maximizes a discounted stream of utility arising from per capita consumption, C, times the number of house-hold members subject to a budget constraint. As to the utility function we assume a logarithmic function $U(C) = \ln C$.

The individual's budget constraint in per capita terms is given by[11]

$$Y(1-\tau) = \dot{K} + C + A + \tau_E E L^{-1} + (\delta + n)K, \quad K(0) = K_0, \tag{7}$$

with Y per capita production, K per capita capital, A per capita abatement activities and E aggregate emissions. $\tau \in (0,1)$ is the income tax rate, $\tau_E > 0$ is the tax on emission and δ is the depreciation rate of capital. L is labour, which grows at rate n. In our model formulation abatement is a private good.[12] The production function is given by

$$Y = BK^{\alpha}\bar{K}^{1-\alpha}D(T-T_0), \tag{8}$$

with K per capita capital, $\alpha \in (0,1)$ the capital share, and B is a positive constant. $D(T - T_0)$ is the damage due to deviations from the normal temperature T_0 and has the same functional form as $D(\cdot)$. $\bar{K}$ gives positive externalities from capital resulting from spillovers. This assumption implies that in equilibrium the private gross marginal returns to capital[13] are constant and equal to $\alpha BD(\cdot)$, thus generating sustained per capita growth if B is sufficiently large. This is the simplest endogenous growth model existing in the economics literature. However, since we are not interested in explaining sustained per capita growth but in the

[11] The per capita budget constraint is derived from the budget constraint with aggregate variables.

[12] There exist some contributions which model abatement as a public good. See for example Ligthart and van der Ploeg (1994) or Nielsen et al. (1995).

[13] With gross return we mean the return to capital before tax and for the temperature equal to the pre-industrial level.

interrelation between global warming and economic growth, this model is sufficiently elaborate.

We should also like to point out that we only consider an emission tax and not other environmental policies such as tradeable permits. We do this because we consider a representative agent. We do not have multiple actors in our study who can trade permits. Therefore we consider the emission tax as the regulatory instrument. However, we are aware that under certain more realistic scenarios permits may be superior to taxation as an environmental policy measure. Permits might become important, in particular when it is difficult to evaluate marginal costs and benefits of abatement so that the effects of an environmental tax are difficult to evaluate. In this case permits that limit the quantity of emissions are preferable.[14]

As concerns emissions of GHGs we assume that these are a by-product of capital used in production and expressed in CO_2 equivalents, so emissions are a function of per capita capital relative to per capita abatement activities. This implies that a higher capital stock goes along with higher emissions for a given level of abatement spending. This assumption is frequently encountered in environmental economics (see e.g. Smulders, 1995, or Hettich, 2000). It should also be mentioned that the emission of GHGs does not affect utility and production directly but only indirectly by affecting the climate of the earth which leads to a higher surface temperature and to more extreme weather situations. Formally, emissions are described by

$$E = \left(a\frac{LK}{LA}\right)^{\gamma}, \tag{9}$$

with $\gamma > 0$ and $a > 0$ constants. The parameter a can be interpreted as a technology index describing how polluting a given technology is. For large values of a a given stock of capital (and abatement) goes along with high emissions implying a relatively polluting technology and vice versa.

The government in our economy is modelled very simply. The government's task is to correct the market failure caused by the negative environmental externality.[15] The revenue of the government is used for non-productive uses, and it does not influence the utility of the household. This implies that government spending does not affect the consumption–investment decision of the household.

The agent's optimization problem can be written as

$$\max_{C,A} \int_0^{\infty} \mathrm{e}^{-\rho t} L_0\, \mathrm{e}^{nt} \ln C \,\mathrm{d}t, \tag{10}$$

subject to (7), (8) and (9). ρ in (10) is the subjective discount rate, and L_0 is labour supply at time $t = 0$ which we normalize to unity and which grows at constant rate n. It should be noted that in the competitive economy the agents neither take into account the negative externality of capital, the emission of GHG, nor the positive externalities (i.e. the spillover effects).

[14] For an extensive treatment of permits and their implementation problems when used as regulatory instruments to correct for market failure, see Chichilnisky (2004).

[15] How the government has to take into account the positive externality is studied in Section 4.

To find the optimal solution we form the current-value Hamiltonian[16] which is

$$H(\cdot) = \ln C + \lambda_1((1-\tau)BK^{\alpha}\bar{K}^{1-\alpha}D(\cdot) - C - A - \tau_{\mathrm{E}}L^{-1}a^{\gamma}K^{\gamma}A^{-\gamma} \\ -(\delta+n)K), \tag{11}$$

with λ_1 the shadow price of K. Note that we used $E = a^{\gamma}K^{\gamma}A^{-\gamma}$.

