ELSEVIER

Journal of Economic Behavior & Organization
Vol. 66 (2008) 335–346

JOURNAL OF
Economic Behavior
& Organization

www.elsevier.com/locate/econbase

An economic model of work-related stress☆

Alfred Greiner *

Department of Business Administration and Economics, Bielefeld University, P.O. Box 100131, 33501 Bielefeld, Germany

Received 6 September 2005; received in revised form 11 December 2006; accepted 11 December 2006
Available online 16 December 2006

Abstract

In this paper we present an economic model of optimal consumption and labor supply where we assume that working may generate stress that affects the well-being of the representative individual. As to stress we posit that it is influenced by cumulated past labor and capital. The latter reflects the fact that work-related stress evolves gradually over time and that it is more likely to occur in modern societies. Using optimal control theory we demonstrate that sustained cycles may result. Further, we numerically compute the global optimal value function and give a representation of the limit cycle.

JEL classification: C61; D91; J22

Keywords: Work-related stress; Intertemporal optimization; Limit cycles

1. Introduction

According to the European Agency for Safety and Health at Work, work-related stress affects 28 percent of workers in the European Union (EU) and is the second most common health problem related to work, after back pain (Cox et al., 2000, p. 10). On the individual level, the consequences of stress are that the person's general quality of life as well as his well-being are reduced. For some people who experience work-related stress the consequences may be more drastic, implying that stress negatively affects their health. Typical health problems caused by work-related stress are, for example, insomnia, constant tiredness, high blood pressure and nervous twitches, just to mention a few.

☆ I thank a referee for detailed and insightful comments on an earlier version.
* Tel.: +49 521 106 4859; fax: +49 521 106 89005.
E-mail address: agreiner@wiwi.uni-bielefeld.de.

0167-2681/$ – see front matter © 2006 Elsevier B.V. All rights reserved.
doi:10.1016/j.jebo.2006.12.001

In addition to these individual problems work-related stress also causes costs for society. The European Commission estimates that costs due to work-related stress in the EU amount to at least 20 billion Euro annually (European Commission, 2002). For France, Bejean et al. (2003) have estimated that work-related stress cost between 830 and 1656 million Euro in 2000, which represented 13–26 percent of total spending of the social security occupational illnesses and work injuries branch. Besides direct health costs work-related stress leads to costs due to absenteeism and raising individuals' quitting behavior, causing costs for firms. Leotaridi and Ward (2002) find that individuals experiencing stress are 25 percent more likely to hold intentions to quit or being absent from work than those without work-related stress.

As concerns stress, one has to point out that stress can also be favorable to a person's well-being, and a certain amount of stress is even needed in order to remain healthy and alert. Therefore, in the psychological literature one finds the distinction between two forms of stress, between the so-called eustress and the distress. Eustress is the positive form of stress that is beneficial to an individual. This kind of stress or pressure is stimulating and enhances performance. However, when stress or pressure becomes too large such that the individual perceives himself unable to cope successfully with a situation, he is subject to distress, the negative form of work-related stress (as to the distinction between eustress and distress see Selye, 1974 or Cooper and Cartwright, 1996). The latter form of stress, distress, is perceived as negative by a person and may lead to the sort of individual health problems mentioned above.

In general, work situations are experienced as stressful when the demands made on the person do not match the resources available (in the individual or provided by the organization) or do not meet the person's needs and motivation. This can also serve as a definition for (di-)stress. As concerns the causes of work-related stress, Levi (1984) summarizes the factors under four headings, which are then differentiated further: quantitative overload, qualitative underload, lack of control over work and lack of social support.

In this paper, we will focus on the first and the third factor. One of our assumptions will be that stress arises as individuals simply have too much work, a fact that seems to be of relevance particularly for Japan (see Leotaridi et al.). We refrain from modelling the second factor, qualitative under- or overload as a possible source of stress. Qualitative underload means that the individual's work is not demanding, so he may be bored by his work; overload simply means that the work is too difficult. The other factor generating stress is the lack of control over work that is perceived as a threat to individual freedom, autonomy and identity. We take account of this factor because there is strong evidence that machine- and systems-paced work, especially of high rate, is detrimental to psychological and physical health (see, e.g. Cox, 1985 or Bradley, 1989). It should also be pointed out that workload has to be considered in relation to work pace such that it is in particular the interrelation between these two factors that generates stress.

The goal of this paper is to present a formal model that takes into account that work may lead to stress. Our main objective, then, is to study the implications of the model, in particular its dynamics.

The rest of the paper is organized as follows. In the next section we present our model and our modelling of work-related stress. Section 3 studies the dynamics of our main model. In Section 4 we present two variations of the model, and Section 5, finally, concludes.