The necessary optimality conditions are given by

$$\frac{\partial H(\cdot)}{\partial C} = C^{-1} - \lambda_1 = 0, \tag{12}$$

$$\frac{\partial H(\cdot)}{\partial A} = \tau_{\mathrm{E}}L^{-1}a^{\gamma}K^{\gamma}\gamma A^{-\gamma-1} - 1 = 0, \tag{13}$$

$$\dot{\lambda}_1 = (\rho+\delta)\lambda_1 - \lambda_1\left((1-\tau)B\alpha D(\cdot) - \left(\frac{\tau_{\mathrm{E}}}{LK}\right)\gamma a^{\gamma}K^{\gamma}A^{-\gamma}\right). \tag{14}$$

In (14) we used that in equilibrium $K = \bar{K}$ holds. Further, the limiting transversality condition $\lim_{t\to\infty} \mathrm{e}^{-(\rho+n)t}\lambda_1 K = 0$ must hold.

Using (12) and (14) we can derive a differential equation giving the growth rate of per capita consumption. This equation is obtained as

$$\frac{\dot{C}}{C} = -(\rho+\delta) + \alpha(1-\tau)BD(\cdot) - \gamma\frac{\tau_{\mathrm{E}}}{LK}a^{\gamma}K^{\gamma}A^{-\gamma}. \tag{15}$$

Combining (13) and (9) yields

$$E = \left(\frac{\tau_{\mathrm{E}}}{LK}\right)^{-\gamma/(1+\gamma)} a^{\gamma/(1+\gamma)}\gamma^{-\gamma/(1+\gamma)}. \tag{16}$$

Using (5) and (6) from Section 2, with the numerical parameter values introduced and the equations derived in this section the competitive economy is completely described by the following differential equations:

$$\dot{T}(t)c_{\mathrm{h}} = \frac{1367.5}{4}(1-\alpha_1(T)) - 0.95(5.67\times 10^{-8})\left(\frac{21}{109}\right)T^4 \\ +(1-\xi)6.3\ln\frac{M}{M_0}, \quad T(0) = T_0, \tag{17}$$

[16] For an introduction to the optimality conditions of Pontryagin's maximum principle, see Feichtinger and Hartl (1986) or Seierstad and Sydsaeter (1987).

$$\dot{M} = \beta_1 \left(\frac{\tau_E}{LK}\right)^{-\gamma/(1+\gamma)} a^{\gamma/(1+\gamma)} \gamma^{-\gamma/(1+\gamma)} - \mu M, \quad M(0) = M_0, \tag{18}$$

$$\frac{\dot{C}}{C} = -(\rho + \delta) + \alpha(1-\tau)BD(\cdot) - \gamma \left(\frac{\tau_{\mathrm{E}}}{LK}\right)^{1/(1+\gamma)} a^{\gamma/(1+\gamma)} \gamma^{-\gamma/(1+\gamma)}, \tag{19}$$

$$\frac{\dot{K}}{K} = (1-\tau)BD(T-T_0) - \left(\frac{\tau_{\mathrm{E}}}{\mathrm{LK}}\right)^{1/(1+\gamma)} a^{\gamma/(1+\gamma)} \gamma^{-\gamma/(1+\gamma)} (1+\gamma) - \frac{C}{K} - (\delta + n), \quad K(0) = K_0, \tag{20}$$

where $C(0)$ can be chosen by society.

3.2. The dynamics of the competitive economy

First we define a balanced growth path or steady state.

Definition. A balanced growth path (BGP) is a path such that $\dot{T} = 0$, $\dot{M} = 0$ and $\dot{C}/C = \dot{K}/K$ hold, with $M \geq M_0$.

This definition contains several aspects. First, we require that the temperature and the GHG concentration must be constant along a BGP. This is a sustainability aspect. Second, the growth rate of per capita consumption equals that of per capita capital and is constant. Third, we only consider balanced growth paths with a GHG concentration that is larger than or equal to the pre-industrial level. This requirement is made for reasons of realism. Since the GHG concentration has been rising monotonically over the last decades, it is not necessary to consider a situation with a declining GHG concentration.

To study the dynamics of our model we consider the ratio $c \equiv C/K$ which is constant on a BGP. Thus, our dynamic system is given by the following differential equations:

$$\dot{T}(t) = \left(\frac{1367.5}{4}(1-\alpha_1(T)) - 0.95(5.67\times 10^{-8})\left(\frac{21}{109}\right)T^4\right)c_{\mathrm{h}}^{-1} + \left((1-\xi)6.3\ln\frac{M}{M_0}\right)c_{\mathrm{h}}^{-1}, \quad T(0) = T_0, \tag{21}$$

$$\dot{M} = \beta_1 \left(\frac{\tau_{\mathrm{E}}}{LK}\right)^{-\gamma/(1+\gamma)} a^{\gamma/(1+\gamma)} \gamma^{-\gamma/(1+\gamma)} - \mu M, \quad M(0) = M_0, \tag{22}$$

$$\dot{c} = c\left((n-\rho) - (1-\alpha)(1-\tau)BD(\cdot) + \left(\frac{\tau_{\mathrm{E}}}{LK}\right)^{1/(1+\gamma)} a^{\gamma/(1+\gamma)} \gamma^{-\gamma/(1+\gamma)} + c\right), \tag{23}$$

where $c(0)$ can again be chosen freely by society.