2. The structure of the model

Our model consists of a representative household with a utility function that positively depends on consumption at time t, $C(t)$, and negatively on labor, $L(t)$. The latter models the preference for leisure, as usual in economics. We should also like to point out that we treat labor as a control variable, a fact that is quite common in economics. Another possibility would be to treat labor as a stock that can be controlled by a hiring or firing rate. The latter approach, however, is not pursued in this paper.

In addition, we make the assumption that utility depends on work-related stress, $S(t)$. As concerns the effect of stress on utility we posit that there is an optimum level of stress, denoted by S^*. Deviations from this level negatively affect utility. This holds both for levels of stress that exceed S^* as well as for levels smaller than S^*. Thus, we take into consideration that, for small values of stress, a rise in stress may well have a positive effect on a person's well-being (see, e.g. Cooper and Cartwright, 1996, pp. 6–8). In this case we speak of eustress. If stress becomes too large (i.e. if it exceeds the level S^*), well-being declines with stress. In this case, we speak of distress as already mentioned in Section 1.

As to the utility function $U(\cdot)$ we assume[1] that it is separable in C, L and S and a root function of C, linear in L and negative quadratic in S. Thus, the utility function is given by

$$U(C, L, S) = \sqrt{C} + (\bar{L} - L) - a(S - S^*)^2, \tag{1}$$

with $S^* > 0$, $\bar{L}$ the maximum available labor supply, and $a > 0$ is a constant. Note that the utility function is strictly concave in C and S and linear in L.

As to the constraints the first is the usual budget constraint stating that production is spent for consumption and saving; that is

$$\dot{K} = K^{\alpha} L^{\beta} - C - \delta K. \tag{2}$$

K is the capital and equals cumulated past investment, and $\delta > 0$ is the depreciation rate. $K^{\alpha} L^{\beta}$ is the production function with $\alpha \in (0, 1)$ the capital share and $\beta \in (0, 1)$ the labor share.

The second constraint describes the stress variable S. Stress is a function of cumulated past labor. Thus, we take into account that work-related stress is caused by labor and, second, that it is the cumulative effect of labor that leads to stress. The latter seems to be important because working overtime one or two times a month does not necessarily lead to stress. However, if this occurs more often, stress is likely to occur. In addition, we assume that the effect of labor on stress is the stronger the higher the capital stock is by assuming a multiplicative relationship. Thus, the differential equation describing the evolution of stress over time is given by

$$\dot{S} = L\,K - \eta S, \tag{3}$$

with $\eta > 0$ reflecting the ability to recover or, more generally, the fact that the experience of stress declines over time.

We assume a multiplicative relation between labor and capital because, as mentioned in Section 1, machine- and system-paced work is detrimental to health since it forces people to work in accordance with the machines. This deprives them of their control over work, which implies that their personal needs are often left behind. It is in particular the interaction between labor and capital that causes work-related stress, so an additional unit of work has stronger effects on stress in societies with larger capital stocks compared to societies with smaller capital stocks. This holds because in modern economies work is often alienated due to the fact that people have to communicate with machines rather than with human beings. In addition, it must be stated

[1] In the following we delete the time argument t.

that work-related stress is a phenomenon that occurs primarily in developed countries that are characterized by high capital stocks compared to countries, say, 100 or 200 years ago. Thus, our modelling of Eq. (3) seems to be justified.

The intertemporal optimization problem, then, is to choose consumption and labor such that the discounted stream of utility over an infinite time horizon is maximized subject to the two constraints (2), (3) and $L \leq \bar{L}$. Denoting by $\rho > 0$ the subjective discount rate, the formal problem is

$$\max_{C,L} \int_0^\infty \mathrm{e}^{-pt}(\sqrt{C} + (\bar{L} - L) - a(S - S^*)^2)\,\mathrm{d}t, \tag{4}$$

subject to (2), (3) and $L \leq \bar{L}$.

Optimality conditions are derived from the Lagrange function $\mathcal{L}$, which is given by

$$\begin{aligned}\mathcal{L} = \sqrt{C} + (\bar{L} - L) - a(S - S^*)^2 + \lambda_1(K^\alpha L^\beta - C - \delta K)\\ + \lambda_2(LK - \eta S) + \gamma(\bar{L} - L),\end{aligned} \tag{5}$$

with λ_1 and λ_2 denoting costate variables or shadow prices of K and S and γ is the Lagrange multiplier.

The necessary optimality conditions are obtained as

$$\frac{\partial \mathcal{L}}{\partial C} = 0.5C^{-0.5} - \lambda_1 = 0, \tag{6}$$

$$\frac{\partial \mathcal{L}}{\partial L} = -1 + \lambda_1 \beta L^{\beta-1} K^\alpha + \lambda_2 K - \gamma = 0, \tag{7}$$

$$\dot{\lambda}_1 = (\rho + \delta)\lambda_1 - \lambda_1 \alpha K^{\alpha-1} L^\beta - \lambda_2 L, \tag{8}$$

$$\dot{\lambda}_2 = (\rho + \eta)\lambda_2 + 2a(S - S^*), \tag{9}$$

$$\gamma \geq 0, \quad \gamma(\bar{L} - L) = 0. \tag{10}$$

In addition we require that the transversality condition $\lim_{t\to\infty} \mathrm{e}^{-pt}(\lambda_1 K + \lambda_2 S) = 0$ must be fulfilled. In the following, we assume that maximum available labor is sufficiently large such that the constraint $L \leq \bar{L}$ is not binding, implying that $\gamma = 0$ holds.