To study the dynamics of our model we resort to numerical simulation. We start with a description of the parameter values we employ in our numerical analysis.

We consider one time period to comprise one year. The discount rate is set to $\rho = 0.03$, the population growth rate is assumed to be $n = 0.02$, and the depreciation rate of capital is $\delta = 0.075$. The pre-industrial level of GHGs is normalized to one (i.e. $M_0 = 1$) and we set $\gamma = 1$. ξ is set to $\xi = 0.23$ (see Section 2). The income tax rate is $\tau = 0.15$, and the capital share is $\alpha = 0.45$. This value seems to be high. However, if capital is considered in a broad sense meaning that it also comprises human capital, this value is reasonable. B is set to $B = 0.35$, implying that the social gross marginal return to capital is 35 percent for $T = T_0$.

As to τ_E/LK we set $\tau_E/LK = 0.001$, and a is set to $a = 1.65 \times 10^{-4}$. τ_E/LK and a determine the ratio of abatement per capital stock, which is given by $A/K = (a\tau_E/LK)^{0.5}$. With these values A/K takes the value $A/K = 4.1 \times 10^{-4}$. For example, in Germany the ratio of abatement spending to capital in 2000 was 9.7×10^{-4} (see Institut der deutschen Wirtschaft, 2003, Tables, 2.11 and 8.7). Below, we will analyze how different values for τ_E/LK affect the dynamics of our model. As concerns the damage function $D(\cdot)$ we assume the function

$$D(\cdot) = \left(a_1(T - T_0)^2 + 1\right)^{-\psi}, \tag{24}$$

with $a_1 > 0$, $\psi > 0$. As to the numerical values of the parameters in (24) we assume $a_1 = 0.04$ and $\psi = 0.05$. These values imply that a rise of the surface temperature by 3 (2, 1) degree(s) implies a damage of 1.5 (0.7, 0.2) percent. The IPCC estimates that a doubling of GHGs, which goes along with an increase of the global average surface temperature between 1.5 and 4.5 °C, reduces world GDP by 1.5–2 percent (see IPCC, 1996, 218), so that our choice for the parameters seems justified.

As to the albedo, $\alpha_1(T)$, we use a function as shown in Fig. 1. We approximate the function shown in Fig. 1 by a differentiable function. More concretely, we use the function

$$1 - \alpha_1(T) = k_1 \left(\frac{2}{\Pi}\right) \mathrm{ArcTan}\left(\frac{\Pi(T - 293)}{2}\right) + k_2. \tag{25}$$

k_1 and k_2 are parameters that are set to $k_1 = 5.6 \times 10^{-3}$ and $k_2 = 0.2135$. Fig. 2 shows the function $(1 - \alpha_1(T))$ for these parameter values.

With (25) the pre-industrial average global surface temperature is about 287.8 K (for $M = M_0$) and $1 - \alpha_1(\cdot) = 0.2083$. For $T \to \infty$ the expression $1 - \alpha_1(\cdot)$ converges to $1 - \alpha_1(\cdot) = 0.2191$ which corresponds to an increase of about 5 percent.

To get insight into our model we first note that on a BGP the GHG concentration and the average global surface temperature are completely determined by the emission tax rate τ_E/LK This holds because this ratio determines optimal abatement spending via (13). The global surface temperature on the BGP, then, gives the ratio of consumption to capital and the balanced growth rate, g. Solving (22) = 0 with respect to M and inserting the result in (21) = dT gives a function as shown in Fig. 3.

One realizes that there are three solutions for dT = 0. Table 2 gives the steady state values for T^* and c^* and the balanced growth rate, g, as well as the eigenvalues of the Jacobian matrix corresponding to (21)–(23).[17]

Table 2 shows that the first and third long-run steady states (I and III) are saddle point stable, while the second is unstable, with the exception of a one-dimensional stable manifold.

[17] The * gives steady state values.

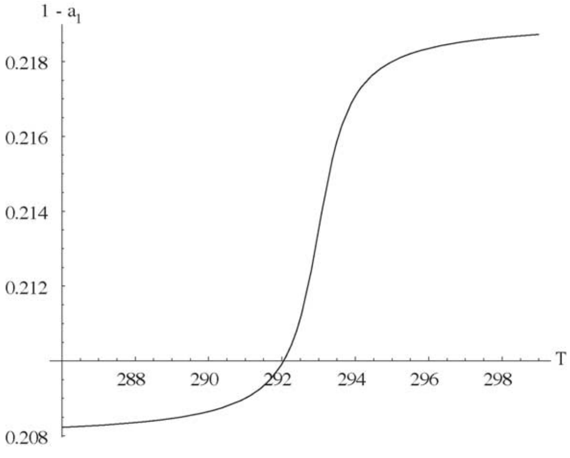

Fig. 2. Albedo as a smooth function of the temperature.