With $\gamma = 0$, we get from (6) and (7) optimal consumption and optimal labor supply as functions of the costate variables and of state variables as

$$C = 0.25\lambda_1^{-2}, \qquad L = \left(\frac{\beta \lambda_1 K^\alpha}{1 - \lambda_2 K}\right)^{1/(1-\beta)}. \tag{11}$$

Inserting (11) in $\dot{K}$, $\dot{S}$, $\dot{\lambda}_1$ and $\dot{\lambda}_2$ gives an autonomous system of differential equations in the state variables K and S and in the costate variables λ_1 and λ_2. This system is given by

$$\dot{K} = K^\alpha L(K, \lambda_1, \lambda_2, \cdot)^\beta - C(\lambda_1) - \delta K, \tag{12}$$

$$\dot{S} = L(K, \lambda_1, \lambda_2, \cdot)K - \eta S, \tag{13}$$

$$\dot{\lambda}_1 = (\rho + \delta)\lambda_1 - \lambda_1 \alpha K^{\alpha-1} L(K, \lambda_1, \lambda_2, \cdot)^\beta - \lambda_2 L(K, \lambda_1, \lambda_2, \cdot), \tag{14}$$

$$\dot{\lambda}_2 = (\rho + \eta)\lambda_2 + 2a(S - S^*). \tag{15}$$

3. The dynamics of the model

3.1. The analytical model

Eqs. (12)–(15) completely describe the dynamic behavior of our model. We are interested in the dynamics around a rest point or stationary point of this system, in particular in the question of whether the model converges to the rest point or whether it may generate cycles for example. To do so we first assume that a unique rest point exists for the analytical model and compute the Jacobian matrix evaluated at the rest point.

The Jacobian is given by

$$J = \begin{pmatrix} \alpha K^{\alpha-1}L^{\beta} + K^{\alpha}\beta L^{\beta-1}L_K - \delta & 0 & \beta K^{\alpha}L^{\beta-1}L_{\lambda_1} - C_{\lambda_1} & K^{\alpha}\beta L^{\beta-1}L_{\lambda_2} \\ L + KL_K & -\eta & KL_{\lambda_1} & KL_{\lambda_2} \\ -a_{31} & 0 & a_{33} & -a_{34} \\ 0 & 2a & 0 & \rho + \eta \end{pmatrix},$$

with

$$a_{31} = \lambda_2 L_K + \lambda_1((\alpha-1)\alpha K^{\alpha-2}L^{\beta} + \alpha\beta K^{\alpha-1}L^{\beta-1}L_K),$$
$$a_{33} = \rho + \delta - \alpha K^{\alpha-1}L^{\beta} - \lambda_2 L_{\lambda_1} - \alpha\beta K^{\alpha-1}\lambda_1 L^{\beta-1}L_{\lambda_1},$$
$$a_{34} = \lambda_2 L_{\lambda_2} + \alpha\beta K^{\alpha-1}\lambda_1 L^{\beta-1}L_{\lambda_2} + L.$$

L_k, C_k denote the derivative of L and C with respect to variable k, $k = K, \lambda_1, \lambda_2$. From (11) it can easily be seen that the derivatives have the following signs:

$$C_{\lambda_1} < 0, \quad L_{\lambda_1} > 0, \quad L_{\lambda_2} > 0, \quad L_K >=< 0 \quad \mathit{for} \quad \lambda_2 >=< -\alpha K^{-2\alpha-1}(1-\lambda_2 K). \tag{16}$$

The eigenvalues of that matrix are given by

$$\mu_{1,2,3,4} = \frac{r}{2} \pm \sqrt{\left(\frac{r}{2}\right)^2 - \frac{W}{2} \pm \sqrt{\left(\frac{W}{2}\right)^2 - \det J}},$$

with W defined as

$$W = \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{22} & a_{24} \\ a_{42} & a_{44} \end{vmatrix} + 2\begin{vmatrix} a_{12} & a_{14} \\ a_{32} & a_{34} \end{vmatrix}, \tag{17}$$

where a_{ij} is the element of the ith row and jth column (see Dockner and Feichtinger, 1991).

Looking at the formula for the eigenvalues, one immediately realizes that the eigenvalues are symmetric around $\rho/2$. Since $\rho > 0$ holds, this implies that the system is never completely stable (in the sense that all eigenvalues have negative real parts) but can only be saddle point stable. From an economic point of view, convergence to the stationary state means that all variables are constant in the long run. That is there are constant levels of consumption and labor supply and, as a consequence, a constant capital stock and a constant level of stress. The transitional behavior of the variables in case of saddle point stability is characterized by unimodal time paths if the eigenvalues are real. If the eigenvalues are complex conjugate, however, the variables are characterized by cyclical oscillations until the stationary point is reached. This means that both the capital stock as well as the level of stress show oscillations over time, however, with declining amplitudes until the stationary point is reached asymptotically.

Besides convergence to the stationary state in the long run, the system may show persistent endogenous cycles. This behavior can be observed if the dynamic system (12)–(15) undergoes a Hopf bifurcation. A Hopf bifurcation states the following (for a complete statement of the Hopf bifurcation theorem, see, e.g. Hassard et al., 1981): Assume that we continuously vary a parameter, say the discount rate, and that for a certain critical value of that (bifurcation) parameter two eigenvalues become purely imaginary. Assume in addition that the crossing speed of the eigenvalues is non-zero as the bifurcation parameter is varied. Then, there exist stable or unstable limit cycles that occur for values of the bifurcation parameter that are larger or smaller than the critical parameter value for which two eigenvalues are purely imaginary.

Let us find out whether persistent cycles may occur in our model. From the formula of the eigenvalues (see, e.g. Dockner and Feichtinger) we know that $W > 0$ is a necessary condition for two purely imaginary eigenvalues and, thus, for the emergence of a Hopf bifurcation that leads to stable limit cycles. Looking at the constant W we see that only the expression $a_{11}a_{33} - a_{31}a_{13}$ may become positive. Using the fact that $a_{11} + a_{33} = \rho$ holds (cf. Feichtinger and Hartl, 1986, p. 134) we may write $a_{11}a_{33} - a_{31}a_{13}$ as

$$\begin{aligned} a_{11}a_{33} - a_{31}a_{13} = & (\alpha K^{\alpha-1}L^{\beta} + K^{\alpha}\beta L^{\beta-1}L_K - \delta)(\rho + \delta - \alpha K^{\alpha-1}L^{\beta} - K^{\alpha}\beta L^{\beta-1}L_K) \\ & + (\lambda_2 L_K + \lambda_1((\alpha-1)\alpha K^{\alpha-2}L^{\beta} + \alpha\beta K^{\alpha-1}L^{\beta-1}L_K)) \\ & \times (\beta K^{\alpha}L^{\beta-1}L_{\lambda_1} - C_{\lambda_1}). \end{aligned} \tag{18}$$

For $\delta \geq \alpha K^{\alpha-1}L^{\beta} + K^{\alpha}\beta L^{\beta-1}L_K$ the first term in Eq. (18), $a_{11}a_{33}$, is negative. If the marginal product of capital is smaller than the depreciation rate at the steady state, we say that there is negative growth at the steady state. As concerns the second term, $-a_{31}a_{13}$, it is difficult to make a clear statement. However, it is seen that a positive λ_2 makes a positive sign of the second term more likely.[2] Further, a positive λ_2 and a high subjective discount rate make it more likely that the first term is positive, too. Thus, a Hopf bifurcation leading to limit cycles is more likely for a positive value of λ_2 together with a high discount rate. It should be noted that λ_2 at the stationary point is positive (negative) if the level of stress is lower (higher) than S^*.

From an economic point of view, the conditions leading to persistent cycles can be interpreted such that these oscillations may occur when the individual's stress level is smaller than S^* (that is when the individual experiences eustress in his work, and when he is impatient, the latter being reflected by a high subjective discount rate). These conditions state that a rise in the level of stress raises the individual's well-being, suggesting that he identifies himself with his work and may even be enthusiastic in his work. Since the individual is impatient, he works a lot at nearby time periods, thus raising the level of stress and his well-being as he is in the eustress range. However, with a rising level of stress the marginal value of additional stress, its shadow price, declines. A declining shadow price of stress leads the individual to reduce his work supply, generating a decline in the stress level. This goes on until the shadow price of stress rises again, due to the decline in the level of stress, thus, leading to persistent cycles. It should be mentioned that cyclical labor supply implies oscillations in the income and also in consumption.

[2] Recall that the signs of L_i and C_i are given in (16) and that $\lambda_2 > 0$ implies $L_K > 0$, since $1 - \lambda_2 K > 0$ must hold for L to be real.