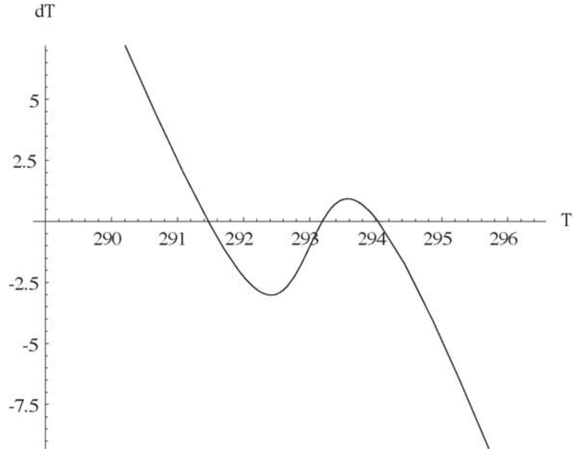

Fig. 3. Multiple steady states in the long-run.

Thus, there are two possible long-run steady states to which the economy can converge. The first one implies a temperature increase of about 3.7° and a balanced growth rate of about 2.6 percent; the other BGP corresponds to a temperature increase of about 6.2° and a balanced growth rate of about 2.2 percent. $1-\alpha_1(\cdot)$ takes the value 0.2093 for $T^* = 291.5$

Table 2
Steady state values, balanced growth rate and eigenvalues for the competitive model with $\tau_E/LK = 0.001$

Steady state	T^*	c^*	g (percent)	Eigenvalues
I	291.5	0.1697	2.6	−4.99, 0.17, −0.1
II	293.2	0.167	2.3	4.76, 0.167, −0.1
III	294	0.1657	2.2	−3.55, 0.166, −0.1

and 0.2171 for $T^* = 294$ showing that the quantitative decrease in the albedo does not have to be large for the occurrence of multiple equilibria. Our result suggests that there exists a threshold such that the initial conditions determine whether it is optimal to converge to steady state I or III.

3.3. Robustness and comparative static results

The last section demonstrated that there may exist a threshold for the competitive economy that determines whether it is optimal to converge to the long-run equilibrium that corresponds to a relatively small rise in the temperature or to the one with a large temperature increase. Here, we want to address the question of how robust this result is with respect to the emission tax ratio τ_{E}/LK. Further, we want to undertake some welfare considerations for the economy on the BGP.

We should also like to point out that in the very long-run when fossil fuels will be exhausted, the problem of global warming does not exist any longer. However, our approach models an economy where fossil fuels are an important input factor in the production process. Studying the model along the BGP, then, implies that the economy is successful in stabilizing emissions at a constant but higher level and that the convergence speed is sufficiently high. Of course, the BGP is only reached for $t \to \infty$ but, nevertheless, the BGP may be a good approximation if the deviations from the BGP are only small.

Varying the emission tax rate τ_{E}/LK affects the position of the dT curve in Fig. 3, thus determining the equilibrium temperature and, possibly, the number of equilibria. A rise in τ_{E}/LK shifts the dT curve downward and to the left implying a decrease of the temperature(s) on the BGP. Further, for a sufficiently high value of τ_{E}/LK only one equilibrium exists. For example, raising τ_{E}/LK to $\tau_{\mathrm{E}}/LK = 0.0011$ gives a unique long-run BGP with a steady state temperature of 291.8 K. This equilibrium is saddle point stable (two negative real eigenvalues). Reducing τ_{E}/LK to $\tau_{\mathrm{E}}/LK = 0.0008$ also gives a unique BGP with a steady state equilibrium temperature of 294.8 K. This equilibrium is also saddle point stable (two negative real eigenvalues). This demonstrates that the government choice of the emission tax ratio is crucial as concerns the long-run outcome. This holds for both the temperature in equilibrium and for the dynamics of the system.

Presuming the uniqueness of the steady state, we can concentrate on welfare considerations. We will limit our investigations to the model on the BGP although this can only be an approximation. Welfare on the BGP is given by

$$J = \int_0^{\infty} \mathrm{e}^{-(\rho-n)t} \ln(c^* K^* \mathrm{e}^{gt})\, \mathrm{d}t, \tag{26}$$

which shows that welfare in steady state positively depends on the consumption ratio, c^*, on the balanced growth rate, g, which is determined endogenously, and on K^* which we normalize to one (i.e. $K^* = 1$). From (19) and (23) one realizes that τ_{E}/LK has a negative direct effect on c^* and on g and a positive indirect effect by reducing the equilibrium surface temperature which implies smaller damages. This suggests that there exists an inverted U-shaped curve between the emission tax ratio and the growth rate and welfare. To see this more clearly we calculate the balanced growth rate, c^*, and the average global surface temperature for different values of τ_E/LK and for different damage functions. The results are shown in Table 2. As to the damage function we use the parameter values from the last section, $a_1 = 0.04$, $\psi = 0.05$, and, in addition, $a_1 = 0.03$, $\psi = 0.03$. Setting $a_1 = 0.03$ and $\psi = 0.03$ implies that a rise of the surface temperature by 3 (2, 1) degree(s) implies a damage of 0.7 (0.3, 0.09) percent of world GDP.

First, we can see from Table 2 that the balanced growth rate, g, and the consumption share, c^*, react in the same manner to changes in the emission tax ratio τ_E/LK so that maximizing the balanced growth rate also maximizes welfare. Further, Table 2 confirms that there exists an inverted U-shaped curve[18] between the emission tax ratio and the balanced growth rate and welfare. For the higher damage ($a_2 = 0.04$, $\psi = 0.05$) it is optimal to choose the emission tax rate so that the temperature remains at its pre-industrial level, implying that the damage is zero. For a lower damage corresponding to the temperature increase ($a_2 = 0.03$, $\psi = 0.03$) the balanced growth rate is maximized for a value of τ_E/LK that gives an average surface temperature exceeding the pre-industrial level. In this case, accepting a deviation from the pre-industrial average global surface temperature has positive growth and welfare effects in the long-run.

4. The social planner's problem

In formulating the optimization problem, a social planner takes into account both the positive and negative externalities of capital. Consequently, for the social planner the resource constraint is given by

$$\dot{K} = BKD(T - T_0) - C - A - (\delta + n)K, \quad K(0) = K_0. \tag{27}$$

Then the optimization problem is

$$\max_{C,A} \int_0^{\infty} \mathrm{e}^{-\rho t} L_0 \, \mathrm{e}^{nt} \ln C \, \mathrm{d}t, \tag{28}$$

subject to (27), (5), (6) and (9), where $D(\cdot)$ is again given by (24).

To find necessary optimality conditions we formulate the current-value Hamiltonian which is

$$\begin{aligned} H(\cdot) = {} & \ln C + \lambda_2(BKD(T - T_0) - C - A - (\delta + n)K) + \lambda_3(\beta_1 a^{\gamma} K^{\gamma} A^{-\gamma} - \mu M) \\ & + \lambda_4 (c_h)^{-1} \left(\frac{1367.5}{4}(1 - \alpha_1(T)) - (5.67 \times 10^{-8}) \left(\frac{19.95}{109} \right) T^4 \right. \\ & \left. + (1 - \xi) 6.3 \ln \frac{M}{M_0} \right), \end{aligned} \tag{29}$$

with $\alpha_1(T)$ given by (25). λ_i, $i = 2, 3, 4$, are the shadow prices of K, M and T respectively and $E = a^{\gamma} K^{\gamma} A^{-\gamma}$. Note that λ_2 is positive while λ_3 and λ_4 are negative.

[18] We calculated more values that we, however, do not show here.

The necessary optimality conditions are obtained as

$$\frac{\partial H(\cdot)}{\partial C} = C^{-1} - \lambda_2 = 0, \tag{30}$$

$$\frac{\partial H(\cdot)}{\partial A} = -\lambda_3\beta_1 a^{\gamma} K^{\gamma} \gamma A^{-\gamma-1} - \lambda_2 = 0, \tag{31}$$

$$\dot{\lambda}_2 = (\rho+\delta)\lambda_2 - \lambda_2 BD(\cdot) - \lambda_3\beta_1\gamma a^{\gamma} K^{\gamma-1} A^{-\gamma}, \tag{32}$$

$$\dot{\lambda}_3 = (\rho-n)\lambda_3 - \lambda_3\mu - \lambda_4(1-\xi)6.3c_{\mathrm{h}}^{-1}M^{-1}, \tag{33}$$

$$\begin{aligned}\dot{\lambda}_4 = {} & (\rho-n)\lambda_4 - \lambda_2 BKD'(\cdot) + \lambda_4(c_{\mathrm{h}})^{-1}341.875\alpha_1'(\cdot) \\ & + \lambda_4(5.67\times10^{-8})\left(\frac{19.95}{109}\right)4T^3(C_{\mathrm{h}})^{-1},\end{aligned} \tag{34}$$

with $\alpha_1' = -k_1(1+0.25\Pi^2(T-293)^2)^{-1}$. Further, the limiting transversality condition $\lim_{t\to\infty} \mathrm{e}^{-(\rho+n)t}(\lambda_2 K + \lambda_3 T + \lambda_4 M) = 0$ must hold.

Comparing the optimality conditions of the competitive economy with that of the social planner demonstrates how the government has to set taxes in order to replicate the social optimum. Setting (13) = (31) shows that τ_{E}/LK has to be set such that $\tau_{\mathrm{E}}/LK = \beta_1(-\lambda_3)/\lambda_2 K$ holds. Further, setting the growth rate of per capita consumption in the competitive economy equal to that of the social optimum gives $\tau = 1-\alpha^{-1}$.