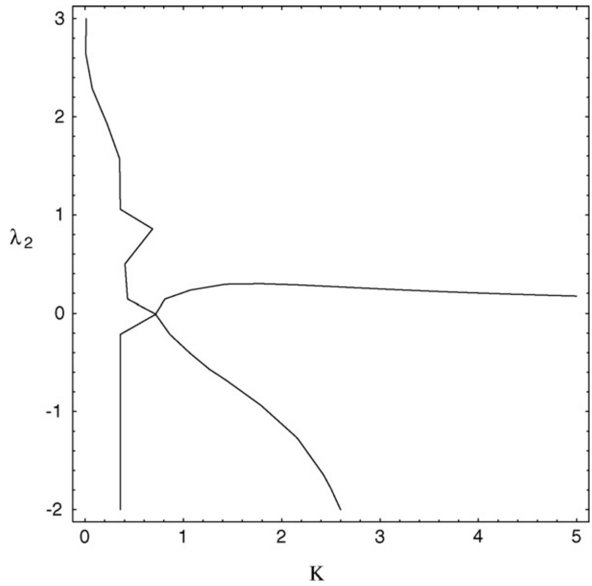

Fig. 1. $\dot{K}=0$ curve (monotonically falling) and $\dot{\lambda}_1=0$ curve (first rising, then declining) in the (K–λ_2) plane.

In order to gain additional insight into our model and to prove the existence of persistent cycles, we next present a numerical example.

3.2. A numerical example

To study our model numerically, we assume a constant returns to scale production function with a capital share of 30 percent and a labor share of 70 percent (i.e. $\alpha=0.3$ and $\beta=0.7$). δ and η are set to $\delta=0.075$ and $\eta=0.05$. S^* is set to $S^*=3$, $a=0.1$ and the subjective discount rate ρ serves as bifurcation parameter.

Before we study the time paths of the variables of our model, we address the question of existence and uniqueness of a stationary state. To do so, we first set $\rho=0.05$. With this parameter value we solve (13) = 0 with respect to S and insert the resulting[3] $\bar{S}$ (K, λ_2, λ_2, ·) in (12), (14) and (15) and, then, solve (15) = 0 with respect to λ_1 yielding $\bar{\lambda}_1$ (K, λ_2, ·). Inserting $\bar{\lambda}_1$ (K, λ_2, ·) in (12) and (14) and solving (12) = 0 and (14) = 0 with respect to K and λ_2 gives the rest point of the dynamic system. Fig. 1 shows the $\dot{K}=0$ and $\dot{\lambda}_1=0$ curves in the (K–λ_2) plane demonstrating that there exists a unique rest point for our model. Varying the discount rate with $\rho=0$ as lower bound and $\rho=0.35$ as upper bound does not change the qualitative outcome: there always exists a unique rest point. Further, it should be noted that for about $\rho=0.054$ we get $\bar{\lambda}_2=0$ and $\bar{S}=S^*=3$ whereas for $\rho<(>)$ 0.054, $\bar{\lambda}_2$ is negative (positive) and $\bar{S}$ is larger (smaller) than $S^*=3$.

Next, we analyze the local dynamics at the rest point for different values of the discount rate by computing the eigenvalues of the Jacobian matrix. It turns out that for $\rho\in(0, 0.3083)$ the eigenvalues are complex conjugate with two having negative real parts and two having positive real parts. This implies that the model is characterized by saddle point stability with

[3] The '‾' denotes values at the rest point.

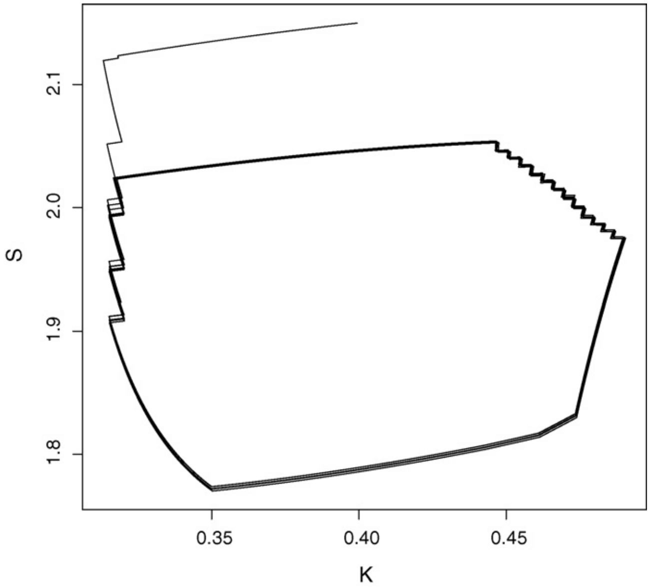

Fig. 2. Limit cycle in the (K–S) plane.

a two-dimensional stable manifold. For $\rho_{\text{crit}} = 0.3083432$ the differential equation system undergoes a supercritical Hopf bifurcation giving rise to stable limit cycles.[4] The limit cycles occur for an interval with strictly positive measure of the discount rate where the discount rates are slightly larger than the critical value ρ_{crit}. If ρ is increased further, all real parts of the eigenvalues become positive, implying that the system becomes unstable.