This result shows that the emission tax per aggregate capital has to be set such that it equals the effective price of emissions, $-\lambda_3\beta_2$, divided by the shadow price of capital times per capita capital, $\lambda_2 K$, for all $t\in[0,\infty)$. This makes the representative household internalize the negative externality associated with capital. Further, it can be seen that, as usual, the government has to pay an investment subsidy (or negative income tax) of $\tau = 1-\alpha^{-1}$. The latter is to let the representative agent to take into account the positive spillover effects of capital. The subsidy is financed by the revenue of the emission tax and/or by a non-distortionary tax, such as a consumption tax, or a lump-sum tax.

From (30) and (31) we get

$$\frac{A}{K} = (c(-\lambda_3)\beta_1\gamma a^{\gamma})^{1/(1+\gamma)}, \tag{35}$$

with $c \equiv C/K$. Using (35), (30) and (32) the social optimum is completely described by the following system of autonomous differential equations:

$$\dot{C} = C(BD(\cdot) - (\rho+\delta) - ((C/K)(-\lambda_3)\beta_1\gamma a^{\gamma})^{1/(1+\gamma)}), \tag{36}$$

$$\dot{K} = K\left(BD(\cdot) - \frac{C}{K} - \left(\left(\frac{C}{K}\right)(-\lambda_3)\beta_1\gamma a^{\gamma}\right)^{1/(1+\gamma)} - (\delta + n)\right), \quad K(0) = K_0, \tag{37}$$

$$\dot{M} = \left(\frac{C}{K}\right)^{-\gamma/(1+\gamma)}(-\lambda_3)^{-\gamma/(1+\gamma)}\beta_1^{1/(1+\gamma)}\gamma^{-\gamma/(1+\gamma)}a^{\gamma/(1+\gamma)} - \mu M, \quad M(0) = M_0 \tag{38}$$

$$\begin{aligned}\dot{T} &= c_{\mathrm{h}}^{-1}\left(341.875(1 - \alpha_1(T)) - 5.67 \times 10^{-8}\left(\frac{19.95}{109}\right)T^4 + 6.3(1 - \xi)\ln\frac{M}{M_0}\right),\\ &\quad T(0) = T_0,\end{aligned} \tag{39}$$

$$\dot{\lambda}_3 = (\rho - n)\lambda_3 + \lambda_3\mu - \lambda_4(1 - \xi)6.3c_{\mathrm{h}}^{-1}M^{-1}, \tag{40}$$

$$\begin{aligned}\dot{\lambda}_4 &= (\rho - n)\lambda_4 - B\frac{K}{C}D'(\cdot) + \lambda_4(c_{\mathrm{h}})^{-1}341.875\alpha_1'(\cdot)\\ &\quad + \lambda_4\left(5.67 \times 10^{-8}\left(\frac{19.95}{109}\right)c_{\mathrm{h}}^{-1}4T^3\right).\end{aligned} \tag{41}$$

As for the competitive economy a BGP is given for variables T^*, M^*, λ_3^*, λ_4^* and c^* such that $\dot{T} = \dot{M} = 0$ and $\dot{C}/C = \dot{K}/K$ holds, with $M > M_0$. It should be noted that $\dot{T} = \dot{M} = 0$ implies $\dot{\lambda}_3 = \dot{\lambda}_4 = 0$.

To study the dynamics we proceed as follows. Since $\dot{C}/C = \dot{K}/K$ holds on the BGP, we get from (37) and (36) $c^* = \rho - n$. Next, we set $\dot{M} = 0$ giving $M = M(\lambda_3, \cdot)$. Inserting $M = M(\lambda_3, \cdot)$ in $\dot{\lambda}_3$ and setting $\dot{\lambda}_3 = 0$ yields $\lambda_4 = \lambda_4(\lambda_3, \cdot)$. Using $M = M(\lambda_3, \cdot)$ and $\lambda_4 = \lambda_4(\lambda_3, \cdot)$ and setting $\dot{T} = 0$ gives $\lambda_3 = \lambda_3(T, \cdot)$. Finally, inserting $\lambda_3 = \lambda_3(T, \cdot)$ in $\dot{\lambda}_4$ gives a differential equation that only depends on T and a T^* such that $\dot{\lambda}_4 = 0$ holds and gives a BGP for the social optimum.

For the parameter values employed in the last section with $a_1 = 0.04$, $\psi = 0.05$ in the damage function shows that there exists a unique BGP that is saddle point stable (two negative real eigenvalues). The temperature and the GHG concentration are $T^* = 287.9$ and $M^* = 1.02$, implying a temperature increase of 0.1°.

However, this result depends on the damage function. For extremely small damages going along with global warming we get a different outcome. For example, with $a_1 = 0.004$, $\psi = 0.004$ a temperature increase of 3° reduces world-wide GDP by merely 0.014 percent. With theses values we get three equilibria where two are saddle point stable and one is unstable. The temperatures on the BGPs are $T_1^* = 292$, $T_2^* = 294.3$ and $T_3^* = 295.4$. The eigenvalues of the Jacobian matrix, μ_i, $i = 1, 2, 3, 4$, corresponding to (38)–(41) are $\mu_{11} = 3.37$, $\mu_{12} = -3.36$, $\mu_{13} = 0.31$, $\mu_{14} = -0.3$ for $T = T_1^*$, $\mu_{12} = 4.7$, $\mu_{22} = -4.69$, $\mu_{23} = 0.005 + 0.12\sqrt{i}$, $\mu_{24} = 0.005–0.12\sqrt{i}$ for $T = T_2^*$

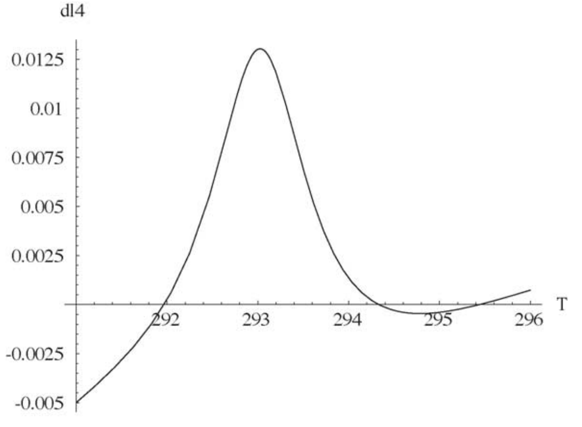

Fig. 4. Multiple equilibra in the social optimum for $a_1 = 0.004$, $\psi = 0.004$.

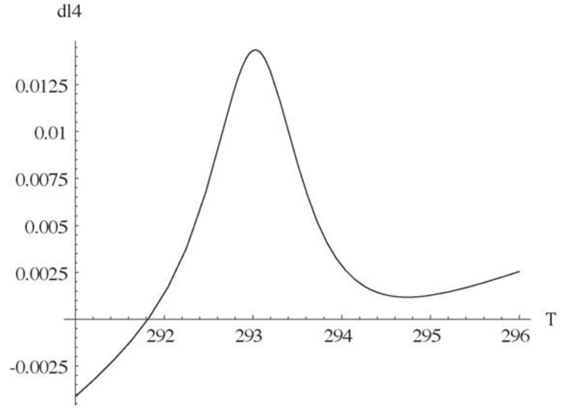

Fig. 5. Unique equilibrium in the social optimum for $a_1 = 0.004$, $\psi = 0.005$.

and $\mu_{34} = 6.34$, $\mu_{32} = -6.33$, $\mu_{33} = 0.07$, $\mu_{34} = -0.06$ for $T = T_3^*$. If the damage of the temperature increase is slightly larger, then the long-run BGP is again unique. Setting $a_1 = 0.004$, $\psi = 0.005$ we get $T^* = 291.8$, and this equilibrium is saddle point stable. Figs. 4 and 5 illustrate the two situations.

5. Conclusions

This paper has analyzed the dynamics of a simple endogenous growth model with global warming. Taking into account that the albedo of the earth depends on the average global surface temperature we could demonstrate that the competitive economy may be characterized by multiple long-run BGPs. In this case, the long-run outcome depends on the initial conditions of the economy.

Table 3
Balanced growth rate, c^* and T^* for different values of τ_E/LK with $a_1 = 0.04$, $\psi = 0.05$ and $a_1 = 0.03$, $\psi = 0.03$, respectively

$a_1 = 0.04$, $\psi = 0.05$				$a_1 = 0.03$, $\psi = 0.03$			
τ_E/LK	g	c^*	T^*	τ_E/LK	g	c^*	T^*
0.0011	0.0260	0.1702	291.2	0.0011	0.0273	0.1718	291.2
0.004	0.0280	0.1728	287.8	0.0035	0.0281	0.1729	288.4
0.0055	0.0277	0.1725	287.0	0.0042	0.0280	0.1728	287.8

We should like to point out that the change in the albedo does not have to be large to generate this outcome. Our example showed that even a quantitatively small decrease in the albedo may generate multiple equilibria. It is the existence of the feedback effect of a higher temperature influencing the albedo of the earth that leads to this result. Further, other feedback effects, for example a change in the ratio of emitted to reflected radiative flux, leading to non-linearities would produce the same qualitative outcome.

The government plays an important role in our model because the choice of the emission tax ratio affects not only the temperature change in equilibrium but also the dynamics of the competitive economy, so the emission tax ratio, and thus the abatement share, is crucial as to whether the long-run BGP is unique or whether there exist several BGPs. The social planner's problem is characterized by a unique BGP for plausible damages going along with global warming. However if the damages caused by the temperature increase are very small, the social optimum may also generate multiple equilibria and possibly thresholds.