Our analysis thus far has used necessary optimality conditions and characterized the local dynamics around the stationary state. To get an idea about the global dynamics of the optimally controlled system, we numerically compute the optimal value function by solving the Hamilton–Jacobi–Bellman equation. This method gives the full global information about the optimal value function that, for its part, yields the optimal control in feedback form (a detailed description of the algorithm we use is given in Grüne, 1997 and in Grüne, 2004).

In particular, we are interested in the question of whether persistent cycles may turn out to be the optimal solution. Therefore, we set the subjective discount rate ρ to $\rho = 0.3084$, which is slightly larger than the critical value ρ_{crit} where a Hopf bifurcation was detected. The calculation of the optimal controls confirm that persistent cycles turn out to be the optimal solution. Fig. 2 shows the convergence of the optimal path to the limit cycle in the (K–S) plane.

4. Variations of the model

In this section we present two modifications of our main model and address the question of whether the emergence of endogenous cycles may also be possible in those models.[5] The first

[4] The bifurcation analysis was done with LOCBIF (see Khibnik et al., 1993).

[5] This section was motivated by the referee's comments.

modified model we consider assumes that labor is not a control variable but exogenously fixed. The second variation asserts that labor is exogenous and that stress affects production.

4.1. Exogenous labor supply

From an economic point of view, the assumption of an exogenously given labor supply can be justified by arguing that the amount of work is determined by institutional arrangements. Assuming that labor is given exogenously, utility can be written as a function that only depends on consumption and on work-related stress. Then, utility is given by $U=U(C, S)$ with $U_C(\cdot)>0$, $U_S(\cdot)<0$ and $U(\cdot)$ concave in C and S jointly and strictly concave in C and in S.

The optimization problem is then written as

$$\max_{C} \int_0^{\infty} \mathrm{e}^{-pt} U(C, S)\,\mathrm{d}t, \tag{19}$$

subject to (2) and (3).

Forming the current-value Hamiltonian, maximizing with respect to C, the following autonomous differential equation system can be derived, which completely describes the model:

$$\dot{K} = K^{\alpha} L_{\mathrm{e}}^{\beta} - C(S, \lambda_3) - \delta K, \tag{20}$$

$$\dot{S} = L_{\mathrm{e}} K - \eta S, \tag{21}$$

$$\dot{\lambda}_3 = (\rho + \delta)\lambda_3 - \lambda_3 \alpha K^{\alpha-1} L_{\mathrm{e}}^{\beta} - \lambda_4 L_{\mathrm{e}}, \tag{22}$$

$$\dot{\lambda}_4 = (\rho + \eta)\lambda_4 - U_S(\cdot), \tag{23}$$

with λ_i, $i=3, 4$, the costate variables of K and S in this model, respectively, and L_{e} denoting the exogenous labor supply. Further, the transversality condition $\lim_{t\to\infty} \mathrm{e}^{-pt}(\lambda_3 K + \lambda_4 S)=0$ must hold again.

In optimum, consumption is determined by the maximum principle, $U_C(C, S) - \lambda_3 = 0$, giving C as a function of S and λ_3. The derivatives are immediately obtained as $\partial C/\partial S = -U_{CS}/U_{CC}$ and as $\partial C/\partial \lambda_3 = 1/U_{CC}$. With this result, it is straightforward to calculate the Jacobian as

$$J = \begin{pmatrix} \alpha K^{\alpha-1} L_{\mathrm{e}}^{\beta} - \delta & \dfrac{U_{CS}}{U_{CC}} & \dfrac{-1}{U_{CC}} & 0 \\ L_{\mathrm{e}} & -\eta & 0 & 0 \\ -(\alpha-1)\alpha K^{\alpha-2} L_{\mathrm{e}}^{\beta} \lambda_3 & 0 & \rho + \delta - \alpha K^{\alpha-1} L_{\mathrm{e}}^{\beta} & -L_{\mathrm{e}} \\ 0 & \dfrac{-(U_{CC} U_{SS} - U_{CS}^2)}{U_{CC}} & -\dfrac{U_{CS}}{U_{CC}} & \rho + \eta \end{pmatrix}.$$

In the last section we pointed out that a positive sign of W, as defined in (17), is necessary for the emergence of sustained cycles. Looking at the Jacobian matrix of the model in this subsection, we see that there are two mechanisms that may lead to a positive W generating persistent cycles. The first is positive growth at the steady state, meaning that the marginal product of capital at the steady state exceeds the depreciation rate (i.e. $\alpha K^{\alpha-1} L_{\mathrm{e}}^{\beta} > \delta$ holds). This together with a sufficiently high discount rate may make the term $a_{11} a_{33}$ in W positive.