Overall, as concerning government actions, we obtain in our model results similar to those in the growth literature.[19] A zero emission tax is not necessarily welfare improving. Since there are negative externalities arising from private activities, a government emission tax increases incentives for private abatement activities. As we have demonstrated in Table 3, by presuming a unique steady state there exists a welfare, or growth maximizing, emission tax, and the optimal emission tax is not necessarily zero. However, we want to note that if the presumption of a unique steady state does not hold, there could occur bifurcations to multiple equilibria. In this case, however, as demonstrated in Griine et al. in this volume, the problem of an 'optimal' tax scheme is more difficult to treat, since the welfare effects of tax schemes are more complex and can only be studied numerically.

References

Barro, R.J., 1990. Government spending in a simple model of endogenous growth. Journal of Political Economy 98, S103–S125.

Broecker, W.S., 1997. Thermohaline circulation, the Achilles heel of our climate system: will man made CO_2 upset the current balance? Science 278, 1582–1588.

Chichilnisky, G., 2004. Three Pegram Lectures. Business School, Columbia University. http://www.chichilnisky.com/lectures.htm.

Deke, O., Hooss, K.G., Kasten, C., Klepper, G., Springer, K., 2001. Economic impact of climate change: simulations with a regionalized climate-economy model. Kiel Institute of World Economics, Working Paper No. 1065.

[19] See Barro (1990).

Deutsch, C., Hall, M.G., Bradford, D.F., Keller, K., 2002. Detecting a potential collapse of the North Atlantic thermohaline circulation: implications for the design of an ocean observation system. Mimeo. Princton University.

Feichtinger, G., Hartl, R.F., 1986. Optimale Kontrolle okonomischer Prozesse: An-wendungen des Maximumprinzips in den Wirtschaftswissenschaften. de Gruyter, Berlin.

Gassmann, F., 1992. Die wichtigsten Erkenntnisse zum Treibhaus-Problem. In: Schweiz-erische Fachvereeinigung fur Energiewirtschaft (Ed.), Wege in eine CO_2—arme Zukunft. Verlag der Fachvereine, Zurich, pp. 11–25.

Harvey, D.L.D., 2000. Global Warming—The Hard Science. Prentice-Hall, Harlow.

Henderson-Sellers, A., McGuffie, K., 1987. A Climate Modelling Primer. Wiley, Chichester.

Hettich, F., 2000. Economic Growth and Environmental Policy. Edward Elgar, Cheltenham.

Institut der deutschen Wirtschaft, 2003. Deutschland in Zahlen. Koln.

IPCC, 2001. Climate Change 2001: The Scientific Basis. IPCC Third Assessment Report of Working group I. http://www.ipcc.ch.

IPCC, 1996. In: Bruce, J.P., Lee, H., Haites, E.F. (Eds.), Climate Change 1995: Economic and Social Dimensions of Climate Change, Contribution of Working Group III to the Second Assessment Report of the IPCC. Cambridge University Press, Cambridge, pp. 40–78.

Keller, K., Tan, K., Morel, F.M., Bradford, D.F., 2000. Preserving the ocean circulation: implications for the climate policy. Climate Change 47, 17–43.

Kemfert, C., 2001. Economy-Energy-Climate Interaction. The Model WIAGEM. Fon-dazione Eni Enrico Mattei. Nota di Lavoro 71, 2001.

Ligthart, J.E., van der Ploeg, F., 1994. Pollution, the cost of public funds and endogenous growth. Economics Letters 46, 339–348.

Nielsen, S.B., Pedersen, L.H., Sorensen, P.B., 1995. Environmental policy, pollution, unemployment, and endogenous growth. International Tax and Public Finance 2, 185–205.

Nordhaus, W.D., Boyer, J., 2000. Warming the World. Economic Models of Global Warming. MIT Press, Cambridge, MA.

Peck, S., Teisberg, T.J., 1992. CETA: a model for carbon emissions trajectory assessment. Energy Journal 13, 55–77.

Popp, D., 2003. ENTICE: endogenous technical change in the DICE model of global warming. NBER Working Paper No. 9762.

Roedel, W., 2001. Physik unserer Umwelt—Die Atmosphare. Springer-Verlag, Berlin.

Schmitz, G., 1991. Klimatheorie und -modellierung. In: Hupfer, P. (Ed.), Das Klimasys-tem der Erde: Diagnose und Modellierung, Schwankungen und Wirkungen. Akademie Verlag, Berlin, pp. 181–217.

Seierstad, A., Sydsaeter, K., 1987. Optimal Control with Economic Applications. North-Holland, Amsterdam.

Smulders, S., 1995. Entropy, environment, and endogenous growth. International Tax and Public Finance 2, 319–340.

Tol, R.S.J., 1999. Spatial and temporal efficiency in climate policy: an application of FUND. Environmental and Resource Economics 14, 33–49.