The second mechanism that can generate cycles is a positive effect of stress on the marginal utility of consumption (i.e. if $U_{CS}>0$ holds), in this case, the term $2a_{12}a_{34} = 2(-L_{\mathrm{e}})U_{CS}/U_{SS}$ also becomes positive. A positive effect of stress on the marginal utility of consumption can be expected as long as the individual experiences eustress in his work. This holds because eustress is the positive form of stress that raises the individuals well-being. Consequently, if the individual experiences eustress it is also likely that the benefit of additional consumption rises with more eustress. However, if the individual is in the range of distress, the negative form of stress, then a negative sign of U_{CS} is to be expected. This holds because in this case he experiences work as a burden and additional stress reduces his well-being. Therefore, the individual will also not be able to enjoy additional consumption.

Comparing the mechanisms leading to cycles in the model of this subsection with our main model of the last section, we see that they are basically the same, in both models, positive growth at the steady state can generate cycles, and these are more likely as long as the individual experiences eustress (i.e. the positive form of work-related stress).

In the next subsection, we study this model with the additional assumption that work-related stress affects the productivity of the individual.

4.2. *Stress affecting productivity*

Here, we assume that work-related stress affects both the well-being of the individual as well as his productivity. This is certainly justified because people who are satisfied with their work will be more productive than those who are stressed in their work. This is also confirmed by psychology. For example, Fig. 1 in Cooper and Cartwright (p. 7) shows that 'performance' is an inverted U-shaped function of 'demands or pressure'. First, performance rises as demands increase, reaches a maximum, and then declines as demands are further increased. To model this fact, we assume that output now is given by

$$Y = K^{\alpha} L_{\mathrm{e}}^{\beta} F(S - S^{*}), \tag{24}$$

with $F(\cdot) > 0$, $F'(\cdot) >=< 0$ for $S <=> S^{*}$ and $F''(\cdot) < 0$. The function $F(\cdot)$ gives the effects of work-related stress on productivity. As long as the individual experiences eustress (i.e. as long as $S < S^{*}$), a rise in stress raises productivity as well as well-being. For $S = S^{*}$ productivity reaches its maximum before it declines with further increasing stress when the individual is in the range of distress (i.e. for $S > S^{*}$).

The optimization problem, then, is to maximize (19), subject to (2) and (3), with production in (2) given by (24). Forming the current-value Hamiltonian, maximizing with respect to C, we can derive the following autonomous differential equation system:

$$\dot{K} = K^{\alpha} L_{\mathrm{e}}^{\beta} (S - S^{*}) - C(S, \lambda_5) - \delta K, \tag{25}$$

$$\dot{S} = L_{\mathrm{e}} K - \eta S, \tag{26}$$

$$\dot{\lambda}_5 = (\rho + \delta)\lambda_5 - \lambda_5 \alpha K^{\alpha-1} L_{\mathrm{e}}^{\beta} F(S - S^{*}) - \lambda_6 L_{\mathrm{e}}, \tag{27}$$

$$\dot{\lambda}_6 = (\rho + \eta)\lambda_6 + \lambda_5 F'(S - S^{*}) - U_S(\cdot), \tag{28}$$

with λ_i, $i = 5, 6$, the costate variables of K and S in this model, respectively, and L_{e} denoting the exogenous labor supply. Further, the transversality condition $\lim_{t\to\infty} \mathrm{e}^{-pt}(\lambda_5 K + \lambda_6 S) = 0$ must hold again.

The Jacobian at the steady state can be computed as

$$J = \begin{pmatrix} \alpha K^{\alpha-1} L_{\mathrm{e}}^{\beta} F(\cdot) - \delta & K^{\alpha} L_{\mathrm{e}}^{\beta} F'(\cdot) + \dfrac{U_{CS}}{U_{CC}} & -\dfrac{1}{U_{CC}} & 0 \\ L_{\mathrm{e}} & -\eta & 0 & 0 \\ a_{31} & a_{32} & a_{33} & -L_{\mathrm{e}} \\ a_{41} & a_{42} & a_{43} & \rho + \eta \end{pmatrix},$$

with

$$a_{31} = -(\alpha - 1)\alpha K^{\alpha-2} - L_{\mathrm{e}}^{\beta} F(\cdot)\lambda_5, \qquad a_{32} = -\alpha K^{\alpha-1} L_{\mathrm{e}}^{\beta} F'(\cdot)\lambda_5,$$
$$a_{33} = \rho + \delta - \alpha K^{\alpha-1} L_{\mathrm{e}}^{\beta} F(\cdot), \qquad a_{41} = -\alpha K^{\alpha-1} L_{\mathrm{e}}^{\beta} F'(\cdot)\lambda_5,$$
$$a_{42} = -K^{\alpha} L_{\mathrm{e}}^{\beta} F''(\cdot)\lambda_5 - \frac{(U_{CC}U_{SS} - U_{CS}^2)}{U_{CC}}, \qquad a_{43} = -K^{\alpha} L_{\mathrm{e}}^{\beta} F'(\cdot) - \frac{U_{CS}}{U_{CC}}.$$

Looking at the terms in W, we immediately see that the same effects as in the last subsection can lead to sustained cycles. This was to be expected, and we do not comment any further on these two mechanisms.

However, there is an additional factor that can bring about cycles. Assume that the necessary conditions for cycles in the model of the last subsection are not fulfilled so that cycles are excluded in that model. Then, it is nevertheless possible that the extended model of this subsection produces limit cycles. Looking at W we realize that the term $2(-L_{\mathrm{e}})K^{\alpha} L_{\mathrm{e}}^{\beta} F'(\cdot)$ can become positive. This term will be positive when the individual experiences distress ($S > S^*$) because then $F'(\cdot)$ is negative. Leaving aside positive growth at the steady state for the moment as mechanism leading to cycles, eustress is not necessary for cycles. Instead, they may also occur when the individual experiences distress. The reason for this outcome is that the assumption of stress affecting the productivity of the individual brings an additional nonlinearity into the model that can lead to cycles.

Thus, even in the case when the previous model could not produce sustained cycles, we can get cyclical behavior of optimally controlled variables when we take into account that work-related stress can affect the productivity of the individual.

5. Conclusion

Work-related stress is an important phenomenon affecting the well-being of individuals, but it has not been studied theoretically within a formal economic model, as far as we know. This paper has presented a first framework within which we studied the time paths of relevant variables taking into account work-related stress. We could show that persistent endogenous cycles may turn out to be the optimal solution. This implies that it can be optimal to work more for certain times followed by periods with less work and, for example, to take a sabbatical. It should be noted that one mechanism generating sustained cycles was eustress, the positive form of work-related stress. Formulated differently, if people draw satisfaction out of their work and do not only consider work as a means to make their living, this may lead to optimal cyclical labor supply.

As concerns our assumptions it must be stressed that our model is just one possible formulation of work-related stress; other formulations and extensions may be relevant and feasible as well. For example, one could imagine that labor is treated as a state variable that can be controlled by a hiring rate, as mentioned in Section 2. Further, the lack of social support is also a factor that can lead to stress (cf. Levi, 1984 or Giebels and Janssen, 2004), so that possible interactions between working individuals (e.g. between subordinate and superior) could be a useful extension. These aspects, however, are beyond the scope of this paper and could be taken into account in future studies.

References

Bejean, S., Sultan-Taib, H., Wolff, N., 2003. Modelling the Social Cost of Work-Related Stress. LEG-Document de travail, No. 2003-11. Universite de Bourgogne.

Bradley, G., 1989. Computers and the Psychological Work Environment. Taylor and Francis, London.

Cooper, C., Cartwright, S., 1996. Mental Health and Stress in the Workplace. A Guide for Employers. HMSO, London.

Cox, T., 1985. The nature and measurement of stress. Ergonomics 28, 1155–1163.

Cox, T., Griffiths, A., Rial-Gonzalez, E., 2000. Research on Work-related Stress. European Agency for Safety and Health at Work, Luxembourg.

Dockner, E.J., Feichtinger, G., 1991. On the optimality of limit cycles in dynamic economic systems. J. Econ. 53, 31–50.

European Commission, 2002. Guidance on Work-Related Stress—Spice of Life or Kiss of Death. Office for Official Publications of the European Communities, Luxembourg.

Feichtinger, G., Hartl, R.F., 1986. Optimale Kontrolle ökonomischer Prozesse: Anwendungen des Maximumprinzips in den Wirtschaftswissenschaften. De Gruyter, Berlin.

Giebels, E., Janssen, O., 2004. Conflict stress and reduced wellbeing at work: the buffering effect of third-party help. IACM 17th Annual Conference Paper. http://ssrn.com/abstract=602063.

Grüne, L., 1997. An adaptive grid scheme for the discrete Hamilton–Jacobi–Bellman equation. Numerische Mathematik 75, 319–337.

Grüne, L., 2004. Error estimation and adaptive discretization for the discrete stochastic Hamilton–Jacobi–Bellman equation. Numerische Mathematik 99, 85–112.

Hassard, B.D., Kazarinoff, N.D., Wan, Y.-H., 1981. Theory and Applications of Hopf Bifurcation. Cambridge University Press, Cambridge.

Khibnik, A.I., Kuznetsov, Y.A., Levitin, V.V., Nikolaev, E.V., 1993. Continuation techniques and interactive software for bifurcation analysis of ODE's and iterated maps. Physica D (Nonlinear Phenomena) 62, 360–371.

Leotaridi, R.M., Ward, M.E., 2002. Dying to work? An Investigation into Work-Related Stress, Quitting Intentions and Absenteeism. IZA Discussion Paper, No. 493. University of Bonn, Bonn.

Levi, L., 1984. Stress in Industry: Causes, Consequences and Prevention. Occupational Safety and Health Series, No. 51. International Labour Office, Geneva.

Selye, H., 1974. Stress, Bewältigung und Lebensgewinn. Piper & Co., München.